# Mathematical Knowledge for Teaching:
# Exploring its Transferability and Measurement in Ghana

By

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## Dedication

To my parents,

Mr Ofosu Daaku and Mrs. Theodora Daaku

My husband and children,

George, Kenneth and Florence

# Acknowledgements

I will give thanks to the Lord with my whole heart; I will recount all of your wonderful deeds.

*—Psalm 9:1*

I feel truly blessed to have reached this stage of my life. This dissertation could not have been written without the support, encouragement, and guidance of my committee, colleagues, friends, and family.

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# ABSTRACT

This dissertation examines the extent to which a U.S.-developed theory of teacher knowledge, Mathematical Knowledge for Teaching (MKT), is applicable in a Ghanaian context. To address the problem of Ghanaian students' poor student achievement in mathematics, caused in part by the quality of teaching, this dissertation is grounded in the premise that teacher knowledge influences teaching quality and consequently impacts students' learning. Progress made on MKT in the U.S. to help advance the quality of teachers' training motivates this study to examine whether the theory and measurement of MKT can support similar improvement of teaching quality in Ghana. This study therefore focuses on the question of the transferability of the MKT concept and its measures. If MKT is applicable in the Ghanaian context, then an adaptation of the MKT measures could serve as a diagnostic tool that could be used to design and evaluate professional development of teachers and to study MKT in Ghana. Specifically, this study is guided by the question: "*To what extent can empirically derived U.S.-developed measures of MKT be used to study MKT held by a sample of primary teachers in Ghana." In particular,* do the U.S.-developed Ghanaian-adapted MKT measures validly measure MKT in Ghana? I address this with the following two sub-questions.

a. What is the relationship between teachers' MKT scores and their reasoning about their responses to the adapted MKT measures?

b. What is the relationship between teachers' MKT scores and the mathematical quality of their instruction?

This study first adapts U.S-developed measures of MKT to make them usable in Ghana, without altering their substantive content. The adapted measures were then administered to 60 conveniently sampled practicing teachers. Three fourth-grade teachers among them were selected for in depth analysis to examine the validity of their MKT scores in two ways. First, they were interviewed to determine the consistency of their reasoning with their mathematical knowledge as assessed; and second, two consecutive mathematics lessons are analyzed for the mathematical quality of the instruction. Findings from the three teachers were extended to three sixth-grade teachers to determine the consistency of these findings.

Results from the analysis of the data show that although the MKT construct is valid in principle in the Ghanaian context, there is strong evidence to suggest that the instruments that assess MKT and the mathematical quality of instruction need further adaptation to suit the Ghanaian context.

# Chapter 1: Introduction and problem context for the study

## Rationale and problem context of this study

In a comparative study of students' achievement in 8th grade mathematics in 45 countries, Ghana ranked 44$^{th}$ (Anamuah-Mensah, Mereku, & Asabere-Ameyaw, 2004) illustrating that Ghana's students perform poorly on accepted measures of mathematical skill. Anamuah-Mensah et al. (2004) attribute Ghanaian students' performance to the quality of mathematics teaching at that level, positing that students were only able to answer questions that required recall of facts and procedures, and not deep conceptual knowledge of mathematics. These problems are not unique to Ghana. In a 2006 UNESCO report on teachers, the authors make a strong claim that in order to improve student's opportunities to learn and their outcomes, a strong focus on teacher quality is essential (UNESCO, 2006).

There are many ideas about what constitutes teacher quality such as characteristics of teachers, or where and how teachers are educated. Although some researchers agree that teachers' "basic academic skills and in-depth content knowledge are predictors of student achievement" (UNESCO, 2006, p. 76), current data on African teachers are vague proxies for such knowledge[1]. Accordingly, UNESCO described such data as not useful in making helpful

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[1] Data from SACMEQ (2004), Data Archive for the SACMEQ I and SACMEQ II Projects. Paris: IIEP-UNESCO and TIMSS(2003, 2004) data sets.

recommendations on the use of scarce resources to improve teacher quality (UNESCO, 2006).

There is an increasing sense that one crucial aspect of teacher quality is the way in which teachers know their content and the extent to which this knowledge can be effectively deployed in teaching. Earlier research has suggested that proxies such as the number of courses that teachers have taken or teachers' degrees could identify teacher knowledge, but in the last three decades, there has been increasing agreement that proxies alone are inadequate. Instead, a certain kind of knowledge is required in order to teach effectively.

Different conceptions of what teachers need to know in order in order to teach effectively have been proposed. According to Shulman's conceptualization, teachers need to have content knowledge, general pedagogical knowledge, curriculum knowledge, pedagogical content knowledge, knowledge of learners and their characteristics, knowledge of educational contexts, as well as knowledge of educational ends, purposes, and values(Shulman, 1986). More recently, researchers have proposed other conceptualizations such as the Knowledge Quartet (foundation, transformation, connection, and contingency) - conceptualized by Rowland, Huckstep, and Thwaites (2005). Others include Senk and colleagues' classifications of mathematical curricular knowledge, knowledge of planning for mathematics teaching and learning, and enacting mathematics for teaching and learning (Senk et al., 2008); Blum and Krauss' classifications of tasks and multiple solutions, misconceptions and difficulties, and explanations and representations (Blum & Krauss, 2008); and Schmidt and colleagues' curricular knowledge, instructional planning, and student learning (Schmidt et al., 2008). Although the different conceptions of teachers' mathematical knowledge are distinct, there are overlapping properties such as the inclusion of some component of content knowledge. However, the particular nature of content knowledge needed for teachers to teach effectively is unclear. Progress has been made at the University of Michigan where the Mathematical Knowledge for Teaching (MKT) research group has reframed the problem of teacher knowledge by focusing not on teachers' knowledge of curriculum, or mathematical content. Rather, they have drawn their attention to the mathematical knowledge demands of engaging in fundamental tasks of teaching (Ball, Thames, & Phelps, 2008). The MKT research group studied records of practice of U.S. teaching, identified routine tasks of teaching mathematics, and analyzed the mathematical demands of such tasks. They have subsequently proposed the theory of "mathematical knowledge for teaching" (MKT) defined as "the mathematical knowledge needed to carry out the work of teaching" (Ball et al., 2008, p. 397).

The problem of poor student achievement had led to the need for well-tested measures of mathematical teaching knowledge, as a critical factor contributing to teaching quality. In the U.S., it has been demonstrated that it is possible to measure MKT via a written test that teachers can take and that the measure has high validity with regard to the mathematical quality of their instruction (Hill et al., 2008). It has also been shown in the U.S. that students whose teachers score high on this test tend to perform better on standardized tests (Hill, Rowan, & Ball, 2005).

That there has been progress on measuring this kind of knowledge in the U.S. holds promise for identifying that knowledge among teachers in other countries. In fact, some studies have been done in Ireland, Norway, Indonesia, and Korea in this regard. Still, it is far from clear that the concept of MKT travels across cultural lines. The need for improvement in teacher quality in Ghana makes it critical to investigate whether the progress made on MKT can help advance the quality of teachers' training. This study therefore focuses on the question of the transferability of the MKT concept and its measures. If MKT can be transferable to the Ghanaian context the MKT measures could serve as a diagnostic tool that could be used to design and evaluate professional development of teachers and a useful tool to study MKT in Ghana.

In the next section, I present a description of the educational context in Ghana. This contextual information will provide a background understanding of teaching mathematics in Ghana and also describe the similarities between Ghana and U.S. that warrant the use of MKT as a theory for the study of Ghanaian teachers' capabilities.

## Context of education in Ghana

The current Ghanaian education system was instituted in September 2007. Basic education (primary and junior secondary school (JSS)) in public schools is tuition free and mandatory for all children of school-going age. The educational system in Ghana currently has two years of pre-school, six years of primary school, three years of junior secondary school (JSS), and four years of senior secondary school (SSS). Completion of senior secondary school provides students with opportunities to enter into a variety of tertiary institutions, such as polytechnics, technical schools, colleges, and universities.

In curriculum development, Ghana is dissimilar to the U.S. In the U.S., states and district level officials determine the school curricula although professional associations recommend content standards. The National Council for Teachers of Mathematics (NCTM), for instance, proposes content standards in mathematics (NCTM, 2000). In Ghana however, the Ministry of Education (MOE) and the Ghana Education Service (GES) regulates the curriculum at all school levels. Ghana is a multilingual country where more than forty mutually unintelligible indigenous languages are spoken. The language of instruction is English although public school students can be instructed in the local language for the first three years after which most instruction is in English (Wilmot, 2008). Data from the Ministry of Education[2] report that about 22% of the 16,410 primary schools are private schools and 16% of the school-going children attend private schools. Overall, the net enrollment ratio[3] is 78%. The pupil to teacher ratio is 34:1 in public schools and 26:1 in private schools and about 20% of teachers are employed in private schools.

Public schools are funded and owned by the government while private schools are owned and operated by individuals, and some religious institutions. Some

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[2] Retrieved from http://www.moess.gov.gh/EMIS/EMIS%20BASIC%20EDUCATION%20DATA/Basic/2006-2007/Basic%20School%20Report%202006-2007%20No.1.pdf on March 12, 2009

[3] Net enrollment ratio is the ratio of the number of children of official school age (as defined by the education system) enrolled in school to the number of children of official school age in the population.

private schools are unregistered by the government and generally charge less tuition than registered schools. Recent research reported the range of private primary school tuition ranged from under 1 dollar a term to 700 dollars a term (about 12 weeks) (Tooley, Dixon, & Amuah, 2007). Teaching and learning resources in private schools range widely from very few teaching and learning resources to a few high-private schools where books, computers, science laboratories and extra-curricular activities abound. Most public schools do not have basic building facilities and others lack resources such as books, furniture or electricity, and are scarcely equipped with some tables and chairs, and a blackboard.

Primary school subjects include Mathematics, English Language, Ghanaian Language, Integrated Science, Agriculture and Environmental Studies, Vocational Skills, Life Skills, French and Religious and Moral Education.

**School Mathematics in Ghana**

The Ministry of Education determines the content of the school mathematics curriculum and must approve all textbooks prior to their use in schools. As such, most of the topics across different textbooks are closely aligned to the mathematics content expectations as outlined in the syllabus.

School mathematics content in the primary school is focused on Number, Shape and Space, Measurement, Collecting and Handling Data, Problem Solving, and Investigation with Numbers (Ministry of Education, 2001). In the 2001 version of the primary school mathematics syllabus, the Ministry of Education requires teachers to “ensure maximum pupil participation,” avoid rote learning and drill-oriented methods, and rather “emphasize participatory teaching and learning” (p. ix). Teachers are also asked to help students develop among other things, “knowledge and understanding as exhibited by students’ ability to explain, summarize, rewrite, paraphrase, give examples, generalize, estimate, or predict consequences based on a trend” (Ministry of Education, 2001, p. ix). The syllabus urges teachers to help students to be able to apply their knowledge to new and unfamiliar situations, analyze by differentiating, comparing, separating; synthesize by combining, compiling, devising, planning, and revising; and evaluate by appraising, make comments or judgments, comparisons, contrasting, criticizing, and justifying among others. Teachers are cautioned that students struggle most with evaluation of claims that is the highest form of thinking and learning (Ministry of Education, 2001, p. ix).

These guidelines show that at the policy level, Ghanaian teachers are expected to eschew rote learning and teach in ways that permit students to engage with the mathematics. These values are consistent with NCTM-reform ideas currently being promoted in the U.S. (NCTM, 2000). The NCTM asks teachers to engage students in “serious mathematical thinking” (p. 18) and encourage student collaboration, justification of their thinking through the use of worthwhile mathematical tasks, which are seen as important for the children’s mathematical learning (NCTM, 2000).

**Background of primary mathematics teachers in Ghana**

Two universities and 38 teacher-training colleges in Ghana are charged with the preparation of pre-service initial teacher training. Comprehensive studies by the

Multi-Site Teacher Education Research (MUSTER) project in Ghana examined the quality and supply of teachers for basic schools with a focus on teacher training colleges (Akyeampong, 2003). Using interviews, surveys and classroom observations, Akyeampong (2003) also reports teacher-training institutions are unable to attract candidates with high academic backgrounds. About 70 to 80% of the teachers had received no prior formal teaching experience in teaching and were educated mostly in urban areas. Akyeampong argues that this could imply insufficient professional capital as well as little appreciation for the challenges of teaching in rural areas.

The teacher training in Ghana is a three-year program with 33 weeks per year. The curriculum comprises Foundation Academic and Introductory Studies in Education, Educational Studies, Curriculum Studies integrated with methodology, and Practicum and other practical activities. Akyeampong (2003) reported that teacher educators primarily lectured and “dictated notes” (p. viii) with few opportunities for collaborative work. Colleges generally lacked adequate instructional materials such as textbooks.

Graduates of the teacher training programs are typically assigned to teach in public schools. Private school teachers have a range of qualifications. Some of the teachers are graduate teachers and some of them are not. The graduate teachers either have a bachelor in education degree or have a bachelor’s degree in a non-education field but have a diploma in education.

In this section, I have provided a broad description of education in Ghana and an overview of school mathematics in Ghana. I now turn to a more detailed comparison of the U.S. and Ghana contexts to justify the selection of MKT as a theory worth investigating in Ghana.

## Primary teaching is mathematically demanding work

The MKT research group has developed the MKT construct through a detailed study of U.S. teaching and the study of relevant literature (Ball, 1999; Ball & Bass, 2000; Ball, et al., 2008). They argue that knowing the mathematics students are expected to learn is inadequate to be able to teach mathematics effectively.

| Student A | Student B | Student C |
| --- | --- | --- |
| 35<br>x 25<br>125<br>+75<br>875 | 35<br>x25<br>175<br>+700<br>875 | 35<br>x 25<br>25<br>150<br>100<br>+600<br>875 |

Figure 1.1: Student solutions for 35 x 25 (Ball & Bass, 2003, p. 7)

For instance, a teacher who knows how to multiply 25 x 35 in a particular way to get an answer of 875 may not be able to explain that alternative strategies that yield the same result can be generalized to all whole numbers (see Figure 1.1). This process, they argue is fundamentally a mathematical, not pedagogical question. To support their claim, they argue that a teacher's ability to determine the strategy used in B for example, does not mean the teacher can correctly unpack the strategy used in A or C. They posit.

Take solution (A) for instance. Where do the numbers 125 and 75 come

> from? And how does 125 + 75 = 875? Sorting this out requires insight into place value (that 75 represents 750, for example) and commutativity (that 25 × 35 is equivalent to 35 × 25), just as solution (C) makes use of distributivity (that 35 × 25 = (30 × 20)+ (5 × 20)+ (30 × 5)+ (5 × 5). Even once the solution methods are clarified, establishing whether or not each of these generalizes still requires justification.
>
> Significant to this example is that a teacher's own ability to solve a mathematical problem of multiplication (35 × 25) is not sufficient to solve the mathematical problem of teaching--to inspect alternative methods, examine their mathematical structure and principles and to judge whether or not they can be generalized (Ball & Bass 2003, p. 7).

From the above, Ball and Bass (2003) show that knowing just what students are expected to know is not sufficient to engage in the *mathematical* work involved in the teaching of primary mathematics. The work of evaluating student strategies is but one task in the identified mathematical tasks of teaching. The MKT research group argue that MKT is needed for teachers to engage in the tasks of teaching such as selecting representations for use in class, posing good mathematical questions, assessing student learning, and representing mathematical ideas (Ball & Bass, 2003; Ball et al., 2008).

Given the progress that has been made on the construct of MKT in the U.S. a crucial question to which I now turn is the extent to which these ideas offer resources for untangling the idea of teacher knowledge in Ghana. That is, *to what extent can the theory of MKT as conceptualized in the U.S. be similar to MKT if it is conceptualized in the Ghanaian context*?

## Is MKT-U.S. the same as MKT-Ghana?

Is it reasonable to hypothesize that the theory of MKT is applicable in Ghana?

Given that there are similarities in primary mathematics content and teaching expectations in both Ghana and the U.S., are the mathematical knowledge, sensibilities, and habits of mind needed to teach mathematics effectively in Ghana similar to that of the U.S.?

In the Ghanaian primary school mathematics syllabus, for example, the Ministry of Education requires teachers to “ensure maximum pupil participation”; minimize rote learning and drill-oriented methods; and rather “emphasize participatory teaching and learning” (year, p. ix). Teachers are also asked to help students develop among other things, “knowledge and understanding as exhibited by students’ ability to explain, summarize, rewrite, paraphrase, give examples, generalize, estimate, or predict consequences based on a trend” (Ministry of Education, 2001, p. ix). As explained earlier, the primary mathematics syllabus also urges teachers to help students to engage in higher order thinking such as apply their knowledge to new and unfamiliar situations, analyze by differentiating, comparing, separating; synthesize by combining, compiling, devising, planning, and revising; and evaluate by appraising, make comments or judgments, comparisons, contrasting, criticizing, and justifying among others” (Ministry of Education, 2001).

These guidelines show that at the policy level, Ghanaian teachers are expected to limit rote learning and teach in ways that permit students to engage with the mathematics. These values are consistent with NCTM-reform ideals currently being promoted in the U.S. (NCTM, 2000).

The mathematics content taught in the U.S. and Ghana includes similar content such as the development of Number, Shape and Space Geometry, Measurement, Sets, Relations and Functions, Data, and Problem Solving. For instance, Figure 1.2 shows the comparison of some content in both countries.

| Sample of Math Content in Ghana[4] | Sample of Math Content in the U.S. (Michigan)[5] |
|---|---|
| Multiplying 4-digit numbers by 1-digit numbers<br>Multiplying 3-digit numbers by 2-digit numbers<br>Solving word problems involving multiplication and division | N.FL.05.04 Multiply a multi-digit number by a two-digit number; recognize and be able to explain common computational errors such as not accounting for place value.<br>N.FL.05.05 Solve applied problems involving multiplication and division of whole numbers. |
| Adding and subtracting fractions with different denominators<br>Multiplying a fraction by a whole number<br>Find a fraction of a given whole number<br>Dividing a fraction by a whole number<br>Comparing fractions with the same and different denominators | Multiply and divide fractions<br>N.ME.05.12 Find the product of two unit fractions with small denominators using an area model.<br>N.MR.05.13 Divide a fraction by a whole number and a whole number by a fraction, using simple unit fractions.<br>Add and subtract fractions using common denominators<br>N.FL.05.14 Add and subtract fractions with unlike denominators through 12 and/or 100, using the common denominator that is the product of the denominators of the 2 fractions, |
| Measuring capacity of containers in litres and milliliters<br>Estimating and verifying the capacity of containers in milliliters<br>Writing capacities in given litres and millilitres using decimal notation | Know, and convert among, measurement units within a given system M.UN.05.01 Recognize the equivalence of 1 liter, 1,000 ml and 1,000 cm3 and include conversions among liters, milliliters, and cubic centimeters.<br>M.UN.05.02 Know the units of measure of volume: cubic centimeter, cubic meter, cubic inches, cubic feet, cubic yards, and use their abbreviations (cm3, m3, in3, ft3, yd3).<br>M.UN.05.03 Compare the relative sizes of one cubic inch to one cubic foot, and one cubic centimeter to one cubic meter.<br>M.UN.05.04 Convert measurements of length, weight, area, volume, and time within a given system using easily manipulated numbers. |

Figure 1.2: Comparison of some 5th grade primary mathematics content in Ghana and U.S.

[4] Content from 2001 Ghana Primary School Syllabus.
[5] Michigan Grade Level Content Expectations were downloaded from http://www.michigan.gov/documents/MathGLCE_140486_7.pdf on October 25, 2010

Although there are similarities between MKT in Ghana and MKT in the U.S. as shown by the mathematics content as well as the teaching expectations, differences exist that might influence the nature of MKT in Ghana. A closer examination of the mathematics content in the textbooks shows that subtle but significant differences exist in the nature of student expectations. Although a thorough examination of curriculum in both country contexts is beyond the scope of this study, it is clear that some differences such as the use of metric units in Ghana and imperial units in the U.S. and the depth of mathematics content as shown Figure 1.2 may impact the nature of MKT in Ghana. For example, in measurement, the Ghanaian syllabus requires students to measure and estimate the capacity of containers in litres and millilitres. In addition, students are expected to be able to write the capacities using the specified notation. The U.S. however goes beyond writing, measuring, and estimating. Students are required to convert among measurement units both in imperial and metric units, and compare relative sizes across different units. This comparison shows that the depth of knowledge about measurement expected in the U.S. is deeper than in Ghana, at least at the 5$^{th}$ grade level.

My knowledge of Ghanaian culture and traditions also suggests that classroom teaching practices currently promoted in the U.S. by NCTM (2000) and in Ghana by the Ministry of Education (2001) such as the increased use of communication in mathematics classrooms might be distinctly different in the U.S. and in Ghana. This is because of the perception of the teacher as a figure of authority in Ghana, an adult who commands respect and who has the authority to permit students to speak in class. In addition, the general lack of educational resources such as books, school furniture, and manipulatives, I argue, negatively impacts the implementation of what might be perceived in other cultures as a routine task of teaching, such as selecting and using manipulatives. Preliminary viewing of classroom teaching from my data suggests that some teachers do not have the luxury of space to move around a classroom to see what all students are doing; this hinders them from effectively analyzing student work during a lesson. Such challenges compounded by classroom sizes of 65 and above, present teachers with different concerns that may not be primarily mathematical. How then might the differences that exist between teaching in Ghana and teaching in the U.S. impact the nature of MKT in Ghana?

While acknowledging the differences, I hypothesize that the theory of MKT is substantially similar to the theory of MKT in Ghana. This is because most of the routine tasks of teaching identified by Ball et al. (2008) are also routine tasks of teaching mathematics. These include presenting mathematical ideas, responding to students' "why" questions, explaining mathematical goals and purposes to parents, linking representations to underlying mathematical ideas and to other representations, choosing and developing usable definitions, and giving or evaluating mathematical explanations (Ball, et al., 2008, p. 400). The differences between the tasks, I hypothesize, would exist in *how* teachers in either country execute such tasks. For instance, the problem contexts that teachers need to develop must be relevant to students' experiences and may not be relevant to students from other cultures.

In addition, I expect that some issues could influence the construct of MKT in Ghana. The first issue pertains to the testing conditions in both countries. In the U.S., teachers' MKT were assessed via a paper and pencil test administered via mail whereas in Ghana, teachers were assessed in test-like conditions in a classroom. Teachers in Ghana did not have multiple opportunities to think about the test items over a period of time. Instead, they had to complete the test within a more constrained time frame and under stringent test conditions. In addition, the different multiple-choice formats of the test might be challenging for the teachers in the Ghanaian sample who might not be familiar with the different multiple-choice formats. These testing conditions could influence the MKT scores of the teachers.

The second possible issue regards the use of language. Ghana is similar to most African countries because the language of instruction is usually different from the native language. Earlier work (Cole, 2009) demonstrated the possibility that the nuances of language could potentially influence the transfer of MKT, in particular, in the assessment of technical mathematical language used in instruction. This dissertation will examine the extent to which language might influence the transfer of MKT in Ghana.

The third issue is one of critical importance to the transfer of MKT in Ghana. This issue examines the extent of cultural congruence of the tasks of teaching. As hypothesized earlier, I expect differences in *how* the tasks of teaching are enacted in Ghana but in addition, there might be larger issues pertaining to the Ghanaian sample's familiarity with paper and pencil tests that assess their mathematical knowledge. This issue does not imply that teachers in Ghana do not know how to take a paper and pencil test, instead, it calls attention to tests of this nature being unfamiliar to teachers because teachers might not have their knowledge assessed at any given time. I expect the dissertation will highlight if and how the differences that exist between the U.S. and Ghana impact the construct of MKT in Ghana.

## Research questions

Given the problem of poor student achievement in Ghana and an increased focus on improving teacher quality as a possible intervention for improving students' performance, this dissertation is an attempt to improve teacher quality in Ghana by examining mathematical knowledge for teaching in the Ghanaian context. Because the construct of MKT and its measurement has been developed primarily in the U. S., I ask, *To what extent can empirically derived U.S.-developed measures of MKT be used to study MKT held by a sample of primary teachers in Ghana?*

To explore the degree to which MKT, a U.S. practice-based, empirically derived theory could be applicable in Ghana, I first ask *how* empirically-derived U.S.-developed measures of MKT can be used to study MKT in Ghana. I undertake some general and school-cultural as well as mathematical (Delaney et al., 2008) adaptations to make the measures functional with Ghanaian teachers. I refer to them as "Ghanaian-adapted MKT measures." I elaborate on these adaptations in Chapter 3.

The second step is to determine whether the mathematical knowledge for teaching demands of teaching in Ghana are comparable to the mathematical knowledge demands identified in the MKT measures. The following sub-questions are therefore used:

Do the U.S.-developed Ghanaian-adapted MKT measures validly measure MKT in Ghana?

c. What is the relationship between teachers' MKT scores and their reasoning about their responses to the adapted MKT measures?
d. What is the relationship between teachers' MKT scores and the mathematical quality of their instruction?

To determine whether the "Ghanaian-adapted" MKT measures are indeed measuring Ghanaian teachers' MKT, I will explore the extent to which teachers' responses to the Ghanaian-adapted measures reflect their mathematical knowledge using cognitive interviews. With these interviews, I investigate whether teachers are correctly responding to the measures because of correct or incorrect mathematical thinking and whether they are incorrectly responding to the measures because of incorrect or correct mathematical thinking. This would determine whether teachers' scores on the measures could be used as a true measure of the MKT knowledge as assessed. Figure 1.3 summarizes the research questions of the study (or presents the research questions of the study graphically).

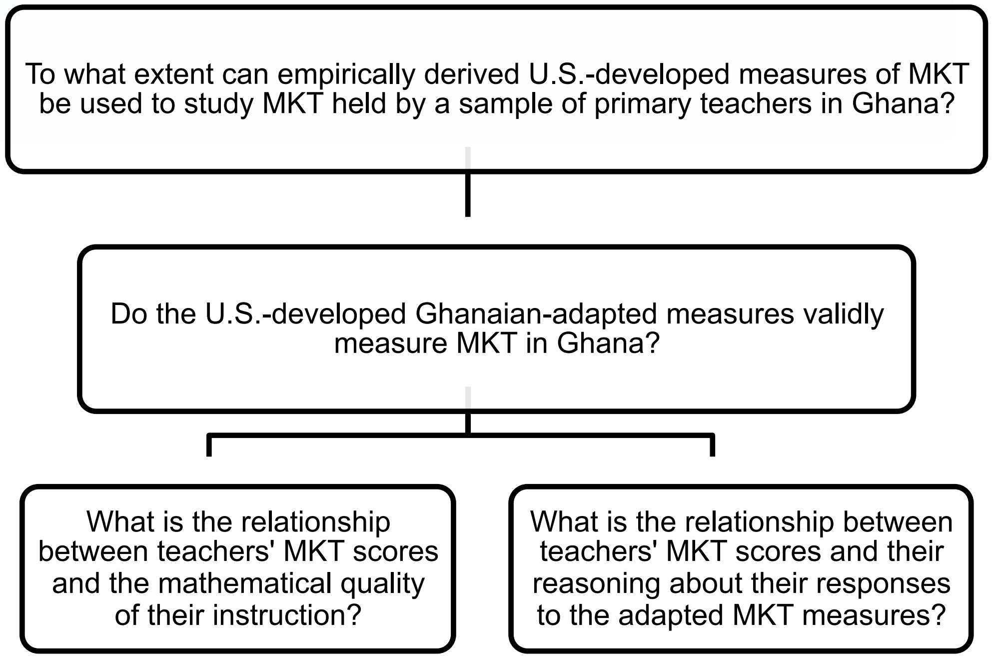


Figure 1.3: Research questions of the proposed study

## Significance of the study

This exploratory study is designed to examine the applicability of a US-developed construct, MKT, in a Ghanaian setting. It thus utilizes the U.S. construct of MKT to investigate the MKT of teachers in Ghana. This is an issue of concern to the African mathematics education community, the Learning Mathematics for Teaching (LMT) project, as well as the mathematics education community in general. The mathematics education community would also benefit from this work because it adds to the literature about primary teachers' mathematical knowledge and broadens the opportunities for researchers in Africa to study teachers' mathematical knowledge by studying mathematics teaching.

This work provides a broader international dimension to the work of the LMT project. The MKT measures have been used in the US (Hill, Rowan, & Ball, 2005), Ireland (Delaney, 2008), Norway (Mosvold & Fauskanger, 2009), Indonesia (Ng, 2009) and Korea (Kwon, 2009) but have not been used in Africa. I expect this study to break the ground for the discussion of primary teachers' mathematical knowledge in Ghana and in Africa more broadly. Consequently, this study paves the way for other African countries to explore the use of MKT measures in their country contexts.

Methodologically, this study adds to the existing literature on adaptation of measures developed in one context and used in other contexts and provides more guidelines for use in Ghana; it also has broad implications for other African countries. Some researchers have been accused of "uncritical transfer" (Crossley & Watson, 2003) of theories developed for use in one context and applied in another. An understanding of teachers' reasoning used to respond to the MKT measures is needed to provide some understanding of the extent to which the theory is applicable in Ghana.

This study has the potential to inform educational policy in Ghana by suggesting ways to modify or adapt the content of mathematics teacher education and professional development programs to address the needs of the mathematics education in Ghana. In studying the use of these measures with a small sample of teachers in Ghana, this research could lay the groundwork for a broader national study to examine what teachers in Ghana know and are able to do. In addition, professional development opportunities could be designed to bridge the gap between what teachers know and what teachers need to know.

## Boundaries and limitations of the study

The study of Ghanaian teachers’ mathematical knowledge for teaching is an important contribution to the literature on primary teachers’ knowledge. However, this exploratory study has four main limitations.

First, the data from this study are not drawn from a random sample thus no claims can be made about Ghanaian teachers’ MKT in general. Second, the data are neither designed nor intended to identify good and bad teachers. It is also not an attempt to compare Ghanaian teachers’ knowledge to that of their Irish or U.S. counterparts. The purpose of this study is to explore the degree to which US-developed MKT measures adapted for Ghana could determine Ghanaian teachers’ MKT.

Although student learning is an important consideration, the third limitation is the unavailability of student learning data. The only evidence about student learning can be inferred from public displays of student learning in the classroom. These could include a student’s board work, students’ responses to teacher questions, and student-initiated comments or questions. This study would therefore not attempt to describe student learning as a consequence of teacher knowledge or as a consequence of mathematical instruction.

A fourth limitation is that this research utilizes U.S.-developed video codes that measure Mathematical Quality of Instruction based on the study of U.S., not Ghanaian instruction. Similar to work undertaken by Delaney (2008), the MQI video codes were not specifically adapted for use in Ghana. Further research beyond the scope of this study is the need to identify MQI in a Ghanaian setting using Ghana-adapted or developed video codes that might be a better measure of MQI.

## Organization of the dissertation

Chapter 2 provides the historical context and background for this study. It sets out the studies of teacher knowledge, discussing the various conceptions of teacher knowledge and provides a justification for the selection of MKT as the theory of study in this dissertation study. As a study of Ghanaian teaching, this chapter also provides a bird's eye view of the research terrain in Africa by examining selected educational research studies in Africa and situates this dissertation study as filling a gap identified in the literature on research in Ghana and Africa more broadly.

Chapter 3 provides a detailed description of the methods used to study MKT in Ghana. These include IRT analysis of multiple-choice items, grounded theory, cognitive interviews and video coding. The chapter also provides an account of the measures adaptation, data collection and analysis techniques.

Chapter 4 presents the findings of the study. The first part of Chapter 4 is focused on the quantitative analysis of the MKT scores of the sample, and a qualitative examination of items classified as easy and difficult for the teachers sampled. The second part examines the first validation study that investigates the relationship between teachers' MKT scores and their reasoning about the items and an examination of the relationship between the MKT scores and interview findings. The third part reports on the findings from the video study. These findings will determine the extent to which teachers' MKT scores could be used to determine the mathematical quality of their instruction, and results from the MKT administration, interview scores, and video study will be triangulated to investigate the existence of any relationships among them.

Chapter 5 presents the discussion of the findings in the context of the research question, and draws on other sources of data such as the syllabus and textbooks to explain the findings of the study.

Chapter 6 outlines the theoretical, methodological, and practical implications of the study and concludes with further suggestions for further research.

# Chapter 2: Literature review

The aim of this study is to examine how U.S. practice-based measures of teachers' knowledge can effectively be used to determine Ghanaian teachers' mathematical knowledge for teaching (MKT). A fundamental assumption of this study is that teachers' mathematical knowledge impacts the quality of their mathematics teaching and hence influences students' opportunities to learn mathematics (Charalambous, 2008; Hill, Blunk, Charalambous, et al., 2008). The outcome of this study is important because Ghanaian students' primary mathematics learning opportunities make them unable to compete in the fast changing world (Anamuah-Mensah et al., 2004) and a closer study of Ghanaian teachers' mathematical knowledge is a useful site for addressing this problem.

Primarily, this chapter aims at providing a theoretical justification for the selection of Mathematical Knowledge for Teaching (MKT) as the focus of this study against alternative theories of teacher knowledge. A secondary goal of this chapter is to provide a bird's eye view of the research terrain in Africa and situate this study within the larger spectrum of research efforts in Africa.

This chapter is organized into six main sections. The first section highlights the development of theories of teachers' mathematical knowledge and provides a historical framing of how research in this area has evolved to its current state. I then turn specifically to the theory of MKT and describe how MKT is conceptualized and measured. This section thus answers the question: What do teachers need to know in order to teach effectively? In the second section, I compare other theories of teacher's mathematical knowledge and highlight how each of the theories conceptualizes teachers' mathematical knowledge. I also examine how these theories compare to MKT in their content as well as their measurement. The third section is focused on studies that have shown teacher knowledge to be important for what children learn. As such, this section answers the question: How can we know that teachers' knowledge is important for students' mathematical learning opportunities?

Most of the conceptions of teacher knowledge were initially developed in the U.S. The fourth section examines international studies of teacher knowledge to determine what was learned from such studies and how such studies can inform an investigation of MKT in Ghana. An important component of this study is the adaptation of a U.S. developed construct, MKT, in a Ghanaian setting. The fifth section thus reviews studies of adaptation and measurement of MKT in international contexts. These studies highlight the progress and challenges faced in the use of MKT measures in other national contexts, and what can be learned that could be used in both this and future studies.

The sixth section addresses the secondary goal of this chapter. It presents education studies in Africa more broadly, and Ghana in particular. This section highlights the nature of research questions being addressed in Africa and the state of the field, with a particular focus on teacher knowledge, and it identifies gaps that remain to be filled in research about Africa. This suggests potential contributions this study could make to the field.

## Content knowledge for teaching--Historical development

This section presents a historical development of the research on teachers' content knowledge. This research provides an important foundation to this dissertation by simultaneously situating the theory of MKT in the larger body of research on teacher knowledge and acknowledging the growth in the field to date.

About 45 years ago, two crucial reports, the *Equality of Educational Opportunity Survey* (Coleman et al., 1966) and the *Inequality* project (Jencks, et al., 1972) reported that student learning was only marginally influenced by teachers and schooling. Since then, several researchers investigated how teachers and schooling can make a difference for student learning. Several studies, generally referred to as the *educational production function* studies, examined the relationships between teacher knowledge and student achievement. These studies, typically used proxies for teacher knowledge such as the number of undergraduate or graduate level mathematics courses taken, degrees earned, and performance on exams (Begle, 1976; Goldhaber & Brewer, 2001; Monk, 1994; Rice 2003).

For instance, Begle (1976) examined the role of teacher knowledge on student performance. He conducted a meta-analysis of research conducted over a 16 year period, of how three proxies – the number of content courses teachers had taken at the level of calculus or beyond; the number of mathematics methods courses they had taken; and whether their undergraduate major or minor was mathematics – influenced student performance. His findings were ambiguous. He first found that the number of content courses taken was positively related to student performance in only 10% of the studies, and negatively related to student achievement in 8% of the studies. Secondly, he found that taking mathematics methods courses yielded positive effects in 24% of the studies and negative effects in 6% of the cases. In the third case, having a major or minor in mathematics was positively related to student performance in 9% of the studies and negatively related in 4% of the cases. These findings also showed that the significant effects were associated with students' computational fluency. Skills such as comprehension, application, or analyses deemed more cognitively demanding were observed less frequently. Begle thus concluded that teachers' subject matter knowledge was not as "powerful" (p. 54) as had previously been assumed and called for research that would not focus on teachers and their characteristics (Begle, 1979).

In spite of Begle's call to shift the focus of research away from teachers, Monk (1994) used data from the Longitudinal Survey of American Youth to study the effects of teacher knowledge on student learning. Monk found that the number of undergraduate and graduate mathematics courses that teachers had taken influenced students learning but only up to a point. In fact, every additional content course that teachers had taken accounted for a 1.2% increase in students' test score[6] but after five courses, each additional course predicted only a 0.2% increase and these findings were not statistically significant. These findings showed that the effects were only significant for students in advanced mathematics courses and not for remedial courses. In addition, undergraduate mathematics methods courses produced higher effects than mathematics content courses (Monk, 1994). Later studies by Rice (2003) also yielded similar mixed findings.

Overall, the aforementioned production function studies did not provide a definitive answer for how teacher knowledge as measured by proxies such as the number of courses a teacher takes influences student learning (cf. Ball, Lubienski, & Mewborn., 2001; Fennema & Franke, 1992; Hill, Sleep, Lewis, & Ball, 2007). The production function studies incorrectly assumed that the proxies were accurate depictions of teacher knowledge. As Cohen and colleagues (2003) point out, it is not just what teachers know, but *how* that knowledge is put to use:

> Knowledge counts in several ways. Teachers who know a subject well, and know how to make it accessible to learners, will be more likely to make good use of a mathematics text, to use it to frame tasks productively and use students' work well, than teachers who don't know the subject, or know it but not how to open it to learners (Cohen, Raudenbush, & Ball, 2003, p. 125).

In addition to the production function studies, other researchers examined how identified teaching behaviors influenced student performance. These studies, typically called the *process-product* studies, examined a variety of behaviors that occurred inside the classroom such as time on task, wait time, classroom

[6] Student scores were obtained from U.S. standardized assessments developed by the National Assessment of Educational Progress

management and organization, curriculum pacing, and question posing (Brophy, 1986; Brophy & Good, 1986; Good, 1996; Reynold & Muijs, 1999). Although the process-product studies were a remarkable contribution to education research, they did not investigate the extent to which teachers' knowledge influences teachers' enactment of these behaviors. In addition, their focus was not on particular school subjects. These studies led to Shulman's (1986) call for a focused attention on teachers' content knowledge.

### Shulman's professional knowledge domains

In the mid-eighties, Shulman (1986) drew attention to the importance of content knowledge in teaching. He identified as the "missing paradigm," (p. 6) the absence of research on teachers' knowledge of content and the role such content knowledge played in instruction (Shulman, 1986). Shulman called for a shift in the focus of how teachers are assessed from its overemphasis on how to teach (methods) to a greater stress on that which is taught (content). Shulman and his colleagues identified the categories of knowledge listed in Figure 2.1.

- Content knowledge
- General pedagogical knowledge, with special reference to those broad principles and strategies of classroom management and organization that appear to transcend subject matter
- Curriculum knowledge, with particular grasp of the materials and programs that serve as “tools of the trade” for teachers
- Pedagogical content knowledge, that special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding
- Knowledge of learners and their characteristics
- Knowledge of educational contexts, ranging from workings of the group or classroom, the governance and financing of school districts, to the character of communities and cultures
- Knowledge of educational ends, purposes, and values, and their philosophical and historical grounds

(Shulman, 1987, p. 8)

Figure 2.1*.* Shulman’s major categories of teacher knowledge

Proposing a new theoretical framework for teacher knowledge, Shulman suggested three domains of content knowledge: subject matter content knowledge, pedagogical content knowledge, and curricular knowledge that together comprise Shulman’s “missing paradigm”. The focus on teachers’ content knowledge was revived and several studies followed Shulman’s work.

### Development of MKT

Germane to this study is the theory of MKT, which is the result of a careful study of teaching practice using a disciplinary mathematics lens. This section presents the progress of MKT in the last 10 years. In 2000, Ball & Bass claimed that “although conceptions of what is meant by “subject matter knowledge,” as well as valid measures thereof, have been developing, we lack an adequate understanding of what and how mathematical knowledge is used in practice" (p. 86).

Ball and Bass were part of the University of Michigan's Mathematics Teaching and Learning to Teach project (MTLT) and the Learning Mathematics for Teaching project (LMT) that spent about 15 years studying mathematics teaching and the mathematics *used* in teaching (Ball & Bass, 2003; Ball et al., 2008). By examining the records of teaching of an accomplished teacher, the MTLT project investigated the mathematical knowledge demands of teaching from which they developed a number of "testable hypotheses" (p. 390) about the domains of mathematical knowledge for teaching (MKT). The LMT project then developed measures that assessed these identified knowledge domains.

The construct of MKT was developed in two significant ways. First, The MKT research group studied teaching practice to identify the mathematical entailments of engaging in the work of teaching. Ball (1999) describes it as follows:

> We seek to analyze how mathematical and pedagogical issues meet in teaching – at times intertwining, at times mutually supporting, and at times creating conflicts. Through analyses of mathematics in play in the context of teaching, the project extends and challenges existing assumptions of what it is about mathematics that primary teachers need to know and appreciate, and where and how in teaching such understandings and appreciation are needed (p. 28).

Thus this work goes beyond knowing particular mathematical content, such as how to do multi-digit multiplication, it also involves the mathematical demands of planning for a lesson, listening to children, asking questions, and helping to develop, validate, and justify mathematical definitions, claims, and methods (p. 28).

The second way the construct of MKT was developed involved an in depth analysis of mathematics education literature that studied the work of teaching. The MKT research group founded their work on two main bodies of literature that were inspired by Shulman's (1986) work: "one on teachers' subject matter knowledge and its role in teaching; and the second on the interplay of mathematics and pedagogy in teaching and teachers' learning" (Ball, 1999, p. 22). The first body of literature examined teachers' knowledge of specific subjects such as history (Wineburg, 1996), English (Grossman, 1990), mathematics (Wilson, 1988) and science (Carlsen, 1988). The second body of work was more focused on mathematics education (Borko, Eisenhart, Brown, Underhill, et al., 1992; Simon, 1993; Thompson, 1984).

The theory of MKT is informed by a study of teaching practice; in this sense the theory of MKT proposed by Ball is *practice-based* (Ball & Bass, 2003). As such, a distinction between construct and theory is crucial.

Simpson describes a theory as "a scheme or a system of ideas or statements held as an explanation or account of a group of facts or phenomena" (Simpson, 2004). The MKT research group's theory of MKT posits that teachers of mathematics need to know mathematics that is entailed in and demanded by the work of teaching. As such, this theory of MKT is "*etic*" in nature (Pike, 1954). This means "it describes a generalized approach to and belief about knowledge that can be related to all countries" (Delaney, 2008, p. 22). If that is the case, researchers in other country contexts such as Korea, Ghana, Ireland, Indonesia, and Norway could study the work entailed in teaching to identify the mathematical knowledge demands. As Ball et al. (2008), aptly describe,

> On the one hand, the generality of our results may be limited because our data are limited to only a few classrooms all situated in the U.S. context. On the other hand, our results are likely to be broadly applicable because our conception of the work of teaching is based, not on a particular approach to teaching, but on identifying fundamental tasks entailed in teaching (p. 396).

A construct is however not as generalizable. Pike argues that a construct is context bound and holds true in a particular setting at a particular time (Pike, 1954). Ho and Cheung (2007) also argue that behaviors are culture-specific thus researchers need to develop theories and measures sensitive to local contexts. As such, the construct of MKT as a U.S. practice-based and empirically derived is *emic* in nature. And so the work done by the MKT research group has both emic and etic dimensions. While the theory of MKT has the potential to be generalizable (See Cole, 2009; Delaney, 2009; Kwon, 2009; Ng, 2009), the construct of MKT might be more closely tied to contexts. This study is therefore well situated to examine this phenomenon.

I now turn to the process of development of the U.S. construct of MKT. Central to Ball et al.'s argument about the theory of MKT are three main points, as Ball asserts:

> 1. Much of the work of teaching is mathematical in nature with significant mathematical demands… although our examples are drawn from the context of teaching, the *mathematical* knowledge needed to engage them stands on its own as a domain of understanding…
> 2. … the mathematical knowledge… identified here has a relevance to teaching that is often missing from discussions about the mathematics needed by teachers. By identifying mathematics in relation to specific tasks in which teachers engage, we establish its

> relevance to what teachers do… our practice-based conceptualization of content knowledge for teaching provides an additional way of building bridges between these two worlds[7]: it does so by defining knowledge in broad terms, including skill, habits of mind, and insight and by framing knowledge in terms of its use…
>
> 3. We suspect that many of these insights extend to the knowledge teachers need in other subjects as well.
>
> (Ball, et al., 2008, pp. 398-399)

To illustrate MKT as an important conception of teacher knowledge Ball and colleagues use an example of a multi-digit subtraction problem: 307 – 168 written in vertical form. They argue that teachers must be able to know how to do the subtraction problem but more importantly, this ability is not sufficient for teaching multi-digit subtraction. For instance, some students might solve the problem in the following way:

$$\begin{array}{r} 307 \\ -\,168 \\ \hline 261 \end{array}$$

In the above example, the student may have calculated the differences between the numbers in each column by subtracting the smaller digit from the larger digit. The MKT research group argues that any teacher might be able to see this solution as incorrect. However, “skillful teaching requires being able to size up the source of a mathematical error” (p. 379) and the process of sizing up must be done “in-the-moment” of teaching.

Another error that a student could make is to solve the problem in the following way:

[7] The two worlds are the academic world of disciplinary knowledge and the practice world of teaching.

$$
\begin{array}{r}
307 \\
-\ 168 \\
\hline
169
\end{array}
$$

In this example, the student correctly traded 1 hundred from the hundreds column and was able to represent the 7 in the ones column as 17, subtracted 8 from 17 to yield 9. The student then "brought down" the 6 in the tens column and then subtracted 1 from the remaining 2 in the hundreds column to get 1. The MKT research group suggests that this kind of error analysis needs to be "efficiently and fluently" (p. 397) carried out by the teacher. Again, a teacher should be able to correctly identify the source of the second error, and be able to see that the source of the errors in each case is different. In particular, a determination that each of the answers is incorrect is not sufficient knowledge for a teacher to be able to effectively teach multi-digit subtraction.

The MKT research group argues that error analysis of this nature is unique to the work that teachers do and posit that this kind of mathematical knowledge and reasoning is not likely to be encountered by other adults who are not teachers. In addition to error analysis, Ball and her group argue that other mathematical tasks of teaching, such as analyzing students' non-standard approaches to problem solving, presenting mathematical ideas, linking representations to underlying ideas and to other representations, and giving or evaluating mathematical explanations require mathematical knowledge, skills, and habits of mind that are unique to teaching (Ball et al., 2008).

The above example illustrates the practice-based nature of MKT; it is both derived *from* and used *in practice.* If the nature of student errors in multi-digit subtraction in Ghana is similar to the errors in the U.S., one could argue that the mathematical knowledge demands of teaching subtraction in both countries are similar.

### *Domains of MKT*

The MKT research group posits that MKT both builds on and refines Shulman's work. They group MKT broadly into subject matter content knowledge, and pedagogical content knowledge (see Figure 2.2). Content knowledge, or subject matter knowledge includes Common Content Knowledge (CCK): mathematical knowledge held in common with other professional users of mathematics, such as the knowledge of how to carry out a division calculation. Another domain of subject matter knowledge is horizon knowledge (HCK): defined to include the knowledge of mathematics that students are likely to encounter in future. For example, teaching students the meaning and use of the equal sign in lower primary grades prepares them for later work in algebra. The third domain of subject matter knowledge is what they term Specialized Content Knowledge (SCK). This is the "mathematical knowledge and skill uniquely needed by teachers in the conduct of their work" (Ball et al., 2008, p. 400). Some of the tasks of teaching include analyzing students' responses, both correct and incorrect. Correct responses need to be analyzed to investigate whether the strategy used can be generalized for all cases, identify boundaries within which particular strategies would work, and incorrect responses need to be analyzed mathematically to determine not only where intervention is needed, but an identification of what students are doing mathematically (Ball et al., 2008).

Pedagogical content knowledge can also be subdivided into three domains: knowledge of content and students (KCS), which is an amalgam of knowledge about the content with knowledge about the students; knowledge of content and teaching (KCT), which is a combination of knowledge about the content and knowledge about teaching; and knowledge of content and curriculum. The latter domain, they are cautious to mention, is still under development thus could be represented differently as more work is done (Ball et al, 2008).

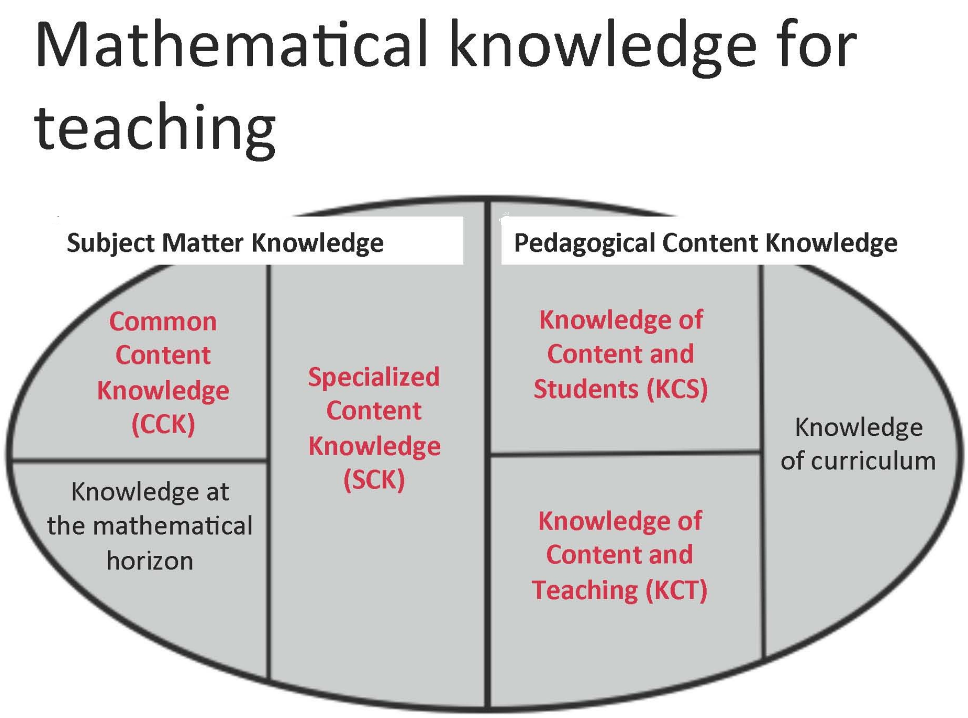


Figure 2.2. Mathematical Knowledge for Teaching. Presented by Ball, Bass, Sleep, & Thames (March10, 2006) at the Eighth Annual Chicago Symposium Series on Excellence in Teaching Mathematics and Science: Research and Practice, Chicago, IL.

### *Measuring MKT*

The work of the MKT research group was foundational for the development of measures to assess these domains of mathematical knowledge (Hill, Schilling, & Ball, 2004) and was a significant contribution to the field. These were multiple-choice tests purported to measure MKT. The questions were contextualized in the work that teachers routinely do and were developed to measure teacher knowledge in: number and operations; patterns, functions, and algebra; and geometry. The items were written by teams comprising mathematicians, mathematics educators, teachers, psychometricians, and other researchers (Bass & Lewis, 2005). A sample item is given in Figure 2.3:

7. Which of the following story problems could be used to illustrate $1\frac{1}{4}$ divided by $\frac{1}{2}$? (Mark YES, NO, or I'M NOT SURE for each possibility.)

| | Yes | No | I'm not sure |
|---|---|---|---|
| a) You want to split $1\frac{1}{4}$ pies evenly between two families. How much should each family get? | 1 | 2 | 3 |
| b) You have \$1.25 and may soon double your money. How much money would you end up with? | 1 | 2 | 3 |
| c) You are making some homemade taffy and the recipe calls for $1\frac{1}{4}$ cups of butter. How many sticks of butter (each stick = $\frac{1}{2}$ cup) will you need? | 1 | 2 | 3 |

Figure 2.3. Item 7 on LMT mathematics released items[8]

[8] This item was downloaded from http://sitemaker.umich.edu/lmt/files/LMT_sample_items.pdf on 02/10/2011.

The item probes an example of the specialized knowledge that The MKT research group suggest is unique to teaching. Many educated adults are likely to be able to calculate $1\frac{1}{4} \div \frac{1}{2}$. However, only teachers need to know a variety of problem contexts to illustrate $1\frac{1}{4} \div \frac{1}{2}$. In addition, teachers are required to be able to size up examples of problem contexts to determine whether any student-produced examples correctly illustrate the problem and the cases for which such examples may not be applicable. Teachers are also required to tell the difference between problem contexts that illustrate $1\frac{1}{4} \div \frac{1}{2}$ and ones that illustrate $1\frac{1}{4} \div 2$ and be able to identify this difference when students propose such examples.

Pre and posttests using the measures on a sample of teachers at the California Professional Development Institute (Hill & Ball, 2004) showed growth in teachers' MKT. Later work by Hill (2007) on a nationally representative sample of teachers showed that U.S. middle school teachers demonstrated a stronger understanding of number concepts than they did of algebra. While these studies showed that the items could be used at scale, later studies tested the validity of the measures by relating teachers' scores on these measures to the mathematical quality of their instruction and to their students' learning gains.

Hill, Rowan, and Ball (2005) found that teachers with high levels of MKT as measured by their instruments, had students who had, over a year, the equivalent of more than two weeks of additional mathematical instruction, and the differences between the high and low achieving teachers was significant (Hill, Rowan, Ball, 2005). Thus the theory of MKT is useful in identifying the kind of mathematical knowledge that is useful for teaching, but also positively correlated with student achievement.

In addition, Hill and colleagues (2008) found that teachers with high levels of MKT taught in qualitatively different ways from teachers who did not score as high. In addition, teachers with low scores also tended to have more mathematical errors in their lessons (Hill et al., 2008).

MKT has been shown to possess three main features: (1) the theory of MKT is developed *in practice* and is relevant to what teachers do; (2) the theory uses a disciplinary lens to analyze teaching tasks and is thus *mathematical* in nature; and (3) the development of multiple-choice items enables the use of the MKT measures at scale. These three points provide a good justification for the potential use of the MKT measures in Ghana. The study of this dissertation is based on the assumption that with some cultural adaptations, the MKT measures could be used in Ghana. As such, if the adapted measures are not sensitive to the context in Ghana, there are significant implications for the study of teachers' knowledge in Ghana and Africa more broadly. The second reason is that the multiple-choice nature of the items provides the opportunity to study Ghanaian teachers' MKT at scale, if the items are shown to be a valid measure of MKT in Ghana.

Given that the theory of MKT has been shown to be important for instruction and for student learning, I now turn to other theories of teachers' mathematical knowledge and compare these conceptualizations with MKT. These comparisons illustrate how the different conceptions of teachers' knowledge compare to each other and ultimately to Shulman's (1986) theory by outlining the extent to which content knowledge and pedagogical content knowledge are conceived; if and how each theory is measured; and how close the conception of teacher's mathematical knowledge is to the practice of teaching and to disciplinary mathematics.

## Other conceptualizations of teachers' mathematical knowledge

There are different widely held views about the kind of mathematical knowledge needed for teaching. This review will however focus on conceptions of teacher knowledge that have been theoretically grounded and/or have developed measures to capture this knowledge. Researchers have developed different theoretical frameworks for categorizing sub-domains of mathematical knowledge. These include Senk and colleagues' classifications of mathematical curricular knowledge, knowledge of planning for mathematics teaching and learning, and enacting mathematics for teaching and learning (Senk, et al., 2008); and Blum & Krauss' classifications of tasks and multiple solutions, misconceptions and difficulties, and explanations and representations (Blum and Krauss, 2008). Figure 2.4 highlights a comparison of the different conceptions of teacher knowledge with Shulman's main theories of content knowledge and pedagogical content knowledge. The figure also highlights any components of the conception that is distinct from content knowledge and pedagogical content knowledge, the extent to which the theory is grounded in teaching practice and disciplinary mathematics, and how the conception is measured.

Figure 2.4 begins with Shulman’s groundbreaking conceptions, followed by Rowland and others’ conceptions, Senk and others’, Baumert and others, and finally Ball and colleagues’ theory of MKT, which is the theory under study in this dissertation.

| | Content knowledge | Pedagogical content knowledge | | Distinct from PCK and CK | Grounded in Teaching practice | Grounded in the discipline of mathematics | Measurement |
|---|---|---|---|---|---|---|---|
| Shulman, 1986, pp 9-10 | Content knowledge | Curricular knowledge | Pedagogical content knowledge | General pedagogical knowledge | No | No | No |
| | Amount and organization of knowledge, per se, in the mind of the teacher | Full range of programs designed for teaching particular subjects | Particular form of content knowledge that embodies aspects of content germane to its teachability | Knowledge of educational contexts | | | |
| | Understanding the structures of the subject matter in substantive and syntactic ways (Schwab, 1978) | Variety of instructional materials available in relation to those programs | Most useful forms of representations | Knowledge of educational ends, purposes, and values and their philosophical and historical grounds | | | |
| | Knowedge of the what and the why of the content | Set of indications and contraindications for the use of particular curriculum materials | Most powerful analogies, illustrations, examples<br>The ways of representing and formulating the subject that make it comprehensible to others<br>Understanding of what makes particular topics easy or difficult<br>Student conceptions and preconceptions and misconceptions | Knowledge of learners and their characteristics | | | |
| | **Content knowledge** | Pedagogical content knowledge | | Distinct from PCK and CK | Grounded in Teaching practice | Grounded in the discipline of mathematics | Measurement |
| Rowland et al., 2005, pp 260- 263 | Foundation | Connections | Transformation | Contingency | **Yes,** in the study of videos of teaching | Yes | No |
| | Knowledge and understanding of mathematics per se, | The coherence of the planning or teaching displayed across an episode | Knowledge-in-action as demonstrated both in planning to teach and in teaching itself. | Preparedness to deviate from initial teaching agenda | | | |
| | Knowledge of significant tracts of literature and thinking which has resulted from systematic enquiry into the teaching and learning of | Connected knowledge for teaching (Ball, 1990)<br>Profound understanding of fundamental mathematics (Ma, 1999) | Based on Shulman's idea of teachers transforming content knowledge into "pedagogically powerful" ways | Ability to think on one's feet | | | |
| | Espoused beliefs about math (Rowland et al., P.260) | Management of discourse<br>Sequencing of topics of instruction within and between lessons<br>Knowledge of structural connections within mathematics and awareness of the relative cognitive demands of different topics and tasks | | | | | |

Figure 2.4. Comparisons of other theories with Shulman (1986)

| Senk et al., 2008, p.5 | **Content knowledge** | Pedagogical content knowledge | | Distinct from PCK and CK | Grounded in Teaching practice | Grounded in the discipline of mathematics | Measurement |
|---|---|---|---|---|---|---|---|
| | Enacting mathematics for teaching and learning | Mathematical curricular knowledge | Knowledge of planning for mathematics teaching and learning | No | No, teacher knowledge determined from curriculum; school content based on TIMSS | Yes | Yes (multiple-choice, complex multiple-choice, constructed response) |
| | Analyzing or evaluating students' mathematical solutions or arguments | Establishing appropriate learning goals | Planning or selecting appropriate activities | | | | |
| | Analyzing the content of student questions | Selecting possible pathways and seeing connections within the | Choosing assessment formats | | | | |
| | Explaining or representing mathematical concepts or procedures | Identifying the key ideas in learning programs | Predicting typical students' responses, including misconceptions | | | | |
| | | Knowledge of mathematics curriculum | Planning appropriate methods for representing mathematical ideas | | | | |
| | | | Planning mathematics lessons | | | | |

| Baumert et al., 2010, p. 142 | **Content knowledge** | Pedagogical content knowledge | Distinct from PCK and CK | Grounded in Teaching practice | Grounded in the discipline of mathematics | Measurement |
|---|---|---|---|---|---|---|
| | Content knowledge | Pedagogical content knowledge | No | None but some of their items are grounded in the work that secondary teachers | No | No, their questions are open-ended |
| | Profound mathematical understanding of the mathematics taught at school | Knowledge of mathematical tasks as instructional tools | | | | |
| | | Knowledge of students' thinking and assessment of understanding | | | | |
| | | Knowledge of multiple representations and explanations of mathematical problems | | | | |

| Ball et al., 2008 | **Content knowledge** | | | Pedagogical content knowledge | | | Distinct from PCK and CK | Grounded in Teaching practice | Grounded in the discipline of mathematics | Measurement |
|---|---|---|---|---|---|---|---|---|---|---|
| | *Common content knowledge* | *Specialized content knowledge* | *Knowledge of math at the horizon* | *Knowledge of curriculum* | *Knowledge of content and teaching* | *Knowledge of content and students* | No | Yes, in the study of classroom teaching | Yes | Yes, measurable at scale with multiple -choice items. |
| | The mathematical knowledge and skill used in settings other than teaching | The mathematical knowledge and skill unique to teaching | An awareness of how mathematical topics are related over the span of mathematics included in the curriculum. | The mathematical knowledge and skill used in settings other than teaching | Knowledge that combines knowing about teaching and knowing about mathematics | Knowledge that combines knowing about students and knowing about mathematics | | | | |

Figure 2.4. Comparisons of other theories with Shulman (1986) continued.

There are some marked conceptual differences between the different conceptions about what teachers need to know to teach effectively. In addition to Shulman (1986) and Ball and others (2008), other theories have been proposed. I now turn to how these theories compare with Shulman's theory.

Rowland and his colleagues (2005; 2008), similar to Ball and colleagues, studied videos of pre-service primary mathematics teachers and used a grounded approach to develop their domains of knowledge identified the Knowledge Quartet: Foundation, Transformation, Connection, and Contingency. Rowland and colleagues' conception of teacher knowledge includes "beliefs" as a component of foundational knowledge. The inclusion of beliefs distinguishes Rowland and others' conception from Shulman's (1986) theories, which does not include a category for beliefs. In addition, they also propose *contingency* as a component of teacher knowledge. They define *contingency* to mean "a teachers' preparedness to deviate from their teaching agenda" or "the ability to think on one's feet" (Rowland et al., 2005, p. 263). Beliefs and contingency are distinct from Ball and others' MKT.

Another difference between Rowland and his colleagues' conception of teacher knowledge and Ball's MKT is in their measurement of teacher knowledge. Rowland and others do not report of ways to measures the Knowledge Quartet, in fact, their research only provides an account of their conceptualization of teacher knowledge but not how their conception is measured which is another significant distinction from MKT.

In association with the Australian Council for Educational Research, the Teacher Education and Development Study in Mathematics (TEDS-M) research team is focused on the study of primary and lower secondary school mathematics teacher preparation in 18 countries to identify the nature and extent of their knowledge for teaching. Senk and her colleagues at TEDS-M's conceptualization of teacher knowledge might perhaps be the most closely matched to Ball's MKT. Their "enacting mathematics for teaching and learning" possesses features such as explaining or representing mathematical concepts or procedures, responding to unexpected mathematical issues, and analyzing or evaluating students' mathematical solutions or arguments, which are similar to Ball's conception of Specialized Content Knowledge. In addition, Senk and colleagues (2008) identify conceptual distinctions between content knowledge and pedagogical content knowledge similar to both Ball's and Shulman's. There are some fundamental differences between the two conceptions though. First, Senk and others identify teacher knowledge not by a study of the mathematical demands of teaching, but by the content of school mathematics. Second, their school content is based on the Trends in International Mathematics and Science Study (TIMSS), and third, they do not assess the validity of their measures by studying teachers' mathematical quality of instruction.

The COACTIV project under the leadership of Baumert and his colleagues in Germany has focused on the mathematical knowledge of secondary (high) school mathematics teachers. Baumert et al.'s (2010) description of teachers' mathematical knowledge for teaching showed the knowledge domains as distinctly Content Knowledge (CK), and Pedagogical Content Knowledge (PCK). It is important to note that Baumert and his colleagues did not develop a theoretical framework for teachers' content knowledge. Their conception was based on Shulman's (1986) theory. These scholars describe PCK to include the knowledge of mathematical tasks and tools and the knowledge of student thinking and assessment of mathematical understanding. CK, they conceptualize as similar to Ma's (1999) as a profound mathematical understanding of school mathematics. In addition, their measurement instruments included tasks that appeared to assess Ball et al.'s Specialized Content Knowledge (SCK).

It is valuable to look at these theories because they provide the scope of research on teacher knowledge and the extent to which progress has been made in conceptualizing teacher knowledge and developing ways of measuring it. In spite of the progress in the field, none of the aforementioned conceptions of teacher knowledge is simultaneously grounded in the practice of teaching, is measurable at scale, and grounded in the discipline of mathematics. Shulman's (1986) seminal work contributes much to the work on teacher knowledge but is not grounded in teaching practice or disciplinary mathematics, and does not provide measures to assess his conception of teacher knowledge. Although Rowland and others (2008) conception is grounded in the teaching practice and in disciplinary mathematics, they do not provide measures for assessing their conception of teacher knowledge. Similarly, Senk and others' (2008) conception is grounded in the discipline of mathematics and is measurable, they do not ground their work in teaching practice while Baumert and his colleagues (2008) do not ground their work in teaching practice or disciplinary mathematics and do not have measures to assess their theory of teacher knowledge.  Given that Ball and colleagues' MKT is grounded in the discipline of mathematics while grounded in teaching practice, and they have developed measures to assess this theory at scale (Charalambous, 2008; Ball et al, 2008) MKT is well suited as the theory of teacher knowledge to study in Ghana.

#### Form and content of teachers' mathematical knowledge

Other theories of teacher knowledge are focused on the structure and content of teacher knowledge (Sherin, Sherin, & Madanes, 2000). Knowledge structures examine how knowledge is "organized and presented in a teacher's mind" (Sherin et al., 2000, p. 364). Such studies include ideas such as knowledge packages (Ma, 1999) and agendas, scripts, and routines (Leindhardt, Putnam, Stein, & Baxter, 1991). Studies of the content of teacher education describe the uses of teachers' knowledge and what that knowledge is about. Shulman's conceptualization of teachers' knowledge is one example and mathematics-for-teaching (Davis & Simmt, 2006) is another.

In a study of practicing teachers, Davis and Simmt (2006) proposed four components of mathematics-for-teaching: mathematical objects, curriculum structures, classroom collectivity, and subjective understanding.

Using Ball and Bass methodology of "job analysis," Davis and Simmt study teachers' efforts to learn mathematics and not the study of teachers' enactment of mathematics teaching. Drawing on complexity science, Davis and Simmt argue that the identified components of mathematics-for-teaching are not as distinct from each other as they are nested in each other.

Because the boundaries of complex systems are not distinct, Davis and Simmt argue that for teachers, “knowledge of established mathematics is inseparable from knowledge of how mathematics is established” (p. 297) and illustrate the relationship among the different components of mathematics for teaching as intertwined represented in Figure 2.5.

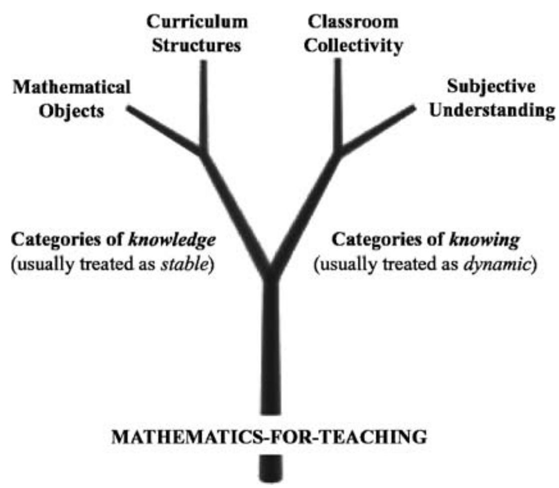


Figure 2.5. Perceived relationships among some aspects of teachers' mathematics-for-teaching (Davis & Simmt, 2006, p. 298).

In their description of the first component, mathematical objects, Davis and Simmt showed that teachers needed to have “access to the web of interconnections that constitute a concept” (p. 301). As such, the development of teachers' ideas of multiplication as a variety of representations such as repeated addition, equal grouping, number-line hopping, sequential folding, ratios and rates, and area-producing showed the complexity of the concept in that multiplication was not the sum of the different interpretations developed by the teachers but rather, ideas that were “incorporated into existing ideas” (p. 301).

Davis and Simmt (2006) describe curriculum structures, the second component of mathematics-for-teaching, as a transformative *recursive elaboration* that produces successive iterations of an idea that “fundamentally transforms the original form”(p. 308). Thus, one could conceive of multiplication as developing from the use of arrays to illustrate whole number multiplication, area models to represent real number multiplication, and grid-based arrays to show the multiplication of whole numbers, decimals, mixed numbers and binomials.

The third component, collective dynamics is based on the premise that mathematics-for-teaching occurs in contexts and cannot be mastered by an individual, as such the examination of “how others might be engaged in productive collectivity” (p. 309) is important. Thus, they focus on the following features of co-activity that promote learning: internal diversity, internal redundancy, decentralized control, enabling constraints, and neighbor interactions.

Subjective understanding is Davis and Simmt’s fourth component of mathematics-for-teaching. Although they do not fully encounter this component in their work with teachers, they recognize the importance of teachers’ individual knowledge. Mathematical knowing, they claim “is rooted in our biological structure, framed by bodily experiences, elaborated within social interactions, enabled by cultural tools, and part of an ever-unfolding conversation of humans and the biosphere” (Davis & Simmt, 2006, p. 315).

Comparing Davis and Simmt's theory of teacher knowledge with Ball's, the basic philosophical differences are illustrated in how Davis & Simmt's mathematics-for-teaching components are nested in complex and dynamic ways while Ball's conceptions are identified as distinct. Although Davis and Simmt use tasks and ideas from Ball and Bass, their unit of analysis is what the group does whereas Ball's unit of analysis is what the teacher does. In addition, they propose that teachers' mathematical knowledge for teaching is not necessarily *more*, but *different.* The MKT research group argues that MKT is *more and different* (Ball et al, 2008, p. 396).

Regardless of how teacher knowledge is conceptualized, a desired outcome of schooling is student learning which is also dependent on instruction (Hill et al., 2005). As such, although there may be philosophical differences in how teachers' mathematical knowledge is conceptualized as reviewed above, it is important to examine how teachers' knowledge impacts instruction.

## Teacher knowledge and its influence on instruction

Studies of the relationship between teachers' knowledge and their enactment is an essential part of this dissertation because a critical aspect of this study involves the establishment of the validity of the MKT measures for use in Ghana by examining the relationship between teachers' MKT scores and their mathematical quality of instruction. The past twenty years have seen a number of studies that have examined teachers' mathematical knowledge and how it is utilized in the work that teachers do (e.g. Ball, 1990; Ball et al., 2001; Borko et al., 1992; Leinhardt & Smith, 1985; Simon, 1993). These studies have been conceptually organized as “affordance”[9] studies and illustrate how teachers’ robust mathematical knowledge positively impacts their classroom instruction, and “deficit” studies show how teachers’ inadequate mathematical knowledge negatively influences their teaching practices.

**Affordance studies**

Lloyd & Wilson (1998) report on a study of the implementation of a reform curriculum by a high school teacher, Mr. Allen, in a functions lesson. Lloyd and Wilson describe a conception as “a person's general mental structures that encompass knowledge, beliefs, understandings, preferences, and views” (p. 249). Mr. Allen has 14 years teaching experience and data from interviews and baseline and post-lesson interviews show that he has a well-developed understanding of the concept of functions. Some qualities of good mathematical instruction that Mr. Allen exhibited included an appropriate definition of functions, an emphasis on the conceptual nature of functions, and the use of good classroom discussions that probed student thinking by asking for explanations and meaning. This study provides examples of how teachers’ knowledge influences instruction. This dissertation also investigates the extent to which this relationship is influenced by teachers’ MKT scores.

Another seminal body of literature that demonstrates the influence of teachers’ content knowledge on their instruction is the study of two expert teachers’ year’s long worth of teaching data (Ball, 1991; Lampert, 1986, 1989, 1990, 2001;

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[10]As characterized by Hill et al. , 2008 to identify studies that show how teachers’ mathematical knowledge positively influences instruction.

Leinhardt & Steele, 2005). Lampert (2001) illustrated her rich understanding of mathematical content knowledge in very explicit ways. In planning her lessons, Lampert considered the topics in the larger frame of the mathematical structures in which the topics was a part; as a result, she considered topics such as "division and remainders, fractions and decimals, and rate and ratio" (p. 220) as part of multiplicative structures.

In her instruction, she adapted her tasks skillfully to suit her students and maintained the cognitive demands of the tasks to ensure students productive mathematical experiences (cf. Leinhardt & Steele, 2005). Lampert's work provided many instances of the rich application of mathematical knowledge in instruction and the explicit attention to the work of teaching contributed to the development of the construct of MKT.

Another important study that carefully explicated how a teachers' rich and profound mathematical knowledge is usefully employed in teaching is the study of Ball's third grade classroom (Ball, 1992; 1993; Ball & Bass, 2000). In one study, Ball and Bass (2003) report on a subtraction lesson where students were working on the following task:

> Joshua ate 16 peas on Monday and 32 peas on Tuesday. How many more peas did he eat on Tuesday than he did on Monday?" (p. 91).

In the students' presentation of their answers, three main strategies were shown. First, Sean counted up on the number line from 16 until he got to 32. Second, Betsy used bean sticks to build 16, then built 32 and matched the number of peas eaten on Monday with the number eaten on Tuesday to get the result. The third strategy involved Mei disagreeing with Betsy's method, arguing that you could just subtract 16 from 32 to get the result. Of the six different strategies suggested in that lesson, Ball carefully explained each strategy to the students and provided the students with the similarities and differences across the methods. This demonstrated her deep conceptual understanding of the mathematics and how her knowledge was informing her instruction (Ball & Bass, 2000).

These affordance studies demonstrate the importance of teachers' deep conceptual mathematical knowledge and how this knowledge is exhibited in instruction and therefore matters for students' mathematics learning opportunities. The particular components of rich mathematical instruction such as the use and connection of representation, use of mathematical definitions, the production of mathematical explanations and the rich use of mathematical language inform the construct of MKT. These components of rich mathematical instruction have also been identified both from the study of instruction and relevant mathematics education literature to be components of rich mathematical instruction. In addition, this dissertation informs the codes that measure mathematical quality of instruction and shows specific ways in which high MKT might inform instruction (see Hill, et al., 2008).

**Deficit studies**

While the aforementioned studies illustrate how teachers' expert knowledge enhanced students' mathematical learning experiences, some teachers' limited knowledge constrained their mathematical instruction in ways that negatively impacted students' mathematical learning opportunities (Cohen, 1990; Peterson, 1990; Schifter, 2001).

Cohen (1990) reports on the case of Mrs. Oublier, a teacher, who, in the wake of the educational reforms that proposed teaching for understanding, enthusiastically adopted the reforms and declared them a success in her classroom. In her lessons, Cohen observed that Mrs. Oublier:

> had adopted innovative instructional materials and activities, all designed to help students make sense of mathematics. But Mrs. O seemed to treat new mathematical topics as though they were a part of traditional school mathematics. She used the new materials, but used them as though mathematics contained only right and wrong answers. She has revised the curriculum to help students understand math, but she conducts the class in ways that discourage exploration of students' understanding. (p. 312).

In an observed lesson, Mrs. O asked students to create number sentences other than 10+4 = 14. Cohen noted that although students came up with different number sentences, Mrs. O only publicly acknowledged the correct ones by writing them on the board. In all cases, Mrs. O did not ask students to explain their thinking or justify their answers and there was no evidence of students' understanding due to the didactic nature of her instruction. In another instance, Mrs. O asked the students to estimate how many paper clips would be needed to span the length of the desk. Cohen reports that students did not have clear view of the unit of measurement and as a result, students did not have a good frame of reference for the task. Thus, the students' answers "lacked mathematical discrimination. Estimates that were close to three times the actual answer, or one third of it, were accepted by the class and the teacher as reasonable" (p. 319). Cohen suggests that her "relatively superficial knowledge of this subject insulated her from even a glimpse of many things she might have done to deepen students' understanding" (p. 322).

Heaton (1992) also studied the teaching of “Sandra Stein” and reported that Sandra, like Mrs O., lacked mathematical sense making and used an inappropriate metaphor for teaching inverse functions such as such as shifting gears in a car. Heaton comments:

> The car analogy raises questions about what a model for the concept of inverse needs to represent. The car analogy represents the directional movement within the function but does not represent the inverse relationship. Shifting gears does not really give the feeling of undoing, which is the essence of the concept of inverse function. ( p. 157)

Heaton also comments on Sandra’s lack of focus on the word “inverse” in her lesson although the word was a central part of her lesson among other oversights (Heaton, 1992).

In another study, Stein and colleagues (1990) provide evidence of yet another teachers’ inadequate understandings of functions made evident by his use of poor analogy for functions, unlike Mr. Allen in Lloyd and Wilson’s (1998) study. Again, Putnam (1992) also reported on the case of fifth-grade teacher who did not productively use the class time in a lesson on averages. For instance, she spent a good length of time having students conduct a survey only to find that the survey results would not be useful in their lesson (Putnam, 1992).

Borko and colleagues also examined the relationship between pre-service teachers’ knowledge and instruction (Borko, Eisenhart, Brown, Underhill, Jones, & Agard, 1992; Eisenhart, Borko, Underhill, Brown, Jones, et al., 1993). Eisenhart and colleagues (1993) studied pre-service teachers’ knowledge in a fieldwork placement. One such teacher, Ms. Daniel, exhibited superficial knowledge of fractions that was mainly procedural in nature. Using interview data, these scholars showed how her selection of tasks from her curriculum did not provide her students with opportunities to construct the meaning of fractions. Observations of her teaching also showed she was unable to provide meaningful explanations to students about the formula for dividing fractions or an appropriate example for fraction division. Her limited understanding of fractions also hindered her ability to adequately reflect on her teaching (Eisenhart et al., 1993).

In her work, Deborah Schifter suggests “It has not been primarily through reflecting on her own teaching or by observing *fluent* teaching in action, that [she] has come to recognize much of what teachers still need to learn… knowledge and skills become visible by their absence” and upon becoming visible, you can “turn back to illustrations of effective teaching to see them in place” (Schifter, 2001, p. 116). Thus “affordance” and “deficit” studies highlight the important features of good mathematical instruction and inform the video codes that describe mathematical quality of instruction (Hill, et al., 2008). These video codes not only identify the quality of features of effective mathematics instruction such as the use of mathematical language, explanations and definitions, but also the errors that negatively impact students’ learning opportunities.

The findings from the dissertation will be compared with some of these affordance and deficit studies to determine how the study conducted in Ghana compares with these studies conducted in the U.S. For instance how do teachers with particular levels of MKT explain mathematical ideas, use mathematical representations or what is the nature of errors that are identified in their mathematics lessons. I now turn to a review of studies of teacher knowledge internationally. Although none of these studies were conducted in Ghana, an analysis of teachers' knowledge globally informs this study.

### International studies on teacher's mathematics knowledge

As an international comparative study, there is the need to examine non-U.S. studies on teacher' mathematical knowledge to determine how these studies might inform this exploratory study of transfer of MKT. There are a number of comparative studies about teacher knowledge, especially between U.S. teachers and their Asian counterparts (An, Kulm, & Wu, 2004; Andrews & Hatch, 2000; Gorgorió & Planas, 2001; Ma, 1999). In *Knowing and Teaching Primary Mathematics,* Ma (1999) compared the mathematical understandings of small groups of primary school teachers in China and the U.S. Her study showed that even though her sample of Chinese primary teachers are relatively less educated than their American counterparts, the Chinese primary teachers held mathematical knowledge in ways that had more depth than those of the American teachers sampled. Using concepts such as subtraction with and without grouping, and division, Ma explicated that "profound understanding of fundamental mathematics" (p. 118) is essential to the teaching of primary mathematics and that some Chinese teachers had this kind of understanding of mathematics that American teachers in the sample appear to lack (Ma, 1999). While Ma's study did not use a representative sample of Chinese and U.S. teachers in her study, her study used instruments from Michigan State University that, while open-ended, were similar to the MKT measures, because they contained authentic teaching tasks. Similar to Ma's findings, An, Kulm, and Wu

(2004) also studied a small number of U.S. and Chinese teachers. Using data from a survey, classroom observations and interviews, An, Kulm, and Wu concluded that Chinese teachers had a deeper conceptual understanding of mathematics than U.S. teachers. Studies such as An et al. (2004) and Ma (1999) cannot be generalized to teachers in either country because of their small sample sizes.

An ongoing 18-country cross-national study by researchers at Michigan State University is focused on the level and depth of the mathematics and related teaching knowledge attained by prospective primary and lower secondary teachers. This study investigates the knowledge that enables prospective teachers to teach in the kind of demanding curriculum currently found in higher achieving countries (Senk, Peck, Bankov, & Tatto, 2008; Tatto et al., 2008). A study of this nature would require an assumption of similarity across countries of the content of mathematical knowledge and the nature of mathematical knowledge needed to teach effectively. Although these studies employ useful tools to measure teacher knowledge, none of them are focused on teaching practice in Ghana. This study however, is interested in understanding how the measures developed in the U.S. can be used, and what can be learned from their use, in Ghana to assess teachers' mathematical knowledge for teaching. I take up this issue in chapter 3.

## Process of adaptation of measures

Cross-national studies, especially ones that compare developed and developing countries using measures or instruments developed in an industrialized country, and applied or used in non-industrialized country raise important methodological questions (Heyneman & Loxley, 1983). This dissertation is based on the use of U.S.-developed theory and measures of mathematical knowledge in Ghana. The need to exercise caution in making conclusions based on international studies cannot be overemphasized as results from such studies could be misguided. The importance of relevant contextual information is crucial (Keitel, & Kilpatrick, 1999; Lee et al., 2005; Noah, 1984).

In addition to providing contextual information, Schmidt and his colleagues (1996) reported that teaching and content are “embedded in culture” (p. 3). In a study comparing teaching in six countries, they explained the process under which instruments were developed and used in the TIMSS study. As the work evolved, they report, the non-U.S. members regularly questioned conceptions that the U.S. members had taken for granted as they worked together to find constructs that would capture the essence of their work. Other studies in which Chinese teachers’ knowledge was compared to U.S. teachers’ knowledge (Ma, 1999; An, Kulm, & Wu, 2004), did not explain how their measures were adjusted to be applicable in the Chinese contexts.

**Using MKT in other contexts-adaptations issues**

Researchers have used the MKT measures in Ireland, South Korea, Norway, and Indonesia (Delaney, 2009; Kwon, 2009; Mosvold & Fauskanger, 2009; Ng, 2009. Although the countries are very different from Ghana, there are some fundamental similarities in each of the country contexts such as the nature of mathematical knowledge that children are expected to know. I now examine the adaptation issues encountered by these researchers in their use of the MKT measures in a cross-cultural context.

A study by Delaney and colleagues (2008) utilized the MKT measures and adapted them for use in the Irish context. In their work, these researchers reported that the process of adapting the measures was laborious and involved multiple levels of analysis by a focus group comprising mathematics educators, a mathematician and teachers in Ireland. The Irish teachers were asked to comment on the validity of the items, that is, whether the mathematics in the items are likely to be taught in Irish classrooms to which some teachers reported some items unfamiliar. Delaney and others report that some teachers reported being unsure of the meanings of terms like tetrahedron and tessellations. A very important contribution from this research was the proposal of categories for adaptation: changes related to general cultural context, such as changing names in the items to reflect Irish names and spellings; changes related to school cultural context such as the change of text to textbook, students' papers to pupils' work; changes related to the mathematical substance such as inch to centimeters, polygon base to polygon face (Delaney et al., 2008).

Following Delaney and colleagues (2008), Mosvold and Fauskanger (2009) translated the MKT measures into Norwegian and reported on the process and challenges they encountered. For instance, in addition to the categories suggested by Delaney et al., (2008), they proposed two more: changes related to the translation from American English into Norwegian; and changes related to political directives. Some of their changes were similar to Delaney and others', such as *students* were changed to *pupils;* however, there were more distinct changes unique to Norway such as the use of a decimal comma instead of the decimal point and the format of measures. Also, although polygon translates to *polygon,* and congruent to *kongruent*, the Norwegian curriculum did not use those terms, instead, polygon was translated to *mangekant* meaning multi-edge, and congruent was changed to *helt lik* meaning “exactly the same”. These changes, Mosvold and Fauskanger report, could make the items easier for the Norwegian teachers (Mosvold & Fauskanger, 2008).

Ng (2009) also used the MKT measures to evaluate primary teachers’ geometry and number concepts and operations knowledge after a professional development program in Indonesia. Similar to Mosvold and Fauskanger (2009), he translated the measures into Indonesian before administering them to the teachers. Some contextual issues Ng encountered including the use of *Tetris* video game as a context for one of the games. Ng reported that not all Indonesian teachers would be familiar with the game. Ng also encountered some mathematical contextual issues such as the use of “polygon” was restricted to the college level; as such, Ng used a term that meant “multi-sided 2-D shape” in parenthesis following each use of “polygon”. In addition to differences in the general language use and the mathematical language use between Indonesia and the U.S., there are also differences in instructional practices and representation as well as the sequencing of mathematical topics in primary school. These concerns, according to Ng, might influence the validity of the findings. Notwithstanding these concerns, Ng reported that after administering the items to Indonesian teachers in his sample, a comparison of the item difficulties of the measures for teachers in Indonesia to teachers in the U.S. showed a relatively strong correlation (0.817) although the Indonesian sample found the items to be relatively more difficult than the U.S. teachers sampled (Ng, 2009).

In a recent study, Wilmot adapted and used measures of secondary teachers' knowledge of Algebra in Ghana. This study investigated how conceptualizations of the Knowledge of Algebra for Teaching (KAT) project at Michigan State University could be corroborated using data from Ghanaian secondary school teachers. Similar to this study, Wilmot adapted the items to suit the Ghanaian context for instance, the problem

> At a storewide sale, shirts cost $8 each and pants cost $12 each. If S is the number of shirts and P is the number of pants bought, which of the following is a meaning for the expression 8S + 12P?

was changed to become

> "At a storewide sale, shirts cost ¢80000 each and a pair of trousers cost ¢120000 each. If S is the number of shirts and P is the number of trousers bought, which of the following is a meaning for the expression 80000S + 120000P?" (Wilmot, 2008, p. 76)

Wilmot reported that these changes were necessary to make the item more culturally relevant. As such terms such as "pants" were changed to "trousers" and the prices in the question were changed to reflect the current price in Ghana at the time (Wilmot, 2008).

These studies highlight the importance of careful adaption and consideration of contexts before measures can be used cross-nationally. Differences in countries might be as obvious as the language but could be subtle such as the use of different words to represent "polygon" in Indonesia and Norway as a result of the school cultures.

These studies show that although the adaptations are complex, there is evidence that the MKT measures are usable in other country contexts. As such, careful adaptation and rationale for the adaptation of the U.S.-developed measures is a critical component of this study.

I now broadly examine the field of research in Africa and situate this dissertation study in Ghana's context.

## Educational studies in Africa

Educational research in Africa is varied and in multiple dimensions. Some studies include an examination of language policies in teaching (Adler, 1995; Setati, 2003, 2005), others examine the nature and features of pre-service teacher education (Akyeampong, 2002; Akyeampong & Stephens, 2002; Amedeker 2005) and others also study general aspects of instruction (Opoku-Amankwa, 2010; Schoeman, 2005; Steyn, & Plessis, 2007). These studies will present a broad view of educational research in Africa in general and Ghana in particular. While this approach may not directly inform the question under study, there are benefits to be derived from a contextual view of the research terrain in Africa by the identification of the gaps presented by research on specific themes. These themes include mathematical knowledge, teacher education, and teaching in multilingual contexts.

**Mathematical knowledge in Africa**

In their study, Lee, Zuze, and Ross, (2005) examine student literacy and mathematics achievement between and within schools, whether or not this achievement is linked to students' backgrounds, and if school characteristics are associated with effectiveness. The study utilizes data from the Southern Africa Consortium for Monitoring Educational Quality (SACMEQ) II. With data from Botswana, Kenya, Lesotho, Malawi, Mauritius, Mozambique, Namibia, Seychelles, South Africa, Swaziland, Tanzania, Zanzibar, and Zimbabwe, the data include descriptive data about "(a) student characteristics and learning environment, (b) teachers' characteristics and viewpoints, (c) principal's characteristics and viewpoints, (d) equity in the allocation of human and material resources, and (e) achievement of students and teachers" (p. 214).

Using varied hierarchical linear (HLM) models to analyze their data, their findings indicated interesting differences among countries by socio-economic status (SES) and by literacy achievement. Mauritius and Seychelles were typically in the high levels of SES and achievement, with countries such as Uganda, Malawi, Tanzania, Zanzibar, and Mozambique in the low levels. There was also considerable variability within students in South Africa but less so in Lesotho and Malawi. Although Lee et al. (2005) did not focus on teachers in their research, an interesting observation was that teachers sampled in the SACMEQ II data were administered a survey as well as a literacy test. The instrument used to measure teachers' knowledge in this data was the same instrument administered to students.

Wilmot's research cited earlier examined how Ghanaian high school teachers' Algebra knowledge as determined by the measures, was related to their students' performance. Although he found that student performance was positively related to the teachers' advanced mathematical knowledge, though not significantly, Wilmot's study concluded that the data in Ghana did not support KAT's identified knowledge categories (Wilmot, 2008).

Wilmot's study is especially significant because it includes a fundamental component of my study: the adaptation and use of U.S. developed measures in Ghana. Wilmot however focused on high school teachers. In addition, he implicitly assumed that the adapted measures were valid for use in Ghana. An extension of his work that is outside the scope of my study is his study of how pre-service and practicing teachers' performance was related to their students' performance.

### Teacher education research

There are several studies on teacher education in Africa. I now examine research on teacher education broadly and mathematics teacher education in particular. The Multi-Site Teacher Education Research Project (MUSTER) was a four-year research project, funded by the British Department of International Development and was a partnership among university departments in Ghana, Lesotho, Malawi, Trinidad and Tobago, and South Africa. Their work in Ghana focused on the development of teacher education in Ghana[10]. The MUSTER Project's goals in

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[10] For more details on the MUSTER project, please visit http://www.sussex.ac.uk/education/1-4-25-8.html

Ghana included an exploration of the entry into, the experience within, the influence or effects of, the costs of, and the justification for teacher education programs in Ghana. The Ghana country report by Akyeampong (2003) provided detailed contextual information about the Ghanaian education system and history, evolution of educational policy, and the structure of teacher education in Ghana. Some findings indicated that teacher educators routinely used the authoritarian and transmission model of instruction with little opportunities for student-to-student interactions. Teacher educators also placed little value on adaptive behaviors in teaching, only expecting their students to show the specific practices that they have been taught. Results indicated a lack of commitment on the part of educational administrators at the national level to support beginning teachers. As a result, the first three years of teaching was found to be very challenging for novice teachers. The paper concludes with recommendations for the government to take more seriously, the importance of good quality teachers to the attainment of the country's educational goal of providing access to all (Akyeampong, 2003).

In a study of the conceptualization of teacher education in Sub-Saharan countries, Akyeampong (2002) called for the need to reform teacher education programs to make them more "culturally sensitive and relevant to local needs" (p. 2). In his study of Ghana, Malawi, Lesotho, and South Africa, he identified features of teaching and learning in Sub-Saharan African countries to be primarily the authoritarian transmission model of teaching (see also Akyeampong, Pryor, & Ampiah, 2006; Kanu, 1996; Tabulawa, 1997) and claims this model has been very difficult to change due to the social values that exist in the culture, the influence of external examinations, and the presentation of material in textbooks. Akyeampong suggested that if teachers are required to be flexible and adapt their instruction to suit their contexts, teacher education had a role to play to effect this change. He further suggested that teacher education needed to focus on practice and to recognize the contextual nature of teaching experiences so pre-service teachers are provided with experiences that will give them leverage when they graduate (Akyeampong, 2002).

The aforementioned studies highlight the challenges faced by teacher education in Africa as a whole. These studies provide some contextual information about the research site and offer some basis for interpreting the findings. I now examine specific studies and reports on mathematics teacher education in particular.

A monograph that outlines the state of mathematics education in twelve African countries including South Africa, Malawi, Kenya, Tunisia, and Zimbabwe show that African countries' colonial history has played a critical role in the development of teacher education in Africa. Zambia for instance was under colonial rule from 1890 to 1963 and teacher education was markedly different than in the post-colonial era (Tabakamulamu, Haambokoma, Nalube, 2007). This was similar to the history of teacher education in Botswana (Chakalisa, Garegae, Setlhare, & Kaino, 2007).

Owino, Mwathi, O'Connor, Marigi, and Gitau (2007) report that Kenya identified the goals of education as a tool "to be used as training for power judgment, logical thinking and the clear expression of ideas” (p. 91) and these ideals informed mathematics teacher education which was seen as an “appropriate tool to propagate this objective” (p. 91). As such mathematics was deemed a compulsory subject in school. Owino and colleagues report that although the number of years of primary schooling was changed from seven to eight, there was no corresponding change in teacher education. They also report that a pass in mathematics is not a pre-requisite for admission into teacher education college. Teacher education in Kenya is a two-year program with three sections of field placement (also referred to as teaching practice). Mathematics is a required subject in the first year but optional in the second year of training. Teachers who do not enroll in mathematics for the second year can only teach “lower primary classes”. The content of the first year of mathematics include number concepts, numeration system, whole numbers, operations on whole numbers, preparations for teaching, fractions, decimal and percentages, geometry and measurement. The second year content includes indices, integers, geometry, algebra, statistics, measurement, ratio and proportion, business arithmetic and scale drawing.

Owino and colleagues contend that the criteria for recruitment of primary teachers have been blamed for the low performance of students in secondary school mathematics. They argue that inadequate teacher knowledge yields behaviorist approaches to teaching which yields instrumental understanding of mathematics (Owino et al., 2007). The recruitment of qualified candidates into teaching is a persistent problem in African countries and has consequences for students’ classroom learning opportunities (Akyeampong, 2003; Anamuah-

Mensah, Mereku, & Asabere-Ameyaw, 2004). A larger problem is the retention of quality teachers in mathematics. Studies report teachers are lost to high mortatility due to HIV/AIDS in Swaziland (Dlamini, 2007), Lesotho (Polaki, Morobe & Mpalami, 2007), and Botswana (Chakalisa et al., 2007) as well as poor working conditions and brain drain in other countries.

Other studies of teacher education not focused on the structure and history of teacher education was illustrative of teacher identities as well as instructional assessments in teacher education institutions. Akyeampong & Stephens (2002) for instance studied the identities of pre-service teachers in Ghana. 100 randomly selected pre-service teachers were administered a survey and 18 of them provided written autobiographies for analysis. Results indicated that a majority of the students were between 17 and 29 years old, had very weak mathematics and English language scores, and had parents who were either teachers or farmers or had university degrees. The pre-service teachers reported that they did not want to be posted to localities where their services are needed most. Their expectations for the teaching profession were varied: most of them had extrinsic reasons for wanting to become a teacher such as job security and social mobility. This study is relevant because it provides some contextual information about who goes into teaching in Ghana. The small sample size however indicates that the results of this study must be put in perspective.

Another study of pre-service teachers in Ghana focused on the outcome of exposing one of two groups of female pre-service teachers to alternative forms of assessment such as the use of journals and portfolios. They report that the alternate form of assessment involved using interviews and surveys yielded significant improvement in the experimental as well as more positive attitudinal changes. Eshun & Abledu thus suggest the implementation of intervention programs to improve the attitudes and achievement of female pre-service teachers (Eshun & Abledu, 2001).

In a study of assessments in three teacher education sites, Adler and Davis (2006) investigated how "formal evaluative events" or assessments in three higher education sites mathematics courses for teachers, privilege compressed or unpacked mathematics in their administration.

Using data from four teacher education sites, Adler & Davis report that there was "a prevalence of compressed mathematical tasks" (p. 290) that did not require students to unpack or decompress their mathematics knowledge. Adler and Davis conclude with a call for scholars in mathematics and scholars in mathematics education to negotiate the value they place on the other's field and how these will translate into courses for teachers. They propose a hypothesis that the conflict between the two fields could be due to the structure and grammar associated with each field: mathematics is more structured and has strong grammar whereas teaching is not.

These studies illustrate the complex histories, structure of teacher education, and some of the challenges faced in teacher education in Africa. The problem of teacher recruitment is particularly pertinent to the study of teacher knowledge in Ghana and Africa more broadly. In Ghana for example, recruitment affects the quality of teachers, and in addition, there are challenges associated with the availability of basic educational resources (Anamuah-Mensah et al., 2004). These factors influence the quality of instruction more broadly and consequently the quality of mathematics teaching. A thorough understanding of the research context helps to situate the study and informs the discussion of the findings from this study. I now turn to examine some studies that highlight the challenges associated with teaching in multilingual contexts.

**Teaching in multilingual contexts**

In a study of instruction, Adler (1995) used data from six teachers in 3 different teaching contexts, to explore “the dynamics of multilingualism and the teaching and learning of mathematics” (p. 264), recognizing the complexity involved in teaching mathematics in a language that is also a language under study. Situating the study in South Africa’s unique history provided a background for the differences and disparities that exist in schools. Adler “listened to the data” using teacher interviews and was able to glean information about the interaction between the teacher, the learners, the teacher’s knowledge and pedagogy. Her findings indicate that the teachers of multilingual students have developed methods of teaching that she describes as multilingual. That is, the teachers are able to develop ways of teaching to facilitate the learning of their multilingual students such as how and when to use vernacular in class, being deliberate about language use in English and in vernacular.

Setati (2003) examined the role of language in multilingual classrooms in South Africa. She argued that the choice of English as the language of teaching and learning in South Africa had implications for the role that language plays in the classroom. Using the complex history of South Africa to provide a historical framing of the choice of English as the official language in schools, Setati explains of the importance of language in mathematics as it relates to students' learning and communication of mathematical ideas, especially in bilingual classrooms. Setati examined the conditions under which code-switching occurs as it relates to the use of language as an instrument of power and its implications for the learners of mathematics learning that students. In another study, Setati (2005) explores the role of language in a multilingual classroom in South Africa. She investigates the kinds of languages teachers use and the purposes for which they do, and the kinds of discourses that are privileged. She found that the use of procedural discourse was the discourse of assessment and that Setswana was the language of contextual discourse while English was the language of authority, assessment and mathematics. The positioning of language in multilingual classrooms has implication for the teaching of and opportunities for students to effectively engage with mathematics.

Recognition of the role that language plays in this study work will provide the needed impetus to adapt the measures to suit the Ghanaian context. I argue that the role of language could also factor in to how teachers will read and make sense of the items.

In a study in Ghana, Opoku-Amankwa (2009) reported on the effect of the "English-only" language in education policy on pupil's classroom communicative practices and learning in general. As an ethnographic case study, Opoku-Amankwa used data from a fourth grade classroom observation, interviews, and focus groups with students to understand the impact of the English-only policy on how children from different tribes and hence speak different languages learn in the classroom. Findings indicated that some students did not understand the English language. Where students contributed, there were many instances of "safe talk" (Chick, 1996) where students replaced one work in a sentence to yield a different response. For instance, the teacher asked students to form sentences with the word "excuse me" and some examples from the students were: "Excuse me, can I take your pencil?", "Excuse me, can I take your pen?", "Excuse me, can I take your book?". These examples were accepted as correct by the teacher and did not permit the students to take risks in class. Opoku-Amankwa argues that "participants in our case study are colluding in an elaborate pretense: the teacher pretends to be teaching and the pupils pretend to be learning" (p. 130). In addition, the students never asked questions or initiated talk in the lesson. In his interactions with students, Opoku-Amankwa reported that students expressed interest in speaking their native language because they could express themselves more effectively in their local language.

Teaching is a complex activity and teaching in a multilingual setting presents more complex dynamics that require constant negotiation by the teacher and the students. These studies show that teaching in multilingual settings presents serious challenges to teachers and students alike and the resulting consequence is the implication of teacher knowledge in these contexts.

## Summary

In this chapter, I have outlined the historical development of content knowledge for teaching and provided a theoretical justification for my selection of MKT as a theory of study over other theories of teacher knowledge. I explained the three qualities of MKT as disciplinary based, grounded in teaching practice, and measurable at scale. In addition, MKT has been shown to be important for student learning. It has been demonstrated that MKT is applicable in Ireland and other studies show possible applications in Indonesia, Norway and Korea. The *affordance* and *deficit* studies highlight important ways that teacher knowledge influences instruction. Given the problem of poor student achievement in Ghana and the consideration of improving teacher knowledge as an important aspect of teacher quality, MKT is an important theory that could positively enhance teachers' knowledge and consequently, students' learning opportunities.

The research on Africa provided some contextual information to help readers understand the nature of teacher education in some African countries, the quality of teachers and teacher preparation programs, and the challenges associated with teaching in multilingual contexts. The studies reviewed in this chapter show that there have been few studies of the mathematical knowledge of primary teachers in Africa. The measures used in these studies were limited in scope although the teachers were sampled from 13 African countries (Lee et al., 2005). Wilmot (2008) was perhaps the closest in scope to this study but his focus was on high school teachers' algebra knowledge. Theoretically, this study thus fills the gap of examining primary teachers' mathematical knowledge *for teaching* that has implications for Ghanaian teachers' and African teachers in general.

Methodologically, this study contributes to the studies of adaptation and transfer of MKT in non-U.S. contexts. This study provides a basis of comparison of issues of adaptation of the MKT measures and the measurement of MQI in non-U.S. contexts.

This study addresses a substantial gap in the literature: the application and transfer of MKT in Ghana. By carefully adapting the MKT measures for use in Ghana, and attending to the elemental and construct validity of the measures, this study takes a crucial step of studying MKT in Africa more broadly but Ghana in particular.

# Chapter 3: Methods

Given the problem of poor student achievement in Ghana and an increased focus on improving teacher quality as a possible option of improving students' performance, this study examines teacher knowledge by investigating the mathematical knowledge for teaching in the Ghanaian context. This study was not designed to examine the mathematical knowledge of teachers, rather, it focused on the extent to which the U.S.-developed instrument can be a valid measure of the teachers' knowledge in Ghana. This dissertation is an empirical validation of the transferability of a U.S.-developed construct, MKT, in a Ghanaian setting and aimed to examine how the study of a non-representative sample of teachers' MKT could inform the extent to which the U.S. construct of MKT and its measurement is applicable in Ghana. To do this, I analyzed survey data from 60 teachers and interview and classroom teaching data from 6 teachers. This chapter describes the methods and analysis used.

## Design of study

The orienting question of this dissertation study is "To what extent can empirically derived U.S.-developed measures of MKT be used to study the MKT held by primary teachers in Ghana?" With this question, I explored the overall utility of the MKT measures in Ghana. To determine the extent to which the U.S.-developed measures could be used to study Ghanaian teachers' MKT, this study was divided into four phases. The first phase was a process of adapting the MKT measures for use in Ghana. This involved modest adaptations of the MKT instrument that accounted for the general as well as the school culture in Ghana, while maintaining the mathematical substance of the measures. The second phase was the administration of the MKT measures to 60 teachers in Ghana and an analysis of the difficulties of the individual items. These analyses would determine the knowledge level of teachers for which the test might be best suited. The third phase of the study involved the use of in depth (cognitive) interviews to determine the validity of selected teachers' scores on the measures, specifically whether teachers answer an item correctly using correct reasoning or whether teachers answer an item incorrectly using incorrect reasoning. The fourth phase of the study examined another source of validation of the measures. This involved a study of two consecutive mathematics lessons from each of the selected teachers to assess the relationship between the teachers' MKT scores and the mathematical quality of their instruction.

In the data analysis for this study, I drew on literature on the development of MKT, cross-national studies of the application of MKT, and a mathematical perspective to analyze mathematics teaching in Ghana. In my analysis of the MKT survey scores, teacher interviews and video of mathematics teaching in Ghana, I utilized Item Response Theory (IRT), a disciplinary mathematics perspective, and video lesson analysis.

There are seven sections. The first section in this chapter outlined the design of this study; the second section provides the rationale for the adaptation of the measures and a snapshot of the adaptations that were undertaken. The third section provides an overview of the data sources and the fourth section outlines the process of data collection, recording, and analysis of the survey data. The fifth section provides the data collection and analyses of the interview and describes the process of assessing elemental validity and the sixth section provides the data collection and analyses of the video data. The seventh section outlines the assessment of another source of validity of the measures by examining the construct validity.

## Adaptation of measures

To guard against what Crossley and Watson (2003) call "uncritical transfer" of research findings from U.S.-based research into African contexts, the MKT measures needed to be adapted to suit the Ghanaian contexts in two ways:

(a) Accessibility: Can Ghanaian teachers understand the literal meaning of the questions the survey is asking?

(b) Face validity of the instrument: Are the contexts embedded in the items relevant to and authentically representative of mathematics teaching in Ghana?

To address these concerns, a focus group was conducted to adapt the measures in the survey to suit the Ghanaian context. The focus group was comprised of: an experienced mathematics educator and curriculum developer; an experienced classroom teacher with at least 10 years teaching experience in primary and middle grades[11]; the author (a Ghanaian-trained teacher); and a research assistant who had no experience teaching in Ghana but extensive mathematics

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[11] With one exception, the members of the focus group's experiences were gained in Ghana

teacher education experience in an Asian country. The focus group examined each question in the survey and addressed each of the concerns outlined above. Following the methodology pioneered by Delaney and colleagues when studying teachers in Ireland, (Delaney et al., 2008), this study attempted adaptations to the LMT survey in ways that did not compromise the cognitive demands of the tasks. As such, every effort was made to ensure that the mathematical content of the tasks was not changed. Changes that were made include general cultural changes such as changing proper names to make them more culturally relevant, and changing words such as “brownies” and “pizzas” were changed to “cakes” and “bread” respectively (see Table 3.1). In addition, as a response to the current policy environment in Ghana, the mathematics educator/curriculum developer in the focus group recommended that items reflect mathematical conventions that the Ghanaian community values. In response to that, mathematical changes included formatting changes such as writing fractions as $\frac{1}{4}$ instead of $^1/_4$ were made.

Although the MKT measures are in English, there are a number of native languages spoken in Ghana and the focus group was conducted on the premise that all teachers surveyed would have sufficient comprehension of English to be able to answer the items. To improve the accessibility of the measures, the focus group suggested changes to some educational context words and expressions such as “professional development” were changed to “in-service training”. This is in concert with researchers such as Hambleton (1994) who suggest that the unfamiliar names and contexts might be distracting to the test takers.

Additionally, Yen (1993) further cautions that such distractions could negatively influence teachers' performance on the items. Table 3.1 provides the adaptations made.

In terms of the face validity of the instrument, the focus group agreed that tasks were relevant to teaching in Ghana and consequently recommended no changes to the contexts embedded in the items, and did not suggest the exclusion of any items. Another source of validation of the contexts was established when teachers were interviewed. All the teachers agreed that the contexts were within boundaries for our consideration as work that teachers routinely engage in.

Table 3.1. Summary of Adaptation of MKT Measures

| Category | Item # | U.S. Form | Ghanaian-adapted form |
|---|---|---|---|
| General Cultural | 1 | Ives | Ofosu |
| | 2 | Kwon<br>Huang | Quaye<br>Lartey |
| | 3 | Lopez<br>Nina<br>Mandy<br>Rosa | Amey<br>Nana<br>Mary<br>Rose |
| | 4 | Lee | Ocloo |
| | 5 | Sanchez | Amo |
| | 6 | Siegel<br>Valencia | Kyere<br>Quaye |
| | 7 | Pizzas | Cakes |
| | 8 | Chambreaux | Baah |
| | 9 | Webb | Baba |
| | 10 | Stephenson | Twum |
| | 12 | West | Otu |
| | 13 | Walker | Kpodo |
| | 15 | Ng | Donkor |
| | 16 | Stone | Amponsah |
| | 20 | Wise<br>Pizza<br>Farmer Brown<br>Brownies | Opare<br>Cake<br>Opanyin Addo<br>Bread |
| | 21 | Callahan<br>Todd | Amo<br>Kwame |
| | 22 | Violetta | Amponsah |
| | 23 | Wright<br>Julie<br>Jeff | deSouza<br>Adwoa<br>Nii |
| | 25 | Washington<br>Billie | Otoo<br>Ato |
| | 26 | Geoffrey<br>Heather | Fiifi<br>Hannah |
| | 27 | Jackson | Twumasi |
| | 28 | Jackson | Ofori |
| | 29 | Susan<br>Davis | Ablah<br>Okyere |
| | 30 | Teva | Oppong |
| | 31 | Martin | Okpoti |
| | 32 | Yolanta | Ohene |
| | 33 | Bourlin | Mensah |
| | 34 | Hernandez | Andah |

| | | | |
|---|---|---|---|
| | 35 | Williams | Annan |
| | 36 | Jones | Dontwi |
| **Category** | **Item #** | **U.S. Form** | **Ghanaian-adapted form** |
| **School Cultural** | 3 | Professional development program | In-service training workshop |
| | 5 | District-sponsored professional development | Cluster-based in-service training |
| | 6 | Practice state mathematics exam | School educational assessment |
| | 20 | Quiz | test |
| | 22 | Papers | work |
| | 28 | State assessment<br>Mini-lessons<br>Quiz | Promotion exam<br>Extra classes<br>test |
| | | | |
| **Mathematical** | 4 | ¼ | $\frac{1}{4}$ |
| | 7 | Dollars | cedis |
| | 9 | $^{6}/_{10}$ of a dollar<br>$^{4}/_{5}$ of a dollar<br>money | $\frac{6}{10}$ of a cake<br>$\frac{4}{5}$ of a cake<br>cake |
| | 19 | Large numbers | Three-digit numbers |
| | 21 | Traditional algorithm<br>Conventional algorithm | Short multiplication algorithm<br>Conventional short algorithm |
| | | | |
| **Other** | 2 | One tenth | tenths |
| | 7 | Illustrating | showing |
| | 13 | Why this is true | why this is true |

## Data sources

This study utilizes three main sources of data from Ghana (See Figure 3.1). The first is the responses of 60 Ghanaian primary teachers to a survey of primary mathematics knowledge developed from U.S. teaching practice. The survey was adapted for use in Ghana using guidelines suggested by Delaney (see table 3.1) and colleagues (2008). The second source of data is interview audio, interview notes and transcribed interview responses from 6 teachers who completed the primary mathematics survey. The third source of data is 12 videotaped and transcribed mathematics lessons taught by the 6 teachers interviewed. The 6 teachers were selected using a multiple case-study approach (Yin, 2006). In addition to the main data sources, other sources of data such as the government mathematics syllabus and government-approved textbooks are analyzed to a lesser extent to help understand the data better and provide explanations for the findings. The data collection and analysis are described in Figure 3.1.

| **Orienting question:** *To what extent can empirically derived U.S.- developed measures of MKT be used to study the MKT held by primary teachers in Ghana?* | | | |
|---|---|---|---|
| **Guiding questions** | **Data** | **Analysis/technique** | **Goals** |
| How can empirically derived U.S.- developed measures of MKT be used to study MKT in Ghana? | US-developed MKT measures | Adaptation of measures to suit general cultural context, school cultural context, and mathematical substance (Delaney et al., 2008) | Develop Ghanaian-adapted MKT measures for use in Ghana |
| Do the U.S.- developed Ghanaian-adapted MKT measures validly measure MKT in Ghana? | Survey forms with 43 Number of Operations items and 13 Patterns, Functions, and Algebra items completed by 60 teachers | IRT scaled scores<br>Item difficulties on a 1-parameter model<br>Qualitative analysis of relatively easy and relatively difficult questions (Delaney, 2008; Hambleton et al., 1991) | Identify features of very easy and very difficult items.<br>Investigate the knowledge level that the MKT measures are best suited for in Ghana. |
| What is the relationship between teachers' MKT scores and their reasoning about their responses to the adapted MKT measures? | Responses to Cognitive interviews administered to 6 selected teachers | Coding of teacher responses for consistency with MKT scores<br>Investigation of teacher responses to a subset of items using a multiple case-study approach (Yin, 2006) | Are teachers reasoning about the items in expected ways? For instance, are correct answers the result of correct reasoning and incorrect answers the result of incorrect reasoning? |
| What is the relationship between teachers' scores and the mathematical quality of their instruction? | 6 video lessons taught by the 6 selected Ghanaian teachers. | Video coding of lessons for mathematical quality of general and technical language use, explanations, mathematical representations, and an examination of teacher errors (Erickson, 2006; Hill et al, 2008) | Investigating the mathematical knowledge demands of using language effectively and choosing and using representations in observed lessons.<br>Examining the nature of teacher errors in the lessons?<br>What is the mathematical quality of instruction for the relatively high-scoring teachers? What is the mathematical quality of instruction of low-scoring teachers? |

Figure 3.1. An overview of data collection and analysis techniques.

## Data collection: Survey

### Instrument

To measure MKT in Ghana, I selected the LMT Primary Number Concepts and Operations form B_01 that had previously been administered to U.S. and Irish teachers (Delaney, 2008; Hill & Ball, 2004; Hill et al., 2004). This form was selected because the mathematics content of Elementary Number Concepts and Operations section was similar to the primary content in Ghana. A complete form was used in this study first, to reduce researcher bias that could arise in selecting items from multiple forms and second, to facilitate comparison with previous studies. The items on form B_01 were in the identified MKT domains of CCK, SCK, and KCS and comprised items in number and operations and patterns, functions and algebra strands. There were three constructed response items included to assess teachers' reasoning broadly but due to the length of time it took to complete the test, many teachers did not respond to those items. This dissertation will be limited to the study and discussion of the Elementary Number Concepts and Operations sections of the form as those questions were used as the interview items. Table 3.2 provides a summary of the form by strand and MKT sub-domain.

Table 3.2. Breakdown of LMT Survey Items by Curriculum Strand, and by Sub-domain

| | Number and operations | Patterns, functions, and algebra | Total |
|---|---|---|---|
| **SCK*** | 10 | 5 | 15 |
| **CCK**** | 15 | 8 | 23 |
| **KCS***** | 18 | - | 18 |
| **Total** | 43 | 13 | 56 |

*SCK: Specialized content knowledge

** CCK: Common content knowledge

***KCS: Knowledge of content and students

Overall, there were 29 individual items and 7 item had a total of 27 testlets[12] this yielding 56 individual items assessed in the MKT survey. The interview items comprised 12 individual multiple-choice items and three items had a total of 12 testlets yielding 24 individual items.

## Sample

Teacher recruitment was different in public and private schools. To recruit public school teachers for the survey, public school district officials were contacted seeking their permission to conduct research in the schools. School principals in the district were then contacted and I met with each principal in each school to explain the goals of the research. I then met 4th to 6th grade teachers in that school to request their voluntary participation. Teachers who opted to be part of the survey were then included. In private schools however, I contacted either the principal or directors of the school, and explained the goals of the study to them.

[12] See Figure 5.1 for example of an item with a testlet

If the school leadership granted permission for the research, I then spoke to the teachers in that school. In all cases, eligible private school teachers agreed to be part of the survey.

The sample comprised 25 teachers from 8 public schools and 35 teachers from 7 private primary schools (See Table 3.3). All the schools in this convenience sample were located within a 10-mile radius with the exception of one that was about 30 miles away from the data collection region. This school was selected to balance the distribution of schools in the sample to include a high-tuition paying private school. All the schools were located in the southern part of Ghana and were classified as either public or private depending on the leadership of the school. The private schools were further classified as high and low-medium depending on the tuition that students were required to pay per term of about 12 weeks.

Table 3.3. Distribution of Teachers in the Sample by School Type

| | **Tuition level/term in U.S. Dollar equivalents** | **Number of schools** | **Number of teachers in the sample** |
|---|---|---|---|
| **Public** | free | 8 | 25 |
| **Private – low to medium** | Between 50 and 150 | 4 | 15 |
| **Private – high** | Between 150 and 500 | 3 | 20 |
| **Total** | | 15 | 60 |

### Survey administration design

In all but one school, I was present for the administration of the survey. In those cases, the survey was administered in test-like conditions. Teachers could neither confer with each other nor use calculators during the test. All teachers took the test in similar conditions. In the one case where the school administrator did not allow us to be present for the test, the survey was given to the principal and I returned for the completed survey a week later. Because I was not present for the test administration, I did not select any teachers from that school for further study. It is important to note that the test was administered in the U.S. differently. The survey was mailed to teachers to complete and after completing the survey, the U.S. teachers mailed their responses to the MKT survey group. The differences in testing conditions could influence the findings of this study as the test was not administered under similar conditions as Ghana and the U.S and assessing the performance of the measures across both countries must be interpreted in this context.

Prior to any survey administration, teachers were given the opportunity to not participate in the study. All teachers who took the survey signed consent forms. There was 100% response rate for a few reasons: first, the researcher was present for most of the administrations, second, teachers were given the option to not participate in the survey so teachers who did not want to volunteer to be a part of the study were excluded, and third, the school leadership had agreed to be part of the study.

During the survey administration teachers were asked to solve the questions independently. I was present for all but one of the administrations but no teacher asked clarifying questions about the survey content. This could indicate a successful adaptation of the instrument or a cultural reluctance to question the authority of the instrument. There was no time limit for teachers to work on the survey but in general, teachers needed about 3 hours to complete the survey. All respondents were compensated[13] for their time. Some teachers had to leave the testing area prior to completion of all the items but all teachers completed the multiple-choice section of the survey that was used in the analysis of this dissertation.

#### Demographics of respondents

Of the 60 teachers, 45% (27) were female and 55% (33) were male. Seventy-three percent reported some formal training in teacher education prior to teaching, 10% had no formal teacher education, 10% were qualified secondary school teachers[14], and 10% reported having neither a bachelor's degree nor teacher education diploma. Table 3.4 shows the teaching experience of the teachers in the sample. Under a third of the teachers sampled had between 2 and 5 years of teaching experience, and about 37% had between 6 and 20 years of teaching experience.

#### Recording data

Teachers' responses to the survey were recorded in Microsoft Excel. Where no answer was evident, or non-response, 0 was recorded. Invalid scores were also recorded as 0.  Teachers' responses were scored as invalid if the teacher's selection could not be interpreted. For instance, a teachers' selection of either two options in the same stem instead of one, or selecting the main test item

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[13] Teachers who took the survey were given $10 and teachers who were later interviewed and videotaped received an additional $40
[14] Secondary and high school are used interchangeably

instead of the answer options as a response was scored as an invalid response. The purpose of this study was not to report on teachers' performance; as such, responses were scored using the answer key developed by the Learning Mathematics for Teaching project. Responses to the demographic section of the data were treated differently. Information about the school's status such as private or public, or the level of tuition was the same for all teachers in the school, so consistent entries were made regardless of what teachers recorded in 3 cases. For instance if the teacher was in a public school but indicated otherwise, I entered public school for the teacher, if the teacher indicated that their public school was fee-paying, I changed that to indicate that the tuition was free for public schools.

Table 3.4. The Number and Percentage of Teachers in the Study by Years of Teaching Experience

| Teaching experience | Number of teachers | Percentage |
|---|---|---|
| **0 to 1** | 13 | 22 |
| **2 to 5** | 17 | 28 |
| **6 to 10** | 10 | 17 |
| **11 to 20** | 12 | 20 |
| **21 or more** | 7 | 12 |
| **Total** | 59[15] | 99 |

**Missing data**

Teachers' consent to participate in the study was interpreted to mean they would attempt to answer all the questions. The items on the measures were not deemed "compulsory" or important, neither were there high stakes associated

[15] Only one teacher did not report his number of years teaching.

with teachers' participation. Teachers were explicitly informed that they were under no obligation to answer each question (consent letter to teachers, Appendix A). As such, teachers' responses to the items were varied from one form where 24 out of the 56 items were attempted to 8 forms on which all 56 items were attempted.

There were three main reasons why some teachers did not complete the survey. These reasons were identified from the multiple sources of data collection. First, the items were too difficult for the teacher, second, the teacher accidently skipped a page, third, in some test sites, participants did not have enough time to complete the survey because they had to go and teach their respective classes. Interpreting missing data is important because there are implications for describing teacher knowledge. A very intelligent teacher might choose to not respond to a question because they know enough about the content to know that they cannot answer the question correctly (De Ayala, Plake, & Impara, 2001). Such teachers might be able to adequately interpret the answer choices available to make a decision that they do not know the answer to the problem and would not guess a response. The distinction among items that were not attempted because teachers had to leave early, items that were not attempted due to their perceived difficulty, and items that were not attempted because teachers did not want to guess is important. However, all three categories of items are treated as omitted, which might cause an underestimation of teachers' MKT levels reported. It is important to note that there were no distinguishing characteristics of teachers in any particular group and in four cases, there was no data about the teachers' background to inform any analysis.

**Data analysis**

The MKT survey is unique because the test items are grounded in the work of teaching. As a result, teachers' performance on the MKT measures would provide initial data about the prospects of measuring Ghanaian teachers' MKT via the measures. The teachers' MKT would be analyzed via a 1-parameter IRT model. Due to the relatively small sample size, 1-parameter models are recommended for analysis (Harris, 1989).  Because the MKT items vary in difficulty, reporting the number of items answered correctly might not give a valid measure of teachers' MKT proficiency and would not facilitate a good comparison of teachers' MKT via scores. The use of IRT scores to report teachers' performance on the items addresses the challenges posed by reporting raw scores (Bock, Thissen, & Zimowsky, 1997). IRT utilizes a scale with mean 0 and standard deviation 1 to estimate MKT proficiency. IRT also utilizes the same scale to estimate the difficulties of specific items. This means that an average item has a difficulty of 0 implying that a person of average proficiency has a 50% chance of responding to an item correctly (Hambleton, Swaminathan, & Rogers, 1991) thus an easy item would have an item difficulty of -3 and a difficult item, +3. Bilog-MG version 3 IRT software (Zimowski, Muraki, Meslevy, & Brock, 2003) was used to estimate the proficiencies and item difficulties. In addition to the item level analysis that IRT allows for, important information that this analysis will provide is a determination of the knowledge level of teachers that the test might be best suited for. For example, the analysis could show that the MKT measures might be better suited to identify teachers with high levels of MKT and not a reliable measure for teachers with low levels of MKT. Such a finding would require the future use of items with easier levels of difficulty to better discriminate among the low to medium scorers.

## Data collection: Interview study

### Interview protocol

The items selected for the interview were a subset of the Number Concepts and Operations 2001 scale. There were 24 questions based on 15 items from the Elementary Number and Operations scale. All the questions tested teachers' mathematical content knowledge that was either "common" or "specialized" (see Ball, Thames, Phelps, 2008). The interview protocol was designed to capture teachers' reasoning about the questions. As such, in addition to the items as represented in the original MKT measures, probing questions such as "Why do you think B is the answer; Why is A not the answer; Do you use such representations in your classroom; and, Is this scenario likely in your classroom", were included. These probes were meant to understand not only teachers' thinking and reasoning about the item, but also the relevance of the context embedded in the items.

### Sub-Sample for further analysis

From the sample of 60 teachers, 10 teachers from five schools (two public and three private) were selected for an analysis of their interviews and mathematics lessons. The 10 teachers included two teachers from each school and represented the highest and lowest scorer from each of the schools. No medium scorers were selected to determine the extent to which teachers on the high and low spectrums of MKT reasoned about the items and how these teachers' instruction was related to their MKT scores. This was because this exploratory study was primarily investigating how high and low teachers reasoned and taught and the use of medium knowledge teachers could introduce some noise into the comparison.

From this sample, six teachers were selected for further analysis. The selected cases were chosen primarily because they were teachers at similar grade levels but showed a variety in their test scores. A description of the teachers in this sample is provided in Table 3.5.

Table 3.5. Distribution of Interview Teachers by Grade Level and Raw MKT Score

| Teacher name | Public/Private school | Grade level | Raw MKT score out of 56 |
|---|---|---|---|
| **Fiifi**[16] | Private | 4 | 6 |
| **Kwaku** | Private | 4 | 14 |
| **Gyidi** | Public | 4 | 30 |
| **Afua** | Public | 6 | 29 |
| **Ewusi** | Private | 6 | 40 |
| **Joojo** | Private | 6/JSS 1 | 29 |

[16] All names are pseudonyms to protect the identities of the participants.

### Procedure

Teachers were informed that the process was entirely voluntary and if they were unable or unwilling to answer any question, they should say skip and the researcher would not coerce them to respond to the question. Interviews were routinely carried out at a location convenient to the teachers, and so locations ranged from the teachers' schools to the researcher's home. All teachers were compensated for their time, and if required, transportation was provided to teachers to their homes or schools. Teachers did not have access to their survey scores during the interview. Each teacher was asked to read each question out loud and to think aloud as he/she solved each item. All interviews were audio recorded and were transcribed verbatim with no grammatical corrections made to maintain their integrity. I however included punctuations as needed to guide the reader.

### Interview analysis

To gain a deeper understanding of the validity of the results from the LMT survey, it was important to determine how consistent these findings were in terms of how teachers reasoned about each question they answered. The transcripts of the interviews were coded to determine whether teachers' correct responses in the survey were attributed to correct reasoning, or conversely, if teachers' incorrect responses were due to incorrect mathematical reasoning.

In addition, the relevance of the problem context was probed. This provided more information about the applicability of the problem contexts to Ghana. For instance if the problem asked teachers to evaluate alternative student solutions for a subtraction problem and in all cases, strategies for solving subtraction problems are teacher-initiated, then the nature of mathematical work involved in the teaching of subtraction might be different in Ghana than the U.S.

**Assessing validity of MKT scores via examining teachers' reasoning**

To assess the validity of the utility of the MKT measures, the selected teachers were interviewed to gain an understanding of the cognitive processes that underlie their responses to the MKT items (Hill et al., 2007; Tourangeau, 1984). Cognitive interviews help assess how the respondents understood the questions as well as the mathematics of the tasks, how they formulated their answers, the level of knowledge they needed to answer a question accurately, and any other reactions to the questions that may not have been anticipated. Teachers were asked to read the question out loud, and talk through the solution, a method referred to as "concurrent think-aloud" (Sudman, Bardburn, & Schwarz, 1996). Due to space limitations, four of the 15 interview questions,[17] two relatively easy, and two relatively difficult, based on the item difficulties, are reported here.

The cognitive interview therefore, provides the researcher with the data to either accept the MKT scores as a valid and true representation of teachers' knowledge about number concepts and operations, or to raise questions about the nature of adaptation needed for the instruments to measure what they are intended to measure. The interview data were coded using the codes of Hill and colleagues

[17] Sample released items can be seen at http://sitemaker.umich.edu/lmt/files/LMT_sample_items.pdf (accessed on December 4, 2010).

(2007) for consistency (whether interview responses were aligned with survey responses) to establish a relationship between teachers' answers for the multiple-choice items and their reasoning for that answer as well as their thinking about the alternative answer choices in the item.

**Consistency:** teachers' reasoning for a particular item should be consistent with the multiple-choice answer they selected on the written test. A teacher's selection of a correct answer on the test should imply that the teacher knows the mathematics of the item and can correctly provide the rationale for choosing that answer and not others during the interview. Conversely, a teacher's selection of an incorrect answer on the written test should reflect faulty thinking or a lack of the knowledge to answer that particular item correctly during the interview.

**Justification:** This code was designed to record teacher's mathematical reasoning about an item. To answer a question correctly, teachers could draw on some or all of the following: mathematical definitions, the use of examples and counterexamples, memorized rules or algorithms, or other mathematical reasoning such as inference or deductions to justify their response. It was also possible for teachers to answer a question correctly by drawing on non-mathematical reasoning such as guessing, employing test-taking strategies such as eliminating some responses or other kinds of non-mathematical thinking. Transcripts were coded as justified if the teacher selected the correct response and provided the mathematical reasoning to support the answer, or if the teacher selected the incorrect response and did not show any evidence of mathematical reasoning about the task to support their selection of the incorrect response.

Similarly, transcripts were coded as unjustified if a teacher's correct response could not be explained by the use of correct mathematical reasoning. Transcripts were also coded as not justified when an incorrect response did not imply a lack of knowledge about the specific task being assessed. Two researchers, including the author, coded the first three interviews (50%) and a 95% inter-rater agreement was reached. The second three interviews were coded by the author only.

### Data collection: Video study

Another source of validation of the LMT instrument as a reliable measure of teachers' MKT in Ghana, this analysis examined the competence with which the six teachers performed routine mathematical teaching tasks such as providing explanations, selecting and using mathematical representations, and using mathematical language, and the degree to which the quality of enactment of these tasks was related to teachers' MKT scores. Studies have shown teachers' knowledge can influence teaching (Fennema & Franke, 1992; Hill et al., 2008; Leinhardt & Smith, 1985) however, research is still growing on how teachers' knowledge gets used in teaching. Hill et al. (2008) have established relationships between teachers' MKT and the mathematical quality of instruction, and have further investigated what MKT affords instruction and how the lack of MKT constrains instruction. This analysis examines how the six selected teachers' MKT scores are related to their mathematical quality of instruction (MQI). Hill et al. describe "mathematical quality of instruction" as:

> a composite of several dimensions that characterize the rigor and richness of the mathematics of the lesson, including the presence or absence of

> mathematical errors, mathematical explanation and justification, mathematical representation, and related observables.
> (Hill et al., 2008, p. 431).

Table 3.6 shows the five major themes that comprise MQI[18]. The MKT research group has identified several tasks of teaching that require mathematical knowledge (Ball et al., 2008). Delaney and colleagues (2008) have also identified similar tasks in research in Ireland. This research study would examine how the 6 selected teachers' MKT is related to their use of three specific elements in mathematical quality of instruction (MQI): mathematical representations, use of mathematical language, and their use of mathematical explanations in their mathematics lessons. These three aspects out of five features of MQI were selected because Hill and others (2008) have shown that teachers with high levels of MKT teach in qualitatively different ways with respect to these elements. The two elements of MQI not selected for analysis were developing mathematical generalizations and developing multiple procedures and solution methods. These features were not selected because a preliminary examination of the data indicated minimal presence of these features across the data.

Teachers with low levels of MKT have more errors and oversights in their lessons than teachers with high levels of MKT. As such, the nature of errors in each lesson would be also examined to investigate the relationship between the selected teachers' MKT and the general mathematical quality of their instruction pertaining to the named elements of MQI.

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[18] A detailed explanation of the literature that undergirds the elements of MQI are in Hill et al, (2008) and Learning Mathematics for Teaching, (2010)

Table 3.6. Elements of Mathematical Quality of Instruction

| |
|---|
| 1. Richness of the mathematics—the use of multiple representations, linking among representations, mathematical explanation and justification, and explicitness around mathematical practices such as proof and reasoning; |
| 2. Mathematics errors—the presence of computational, linguistic, representational, or other mathematical errors in instruction; <br> a. Contains subcategory specifically for errors with mathematical language |
| 3. Connecting classroom practice to mathematics—the degree to which classroom practice is connected to important and worthwhile mathematical ideas and procedures as opposed to either non-mathematical focus, such as classroom management, or activities that do not require mathematical thinking, such as students following directions to cut, color, and paste, but with no obvious connections between these activities and mathematical meaning(s); |
| 4. Responding to students inappropriately—the degree to which teacher either misinterprets or, in the case of student misunderstanding, fails to respond to student utterance; |
| 5. Responding to students appropriately—the degree to which teacher can correctly interpret students' mathematical utterances and address student misunderstandings; <br> (Hill, Blunk, Charalambous, Phelps, Sleep, Ball, 2008, p. 437) |

**Sub-sample**

The same teachers selected for the cognitive interviews (see Table 3.5) also participated of the video study. There were two female and four male teachers in the sample. One teacher did not report his number of years teaching but of the five remaining teachers, there was a range of teaching experience from five years to 18 years. Table 3.7 shows the mathematical content taught by the teachers in the sample. The three fourth-grade teachers taught collecting and organizing data, fractions, and properties of basic operations.

Table 3.7. Distribution of Video Teachers by Grade Level and Content Taught in Lesson

| Teacher name | Gender | Public/Private | Grade level | Teaching experience (years) | Lesson domain | Mathematical content in lesson |
|---|---|---|---|---|---|---|
| **Fiifi** | Male | Private | 4 | not available | Data | Collecting data and recording results |
| **Kwaku** | Male | Private | 4 | 5 | Number and Operations | Fractions |
| **Gyidi** | Female | Public | 4 | 10 | Number and Operations | Properties of basic operations |
| **Afua** | Female | Public | 6 | 18 | Number and Operations | Multiplying a 5-digit number by a 1-digit number by Napier's method |
| **Ewusi** | Male | Private | 6 | 18 | Money | Solving simple problems involving profit and loss |
| **Joojo** | Male | Private | 6/JSS1 | 10 | Number | Decimal fractions |

**Procedure**

The classes observed were the ones planned for the day; lessons where tests or reviews were planned were avoided. A video camera was placed at the back of the room and was focused on the teacher. In all cases, the students were seated in rows facing the board so students' faces were not visible. Parental consent was obtained to videotape students and students for whom we did not receive consent were seated outside of the range of the camera. An audio recorder was also placed in front of the room to capture the teacher's voice to complement the video recording and also to serve as back up audio.

**Video analysis**

Two mathematics lessons per teacher were recorded and each lesson was divided into seven-minute clips (Learning Mathematics for Teaching, 2006). Three graduate research assistants, one of whom was familiar with the coding process, were recruited to code the videos. I trained the two students who did not have experience in the use of the LMT codes. After the training, each graduate assistant was asked to code a minimum of 3 videos individually and I met with each of them separately to reconcile our codes. Each video was coded by at least 2 people, including me. The reliability measures for the data are provided in Table 3.8. Three pairs of graduate assistants coded 2 videos and the lesson within each pair that had the highest reliability was selected for analysis.

Table 3.8. Reliability Levels of Video Codes for Videos with Highest Reliabilities

| Lesson name | Coding pair | Inter-rater reliabilities* |
|---|---|---|
| Fiifi 1 | 1, 2 | 0.75 |
| Fiifi 2 | 1,3 | 0.74 |
| Gyidi 1 | 1,3 | 0.84 |
| Gyidi 2 | 1,4 | 0.73 |
| Kofi 1 | 1,4 | 0.67 |
| Kofi 2 | 1,2 | 0.56 |

* inter-rater reliabilities calculated as percentage of agreement between raters. This was calculated as the number of codes for which both coders agreed divided by the total number of codes multiplied by 100.[19]

[19] Reliability refers to the extent to which a construct is measured consistently (Cronbach, 1990). An inter-rater reliability of 1 means that there was 100% agreement between raters, conversely, a rating of 0 means there was no agreement on any code between raters. In general, acceptable levels of reliability require a minimum level of 0.7.

### Video coding process

The video coding process was very rigorous. LMT spent many years refining the video codes and addressing the validity and reliability of the video coding rubric. The coding glossary (see Appendix C) was refined many times in the last 6 years as the LMT group used evidence from the study of instruction to refine the theory of MKT and the instruments used to measure it. During the coding process, a trained coder would watch the entire lesson first, usually making notes on the transcript. The next step involved watching the lesson again, this time in 7-minute intervals[20] and code for enactment and the level of error. For enactment, the code was 1, 2, or 3, depending on the level of proficiency with that feature and for errors, 1 was assigned for error-free segments and 2 or 3 was assigned depending on the degree of error observed in the lesson. In each case, the coder made detailed notes providing rationale for the assignment of a rank of 1, 2, or 3 for a particular feature. For instance, if a teacher was using mathematical language in careful ways, attending to the meaning of the word, and if applicable, being explicit about the use the word in mathematical and non-mathematical settings, the language category will be assigned a 3, the highest possible score. In the same way, a coder would give a rank of 1 if the segment was error free and if there was an error, the coder will make a determination first if the error was a major mathematical error or oversight, an imprecise use of language or

---

[20] Earlier versions of the coding rubric required watching the video in 5-minute intervals. The time was changed to 7 minutes to ease the burden of coders and to facilitate comparison with other lesson observation protocols.

notation, or a lack of clarity. The coder would then gauge the level of seriousness of the error and assign a 2 or 3 with 3 for the most severe error.

***Structure of MQI codes***

The first three codes were broadly categorized to determine for each segment: Whether firstly, the format of the segment was mostly active instruction, small group or individual work, or both; secondly, whether the segment was mostly teacher directed or if there was whole group discussion; and thirdly, whether the time spent in that segment was focused on mathematics.

There were also four meta-codes for richness of mathematics, working with students and mathematics errors and imprecision, and student cognitive demand. Each of the meta-codes comprised sub-codes, for instance, the richness meta-codes comprised representation, multiple procedures and solution methods, explanations, developing mathematical generalizations, and mathematical language. In addition, the coder is asked to assign the level at which the teacher equitably engaged with students, and communicated high expectations and potential. Each coder was also required to make an estimate of the MQI of the lesson and a lesson-based guess at the teacher's MKT and other meta-codes. In each case, the coder would draw on their mathematical knowledge and the coding glossary to make detailed notes to support their codes.

The current version of the codes requires that for the mathematical features, if the feature is present and incorrectly enacted, a code of 1 be assigned. Also, a code of 1 is assigned if the feature is present and weakly used. For instance if there are no mathematical explanation in a lesson or if little mathematical language is used, a code of 1 is assigned for each feature respectively. For the purposes of this study, I redefined some codes to align them with the design of this study, as follows. First, per the glossary, a value of 1 in the codes for mathematical features (except errors) could mean that the feature was completely absent, or that the feature was not enacted appropriately. For this study, I wanted a clear distinction between features that were enacted, even poorly, and features that were absent. A code of 1 conflated the two features so I redefined 1 to indicate the absence of the feature; 2 for poor or medium enactment with a code of 2 in errors for poor enactment, and a code of 3 for correct, and explicit attention to the feature. To compensate for conflating the poor and medium enactment of the feature, the error codes were used to capture poor enactment. For example, a poor or incorrect use of a mathematical representation was given a 2 for representation to show that the mathematical feature was present, and the error feature was coded at 2 or 3 depending on the gravity of the error.

Second, the code “linking or connecting” was designed to record the linking and connecting among different representations of mathematical ideas or procedures, and across representations. This code does not explicitly record the use of mathematical representations. For instance, using a graph to show the distribution of birth months in a class of 25 students, or the use of oranges to represent a fractional whole or part would not be recorded as the use of representations unless the representation was explicitly linked to another representation. I thus redefined the code to also include the use of a representation to demonstrate a mathematical idea as well as the connecting and linking of representations.

The coding process identified the presence or absence of a mathematical feature such as the use of technical language, and the appropriateness or inappropriateness of the presence or absence of that feature. After individually coding a complete lesson video, the two coders met to reconcile their codes and the reconciled code was recorded as the final agreed-upon code.  Then a lesson summary was created that recorded salient points about the lesson highlighting mathematically high and low points in the lesson. Figure 3.2 shows a portion of a blank video coding instrument (see Appendix D and E for a blank and completed coding sheet respectively). The calculation of inter-coder agreement was calculated to be the number of codes for which there was agreement between coders divided by the total number of codes. The inter-coder agreement for all three cases showed a range of 0.57 to 0.89, two of which are were quite low (Miles & Huberman, 1994). Both low reliabilities were associated with one case, Kofi. Kofi's low reliability was not surprising because both his lessons showed very similar reactions from coders. These findings will be discussed in the discussion sections of this dissertation. When I dropped his case, the inter-rater reliabilities were at 0.72 and higher.

| | Chapter 1 | | | Chapter 2 | | |
|---|---|---|---|---|---|---|
| | Rater 1 | Rater 2 | Reconciled | Rater 1 | Rater 2 | Reconciled |
| **Code on First Watch** | | | | | | |
| **Format of the segment** | | | | | | |
| Active instruction (0) or small group/partner/individual work (1), both (2) | | | | | | |
| **Mode of Instruction (1 = None, 2 = Some, 3 = Most/All)** | | | | | | |
| Direct instruction | | | | | | |
| Whole-class discussion | | | | | | |
| Working on applied (real-world) problems | | | | | | |
| **Time on Task (0 = No, 1 = Yes)** | | | | | | |
| Code here for whether the focus is on mathematics content during half or more of the segment (3.5 minutes or more total) | | | | | | |
| **Second Watch: "Quailty" Codes** | | | | | | |
| **Richness of the Mathematics (1 = Low, 2 = Mid, 3 = High)** | | | | | | |
| Representations | | | | | | |
| Multiple procedures or solution methods | | | | | | |
| Explanations | | | | | | |
| Developing mathematical generalizations | | | | | | |
| Mathematical language | | | | | | |
| Overall richness of the mathematics | | | | | | |

Figure 3.2. Portion of blank template of LMT video coding instrument

### *Quantitative analysis of video codes*

Because the lessons were of different lengths (43 to 87 minutes), in order to facilitate comparisons across lessons, each code was averaged to account for the length of the lesson. For example, if Kofi's first 7-minute clip has 14 codes in 9 clips, then, Kofi, on average, has 1.56 (14/9) codes per clip. These averages were used to represent the overall score for each specific feature. Microsoft Excel was used to determine the means of each of the coded features of MQI and SPSS was used to calculate the correlations between the MKT scores and the MQI means respectively. For instance, across the 6 cases, lowest to highest scores for overall richness of the mathematics ranged from 1.1 to 2.3. The range represents the variation of scores among the nine lessons from these six cases. Higher scoring lessons feature more and better use of mathematical features such as explanations, representations, and multiple procedures and solutions than lower scoring lessons, and are thus likely to have a higher mathematical quality than lower scoring lessons.

To assess the validity of the MKT measures using MQI, I calculated the Spearman correlations of the six teachers' MKT survey scores and the meta-codes of overall richness of mathematics, overall working with students and mathematics, and overall errors and imprecision. These correlations helped to explain the extent to which the assumption that teachers' with high MKT teach in qualitatively superior ways than teachers with low MKT (see Hill et al., 2008; Hill, Schilling, & Ball, 2004) holds true with the Ghanaian sample.

The afore-mentioned sections outlined the design of the study and described the methods used to inform this dissertation study. I now turn to provide a rationale for validation and explain the different approaches to validation.

## Using MKT measures in Ghana: An interpretive approach

This study of transfer of MKT in Ghana can be summarized using the interpretive argument utilized by the MKT research group (Schilling & Hill, 2007) and Delaney (2008). Schilling and Hill posit that assessing the validity of a test addresses two important issues. First, validity determines whether the test "provides information of interest to test consumers" (p.70) and second, "whether scores generated by the test assist in making good decisions" (p.70). Despite its importance, research has suggested that validation is the most unsatisfactory aspect of test development (Messick, 1988). Kane (2001) addressed this problem with the development of an argument-based approach to validity consisting of two stages. First, the *formative* stage occurs where an interpretive argument is developed that states explicitly the assumptions and inferences associated with the proposed interpretation of the test. The second stage is the *summative* stage where the validity argument is developed and the corresponding interpretive argument is evaluated and if possible, reformulated taking in account the empirical evidence (Schilling & Hill, 2007).

Schilling and Hill used Kane's argument to investigate the validity of MKT measures developed for and used in the U.S. and to gain new information about the measures. They identified three assumptions and related inferences: elemental, structural, and ecological. While acknowledging the importance of all three assumptions and related inferences, this dissertation focused only on the elemental and ecological assumptions.

Elemental refers to the " The performance of specific test items, including consistency of subjects' thinking (and knowledge) with items because they address the constituent elements upon which the test is based" (p.73); and ecological refers to the "external structure of the test, including the relationship of the test scales with external variables" (p.71) because they place the test in context and like biological concerns… the inter-relationships are often complex and specific to a particular test" (p.73).

The interpretive argument and their related inferences for the use of MKT measures in Ghana are therefore the same as Delaney's modification of Schilling & Hill's (2007) assumptions:

> *Elemental assumption*: Teachers draw on mathematical knowledge used in teaching to respond to the questions.
> *Inference (a):* When responding to the items, teachers use mathematical knowledge used in teaching and their general mathematical knowledge.

> *Inference (b)*: Items on the test relate to activities in which teachers regularly engage (or in which they regularly need to engage)
>
> *Ecological Assumption:* The MKT multiple-choice measures captured the mathematical knowledge teachers need to teach mathematics effectively.
> I*nference*: Teachers' scale scores on the measures are related to the quality of the teachers' mathematics instruction. Higher scale scores are related to more effective mathematics instruction and lower scale scores are related to less effective mathematics instruction.
>
> (Delaney, 2008, p. 147)

These assumptions will be evaluated in the discussion of findings.

## Assessing construct equivalence

This chapter has outlined so far, the methods I used for the data collection and analyses of survey, interview, and video data. These data together provide a valuable source of evidence to determine the validity of the MKT scores of the Ghanaian teachers sampled and are also similar to the development of the theory of MKT in the U.S. (c.f. Ball, 1993, Hill et al, 2004; Hill et al., 2008). A critical component of this study is the attention to construct validity.

Researchers agree that the validity of cross-national research is threatened by the inattention to construct equivalence (Adler, 1983; Singh, 1995). Construct equivalence determines that "a given construct serves the same function and is expressed similarly (i.e., in terms of attitude and behaviors) in different cross-national contexts" (Singh, 1995, p. 603).

To determine the equivalence of the MKT measures, Singh (1995) suggests that three tests of equivalence be conducted before data collection in cross-national studies (see Delaney's adaptation of Singh's (1995) steps in Figure 3.2). These tests are *functional equivalence,* to determine whether the MKT construct serves the same function in Ghana and the U.S.; *conceptual equivalence*, to determine whether the MKT construct is expressed in similar attitudes or behaviors in the U.S. and Ghana; and *instrument equivalence* which determines whether the MKT items, response categories, and questionnaire stimuli are interpreted identically in Ghana and the U.S. (Singh, 1995, pp. 603-606).

This dissertation uses a non-representative sample of data; as such, I investigate the functional, conceptual, and instrumental validity of the MKT items. The assessment of functional, conceptual, and instrumental validity, as well the findings from the analyses of the survey, interview, and video data would collectively inform the utility of the MKT measures in Ghana and determine the nature of items that might be used to assess Ghanaian teachers' MKT at scale.

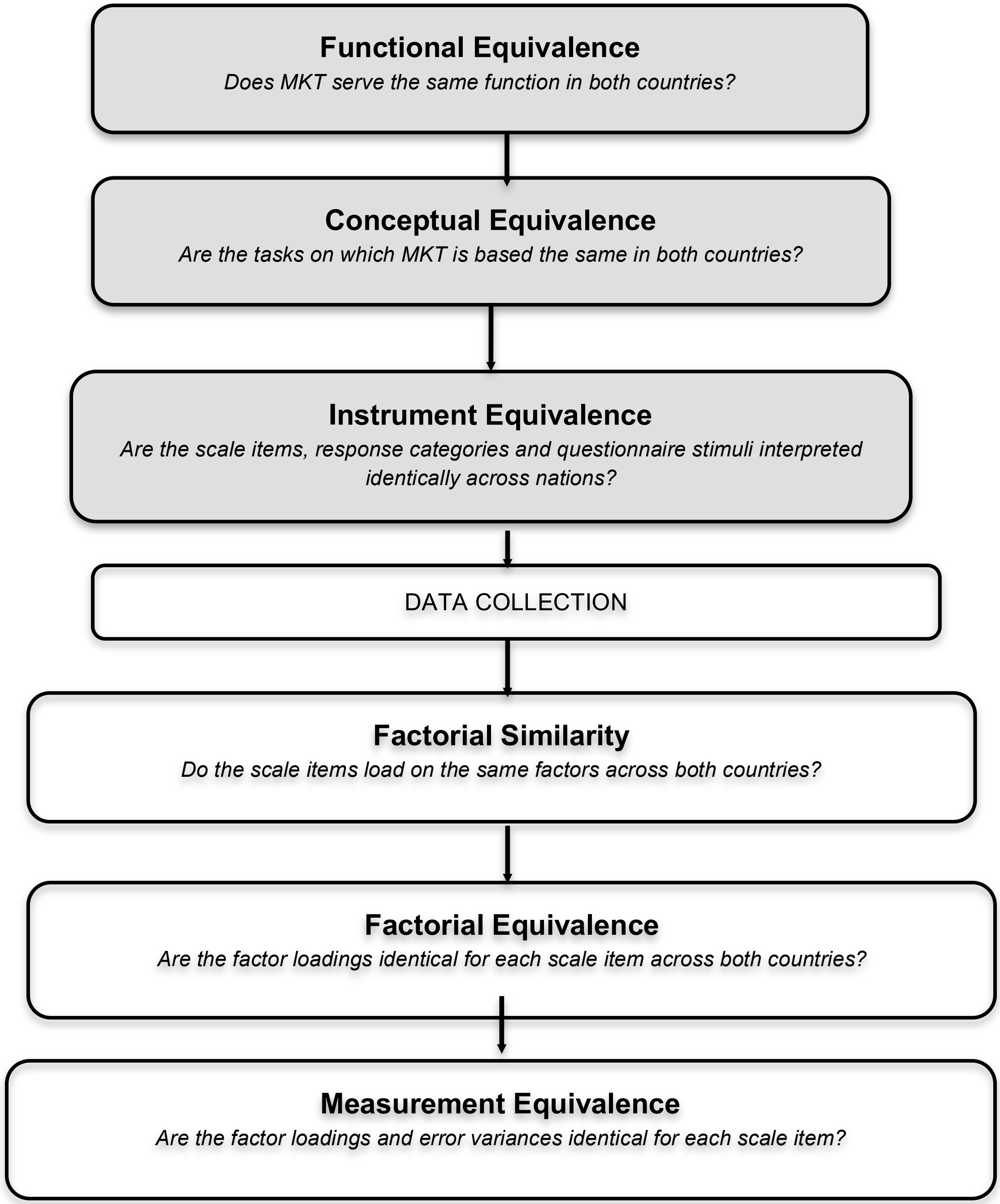



Figure 3.3. Steps to establishing construct equivalence (from Delaney, 2008, p. 72, adapted from Singh, 1995)

***Methodological limitations***.

Several limitations to this study were outlined in Chapter 1. In addition, the nature of the design and method of analysis presented the following limitations:

1. There were no follow-up interviews with teachers to determine their mathematical and pedagogical intentions for the lesson. As a result some inferences about teacher behavior may not be a reflection of their mathematics teaching in general since any investigation of their mathematical quality of instruction cannot and does not take into account the rationale for teachers' decision making, which is an important component of teaching and thus limits the extent to which teachers' mathematical instruction could be fully explained in this study.
2. The LMT-developed coding rubric did not capture all aspects of instruction hence, it was not nuanced enough to distinguish features of mathematics instruction that may be "cultural" in nature. I thus rely on my knowledge of the Ghanaian culture to make inferences about features that might be inherently cultural but more broadly based on the availability or otherwise of educational resources. For instance, features of instruction such as the use of blackboards and the use of exercise books and jotters are not captured by the MQI codes but were central to the mathematics teaching in all classrooms. The use of blackboards could be a consequence of the economic characteristics of Ghana where there is very little if any technology available for use in classrooms. The lack of attention of the MQI rubric to blackboard use for example, provides an incomplete image of the mathematics instruction in Ghana.

3. Teachers taught different content and the nature of the content in some cases had implications for the mathematical quality of the lesson and the validity study as a whole. Kofi's lesson was focused on fractions, which provides a natural use of a variety of mathematical representations, whereas Fiifi's lesson on data representation implied the use of tables and charts as representations to the exclusion of other forms of representation that may have enhanced the MQI.
4. The MQI codes were primarily designed for conceptually focused classrooms thus classrooms where procedures are emphasized may be disadvantaged. Lessons that privileged the use of mathematical procedures to develop mathematical skills were less likely to rank high than lessons designed for the development of mathematical concepts. The MQI instrument's bias in favor of conceptually focused classrooms could underrepresent the mathematical quality of such lessons.
5. Of the four video coders, two had never been trained in the use of the LMT video coding instrument. Given this, the initial coding cycles showed low levels of agreement. The levels of agreement improved after their first coding but might have been higher had the coders had more experience coding video in general, and using the coding instrument in particular. This presents a limitation for the study because it is possible that the use of more experienced coders might have improved the levels of agreement between the raters of the video data. Future studies could benefit from a

re-analysis of the data by more experienced coders of instruction to determine the robustness of the findings presented in this dissertation.

### Generalizability

Generalizability refers to the extent to which findings from this study can be extrapolated to a larger population. In this study the immediate population of interest is teachers in Ghana, but the larger population of teachers in African countries and possibly other developing countries, could benefit from the findings of this study.

Guba argues that findings from qualitative studies should not be considered definitive, but as “working hypotheses for future testing” (cited in Patton, 2002, p. 583). This study is an example of a working hypothesis to determine how and in what particular ways the theory of MKT could be applied in Ghana. I will address the issues of generalizability according to the population used, the settings for the study, and the outcomes of the study (Schofield, 2002).

*Population:* the 60 teachers were selected from a sample of primary level teachers in the Greater Accra region of Ghana. Although the sample is a convenience sample of teachers, I selected teachers from different types of schools (public and private) and within the private schools and from a range of schools (from low fee-paying schools to high fee-paying schools). This means that the findings of the study may be applicable to teachers in the Greater Accra region or the larger cities within Ghana. The cases selected for the interview and video studies were also drawn from different school types to account for the possible range of teachers in Ghana. In spite of these careful considerations, this exploratory study is not designed to find the MKT of “teachers in Ghana” more broadly. This study will inform the extent to which findings from this study could be applied to different kinds of teachers from different kinds of schools in Ghana in particular, and Africa more broadly.

*Settings:* The teachers in this study were required to take a survey (test), be interviewed about the test, and have two of their mathematics lessons videotaped. These teachers had never experienced any of these situations before. Their expectation of participating in a survey did not include “doing a mathematics test” that required their intellectual commitment for about 150 minutes. These unfamiliar settings could influence teachers’ responses to the items, the nature of lessons they choose to teach, and how they responded to the interview items. It is likely that some of the teachers may teach in qualitatively better or worse ways on other days and that the videotaped lessons were “staged” for the benefit of the visitors. Although there is little to suggest that this happened, this is a possibility that must be considered.

It is worthy to consider whether the results reported in Chapter 4 and the discussions in Chapter 5 would have been different if teachers took the survey in the morning and not after school, or if their interviews had occurred in their classrooms and not a neutral building or even my home. Given these conditions, my analysis of the data assumes that all testing conditions were similar. However, my interpretation of the findings will explore the extent to which these settings influenced the transfer of MKT in Ghana.

*Outcome:* The findings of this study are not designed to be applicable to all teachers in Ghana, Africa, or developing countries. Instead, findings will inform the administration of the MKT measures at scale in Ghana and possibly other Sub-Saharan countries in Africa. This study focused on the theory of MKT and examined teachers' reasoning about some of the tasks, and examined the relationship between teachers' MKT scores and their enactment of particular features of MQI-language, explanations, representations, and errors. It is not clear that if the research had focused on other components of MQI the results would be the same.

Given the considerations of populations, settings and outcomes, the findings and conclusions of this study need to be interpreted "in context". Chapter 5 will explore possible future research directions that might improve the extent to which these findings could be deemed generalizable.

## Summary

In this chapter, I outlined the different methods used to address the research question: To what extent can empirically derived U.S.-developed measures of MKT be used to study the MKT held by primary teachers in Ghana. I demonstrate how the research question is answered using sub questions (See Figure 3.1) that address the different phases of the research. As an empirical validation study, examining conceptual, instrumental, and functional validity assesses the validity of the MKT construct. The elemental validity of the measures is assessed using cognitive interviews and the mathematical quality of instruction is assessed using video codes. These analyses would determine the validity of the MKT scores in Ghana. I now turn to the findings in Chapter 4.

# Chapter 4: Findings

## Introduction

This study is designed to examine the validity of U.S.-developed measures of MKT in a Ghanaian context. In Chapter 1, I outlined the importance of this study to contribute to teacher knowledge--in particular, teacher knowledge in Ghana, and the need for measures that are sensitive to Ghanaian teaching practice. Chapter 2 provided the theoretical justification for the selection of MKT as the theory of study and not other theories. It also provided examples of educational studies in Africa more broadly and Ghana in particular. I explained how the research questions would be investigated and analyzed to inform the research study in Chapter 3. The findings of the study, presented in this chapter, show measured promise for the use of the MKT measures in Ghana. From this sample, there is evidence to suggest that the MKT measures are positively correlated to teachers' reasoning about the mathematical tasks, and to the mathematical quality of their instruction.

In the first section of this chapter, I provide an overview of the performance of the LMT measures administered to a convenience sample of 60 Ghanaian teachers. As stated earlier, this research was not intended to study Ghanaian teachers' knowledge, but to examine how the items performed in Ghana by focusing on the item difficulties of the measures. A closer look at the features of items that were shown to be relatively easy and relatively difficult provides information about the effectiveness of the measures of MKT in Ghana. This section concludes with a comparison of Ghana and U.S item difficulties to provide an overall picture of how the items performed in Ghana.

The second section of this chapter reports on the interview study designed to examine the utility of the MKT measures in Ghana. I investigate the reasoning of three fourth grade teachers about selected MKT items to determine the cognitive resources teachers used to answer the items. I first report the findings on a case-by-case basis, providing some background information about each case. These analyses determine the extent to which each case's MKT scores are reflective of their mathematical understandings. I then present a cross-case analysis of the selected teachers where I outline common patterns identified in the cases.

The third major section of this chapter reports on the video study designed to assess the mathematical quality of instruction of the selected teachers by examining their classroom teaching. Similar to the previous section, I first present the case-by-case findings. I then present the cross-case findings to investigate patterns that emerge in the data and relationships that might exist between teachers' MKT scores and elements of MQI.

## Reporting the performance of MKT measures in Ghana by examining item difficulties

The LMT Elementary Number Concepts and Operations form was administered to 60 primary level teachers in Ghana and scored using the answer key provided by the LMT project. Table 4.1 shows the range of the equated item difficulties for the Ghanaian teachers in this sample compared to U.S. teachers. Appendix B provides the item difficulties for the 56 items administered to the 60 Ghanaian teachers sampled. This comparison suggests the presence of other factors that may not have been anticipated in the measures adaptation. Table 4.1 for instance shows that overall, the items were more difficult for the Ghanaian teachers sampled than for U.S. teachers in general.

Table 4.1. Summary Item Difficulties for Ghanaian Sample and U.S. Teachers

| Level of difficulty | Difficulty range | Number of items for Ghana | Number of items for U.S. |
|---|---|---|---|
| Easy | less than -2.0 | 4 | 5 |
| Somewhat easy | between -2.0 and 0 | 21 | 42 |
| Somewhat difficult | between 0 and 2 | 22 | 9 |
| Difficult | greater than 2 | 9 | 0 |

Although this study is not designed to compare Ghanaian teachers to their U.S. counterparts, the comparison to the U.S. item difficulties serves the purpose of illustrating the extent to which the items were easier or more difficult for the Ghanaian teachers. More importantly, this assessment determines the type of teacher these measures may be suited for. For instance items that have very high difficulties are best suited for teachers with high MKT levels and do not effectively discriminate among teachers with medium and low difficulties.
A more detailed comparison of the individual items in Table 4.2 show that although the items were more difficult for the Ghanaian sample, some items were easier for the Ghanaian sample than the U.S. sample. I now turn to a closer examination of the items that the Ghanaian sample found to be relatively easy.

Table 4.2. Difficulty Estimates for Number and Operations Content Knowledge Items with Equated U.S. and Ghanaian Difficulty Estimates.

| Item number on Ghanaian form | Equated Ghanaian difficulties | U.S. difficulties |
|---|---|---|
| | ***Easy Items*** | |
| 18B | -3.86 | -3.08 |
| 23 | -2.571 | 0.767 |
| 11 | -2.221 | -1.092 |
| 30 | -2.063 | -1.532 |
| | ***Difficult items*** | |
| 18E | 2.222 | 0.953 |
| 26 | 2.388 | 0.735 |
| 3 | 2.389 | -0.311 |
| 13B | 2.568 | 0.757 |
| 34C | 2.568 | 0.651 |
| 27C | 2.765 | 0.815 |
| 19 | 3.226 | 0.723 |
| 20A | 3.508 | 0.648 |
| 12 | 4.267 | 1.05 |

**Relatively easy items**

The relatively easy items had item difficulties less than -2. There were only 4 items (7%) in this category. There were six items (~10%) with item difficulties less than -1.5. A closer look at the nature of the items might provide some explanation for this. The easiest question (difficulty = -3.86) required teachers to indicate whether the statement “any number times 0 is 0” is true or false. Another easy item (difficulty = - 2.57) asked teachers to examine three addition word problems to determine which of the problems students might find easiest. In two of the problems, there was a missing addend and one of the problems had both addends. Data from U.S. teachers showed that U.S. teachers also found the former question easy (difficulty = -3.08), the latter question was not as easy (difficulty = 0.77).

**Relatively difficult items**

The most difficult item (difficulty = 4.27) asked teachers to assess different students' explanation for why the rule for reducing fractions by multiplying the numerator and denominator by a common factor works. Of the 60 teachers sampled, only three correctly answered the question, Most of the responses explained the process of cross multiplication, but did not provide an explanation of why it works. In the U.S. sample, the item was not as difficult (difficulty = 1.05). In another difficult question (difficulty = 3.51), teachers were asked to determine if any of three word problems was an accurate representation of a fraction subtraction task where each of the fractions was a unit fraction. The task was of the form: I have a fraction of a whole to start. A second person takes a fraction of my fraction, how much of the whole remains. As the task shows, there are two distinct wholes. The first fraction is the fraction of the original whole. The second fraction is a fraction of the first fraction and so cannot be interpreted as a fraction of the same whole as the first fraction. Teachers' responses illustrated confusion about the second whole identified in the problem.

Examining item difficulties show how the MKT measures fared with the Ghanaian teachers sampled. This analysis provides important information to determine the knowledge level of teachers in Ghana the test might be best suited for. I now turn to a broader analysis of the comparison of the U.S. and Ghana item difficulties using a scatter plot.

**Comparing Ghana and U.S. item difficulties**

To validate the adapted MKT measures in Ghana, data from 60 Ghanaian teachers were analyzed and compared to data obtained from 599 teachers in the United States who participated in California's Mathematics Professional Development Institutes (see Hill et al., 2004).

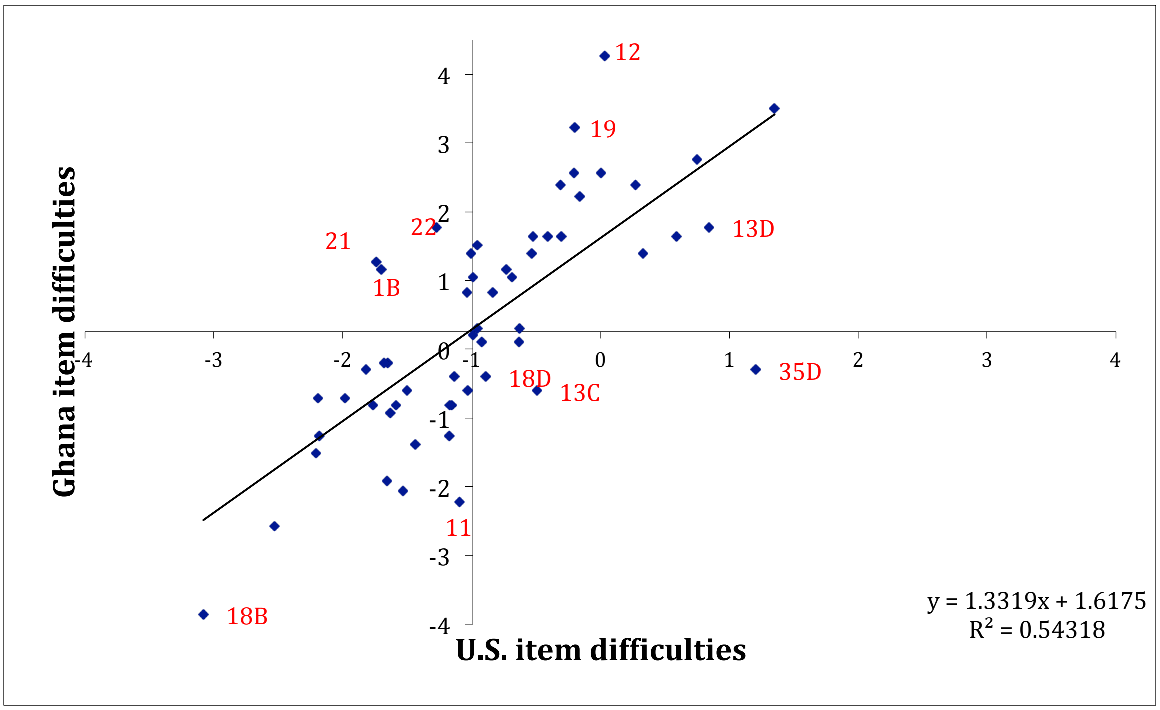



Figure 4.1. A regression line fitted to a scatter plot of the relative difficulties of MKT items administered in Ghana and U.S.

Figure 4.1 shows that most of the Ghana item difficulties were positively correlated with the U.S. difficulties (r =0.737). This means the higher the difficulty of the item in the U.S., the higher the difficulty in Ghana. This is illustrated in Quadrants 1 and 3 where trend line demonstrates that most of the items are reflective of the correlation. The items closest to the trend line are more reflective of the relationship between the U.S. and Ghana item difficulties than items farther away from the trend line.

There were some tasks that the Ghanaian sample found easier than the U.S. sample. Question 35D in Quadrant 2 for instance, asked teachers to determine whether a given sentence was the rule of a defined pattern. Question 11 in quadrant 3 required teachers to identify a student error in a prime factorization task. Question 18B, also in Quadrant 3, required teachers to determine whether or not the statement any number multiplied by zero is zero was true or false. Questions 21 and 22 in quadrant 4 were much more difficult for the Ghanaian sample than for the U.S. sample, with item difficulty differences greater than 3. Question 21 required teachers to identify a student error in a three-digit by one-digit multiplication task and question 21 asked teachers to determine Of the 56 items asked in the survey, there were 35 items where the differences in item difficulty between Ghana and the U.S. were greater than 1. Of this number, 17 items had differences greater than 2 and 3 had differences greater than 3.

Table 4.3. Largest Differences in Item Difficulties between Ghana and the U.S.

| Item Number | Item difficulty (U.S.) | Item Difficulty (Ghana) | Difference (Ghana - U.S.) | Work of teaching in item |
|---|---|---|---|---|
| 21 | -1.74 | 1.27 | 3.01 | Evaluating student's alternative subtraction algorithm |
| 22 | -1.27 | 1.78 | 3.05 | Evaluating and identifying student's thinking on the meaning of the equal sign |
| 19 | -0.20 | 3.27 | 3.47 | Evaluating alternative subtraction algorithms |
| 12 | 0.04 | 4.27 | 4.23 | Explaining equivalent fractions |

Table 4.3 shows that the items with the largest differences were all tasks that the Ghanaian sample found more difficult than the U.S. sample. The tasks of teaching assessed in the items ranged from evaluating alternative algorithms for subtraction and multiplication, evaluating student methods, and explaining equivalent fractions. Question 19 in Quadrant 4 for instance was very difficult for the Ghanaian sample (difficulty= 3.23) but not for the U.S. teachers (difficulty= -0.20. This question asked teachers to evaluate three alternative subtraction strategies to determine which of the strategies could be used to subtract any two whole numbers. These findings could mean that in this sample, tasks of teaching

such as evaluating alternative subtraction algorithms, and evaluating student thinking of the meaning of the equal sign are unfamiliar to the Ghanaian teachers sampled.

### Assessing elemental validity: Association between teachers' MKT scores and their reasoning about the items.

The second research question of this study examined the relationship between teachers' MKT scores as assessed and their reasoning about the items. This section examined a case-by-case analysis of the association between individual teachers' MKT scores and their reasoning about the items. I now present an overview of the interview study, providing a description of the relatively easy and relatively difficult tasks. I will then turn to the results of a cross-case analysis of the three cases, seeking to investigate the existence of a relationship between their MKT and their reasoning about the Number and Operations items in the measures.

As explained in the previous chapter, the interviews comprised the Number and Operations items in the Elementary Number Concepts and Operations 2001 form B developed by LMT. The interviews provided some evidence of teachers' knowledge as shown by the extent to which teachers were able to think about and understand the alternatives proposed in the questions.

If the elemental assumption holds, then the teachers' mathematical reasoning for each item would be consistent with their survey scores on the individual items. Thus, high-scoring teachers would be expected to reason correctly about the mathematics of the tasks they answered correctly, justifying their high MKT score. Conversely, low-scoring teaching teachers would be expected to be unable to reason correctly about the tasks they answered incorrectly, hence justifying their low MKT scores (see Figure 4.2). Although there were 24 items in the interview, time and space limitations required a careful selection of questions for analysis. For example, the easiest question in the interview that asked teachers to determine whether the statement "any number multiplied by 0 equals 0" is true or false was not selected for analysis. This is because an examination of teachers' reasoning for their responses to that item did not provide enough data for analysis.

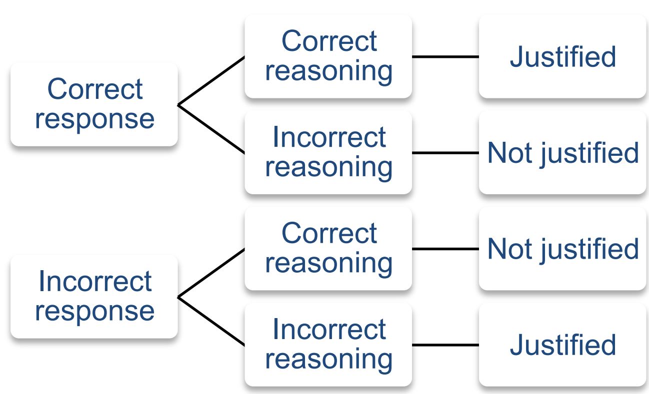



Figure 4.2. Process of justification of interview findings

In most cases, teachers demonstrated knowledge of the rule as a fact, and the item was not assessing the different reasons why that answer was true as did other items that asked teachers to explicitly evaluate explanations about given facts. As such, other items that provided more substantive data for analysis were selected. Table 4.4 shows the interview items ranked by item difficulties.

Table 4.4. Selected Interview Items by Item Difficulty for U.S. and Ghana

| | **Mathematical content** | **Content of task** | **Item difficulty (Ghana)** | **Item difficulty (US)** |
|---|---|---|---|---|
| **Relatively easy items** | Prime factorization | Analyzing student error in factor tree | -2.22 | -1.10 |
| | Fraction comparisons | Evaluating student explanations | -1.27 | -1.18 |
| **Relatively difficult items** | Subtraction | Evaluating alternative subtraction methods | 3.23 | -0.20 |
| | Fractions | Explaining equivalent fractions | 4.27 | 0.035 |

This cognitive interview was designed to determine whether teachers' reasoning about their answers was consistent with the correctness or otherwise of their interview responses. It was important to determine whether teachers' responses (survey and interview) truly reflected their mathematical knowledge being assessed or could be accounted for by other test taking strategies such as guessing or eliminating options from the multiple-choices available in the answer. A teacher could select an incorrect response due to misinterpreting the context of the task or a lack of the mathematical knowledge assessed in the item. Another reason for an incorrect response could be due to the teacher's correct use of his/her professional knowledge to justify the selection of an incorrect option. The cognitive interview is designed to examine these different options and determine, to the extent possible, the source of teachers' reasoning for their responses. I now turn to a description of the selected items.

**Relatively easy items**

The first relatively easy item asked teachers to identify a possible explanation for a student's incorrect prime factorization of 180 using a factor tree. In this question, the student did not factor 9 so the final prime factorization as illustrated by the factor tree was 9 x 2 x 2 x 5.

A teacher solving this problem could use learned rules about factors such as finding the smallest prime factor of the largest product and keep factoring the largest product until you have a product of primes only. A teacher could correctly solve the problem by choosing the correct multiple-choice answer, but it would be unclear whether teachers' knowledge about prime factorization is consistent with mathematical ways of knowing about primes. For instance a teacher could use test-taking strategies to give the correct response or a teacher could guess the answer. An incorrect interview response to this question could be due to the teachers' limited knowledge on prime factorization, or could be due to the teachers' incorrect interpretation of the task. The interview is designed to determine how teachers' reasoning about this item is consistent with their response and what other factors may account for the teacher's response.

The second relatively easy question asked teachers to evaluate different student explanations for comparing two proper fractions. In this case, a teacher could correctly answer the question by drawing on memorized procedures such as comparing the size of the numerators or denominators, finding a common denominator, comparing with a benchmark fraction such as a whole, or zero as a basis for comparison. A teacher's incorrect response could be due to a limited understanding about comparing fractions, a misinterpretation of the different multiple-choice options, or could be due to a teachers' utilization of sound mathematical thinking drawing on student thinking in his/her context, materials from the curriculum, and other professional knowledge to justify the selection of the incorrect response. A teacher's correct answer in the interview must accurately discriminate among the different student explanations in the item, drawing explicitly on the mathematics in each explanation. A summary of how the assessment of teachers' interview responses to the easy questions is provided in Table 4.5.

Table 4.5: Assessment of Teachers' Interview Responses to Easy Items

| | **Prime factorization item** | **Comparing two proper fractions** |
|---|---|---|
| Fiifi | Correct | Correct |
| Kofi | Correct | Incorrect |
| Gyidi | Correct | Correct |

I now describe the relatively difficult items after which the findings will later be presented separately for each teacher.

**Relatively difficult items**

The first relatively difficult question asked teachers to examine three student methods for subtracting two 3-digit numbers and determine which of the three methods could be used to subtract any two whole numbers. Each of the strategies was valid for subtracting any two 3-digit numbers. Teachers could draw on their understanding of place value to explain each of the different strategies or employ test-taking skills to eliminate the strategy that might seem improbable to them. In this task, all three methods are valid subtraction strategies as such, any selection of less than three of the options provided will yield an incorrect response. This item does not discriminate between teachers who know two correct strategies from teachers who know only one correct strategy. It is important to note that the three subtraction strategies in the item were all correct strategies for subtracting any two whole numbers. As such, a teacher who selected two of the three strategies, although incorrect in the context of the problem, could be construed as having more knowledge or mathematical understanding than a teacher who selected one strategy. Consequently, this item does not discriminate among teachers with less than high levels of knowledge about alternative subtraction methods.

The second question asked teachers to evaluate 5 different explanations for *why* reducing a fraction by dividing the numerator and denominator by the same number produces an equivalent fraction to determine which of the explanations is best. In that task, multiple-choice options included different examples of the task that restated the problem in different forms, as well as other descriptions of the process of simplifying fractions; however, neither offered an explanation of *why* the process works. A teacher's correct response could be due to a teacher's understanding of the mathematical processes that explain why reducing a fraction yields an equivalent fraction. Other possible explanations for a correct response could be a teachers' guessing of the answer or the use of test-taking strategies. A teachers' incorrect response to this task could be due to a lack of conceptual knowledge about why the mathematical procedure works. Other incorrect responses could be due to a misinterpretation of the context of the question and a lack of clarity about what the question was demanding. In this task, the stem of the question contained 90 words on seven lines unlike other tasks that were less verbose. The literacy demand of the task may contribute to teachers' engagement with the task and might contribute to how teachers perceived the item. Table 4.6 shows that all three cases incorrectly solved the selected difficult items.

Table 4.6: Assessment of Teachers' Responses to Difficult Items

| | **Alternative subtraction strategies** | **Why reducing a fraction works** |
|---|---|---|
| **Fiifi** | Incorrect | Incorrect |
| **Kofi** | Incorrect | Incorrect |
| **Gyidi** | Incorrect | Incorrect |

As explained in Chapter 3, the teachers were selected primarily because they taught at similar grade levels and their scores were sufficiently different to warrant an exploration. Information about each of the teachers is provided in Table 4.7. The raw MKT score for Number and Operations is reported as well the IRT scores[21]. LMT used IRT scores to create a scale of MKT proficiency. The mean is 0 and the range is between -4 and +4. A score of -4 is a teacher with exceptionally low MKT and a score of +4 is a teacher with exceptionally high MKT. Hill (2007) reports that in general, the average teacher will answer an item correctly 50% of the time. Of the 3 cases, Fiifi scored the lowest ranking about 3 standard deviations below the mean.

[21] The IRT scores were obtained from the LMT conversion tables.

Table 4.7. Overview of Cases

| Name | Grades taught | Years of teaching | Hours of mathematics professional development | MKT score | |
|---|---|---|---|---|---|
| | | | | MKT Survey (out of 24) | Equated IRT score |
| **Fiifi** | 4 | n/a | n/a | 3 | -3.010 |
| **Gyidi** | 4 | 10 | 1 | 11 | -1.111 |
| **Kofi** | 4 | 5 | 0 | 4 | -2.740 |

## Association between MKT scores and reasoning about selected items: Case-by-case analysis

### The case of Fiifi

Fiifi taught in a high tuition-paying private school. His fourth grade class had 35 students. Fiifi described mathematics as a way of using numerals and numbers and said mathematics is encountered in daily activities such as buying and selling. Video data from his mathematics teaching showed that Fiifi appeared to be a firm teacher who did not show any emotional connection with his students. In the response to the survey, he did not report his number of years teaching but reported having a weak knowledge of the mathematics he needed to teach, an average level of knowledge about number and operations, and a weak knowledge of mathematics overall.

In the interview, Fiifi appeared uncooperative and was unwilling to answer some of the questions. In the MKT survey, he correctly answered 6 items, 3 of which were in the Number and Operations strand. In the interview, he answered 9 items correctly.

If the elemental assumption holds, it is expected that Fiifi will exhibit poor or limited understanding of the mathematics in the tasks he incorrectly responded to, showing that his MKT scores are truly reflective of his knowledge. I now present Fiifi’s reasoning about the four described tasks, two relatively easy and two relatively difficult.

In the prime factorization task, Fiifi identified the correct answer, staying consistent with his response in the survey. He believed his choice was correct saying “9 you can still bring it down 3 times 3, so in the end you are going to get 3 times 3 times 2 times 2 times 5” (CI060501, p.3). This reasoning was correct because he identified the student’s error. When asked about the other alternatives in the question, Fiifi did not display much depth in his mathematical understanding. For instance, one option suggested that the student should have begun with 10 times 18 instead of 9 times 20. In this case, Fiifi mentioned that “10 time 18 won’t give you 180” (CI060501, p. 3).

The second relatively easy question required Fiifi to evaluate which one of five different student explanations for comparing the fractions 5/9 and 3/7 was correct. In the interview, Fiifi initially selected an incorrect response: “3/7 is bigger because the sizes are bigger”. In his explanation for this, he said:

> I use example as maybe an orange. You get one piece of orange, now you cut the first one into sevenths, then the second one also you cut it into ninths. The first sevenths, you pick 3 out of it. Already, the 7 will be, the pieces will be bigger than the ninths. So 3 out of it, 3 out of the sevenths would be bigger than 5 out of the 9.
> (CI060501, p.1)

When Fiifi examined the other options, he discounted them all but when read the correct answer that compared both fractions to a benchmark fraction of ½ to conclude that 5/9 is greater than 3/7, he changed his answer saying:

> because I think it is the correct answer, because if you look at it, half of 9 is 4 and a half and half of 7 also will be 3 and half so with this one, this one is

> more than half so I think I will agree that 5/9 is greater because it is greater than one half. (CI060501, p.2)

Thus for the above relatively easy questions, Fiifi, correctly answered the question in the interview, correctly justifying his response for each. In the survey however, he selected an incorrect response, thus rendering his interview response inconsistent with his survey scores.

The first difficult question required Fiifi to explain each of 3 alternative subtraction methods to determine which of the methods could be used to subtract any two whole numbers. For two of the strategies, Fiifi said, “this is too confusing for children” and of the third, said, “sometimes in our book, we use the same formula”, drawing on his curriculum and memorized rules. He selected the option he was familiar with but did not demonstrate an understanding of any of the two other strategies in the item. His survey response was inconsistent with his interview response because he selected different answers, each of which was incorrect.

The second difficult item required an explanation for why reducing fractions by dividing the numerator and denominator by the same number works. In this question, Fiifi quickly selected his response as “this works because for example, ¾ is the same amount as 12/16 only with smaller numbers” (CI060501, p.4) but changed his response to “this works because you divide the top and bottom number by the same number so the new fraction has to be the same amount” saying:

> …because I agree that the new fraction has to be the same amount. If we take D for example, the 12 over 6, you will use the same number that is 4 to

divide the numerator and the denominator and it will give you the 3 over 4 which is the same.
(CI060501, p.4).

Fiifi's interview response demonstrated that his answer was a general case of the task and his initial selection was a specific case of the task; neither of which provided an explanation for why the process worked. In examining the alternatives, he quickly discounted the correct answer, saying "you are not dividing by 1". In the survey, Fiifi's response was also incorrect.

Table 4.8. Fiifi's Interview Findings

| | Description of item | Correct/Incorrect | Justified (Yes/No) | Consistent (Yes/No) |
|---|---|---|---|---|
| **Relatively Easy** | Prime factorization | Correct | Yes | Yes |
| | Fraction comparisons | Correct | Yes | No |
| **Relatively Difficult** | Subtraction | Incorrect | Yes | No |
| | Fractions: why reducing fractions works | Incorrect | Yes | Yes |

Examining the interview responses for all the Number and Operations items, Fiifi's correct answers to the relatively easy questions during the interview were mostly consistent with his survey scores and he provided some mathematical justification for his interview responses (see Table 4.8). Although his interview response to the fraction comparison task was inconsistent with his survey scores, his initial response in the interview was the same as his survey response. Fiifi's performance and engagement with the more difficult items was more telling. His responses to the more difficult questions in the interview were all incorrect and his inability to provide mathematical reasoning for his answers justified his incorrect response. Again, although his response to the alternative subtraction strategies task was inconsistent with his survey score, both responses were incorrect.

A look at Fiifi's responses to the Number and Operations survey items proved interesting. Fifty percent (12 items) of Fiifi's survey responses were scored as invalid because of his selection could not be unambiguously interpreted. In addition, a comparison of his survey and interview scores showed that Fiifi's interview responses were consistent with his survey responses only 13% (3 items) of the time. This means that for the Number and Operations items, Fiifi selected the same response in both the survey and the interview only for three items.

**The case of Gyidi**

Gyidi taught fourth grade in a public school and at the time of the study had 10 years teaching experience. She describes herself as not liking mathematics and having no confidence in her knowledge of the mathematics she needs to teach and hence did not consider herself a "master" mathematics teacher. In the MKT survey, she correctly 11 Number and Operations items. In the interview, she answered 9 of the 24 items correctly. During the interview, Gyidi exhibited a willingness to engage with the items but declined to respond to four questions. If the elemental assumption holds, it is expected that Gyidi would exhibit an average understanding of the mathematics in the task, showing that her MKT score is truly reflective of her knowledge. I now present Gyidi's reasoning about the selected easy and difficult tasks.

In the prime factorization task, Gyidi selected the correct option in the interview explaining:

> because he should have continued with the 9, it should have been 3 and 3 but since he has left the 9 here, he was thinking it's a prime number so he's left it there. (CI061001, p.3)

In the interview, Gyidi correctly justified her selection of the correct response but when asked if the student should have started the factor tree with 10 and 18, she said that using 10 and 18 would produce a longer answer.

In the fraction comparison task, Gyidi selected an incorrect response. Her discussion of the task was telling. She first selected the option that "3/7 is bigger because the pieces will be bigger" (CI061001, p.1), explaining:

> Gyidi: 3/7 will be greater because the pieces will be bigger. With 3/7, let's say it's a whole number and it has been split into 7 portions, you pick 3 out of it. And here too a whole number divided into 9 portions and picking 5, I think 3/7 will be bigger than [5/9]
>
> (CI061001, p.1)

As she explored the other alternatives, she changed her response to select another incorrect response: "they are equal because each is missing 4 pieces from the whole" (CI061001). The following discussion then ensued:

> Yaa: Now you said that B is true and D is the answer so which one would you go with? If you have to choose one
> Gyidi: I wouldn't like, I knew 5/9 and 3/7 are not equal. I'm thinking they are not equal, B says they are equal because, that is what maybe a child will think but I am not thinking.
> Yaa: So then do you think it's a correct explanation or not?
> Gyidi: It's correct or? I will say it's correct
> Yaa: now you have to choose the right answer
> Gyidi: I have to choose the right answer, yes, me I chose D, I said 3/7

is bigger because the pieces are bigger.

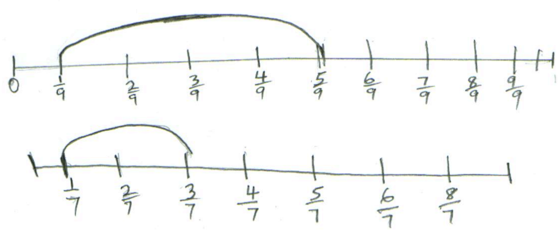
Yaa: So your students have said, you have 2 students and your class is arguing, one group is saying that they think 5/9 and 3/7 is equal because each fraction is missing pieces from the whole and the other group is saying that they think that 3/7 is greater because the pieces will be bigger. What would you do?

Gyidi: I will prefer using the number line. We will use the number line, draw it on the board.

Yaa: can you show me?

Gyidi: if this is a whole, let's take this to be a whole. I'm dividing this into 9 portions. This is the whole, this is also a whole number. Divide this into 7 portions. So this is 5/9 and this is 1, 1 ninth, 2, 3, 4, 5, 6,7 8. This one is 1, 2, 3, 4,5. One seventh, 2 seventh, 3 seventh, 4 seventh, 5 seventh. Ok, then my answer is wrong. 5/9 is bigger than because on the number line, you see that 5/9 is bigger than 3/7.

[Gyidi draws the following]




Yaa: so which one are we going with?

Gyidi: I will pick A, 5/9 is greater than… I will pick E.

Yaa: why?

Gyidi: you will see that this is more than ½, this is more than ½ while 3/7 is less than ½.

Yaa: then it means of the B and the D you've

Gyidi: now I have picked E.

(CI061001, pp.1-2)

Gyidi's made sense of each of the multiple-choice options individually, in most cases drawing representations to explain her thinking, but when pushed to think about the option in the context of the question, she seemed to waver in her thinking. Gyidi finally selected the correct response, which happened to be consistent with her survey score, and was mathematically justified. It is interesting to note that when the problem was re-contextualized, Gyidi appeared better able to find the tools to solve the problem correctly and did not appear to waver.

Gyidi incorrectly responded to both relatively difficult tasks during the interview. In the alternative algorithms for subtractions, Gyidi's responses could not be discounted as exhibiting a lack of knowledge. For method A, Gyidi was able to explain the first strategy but was unable to make the connection between the addition and subtraction problem, saying

> "I don't know what he was trying to do, he was asked to subtract and he was just trying to add 40, ermm, 4 plus 40 plus 500 plus 32 and actually managed to get the answer but that work seems to be too complicated" (CI061001, pp.3-4).

Gyidi called method B "perfect" and described it as simpler and easier. For method C, she identified the strategy as similar to method A and described it as complex. Gyidi did not believe methods A and C to be valid because if students add 4, 40, etc, they would not get the right answer. This shows that Gyidi could identify the strategies in methods A and C but could not abstract beyond the particulars of the task to determine the extent to which the strategy was applicable to all whole numbers.

In the fraction reduction task, Gyidi's interview response was inconsistent with her survey response and both responses were incorrect. She selected "because you making the numerator and denominator smaller by the same amount" using numerical example 3/6 and divided the numerator and denominator by 3 to reduce it to ½ to justify her answer. Although this response made sense, it did not explain *why* the rule worked.

Table 4.9. Gyidi's Interview Findings

| | Description of item | Correct/Incorrect | Justified (Yes/No) | Consistent (Yes/No) |
|---|---|---|---|---|
| **Relatively Easy** | Prime factorization | Correct | Yes | Yes |
| | Fraction comparison | Correct | Yes | Yes |
| **Relatively Difficult** | Subtraction | Incorrect | Yes | Yes |
| | Fractions: why reducing fractions works | Incorrect | Yes | No |

Table 4.9 summarizes Gyidi's interview findings. In three cases, Gyidi's correct responses to the interview questions were mathematically justified by her explanations and consistent with her survey responses. Her incorrect responses to the interview questions were also indicative of her lack of depth of mathematical knowledge about subtraction, the connection between subtraction and addition, and her understandings about why reducing fractions work. Gyidi's MKT score was about the middle and so it is not entirely surprising that she was unable to correctly answer the difficult interview questions. In her overall responses to the survey questions, she scored 2 points less than she did for the same questions in the survey.

Gyidi's survey responses to the items were telling. Of the three fourth grade teachers, Gyidi had the highest number of consistent responses. None of her survey responses were scored as invalid and her interview responses were consistent with her survey responses 63% (15 items) of the time. That means that for 15 items, her interview and survey responses were the same.

**The case of Kofi**

Kofi taught a fourth grade class in a low to medium tuition-paying private school. He had 5 years teaching experience at the fourth grade level. In the MKT survey, he correctly answered 14 items, four of which were in the Number and Operations strand. In the interview, he answered 13 items correctly out of 24. If the elemental assumption holds, it is expected that Kofi's low MKT score implies he would not be able to provide good mathematical reasoning for his responses to the interview questions confirming his low MKT score.

In response to the prime factorization task, Kofi correctly identified the reason why the problem was incorrect, justifying his interview response saying that the student may have forgotten that "2 numbers can be multiplied" to produce 9 "apart from 9 and 1".

For the fractions comparison task, Kofi's reasoning was incorrect. He selected the option "5/9 is greater than 3/7 because 5 is greater than 3" (CI061301, p.1). When asked to consider the other options, he said:

> Kofi: the (b), they are equal because each is missing four pieces from the whole. Ok, what he was talking about, the person did not consider the denominators. He was looking at the 5 which 9 is bigger than it 4, and the 7 which 3 is bigger than it 4. Having looked at this, we normally based on the numerator. The bigger the numerator is, the greater the portion becomes. For that matter, since 5 is bigger than the 3, the 5 should be greater than the 3.
>
> Yaa: what about the next one?
>
> Kofi: let me ok, this one, the 5 over 9. They said the 5/9 is greater because it is more than one-half, while 3 over 7 is less than one half. The fact that 5/9 is greater than ½ and the 3 over 7 also is less than half, it is true, but then, the denominators are not the same, the denominators are not the same and while the denominators are not the same, then you have to find a denominator of which the two of them can divide it.
>
> (CI061301, p.1)

Kofi exhibited a misconception about comparing fractions, that “The bigger the numerator is, the greater the portion.” His misconception directly contradicted his statement that if the denominators are not the same, then you have to find a common denominator in order to compare the fractions. Kofi correctly answered the question in the survey, thus rendering his interview response to that question inconsistent. Responding to the alternative subtraction question, Kofi had some interesting thoughts.

For Method B, he correctly explained the strategy saying,

> Kofi: Here, the one they are subtracting from the bigger number, they keep them in place of value by expanding the 3 to be 300, then the 5 to be 50, then, the 6 will be what? Then all the time, when they take the 3 for instance, expand it to 300, they subtract it from the number they are going to subtract, they will arrive at unknown, they will arrive at answer. Then they goes on to the tens, they expand the tens, which is 50, then they subtract it from that answer, to arrive at the next answer. Then they come to the ones, and take the ones. So that’s what arrive them at this one so this one…
>
> (CI061301, p.2)

His assessment of the strategy was different, he said,

> Kofi: ok, this one, they call something approximately. Whenever they want to subtract, the one they want to subtract, they make it approximately, so this one is 356 and approximately to be exact, it is 360, then after they have subtracted, they will arrive at answer. Then the 360, if you want to make it approximately, that would be 400 and that 400 can also be subtracted to arrive at the right answer.
>
> Yaa: I get what you are saying but you are talking about the subtrahends, the bottom numbers, but the minuends, they are not the same number so what is happening there?
>
> Kofi: ok, the first one also, this one, in the bigger number, in the first place, they increase the first number by 4, that’s ones, they increase the ones by 4, so if you look at the ones, it was 4 but they added another 4 arriving 6 before they made this one approximately, having subtracted, then, they also increased, they moved from the ones to the tens, they also increased the tens to 4, so that means, that from here to here, the ones has been increased to 4 and this place too the tens has been

> increased to 4 and the one they are subtracting, and the one they are subtracting, they also made it approximately so that by 6 and this one is not constant because eh. This is not a, this shouldn't be a method
> (CI061301, p.2)

In Kofi's attempt to explain method C, he paid attention to the numbers' place value and tried to make sense of each of the strategies. Kofi was not convinced that method C was a valid method of subtraction. As such, he selected method B as the only valid method, which was also consistent with his survey response.

In the simplifying fractions task, Kofi selected the option: "this works because you divide the top and the bottom by the same number so the new fraction has to be the same amount". He justified his selection by providing an example of reducing 2/4 to 1/2. Instead of explaining why the procedure worked, Kofi provided more instances of the process and thus failed to correctly answer the question. Kofi's selected response was also consistent with his survey scores, that is, he selected the same incorrect response in both the survey and the interview.

Table 4.10. Kofi's Interview Findings

| | Description of item | Correct/Incorrect | Justified (Yes/No) | Consistent* (Yes/No) |
|---|---|---|---|---|
| **Relatively Easy** | Prime factorization | Correct | Yes | Yes |
| | Fraction comparison | Incorrect | Yes | No |
| **Relatively Difficult** | Subtraction | Incorrect | Yes | Yes |
| | Fractions: why simplifying fractions works | Incorrect | Yes | Yes |

*Consistency measures whether the teacher's answer choice for the survey and interview are the same and does not apply to the correctness of their response.

Kofi's interview findings are summarized in Table 4.10. For the four tasks, it appears that with one exception, Kofi's interview scores and reasoning was consistent with his mathematical knowledge for teaching. Overall, Kofi's low MKT score had the potential to mean that he would get a low score in the interview, and would not have a solid understanding of the mathematics of the tasks. Although Kofi's performance in the selected tasks was predictable, his overall performance in the interview was better than his performance in the survey and warrants further study. Looking at the overall interview with Kofi, his interview score was three times his survey scores. Examining all of Kofi's survey responses showed that 50% (12 items) of his survey responses were scored as invalid. Of the valid survey responses, Kofi selected the same response in the survey and interview only 21% (5 items) of the time.

## Association between MKT scores and reasoning about selected items: Cross-case analysis

In this section, I examine the nature of teacher reasoning associated with their responses. I consider the possible range of explanations for a correct or incorrect response, and locate each of the cases within that spectrum.

### Correct responses

A teacher's correct response could be due to the correct application of learned rules, correct mathematical reasoning associated with some level of conceptual understanding of the mathematics of the task, familiarity with the context of the task based on experience, or ability to connect the task to their professional experience. Conversely, a teacher could correctly respond to the item by utilizing test-taking skills such as eliminating improbable answers or guessing. Figure 4.3 shows the evidence for teachers' reasoning.

| | **Fiifi** | **Gyidi** | **Kofi** |
|---|---|---|---|
| **Prime factorization** | Application of learned rules/formulae | Knowledge of the mathematical concept/process | Knowledge of the mathematical concept/process |
| **Fraction comparison** | Process of elimination | Process of elimination | |

Figure 4.3. Evidence of reasoning for correct interview responses.

Figure 4.3 shows that for Fiifi, Gyidi, and Kofi, their correct interview responses may not fully represent deep conceptual understanding of mathematics. In the prime factorization task, Fiifi explicitly drew on the application of rules and formula to answer the question while Gyidi and Kofi relied on their understanding of the process of prime factorization. In all three cases, there was no evidence for the use of test taking skills or guessing to indicate that the teachers' scores for the prime factorization task were not a valid measure of their knowledge as assessed. On the other hand, Fiifi and Gyidi used the process of elimination to determine the correct response for the fraction comparison task. While Fiifi and Gyidi selected incorrect options as their response, the interview process provided them with opportunities to examine the other answer choices to justify their selection of a "correct" response. This process enabled them to change their response, for Gyidi, more than 3 times. As such there is no evidence that Gyidi and Fiifi's correct response to the fractions comparison task is representative of their mathematical knowledge, at least for the two items described here.

**Incorrect responses**

As a cross-cultural validation study, it is important to examine incorrect responses to determine whether these responses are a result of incomplete or incorrect mathematical reasoning, lack of knowledge, or cross-cultural issues such as a lack of familiarity with the problem context. These findings are summarized in Figure 4.4.

| | Fiifi | Gyidi | Kofi |
|---|---|---|---|
| **Alternate subtraction algorithms** | Unfamiliarity with context/answer options<br><br>Lack of deep conceptual understanding of subtraction | Unfamiliarity with context/answer options<br><br>Lack of deep conceptual understanding of subtraction | Unfamiliarity with context/answer options<br><br>Lack of deep conceptual understanding of subtraction |
| **Fraction comparison** | | | Lack of deep conceptual understanding of fractions |
| **Why reducing a fraction works** | Lack of deep conceptual understanding of fractions | Lack of deep conceptual understanding of fractions | Lack of deep conceptual understanding of fractions |

Figure 4.4. Evidence of reasoning for incorrect interview responses.

Figure 4.4 shows that all the teachers demonstrated a lack of deep conceptual understanding of fraction equivalence and subtraction that justified their incorrect responses. In addition, the three teachers were unfamiliar with the problem context and some of the answer options for the alternative subtraction tasks. Kofi also demonstrated a misconception about his knowledge of fraction comparisons. This analysis of the incorrect responses to the selected interview questions show that teachers' incorrect responses were due to factors such as the lack of knowledge but also due to a lack of familiarity of the content of the tasks which have implications for the validity of the instrument's use in Ghana.

**Summary of findings from interview validation study**

The purpose of the interview study was to examine the extent to which teachers' MKT scores could be validated by examining their reasoning about the MKT items. Teachers' MKT scores are one way of assessing their mathematical knowledge for teaching. Their reasoning about mathematical tasks is another source of data for teachers' knowledge. To determine if the MKT measures truly measure teachers' knowledge in Ghana or whether there are other factors that might inhibit their use in Ghana, I now test the elemental assumption that the MKT scores reflect teachers' mathematical knowledge for teaching and not unrelated factors such as test taking strategies. I also examine the inference from the elemental assumption that teachers' reasoning for a particular item will be consistent with their response for that item in the survey.

The above sections have provided findings from three fourth grade teachers Fiifi, Gyidi, and Kofi. To test this inference however, I will use data from all 6 teachers to determine the strength of the relationship between MKT and teachers' reasoning. These teachers are only representative of the teachers in the sample and not a representative sample of teachers in Ghana and so no inferences can be made about these findings to the larger Ghanaian population of teachers.

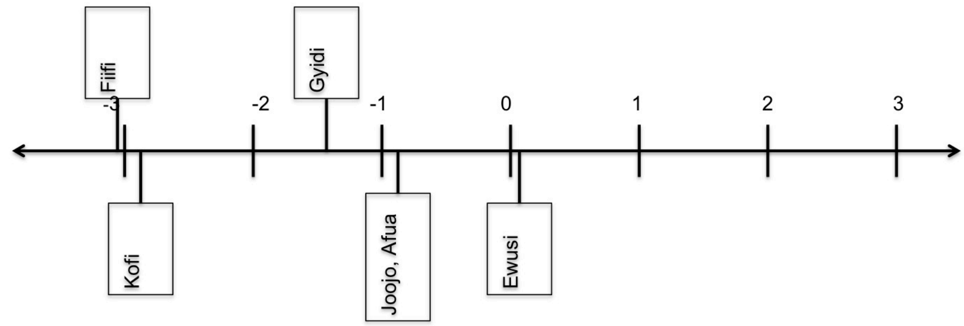
I now compare the IRT scores (from -4 to +4) of the 6 teachers based on their interview results in the Number and Operations form 2001_B matched with the percentage of consistent responses in the survey and interview as well as the percentage of invalid entries in the MKT survey in table 4.9.

Table 4.11: MKT Survey Scores Interview Scores, and Percentages of Consistency and Invalid Scores

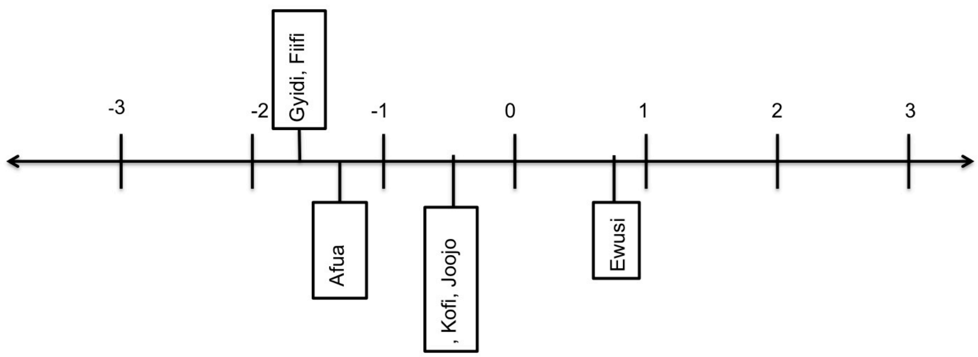
| Teacher | Grade level | MKT score (survey) | % Invalid survey scores | MKT score (interview) | % Consistency* (Survey and interview) |
|---|---|---|---|---|---|
| Ewusi | 6 | 0.0140 | 0 | 0.777 | 79 |
| Joojo | 6/JSS 1 | -0.891 | 17 | -0.671 | 63 |
| Afua | 6 | -0.891 | 0 | -1.333 | 79 |
| Gyidi | 4 | -1.111 | 0 | -1.552 | 63 |
| Kofi | 4 | -2.740 | 50 | -0.671 | 21 |
| Fiifi | 4 | -3.010 | 50 | -1.552 | 13 |

*Consistency measures whether the teacher's answer choice for the survey and interview are the same and does not apply to the correctness of their response.

For the interview findings to be a true reflection of the teachers' knowledge as assessed by the MKT measures, it is expected that a high percentage of consistent items (survey to interview) and a low percentage of invalid scores. Ewusi, Afua and possibly Gyidi appear to share these features whilst Joojo, Fiifi, and Kofi do not.




Teachers ordered according to their IRT scores on the MKT survey (scored from -4 to +4; teachers not placed to precise scale)




Teachers ordered according to their IRT scores on the MKT interview (scored from -4 to +4; teachers not placed to precise scale)

Figure 4.5. Comparison of MKT survey and interview scores

The visual comparison of the six cases in Figure 4.5 show the teachers ranked by their IRT scores for the survey and the interview. The line shows the spread of the teachers from -4 to 4 where teachers closer to -4 have low levels of MKT and teachers closer to +4 have high levels of MKT.

The comparison shows that Fiifi notably improved his score from the survey to the interview but still remained the lowest scorer. Similarly, Ewusi improved his score somewhat and remained the highest scorer. Kofi, however, made a remarkable improvement from his score of almost 3 standard deviations below the mean to almost one half standard deviation below the mean IRT score. Joojo and Gyidi made slight improvements while Afua scored slightly less than she did in the survey. While the statistical significance of these changes are beyond the scope of this study, these changes show that, teachers' responses to the survey may not be wholly attributed to their mathematical knowledge.

As a validation of the MKT survey, the interview study findings seem to indicate that the MKT measures could be used to determine teachers with high MKT (e.g. Ewusi) but may not be a reliable measure for teachers' with medium to low MKT (such as Fiifi or Gyidi). This is because the interview study provided opportunities for low scorers such as Kofi and Fiifi to demonstrate their level of knowledge about tasks that they had previously been scored as invalid, thus improving their performance by a large margin in the interview. This implies that their MKT scores was not a true representation of their MKT about Number and Operations.

## Assessing the ecological assumption: Association between teachers' MKT scores and their mathematical quality of instruction (MQI)

The third research question of this study examines the relationship between teachers' MKT scores as assessed and their MQI. This will determine whether teachers' with relatively high MKT scores teach in qualitatively better ways than teachers with lower MKT scores. Information about each case pertaining to their school and classroom contexts is presented before the findings to help situate the reader in the teachers' school context.

## Case-by-case analysis of relationship between MKT and MQI

### *The case of Fiifi*

Fiifi teaches in a K to junior high (middle) school situated in the city. The school boasts many modern amenities such as a well-stocked library, strong music and arts programs, computer (ICT) services and internet access. The school is considered one of the best primary schools in Ghana with most parents registering their children when they become pregnant. Children are usually driven to school from different parts of the capital. Tuition is generally very high.

Fiifi's IRT scores for the Number and Operations items in the survey was -3.010, which was the lowest in the sample. If Fiifi's MKT score is valid, it is expected that his MQI will be qualitatively lower than Gyidi or Kofi who obtained higher IRT scores on the survey.

As indicated earlier, Fiifi teaches in a self-contained classroom of 35 4$^{th}$ graders. The two consecutive mathematics lessons observed were focused on data collection and reading graphs. Fiifi identified the lesson objectives as:

1. Prepare for the idea of collecting data from simple experiments
2. To learn how information can be set out in tables and graphs, and
3. To learn to read graphs of data.

In the first lesson, Fiifi first introduced the topic to the class and asked students for their birth months which one student recorded on a table drawn on the board. Students were subsequently asked to show that information in a graph sheet. To conclude that day's lesson, Fiifi gave the students another task that required students to use raw data scores obtained by students in a math quiz to construct a tally chart and then graph the information. The class ended with 3 students showing examples of their graphs to the whole class. The second day's lesson began with a quick review of the previous day's lesson. Fiifi then asked each student to come to the front of the class, throw 2 dice on a table, and record the sum on a chart drawn on the blackboard. The students then drew a graph using the data they had generated and the lesson concluded as students were independently working on the task as Fiifi circulated the room.

Fiifi's lessons showed some mathematical strengths. He had clearly identified the learning goals and used several representations such as tables and graphs to show the data collected (children's birth months and sum of dice). The lesson included the use of mathematical language such as bar graph and data. In spite of these strengths, Fiifi's lesson highlighted some significant mathematical errors and oversights. For instance, in the first observation, Fiifi asked the students to use a scale of 1:5 saying: "So I think we can group them in fives.  So we have five here.  We start out with zero, five, ten, fifteen…" (YCVSF_1, p.6) to show the number of students for each month. "Grouping" in fives was not useful because the highest possible number for each month was 8. In addition, Fiifi insisted that students record the numbers as multiples of 5 (see Figure 4.6).

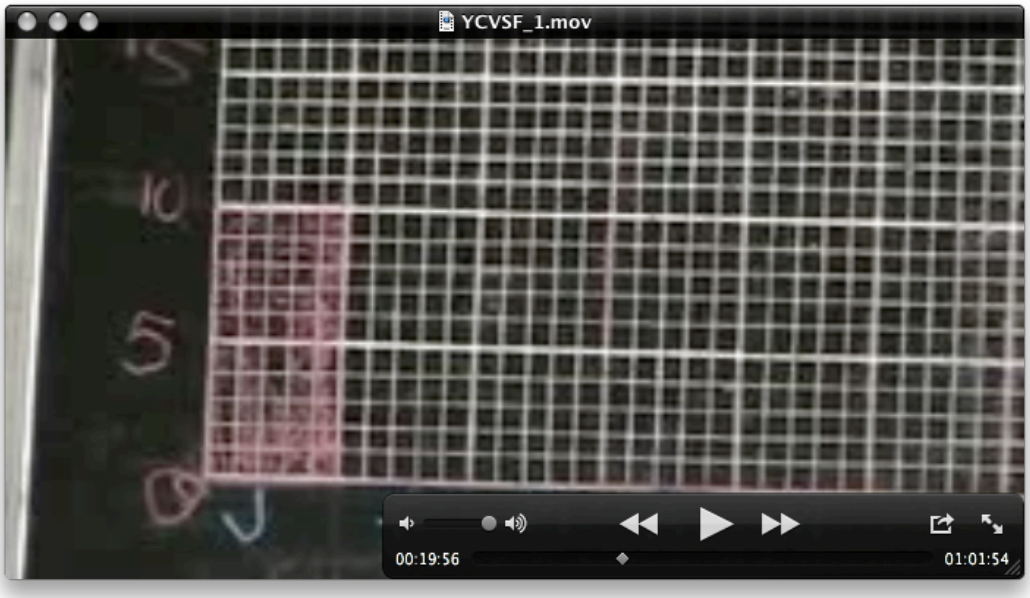



Figure 4.6. Screen shot of Fiifi's representation of data (YCVSF_1, 00:19:56)

As a result, although there were two students born in January, Fiifi's recording indicates that there were 10 students born in January (see Figure 4.6). Also on the first day, after students' recording students' birth month in the tally chart, Fiifi noticed that there was no entry for September and told the class that he was born in September so instead of 0, they should all put 1. This was problematic because the task was clear that the data they were recording was of pupils' birth months and deprived students of the opportunity of drawing a graph where one value was 0. In addition, Fiifi informed the students that the y-axis (which he called "this side") showed the age of students and not the number of students. Another significant problem identified in Fiifi's lesson was his use of mathematical language. The second task in the first day's lesson required students to solve the problem in Figure 4.7 below:

*Classwork*

*Listed below are the marks out of a total of 10 scored by each pupil in primary 4.*

*6,4,7,5,4,4,3,2,9,1,8,3,5,4,8,10,4,6,*
*4,7,8,3,5,4,2,5,6,9,4,1,5,4,6,4,7,3*

1. *What is the lowest mark scored?*
2. *How many pupils scored this mark?*
3. *How many pupils in all are in this Primary 4?*
4. *What is the mark obtained by the top pupil?*

Figure 4.7. Task from Fiifi’s day 2 lesson

In Ghana, the word “marks” typically referred to scores. Fiifi however used the word in 3 different ways. He said:

Fiifi: In your book, we are going to draw a table like this. So for (1)
example, let’s check up and see how many pupils scored only (2)
one **mark**. Count the numbers there. Listed below are the (3)
**marks** out of a total of ten scored by each pupil in primary (4)
four… these were the **marks** each of the pupils scored. (5)
And we are going to use the strokes just like we (6)
used to prepare this graph here. So how many children scored (7)
only one **mark**?...... How many children scored only one? (8)
Come and write on the board let me see. Yes? Count the (9)
number of ones you see in this table. The **marks** scored, you (10)
have it over there and I said only one **mark**? How many pupils (11)
or children scored only one **mark**? (12)
Theresa. (13)
Theresa: Two. (15)
Fiifi: Two. You have one here, and then the other one here. So we (16)
are going to **mark** two strokes like this. Number of pupils who (17)
scored this **mark** is two. And you will write two here. Now let’s (18)
try the next one. How many pupils scored two? Two **marks**… (19)
Very good, we will **mark** two strokes…. (20)
YCVSF_1, 36:00 -39:07

This excerpt shows that Fiifi used the work “marks” as indicated in the task first, to show the scores that students had (lines 3, 4,5, 10,11,12 18); second, he used it to refer to the actual score of 1 as in “one mark” (lines 8, 19) and third, to show the verb ‘to mark’ a stroke (line 17). These different uses of the noun and verb forms of “mark” could have been confusing for a classroom of students for which English is a second language.

Some errors were observed on the second day in the dice activity. No student recorded a sum of 12. When selecting the scales for the axes, Fiifi asked students to not include 12 on the y-axis because no one threw that sum. This was considered problematic because by making the decision to exclude 12, Fiifi did not give the students the opportunity to use zero as an occurrence on their bar graph. Fiifi’s language use was also problematic. He referred to the axes on both days as “this side” without drawing attention to the name of the line or using descriptors such as vertical or horizontal.

I now turn to report on the findings for Gyidi, whose IRT score was -1.111 which means that she was less than one standard deviation from the mean.

***The case of Gyidi***

Gyidi teaches in a K-5 public school located in the outskirts of the capital city, Accra. This school is tuition-free and students generally walk from the local community to attend the school. The school building is structured as a long row of classrooms, with either concrete walls or wooden planks separating each room. Due to the lack of space, between three and four students share a desk designed for one or two people. Class sizes range from the high sixties to low eighties and two teachers are assigned to each class. Gyidi’s class had 72 students. In general, one teacher leads the instruction for specified subjects. The thin walls provide little insulation from the sounds of the environment or other classrooms. During parts of Gyidi's lessons you could distinctly hear most of the sounds from both neighboring classrooms as well as the ringing of bells by peddlers on bicycles, and the sounds of free-range livestock (goats and chickens). Students in the fourth grade class were apparently used to this and did not appear to be bothered by the loud interactions and songs that were a constant feature during instruction in that school.

Gyidi's lessons over both days focused on the properties of basic operations. The syllabus identifies the objectives of the lesson as:

Students will be able to

1. Use properties of basic operations to complete number sentences to find true or false mathematical sentences
2. Use two or more of the basic operations to write number sentences

(Ministry of Education, 2001, pp. 58-59)

Gyidi's lessons began with students reciting a poem in English and/or a local language. She then introduced the lesson for the day with some review. On the first day, the students were given quick mental tasks such as 2+5+8; 2 x 5 x 0; and 3+5+2. She then reviewed the basic operations of addition, subtraction, multiplication and division; and gave students tasks that involved modeling the tasks with counters[22] on their desks. Students were then given some exercises to identify particular operations or numbers that make some number sentences true.

On the second day, Gyidi started the lesson with a brief review of the previous day's work and had two students stand in front of the class. She demonstrated the commutative property of addition with the two students by giving them

[22] Counters are bottle tops typically made from aluminum materials

different numbers of counters. The class then worked on subtraction, multiplication, and division. Towards the end of the class, Gyidi recorded the following on the board:

1. The order in which we do addition does not matter
2. The order in which we do subtraction matters
3. The order in which we do multiplication does not matter
4. The order in which we do division matters

The lesson concluded with students doing some exercises in their class workbooks.

Gyidi's lessons were generally engaging with students actively involved in the lesson. Overall, the strong points in the lessons were the focused use of representations for the different operations. Although students knew the answers to the questions such as 4 x 3 and 10 ÷ 5, Gyidi insisted students model the tasks with counters. Gyidi was scored high for the use of representations because she used representations in 14 minutes of her first lesson and for 35 minutes in her second lesson. In addition, Gyidi was consistent in her use of technical mathematical language such as number sentence, addition, operations, and sets. Gyidi systematically asked students to explain their thinking. For instance, when she asked the students to fill in the box for 12     4 = 8, the following ensued:

| | |
|---|---|
| Gyidi: | What operation should be in the box to make the sentence correct? Some people are still counting. Yes, tell us |
| Student A: | subtraction |
| Gyidi: | he says subtraction. Is he correct? |

| | |
|---|---|
| All: | yes |
| Gyidi: | yes, so how did you get the sub, what did you do? Tell me what you did before you had the subtraction, before you had the answer 8?... |
| Student A: | 12 subtract by 4 is 8 |
| Gyidi: | so what did you do? I asked you to count 12 and what did you do with the 12 and the 4 |
| Student A: | I removed 4 out of it |
| Gyidi: | they can't hear you. They want to hear you. Those at the back want to hear you. You did, tell me all that you did before you had |
| Student A: | I count 12 |
| Gyidi: | I counted 12 |
| Student A: | I counted 12 and I take 4 out of it and I got the 8 |
| Gyidi: | so is he correct? |
| Students: | Yes |
| Gyidi: | Good |

(YCVSG_2, p.3)

Gyidi generally asked students to describe how they got their answer and was scored high in two segments of the lesson (15 minutes) in the first lesson and high in six segments of the lesson (42 minutes) and medium in one segments of the lesson (7 minutes).

Gyidi's lessons were mostly error-free. Two errors were observed in Gyidi's lessons. In the first case, after students solved 18 - 6 to get 12, Gyidi asked them to use their counters to show 6-18. Most of the students said "it can't be" and when she asked why, one student said "because you can't subtract a bigger number form a smaller number" (YCVSG_2, p.5) and she applauded the student in agreement. In the second case, it was not clear whether Gyidi's understanding of 3 x 4 as 3 sets of 4 was consistent with the conventions as stated in the government-approved syllabus. In the second lesson for instance, Gyidi said the following:

> “Now look at this operation on the board. I want you to do it right now. Use your counters. 3 times 4, 3 sets of what, 4. Do it, I’m coming round to check. 3 sets of 4, you are counting 3 sets of 4 ... 3 sets of 3. 3, 3, 3, how many times, 4 times. I said 3 sets, 3 sets of 4 so 3, 3, 3 four times. Good. This is three sets of 4 good, now put them together and count. The three sets, put them together and count.”
>
> (YCVSG_2, p.6)

Gyidi was consistent in her representation of 3 x 4 as 3 sets of 4 as 3, 3, 3, 3 and 4 x 3 as 4 sets of 3 as 4,4,4.

***The case of Kofi***

Kofi’s school is a private pre-K to junior high (middle) school located in the outskirts of the capital in a new and rapidly developing area of the country. The school has one four-storey building with 16 classrooms, a two-storey building with 4 science laboratories, and three smaller one-storey buildings with other classrooms. Kofi’s lessons focused primarily on fractions. Over the two days, Kofi taught students about the basic concepts of fractions and introduced his students to decimal fractions.

Kofi’s IRT score on the MKT survey was less than two standard deviations from the mean (-2.74) and if his scores are an accurate representation of his MKT, it is expected that Kofi will exhibit a low level of MQI in his lessons. Overall, however, Kofi’s lessons showed remarkable strengths as demonstrated by his MQI scores. Kofi used a variety of representations in his lesson. He showed fractions with set models, area models, and used oranges to demonstrate to his students how one or more oranges could be used as the fraction whole. He also used an example of a local soap (key soap bar) as a context for fractions. Kofi enhanced the mathematical quality of his lessons through the use of a variety of technical mathematical terms such as whole, decimal point, partition, dividend, divisible, and called attention to students' use of language such as referring to 0.25 as not point twenty-five but as point two five, provided other names for fractions such as one-fifth as zero point two or point two. Kofi's lessons were also characterized by the use of explanations. He actively sought explanations from his students, and required them to describe the steps they used to solve a problem. Kofi consistently used students' ideas in his lesson, either utilizing a correct response to explain an idea, or remediating a student's incorrect mathematical thinking. Kofi also generated tasks that increased the cognitive demand for his students. For instance, he used a bar of soap called "key soap[23]" as a correct context for a problem on fractions. Kofi asked his students to label the indicated parts of a number line where the number line was drawn as a representation of one bar soap (see figure 4.8).

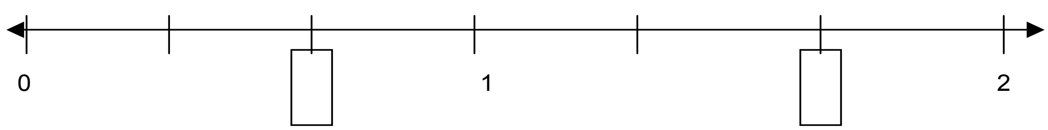



Figure 4.8. Representation of "key soap" on a number line.

In spite of the strengths of his lesson, there were some mathematical errors. For instance, on day 1, Kofi consistently used the idea of "taking away" to represent the fractional part under consideration as exemplified in the following:

| | |
|---|---|
| Kofi: | 25. Ok, now let's look at this orange. How many are they? |
| Students: | 4 |

---

[23] A key soap bar is about 2 feet long

| | |
|---|---|
| Kofi: | ok, take one out of it. How many left? |
| Students: | 3 |
| Kofi: | So one has what? |
| Students: | gone. |
| Kofi: | it has gone out. I have dashed it to Patrick. Ok; so out of four, one has been, what, dashed. So I can also write this one as what? |
| Students: | one over four |
| Kofi: | one out of four, the total were four, but one has been what, out, or one has been dashed to |
| Students: | Patrick |
| Kofi: | In this case, I will write this as one over 4 |

(YCVSI_1, p.2)

In the second day's lesson, Kofi's lesson was focused on converting fractions to decimals and in the process of calculation, he informed students that "you bring 0 from nowhere" to explain the use of 0 in the integer part of the quotient in the division problem. Although these errors were minor, they were mathematically inaccurate and were coded as such.

Perhaps the most significant imprecision Kofi exhibited in his lesson was his use of language. On the first day's lesson on fractions Kofi introduced the name of the fraction parts (numerator and denominator) to his students in the following manner:

| | |
|---|---|
| Kofi: | so even when you come here, and you don't know anything, now you've got to know that the one down is called **denomerator**, the one up is called what, |
| Students: | numerator |
| Kofi: | so when we come here, we said three fourths or I'm going to divide by the numerator by the **denomerator**. The numerator is the one up, and the **denomerator** is what, the one down, which is the same as three divided by four. So here comes in the, here comes the numerator and this is the **denomerator** or this is the numerator, this is the **denomerator** so all the time, the **denomerator** divides the numerator and the numerator is divided by the **denomerator**. is that clear? So before you come here, make sure you state or you give us the |

name of what you are going to deal with, before you solve it. because when you come and you write three divided by four like this, where from it? I hope you are getting me right?

Students: yes sir

(YCVSI_2, p.9)

Although Kofi was incorrect in his pronunciation of “denominator” which could have been attributed to his status as a non-native English speaker, this was troubling because he not only pronounced “denominator” incorrectly, he wrote it on the board as “*denomerator*” for the students.

I now turn to a cross-case comparison of the three teachers to determine the how they enact particular MQI features.

## Cross-case analysis of relationship between MKT and MQI

The three teachers, Fiifi, Gyidi, and Kofi, demonstrated some of the features of MQI in varying degrees. Their differences ranged from the general structure of their lessons to their enactment of particular features of the MQI such as giving explanations, developing generalizations, their use of mathematical language, and their work with students and their class sizes.

### Format and structure of lessons

All 3 teachers were generally “in control” of the delivery of mathematical content and in most cases, there was more teacher-talk than student-talk. Across the three teachers, there were mostly no whole-class discussions where students shared their thinking, and built on one another’s contributions. Gyidi and Kofi provided opportunities for students to share their thinking and Gyidi was more active in discussions with students as the lesson progressed. Kofi on the other hand, required students to come to the board to solve a problem and explain their thinking process to the class. Fiifi worked on real-world problems in his lesson on data collection and representation and Kofi drew on real world contexts to explain fractions by using oranges, the number of students in a class, and a different area model and set model representations of fractions. Gyidi, on the other hand, rarely used real world contexts to support her teaching of properties of basic operations. Appendix G shows a summary of the general structure of lessons for all the six cases.

**Richness of mathematics: using mathematical representations**

The five meta-codes that comprise the "richness" code are explanations, use of representations, mathematical language, multiple procedures and solution methods, and developing mathematical generalizations. The varied nature of the mathematics content taught at the time of data collection lended themselves to the occurrence of some of the mathematical features while some did not. I thus report these findings across the representations and explanations codes that were common across all lessons. These specific features have been shown to be features that teachers with high MKT exhibit in qualitatively better ways than teachers with low MKT (Hill et al., 2008).

Fiifi's lesson on data collection and graphing included the use of representations such as bar graphs and tally charts (mean = 1.5[24]). He also made explicit links between information from the data table and the information in the graphs. Students worked on making graphs and interpreting information from the graphs.

---

[24] Means are rounded to 1 decimal place

The different uses of representations enriched the mathematical quality of his lesson.

Gyidi also used bottle tops (called counters) as manipulatives in her lesson (mean = 2) In teaching properties of basic operations, she required students to use the counters to represent addition, subtraction, multiplication and division problems. In most cases, students knew the answers to her questions such as $4 \times 3$. Her insistence that students use the counters to represent the materials increased the cognitive demand of the task and provided opportunities for students to make connections between their intuitive knowledge of basic facts to the meaning of the operations.

Kofi used representations the most time in his lesson on fractions (mean = 2.3). He brought oranges to school to represent a set of fractions (4 oranges), cut up one orange up to show a whole, used number lines, set models, and area models as different representations of fractions consistently. Students had opportunities to draw the representations and explain their thinking about how the figure is a representation of a fraction.

### Richness of mathematics: explaining mathematical ideas

All teachers utilized some degree of explanations in their lessons. Fiifi's lesson had the least frequency of explanation (mean = 1.1, 1 clip). He rarely required students to explain their thinking and most of his questioning required short answers that were designed to either complete the table on the board, or draw a graph. For instance, after Fiifi drew the graph on the board, he asked students questions about the graph in the following:

| | |
|---|---|
| Fiifi: | …. So now we know the number of people born in each month of the year. You know how many are we in the class? |
| Students: | Twenty-seven. |
| Fiifi: | How many? |
| Students: | Thirty-five. |
| | … |
| Fiifi: | Okay, so now we can put this question down. January, the month January. How many people were born in January if you look at this table? Yes, you? |
| Student: | Two. |
| Fiifi: | So we have two. February, two. February too, two. Okay, then March. Yes? |
| Student: | four |
| Fiifi: | Four. April. Yes? April, June? Four? Yes? |

(YCVSF_1, p.4)

As the above excerpt shows, Fiifi's questions usually required one-word answers and students were not typically asked to explain their thinking or reasoning. Fiifi's questions followed a similar pattern throughout his lessons.

Gyidi elicited explanations in her lesson in a systematic way. She consistently asked students to explain why their responses were correct (mean = 2.1, 7 clips in one lesson and 4 clips in the second lesson). For instance, students were asked to explain why 10 divided by 5 was 2 and 4 multiplied by 3 was 12.

Of the three cases, Kofi's use of explanations was the most consistent (mean = 1.90, with a score of 3 in 4 segments). In his second lesson, he used explanations in his demonstration of the standard form of long division, and explained how different representations showed different fractions. In one case, Kofi gave an explanation in response to two students' questions about the procedure for finding decimal fractions. He said the following:

| | |
|---|---|
| Kofi: | ok, the decimal came because of this point. Whenever you see the point, I'm not talking about full stop, full stop is used? When we are doing English, but when it comes to maths, |

| | |
|---|---|
| | mathematical this thing, and you see this point, it referred to as as decimal point, decimal |
| Students: | point, |
| Kofi: | ok, now I'm done. Who has a question to ask, apart from that I will try another this thing so that you comes to board one after the other you will also try as many as you…yes |
| Student: | so sir if you bring one point if you add another zero, will you bring another point again? |
| Kofi: | No, that is excellent question. The point must be appear only once, are you getting me right? The decimal point is still decimal point so you don't need to repeat the decimal point twice. Once you brought the decimal point once, that is all. So for instance if I must to divide or I must to add zero to this, I will never bring point again because the point is being here, I have brought it. Are you with me? So the point must be appear only once. |

(YCVSI_2, p.4)

Beyond the language comprehension that may arise, Kofi's explanations were routinely detailed and focused on his understanding of the mathematics content. The above excerpt for instance illustrates Kofi's description of the difference between a full stop and a decimal point. In addition, Kofi responds to a student's question asking if another decimal point is required if the conditions for which the first decimal point was needed are present again.

**Working with students and mathematics**

In general, the teachers asked questions, remediated students' mathematical difficulties, and used students' mathematical ideas in their instruction. Gyidi consistently asked questions that required students to explain (mean = 2.1, with a score of 3 in 7 segments a score of 2 in 17 segments in the two lessons) compared to Fiifi (mean = 1.1, with one score of 2 across 14 segments in two lessons) or Kofi (mean = 1.2, with 4 scores of 2 across 22 segments in the two lessons). Kofi systematically remediated students' difficulties (mean = 1.9, with a score of 3 in six segments in 22 segments in the two lessons) compared to Gyidi (mean = 1.2, with a score of 3 once across 17 segments in the two lessons) or Fiifi (mean = 1.1, with two scores of 2 across 14 segments in the two lessons). He also used students' mathematical ideas in instruction (mean = 1.9, with a score of 2 in 4 segments and a score of 3 in 4 segments across both lessons) more than Gyidi (mean = 1.5, with a score of 2 in 8 segments and a score of 3 in one segment across both lessons) or Fiifi (mean = 1.1, with a score of 2, in one segment across both lessons). These mean values are important because they show the differences among the cases of the frequency and quality of their enactment of a particular MQI feature. For instance a 40-minute lesson that showed one instance (in about 7 minutes of instruction) of very high use of student questioning is likely to rank higher than a 90-minute lesson with very little use of student questioning.

Kofi explicitly remediated students' difficulties by addressing and identifying the source of student errors, anticipating their errors and addressing them in his explanations. For instance in his lesson on fractions, he regularly informed students about the different kinds of wholes that a fraction can have as he did in the following:

> Kofi: a fraction, you first look at the total. If the total, for instance, the total means the whole thing. Are you getting me right? If the total is 25, then you pick or some portion is being dashed out, that portion that is being dashed out, out of the total referred to as, as fraction. Which means those that have been gone, or those that have dashed out are part of what, the total of the class. Do you agree with me? so don't confuse yourself whenever you come to fractions because you *think fraction is part of whole number so if it refers to only one object, when one object is being split, then that means that object becomes a fraction, no, it can be group of*

*something*

(YCVSI_1, p.5, italics added for emphasis)

Gyidi also used questions such as “why did you say multiplication, why not division, or addition, or subtraction, why multiplication” (YCVSG_1, p.5) in probing students’ understanding.

**Comparing cases along MQI means**

Examining the format and structure of lessons, richness of mathematical representations and explanations, and working with students and mathematics show some differences among the 3 cases[25]. Gyidi, who had the highest MKT scores (IRT= -1.111) of less than one standard deviation from the mean, had the fewest incidences of errors (mean = 1.06, with a score of 2 in one segment across both lessons) compared to Kofi (mean = 1.2, with a score of 2 in two segments and a score of 3 in one segment across both lessons) and Fiifi (mean = 1.6, with a score of 2 in one segment and a score of 3 in four segments across both lessons). Kofi who scored more than 2 standard deviations below the mean (IRT= -2.74) had the highest average for overall richness of mathematics (mean = 1.9, with a score of 2 in five segments and a score of 3 in five segments across both lessons) compared to Gyidi (mean = 1.8, with a score of 2 in eight segments and a score of 3 in one segment across both lessons) and Fiifi (mean = 1.1, with a score of 2 in one segment and no score of 3 across both lessons). Fiifi whose MKT score was about 3 standard deviations below the mean (IRT= -3.010) had the most error average (mean = 1.6, with a score of 2 in one segment and a

[25] Appendix G shows the MQI means for all 6 cases

score of 3 in four segments across both lessons) compared to Gyidi (mean = 1.1, with a score of 2 in one segment across both lessons) and Fiifi (mean = 1.2, with a score of 2 in two segments and a score of 3 in one segment across both lessons).

It is important to mention here that the length of the lesson and the general format of the lesson influenced the magnitude of the means. This is because there was some range in the proportion of time devoted to active instruction where teachers engaged in rich mathematical features compared to the time for individual student work where not a lot of rich features were recorded. In spite of the differences in the length of time of the different segments of the lesson, there were notable qualitative differences across the lessons that merit the reporting of the different means.

Using the video study alone as a source of validation of the MKT survey use in Ghana, these findings indicate that there could be some associations between MKT scores and MQI for these cases. The next section examines the strength and direction of these relationships.

### Relationship between MKT scores and elements of MQI

The second part of testing the inference of the validity argument involved the quantitative analysis of the correlation of the MQI scores with the MKT scores of the teachers. These correlations would provide the direction and strength of the relationships between teachers' MKT scores and their MQI scores. In this study, although I focused primarily on the three cases (Fiifi, Gyidi, and Kofi), I will report the findings based on data from all six cases. This will help situate the three cases in a larger context and set the ground for thinking about the MKT construct more broadly. Table 4.12 shows the correlations between the teachers' IRT scores. Correlations were calculated between the values obtained from the aggregated codes for the six teachers and the IRT survey scores obtained for all six cases. MQI meta codes were: overall richness in mathematics, overall errors and imprecision, and overall working with students.

Table 4.12. Correlations of MKT Survey Scores with MQI Scores Spearman's rho) for the Six Teachers

| | **Overall richness of mathematics** | **Overall errors and imprecision** | **Overall working with students and mathematics** |
|---|---|---|---|
| **MKT scores (IRT)** | 0.696 | -0.721 | 0.261 |
| **Overall richness of mathematics** | 1 | -0.319 | 0.771 |
| **Overall errors and imprecision** | - | 1 | -0.232 |
| **Overall working with students and mathematics** | - | - | 1 |

In general, the correlations are in the expected directions. That is, teachers' MKT scores are positively related to the overall richness of their mathematics instruction (0.696), overall working with students and mathematics (0.261), and negatively related to the errors identified in their mathematics lessons. The results indicate that the overall richness of the mathematics instruction is more correlated with the IRT scores of the teachers sampled than the overall working with students. The overall richness of the teachers' mathematics instruction is also more correlated to the teachers' MKT than their working with students. As expected, overall errors and imprecision is more correlated to MKT and teachers with higher levels of MKT are less likely to have mathematical errors or imprecision in their teaching and conversely, teachers with low MKT scores are more likely to have more errors in their instruction and less likely to have rich mathematical elements in their lessons.

In spite of the small sample size, these results are in the expected direction and hold promise for identifying elements of the construct of MKT applicable for use in Ghana. In addition, the results seem to indicate that teachers who score highly on the MKT measures are more likely to teach in qualitatively better ways and have fewer errors associated with their instruction compared to teachers who don't score as highly.

**Summary of findings from video validation study**

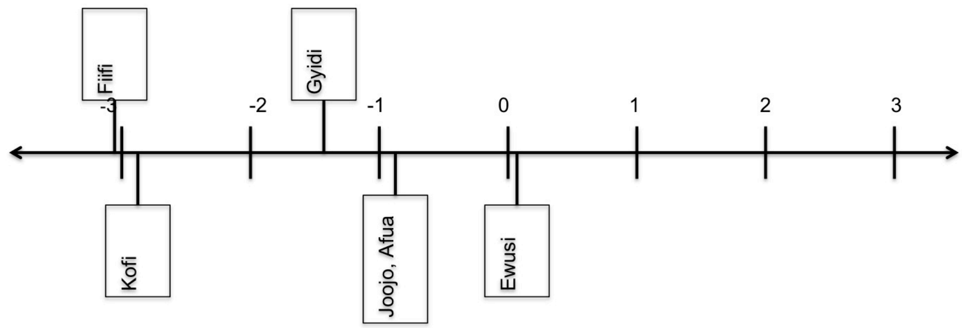
The purpose of the video study was to examine the extent to which teachers' MKT survey scores could be validated by examining mathematical instruction. Teachers' MKT scores are one way of assessing their mathematical knowledge to teach and a careful study of their mathematics instruction is another source of data for assessing teachers' knowledge. As a secondary source of validation to examine the strength of the relationship between teachers' MKT and their instruction, I examined the associations between the teachers' MKT scores, interview scores, and their overall MQI levels as assessed by the coders of the mathematical instruction. As was done above, I will use data from all 6 cases to test this relationship.

Table 4.13. MKT Survey, Interview Scores, Estimates of Teachers' MKT* based on Video Study

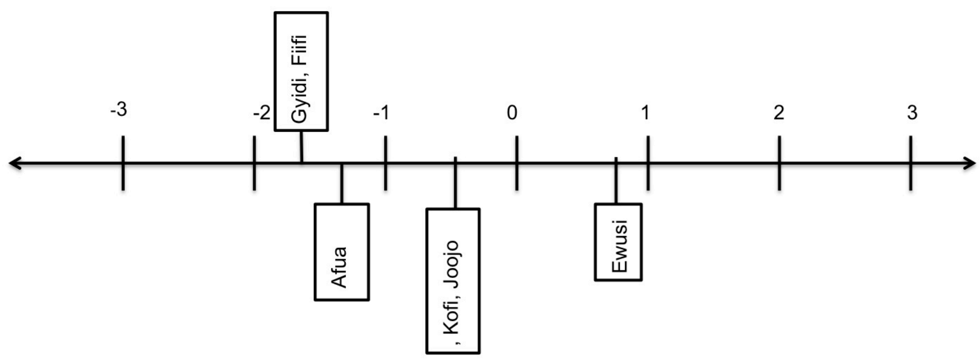
| Teacher | Grade level | MKT survey score (IRT) | MKT interview score (IRT) | Mean estimates of MKT level |
|---|---|---|---|---|
| Ewusi | 6 | 0.0140 | 0.777 | 3 |
| Joojo | 6/JSS 1 | -0.891 | -0.671 | 2 |
| Afua | 6 | -0.891 | -1.333 | 1.5 |
| Gyidi | 4 | -1.111 | -1.552 | 2 |
| Kofi | 4 | -2.740 | -0.671 | 2 |
| Fiifi | 4 | -3.010 | -1.552 | 1 |

*As part of the coding of instruction, the trained video coders estimated teachers' MKT based on the mathematical quality of the instruction they coded. This was on a scale of 1-3 corresponding to low, medium and high.

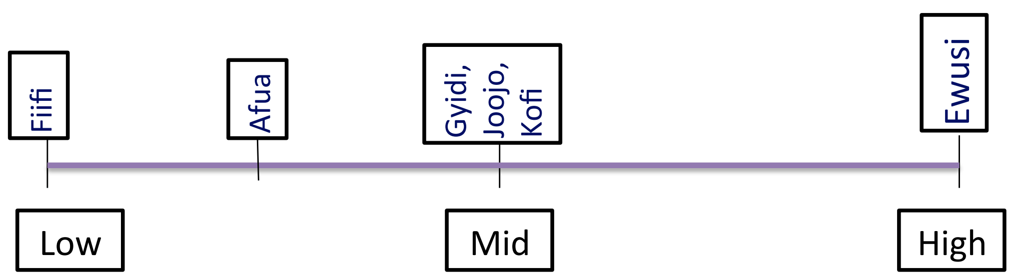
A visual representation of the ordering of the teachers by MKT scores, interview scores, and MKT ranking, not according to scale is shown in Figure 4.9. This representation also confirms the ranking for the highest scorer (Ewusi) and the lowest scorer (Fiifi), with Afua, Joojo, and Kofi in the middle range showing Ewusi's MKT estimate at 3 and Fiifi with the lowest score has an MKT estimate of 1. These findings appear to indicate the consistency of the relationship between the sample's MKT scores and their MQI overall.




Teachers ordered according to their IRT scores on the MKT survey (scored from -3 to +3; teachers not placed to precise scale)




Teachers ordered according to their IRT scores on the MKT interview (scored from -3 to +3; teachers not placed to precise scale)




Teachers in the video study ordered according to their MQI rankings (scored from low to high)

Figure 4.9. Ordering of 6 teachers by MKT scores, interview scores and MQI ranking (not drawn to scale)

The video validation study also yields some interesting observations about features of mathematics instruction that the MQI codes did not capture. I now turn to a description of some of these elements and explore the extent to which they inform MQI in the Ghanaian context.

## Features of instruction not captured by MQI codes

In this study I found four features of instruction that seemed to be important to mathematics instruction in the Ghanaian context but were not adequately captured by the MQI codes. They are, the use of the blackboard to support instruction; use of jotters and exercise books; explicit attention to describing the steps of a solution;; and setting norms for doing mathematical work.

### Use of the blackboard to support instruction

All the teachers observed used blackboards as the means of publicly presenting student work. The boards ranged in size. Some of the boards stretched along the front wall of the classrooms, some were in the middle third of the classroom. In two of the 20 lessons videotaped, two teachers used chart paper to represent examples of a procedure.

As expected, teachers' use of the blackboard was very varied. Gyidi and Fiifi for instance, took about four minutes of class time to write exercises on the board for students to do. Whilst Gyidi gave students individual work to do as she wrote on the board, Fiifi's students sat quietly while Fiifi wrote on the board.

Gyidi and Fiifi also used the board to demonstrate the focus of mathematical content for the day. Fiifi wrote the teaching objectives on the board before the lesson and Gyidi wrote the main mathematical points of her lesson on properties of basic operations towards the end of her lesson. Other teachers' use of the blackboard appeared uncoordinated. Students and teachers erased some parts of the board to free up space to write more mathematical content on the board. Studies in Japan have shown that careful use of the blackboard helps students organize their thinking and their mathematics notes (Yoshida, 2005). In Stigler and Hiebert's (1999) seminal work on comparing mathematics instruction in the U.S., Germany, and Japan, there was overwhelming evidence that Japanese classrooms used blackboards 100% of the time and by the end of the lesson about 83% of the writing on the board remained, compared to 39% in Germany and just under 52% in the U.S. Yoshida identifies the reasons that Japanese teachers use the blackboard as

> 1. To keep a record of the lesson
> 2. To help students remember what they need to do and think about
> 3. To help students see the connection between different parts of the lesson and the progression of the lesson
> 4. To compare, contrast, and discuss ideas students present
> 5. To help to organize student thinking and discover new ideas
> 6. To foster organized students note-taking skills by modeling good organization
>
> (Yoshida, 2005, p. 97)

Given the economic characteristics of Ghana[26] and the absence of high-technological media to support classroom instruction, and in contrast to Japan, Germany, and the US., the blackboard is an essential tool of instruction. Hence, careful use of the blackboard could serve as an important feature of

[26] The United Nations identifies Ghana as a low developing country as characterized by the human development index rank determined by health, education, and living standards indicators (see http://hdr.undp.org/en/media/HDR_2010_EN_Table1_reprint.pdf accessed on July 14, 2011)

mathematical quality of instruction. The MQI codes however, did not account for the use of board work in any of its categories.

Yoshida's reasons for using the blackboard could inform the design of codes to assess the quality of blackboard use in mathematics lessons. For instance, a code about blackboard use could be developed to assess the record-keeping role of blackboards. Other codes could assess the extent to which the teachers' use of the board makes use of student ideas, or student thinking to make connections across mathematical ideas or representations. MQI codes that assess blackboard use could be used to determine the extent to which teachers' with higher levels of utilize their blackboard space and how this compares with teachers with lower levels of MKT. Given the prevalence of blackboard use in Ghana (and possibly other African countries), a careful attention to how mathematics is recorded in public spaces such as the blackboard and the extent to which such uses are related to teachers' MKT and consequently to students' learning opportunities are important.

**Use of jotters and exercise books**

Related to the use of blackboards is the use of jotters and exercise books as record-keeping devices of mathematical lessons. In all the lessons observed, teachers routinely asked students to do some exercises in their jotters and were very explicit in the nature of tasks that are recorded in their exercise books.

Jotters[27] were generally used as the “practice” books where students’ initial engagement with content were recorded. Teachers made decisions about which tasks were recorded in what medium. In most cases, the teacher graded the work in exercise books after school or outside the mathematics lesson and work done in the jotters were usually checked when the teacher circulated in the classroom to monitor students’ individual work.

The MQI codes did not indicate the extent to which teachers’ support of students’ recording of mathematical work in either the jotters or their exercise books was a feature of mathematical quality of instruction. Future studies could investigate the extent to which these features contribute to MQI. This is because the data showed the use of jotters as central to mathematics instruction. This could ensure that teacher preparation and professional development opportunities include elements of record keeping in mathematics classes and support teachers to use their limited resources to optimize students’ learning experiences.

Students’ jotters were generally kept with students whereas exercise books were usually kept at school. Jotters thus provided students, to a large extent, with the record of their mathematical work. Attention to how jotters are organized, what is recorded in them, and how they are utilized could provide insights into the extent to which teachers’ knowledge about mathematics informs the privileging of particular content with particular students in their classes.

---

[27] Jotters are notebooks for taking rough notes, not in their final presentable form.

**Explicit attention to describing the steps of a solution**

Kofi and Ewusi paid careful attention to how students' publicly solved questions on the board. When Kofi asked a student to change $\frac{1}{4}$ to a decimal, the following ensued:

| | |
|---|---|
| Kofi : | Ok, now who can volunteer and then solve it on the board for us. I want you to follow up, follow the steps and then you try it on the board. Yes, you will explain it, you will express yourself. You will come to the board, don't worry. |
| Student: | 4 will not go into 1, so you will write zero, that means it cannot go |
| Kofi: | why 4 cannot go into one, what, actually what are you solving before the 4 cannot go? |
| Student: | four… |
| Teacher: | listen, listen to the question, you must to understand. I'm solving this and this cannot go, why 4 cannot go into 1? |
| Student: | because if 4 go…. |
| Kofi: | listen, it means statement, what is it? Before you solve it, you must to, assuming you are going to teach the little one, what is mean by this? You have to mention, why this one before you tell us this cannot go. |
| Student: | 1 over 4 is the same as 1 divided by 4 so if 4 goes into 1, it cannot be so in mathematics, if it cannot be, you write zero. Then you create your own zero then you bring the point to show that the zero was not there, you create it yourself. 4 goes into 10 8 times, so you write your 8 here. |
| Kofi: | 4 goes into 10 eight times? |

(YCVSI_2,p.5)

As demonstrated in the above clip, Kofi supported the students' description of steps to solve the problem, and required students to use explanations to support each step. Kofi asked students to follow the steps to find the decimal equivalent of ¼ on the board. He was clear in asking them to use explanations. In addition, he interrupted the student's presentation multiple times with guiding questions to help students' description of the steps. In another lesson, Ewusi also constantly supported his students' description of steps in their solution of problems. The process of describing the steps of a procedure was not explicitly captured by the MQI codes.

Kilpatrick, Swafford, and Findell (2001) identified the strands of mathematical proficiency as conceptual understanding, procedural fluency, adaptive reasoning, strategic competence, and productive disposition. They define procedural fluency as “the knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently” (p. 121). They argue that “In the domain of number, procedural fluency is especially needed to support conceptual understanding of place value and the meanings of rational numbers” (p. 121).

This definition of procedural fluency implies that procedures could be used in appropriate or in inappropriate ways to advance or hinder the conceptual development or understanding of a topic. As such, it is an important component of the skills that teachers need to support their children to acquire. The MQI codes however did not account for the extent to which teachers were supporting their students’ description of mathematical steps and how the presence (or absence) of these supports was related to the MKT scores of the teachers. The absence of codes that assess the quality of teachers’ support of students’ mathematical procedures could mean that the elements of instruction as demonstrated in Ghana could be underrepresented and thus not properly accounted for in the assessment of teachers’ MQI.

**Setting norms for doing mathematical work**

Mathematics is a discipline bounded by specified practices and norms that are unique to "doing mathematics". Some of these norms include using definitions, explaining mathematical ideas, reasoning about problems to arrive at a solution, and developing generalizations. Embedded in the practice of these norms are the ways of enactment associated with each norm. For instance, saying that the mathematical sentence 2 x 4 = 8 proves that the statement is true is not a valid mathematical argument. There are prescribed mathematical steps for establishing that something is true.

In the video data from Ghana, there were multiple instances where teachers supported their students' "doing of mathematics" that was not captured by the MQI codes.

These supports range from inserting mathematical language into students' talk, to how to present their solutions. For instance, Kofi drew a set model for 8 balls and asked a student to show 5/8. When one student volunteered, he said:

| | |
|---|---|
| Kofi: | Now count it with them because if you write it in this way, how can they know the total is 8? So I want you to count after you count, that the total was this *[Kofi indicated 8]* and then I have circled this *[Kofi points to 5 of the circles]* |
| Student: | The total was 8, and I circled 5 of them |
| Kofi: | no, you count it with them |
| Students: | one, two, three, four, five, six, seven, eight. |
| Student: | the total was 8 |
| Kofi: | yes and how many have you counted |
| Students: | one, two, three, four, five |

(YCVSI_1, p.9)

The above episode shows an instance of Kofi supporting his students to show how the representation could show 5/8. Ewusi also supported students' presentation of mathematical work by requiring them to use complete sentences in their talk

Ewusi: Yes, come to the board… now tell them, is it a loss or a profit?
Student: It is a loss
Ewusi: why is it a loss?
Student: because the CP[28], he bought the emm, he bought
Ewusi: he bought the item
Student: 15 Gh cedis
Ewusi: he bought the item at
Student: 15 Gh cedis
Ewusi: I want you to say that
Student: he bought the item at 15Gh cedis and he sold it at 11 Gh cedis
(YCVSE_1,p.6)

In another instance, a student solved a problem on the board and even though the answer was correct, Ewusi said:

*Ewusi:* *yes, anything wrong with what she has done? Do you see* anything wrong with it? Yes, Abrefi tell us.
Student: the loss is not
Ewusi: louder, louder
Student: the loss is not in front of the
Ewusi: ok, go and do it
[student goes to the board]
Ewusi: Right, now if you just write CP minus SP, someone may not know what you are talking about so let us know whether it is a loss or what. These things must be there so we know what you are doing.
(YCVSE_2, p.8)

In the above episode, although the students' answer was correct, the student did not use the correct notation to indicate that the answer represented the amount of the loss incurred and Ewusi drew the class' attention to that and the student completed her solution. More specifically, he said, "these things must be there so we know what you are doing". Drawing attention to writing out solutions carefully

[28] CP refers to cost price

and talking through a solution are important features of doing mathematical work. How teachers pay attention to student contributions (written or oral) are important to the mathematical quality of any instruction and these are features that the current version of the MQI codes overlook.

As Lampert argues:

> At every level of schooling, and for all students, reform documents recommend that mathematics students should be making conjectures, abstracting mathematical properties, explaining their reasoning, validating their assertions, and discussing and questioning their own thinking and the thinking of others. These activities do not fit within the tasks that currently define mathematics lessons. Moreover, they require both teachers and students to think differently about the nature of mathematical knowledge. Little research has examined what the intellectually generative sort of mathematical activities espoused … might look like in classrooms or the role that the classroom culture plays in the social construction of a view of mathematical knowledge.
>
> (Lampert, 2001, pp. 32-33).

As Lampert suggests, there is the need for studies that examine how these mathematical practices can be enacted. I concur with Lampert and call for a need to examine *how* teachers can support students' enactment in doing mathematics. These features were all identified and acknowledged by coders to be elements of MQI but were not recorded by the current version of the codes. Earlier versions of the codes included codes for "description of mathematical steps and procedures" but that code was removed. LMT (2010) acknowledges the limitations of these codes and report,

> We do not argue that we have achieved a definitive characterization of MQI. For example, it may be helpful to add codes representing how students grapple with the mathematics (e.g. providing explanations[29], making counter-claims); or to more completely map teacher interactions with content. Yet, we argue that our theorization of this domain is a good

[29] Since their acknowledgement, LMT has introduced some new codes that include student explanations.

> beginning and brings together what have until now been disparate perspectives about what matters, mathematically, in classrooms"
> (LMT, 2011, p. 44)

These analyses therefore have the potential to inform the MQI codes or at least structure the codes to be sensitive to some of the issued identified here. I now conclude the discussion of findings by examining how this study contributes to the establishment of construct validity.

### Establishing construct validity

Using Singh's model for establishing equivalence described in Chapter 3, I now examine the functional, conceptual, and instrumental validity of the MKT items (Singh, 1995). Establishing these validities is an essential step prior to the administration of the measures at scale in Ghana.

*Functional equivalence* asks, "Do the MKT construct serve the same function in both countries?" I use the same logical argument used by Delaney (2008) to establish functional equivalence. To what extent does MKT in both Ghana and the U.S. have the same role (Delaney, 2008; Teune, 1990)? By definition, MKT is "the mathematical knowledge needed to carry out the work of teaching mathematics" (Ball et al., 2008, p. 395). The interpretation of that definition is that MKT "serves the function in every country where mathematics is taught" (Delaney, 2008, p. 73). This is because if mathematics is taught in that country, in this case Ghana, then some knowledge is required to teach it. The nature of the knowledge is not relevant to the functional argument. That the MKT construct serves the same function in Ghana and the U.S. therefore establishes functional equivalence between Ghana and the U.S.

*Conceptual equivalence* asks, “Are the tasks on which MKT is based the same in both Ghana and the U.S.?” This study assumed conceptual equivalence in order to use the measures in Ghana. There are however differences between Ghana and the U.S. in identifiable social, economic and cultural aspects. For instance, the United Nations identify the United States as a highly developed country whereas Ghana is categorized as a low developing country. These economic distinctions have implications for the availability of the basic needs of food, clothing, and shelter in Ghana and the U.S. In education, the availability of resources—financial and human—to support educational endeavors varies greatly across both countries.

In practice therefore, there are likely to be differences associated with *how* these differences are enacted but the fundamental tasks of teaching identified in the mathematical quality of instruction codes in the U.S. were recognizable in the analysis of video data from Ghana. The presence of these identifiable features of MQI such as using mathematical representations, explaining mathematical ideas, and using mathematical language do not compromise the assessment of MQI in the data because the data analysis reported in this dissertation provide some evidence to support a relationship between the Ghanaian samples’ MKT and MQI.

Other tasks of teaching such as use of the blackboard, jotters, as well as description of mathematical steps were identified in the Ghana videos but missing in the U.S.-developed MQI instrument. Further studies are needed to determine the extent to which the inclusion of the other features of MQI identified in Ghana influences MQI overall, and are correlated with MKT scores. This study however seems to indicate some level of conceptual equivalence.

*Instrumental equivalence* asks "Are the scale items, response categories and questionnaire stimuli interpreted identically across Ghana and the U.S.?" No, the process is more complex than teachers' interpretation of the multiple-choice tests. As the MKT scores from the interview show, some of the teachers were unfamiliar with the format of some of the questions and their MKT scores were not representative of their knowledge. Issues of instrument equivalence have also been raised in the use of MKT measures in Norway (Mosvold & Fauskanger, 2009) where the format of the items was also a concern, because Norwegian assessments are primarily open-ended items.

Another issue of interest that pertains to instrument equivalence is the interpretation of teachers' answer choices. In multiple-choice formats, there is usually one correct answer and all other options are "incorrect". Items that by nature might be able to distinguish one level of knowledge from other are not counted as separate from items that do not have that feature. One task that the Ghanaian sample found very difficult was the alternative subtraction task (described in Chapter 4). Teachers' reasoning about the different answer choices could determine one level of knowledge from another. For instance, the current scoring guide for the MKT survey makes no distinction between a teacher who can correctly explain one method from a teacher who can correctly explain two methods. The survey only counts as correct responses of teachers who are able to determine that all three strategies work for all whole numbers. As a study of teacher knowledge, it is important for the survey to make distinctions among different knowledge levels of teachers. A simple remedy to this problem is changing the structure of the item from a “regular” multiple-choice item where one option is the answer to a multiple-choice item with testlets as shown in Figure 5.1. Findings from this dissertation indicate that some teachers’ MKT scores were not an accurate representation of their knowledge about Number and Operations.

The issue of instrument equivalence, especially as it relates to item format could be remediated fairly easily. Future administrations of the MKT measures in Ghana would require training the teachers about the format of the items and this should reduce the number of invalid scores.

To administer the measures with a larger sample requires the use of multiple-choice items. Different formats such as constructive response items that can be easily scored and recorded could be explored in future studies. Given the constraints of time, and resources, the use of multiple-choice items in Ghana is the logical next step.

In addition to Singh’s (1995) model for establishing construct equivalence, another observation from the Ghanaian sample was the length of time teachers took to complete the test. I now elaborate on how this observation might influence the validity of the MKT survey findings.

***Length of time to complete test***

Another concern that may have implications on the use of the items is the length of time it took to complete the test. Wendler and Walker (2006) suggest that test length is related to the content-related validity of the test. In the administration of the LMT survey, all the teachers took between 150 minutes and 180 minutes to complete the test. This may be considered too long (Hambleton et al, 1991) and could have influenced how teachers responded to the items. Similar tests administered to teachers in the U.S. and in Ireland showed that teachers used between 60 and 75 minutes to complete the survey. Psychometricians have recommended that tests should not become a burden for respondents (Hambleton et al, 1991) and it is unclear how the MKT survey's length was a factor in how respondents answered the questions. Wendler and Walker (2006) suggest a test that "must fit into a time window that is small enough to reduce issues of fatigue, inconvenience, and administrative constraints" (p.455). They propose that the test may be shortened to resolve this, or it may be necessary to increase the time for the test. The content of the MKT measures as administered to the Ghanaian sample facilitates a solution to this problem. The whole test form was comprised of two test scales, the Number and Operations scale, and the Patterns, Functions, and Algebra scale. Future administrations of the test would focus on one scale to ease the burden on the respondents.

Overall, the initial steps for establishing construct equivalence have been completed. The next steps involve changes to the instrument prior the use of the MKT measures in Ghana to facilitate a more valid MKT score.

The interpretation of these findings and the degree to which it informs the fundamental goal of this dissertation study of the applicability and transfer of MKT in Ghana is the purpose of Chapter 5.

# Chapter 5: Discussion

In Chapter 1 of this dissertation, I defined the problem of Ghana's poor student achievement in international comparative studies. Ghanaian scholars have attributed poor student performance to a number of issues including the quality of mathematics teaching at the basic level. UNESCO has suggested that one way to improve student outcomes is a strong focus on teaching quality (UNESCO, 2006).

Improving teacher quality has been interpreted in a variety of ways by a variety of education stakeholders including politicians and researchers. However, there is growing agreement that the ways in which teachers know their content knowledge and the extent to which they can effectively deploy this knowledge in teaching is an important aspect of teacher quality that cannot be ignored.

Different conceptions of what teachers need to know in order to be effective mathematics teachers have been proposed (Blum & Krauss, 2008; Rowland et al., 2005; Schmidt et al., 2008; Senk et al., 2008; Shulman, 1986). Despite the differences in the different theories of teacher knowledge, there is agreement that mathematical content knowledge is important; however, the unique nature of content knowledge is unclear.

The MKT research group has made progress by reframing the problem of teacher knowledge by examining mathematics teaching, rather than the curriculum, or mathematics content, or teacher qualifications (Ball & Bass, 2000; 2003; Ball et al., 2008). Their theory of MKT was developed by a careful study of the practice of mathematics teaching, the identification of the routine tasks of teaching, and an analysis of the mathematical knowledge demands of engaging in these teaching routines. The MKT research group also developed measures that can be used at scale to assess this knowledge and determined that teachers with high levels of MKT per their measures taught in qualitatively better ways and their students learned more mathematics than teachers who scored low on their measures (Hill et al., 2005).

To contribute solutions to the problem of poor student mathematical understandings and achievement in Ghana and other developing countries, this study sought to investigate the extent to which the theory of MKT could be applicable to African countries, using Ghana as a case study. Other studies have shown great promise for the use of MKT measures in other country contexts such as Ireland (Delaney, 2008), Korea (Kwon, 2009), and Indonesia (Ng, 2009). Ghana, as an African country, has different social, economic, and resource profiles that require care in the use of U.S.–developed instruments for assessing teacher knowledge. In addition, the MKT measures were developed from the study of U.S. teaching practice and it is unclear whether the transfer of the MKT measures in Ghana is prudent, given that teaching has been shown to be cultural in nature (Stigler & Hiebert, 1999).

This dissertation contributes to the theory of MKT by examining the extent to which the concept and measures of MKT are applicable in a Ghanaian context.

In addition, Ghana's need for improved teacher quality makes it a good case for such a study.

Specifically, this study adapted the U.S.-developed measures of MKT for use in Ghana and examined in three distinct ways the validity and transfer of the MKT construct and its measures scores obtained by the teachers sampled in Ghana. Using quantitative analysis of the teachers' MKT scores to determine the item difficulties, this study determined how the measures were taken up by the Ghanaian sample and what could be learned from the general performance of a convenience sample of 60 teachers. This study also included two validation studies that examined more specifically (1) the relationship between teachers' MKT scores and their reasoning about the items, and (2) the relationship between teachers' MKT scores and the mathematical quality of their instruction.

This chapter is organized in three sections. The first section discusses findings from the administration of the MKT tests in Ghana regarding item difficulties for selected items reported in Chapter 4. The second section focuses on a discussion of the relationship between the MKT scores and teachers' reasoning about the items. The third section discusses the relationship between the MKT scores and the mathematical quality of instruction.

## Examining item difficulties of MKT measures in Ghana

In spite of the invalidity of some survey scores and the literacy demand of the test on the Ghanaian sample, there is evidence to suggest that the MKT measures were more difficult for the Ghanaian sample than for the U.S. teachers (Tables 4.1 and 4.2). The first possible explanation is that the Ghanaian teachers sampled possess less MKT than their U.S. counterparts. Evidence from the qualitative analyses of teacher interviews indicated that some teachers were unwilling to respond to some questions in the interview. This unwillingness could be attributed to them finding the particular item(s) difficult.

Low teacher knowledge could be explained by a variety of factors. Akyeampong (2003) reported on the recruitment of qualified candidates into teaching in Ghana and indicated that the selection standards for teacher candidates were low. Other studies of teacher education in Kenya have attributed the low performance of students to the criteria for the recruitment of primary teachers (Owino et al., 2007). These studies suggest that teachers with low content knowledge in general, not just of mathematics, are not likely to provide good learning environments and opportunities for students to succeed.

Another possible explanation for the low performance of the Ghanaian sample is the format of the items. The MKT items were administered mostly in a multiple-choice format, and some researchers suggest that the structure of the test might present culturally specific challenges. Greenfield (1997) suggests that different cultures emphasize different item formats and it is possible that the Ghanaian teachers' sampled were not completely familiar with a test in this format. Schoenfeld (2007) also argues that multiple-choice items are more difficult than open-ended items. Anecdotal evidence from the Ghanaian teachers sampled indicated that their conception of a "mathematics survey" and the MKT survey as presented was very different. The teachers in general, expected general questions about their mathematical pedagogy and not about their knowledge of mathematics. It is possible that the unexpected “test” as well the multiple-choice format may have influenced their scores. It is unclear the extent to which teachers’ scores were a result of their consistent and thoughtful engagement with the items. As such, it is not clear that all teachers sampled productively engaged with the tasks. From the teachers sampled however, the interview data showed some discrepancies in teachers’ knowledge that the survey did not account for. For instance, there was evidence that some teachers may have guessed some of their responses to the survey and so did not choose to respond to those items in the interview.

Another possible explanation for the low performance is the literacy demand of the MKT items. Although the test was in English, the official language of Ghana, most teachers reported having to read each item multiple times to make sense of what the item was asking. As such, it is unclear whether the teachers found the items difficult because they did not understand the task, or because they did not know the mathematics associated with the task. For instance, the sample released item in Figure 5.1 assesses a teachers’ knowledge of the properties of 0. This item is embedded in a teaching task of evaluating a textbook task. Teachers are required to understand the task of teaching involved, relate it to the task (textbook analysis), and then answer the mathematical question it is asking. By embedding the mathematics inside the task of teaching, it is possible that the literacy demands of reading and understanding the context could serve as a barrier that could inhibit teachers’ productive engagement with the task, thus yielding MKT scores that may not be representative of teachers’ knowledge. In the interview, teachers were required to read the question out loud before selecting their response. Simultaneously reading and hearing the task (in the interview) is different from only reading the task (as was done in the survey). It is not clear how these different modes of engagement with the items influenced teachers' ability to understand the context and the demands of the items.

It is clear that teachers such as Kofi and Fiifi whose responses to some survey items were scored as invalid were able to correctly respond to some of the items. It is also possible that the presence of the interviewer to probe their thinking may have influenced their interview scores. The interviewer used prompts to probe their thinking not only about their selected response but about the other multiple-choice options. As such, some teachers changed their initial response after talking about the other options available.

1. Ms. Dominguez was working with a new textbook and she noticed that it gave more attention to the number 0 than her old book. She came across a page that asked students to determine if a few statements about 0 were true or false. Intrigued, she showed them to her sister who is also a teacher, and asked her what she thought.

Which statement(s) should the sisters select as being true? (Mark YES, NO, or I'M NOT SURE for each item below.)

| | Yes | No | I'm not sure |
|---|---|---|---|
| a) 0 is an even number. | 1 | 2 | 3 |
| b) 0 is not really a number. It is a placeholder in writing big numbers. | 1 | 2 | 3 |
| c) The number 8 can be written as 008. | 1 | 2 | 3 |

Figure 5.1. Sample LMT released item downloaded from http://sitemaker.umich.edu/lmt/files/LMT_sample_items.pdf on May 19, 2011

Another possible explanation for the teachers' performance is that Ghanaian teachers in the sample hold their mathematical knowledge in qualitatively different ways. Teachers in Ghana might possess professional knowledge that is unique to their context. This knowledge might inform their knowledge of possible student explanations for particular topics, common student errors, and the use of particular manipulatives such as counters. Such knowledge might account for the selection of some incorrect responses that are the result of teacher's mathematical knowledge tied to their knowledge of their context and may not be attributed to their lack of mathematical knowledge per se. There are no studies in Ghana that examine the nature of teacher knowledge and so this explanation is yet to be explored.

An attempt to explain teachers' performance on the measures required examining the school mathematics syllabus and textbooks to determine the degree of familiarity the Ghanaian sample might have with the mathematical content being assessed.

A closer look at the items that were easy for the Ghanaian sample revealed interesting features. Of the 56 items in the form, the sampled Ghanaian teachers found 4 items to be relatively easy. The first task asked teachers whether the statement "any number times 0 is 0" is true or false. Information from the primary mathematics syllabus developed by the Ghana Education Service in Ghana showed that the multiplicative property of zero was a stated learning objective for grade 3: The pupil will be able to "state that the product of any number and 0 (zero) is zero" (Ministry of Education, 2001, p. 48).

The second task involved analyzing word problems on addition of natural numbers to determine which of the problems students might find easiest. Again, the syllabus was clear that students were expected to be able to “write addition sentences from word problems and solve them” (Ministry of Education, 2001, p. 20), “solve word problems involving addition or subtraction” (Ministry of Education, 2001, p. 41, 61, 80). These concepts may have been very familiar with the teachers due to its emphasis in the syllabus in classes 2 through 4. Items that were very difficult for the Ghanaian sample were also examined. The first item involved identifying the explanation for why the “cross multiplication” rule works. The mathematics textbooks in Ghana showed that the *process* of cross multiplication was discussed in grade 6 however there was no explanation for *why* the process worked. As such, teachers’ responses to that question were process-oriented and not focused on a conceptual understanding of *why* the process works.

The second difficult task required teachers to look at a number of word problems and determine if any of them could be used to solve a given fraction subtraction statement. The content expectations in the Ghanaian syllabus and primary grade 6 textbook were illuminating. The syllabus showed that students were expected to be able to “add and subtract fractions with different denominators” and “solve and pose word problems involving addition and subtraction of fractions with different denominators” (Ministry of Education, 2001, p. 105). The word problems in the textbook were also very similar. One textbook’s word problems were of the form: “Linda had a piece of cake. She gave Kwaku a third and Moraine half.

What fraction of the cake is left?" (Ghana Publishing Corporation & Blankson-Anaman, 2004, p.19). Other questions in the textbook were somewhat varied but were all focused on the same whole. For instance some of the questions were:

> 2. Mr Abban planted out a field of maize. When harvest came, he had lost 1/10 of the crop to monkeys. A further ¼ of his crop was spoilt by fire. What fraction of the crop did he harvest?
>
> 5. Some of the classes in a school contribute to buy a small present for their head teacher.
> Class 4 gives 1/5 of the total.
> Class 5 gives 2/5 of the total.
> Class 6 gives 3/8 of the total.
> The school secretary gives the rest.
> What fraction of the whole amount does the school secretary give?
> (Ashworth & Wilmot, 2007, p. 27)

The above examples show that the textbooks may not provide teachers and students opportunities to engage in different kinds of tasks for which the whole varies. In fact, all the fraction subtraction word problems from the three texts examined were structurally similar and of the forms "whole minus (A+B+C)", or "[(Whole minus A) minus B] minus C" where A, B, and C are fractions. Another common form of finding the missing addend such as A +B+C+ (unknown) equals whole. These aforementioned tasks utilize different fractions with different denominators of the same whole, thus, textbooks may not offer teachers and students opportunities to discriminate among different tasks where the wholes might be different.

With tasks of this nature, I argue that students might not need to completely understand what the whole is in order to solve the task. Students only need to add the fractions in the problem and subtract the whole (in most cases 1) to get a correct answer. Thus the question may not necessarily assess an understanding of fraction subtraction as a concept but as a process to be carried out. Interestingly, these findings were not unique to Ghana (see Delaney 2008).

**Association between MKT scores and reasoning**

Examining teachers' reasoning about the MKT tasks was illuminating. Although teachers in general reasoned as expected for the easy tasks, their reasoning about the difficult tasks were not as expected. For the alternate subtraction algorithm task for instance, Kofi and Ewusi showed some evidence of understanding more than one strategy but were not convinced that it was generalizable. I examined the primary mathematics syllabus to determine the scope of subtraction content in Ghana. The concept of subtraction was covered in some depth—ranging from the concept of subtraction as comparisons, as take away of symbols, money, and capacity, through regrouping/borrowing, through the use of materials such as multi-base blocks, sticks, or seeds, abacus for multi-digit subtraction problems, word problems. In second grade, pupils are required to use the expanded form to subtract 2 numbers and in third grade, students are introduced to subtraction of fractions. By the fifth grade, students are expected to subtract 4 and 5-digit numbers using abacus, color-coded counters, or the place-value chart (Ministry of Education, 2001).

The syllabus shows that there are expectations for the use of different strategies for subtraction but the strategies do not make use of connections between addition and subtraction. The syllabus content might suggest that although subtraction is taught with a variety of models and using different strategies, it is likely that teaching could be heavily weighted on procedures and the subtraction knowledge package (Ma, 1999) for teachers is not well formed. That is, the knowledge of subtraction is not interconnected with other concepts such as addition. These findings also seem to agree with Akyeampong's (2003) study that reports that in general, candidates recruited into teaching do not have high academic scores, Data from the interview shows that Gyidi's responses to the fractions task and Fiifi's argument that some of the strategies were "too difficult for children" or "too confusing" might be examples of their level of knowledge. Another possible explanation for teachers' low performance is their limited opportunities to engage in content-related professional development. The six teachers interviewed indicated that they had not been in a content-focused professional development in the last 3 years. Of the 60 teachers surveyed, only 10% reported having any form of content-focused professional development. In fact, their professional development was mostly geared towards their compliance with policy directives and general pedagogical issues. Researchers have long stressed the importance of targeted professional development of teachers. Ball and Cohen (1999) argue that most professional development sessions are "spent on sessions and workshops that are intellectually superficial, disconnected from deep issues of curriculum and learning, fragmented, and noncumulative" (p. 3-4). If these findings are somewhat representative of the teachers in the capital city, Accra, then there is a critical need of mathematical professional development at least in the Greater Accra region of Ghana.

Kofi and Fiifi were unique. In spite of their low score in the MKT survey, their interview scores tripled and Kofi moved 2 standard deviations closer to the mean, and Fiifi moved one and a half standard deviations to the mean. A closer look at their MKT survey was telling. Both Kofi and Fiifi provided invalid responses for 12 items. These items were questions of the format similar to figure 5.1. Across the data set of 60 teachers, 8 teachers provided invalid responses and circled the letter (a), (b), (c), or (d) as their response, and not the numbers that correspond to the letter. So in question 1a (from figure 5.1), if the teacher believed that 0 is an even number, he would circle the number 1. By circling (a) the scorer could not determine whether the teachers believed that the statement (a) was true or not, and was scored as an invalid response and no score was awarded. Of the 24 content knowledge items that were assessed in the survey, 12 were of that format. This raises questions about the reliability of the item difficulty values for some of the questions or some test-taking skills influenced the results. In the MKT survey, Fiifi and Kofi had invalid responses for all the questions in the format described above, but the cognitive interview showed that they had some knowledge about the content that the survey did not adequately capture. Kofi's MKT scores therefore moved almost two standard deviations more from the survey to the interview. Although Fiifi's results tripled from the survey to the interview, his score remained low and he remained the lowest ranked in the survey, interview, and also in the assessment of his MQI.

It was surprising that the type of school that a teacher taught in did not necessarily imply a particular kind of mathematical knowledge indeed teachers within the same school obtained both the highest and lowest scores in the survey. Although the sample was very small and not representative, this was very surprising because of the vast differences in resources available in the different types of schools, and because both teachers taught in private schools. This could mean that for these teachers MKT might not be tied to their school contexts, that private schools with high tuition may not necessarily have teachers with high MKT and public schools or private school with low tuition may not necessarily have teachers with low MKT. It is important to investigate this assertion with a larger sample in Ghana to determine if the findings from this study would be different.

### Association between MKT scores and MQI

In the past 20 years, research has examined teachers' mathematical knowledge and how that knowledge is utilized in the work that teachers do (Ball, 1990; Ball et al., 2001; Borko et al, 1992). A*ffordance* and *deficit* studies have shown respectively how robust mathematical knowledge positively impacts classroom instruction and negatively influences instruction.

The qualitative analysis of 3 teachers—Fiifi, Gyidi, and Kofi—supported findings from these studies. Gyidi, who had the highest MKT score of the 3 teachers, also had the highest mean for overall richness in mathematics. She consistently used more mathematical language, was developing mathematical generalizations, and used multiple procedures more than Kofi or Fiifi. When the sample was extended to include all six teachers, the findings were corroborated. Ewusi with the highest MKT score, was consistently rated the highest for MQI; he had no errors in his instruction, and had the highest scores for overall student cognitive demand, and overall richness of mathematics. Ewusi and Gyidi were thus similar to cases in the *affordance* literature (Ball, 1991; Lampert, 2001; Leinhardt & Steele, 2005; Lloyd & Wilson, 1998).

Fiifi was also consistent with findings from the *deficit* studies as his low MKT scores were exhibited in his high number of errors and the absence or weak presence of features of high quality of mathematical instruction. He was thus similar to cases such as “Mrs. Oublier” (Cohen, 1990), “Sandra Stein” (Heaton, 1992), and “Zoe” (Hill et al., 2008). Fiifi’s lesson was problematic not only because of his errors, but because of the absence of features that would have enriched children’s mathematics learning opportunities. For instance he did not once refer to the word “axis” in his lesson on graphs and instead used the expression “this side” while pointing to the axis in question; he spent four minutes of class time writing a task on the board while students sat and watched him with nothing to do; he did not elicit student thinking, and he reduced the cognitive demand of tasks by removing elements of the problem that would make the problem richer for students.

A closer look at the nature of identified in the mathematics lessons showed some differences among the teachers. Fiifi and Kofi demonstrated errors in their mathematical language. As explained above, Fiifi used multiple meanings of the word “marks” in ways that could be seen as confusing and Kofi incorrectly used “denomerator” for denominator in his lesson on fractions. Although both errors were coded as imprecise use of language, Fiifi’s error was more problematic because it exhibited a lack of attention to how the use of language could impact students’ opportunities to engage with the content. Kofi on the other hand, was consistent in his use of “denomerator” in ways that may not significantly impact students’ mathematics learning opportunities.

Evidence from the video study seems to indicate that teachers’ MKT knowledge are positively related with the mathematical quality of instruction. Kofi, with the highest interview score (of the three cases) also demonstrated some positive elements of his instruction but also some important errors in his lessons.

**Examining Kofi’s instruction**

One case, Kofi, was not consistent with findings from the *affordance* or *deficit* studies. His MKT scores from the survey indicated that he had low MKT but the interview scores proved otherwise. If the teachers’ interview scores were now taken as their “true” MKT scores, Kofi now had the highest score of the 3 cases. In their mathematics instruction however, Kofi had the highest mean for the overall richness of mathematics only, compared to Gyidi or Kofi. Gyidi had the highest mean for overall working with students and mathematics (0.9 points difference with Kofi), and for overall student cognitive demand (0.13 difference with Kofi). Kofi’s mathematics lessons were fascinating to watch as he simultaneously demonstrated good mathematical practices but committed major errors in the process. His lessons showed fundamental flaws in his mathematical knowledge. For instance, he told his students that the 0 introduced in finding decimal fractions was taken from “nowhere” and he also told students that 0 represents anything that is impossible. In spite of these flaws, Kofi’s teaching was executed with careful attention to student learning. He consistently used a variety of models in his lesson and modeled norms of “doing mathematics” for his students. These included supporting their explanations in their use of mathematical language. Kofi’s competing positive and negative features in his lesson presented a challenge for video coders who strongly felt one way or another about his teaching. As such, the inter-rater reliabilities for his lessons were not optimal (0.556 and 0.667). Further studies are needed to determine if Kofi is another example of “Noelle” as described in Hill et al. (2008) as a “divergent” teacher.

**Summary**

The orienting research question of this dissertation was: *To what extent can empirically derived U.S.-developed measures of MKT be used to study MKT held by a sample of primary teachers in Ghana?* The analysis of the MKT surveys, interview data, and study of the mathematical quality of instruction of the data showed some promise for the transfer of the MKT construct in Ghana. It is expected that if the MKT construct were fully applicable in Ghana, then the MKT measures could be used as a valid tool to assess teachers’ MKT of Number and Operations in Ghana. This would be have been confirmed by the interview study and the study of teachers’ MQI.

The interview study was designed to determine whether teachers’ correct responses to the MKT survey was a result of correct mathematical reasoning and could exclude other issues such as guessing or the use of test taking skills. Similarly, teachers’ incorrect responses could be explained by the lack of knowledge about the content or some misconception about the concept but not due to correct mathematical reasoning. Findings from the interview study however showed disconfirming cases such as Kofi and Fiifi, whose interview and survey scores were substantially different. In addition, issues of cultural congruence such as teachers’ unfamiliarity with the item content and format, the literacy demands of the test, and the length of time it took to respond to the questions are important issues that affect the use of the MKT measures in Ghana (see Figure 5.2).

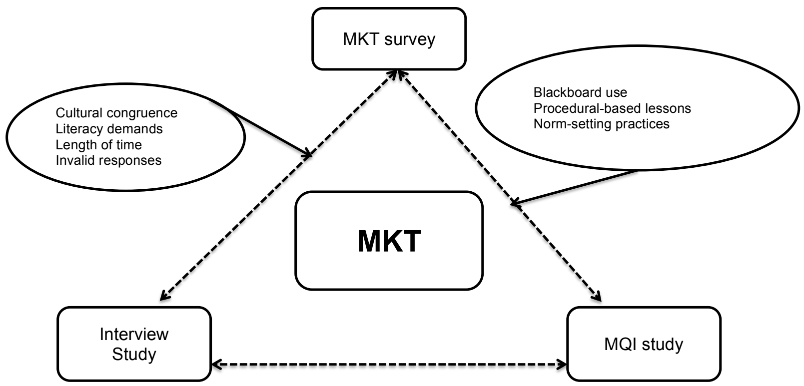



Figure 5.2: Summary of findings for MKT transfer in Ghana

Another source of validation of the teachers' MKT scores was the study of mathematics instruction with a small sample of teachers. This study was designed to assess the strength of the relationship between teachers' MKT scores as assessed and the mathematical quality of instruction (MQI) as determined by the LMT video codes. It was expected that teachers with high MKT scores would teach in qualitatively better ways than teachers with lower MKT scores. The video study provided evidence that the presence of some elements of mathematics instruction in Ghana that were not assessed in the codes. These elements include the use of blackboards, the lack of balanced treatment between conceptually focused and procedurally focused lessons, and the absence of codes to account for explicitly setting norms of mathematical practice. This suggests that there are challenges with the use of the MKT instruments in their current form in Ghana.

In Chapter 6, I present the implications of this study and provide suggestions for further study.

# Chapter 6: Implications and conclusions

This chapter examines the theoretical, methodological, and practical implications of this study and concludes with suggestions for future research.

This dissertation was designed to investigate the extent to which a U.S.-based construct of MKT is applicable in Ghana by examining the use of the measures and the validity of teachers' scores obtained. Although the findings from this study indicate that in general, the Ghanaian teachers sampled found the items to be more difficult than teachers in the U.S., and although the selected sample of teachers were not representative of all the teachers in Ghana, there is evidence to suggest that the theory of MKT is applicable in Ghana, and that teachers with relatively high MKT scores teach in qualitatively better ways and have fewer errors than teachers with relatively low MKT scores.

These findings are essential to the potential use of the MKT measures in other African contexts. Theoretically, this study fills an essential gap that addresses studies of teacher knowledge in Africa and Ghana in particular, a gap highlighted in the survey of literature. There is also a gap in the literature on the application of the theory of MKT in African contexts. This dissertation is a substantial contribution to the theory of teacher knowledge more broadly but has direct implications for the research terrain in Africa and Ghana in particular.

Methodologically, this study contributes to studies of the cross-cultural adaptation of measures, especially into African contexts. In this study, I assumed that the mathematics content in Ghana and the U.S. were relatively similar and undertook adaptations that did not transform the mathematical substance of the tasks and found some important differences especially in the MKT survey and MQI instrument. Other African countries could also utilize this assumption in their adaptation of the MKT or similar measures to determine the extent to which constructs in particular, and theories more broadly can cross cultural lines. Such studies could help determine tasks of teaching and features of mathematical instruction that may be unique to their country contexts and develop ways of assessing such features to improve the use of the instrument in African contexts. Practically, this study has the potential to inform teacher education in Africa and Ghana. Broader professional development to enhance teachers' knowledge of Number and Operations could also designed to strengthen teachers' knowledge in these domains.  Identification of mathematical errors associated with particular content could also provide information about cognitive gaps in teachers' learning, especially in Number and Operations. Knowledge of these gaps could inform the content of teacher preparation and teacher professional development.

## Additional suggestions for further study

In Chapters 4, I made suggestions for further studies that will enrich the findings of this dissertation. The most prominent suggestion that is the ultimate goal of this dissertation is the use of the MKT measures on a larger scale in Ghana and other African countries. This will extend simultaneously the international dimension of the work of the MKT research group and also serve as a tool to diagnose gaps in teacher knowledge that could benefit from professional development. This dissertation relied on a convenience sample of teachers in the southern part of Ghana, showing important challenges to the transfer of MKT to the larger Ghanaian context. Using results from this study to further adapt the MKT instruments, future studies could include clustered samples of teachers from different parts of Ghana to assess their MKT of Number and Operations. Interviews of teachers' reasoning and studies of their mathematical quality of instruction could inform the extent to which the MKT construct and measures are applicable in different parts of Ghana and subsequently, use the instruments at scale.

Another natural extension will be a replication of this study with a different sample of teachers in other regions of Ghana to determine if the particular contexts of the teachers used in this study influenced the results in any particular way. For instance, do teachers in the rural areas hold their mathematical knowledge and teach in qualitatively different ways than teachers in larger cities? To what extent can the MQI instrument be used in schools where there are no classrooms, no chairs, or where four or more grades are taught in the same room? Studies of this nature will enhance the  opportunities to support teachers' mathematics instruction and to a lesser extent, the usability of the MKT instrument as a tool for assessing teacher knowledge more broadly to account for different contextual settings.

Methodologically, this study could be replicated using video coders with similar levels of experience in coding videos of instruction. This will improve the inter-rater reliabilities that were not optimal for Kofi's case. A study of this nature could be compared with these findings to determine how robust the claims about Kofi, the divergent case, were and how lessons such as his could be interpreted. There is also the need for future studies to examine the extent to which language might play in role in teachers' MKT in general and MQI, in particular. This issue is particularly pertinent in non-English speaking countries where adaptation and translation of the MKT measures might be further compounded by issues of translation, interpretation, and assessment. Such studies could examine the use of the MKT instrument in non-English settings such as Benin or Senegal where the MKT instrument would require translation into French before their use. Finally, studies of validation of MKT measures in non-U.S. countries could conduct similar investigations of the mathematical quality of instruction to determine the extent to which teachers MKT scores from their adapted measures are related to the quality of their instruction.

## Conclusions

I would like to first state that the teachers who participated in this study willingly opened their classrooms to me to investigate how this U.S.-developed theory could be adapted for use in Ghana. These data represent a slice of their teaching of a particular topic, to particular students, at a particular point in time. The results of this study should not be interpreted as a verdict on their quality of teaching. The MKT scores, interview data, and videos serve as texts for the study of teaching and the ways in which teaching in Ghana in particular, and Africa more broadly could be improved. This close study of these teachers provides a context for understanding the problem of teacher knowledge and how teacher knowledge could be improved to enhance students' mathematics learning opportunities.

This dissertation is an attempt to understand how examining teachers' mathematical knowledge, in this case MKT, can ultimately improve Ghanaian students' learning of mathematics. It is important to state that attending to the problem of teacher knowledge only addresses part of the larger problem of opportunities to learn mathematics. Without discounting the significance of the contribution this study makes to improve mathematics education outcomes for children in Ghana, and Africa more broadly, it is important to emphasize that other substantial problems exist such as availability of basic teaching and learning resources such as classrooms, desks, chairs, and textbooks. Solving the problem of teacher knowledge is one facet of the problem that is inextricably linked to the other problems facing mathematics education and education in general in Ghana and possibly other African countries.

# Appendices

## Appendix A: Letter of consent to teachers

February, 2008

Dear Teacher,

My name is Yaa Cole and I am a graduate student at the University of Michigan. I am writing to ask for your help with a mathematics survey that investigates the mathematical knowledge that matters for primary school teaching and how teachers develop this sort of mathematical knowledge. The research project, funded in part by the University of Michigan is developing a questionnaire that focuses on mathematical problems that arise in the course of teaching children. The questionnaire will eventually be used to help evaluate and enhance professional development programmes that are meant to improve teachers' ability to solve such problems.

There are two parts to the questionnaire. In the first, I ask you to respond to questions about common mathematics problems in primary school classrooms – for instance examining unusual solutions methods, evaluating students' mathematical statements and determining how to best represent material or generate examples. The second part asks some general questions about your background and teaching. This data will NOT be used to evaluate your own knowledge of mathematics. Instead, I will analyze responses from all teachers participating in this project to identify the best questions for use in future studies of teacher learning and to inform future pre-service and in-service mathematics preparation of teachers. I hope you will be willing to participate because your responses are important and a valued part of the study.

Your response to the 60 – 90 minute questionnaire will remain strictly confidential. Your name will not be attached to the information you provide. You are under no obligation to complete the questionnaire, or to answer all questions presented in it. If you come to a question you do not wish to answer, simply skip it. There are no risks or direct benefits to taking part in this study. You will be asked to sign a form (below) indicating agreement to participate in the study.

If you agree to participate please contact me in one of the following ways: by phone [redacted] [redacted] (home), e-mail: yaacole@umich.edu; or by post [redacted]

[redacted]. Your participation in this project is sincerely appreciated, especially at this busy time of the year. I understand that your time is valuable and as a token of appreciation all mathematics educators who participate in the study will receive a gift token for 10 New Cedis worth of cell phone units.

Thank you for volunteering to participate in this research. Should you have questions regarding your participation, please contact Yaa Cole (yaacole@umich.edu or at [redacted]). You may also contact my advisor for the project, Professor Hyman Bass of the University of Michigan (hybass@umich.edu ). Should you have questions regarding your rights as a research participant, please contact the Behavioral Sciences Institutional Review Board, 540 East Liberty, Suite 202, Ann Arbor, MI 48104-2210, 734-936-0933, email: irbhsbs@umich.edu.

Yours faithfully,

_________________________

Yaa Cole

***You will be given a copy of this information to keep for your records.***

**Statement of Consent:**

Signature:______________________________________ Date: ___________________

Signature of Investigator:_________________________ Date: ___________________

## Appendix B: U.S.-Ghana item difficulties

| Item number on Ghanaian form | U.S. Difficulties (SE) | Equated Ghanaian difficulties (SE) |
|---|---|---|
| 1a | -2.178 (0.232) | -0.298(0.365) |
| 1b | -2.319 (0.246) | 1.157(0.423) |
| 1c | -1.993 (0.215) | -0.196(0.362) |
| 1d | -2.178 (0.232) | -0.710(0.378) |
| 2 | -0.401 (0.145) | 1.644(0.404) |
| 3 | -0.427 (0.145) | 2.389(0.529) |
| 4 | -1.993 (0.215) | -1.266(0.412) |
| 5 | -1.782 (0.199) | -0.817(0.413) |
| 6 | -0.896 (0.155) | 0.825(0.380) |
| 7 | -0.809 (0.153) | -0.605(0.337) |
| 8 | 0.769 (0.153) | 1.393(0.389) |
| 11 | -1.556 (0.184) | -2.221(0.469) |
| 12 | -0.096 (0.143) | 4.267(0.816) |
| 16 | -0.809 (0.153) | 0.306(0.406) |
| 17 | -0.222 (0.143) | 1.644(0.399) |
| 18a | -1.514 (0.182) | -0.605(0.394) |
| 18b | -3.393 (0.418) | -3.860(0.798) |
| 18c | 0.600 (0.149) | 1.644(0.408) |
| 18d | -0.613 (0.148) | 0.399(0.382) |
| 18e | -0.197 (0.143) | 2.222(0.504) |
| 19 | 0.360 (0.145) | 3.226(0.608) |
| 20a | 1.287 (0.173) | 3.508(0.640) |
| 20b | -0.613 (0.148) | 0.105(0.362) |
| 20c | -1.050 (0.160) | -0.926(0.375) |

## Appendix C: MQI Coding Glossary

# MATHEMATICAL QUALITY OF INSTRUCTION (MQI)



This material is based upon work supported by the National Science Foundation under grant numbers EHR-0233456, DUR-0335411 and DRL-0918383 and work supported by the U.S. Department of Education under cooperative agreement number R305C090023.

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*Click to Jump To Code*

**OVERALL LESSON CODES**

# SEGMENT CODES

## Format of the Segment

Indicate the main format in which students worked.

**Please code both formats if at least one minute of each type of instruction occurs**.

If class splits into two or more groups, please code all applicable formats.

Note: If segment is not connected to mathematics, still code the format of the segment.

| Active Instruction | Small Group/Partner/Individual Work |
|---|---|
| Teacher leads discussion or presentation of mathematical material. May be moments where students work individually to solve a problem, but these are brief interludes (under a minute) before return to whole-group work. Whole group does not necessarily imply whole class; class can be split into halves or other fractional components. Key element is extended teacher presentation of mathematical material or posing of mathematical problems to a group. | Teacher divides students into small groups or pairs for work on mathematical problem or task OR students work individually on mathematical problem or task. Typically, teacher circulates among groups or pairs, checking progress. However, teacher may be working on administrative issues, etc. during this time. |

## Classroom Work is Connected to Mathematics

Code here for whether the focus is on *mathematics content* during half or more of the segment (3.5 minutes or more total).

| **No** | **Yes** |
|---|---|
| Focus for majority of the segment is on non-mathematical topics, or student activities that have no clear connections to | Focus is on mathematical content for majority of the segment. |

| | |
|---|---|
| developing mathematical content (e.g., cutting and pasting).<br><br>Examples:<br>• Gathering or distributing materials, other administrative issues<br>• Disciplinary issues that severely impinge upon instructional time<br>• Students doing an activity (cutting, pasting, coloring) that is not clearly connected to mathematics ("bad reform")<br>• Any combination of the above that sums to 3.5 minutes (or half or more of the duration of the segment) | Examples:<br>• Teacher reviews content from a prior lesson; teacher introduces content<br>• Students practice content<br>• Students work on a warm-up problem while teacher takes attendance |

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Note: "Majority" means 3.5 minutes (or half the duration of the segment) or more, unless otherwise noted

| **Mode of Instruction** |
|---|
| Used to code whether mathematical content is supplied in a teacher-directed manner or whether alternative methods are used to develop content. These modes are not exhaustive; that is, segments can receive a "none" for all in some cases. |
| **Direct Instruction** |
| Teacher is in control of delivery of mathematical content. Mathematical content can be correct or incorrect; major feature is high amount of teacher talk and/or control relative to other activities (e.g., student talking, practice, etc).<br><br>Can include, e.g., teacher going over homework, going over problems, presenting new material, reviewing, launching a task, etc. Can be in small group/partner/individual work |

| time. | | |
|---|---|---|
| **None** | **Some** | **Most/All** |
| No direct instruction occurs. | This mode of instruction occurs only for part of the segment or supplements other work featured in the segment – hence, it is *not* the key feature of the segment. | This mode of instruction occurs for the majority of the segment and/or is the *key* feature of instruction featured in this segment. |

| **Whole-Class Discussion** | | |
|---|---|---|
| "The teacher is in charge of the class, just as in direct instruction. However, the teacher is not primarily engaged in delivering information or quizzing. Rather, he or she has students share their thinking, explain the steps in their reasoning, and build on one another's contributions. …. [This mode of instruction] gives students the chance to engage in sustained reasoning." (Chapin et al., 2003, p. 17) **Key feature is that students comment on mathematics of one another's contributions, not just say, "we did it another way."**<br><br>Also, most of transcript will be student voices.<br><br>Whole-class discussion is applied only to active instruction time. | | |
| **None** | **Some** | **Most/All** |
| No whole-class discussion. | This mode of instruction occurs only for part of the segment or supplements other work featured in the segment – hence, it is *not* the key feature of the segment. | This mode of instruction occurs for the majority of the segment and/or is the *key* feature of instruction featured in this segment. |

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| Working on Applied (Real-World) Problems | | |
|---|---|---|
| Teacher and/or students work on applied (real-world) problems.<br><br>Examples of applied (real-world) problems:<br>• Figuring out among four recipes the proportion of orange juice and water that makes a mixture more orangey<br>• Figuring out which is the best phone-call plan among three plans representing a linear, a proportional, and a stepwise function.<br><br>Non-examples:<br>• Mentioning a past contextualized problem but not working on it<br>• Story problems used to illustrate a situation, but that are not actively worked on during the segment.<br><br>Note: Short word problems from math texts DO count in this code and could result in either a mid or a high code, depending on how they are used in the segment. | | |
| **None** | **Some** | **Most/All** |
| No work on applied (real-world) problems is featured in this segment. | This mode of instruction occurs only for part of the segment or supplements other work featured in the segment – hence, it is *not* the key feature of the segment. | This mode of instruction occurs for the majority of the segment and/or is the *key* feature of instruction featured in this segment. |

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# Richness of the Mathematics

This category attempts to capture the depth of the mathematics offered to students. Rich mathematics is either a) focused on the meaning of facts and procedures OR b) focused on key mathematical practices. Rich mathematics allows students to build a conceptual mathematical base and build connections within and among different components of rich mathematics.

For all, element must be substantially correct and clear to count as "mid" or "high." "Mid" constitutes good use of many of these elements; "high" constitutes extraordinary use.

Note: All codes in this category are quality codes. A teacher can get a "high," even if the aspect of instruction you are coding for occurs for only a portion of the segment.

## Linking or Connections

This code refers to teachers' and students' explicit linking and connections:

- Among different representations of mathematical ideas or procedures (e.g., a linear graph and a table both capturing a linear relationship)
- Among different mathematical ideas (e.g., proportionality and linearity; fractions and ratios, etc)
- Across representations and mathematical ideas/procedures (e.g., discussing how linearity is captured in any of the following: a graph, a table, or a mathematical equation)

Note: If links are made but underlying representation/idea is incorrect, do not count as linking and connections.

| Low | Mid | High |
|---|---|---|
| No linking or connections occur. Also code low when connections are **completely** pro forma, e.g. "Yesterday | Links or connections are present, but do not have the features included in "high," or feature these only | Links and connections are present with sustained, careful work on one or more of the following: |

| | | |
|---|---|---|
| we did adding fractions with like denominators, today we will do subtracting fractions with like denominators." | momentarily (e.g., "You can compare ratios the same way you compare fractions" or "You can see that each step in the computation can be seen in this array model here."). | • Detailed discussion of how two mathematical ideas are related to one another<br>• Explicit linking of a representation to the underlying idea it is meant to represent<br>• Explicit linking between multiple representations, showing how they correspond |

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<table>
<tr><th colspan="3">Explanations</th></tr>
<tr><td colspan="3">Mathematical explanations explain why. This includes giving mathematical meaning to ideas or procedures, meaning of steps, or solution methods, e.g.:<br>• Why a procedure works<br>• Why a solution method makes sense<br>• Why an answer is true<br><br>Do NOT code "how" e.g., descriptions of steps here (first I did x, then I did y) or simply providing definitions unless meaning is also attached.<br><br>Examples: the reason for steps in simplifying fractions (dividing by 2/2, for example, is same as dividing by 1; anything divided by 1 is still itself); why particular steps in a complex problem are justified or work to achieve the solution.<br><br>Note: Do not count incorrect or incomplete explanations as explanations.</td></tr>
<tr><th>Low</th><th>Mid</th><th>High</th></tr>
<tr><td>No mathematical explanations are offered by the teacher or students or the "explanations" provided are simply descriptions of steps of a procedure.</td><td>The explanations offered by the teacher or students meet any of the following criteria:<br>• They meet criteria under high, but they are not detailed<br>OR<br>• They are not generalizable (they pertain only to the specific task/problem under consideration)</td><td>The explanations offered by the teacher or students meets the following criterion:<br>• They move beyond particular problems (i.e., they are general explanations, not for particular problems)<br><br>AND meets one or more of the following criteria:<br>• They give meaning to</td></tr>
</table>

| | | |
|---|---|---|
| | OR<br>• They are definitions that contain elements of "why" but that do not fully explain | steps and procedures<br>• They are detailed |

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| Multiple Procedures or Solution Methods | | |
|---|---|---|
| **Multiple procedures or solution methods**<br>• Multiple solution methods for a single problem (including shortcuts)<br>• Multiple procedures for a given problem type<br>Defined as, e.g.:<br>• Taking different mathematical approaches to solving a problem (e.g., comparing fractions by finding a common denominator AND comparing fractions by finding a common numerator)<br>• The teacher/students may solve a (word) problem using two different strategies.<br><br>If the initial strategy(ies) occurred in a prior interval, code the second (or subsequent) interval at this code (e.g., no need to go back and code the initial interval).<br><br>Note: Here you could also include teacher/student mentions multiple different procedures/solution methods even if only one of them is enacted.<br><br>Note: Do NOT code incorrect procedures or solution methods | | |
| **Low** | **Mid** | **High** |
| No evidence of multiple procedures or solution methods for a given problem type. | Multiple procedures or solution methods present, but do not have the features included in "high," or feature these only momentarily (e.g., "this | Multiple procedures or solution methods occur at some length and with special features:<br>• Explicit comparison of multiple |

| | | |
|---|---|---|
| | method is easier than the other" without explicit discussion of why). | procedures/solution methods for efficiency, appropriateness, ease of use, or other advantages and disadvantages<br>• Explicit discussion of features of a problem that cues the selection of a particular procedure<br>• Explicit links between multiple procedures/solution methods (e.g., how one is like or unlike the other) |

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| Developing Mathematical Generalizations | | |
|---|---|---|
| Teacher and/or students **develop** mathematical generalizations by examining instances or examples, then making a general statement (e.g., drawing parabolas $x^2$, $2x^2$, $4x^2$ then making a generalization about shape as coefficient changes).<br><br>Examples of this activity include:<br>• Generalizations of mathematical facts<br>• Generalization of mathematical procedures<br>• "Building up" a mathematical definition or deriving a mathematical property after considering different examples and non-examples (e.g., defining "polygons" after considering different examples and non-examples of polygons)<br><br>Notes:<br>1. Requires at least two examples (either explicitly worked or referred to) from which generalization emerges.<br>2. Code generalizations for only the clip in which generalization emerges/becomes explicit.<br>3. Do NOT code incorrect generalizations.<br>4. Do NOT code when teachers *state* generalizations without first developing them from examples. | | |
| **Low** | **Mid** | **High** |
| No generalizations are developed in this segment.<br><br>Also code "low" for non-mathematical generalizations (e.g., drawing a picture helps solve a word problem). | Students/teacher *develop* a generalization, but the generalization developed is not complete, clear or detailed. | Teacher and/or students *develop* a generalization. The generalization should contain the *mathematical* essence of the work done with regards to a particular task and should be complete and clear. |

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| Mathematical Language | | |
|---|---|---|
| • Fluent use of technical language<br>• Explicitness about mathematical terminology<br>• Supporting students' use of mathematical terms | | |
| **Low** | **Mid** | **High** |
| Teacher does not demonstrate fluency in mathematical language. Teacher uses non-mathematical terms to describe mathematical ideas and procedures AND/OR teacher talk is characterized by sloppy use of mathematical terms.<br><br>If there is little mathematical language used, code here. | Teacher uses mathematical language as a vehicle for conveying content, but has few or none of the special features listed under "high." This is the default code when teacher is using mathematical language neither sloppily nor outstandingly.<br><br>Also code here when segment has both features of "high" but includes some linguistic sloppiness. | Teacher uses mathematical language *fluently*.<br><br>May include explicitness about terminology, reminding students of meaning, pressing students for accurate use of terms, encouraging student use of mathematical language. Density of mathematical language is high during periods of teacher talk. |

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| Overall Richness of the Mathematics | | |
|---|---|---|
| Depth of the mathematics offered to students.<br><br>Note: This is an overall code for each segment/chapter. It is not an average of the above, but an overall estimate of richness | | |
| **Low** | **Mid** | **High** |
| Components or richness are present but are incorrect<br>OR<br>Elements of rich mathematics are not present or are only minimally present | Elements of rich mathematics are present but are used in a conventional way, without special features listed in high. May be characterized by many "mid" codes above, one well-developed "high" along with substantial procedural focus, etc. | Elements of rich mathematics are present, and:<br>a) there is truly outstanding performance in one or more of the elements (even for a brief portion of the segment)<br>OR<br>b) there is a combination of elements that either saturate the segment with mathematical meaning or foster student proficiency with mathematical practices.<br>Two general ways this can happen:<br>• Focus is on meaning via representations linked to one another or to underlying ideas; explanations that |

| | | |
|---|---|---|
| | | generalize; generalizations that are developed from specific examples<br>• Focus is on explicit comparing of solution methods or procedures (e.g., most efficient) but without necessary focus on meaning during this discussion |

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# Working with Students and Mathematics

This category captures whether teachers can understand and respond to students' mathematically substantive productions (utterances or written work) or mathematical errors. By mathematically substantive productions, we mean questions, claims, explanations, solution methods, ideas, etc. that contain substantial mathematical ideas. By students' mathematical errors, we mean those incorrect student productions that offer opportunities for discussing and addressing pertinent mathematical ideas.

## Remediation of Student Errors and Difficulties

With this code, we mean to mark instances of remediation in which student misconceptions and difficulties with the content are *substantially* addressed.

Note: Can be during active instruction or small group/partner/individual work time.

Note: Remediation must have mathematical content.

| Low | Mid | High |
|---|---|---|
| No conceptual remediation occurs for any of the following reasons:<br>• There are no student misunderstandings or difficulties with the content<br>• Remediation is procedural and brief or otherwise non-substantive<br>• The teacher chooses not to remediate<br>• The teacher remediation | Teacher engages in conceptual remediation *briefly* or *occasionally*.<br><br>Also includes very *extensive* procedural remediation. | Teacher engages in conceptual remediation *systematically* and *at length*.<br><br>• Identifying the source of student errors or misconceptions<br>• Discussing how student errors illustrate broader misunderstanding and then addressing those errors.<br><br>Also code high for any instance, however brief, of |

| | | |
|---|---|---|
| is confusing or off-track | | the following:<br>• Anticipating common student errors and providing instruction that helps avoid error<br>• Parsing student productions to separate correct and incorrect thinking. |

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<table>
<tr><th colspan="3">Responding to Student Mathematical Productions in Instruction</th></tr>
<tr><td colspan="3">Student evokes substantive mathematical thought for the class, and teacher understands and responds to it during instruction in mathematically appropriate ways. Student ideas will tend to have features of student explanation/generalization/why question, or a complex description of a solution method. Teacher:<ul><li>Identifies mathematical insight in specific student questions, comments, or work</li><li>Builds instruction on student ideas or methods</li></ul>Note: By “ideas” we do not mean simply answers to problems or pointed questions where teacher has sought a specific, bounded piece of information. They tend to be student explanation, questioning, and reasoning.</td></tr>
<tr><th>Low</th><th>Mid</th><th>High</th></tr>
<tr><td>Routine instruction with no student productions.<br>OR<br>There are student productions, but no evidence that student ideas are “heard,” taken up or used in instruction. Teacher may provide evidence that he/she does not understand student productions.<br>OR<br>Teacher uses student productions but in a way that muddles or confuses the mathematics of the lesson.</td><td>Student productions are present.<br>AND<br>Teacher may engage in features listed under “high” briefly, but instruction generally proceeds without strong use of student mathematical ideas.<br>OR<br>There is evidence that the teacher understands student thinking but chooses not to use it at that time.</td><td>Student productions are present.<br>AND<br>Teacher “hears” what students are saying, mathematically, and responds appropriately during instruction. Students’ mathematical ideas are woven at length into the development of mathematical ideas during the lesson.<br><br>In particular, teacher may comment on student’s mathematical ideas, elicit further student clarification</td></tr>
</table>

| | | |
|---|---|---|
| | | of ideas, ask other students to comment on ideas, expand on and reinforce student utterances, etc.<br><br>Other markers include:<br>• Indentifying key ideas in student statement ("Mark had an interesting idea…")<br>• Highlighting key features of student questions ("Do you see Mark asked a question about whether this would work in all cases?")<br>• Identifying a student with an idea ("Mark's method") |

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| Overall Working with Students and Mathematics | | |
|---|---|---|
| Note: This is an overall code for each segment/chapter. It is not an average of the above, but an overall estimate of the teachers' interactions with the students around the content. | | |
| **Low** | **Mid** | **High** |
| Few substantive interactions between teacher and students. Errors may occur but teacher addresses briefly/procedurally.<br>OR<br>Substantive student mathematical productions or errors do occur, but teacher does not respond to or use those productions<br>OR<br>Teacher responses to student productions lead the lesson off-track | A combination of strong and weak features (e.g., some high-level remediation but teacher ignores students' contributions; very extended procedural remediation but no student productions). | Strong and significant teacher understanding and use of student ideas and errors around the content as evident by *outstanding* performance in one area or solid performance in both. |

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# Errors and Imprecision

This category is intended to capture teacher errors or imprecision in language and notation, uncorrected student errors, or the lack of clarity/precision in the teacher's presentation of the content.

Do not code errors if these errors are captured and addressed within the segment or chapter (in this case, code as "low").

## Major Mathematical Errors or Serious Mathematical Oversights

- Solving problems incorrectly
- Defining terms incorrectly
- Forgetting a key condition in a definition
- Equating two non-identical mathematical terms, etc.

| Low | Mid | High |
|---|---|---|
| Instruction is *clean* of major errors in spoken or written work OR errors that occur are captured and corrected within the segment. | Teacher makes *major errors* either in spoken or written work or teacher neglects to discuss key aspects of a problem (e.g., forgetting a step, forgetting to finish the problem).<br><br>The errors occur in ***part*** of the segment. | Teacher makes *major errors* either in spoken or written work or teacher neglects to discuss key aspects of a problem (e.g., forgetting a step, forgetting to finish the problem).<br><br>The errors occur in ***most*** of the segment. |

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| Imprecision in Language or Notation (Mathematical Symbols) | | |
|---|---|---|
| • Errors in notation (mathematical symbols)<br>• Errors in mathematical language<br>• Errors in general language | | |
| **Low** | **Mid** | **High** |
| Instruction is *clean* of errors in mathematical language, general language, and notation. Any errors made and quickly corrected should also be coded here. | Teacher makes a *small number* of momentary errors in notation, mathematical or general language. | Instruction is characterized of linguistic and notational *sloppiness across the segment* and/or by *major* notational and linguistic errors in even a small number of mathematical terms. |

**Clarification:**

- ***Notation*** includes conventional mathematical symbols, such as +, -, =, or symbols for fractions and decimals, square roots, angle notation, functions, probabilities, exponents. Errors in notation might include inaccurate use of the equals sign, parentheses, or division symbol. By "conventional notation," we do not mean use of numerals or mathematical terms.
- ***Mathematical language*** includes technical mathematical terms, such as "angle," "equation," "perimeter," and "capacity." If a teacher uses these terms incorrectly, code as an error. When the focus is on a particular term or definition, also code errors in spelling or grammar.
- Teachers often use "***general language***" to convey mathematical concepts (i.e., explaining mathematical ideas or procedures in non-technical terms). General language also includes analogies, metaphors, and stories. Appropriate use of terms includes care in distinguishing everyday meanings different from their mathematical meanings. If teacher is unclear in his/her general talk about mathematical ideas, terms, concepts, procedures, code as an error.

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| Lack of Clarity | | |
|---|---|---|
| • Teacher utterances cannot be understood, e.g.:<br>  • Mathematical point is muddled, confusing or distorted<br>  • Language or major errors make it difficult to discern the point<br>  • Teacher neglects to clearly solve the problem or explain content<br>• Teachers' launch of a task/activity lacks clarity (the "launch" is the teacher's effort to get the mathematical tasks/activities into play) | | |
| **Low** | **Mid** | **High** |
| Teacher's presentation of the mathematical content and/or launching of tasks is *clear* and *unambiguous.* | Teacher's presentation of the content and/or launching of tasks is *not clear* for *portions* of the segment. | Teacher's presentation of the mathematical content is *unclear, vague,* or *incomplete* for most of the segment.<br>OR<br>Teacher's work is *muddled* or *confusing* and *severely distorts* the mathematical essence of the content.<br><br>Also, teacher conveys mathematical tasks or problems *incompletel*y or in a *confusing* manner. |

| Overall Errors and Imprecision | | |
|---|---|---|
| Note: This is an overall code for each segment/chapter. It is not an average of the above, but an overall estimate of the errors and imprecision in instruction. | | |
| **Low** | **Mid** | **High** |
| No errors occur. Do not use this code if "mid" or "high" is marked in any category | Brief error or errors generally not serious enough to indicate teacher | Either multiple small errors, **consistent lack of clarity** or one large error to suggest |

| | | |
|---|---|---|
| above. | may lack mathematical knowledge. | that teacher may lack key mathematical knowledge. |

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| Student Participation in Meaning-Making and Reasoning |
|---|
| This code attempts to capture evidence of students' involvement in tasks that ask them to "do" mathematics and the extent to which students participate in and contribute to meaning-making and reasoning. During active instructional segments, this mainly occurs through student mathematical statements: reasoning, explanations, question-asking. During small group/partner/individual work times, this mainly occurs through work on a non-routine task |
| **Students Provide Explanations** |
| Students provide a mathematical explanation for an idea, procedure, or solution.<br><br>Examples:<br>• Students explain why a procedure works<br>• Students explain the procedure they used to solve a particular problem by attending to the meaning of the steps involved in this procedure rather than simply listing those steps<br>• Students explain what an answer means<br>• Students explain why a solution method is suitable or better than another method<br>• Students explain an answer based on an estimate or other number-sense reasoning<br><br>Notes:<br>• Explanations could be initiated by the teacher or self-initiated; they could also be co-constructed with the teacher or constructed individually<br>• Explanations do not have to be complete or correct |

| Low | Mid | High |
|---|---|---|

| | | |
|---|---|---|
| No student explanations are featured in the segment. | The explanations offered are:<br>• Brief<br>• Pertain to a specific problem/task<br>• Do not generalize to key mathematical ideas of the content under consideration<br><br>Examples:<br>• A student explains that ¾ is larger than 3/5 because the denominator in the first fraction is smaller than the denominator in the second fraction.<br>• A student explains that a cube net has 24 right angles because it has 6 square faces. | The explanations offered generalize past specific problems to address key mathematical ideas of the content under consideration.<br><br>Examples:<br>• A student explains that ¾ is larger than 3/5 because the denominator in the first fraction is smaller than the denominator in the second fraction and fractions with smaller denominators correspond to smaller pieces.<br>• A student explains that a cube net has 24 right angles because it has 6 square faces and each square has 4 right angles. |

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| Student Mathematical Questioning and Reasoning |
|---|
| Students engage in mathematical questioning or reasoning, including:<br>• Students provide counter-claims in response to a proposed mathematical statement or idea (whether from another student, the teacher, or a text)<br>• Students ask mathematically motivated questions requesting explanations (e.g., “Why does this rule work?” “What happens if all the numbers are negative?”)<br>• Students provide examples of a phenomena<br>• Students make conjectures about the mathematics discussed in the lesson (e.g., “I’ve been trying to make a triangle with two obtuse angles and I don’t think you can.”)<br>• Students form conclusions based on patterns they identify or on other form of evidence (e.g., “Because the sum of the angles of any triangle is 180 degrees, a triangle cannot have two obtuse angles”)<br>• Students engage in reasoning about a hypothetical or general case (“Because the sum of the angles of any triangle is 180 degrees, a triangle should have at least two acute angle”)<br><br>Note: Students’ productions do not have to be complete or correct. |

| Low | Mid | High |
|---|---|---|
| The segment does *not* feature any of the student behaviors related to this code. | The segment features *one or two* of the student behaviors related to this code. | The segment features *three or more* of the student behaviors related to this code. |

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<table>
<tr><th colspan="3">Enacted Task Cognitive Activation</th></tr>
<tr><td colspan="3">This code refers to the <u>enactment</u> of the task, regardless of the initial demand of the curriculum/textbook task or how the teacher sets up the task for students.<br><br>Notes:<br>• Student confusion does not necessarily suggest that students are engaging with the content at a high cognitive level.<br>• Working on review tasks or on ideas discussed in previous lessons does not necessarily mean that students use lower order thinking skills.<br>• This code should not be confounded with the difficulty of the task or whether it is appropriate for a certain grade-level.</td></tr>
<tr><th>Low</th><th>Mid</th><th>High</th></tr>
<tr><td>Students engage with the content at a low cognitive level.<br><br>Examples of cognitively undemanding activities include:<br>• Recalling and applying well-established procedures<br>• Recalling or reproducing known facts, rules, or formulas<br>• Listening to a teacher presentation with limited student input<br>• Going over homework with little additional</td><td>Students engage with content at mixed level of cognitive activation. May also include:<br><br>• Tasks with variable enactment (high and then low during segment)<br><br>• Direct instruction with substantive student input at certain points<br>• Tasks with middling cognitive demand</td><td>Students engage with content at high level of cognitive activation.<br><br>Examples of cognitively activating activities include when students:<br><br>• Determine the meaning of mathematical concepts, processes, or relationships<br>• Draw connections among different representations or concepts.<br>• Make and test conjectures</td></tr>
</table>

| | | |
|---|---|---|
| student work (e.g., reporting numerical answers)<br>• Unsystematic exploration (i.e., students do not make *systematic and sustained progress in developing mathematical strategies or understanding*) | | • Look for patterns<br>• Examine constraints<br>• Explain and justify |

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| Overall Student Participation in Meaning-Making and Reasoning | | |
|---|---|---|
| This code attempts to capture evidence of students' involvement in "doing" mathematics and the extent to which students participate in and contribute to meaning-making and reasoning.<br>• During ***active instruction segments***, this mainly occurs through student mathematical statements: reasoning, explanations, question-asking.<br>• During ***small group/partner/individual work time***, this mainly occurs through work on a non-routine task.<br><br>Note: This is an overall code for each segment/chapter. It is not an average of the above, but an overall estimate of the student participation in meaning-making and reasoning. | | |
| **Low** | **Mid** | **High** |
| There are only a few or no examples of student participation in meaning making and reasoning. Tasks are largely procedural in nature. Also code here unproductive explorations in which *the majority* of the students are off-track, mathematically. | Students engage with content at *mixed level*. Students may provide substantive explanations or ask mathematically motivated questions. May also include tasks with variable enactment (high and then low during segment). | Students contribute substantially or engage productively in activities that can lead to meaning-making and reasoning. Such contributions are a major feature of the segment, with many student contributions, or extended work on a challenging task. |

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# OVERALL LESSON CODES

| Overall MQI and MKT | | |
|---|---|---|
| These codes are intended to capture the overall mathematical quality of instruction (MQI) and the teacher's mathematical knowledge for teaching (MKT) as suggested by the teacher's work during the lesson. | | |
| **Overall MQI** | | |
| **Low** | **Mid** | **High** |
| Instruction is characterized by combinations of the following:<br>• Systematic teacher errors (mathematical errors, in notation, in language) or lack of clarity around the mathematics<br>• Major teacher conceptual error in a significant portion of the lesson<br>• Unproductive teacher-student interactions around the content (e.g., teacher cannot effectively remediate)<br>• Lack of directionality/unsystematic exploration | Instruction does not have characteristics of "low" and is mostly error-free, but lacks the mathematical richness, appropriate use and discussion of procedures, and the sharp mathematical focus detailed under "high." Examples:<br>• Mostly error-free procedural instruction, perhaps with occasional but not consistent elements of richness<br>• Mainly pro forma interactions with students (inquiry, response, evaluation-type discussion) | Instruction is error-free (save for MINOR slips) and characterized by combinations of the following:<br>• Mathematical richness in terms of explanations, links and connections,<br>• Focus on mathematical practices (developed generalizations, mathematical efficiency) that is sustained and detailed.<br>• Instruction has a clear and sharp mathematical focus and directionality that allows students to develop the important mathematical ideas under consideration |

| | | |
|---|---|---|
| • Lack of connection of classroom activities to mathematical content | | • Instruction is also characterized by at least some productive teacher-student interactions around the content (either working with student ideas/errors OR student participation in mathematical meaning-making). |

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<table>
<tr><th colspan="3">Lesson-Based Guess at MKT</th></tr>
<tr><td colspan="3">How do you think the teacher would score on our MKT assessment?<br><br>Note: Differs from overall MQI in that raters may use more judgment to estimate generally what teacher MKT could be. For instance, lesson captured may be mostly practice, but there may be some evidence of very strong teacher MKT.</td></tr>
<tr><th>Low</th><th>Mid</th><th>High</th></tr>
<tr><td>MKT is estimated as low, as suggested by combinations of the following:<br>• Inappropriate use of representations, mathematical notation and language<br>• Mathematical errors<br>• Inappropriate explanations, descriptions of procedures, or discussion of student ideas and contributions<br>• Teacher puzzlement and incorrect or distorted presentation of the content</td><td>MKT is estimated as mid as suggested by the following:<br>• Instruction is mostly error-free but there is no evidence teacher has ability to provide rich instruction, understands lesson material deeply, or has capacity to work with students' thinking or errors.</td><td>MKT is estimated as high, as suggested by combinations of the following:<br>• Ability to provide accurate and rich instruction<br>• Strong use of mathematical language<br>• Ability to follow and build on mathematical ideas<br>• Ability to unpack the content and make it accessible to students<br>• Ability to identify and remediate student errors and misconceptions</td></tr>
</table>

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| Lesson Level Binary Codes |
| --- |
| All codes should be scored as **Present** or **Not Present** |
| **Orienting** |
| The teacher states the lesson objectives at the beginning of the lesson in a way that is clear and understandable to the students. The teacher may not present objectives before reviewing homework from prior assignment. This can be coded present as long as the objectives are presented prior to beginning the primary instruction of the lesson. |
| **Summarization** |
| At the end of the lesson, the teacher (or student designate) reflects on what the class has accomplished and/or describes how the day's objectives connect to future goals (e.g., teacher provides a summary that restates or paraphrases the objective of the lesson, the work achieved during the lesson and/or previews related concepts that prepare students for the next phase of the learning process) |
| **Checking Broadly for Understanding** |
| Evidence suggests that teacher has systematically gathered information about student understanding in either this or a past lesson. Includes surveying a large percentage of student about answer (thumbs up/thumbs down; whiteboard), several comments on student non-verbal cues, such as puzzled looks; commenting on common student errors; reporting results of self-assessment back to students. |
| **Differentiated Instruction** |
| The teacher provides differentiated instruction, such that different groups of students engage in tasks focused on different content (e.g., one small group works on geometry problems while another works on solving algebraic equations), or similar tasks of differing level of difficulty (e.g., all students find the area of shapes – some shapes are more difficult to work with than others). |

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# Appendix D: Blank reconciling coding sheet

Chapter level codes

| | Chapter 1 | | | Chapter 2 | | | Chapter 3 | | | Chapter 4 | | | Chapter 5 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | Reconciled | 1 | 2 | Reconciled | 1 | 2 | Reconciled | 1 | 2 | Reconciled | 1 | 2 | Reconciled |
| **Code on First Watch** | | | | | | | | | | | | | | | |
| **Format of the segment** | | | | | | | | | | | | | | | |
| Active instruction (0) or small group/partner/individual work (1), both (2) | | | | | | | | | | | | | | | |
| **Mode of Instruction (1 = None, 2 = Some, 3 = Most/All)** | | | | | | | | | | | | | | | |
| Direct instruction | | | | | | | | | | | | | | | |
| Whole-class discussion | | | | | | | | | | | | | | | |
| Working on applied (real-world) problems | | | | | | | | | | | | | | | |
| **Time on Task (0 = No, 1 = Yes)** | | | | | | | | | | | | | | | |
| Code here for whether the focus is on mathematics content during half or more of the segment (3.5 minutes or more total) | | | | | | | | | | | | | | | |
| **Second Watch: "Quality" Codes** | | | | | | | | | | | | | | | |
| **Richness of the Mathematics (1 = Low, 2 = Mid, 3 = High)** | | | | | | | | | | | | | | | |
| Representations | | | | | | | | | | | | | | | |
| Multiple procedures or solution methods | | | | | | | | | | | | | | | |
| Explanations | | | | | | | | | | | | | | | |
| Developing mathematical generalizations | | | | | | | | | | | | | | | |
| Mathematical language | | | | | | | | | | | | | | | |
| Overall richness of the mathematics | | | | | | | | | | | | | | | |
| **Working with students and mathematics (1 = Low, 2 = Mid, 3 = High)** | | | | | | | | | | | | | | | |
| Teacher questioning | | | | | | | | | | | | | | | |
| Remediating student difficulties | | | | | | | | | | | | | | | |
| Uses student mathematical ideas | | | | | | | | | | | | | | | |

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| in instruction | | | | | | | | | | | | | | | |
| Overall working with students and mathematics | | | | | | | | | | | | | | | |
| **"Amount" codes** | | | | | | | | | | | | | | | |
| **Errors and Imprecision (low = error-free, high = error-full, 1 = Low, 2 = Mid, 3 = High)** | | | | | | | | | | | | | | | |
| Major mathematical errors or serious mathematical oversights | | | | | | | | | | | | | | | |
| Imprecision in notation (mathematical symbols) or mathematical language | | | | | | | | | | | | | | | |
| Lack of clarity | | | | | | | | | | | | | | | |
| Overall errors and imprecision | | | | | | | | | | | | | | | |
| **Student cognitive demand (1 = Low, 2 = Mid, 3 = High)** | | | | | | | | | | | | | | | |
| Students provide explanations | | | | | | | | | | | | | | | |
| Student mathematical questioning and reasoning | | | | | | | | | | | | | | | |
| Enacted task cognitive demand | | | | | | | | | | | | | | | |
| Overall student cognitive demand | | | | | | | | | | | | | | | |

**Lesson level codes**

| | insert video name | | |
|---|---|---|---|
| | coder 1 | coder 2 | Reconciled |
| **Equity (1 = Low, 2 = Mid, 3 = High)** | | | |
| Equitable engagement with students | | | |
| Communication of high expectations and potential | | | |
| **Lesson level binary codes (0 = not present, 1 = present)** | | | |
| Orienting | | | |
| Summarization | | | |
| Teacher Pacing | | | |
| Checking broadly for understanding | | | |
| Students engage in sustained individual academic work at least once during the lesson. | | | |
| Students engage in group activities or work with peers at least once during the lesson. | | | |
| The teacher provides differentiated instruction, such that different groups of students engage in tasks focused on different content (e.g., one small group works on geometry problems while another works on solving algebraic equations), or similar tasks of differing level of difficulty (e.g., all students find the area of shapes – some shapes are more difficult to work with than others). | | | |
| **Overall MQI and MKT (1 = Low, 2 = Mid, 3 = High)** | | | |
| Overall MQI | | | |
| Lesson-based guess at teacher MKT | | | |
| **Overall dimensions- whole lesson (1 = Low, 2 = Mid, 3 = High)** | | | |
| Overall richness of the mathematics | | | |
| Overall working with students and mathematics | | | |
| Overall errors and imprecision | | | |
| Overall student cognitive demand | | | |

## Appendix E: Filled reconciled coding sheet showing chapter level codes*

| | Video name | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Chapter 1 | | | Chapter 2 | | | Chapter 3 | | | Chapter 4 | | | Chapter 5 | | | Chapter 6 | | |
| | Rater 1 | Rater 2 | Reconcile | Rater 1 | Rater 2 | Reconcile | Rater 1 | Rater 2 | Reconcile | Rater 1 | Rater 2 | Reconcile | Rater 1 | Rater 2 | Reconcile | Rater 1 | Rater 2 | Reconcile |
| **Code on First Watch** | | | | | | | | | | | | | | | | | | |
| **Format of the segment** | | | | | | | | | | | | | | | | | | |
| Active instruction (0) or small group/partner/individual work (1), both (2) | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| **Mode of Instruction (1 = None, 2 = Some, 3 = Most/All)** | | | | | | | | | | | | | | | | | | |
| Direct instruction | 3 | 3 | 3 | 3 | 3 | 3 | 2 | 2 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| Whole-class discussion | 1 | 3 | 1 | 1 | 3 | 1 | 1 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| Working on applied (real-world) problems | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| **Time on Task (0 = No, 1 = Yes)** | | | | | | | | | | | | | | | | | | |
| Code here for whether the focus is on mathematics content during half or more of the segment (3.5 minutes or more total) | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| **Second Watch: "Quailty" Codes** | | | | | | | | | | | | | | | | | | |
| **Richness of the Mathematics (1 = Low, 2 = Mid, 3 = High)** | | | | | | | | | | | | | | | | | | |
| Representations | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| Multiple procedures or solution methods | 2 | 2 | 2 | 3 | 1 | 3 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| Explanations | 3 | 1 | 2 | 2 | 1 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 2 | 1 | 2 | 1 | 1 | 1 |
| Developing mathematical generalizations | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| Mathematical language | 2 | 3 | 3 | 2 | 3 | 3 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 3 | 2 | 1 | 1 | 1 |
| Overall richness of the mathematics | 2 | 2 | 2 | 2 | 2 | 3 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| **Working with students and mathematics (1 = Low, 2 = Mid, 3 = High)** | | | | | | | | | | | | | | | | | | |
| Teacher questioning | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| Remediating student difficulties | 2 | 1 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 2 | 2 | 1 | 1 | 1 | 1 | 1 | 1 |
| Uses student mathematical ideas in instruction | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| Overall working with students and mathematics | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| **"Amount" codes** | | | | | | | | | | | | | | | | | | |
| **Errors and Imprecision (low = error-free, high = error-full, 1 = Low, 2 = Mid, 3 = High)** | | | | | | | | | | | | | | | | | | |
| Major mathematical errors or serious mathematical oversights | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| Imprecision in notation (mathematical symbols) or mathematical language | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| Lack of clarity | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| Overall errors and imprecision | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| **Student cognitive demand (1 = Low, 2 = Mid, 3 = High)** | | | | | | | | | | | | | | | | | | |
| Students provide explanations | 1 | 2 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 2 | 2 | 1 | 1 | 1 |
| Student mathematical questioning and reasoning | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| Enacted task cognitive demand | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| Overall student cognitive demand | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

***** highlighted cells are cells for which there was no agreement between raters/coders and for which reconciliation was needed.

Lesson ID: YCVSG_2

Memo author: Yaa Cole

Topic of lesson: Properties of basic operations

Narrative description of lesson:

1. Teacher reviewed previous day's lesson with students and asked students to do four exercises in their jotters.
2. Two students were brought to the front of the classroom and teacher demonstrated commutative property of addition by giving each student first 5, then 6 counters for the whole class to find the sum.
3. Students do several exercises in whole group with teacher asking students to use their counters to solve the problems. Teacher also asks for explanations for why the different answers are correct.
4. After about 22 minutes of whole class discussion, teacher asks students to solve some problems in their mathematics classwork books.

Mathematical issues or problems that emerged during the lesson: (Note any major areas that were problematic with 1-2 examples (e.g., language….); also note any particular instances that stood out.)

- Teacher agreed with a student who said "you can't take a bigger number away from a smaller number".

Mathematical strong points of lesson: (Note any major mathematical strengths of the lesson, e.g., linking representations to one another, a nice explanation or justification. If there is special attention to equity via explicitness, please do include this here.)

- Good use of counters to represent operations even when students knew the answer to the questions
- Connecting the different tasks with other problems and with the representations
- Good use of questions to ask for student reasoning
- Good selection of examples for whole group discussion

Other themes or thoughts:
(Other things that we might want to record, as memos to ourselves about future topics to write about, what this video might be good to demonstrate, etc. etc.)

- Teacher worked well with students in whole group format and although the discussion was not student-directed, it was very engaging and students were involved in the lesson

Special notes:

- The lesson began with 52 students in the class and 20 students arrived late. By the end of the lesson, there were 72 students in the class.
- Good use of the board. Students were doing individual work when teacher wrote problems on the board so students were engaged
- The teacher used the board to summarize the main points of the lesson.

## Appendix F: MQI codes for six cases showing aggregate scores for two lessons

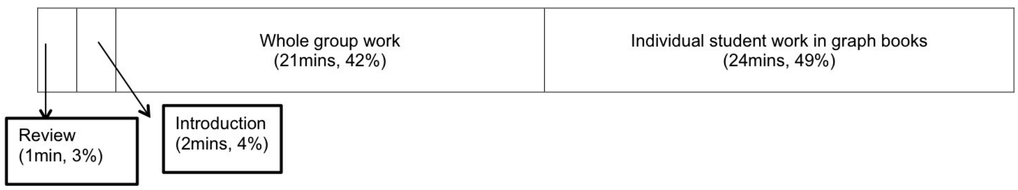
| Teacher | Ewusi | Gyidi | Afua | Joojo | Kofi | Fiifi |
|---|---|---|---|---|---|---|
| | | | | | | |
| IRT score (survey) | 0.01 | -0.89 | -0.89 | -1.11 | -2.74 | -3.01 |
| IRT score (interview) | 0.78 | -0.67 | -0.67 | -1.55 | -0.67 | -1.55 |
| Active instruction (0) or small group/partner/individual work (1), both (2) | 0.67 | 1.31 | 0.78 | 0.00 | 0.43 | 0.69 |
| Direct instruction | 2.17 | 2.26 | 1.56 | 3.00 | 2.61 | 2.50 |
| Whole-class discussion | 2.17 | 1.22 | 1.00 | 1.00 | 1.09 | 1.00 |
| Working on applied (real-world) problems | 3.00 | 1.00 | 1.00 | 1.00 | 1.65 | 2.17 |
| Code here for whether the focus is on mathematics content during half or more of the segment (3.5 minutes or more total) | 0.83 | 1.00 | 1.00 | 1.00 | 0.95 | 1.00 |
| Representations | 1.00 | 1.97 | 1.00 | 1.89 | 2.35 | 1.52 |
| Multiple procedures or solution methods | 1.00 | 1.29 | 1.33 | 1.33 | 1.15 | 1.00 |
| Explanations | 2.67 | 1.59 | 1.33 | 1.78 | 1.89 | 1.06 |
| Developing mathematical generalizations | 2.17 | 1.61 | 1.00 | 1.67 | 1.19 | 1.00 |
| Mathematical language | 2.50 | 2.20 | 1.56 | 2.22 | 2.05 | 1.21 |
| Overall richness of the mathematics | 2.33 | 1.81 | 1.33 | 2.11 | 1.90 | 1.06 |
| Teacher questioning | 2.33 | 2.10 | 1.00 | 1.56 | 1.19 | 1.06 |
| Remediating student difficulties | 1.33 | 1.22 | 1.22 | 1.22 | 1.85 | 1.17 |
| Uses student mathematical ideas in instruction | 2.17 | 1.53 | 1.00 | 1.44 | 1.85 | 1.06 |
| Overall working with students and mathematics | 2.50 | 1.76 | 1.00 | 1.44 | 1.85 | 1.06 |
| Major mathematical errors or serious mathematical oversights | 1.00 | 1.06 | 1.00 | 1.44 | 1.19 | 1.25 |
| Imprecision in notation (mathematical symbols) or mathematical language | 1.17 | 1.06 | 1.00 | 1.56 | 1.20 | 1.42 |
| Lack of clarity | 1.00 | 1.06 | 1.00 | 1.22 | 1.14 | 1.15 |
| Overall errors and imprecision | 1.00 | 1.06 | 1.00 | 1.56 | 1.19 | 1.63 |
| Students provide explanations | 1.83 | 1.41 | 1.22 | 1.33 | 1.32 | 1.00 |
| Student mathematical questioning and reasoning | 1.50 | 1.11 | 1.00 | 1.00 | 1.29 | 1.00 |
| Enacted task cognitive demand | 1.67 | 1.41 | 1.00 | 1.56 | 1.27 | 1.06 |
| Overall student cognitive demand | 1.33 | 1.40 | 1.00 | 1.33 | 1.28 | 1.00 |

# Appendix G: Average timeline of lessons for 6 cases

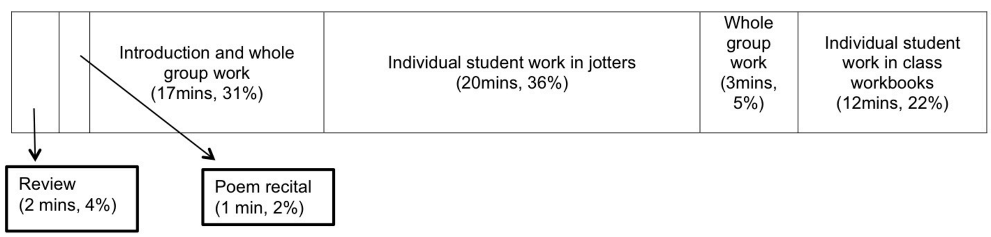
**Average timeline of lessons for each case**

Below is the average timeline for each case. The length of the bar represents the whole lesson (100%) and the segments represent the proportion of the lesson devoted to each segment. In parenthesis is the number of minutes for that segment and the percentage of the whole lesson devoted to the named event. For instance, about 2 minutes and 4% of instructional time in Fiifi's class was spent on the review. The table below provides the average percentage of time spent on each segment and the number of minutes corresponding to that portion of the lesson.

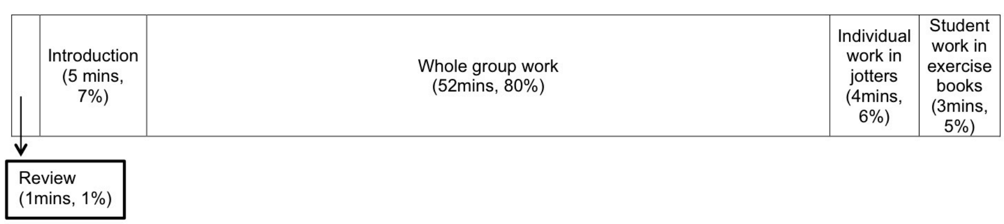
**Fiifi**




**Gyidi**




**Kofi**



**Ewusi**

| | | | | | |
|---|---|---|---|---|---|
| Review (1min, 2%) | Acting out lesson (5mins, 10%) | Introduction (22mins, 44%) | Whole group work (9mins, 18%) | Individual student work in jotters (8mins, 16%) | Whole group work (5mins, 10%) |

**Joojo**

| | |
|---|---|
| Review (5mins, 9%) | Whole group work (53mins, 91%) |

**Afua**

| | | | | | |
|---|---|---|---|---|---|
| Review (6mins, 8%) | Introduction (8mins, 12%) | Whole group work (7mins, 10%) | Individual student work in jotters (11mins, 16%) | Whole group work (7mins, 10%) | Individual student work in exercise books (29mins, 44%) |

| Average Percentage and Average Number of Minutes per segment of lesson for six cases | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Fiifi | # of mins | Gyidi | # of mins | Kofi | # of mins | Ewusi | # of mins | Joojo | # of mins | Afua | # of mins |
| Review | 3% | 1 | 4% | 2 | 1% | 1 | 2% | 1 | 9% | 5 | 8% | 6 |
| Poem recital | 0% | 0 | 2% | 1 | 0% | 0 | 0% | 0 | 0% | 0 | 0% | 0 |
| Acting out lesson | 0% | 0 | 0% | 0 | 0% | 0 | 10% | 5 | 0% | 0 | 0% | 0 |
| Introduction | 4% | 2 | 31% | 17 | 7% | 5 | 44% | 22 | 0% | 0 | 12% | 8 |
| Whole group work 1 | 42% | 21 | | | 80% | 52 | 18% | 9 | 91% | 53 | 10% | 7 |
| Individual student work in jotters | 0% | 0 | 36% | 20 | 6% | 4 | 16% | 8 | 0% | 0 | 16% | 11 |
| Ind student work in graph books | 49% | 24 | 0% | 0 | 0% | 0 | 0% | 0 | 0% | 0 | 0% | 0 |
| Whole group work 2 | 3% | 1 | 5% | 3 | 0% | 0 | 10% | 5 | 0% | 0 | 10% | 7 |
| Individual student work in exercise books | 0% | 0 | 22% | 12 | 5% | 3 | 0% | 0 | 0% | 0 | 44% | 29 |
| Summary/ conclusion | 0% | 0 | 0% | 0 | 0% | 0 | 0% | 0 | 0% | 0 | 0% | 0 |
| Total | 100% | 50 | 100% | 56 | 100% | 65 | 100% | 50 | 100% | 58.00 | 100% | 67 |

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