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ON THE EXCITING FORCES ACTING UPON A SHIP IN REGULAR WAVES IN SHALLOW WATER

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Exciting forces acting on a moving ship in a shallow water can be divided into two parts: The main exciting forces which are calculated according to the Krylov assumption (that is, the ship does not affect water pressure distribution) and the second which takes into account the hydrodynamic forces (damping and inertia) and is called the deffractional component of the exciting forces. Comparison of exciting forces defined according to the Krylov assumption, with the experimental data in an infinite depth, shows that matching takes place when the length of the wave is much greater than the length of the ship.

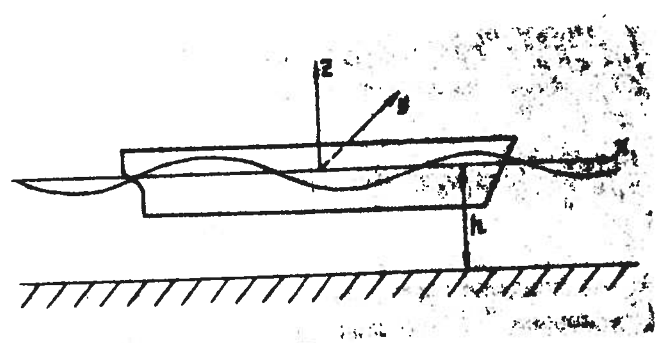
Attempts in taking into account the hydrodynamic forces for actual ships involves many mathematical difficulties even for infinite depth, Reference [1]. In the case of shallow water the wave problem is not solved completely. For approximate methods see References [2] and [3].

In this paper the same assumptions as in References [2] and [3] are used:

1) The hypothesis of plane flow, and

2) The characteristic speed is the speed of the wave profile variation in the vertical direction.

Good agreement of experimental and calculated exciting forces for a wide range of relative frequencies, which was found in Reference [3] allow us to assume that the hypothesis accepted are not too inaccurate.



The following coordinate system has been used (see Figure 1). Let us assume that the non-moving ship is acted upon by a train of progressive waves with a potential

$$\Phi_{w}(x,y,z,t)=\frac{ag}{\sigma}\frac{ch\,k_0(z+h)}{ch\,k_0 h}\sin(k_0x+\sigma t), \tag{1}$$

where a = wave amplitude

$\sigma = \frac{2\pi}{\tau}$ = wave frequency

k_o = the wave number which is defined as the real and positive root of $R_o \tanh k_o h = \frac{\sigma^2}{g}$

Considering the linearity of the problem, we can define the total potential as

$$\Phi_{\sigma}=\Phi_{w}+\Phi, \tag{2}$$

where ϕ = velocity potential as a result of the ship influence when subjected to the action of waves with potential ϕ_w.

The vertical component of the exciting forces is

$$F=\rho\iint_S \frac{\partial\Phi_\sigma}{\partial t}\cos(n,z)\,dS \tag{3}$$

Substituting (2) into (3)

$$F=\rho\iint_S \frac{\partial\Phi_w}{\partial t}\cos(n,z)\,dS+\rho\iint\frac{\partial\Phi}{\partial t}\cos(n,z)\,dS; \tag{4}$$

The first integral corresponds to the main part of the exciting forces and the second integral to the second part (i.e., hydrodynamic).

Looking at the main part of the exciting forces,

$$F_w = \rho a g \iint_S \frac{ch\,\kappa_0 (z+h)}{ch\,\kappa_0 h} \cos(\kappa_0 x + \sigma t) \cos(n, z)\, dS$$

or

$$F_w = \rho a g \int_{-\frac{L}{2}}^{+\frac{L}{2}} \int_\ell \frac{ch\,\kappa_0 (z+h)}{ch\,\kappa_0 h} \cos(\kappa_0 x + \sigma t) \cos(n, z)\, d\ell\, dx,$$

where ℓ = frame section contour.

Using Gauss identity for the inside integral of the above we get

$$F_w = \rho a g \kappa_0 \int_{-\frac{L}{2}}^{+\frac{L}{2}} \int_\ell \frac{sh\,\kappa_0 (z+h)}{ch\,\kappa_0 h} \cos(\kappa_0 x + \sigma t)\, d\omega\, dx - \rho a \int_{-\frac{L}{2}}^{+\frac{L}{2}} \int_{-y_0}^{+y_0} \cos(\kappa_0 x + \sigma t)\, dy\, dx =$$

$$= \rho a g \kappa_0 \int_{-\frac{L}{2}}^{+\frac{L}{2}} \int_\omega \frac{sh\,\kappa_0 (z+h)}{ch\,\kappa_0 h} \cos(\kappa_0 x + \sigma t)\, d\omega\, dx - 2\rho a \int_{-\frac{L}{2}}^{+\frac{L}{2}} y_0 \cos(\kappa_0 x + \sigma t)\, dx, \qquad (5)$$

where w - is the area of the ith frame up to the z waterline.

y_o - is the width of the ith frame up to the design waterline.

Looking at the integral

$$J = \kappa_0 \int \frac{sh\,\kappa_0 (z+h)}{ch\,\kappa_0 h}\, d\omega,$$

and substituting $dw = 2y_z\, d_z$

where y_z is the width of the ith frame up to the z waterline.

We get

$$J = 2\kappa_0 \int_{-T}^{0} \frac{sh\,\kappa_0 (z+h)}{ch\,\kappa_0 h}\, y_z\, dz.$$

Integrating by parts and substituting into (5)

$$F_w = -2\rho a g \int_{-\frac{L}{2}}^{+\frac{L}{2}} \int_{-T}^{0} \frac{ch\,\kappa_0 (z+h)}{ch\,\kappa_0 h}\, y'_z \cos(\kappa_0 x + \sigma t)\, dz\, dx . \tag{6}$$

Introducing a correction coefficient $H_T(x)$ which can be defined as

$$\varkappa_T(x) = -\frac{1}{y_0} \int_{-T}^{0} \frac{ch\,\kappa_0 (z+h)}{ch\,\kappa_0 h}\, y'_z\, dz ,$$

(6) can be written as

$$F_w = 2\rho a g \int_{-\frac{L}{2}}^{+\frac{L}{2}} \varkappa_T(x) \cos(\kappa_0 x + \sigma t)\, dx . \tag{7}$$

Because of the approximation and the closeness of this value of $H_T(x)$ to unity for all frames we can (instead of varying this value), take an average along the ship length.

It is evident that

$$2\varkappa \int_{-\frac{L}{2}}^{+\frac{L}{2}} y_0\, dx = 2\int_{-\frac{L}{2}}^{+\frac{L}{2}} \varkappa_T(x)\, y_0\, dx = -2\int_{-\frac{L}{2}}^{+\frac{L}{2}} \int_{-T}^{0} \frac{ch\,\kappa_0 (z+h)}{ch\,\kappa_0 h}\, y'_z\, dz\, dx = \int_{-T}^{0} \frac{ch\,\kappa_0 (z+h)}{ch\,\kappa_0 h} \int_{-\frac{L}{2}}^{+\frac{L}{2}} 2 y'_z\, dx\, dz . \tag{8}$$

The integrals in (8) are equal to

$$2\int_{-\frac{L}{2}}^{+\frac{L}{2}} y_0\, dx = S ; \qquad 2\int_{-\frac{L}{2}}^{+\frac{L}{2}} y'_z\, dx = \frac{d}{dz} ; \qquad 2\int_{-\frac{L}{2}}^{+\frac{L}{2}} y_z\, dx = S'_z ,$$

where S - is the area of the designed waterline

S_z - is the area of an intermediate waterline whose distance from the designed waterline is z.

From this we can find

$$\varkappa = -\frac{1}{S}\int_{-T}^{0} \frac{ch\,\kappa_0 (z+h)}{ch\,\kappa_0 h} S_z' \, dz, \tag{9}$$

With h → ∞, H approaches the well known correction coefficient for finite draft, used often in ship motions calculations, Reference [4].

$$\varkappa = -\frac{1}{S}\int_{0}^{T} e^{-\kappa z} S_z' \, dz.$$

Assuming a parabolic distribution of the waterplane area

$$S_z = S\left[1 - \left(\frac{z}{T}\right)^n\right],$$

where

$$n = \frac{\chi}{1-\chi}; \qquad \frac{dS}{dz} = -\frac{S}{T}\left(\frac{z}{T}\right)^{n-1}.$$

we get the integral (9) in the form

$$\varkappa = \frac{n}{T}\int_{0}^{T}\left(\frac{z}{T}\right)^{n-1} \frac{ch\,\kappa_0(-z+h)}{ch\,\kappa_0 h} \, dz.$$

Introducing the dimensionless coefficient $\bar{z} = \frac{z}{T}$ and substituting $\kappa_0 = \frac{2\pi}{\lambda}$

$$\varkappa \int_{0}^{1} n(\bar{z})^{n-1} \frac{ch\,A}{ch\,B} \, d\bar{z},$$

where

$$A = 2\pi\frac{T}{\lambda}\left(-\bar{z} + \frac{h}{T}\right) \qquad B = 2\pi\frac{T}{\lambda}\cdot\frac{h}{T}.$$

For given values of χ and $\frac{h}{T}$ we can obtain H for the whole practical range of $\frac{T}{\lambda}$. An example for y = 0.6 and $\frac{h}{T}$ = 1.5; 3 and 5 is given in Figure 2.

When $\frac{h}{T}$ = 5 the curve for H is almost identical to the curve which corresponds to $\frac{h}{T} = \infty$ (see Reference [4]).

The principle part of the exciting force according to Krilov assumption may be defined as follows

$$F_w = 2\varkappa\rho a g \int_{-\frac{L}{2}}^{+\frac{L}{2}} y_0 \cos(\kappa_0 x + \sigma t)\,dx .$$

Introducing the dimensionless parameters

$$\bar{x} = \frac{2}{L}x ; \quad \bar{\kappa}_0 = \frac{L}{2}\kappa_0 = \frac{L}{\lambda}\pi ; \quad \bar{a} = \frac{2}{L}a ; \quad \bar{y} = \frac{2}{B}y .$$

we get

$$F_w = F_{w1}\cos\sigma t + F_{w2}\sin\sigma t ,$$

where

$$F_{w1} = \frac{\gamma B L^2}{4}\varkappa\bar{a}\int_{-1}^{+1}\bar{y}_0\cos\bar{\kappa}_0\bar{x}\,d\bar{x} ; \qquad F_{w2} = -\frac{\gamma B L^2}{4}\varkappa\bar{a}\int_{-1}^{+1}\bar{y}_0\sin\bar{\kappa}_0\bar{x}\,d\bar{x} .$$

To estimate the hydrodynamic part of the exciting force let us assume that the flow, which is characterized by the potential, p, is two dimensional due to its long length and can be described by $\frac{\partial\Phi_w}{\partial z}$. We can also assume that the velocity $\frac{\partial\Phi_w}{\partial z}$ in section i can be replaced by a constant vertical velocity.

$$v_z^i = \left(\frac{\partial\Phi_w^i}{\partial z}\right)_{z=-\frac{T}{2}} .$$

Then for a given transverse section of the ship hull the potential can be represented by

$$\Phi^{i}=\left(\frac{\partial \Phi_{w}}{\partial z}\right)_{z=-\frac{T}{2}} \varphi^{i}(x,y,z),$$

where φ^{i} is the amplitude of the velocity potential for the cylinder with the hull section form oscillating in the vertical direction, with unit value of the speed amplitude.

The hydrodynamic part of the exciting forces taking into account (10) can be written as

$$F_{r}=\rho\iint_{S}\frac{\partial \Phi}{\partial t}\cos(n,z)\,dS=\rho\iint_{S}\left(\frac{\partial^{2}\Phi_{w}}{\partial t\,\partial z}\right)_{z=-\frac{T}{2}}\varphi\cos(n,z)\,dS.$$

Substituting $\lambda^{i}=-\rho\int_{\ell^{i}}\varphi^{i}\cos(n,z)\,d\ell,$ (12)

where ℓ frame section contour

$$F_{r}=-\int_{-\frac{L}{2}}^{+\frac{L}{2}}\left(\frac{\partial^{2}\Phi_{w}}{\partial t\,\partial t}\right)_{z=-\frac{T}{2}}\lambda\,dx. \tag{13}$$

the expression (12) represents the added masses of the above mentioned cylinder. In the case of high frequency λ^{i} to the first approximation be taken as half of the added mass for round cylinder. The frame section contour is taken into account by coefficient ζ_{2}^{i}, which can be found by using V. A. Sobolov's or Prohaska's, Reference [5], graphs as a function of β and $\frac{B}{T}$.

The influence of shallowness for a given ratio $\frac{h}{T}$ may be taken into account by the correction coefficient ζ_{h} (Figure 3), which was calculated for the rectangular frame contour with $\frac{B}{T}=2$. The added mass for this case is defined by

$$\lambda^{i}=0,5\,\zeta_{2}^{i}\,\zeta_{h}\,\pi\rho y_{0}^{2}. \tag{14}$$

For some frame section contours Prohaska's graphs, Reference [5], can be used, where the contour and shallowness are simultaneously taken into account (Figure 4).

The added mass for unit ship length is

$$\lambda^{i} = 0,5\, \zeta^{i}_{2h}\, \pi \rho y_0^2 . \qquad (15)$$

For inland waterway ships with frame contours closely approximating rectangular forms, Figure 5 is used. The added mass is calculated, using formula

$$\lambda^{i} = 0,5\, K^{i} \rho y_0^2 \qquad (16)$$

Numerical analysis has shown that for $\frac{h}{T} > 2$ the difference between the added mass values obtained by the different methods is less than 5%.

The necessity of taking into account the free surface makes the solution more difficult because the corresponding wave problem in the flow (with the resulting complicated velocity distribution in the vertical direction) is practically unsolvable. Taking into account all previous assumptions we can use the graphs of Ursell (Reference [6]) for the wave problem in finite depth (see Figure 6).

The correction coefficient which corrects for the influence of the free surface, in calculating the added mass, can be given as

$$\zeta^{i}_{K} = \frac{1 + K^{i}_{4}}{2} , \qquad (17)$$

where K^{i}_{4} is obtained from Figure 6 for given values of

$$\zeta_0 = \frac{\sigma^2 y_0}{g} ; \quad \frac{h}{T}$$

For intermediate values of $\frac{h}{T}$ which are not included in the graphs, the expression (17) is rewritten as

$$\zeta_K^i = \frac{1 + K^i_{4\,h=\infty} \cdot \varepsilon^i_h}{2} \tag{18}$$

using K_4^i from Figure 6 when $h = \infty$ and

$$\varepsilon_h^i = \frac{K_4^i \quad \text{when } \frac{h}{T} \neq \infty}{K_4^i \quad \text{when } \frac{h}{T} = \infty}$$

obtained from Figure 7 for a given value of ζ_0 and $\frac{h}{T}$

The influence of the frame contour on K_4^i can be estimated using the graphs by Tasai (Reference [7]). Then ζ_K^i becomes

$$\zeta_K^i = \frac{1 + K^i_{42} \varepsilon^i_h}{2} \tag{19}$$

The added mass, taking into account the free surface correction, is obtained using the same formulas (14), (15), and (16) multiplied by ζ_K^i. The influence of three dimensional flow on the value of the added mass is found using the coefficient R_1 (Figure 8) as a function of $\frac{L}{B}$ (Reference [5]).

In lieu of (14) and (17) formula (13), for the hydrodynamic part of the exciting forces, is

$$F_r = -\frac{\pi}{4} R_1 \rho \int_{-\frac{L}{2}}^{+\frac{L}{2}} \left(\frac{\partial^2 \Phi_w}{\partial t\, \partial t} \right)_{z=-\frac{T}{2}} \zeta_2 \zeta_h (1 + K_4) y_0^2 \, dx =$$

$$= -\frac{\pi}{4} R_1 \rho a \sigma^2 \, \frac{sh\, K_0 \left(-\frac{T}{2} + h \right)}{sh\, K_0 h} \int_{-\frac{L}{2}}^{+\frac{L}{2}} \zeta_2 \zeta_h (1 + K_4) \cos(K_0 x + \sigma t).$$

Non-dimensionalizing, as was done for (11), we get

$$F_r = F_{r1} \cos \sigma t + F_{r2} \sin \sigma t, \tag{20}$$

$$F_{r1} = -\frac{\pi\gamma B L^2 R_1 \bar{a} \zeta_B}{32} \frac{sh\, C}{sh\, D}\int_{-1}^{+1} \zeta_2 \zeta_h (1+K_4)\bar{y}_0^2 \cos \bar{K}_0 \bar{x}\, d\bar{x};$$

$$F_{r2} = \frac{\pi\gamma B L^2 R_1 \bar{a} \zeta_B}{32} \frac{sh\, C}{sh\, D}\int_{-1}^{+1} \zeta_2 \zeta_h (1+K_4)\bar{y}_0^2 \sin \bar{K}_0 \bar{x}\, d\bar{x}.$$

The total exciting force according to (4) is defined by the sum

$$F = (F_{w1}+F_{r1})\cos \sigma t + (F_{w2}+F_{r2})\sin \sigma t = \sqrt{(F_{w1}+F_{r1})^2 + (F_{w2}+F_{r2})^2}\cos(\sigma t - \alpha), \tag{21}$$

where $\alpha = \text{arctg}\,\frac{(F_{w2}+F_{r2})}{(F_{w1}+F_{r1})}$ is the phase difference between the exciting force and the waves. The moment of the exciting forces about the OY axis is defined by

$$M_y = -\iint_S (p-p_0)x \cos(n,z)\,dS + \iint_S (p-p_0) z \cos(n,x)\,dS.$$

The second integral is small compared to the first and is neglected. Then using the Lagrange integral,

$$M_y = \rho \iint_S \frac{\partial \Phi_\sigma}{\partial t}\, x \cos(n,z)\, dS \tag{22}$$

This expression for the exciting moment is different from equation (3) for the exciting forces due to the additional x component in the integrand. For this reason, the exciting moment is described by an expression identical to (21)

$$M = (M_{w1} + M_{r1})\cos\sigma t + (M_{w2} + M_{r2})\sin\sigma t =$$

$$= \sqrt{(M_{w1} + M_{r1})^2 + (M_{w2} + M_{r2})^2}\cos(\sigma t - \beta) \tag{23}$$

where

$$M_{w1} = \frac{\gamma B L^3}{8} \varkappa \bar{a} \int_{-1}^{+1} \bar{x} \bar{y}_0 \cos \bar{k}_0 \bar{x}\, d\bar{x};$$

$$M_{w2} = -\frac{\gamma B L^3}{8} \varkappa \bar{a} \int_{-1}^{+1} \bar{x} \bar{y}_0 \sin \bar{k}_0 \bar{x}\, d\bar{x};$$

$$M_{r1} = -\frac{\pi \gamma B L^3 R_2 \bar{a} \zeta_B}{64} \frac{sh\,C}{sh\,D} \int_{-1}^{+1} \zeta_2 \zeta_h (1 + K_4) \bar{x} \bar{y}_0^2 \cos \bar{k}_0 \bar{x}\, d\bar{x};$$

$$M_{r2} = \frac{\pi \gamma B L^3 R_2 \bar{a} \zeta_B}{64} \frac{sh\,C}{sh\,D} \int_{-1}^{+1} \zeta_2 \zeta_h (1 + K_4) \bar{x} y_0^2 \sin \bar{k}_0 \bar{x}\, d\bar{x};$$

R_2 - the three dimensional flow correction (Figure 8)

$$\beta = \operatorname{arctg} \frac{(M_{w2} + M_{r2})}{(M_{w1} + M_{r1})};$$

where β is the phase difference between the exciting forces and the waves.

With the above equations a numerical example was calculated for a Series 60 ship with a prismatic coefficient 0.60 for $\frac{h}{T}$ = 4 and 6 (see Figures 9 and 10). The results are given in terms of a non-dimensional exciting force $\bar{F}$ and moment $\bar{M}$

$$\bar{F} = \frac{\sqrt{(F_{w1} + F_{r1})^2 + (F_{w2} + F_{r2})}}{\gamma a S}; \tag{24}$$

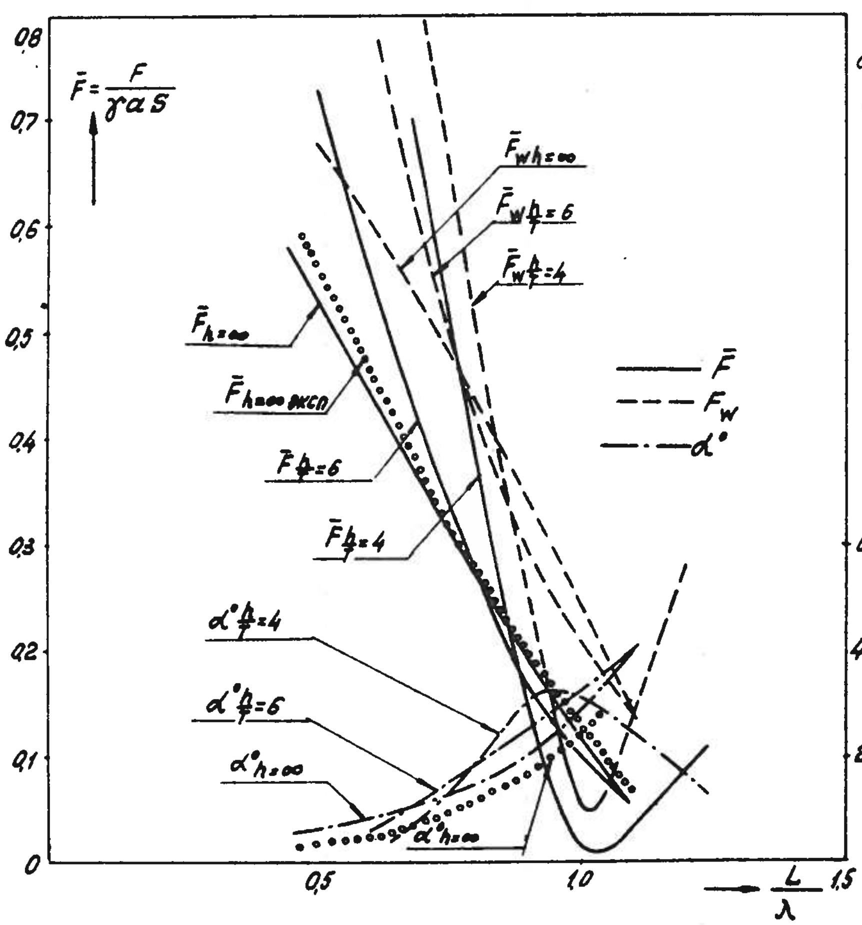
$$\bar{M} = \frac{\sqrt{(M_{w1} + M_{r1})^2 + (M_{w2} + M_{r2})}}{\gamma J_y \alpha_w}; \tag{25}$$

where S = waterplane area of the designed waterline

J_Y = moment of inertia of the same waterplane

$\alpha_\omega = \frac{2\pi a}{\lambda}$ - wave slope

In the same figures, are given the results obtained by Reference [3] and experimental data for infinite depth (Reference [8]). The qualitative influence of shallowness is expressed by the increase of the exciting force for long waves, a decrease in the range of $\frac{L}{\lambda} \approx 1$ and an increase for short waves. For shallow water the role of the hydrodynamic forces reaches a value of 60¢ of the total force for the practical range of $\frac{L}{\lambda} = 1$.



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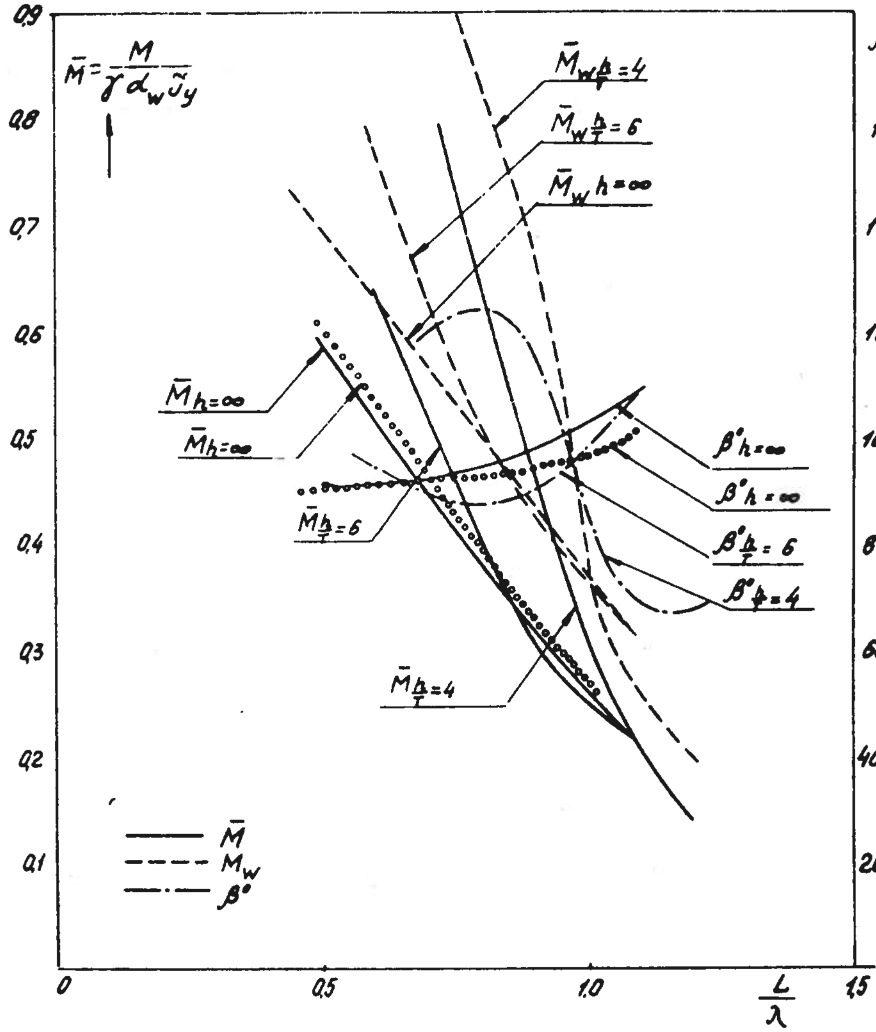


Fig. 10

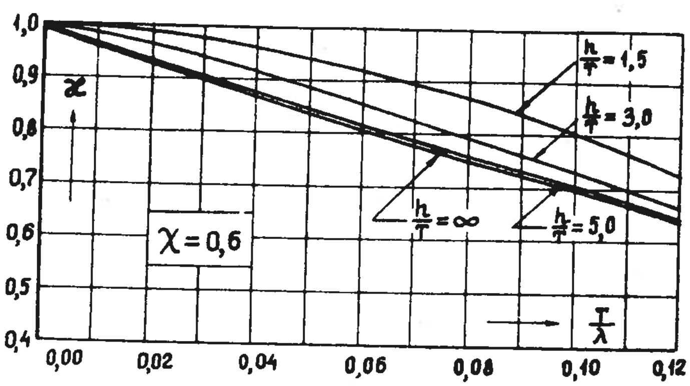


Fig 2

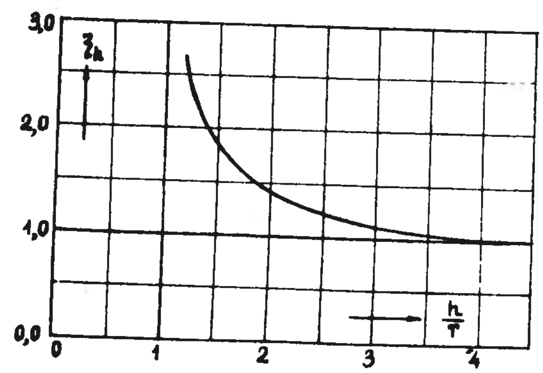
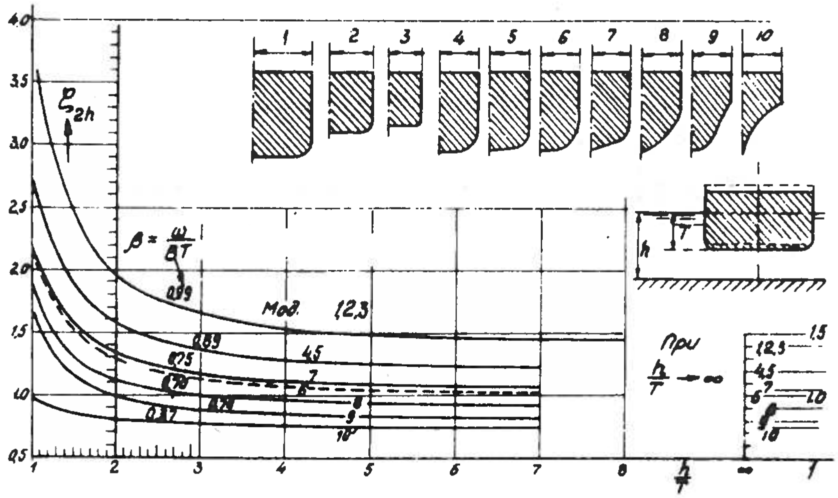


Fig. 3



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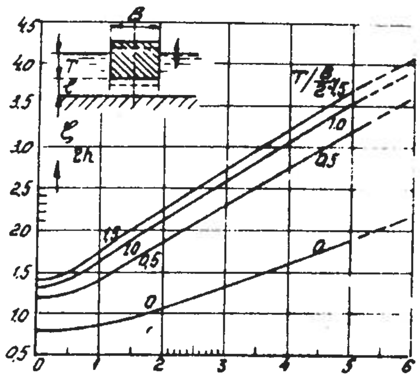

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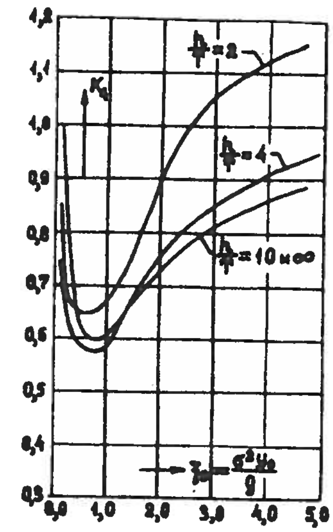

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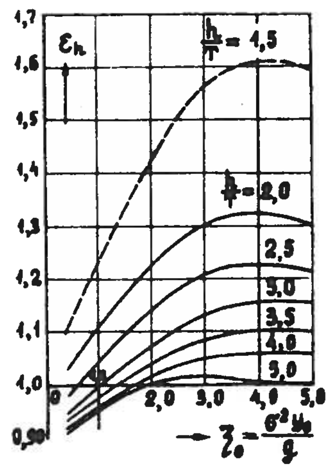

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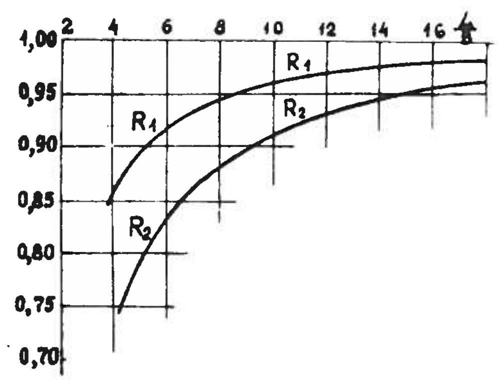

Fiq.8

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