No. 335
May 1997

Extension and Application of ZARNICK: A Nonlinear Simulation Program for High Speed Planing Hulls

by

Carolyn Frank
Bradley Given
Armin W. Troesch

Department of Naval Architecture
and Marine Engineering
College of Engineering
The University of Michigan
Ann Arbor, MI 48109-2145

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(Preliminary Report)

May 1, 1997

Carolyn Frank
Bradley Given
Armin W. Troesch

Department of Naval Architecture and Marine Engineering
The University of Michigan
Ann Arbor, Michigan
(313) 763-6644
e-mail troesch@engin.umich.edu

This work has been sponsored by the Office of Naval Research under Contracts DOG-G-N0014-95-1-1124 and DOG-G-N00014-94-1-0652 with program managers Dr. Thomas Swean and Dr. Edwin Rood, respectively and through the National Coastal Resources Research & Development Institute under Contract MT96.045-731903

Introduction

A nonlinear mathematical model (Zarnick, 1978) has been extended to calculate the motions of a variable deadrise planing boat in waves, using Zarnick's low aspect ratio strip theory. It is assumed that the wavelengths are large relative to boat length and that the wave slope is small. The acceleration in the x-direction is assumed small and therefore set to zero enabling the program to solve for the required thrust. A third order polynomial may be used to vary the deadrise of a hull from bow to stern. Several post processors have been written to analyze the motions and accelerations.

Planing dynamics in a seaway is a complex, nonlinear problem. The Zarnick paper (1978) was the first to formulate the time domain approach for planing hull seakeeping though several authors have duplicated and extended the approach since then (e.g. Payne, 1990 and Keunung, 1992). An extensive bibliography and comparison with experimental data can be found in Payne (1995).

This report describes the validation of the program and a preliminary variable deadrise study. In the deadrise variation comparison, five hull forms are evaluated for reduced drag and operability (i.e. rideability) enhancement.

Validation of Code: Comparison between Zarnick and Savitsky predictions

Values for the centers of pressure, the chine wetted lengths, the friction coefficients, the dynamic lift coefficients and the buoyancy lift coefficients predicted by the Zarnick code (primarily theoretical, e.g. Zarnick, 1978) and Savitsky formulas (primarily empirical, e.g. Savitsky, 1964 or Doctors, 1985) are compared.

Each comparison is made for a given trim angle and nondimensional wetted length. The trim angles varies from 2 to 12 degrees in increments of one-half degrees. The nondimensional wetted lengths ranges from 1.9 to 3.4 depending on the location of the center of gravity and the trim angle. The hull used in the validation comparison is a prismatic hull with a constant deadrise of 20 degrees. Two centers of gravity are used. One is located 18.645 ft. forward of the transom (nondimensionalized by the beam of 7.6 ft.) and 0.691 ft. above the baseline. For the second case, the center of gravity is located 12.645 ft. forward of the transom (also nondimensionalized by the beam of 7.6 ft.) and 0.691 ft. above the baseline). Since the hull is prismatic, the overall length is not a factor in the analysis other than the requirement that the geometric length always exceed the keel wetted length.

The trim angle and wetted lengths are assumed given allowing for the determination of the other hydrodynamics quantities. It is not necessary to iterate the equations of equilibrium in balancing the center of pressure with the center of gravity and drag force, as one would normally do in a typical planing hull calculation (Savitsky, 1964 or Doctors, 1985).

Figure 1 shows the various nondimensional wetted lengths (i.e. λ) and trim angles used for each of the two center of gravity locations. The lcg parameter, λ, is a measure of the mean wetted length. It is the average of the keel wetted length and the chine wetted length normalized by the transom beam. Between the lcg two cases, λ's ranging from1.9 to 3.4 are covered.

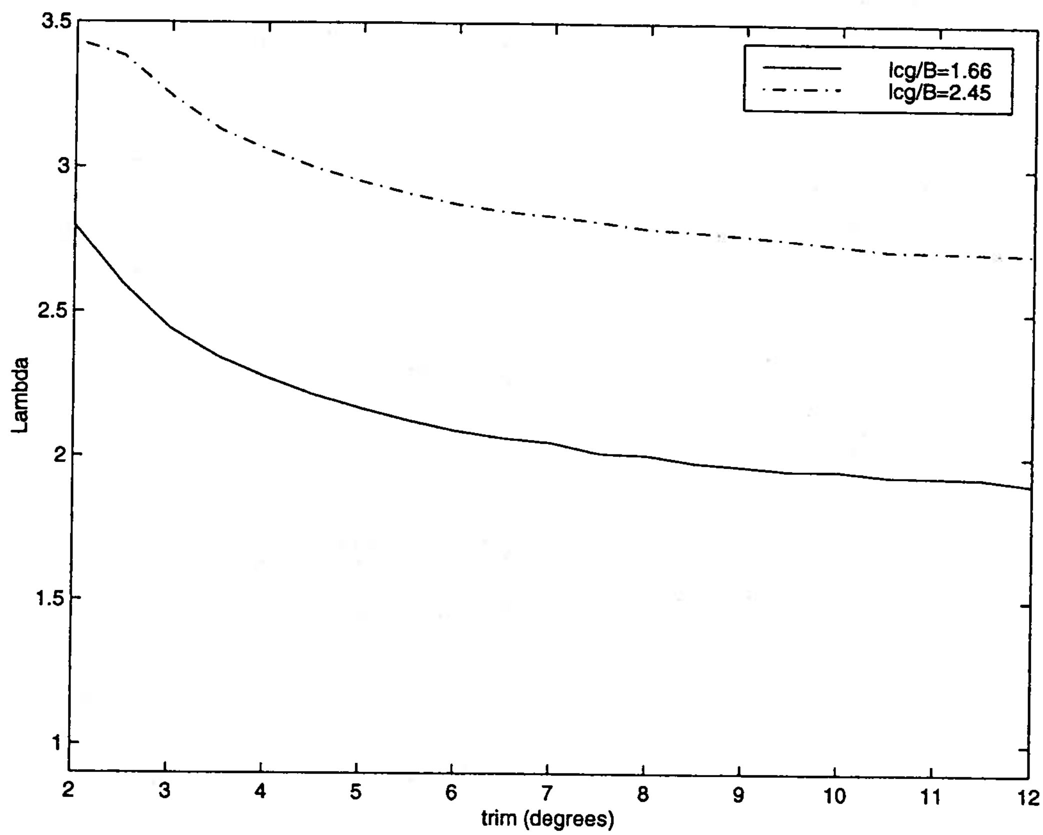

Figure 1: Nondimensional Wetted Length (λ) versus Trim Angle (τ) values spanned by the two lcg conditions

• Zarnick Calculations

The original computer program written by E. Zarnick (Zarnick, 1978) is a time integration simulator, commonly referred to as a "time domain" code. Equations of motion are evaluated and satisfied at incremental time steps, balancing hydrodynamic forces and moments with inertial loads (i.e. D'Alembert forces) and propeller thrust. In the absence of incident waves, the solution of the dynamic equations of motion is similar to the quasi-static iteration method of Savitsky (Savitsky, 1964 or Doctors, 1985). Typically, the analysis starts with the hull weight and geometry and one of two possible performance parameters: either a specified design speed or specified installed horsepower. The desired output is then the required horsepower corresponding to the specified design speed or the maximum speed corresponding to the specified installed horse-power.

From the viewpoint of the Savitsky solution technique, the system of equations are *implicit* if the weight is given as the independent variable and the wetted length and trim are treated as the dependent variables. Conversely the equations become *explicit* if the wetted length and trim are given as the independent variables and the weight (i.e. lift force) becomes the dependent variable. For the validation comparisons of the Zarnick code with the Savitsky empirical values, the wetted lengths and trim angles were prescribed, and the comparisons were made on the resulting centers of pressure, the chine wetted lengths, the friction coefficients, the dynamic lift coefficients and the buoyancy lift coefficients

The keel wetted length is determined in the Zarnick code by evaluating at each station (moving from the bow to the transom) whether the keel intersects the water plane. The keel wetted length is then defined to be the distance along the keel from the transom to the intersection with the water plane. For a given location of the center of gravity (both lcg and vcg) and mean trim angle, the keel wetted length, *klw*, is thus assumed given and so all other calculations are based on this value.

In order to calculate the wetted length parameter λ, the chine wetted length must be determined. Zarnick's method includes a wetting factor which is based upon the theory of Wagner (1931). The chine wetted length, *cwl*, can be calculated either in the Zarnick computer code or from the simple expression based on Wagner's theory shown below:

$$cwl = kwl - \frac{B \tan\beta}{\pi \tan\tau} \qquad (1)$$

where B, β, and τ are the hull beam, deadrise angle and mean running trim angle respectively. The "theoretical" mean wetted length is then given by Equation (2)

$$\lambda = \frac{(cwl + kwl)}{2B} = \frac{kwl}{B} - \frac{1}{2\pi}\frac{\tan\beta}{\tan\tau} \quad . \qquad (2)$$

The center of pressure, *xlcp*, is calculated by dividing the overall moment about the center of gravity by the total vertical force, *Ft*. The total hydrodynamic moment (*NL*), lift force (*FL*) and hydrostatic force (*bf*) are calculated by the Zarnick code:

$$Ft = FL + bf\cos\tau$$

$$xlcp = \frac{NL}{Ft} \qquad (3)$$

This formula gives the location of gravity relative to the center of gravity. In order to get the center of pressure relative to the transom, the location of gravity must be added.

The following formulas were used to calculate the values of the various nondimensional coefficients (where the Zarnick code calculated the lift force, the total moment, the buoyancy force, and the wetted surface area (*wsa*)):

dynamic lift coefficient:

$$cdlift = \frac{-FL\cos\tau}{\frac{1}{2}\rho V^2 B} \tag{4}$$

lift coefficient including buoyancy:

$$cblift = \frac{-FL\cos\tau + bf}{\frac{1}{2}\rho V^2 B} \tag{5}$$

frictional drag coefficient:

$$cfdrag = \frac{DRAG}{\frac{1}{2}\rho (V_m)^2 (wsa)} \tag{6}$$

where

$$V_m = V\left(1 - \frac{cdlift}{\lambda\cos\tau}\right)^{1/2}$$

and the total frictional drag (*DRAG*) is

$$DRAG = \frac{1}{2}\rho(cf)(V_m)^2(wsa)) \tag{7}$$

with

$$cf = \frac{0.455}{(\log_{10}(reyn))^{2.58}} + 0.0004 \qquad \text{and}$$

$$reyn = \frac{\rho V_m (B\lambda)}{\mu}.$$

• Savitsky Calculations

Given the trim angle and nondimensional wetted length from the Zarnick code, the nondimensional coefficients of Equations (4) - (6) were evaluated and compared with similar Savitsky quantities. In order to calculate the nondimensional mean wetted length for the Savitsky formulas, the empirical chine wetted length must be determined.

For the Savitsky λ values, chine wetted length was calculated using the keel wetted length and an empirical formula (Brown 1971 and Martin 1978a,b):

$$cwl = kwl - (0.57 + 0.001\beta)\left(\frac{\tan\beta}{2\tan\tau} - 0.006\beta\right) \tag{8}$$

This empirical calculation of the chine wetted length was used for the calculations of the other values. The following figures show the differences in calculating the chine wetted length when using the empirical formula (Equation (8)) compared with the theoretical formula (Equation (1)).

Figures 2 and 3 show the chine wetted lengths for two given lcg/B ratios (lcg/B=1.66 and 2.45 respectively) when the keel wetted length is fixed and the chine wetted length is calculated using an empirical or theoretical formula. In both cases, the empirical values are larger than the theoretical values by approximately 5 - 10% indicating that the hull wetting as given by Wagner's 1931 theory underpredicts the extent of the jet rise-up.

Experimental observations indicated that the wetted surface does not follow a straight line between the keel intersection point and the chine intersection point as Equation (2) would suggest (Savitsky, 1964 and Savitsky and Brown, 1976). Therefore the empirical mean wetted length is defined, following those authors, as

$$\lambda_{ave} = \frac{(cwl + kwl)}{2B} + 0.03 \tag{9}$$

where 3% of the beam has been added to better match experimental values.

Figures 4 and 5 show the nondimensional wetted lengths, λ and λ_{ave}, for two given lcg/B ratios (lcg/B=1.66 and 2.45 respectively) when the keel wetted length is fixed and the mean wetted length is calculated using theoretical (Equation (2)) or empirical formulas (Equations (8) and (9)). As expected, based upon the previous paragraphs discussion on the chine wetted lengths, the empirical mean wetted lengths are larger than the theoretical wetted lengths by approximately 5%. Since these values are directly used in estimating the frictional drag, there will be a corresponding increase in the empirical frictional drag coefficient, *cfdrag* Equation (6), over the theoretical frictional drag coefficient.

With the trim and mean wetted length known, the nondimensional coefficients given in Equations (4) - (6) can evaluated using Savitsky (1964) empirical relationships. This was done using the following equations.

The flat plate lift coefficient (C_{Lo}) was calculated using the trim angle, the given λ_{ave}, i.e. Equations (8) and (9), and the Froude number ($F_B = V/(\sqrt{gB})$):

$$C_{Lo} = \tau^{1.1}\left(0.0120\lambda_{ave}^{1/2} + 0.0055\frac{\lambda_{ave}^{5/2}}{F_B^2}\right) \tag{10}$$

The deadrise lift coefficient ($C_{Lo\beta}$) was then determined:

$$C_{L\beta} = C_{Lo} - 0.0065\beta C_{Lo}^{0.60} \tag{11}$$

Then the weight was calculated by:

$$W = C_{L\beta}\frac{1}{2}\rho V^2 B^2 \tag{12}$$

The rest of the Savitsky values used in the comparison were calculated using the following equations:

location of center of pressure:

$$xlcp = \lambda_{ave} B\left(0.75 - \frac{1}{5.21\frac{F_B^2}{\lambda_{ave}^2} + 2.39}\right) \tag{13}$$

dynamic lift coefficient:

$$C_{Ld} = 0.012\lambda_{ave}^{1/2}\tau^{1.1} - 0.0065\beta(0.012\lambda_{ave}^{1/2}\tau^{1.1})^{0.60} \tag{14}$$

drag coefficient:

$$D_F = \frac{1}{2}\rho V_m^2 S_F C_F \tag{15}$$

where

$$V_m = V\left(1 - \frac{C_{Lc}}{\lambda_{ave}\cos\tau}\right), \tag{16}$$

$$S_F = \frac{\lambda_{ave}B^2}{\cos\beta}, \tag{17}$$

$$C_F = \frac{0.455}{(\log_{10} reyn)^{2.58}} + 0.0004, \tag{18}$$

and

$$reyn = \frac{\rho V_m(B\lambda_{ave})}{\mu}. \tag{19}$$

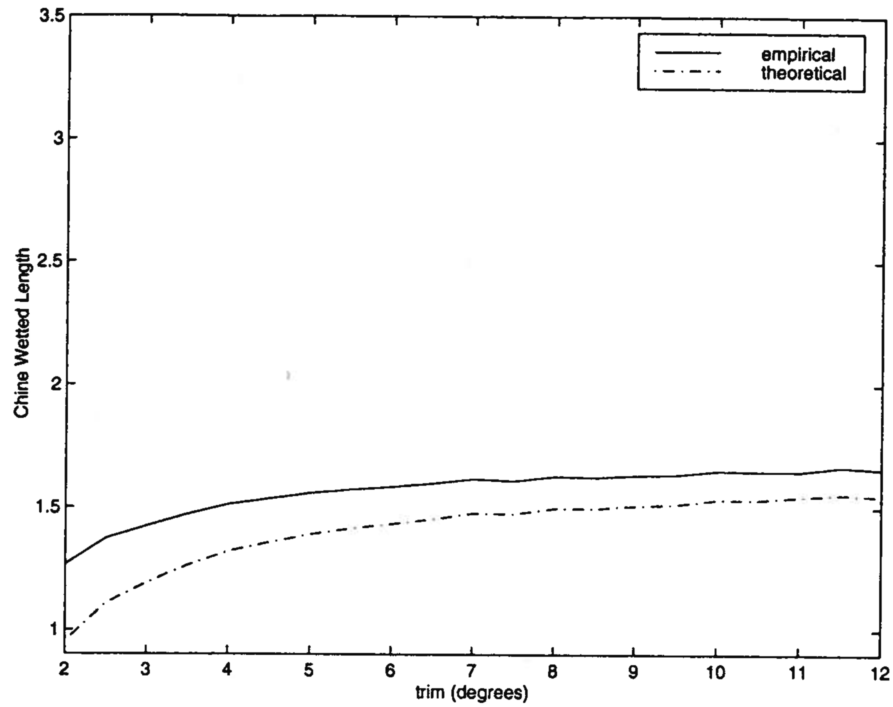

Figure 2: Chine Wetted Length versus Trim Angle (lcg/B=1.66)

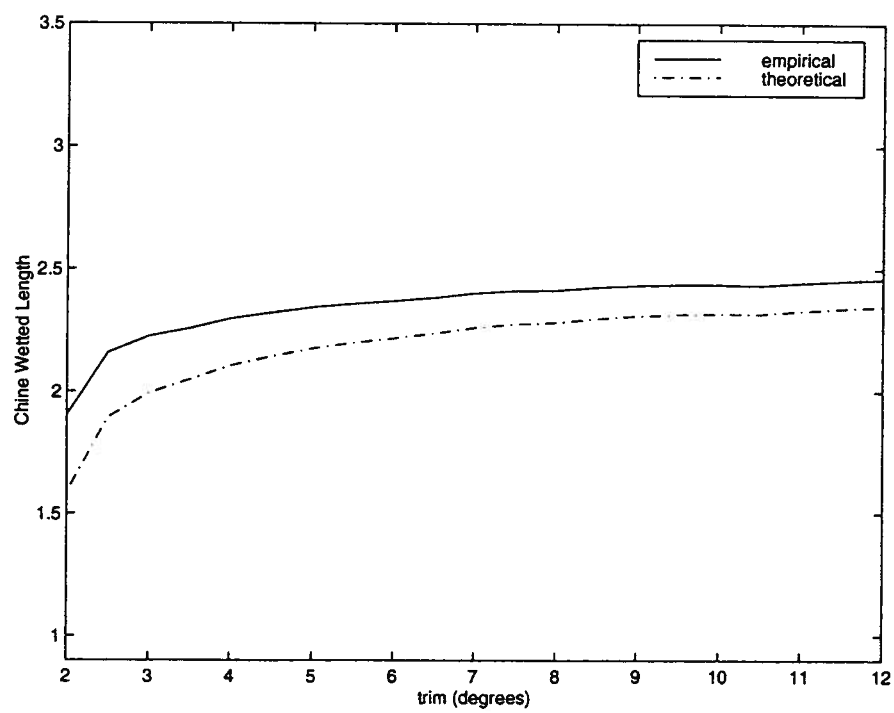

Figure 3: Chine Wetted Length versus Trim Angle (lcg/B=2.45)

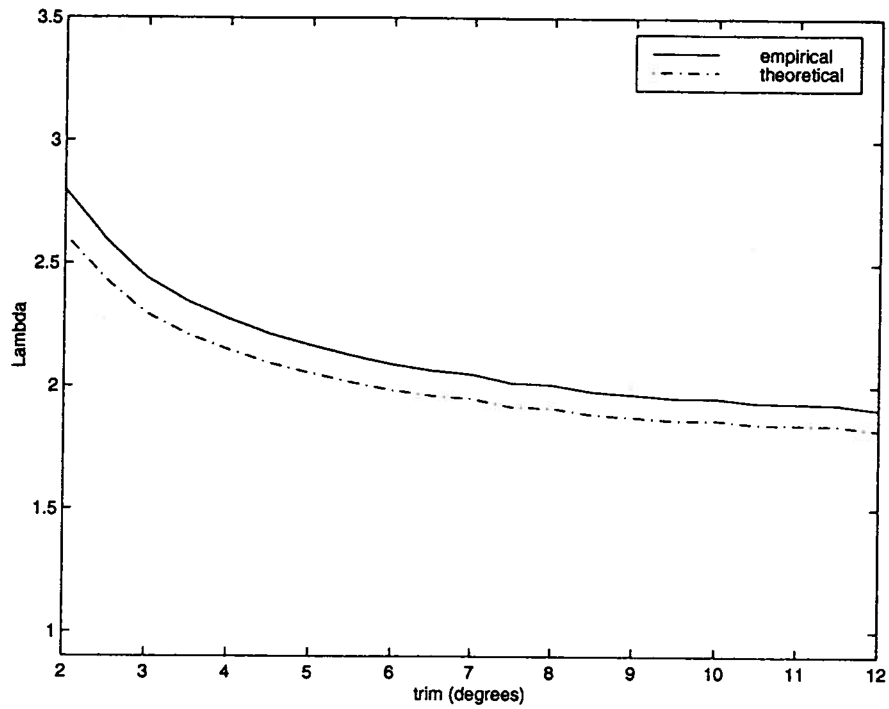

Figure 4: Nondimensional Wetted Length versus Trim Angle (lcg/B=1.66)

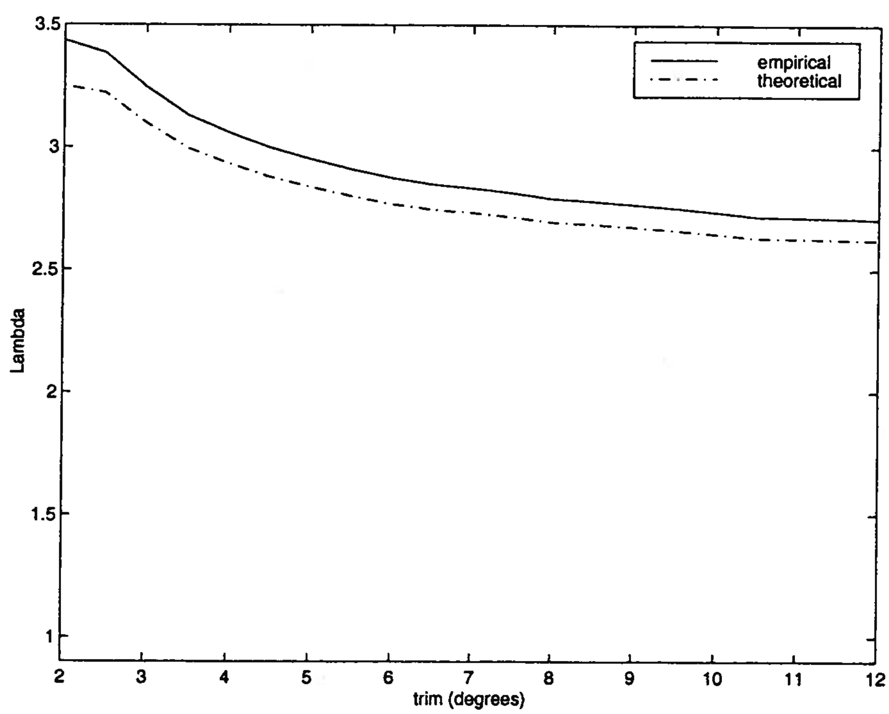

Figure 5: Nondimensional Wetted Length versus Trim Angle (lcg/B=2.45)

Results of Validation Study

The results of the validation study are shown in Figures 6 and 7. Here the ratios of the various hydrodynamic quantities, as calculated by the Zarnick time domain code and the Savitsky method, are compared. Specifically, the quantities plotted are as follows:

center of pressure - Zarnick/Savitsky	(Eq. (3)/Eq. (13))
chine wl - Savitsky/ Zarnick	(Eq. (8)/Eq. (1))
dyn. lift coeff - Zarnick/Savitsky	(Eq. (4)/Eq. (14))
buoy. lift coeff - Zarnick/Savitsky	(Eq. (5)/Eq. (11))
friction coeff - Savitsky/ Zarnick	(Eq. (15)/Eq. (7))

The hydrodynamic ratios are plotted vs. the trim angle for the two lcg/B ratios. Generally the overall predictive capabilities of the time domain code are well within the accuracy levels needed for effective design analysis.

Figure 6, i.e. lcg/B = 1.66, has the largest differences at the smaller trim angles. Specifically, the dynamic lift is overpredicted by Zarnick by approximately 25% - 30% and the total lift, represented by the buoyancy lift coefficient is over predicted by 15% - 20%. For trim angles beyond six degrees, the overprediction is lower, of the order 15% and 12% for the dynamic lift and the buoyancy lift respectively. The center of pressure, as calculated by the two methods, is within 10%. The chine wetted lengths and frictional drag comparisons are significantly closer, with Zarnick under-predicting the empirical values by 2% - 3%, except at the very low trim angles.

In Figure 7, i.e. lcg/B = 2.45, the differences are significantly smaller than those shown by Figure 6 with most the theoretical quantities being within 10% of the empirical values. Based upon Figure 1, this suggests that Zarnick compares well with Savitsky type analysis when the wetted length to beam ratio is greater than three. In addition, the dependence on trim angle is less pronounced; variations in the ratios of theory to experiment are not significantly affected by trim changes. Specifically, the dynamic lift is overpredicted by Zarnick by approximately 7% - 18% and the total lift, represented by the buoyancy lift coefficient overpredicts or underpredicts by a maximum of 3% except at a trim angle of 2 degrees, where Zarnick overpredicts by 7%. The center of pressure, as calculated by the two methods, is within 6% - 11%. The chine wetted lengths show Zarnick under-predicting the empirical values by 2% - 6%. The frictional drag is underpredicted by somewhat more, 2% - 12%.

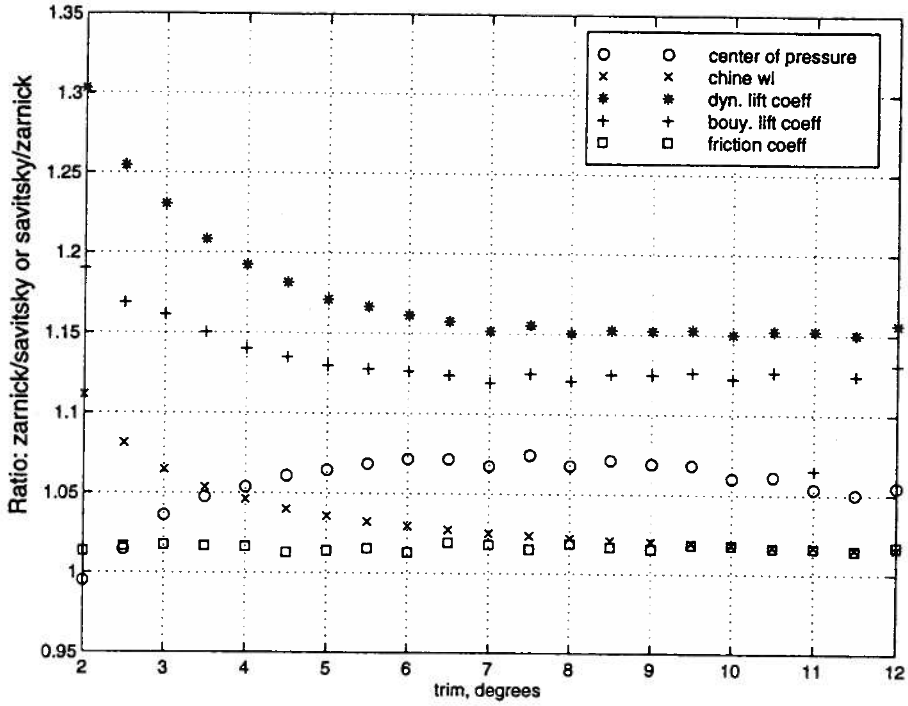

Figure 6. Ratios of Zarnick/Savitsky or Savitsky/Zarnick vs. Trim Angle for lcg/B=1.66

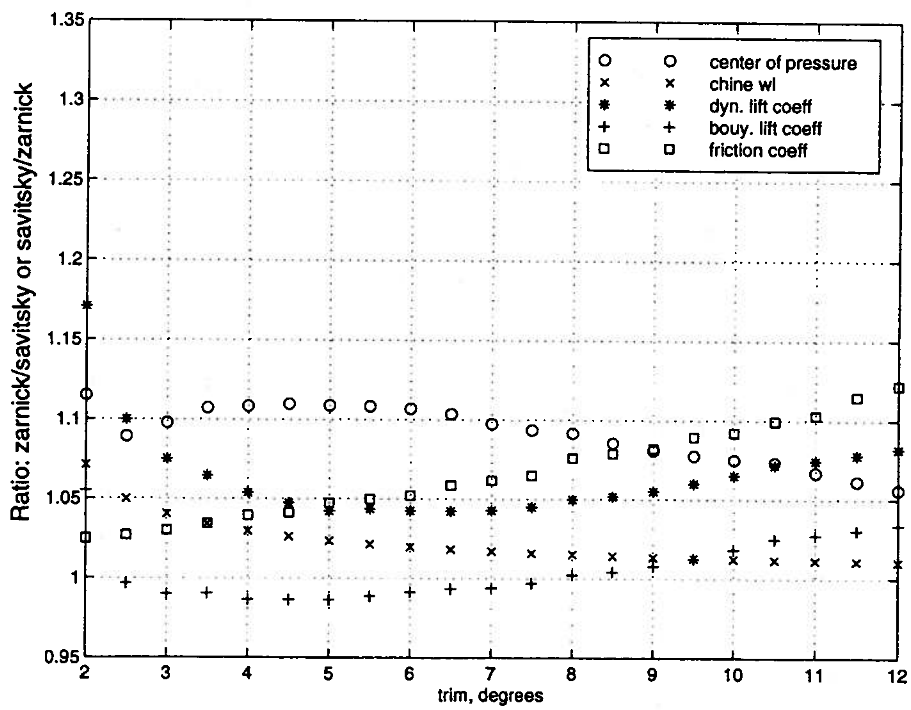

Figure 7: Ratios of Zarnick/Savitsky or Savitsky/Zarnick vs. Trim Angle for lcg/B=2.45

Variable Deadrise Study

A preliminary study of the effects of deadrise variations was undertaken. This study is preliminary in the sense that two of the many hydrodynamic terms in the computer code were normalized by the deadrise at the bow rather than the continuous deadrise distribution over the length. The effect this had on the calculations was numerically or quantitatively significant, but qualitative comparisons were still possible. Those qualitative comparisons are shown below. The code has since been modified and a more extensive deadrise variation study is in progress.

Five hull forms were considered, one parent and four variants. The parent is identified as a "High Speed Planing Boat," i.e. HSPB or Hull 1, and the others are simply named "Hulls 2-5".

A third order polynomial was used to vary the deadrise from bow to stern for Hulls 2 through 4

$$\beta_v(x) = \beta_o(x)(a_o + a_1 x^2 + a_3 x^3) \qquad (20)$$

where β_0 is the deadrise distribution of a standard high speed planing boat (Hull 1). There were the following boundary conditions:

$$\frac{\partial \beta_v}{\partial x} = 0\text{, at x=0 and x=L,}$$

$$\frac{\partial^2 \beta_v}{\partial x^2} = 0\text{, at x=}\frac{L}{2}\text{,}$$

and

$$\beta_v = a_o k\text{, at x=L.}$$

The constants in Equation (20) then can be found as

$$a_1 = 0 \quad a_2 = 3\frac{a_o(k-1)}{L^2} \quad a_3 = -2\frac{a_o(k-1)}{L^3},$$

and the equation for the deadrise variation becomes

$$\beta_v(x) = \beta_o(x)\left[a_o + 3\frac{a_o(k-1)x^2}{L^2} - 2\frac{a_o(k-1)x^3}{L^3}\right]. \qquad (21)$$

The two variables which defined the deadrise for each hull were a_o and k. For Hull 1

(HSPB), $a_o = 1.0$ and $k = 1.0$, for Hull 2, $a_o = 1.1$ and $k = 1.1$, for Hull 3, $a_o = 0.9$ and $k = 1.0$, and for Hull4 $a_o = 1.1$ and $k = 0.8$.

Hull 5 used a fourth order polynomial to vary the deadrise:

$$\beta_v(x) = \beta_o(x)(a_o + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4) \qquad (22)$$

where

$$a_1 = \frac{-7}{216L}(324a_0 - 343 + 19a_o k),$$

$$a_2 = \frac{1}{72(L)^2}(2052a_o - 2401 + 349a_o k,$$

$$a_3 = \frac{-1}{72L^3}(2124a_o - 2401 + 277a_o k),$$

and

$$a_4 = \frac{7}{216L^4}(324a_0 - 343 + 19a_o k).$$

For Hull 5 a_o=0.6 and k=1.6 were chosen.

The various deadrise distributions are shown in Figure 8. Hulls 1 - 4 can be considered as "deep vee" type hulls where Hull 5 has reduced deadrise in the extreme forward stations. If the deadrise in the forebody is averaged in a mean square sense, a measure of the differences in the hull geometries can be determined. The RMS deviation of the forebody deadrise, in degrees,. is shown below in Table I. Hull 1, or the HSPB hull, was used as the baseline hull.

Table 1: Relative RMS Forebody Deadrise Loss or Gain

Hull Type	Loss/Gain (deg)
Hull 1 (HSPB)	0.0
Hull 2	+ 4.8
Hull 3	- 4.1
Hull 4	+ 3.2
Hull 5	- 7.1

It is expected that high deadrise in the bow produces a relatively smooth, but wet ride in rough seas, whereas increased flare, such as the reduced deadrise of Hull 5, will produce a relatively rough but dry ride. It is also expected that lower deadrise in the after areas will promote better lift-to-drag ratios. These "rules of thumb" have been verified by the results shown in the following figures.

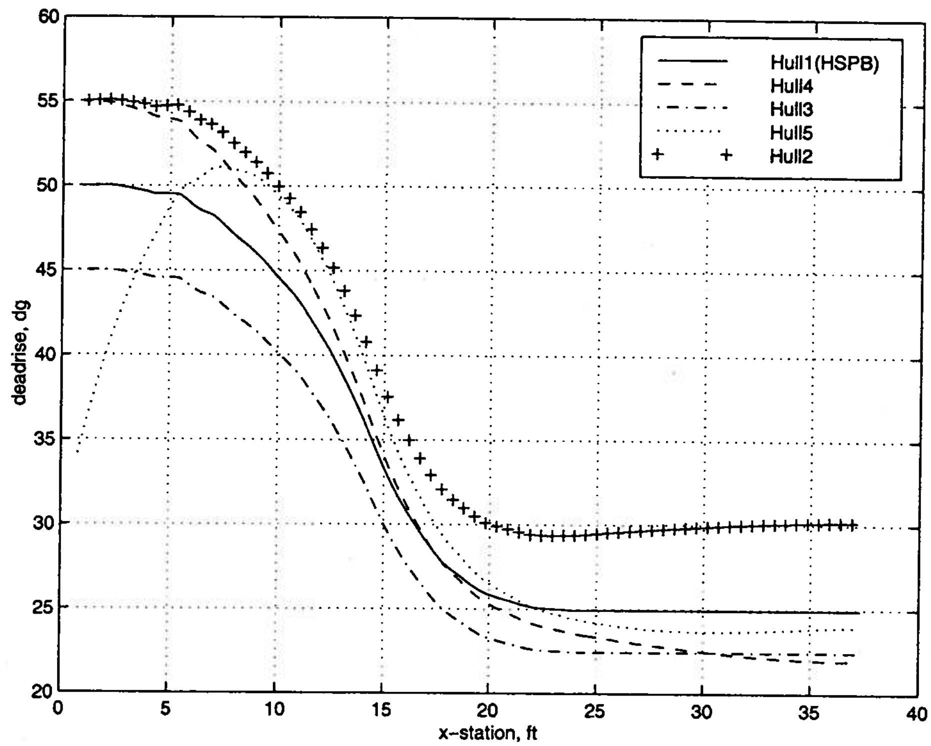

Figure 8: HSPB & Four Other Hulls Deadrise Distribution from Bow to Stern.This figure shows the distribution for deadrise variation of five hulls. Hull 1 is the original HSPB deadrise distribution. Hulls 2 through 4 were varied using a third order polynomial (Equation (21)) and Hull5 was varied using a fourth order polynomial (Equation (22)).

- **RMS Motions and Average Thrust**

In this preliminary study, the RMS motions and average thrust were found in regular waves with a wave amplitude of 0.25 ft. and the x-location of the center of gravity 15 ft. forward of the transom. The HSPB has a length of 37.3 ft., a beam of 7.56 ft. and a weight of 19050 lbs. The results are shown in Figures 9 through 11.

The bow and center of gravity accelerations are shown in Figure 9 and 10 respectively. All hull types show maximum bow acceleration when the wave length is slightly smaller than the boat length. The center of gravity accelerations typically are largest when the wave length is approximately six boat lengths. As expected, the hull with the largest flare, Hull 5, has the highest accelerations over most of the wave length range, the exception being in extremely long waves. While the time domain code does not calculate deck wetness, the results suggest that Hull 5 also may have the driest ride.

The accelerations can be averaged in a mean square sense across the various wave lengths and compared to the parent hull, Hull1 or the HSPB. There are not significant differences in the trends between the bow and center of gravity values. Generally, the hulls with less deadrise forward can have substantially increased accelerations and the hulls with increased deadrise can have reduced accelerations. Approximate values are given in Table II for comparative purposes. In this limited study, Hull 4 has the largest reduction relative to the HSPB, approximately 10%.

Table 2: Relative RMS Accelerations Loss or Gain

Hull Type	Loss/Gain (%)
Hull 1 (HSPB)	0%
Hull 2	- 5%
Hull 3	+ 10%
Hull 4	- 10%
Hull 5	+ 40%

Another measure of relative merit would be the expended power or drag of the different hull forms. The mean thrust required in waves with 0.25 ft. wave amplitude and differing wave length was calculated and the results are summarized in Table III. In Table III, the total drag of each hull was averaged in a mean square sense over the wave length range and compared to the HSPB. Generally, the hulls with less deadrise aft, i.e. Hulls 3, 4, and 5, have reduced drag and the hull with increased deadrise aft, Hull 2, has increased drag.

Table 3: Relative RMS Drag Loss or Gain

Hull Type	Loss/Gain (%)
Hull 1 (HSPB)	0%
Hull 2	+ 5.0%
Hull 3	- 2.0%
Hull 4	- 1.0%
Hull 5	- 1.5%

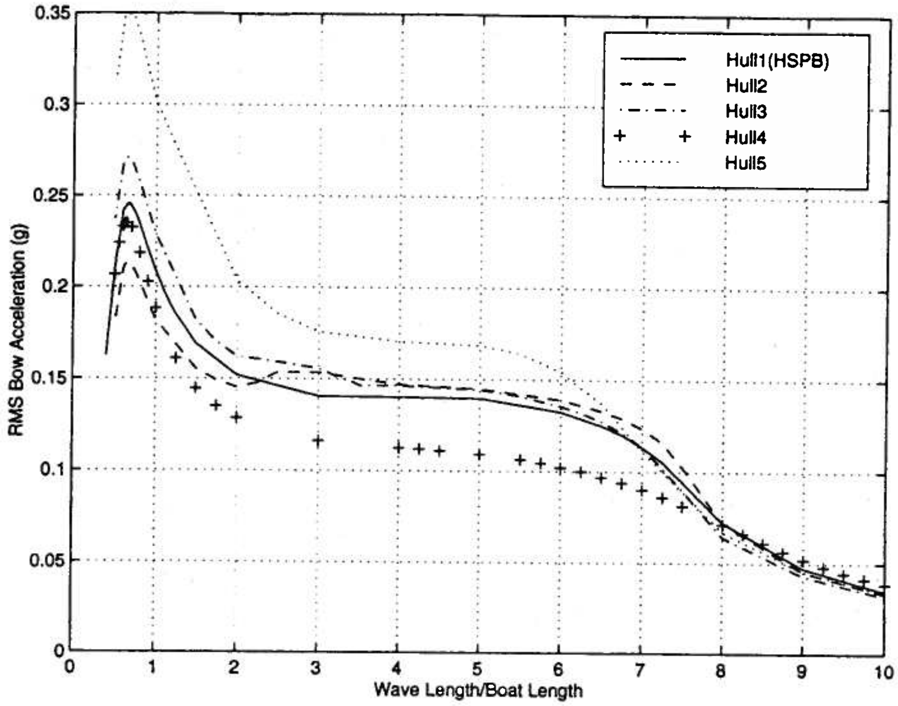

Figure 9: Bow Acceleration in Waves for Speed = 80 ft./sec and Wave Ht=0.5 ft. for HSPB and 4 Varied Hulls

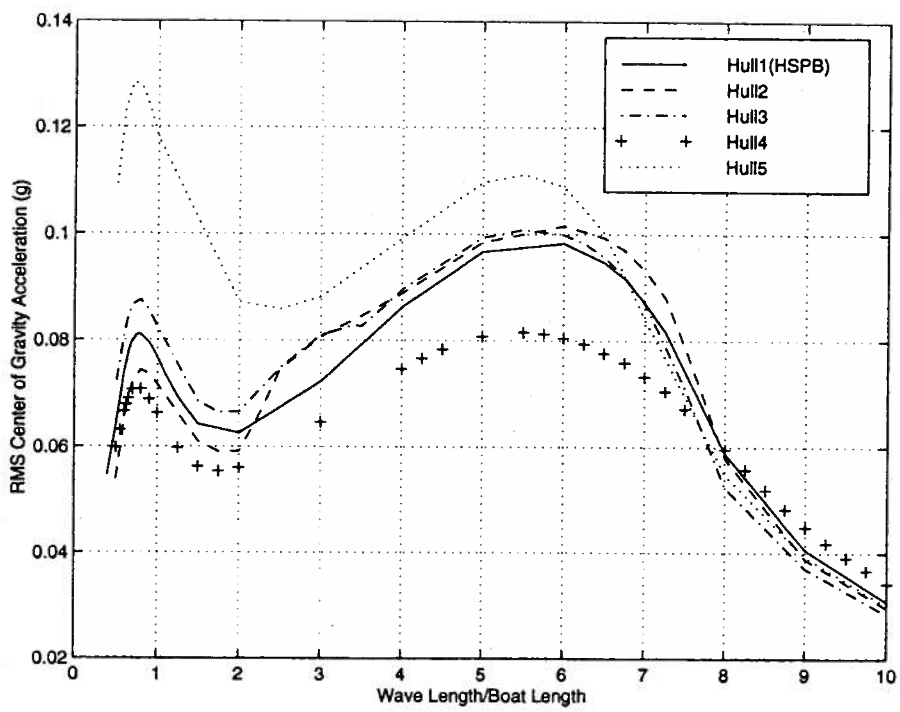

Figure 10: Center of Gravity Acceleration in Waves for Speed=80 ft./sec and Wave Ht=0.5 ft. for HSPB and 4 Varied Hulls

Summary

This report has shown that the time domain code based upon Zarnick's (1978) model is sufficiently accurate to be used in high speed planing hull design. The validation was accomplished through comparisons with experimental results for constant deadrise hull forms (Savitsky, 1964). The extension of the code to include variable deadrise will allow users to conduct optimization analyses of various competing hull shapes. The deadrise variation study of this report is preliminary since two sectional hydrodynamic components were scaled by the bow deadrise angles instead of the distribution of deadrise angles producing errors in modeling. However the comparative study is qualitatively accurate and demonstrates the potential of the corrected code.

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