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THE WAVE RESISTANCE OF A SURFACE PRESSURE DISTRIBUTION IN UNSTEADY MOTION

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TRANSLATORS' NOTE:-

In this paper the symbols used for the hyperbolic functions have the following meanings:

$$sh \equiv \sinh$$

$$ch \equiv \cosh$$

$$th \equiv \tanh$$

We consider the problem of non-steady motion of a surface pressure distribution of arbitrary form $p(x, y)$ over an area Ω. A coordinate system $(0, x, y, z)$ is fixed on the undisturbed surface, where x is in the direction of motion and z is vertically upwards.

The disturbance potential which is caused by this pressure distribution is defined by the solution of Laplace's Equation, $\Delta\phi = 0$ with the following boundary conditions:

On the free surface:

$$\frac{\partial^2\varphi}{\partial t^2} + \frac{\partial\varphi}{\partial z} + v^2\frac{\partial^2\varphi}{\partial x^2} - 2v\frac{\partial^2\varphi}{\partial x\partial t} - \frac{dv}{dt}\frac{\partial\varphi}{\partial x} = -\varkappa\frac{\partial p_u}{\partial t} + \varkappa v\frac{\partial p_u}{\partial x}$$

at $z = 0$. (1)

On the bottom:

$$\frac{\partial\varphi}{\partial z} = 0$$

at $z = -H$. (2)

In expressions (1) and (2) all the variables are non-dimensionalized with respect to l, the half-length of the pressure distribution (see Fig. 1), p_0, the characteristic pressure, and g, the acceleration of gravity. The disturbance potential ϕ can be represented by the following integral form:

$$\varphi(x,y,z,t) = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[A(t)e^{-\sqrt{\alpha^2+\beta^2}\,z} + B(t)e^{\sqrt{\alpha^2+\beta^2}\,z}\right]e^{i(\alpha x+\beta y)}\,d\alpha\, d\beta$$

We consider the case when $\frac{\partial p_u}{\partial t}=0$. Let us assume

$$\varkappa v \frac{\partial p_u}{\partial x} = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d\alpha\, d\beta \iint_{\Omega} \varkappa v \frac{\partial p_u}{\partial x_0}(x_0, y_0)\, e^{-i[\alpha(x-x_0)+\beta(y-y_0)]}\, dx_0\, dy_0 \; .$$

Making the substitutions

$$\alpha = \kappa\cos\theta, \qquad \beta = \kappa\sin\theta \; ,$$

we get

$$\varphi(x,y,z) = \frac{1}{4\pi^2}\int_0^{\infty}\kappa\, d\kappa \int_{-\pi}^{\pi} d\theta\, \left[A(t)e^{-\kappa z} + B(t)e^{\kappa z}\right] e^{i[x\cos\theta + y\sin\theta]\kappa} \tag{3}$$

and

$$\varkappa v \frac{\partial p_u}{\partial x} = \int_0^{\infty}\kappa\, d\kappa \int_{-\pi}^{\pi} d\theta\, \frac{\varkappa v}{4\pi^2}\iint_{\Omega} \frac{\partial p_u}{\partial x_0}\, e^{i\kappa[(x-x_0)\cos\theta + (y-y_0)\sin\theta]}\, dx_0\, dy_0 \; . \tag{4}$$

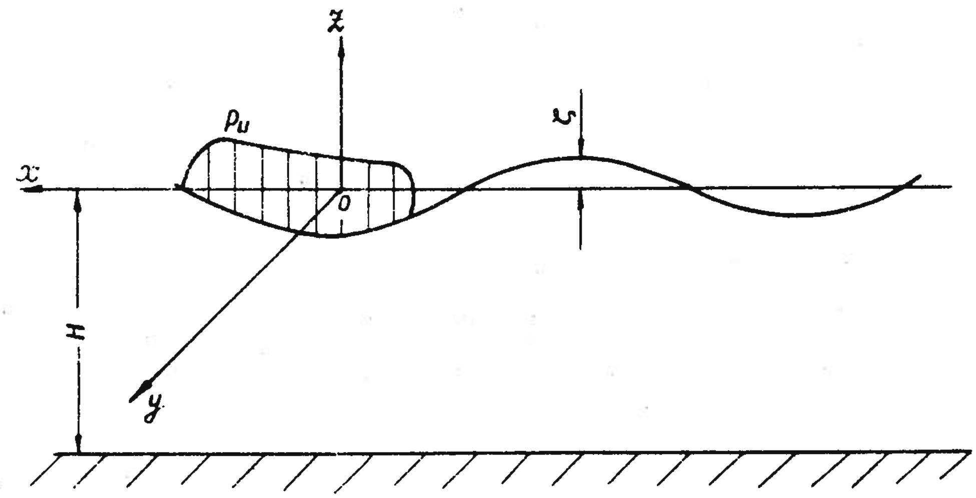

Fig. 1

Substituting (3) and (4) in (1) and (2), and equating the integrands (because the equality must hold for all k and θ), then expressing B in terms of A from equation (2) and substituting in (1), we obtain

$$\frac{d^2A}{dt^2}+\kappa th\kappa H + A\vartheta^2(i\kappa\cos\theta)^2-2\vartheta\frac{dA}{dt}(i\kappa\cos\theta)-A\frac{d\vartheta}{dt}(i\kappa\cos\theta) =$$

$$= (1+e^{2\kappa H})^{-1}\iint_{\Omega}\varkappa\vartheta\frac{\partial p_u}{\partial x_0}e^{-i\kappa(x_0\cos\theta+y_0\sin\theta)}dx_0dy_0 \quad . \qquad (5)$$

Now let us introduce the function

$$A_1=Ae^{-i\kappa\cos\theta\cdot\int\vartheta dt} \quad . \qquad (6)$$

To determine A_1 we note that it satisfies the differential equation

$$\frac{d^2A_1}{dt^2}+A_1\kappa th\kappa H=f(\kappa,\theta,t), \qquad (7)$$

where

$$f(\kappa,\theta,t)=e^{-i\kappa\cos\theta\cdot S}\cdot(1+e^{2\kappa H})^{-1}\iint_{\Omega}\varkappa\vartheta\frac{\partial p_u}{\partial x_0}e^{-i\kappa(x_0\cos\theta+y_0\sin\theta)}dx_0dy_0, \qquad (8)$$

and

$$S=\int\vartheta dt \quad . \qquad (8a)$$

Solving (7) with zero initial conditions,

$$t=0 \qquad A_1=0; \qquad \frac{dA_1}{dt}=0$$

(equivalent to $\varphi=0$ and $\frac{\partial\varphi}{dt}=0$), by the method of variation of the integration constant, we get

$$A_1=\left(\frac{1}{\delta}\int f\cos\delta t\,dt\right)\sin\delta t-\left(\frac{1}{\delta}\int f\sin\delta t\,dt\right)\cos\delta t +$$

$$+\left(-\frac{1}{\delta}\int f\cos\delta t\,dt\right)_{t=0}\sin\delta t+\left(\frac{1}{\delta}\int f\sin\delta t\,dt\right)_{t=0}\cos\delta t, \qquad (9)$$

where $\qquad \delta^2=\kappa th\kappa H$.

Therefore the disturbance potential is

$$\varphi(x,y,z,t) = \frac{1}{4\pi^2}\int_0^\infty \kappa d\kappa \int_{-\pi}^{\pi} d\theta \cdot A_1 \cdot e^{i\kappa\cos\theta\cdot\int v dt} \times$$
$$\times \left[e^{-\kappa z} + e^{\kappa(2H+z)}\right] e^{i\kappa(x\cos\theta + y\sin\theta)} . \tag{10}$$

The wave resistance is obtained from

$$R = -\iint_\Omega p_u(x,y)\,\frac{\partial\zeta}{\partial x}\,dxdy, \tag{11}$$

assuming that R is equal to the horizontal component of the pressure acting on the area Ω. In formula (11) all variables are dimensional. In non-dimensional form it would be

$$\begin{aligned}
&\frac{R}{\rho_0 \ell^2} = -\iint_\Omega p_u(x,y)\,\frac{\partial\zeta}{\partial x}\,dxdy;\\
&\frac{\partial\zeta}{\partial x} = -\varkappa\frac{\partial p_u}{\partial x} + v\frac{\partial^2\varphi}{\partial x^2} - \frac{\partial^2\varphi}{\partial x\partial t} \qquad \text{at} \quad z=0;\\
&\frac{R}{\rho_0\ell^2} = -\iint_\Omega \varkappa p_u\frac{\partial p_u}{\partial x}\,dxdy + \iint_\Omega p_u v\frac{\partial^2\varphi}{\partial x^2}\,dxdy - \iint_\Omega p_u\frac{\partial^2\varphi}{\partial x\partial t}dxdy =\\
&\quad = -J_1 + J_2 - J_3 .
\end{aligned} \tag{12}$$

In the last equation the first term is the horizontal component of the pressure, the second is equal to the resistance during steady motion, and the third term occurs only because of the unsteadiness. We can obtain

$$J_2 = \iint_\Omega \frac{v}{4\pi^2}\int_0^\infty \kappa d\kappa\int_{-\pi}^{\pi} d\theta\cdot A_1\left(1+e^{2\kappa H}\right)(i\kappa\cos\theta)^2\, G(\kappa,\theta)\, e^{i\kappa\cos\theta\cdot s} , \tag{13}$$

$$G(\kappa,\theta) = \iint_\Omega p_u\, e^{i\kappa(x\cos\theta + y\sin\theta)}\,dxdy ; \tag{14}$$

and $$\mathcal{J}_3=\frac{1}{4\pi^2}\int_0^\infty \kappa d\kappa\int_{-\pi}^{\pi} d\theta\,\frac{\partial A_1}{\partial t}\left(1+e^{2\kappa H}\right)e^{i\kappa\cos\theta\cdot S}(i\kappa\cos\theta)\,G(\kappa,\theta)+\mathcal{J}_2 \quad . \tag{15}$$

The wave resistance can be evaluated from

$$\frac{R}{\rho_0\ell^2}=-\iint_\Omega \varkappa p_u\frac{\partial p_u}{\partial x}\,dxdy-\frac{1}{4\pi^2}\int_0^\infty\kappa d\kappa\int_{-\pi}^{\pi}\frac{\partial A_1}{\partial t}\left(1+e^{2\kappa H}\right)\times$$
$$\times(i\kappa\cos\theta)e^{i\kappa\cos\theta\cdot S}\cdot G(\kappa,\theta)\,d\theta \quad . \tag{16}$$

On the basis of (9)

$$\frac{\partial A_1}{\partial t}=c_{10}\,\delta\cos\delta t-c_{20}\,\delta\sin\delta t+\cos\delta t\iint f\cos\delta t dt+\sin\delta t\iint f\sin\delta t dt\,. \tag{17}$$

And from (8)

$$\int f\cos\delta t dt=\int\varkappa v e^{-i\kappa\cos\theta\cdot S}\cdot\cos\delta t\left(1+e^{2\kappa H}\right)^{-1}\iint_\Omega\frac{\partial p_u}{\partial x_0}\,e^{-i\kappa(x_0\cos\theta+y_0\sin\theta)}\,dx_0dy_0dt=$$
$$=\varkappa\left(1+e^{2\kappa H}\right)^{-1}G^*(\kappa,\theta)\int v e^{-i\kappa\cos\theta S}\cos\delta t dt\,; \tag{18}$$

where $$G^*(\kappa,\theta)=\iint_\Omega\frac{\partial p_u}{\partial x}\,e^{-i\kappa(x\cos\theta+y\sin\theta)}\,dxdy \quad . \tag{19}$$

Identically

$$\int f\sin\delta t dt=\varkappa\left(1+e^{2\kappa H}\right)^{-1}G^*(\kappa,\theta)\int v e^{-i\kappa\cos\theta\cdot S}\sin\delta t dt \quad . \tag{20}$$

If the motion started from rest, i.e. $v\to0$, $t\to0$, then, using (8), (9), (18) and (20),

$$\frac{\partial A_1}{\partial t}=\varkappa\left(1+e^{2\kappa H}\right)^{-1}G^*(\kappa,\theta)\;\Phi^*(\kappa,\theta,t), \tag{21}$$

where

$$\Phi^*(\kappa,\theta,t)=\left[\cos\delta t\int v e^{-i\kappa\cos\theta\cdot S}\cos\delta t dt+\sin\delta t\int v e^{-i\kappa\cos\theta\cdot S}\sin\delta t dt\right]; \tag{22}$$

$$\frac{R}{\rho_0\ell^2}=-\iint_\Omega\varkappa p_u\frac{\partial p_u}{\partial x}\,dxdy-\frac{\varkappa}{4\pi^2}\int_0^\infty\kappa d\kappa\int_{-\pi}^{\pi}(i\kappa\cos\theta)G^*(\kappa,\theta)\,G(\kappa,\theta)\;\Phi(\kappa,\theta,t)\,d\theta\,; \tag{23}$$

and

$$\Phi(\kappa,\theta,t)=e^{i\kappa\cos\theta\cdot s}\Big[\cos\delta t\int v e^{-i\kappa s\cos\theta}\cos\delta t\,dt +$$

$$+\sin\delta t\int v e^{-i\kappa s\cos\theta}\sin\delta t\,dt\Big] \quad . \tag{24}$$

Let us now use this solution to calculate the wave resistance for the case of a constant pressure distribution acting on a rectangular area with $\mu = b/l$ moving at a constant speed over deep water. Let us assume that the speed changes according to Fig. 2 below.

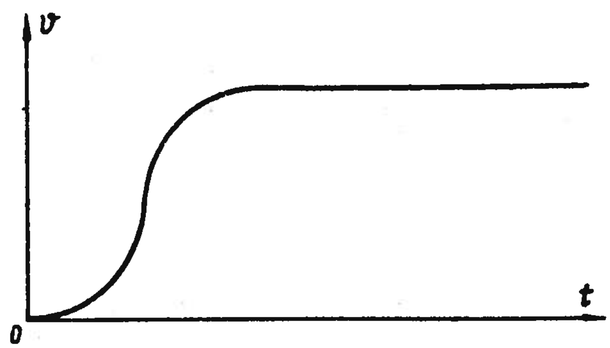

Fig. 2

For $t \to \infty$, the required functions become:

$$\Phi(\kappa,\theta,t)=\frac{v^2(-i\kappa\cos\theta)}{\delta^2-\kappa^2\cos^2\theta\cdot v^2} \quad ,$$

$$G^*(\kappa,\theta)=\frac{4iP_u\sin(\kappa\cos\theta)\sin(\kappa\mu\sin\theta)}{\kappa\sin\theta}+i\kappa\cos\theta\;\bar{G}(\kappa,\theta) \quad ,$$

$$G(\kappa,\theta)=4p_u\;\frac{\sin(\kappa\mu\sin\theta)\cdot\sin(\kappa\cos\theta)}{\kappa\sin\theta\cdot\kappa\cos\theta} \quad , \tag{25}$$

$$GG^*=i\,32\,p_u^2\;\frac{\sin^2(\kappa\mu\sin\theta)\,\sin^2(\kappa\cos\theta)}{\kappa^2\sin^2\theta\cdot\kappa\cos\theta} \quad ,$$

$$\frac{R}{P_0\ell^2} = -\frac{\varkappa}{4\pi^2}\int_0^\infty \kappa\, d\kappa \int_{-\pi}^{\pi} (i\kappa\cos\theta)\cdot\frac{32 p_u^2 \sin^2(\kappa\mu\sin\theta)\sin^2(\kappa\cos\theta)}{\kappa^2\sin^2\theta \;\; \kappa\cos\theta}\;\times$$
$$\times\,\frac{v^2(-i\kappa\cos\theta)}{\delta^2 - \kappa^2\cos^2\theta\cdot v^2}\; d\theta$$

Substituting

$$\lambda = \kappa; \qquad \alpha = \kappa\cos\theta,$$

we obtain

$$\frac{R}{P_0\ell^2} = -\frac{8p_u^2\varkappa}{\pi^2}\int_0^\infty d\lambda \int_{-\lambda}^{\lambda} \frac{i\sin^2(\mu\sqrt{\lambda^2-\alpha^2})\sin^2\alpha\cdot\alpha\cdot\lambda}{(\lambda^2-\alpha^2)\left(\frac{\lambda}{v^2}-\alpha^2\right)}\;\frac{\lambda\, d\alpha}{\sqrt{\lambda^2-\alpha^2}} \quad . \tag{26}$$

The integrand has four poles on the real axis, at $\alpha = \pm\lambda$ and $\alpha = \pm\sqrt{\lambda}/v$ (Fig. 3, below). To calculate the inner integral we use the theorem of residues. For the contour of integration we take the semicircle C_r of infinitely large radius in the upper half of the complex plane α, going underneath the poles as shown. Substituting $\lambda = t^2/v^2$ in (26), we obtain finally, as a particular case of (23), the established formula of B.P. Bolshakov.

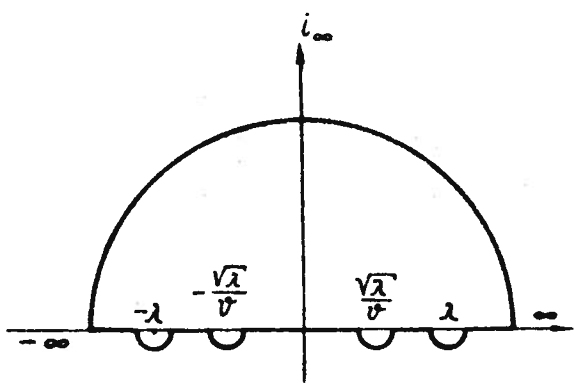

Fig. 3

$$R = \frac{16 p_u^2 v^2}{\pi \gamma g} \int_1^\infty \sin^2 \frac{\kappa t}{2} \sin^2 \frac{\mu \kappa t \sqrt{t^2-1}}{2} \frac{dt}{(t^2-1)^{\frac{3}{2}}} , \tag{27}$$

where
$$\kappa = \frac{2 g \ell}{v^2} .$$

TWO - DIMENSIONAL PROBLEM:-

In this case the velocity potential can be expressed as

$$\varphi(x,z,t) = \int_0^\infty \Big[(A \cos \kappa x + C \sin \kappa x) ch\, \kappa z +$$
$$+ (B \cos \kappa x + D \sin \kappa x) sh\, \kappa z \Big] d\kappa , \tag{28}$$

where A, B, C, D are unknown functions of t and k. For the solution we employ (1) and (2). Assuming $\partial p_u / \partial t = 0$, the right-hand side of (1) can be written as a Fourier integral:

$$\varkappa v \frac{\partial p_u}{\partial x} = \frac{\varkappa v}{\pi} \int_0^\infty d\kappa \int_{-\infty}^{\infty} \frac{\partial p_u}{\partial u} \cos \kappa (u - x) du =$$
$$= \int_0^\infty f_1(\kappa,t) \cos \kappa x \, d\kappa \int_0^\infty f_2(\kappa,t) \sin \kappa x \, d\kappa ,$$

where
$$f_1(\kappa,t) = \frac{\varkappa v}{\pi} \int_{-1}^{1} p_u \kappa \sin \kappa u du ; \qquad f_2(\kappa,t) = - \frac{\varkappa v}{\pi} \int_{-1}^{1} p_u \kappa \cos \kappa u du .$$

After substituting (28) into (2) we obtain the following system of equations:

$$\left.\begin{aligned} A\,sh\,\kappa H &= B\,ch\,\kappa H \\ C\,sh\,\kappa H &= D\,ch\,\kappa H \end{aligned}\right\} ; \tag{29}$$

$$\left.\begin{aligned} B &= A\,th\,\kappa H \\ D &= C\,th\,\kappa H \end{aligned}\right\} . \tag{30}$$

Substituting (28) in (1) and equating the coefficients of cos kx and sin kx we obtain the following system:

$$\left.\begin{aligned} \frac{\partial^2 A}{\partial t^2} - \kappa B - \kappa^2 v^2 A - 2v\kappa \frac{\partial C}{\partial t} - \kappa \frac{dv}{dt} C &= f_1(\kappa, t) \\ \frac{\partial^2 C}{\partial t^2} + \kappa D - \kappa^2 v^2 C + 2v\kappa \frac{\partial A}{\partial t} + \kappa \frac{dv}{dt} A &= f_2(\kappa, t) \end{aligned}\right\} \tag{31}$$

Eliminating B and D from (31) and (30), we have

$$\left.\begin{aligned} \frac{\partial^2 A}{\partial t^2} + \left[\kappa th\kappa H - \kappa^2 v^2\right] A - 2v\kappa \frac{\partial C}{\partial t} - \kappa \frac{dv}{dt} C &= f_1(\kappa, t) \\ \frac{\partial^2 C}{\partial t^2} - \left[\kappa th\kappa H - \kappa^2 v^2\right] C - 2v\kappa \frac{\partial A}{\partial t} - \kappa \frac{dv}{dt} A &= f_2(\kappa, t) \end{aligned}\right\} \tag{32}$$

Multiplying the second equation by i and adding to the first, and introducing

$$A_1 = A e^{i\kappa s}; \qquad C_1 = C e^{i\kappa s}; \qquad s = \int v\,dt$$

we obtain the following differential equation for W_1 which is represented as

$$W_1 = A + i C_1;$$

$$\frac{d^2 W_1}{dt^2} + \kappa th\kappa\, W_1 = f(\kappa, t),$$

where $$f(\kappa, t) = e^{i\kappa s}\left[f_1(\kappa, t) + i f_2(\kappa, t)\right].$$

The solution of this equation with zero initial conditions is

$$W_1 = \frac{\sin\delta t}{\delta}\int f\cos\delta t dt - \frac{\cos\delta t}{\delta}\int f\sin\delta t dt +$$
$$+\left(\frac{1}{\delta}\int f\sin\delta t dt\right)_{t=0}\cos\delta t + \left(-\frac{1}{\delta}\int f\cos\delta t dt\right)_{t=0}\sin\delta t \quad . \tag{33}$$

If the motion starts from rest, the two last terms become zero. Then

$$W_1 = \frac{\sin\delta t}{\delta}\int f\cos\delta t dt - \frac{\cos\delta t}{\delta}\int f\sin\delta t dt \quad . \tag{34}$$

The functions A, C are

$$A = Re(W_1 e^{-i\kappa s}) ; \tag{35}$$

and

$$C = \mathcal{J}m(W_1 e^{-i\kappa s}). \tag{36}$$

The velocity potential is then

$$\varphi(x,z,t) = \int_0^\infty \{Re\,(W_1 e^{-i\kappa s})\,[ch\kappa z + th\kappa H\cdot sh\kappa z]\,\cos\kappa x +$$
$$+\mathcal{J}m(W_1 e^{-i\kappa s})\,[ch\kappa z + th\kappa H\cdot sh\kappa z]\,\sin\kappa x\}\,d\kappa , \tag{37}$$

where W_1 is given by (34) and $s = \int v\,dt$.

The wave resistance is

$$\frac{R}{\rho_0 \ell} = \int_{-1}^{1} p_u \frac{\partial\zeta}{\partial x}\,dx = \int_{-1}^{1} p_u\, x\,\frac{\partial p_u}{\partial x}\,dx + v\int_{-1}^{1} p_u \frac{\partial^2\varphi}{\partial x^2}\,dx -$$
$$-\int_{-1}^{1} p_u \frac{\partial^2\varphi}{\partial x\partial t}\,dx \qquad \text{at} \qquad z = 0 \quad . \tag{38}$$

Substituting (37) in (38) and reversing the order of integration, we obtain

$$\frac{R}{p_0 \ell} = -\varkappa \int_{-1}^{1} p_u \frac{\partial p_u}{\partial x}\, dx - v \int_{0}^{\infty} \kappa^2 Re\, (\Phi \cdot G)\, d\kappa - \int_{0}^{\infty} \kappa \frac{d}{dt} \left[\Im m (\Phi \cdot G) \right] d\kappa, \tag{39}$$

where

$$\Phi(\kappa, t) = W_1 e^{-i\kappa S};$$
$$G(\kappa) = \int_{-1}^{1} p_u e^{-i\kappa x}\, dx. \tag{40}$$

Using these solutions for the case of constant speed, we get Lamb's formula for the resistance:

$$R = \frac{4 p_u^2}{\gamma} \sin^2 \frac{g\ell}{v^2}. \tag{41}$$

The results of the investigation of the wave resistance for the case of a constant acceleration, using (39) and the approximate method of Stationary Phase, are presented in Fig. 4 below. The hump speed for this motion is considerably less than for the case of steady motion.

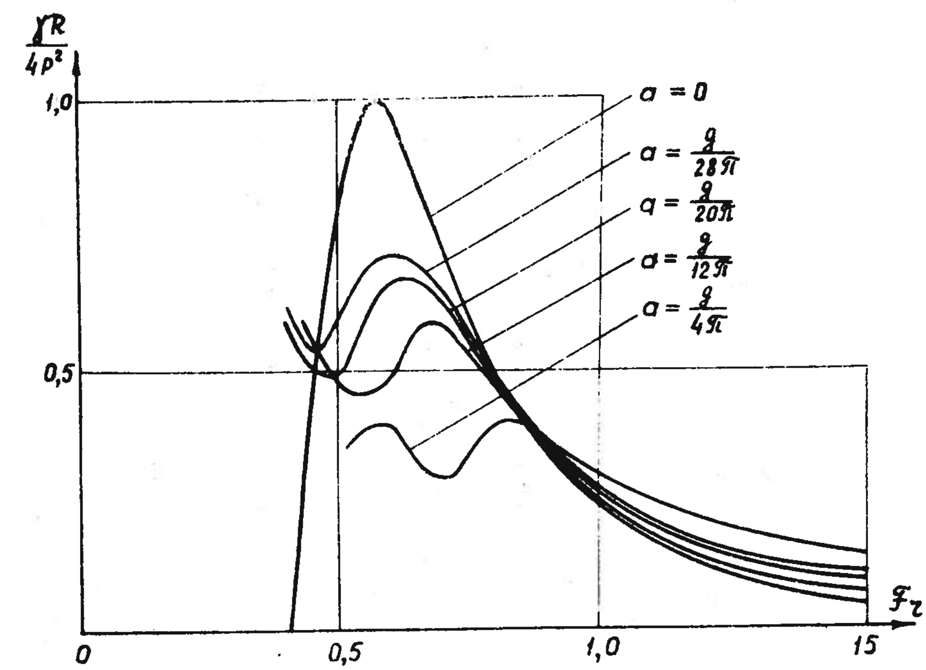

Fig. 4

Formulas (23) and (39) are recommended for the determination of wave resistance for non-steady motion.

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