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THE COST SAVINGS OF MULTIPLE SHIP PRODUCTION

John C. Couch

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THE DEPARTMENT OF NAVAL ARCHITECTURE AND MARINE ENGINEERING

THE UNIVERSITY OF MICHIGAN
COLLEGE OF ENGINEERING

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THE COST SAVINGS OF MULTIPLE SHIP PRODUCTION

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The Cost Savings of Multiple Ship Production

John C. Couch, University of Michigan

Introduction

We are all aware that increased production tends to lower costs, especially in mass production industries. To a lesser degree, this is also true in shipbuilding (See Fig. 1). However, few of the publications on manufacturing cost-quantity relationships, which mathematically express this downward trend, acknowledge their application to the shipbuilding industry. This lack of available information, plus the author's participation in a recent student project related to the problem, provided the impetus for further research which is reported here. Then, the object of this paper is to discuss the relationship between units built and unit costs and to present a theory which, it is found, can be used to estimate the cost savings of multiple ship production. Before discussing cost-quantity relationships applied to shipbuilding, it will be necessary to review briefly the development of these theories in other industries.

General Principle and Current Theories

In a repetitive manufacturing process many units can usually be produced for less apiece than can a few. The principal reasons for this cost reduction are:

1. The existence of non-recurring, preparatory, or tooling charges that are not repeated once production begins.
2. The savings resulting from buying materials in large quantities, and in more efficient sizes and shapes.
3. Increased labor efficiency with succeeding units.

This principle has been recognized for some time, and the first attempts,¹ to the author's knowledge, to express the mathematical relationship between product cost and quantity appeared in 1928. However, the early papers generally oversimplified the problem by ignoring some of the factors that contribute to the resulting cost reductions. The most sophisticated analyses are found in aircraft industry publications and apparently this industry pioneered work in the field. In fact, the first published formulation of the cost-quantity relationship that the author believes applicable to shipbuilding, appeared in the *Journal of Aeronautical Sciences* in 1936.² This is the progress of learning curve theory.

The learning curve theory is a cost-quantity formulation that was developed and used prior to World War II by airframe manufacturers and the U.S. Air Force for estimating the cost of producing airframes.³ The theory, as it is most popularly known, states that as the number of units produced doubles the "cost per unit" decreases by some constant percentage. Two different forms of the theory have developed: the first treats the "cost per unit" as the cumulative average cost of X units; the second treats it as the added cost of producing the Xth unit. In either form, the relationship expressed by the theory is a hyperbolic function that plots as a straight line on logarithmic grid paper (i.e., it is a log-linear relationship). Figures 2 and 3 are the graphical representations of these two forms. Their traditional mathematical formulations are easily understood and the popularity of this theory may be attributed to the simplicity of these formulations and to the ease with which they may be plotted.

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Mathematics of Learning Curve Theory

The first form of this theory, cumulative average cost log-linearity, was originally expounded by T. P. Wright in Ref. 3. He proposed cumulative average log-linearity for both direct labor and material costs, but excluded non-recurring costs. This form of the learning curve theory is formulated as

$$\bar{Y} = aX^b \quad (1)$$

where

\bar{Y} = the cumulative average cost per unit

X = the number of units

a = the cost of the first unit

b = a parameter related to the Slope of the learning curve (it is literally the geometric slope of the line on logarithmic grid paper).

In learning curve terminology, Slope is normally defined as the ratio of the cost per unit for $2X$ units to the cost per unit for X units. That is,

$$\text{Slope} = S = \frac{a(2X)^b}{aX^b} = 2^b \quad (2)$$

when we speak of an 80 per cent learning curve ($S = 0.800$, $b = -0.322$), we mean that the cost per unit decreases 20 per cent every time the number of units is doubled.

Now if Equation 1 holds true for cumulative average cost, the cumulative total for X -units (Y) would be

$$Y = \bar{Y} X = aX^{1+b} \quad (2)$$

and the added cost of the X th unit (y_i) would be

$$y_i = a(X_i^{1+b} - X_{i-1}^{1+b}) \quad (3)$$

The various relationships expressed by this form of the theory are plotted in Fig. 2.

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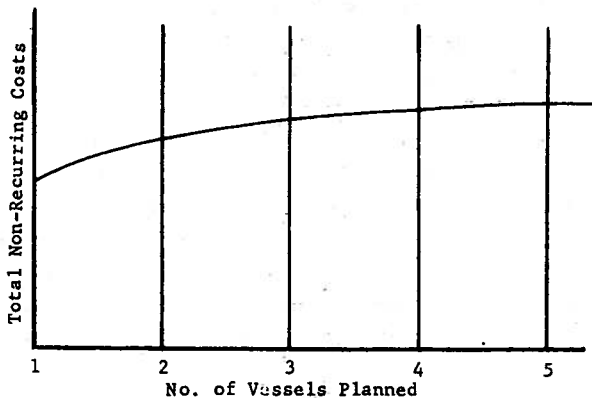


Fig. 4

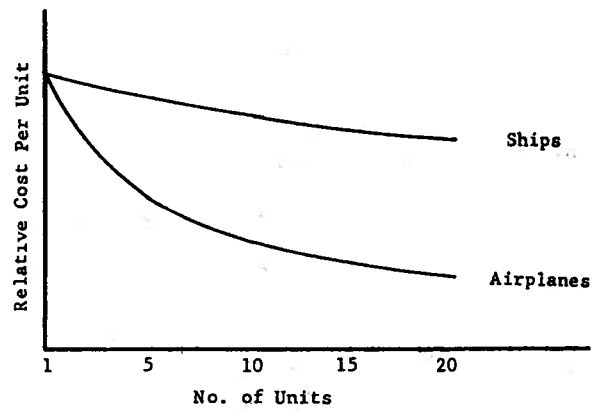


Fig. 1

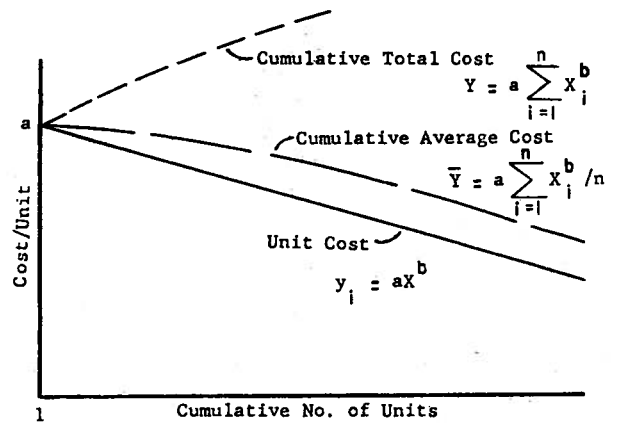


Fig. 2 Cumulative Average Cost Log.-Linearity (Logarithmic grid)

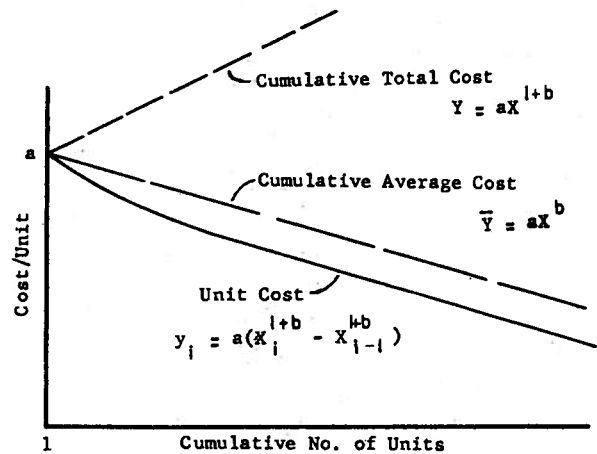


Fig. 3 Unit Cost Log.-Linearity (Logarithmic grid)

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The second form of the learning curve theory was probably originally presented in Ref. 4. This form assumes unit cost rather than cumulative average cost, log-linearity. Again, in Ref. 4, the theory was applied to direct labor costs, exclusive of non-recurring costs. The log-linear unit cost curve is formulated as

$$y_i = aX^b \quad (4)$$

where the symbols have the previously stated meanings. If the Equation 4 holds true for unit costs, then cumulative total and cumulative average costs would be expressed by Equations 5 and 6 respectively:

$$Y = a \sum_{i=1}^n X_i^b \quad (5)$$

$$\bar{Y} = a \sum_{i=1}^n X_i^b / n \quad (6)$$

The relationships expressed by Equations 4, 5 and 6 are plotted in Fig. 3.

In studying the relationships proposed by these two forms of the theory and represented in Figs. 2 and 3, several observations can and should be made. First, if the cost-quantity relationship for a particular product is found to be log-linear it can be either unit cost log-linear, or cumulative average log-linear, but it cannot be both. Second, since no matter how many units are produced, the costs can never reach zero, there must be a limit to the proposed long-linear relationships. Some attempts have been made⁵ to determine where this deviation would begin and have indicated that a leveling off of the curve will take place only after a very large number of units have been produced (i.e., 100 to 1000 or more, depending on the complexity of the product and the experience of the plant). Since comparatively few vessels are involved in peacetime multiple shipbuilding contracts we need not be concerned with this aspect of the problem. The third observation that can be made is that in order to assume a log-linear relationship between costs per unit and cumulative output, we assume that plant facilities and product design remain more or less constant. That is, no significant changes are made in the plant's production facilities and no major product design changes occur.

Obviously, to be able to estimate costs of succeeding units by means of the learning curve

technique, once the cost of the first unit is estimated, we must know what Slope is applicable to the particular product and plant concerned. This can only be determined from evaluation of the past cost records of that plant—the accuracy of the chosen Slope depending on the extent and accuracy of the company's records of costs of similar products. Because of the many studies done and the wealth of empirical data available to airframe manufacturers, they have made considerable progress in predicting reliable Slopes for different types of aircraft.

The Slope of a given learning curve will depend on several factors, the most important of which are the particular plant's experience and the product's complexity. The more experienced plants and/or the less complex products will dictate rather flat learning curves (greater per cent Slopes) since their increase in overall production efficiency stands to improve less with succeeding units than if the plant were inexperienced and/or the product relatively complex. Another factor that will influence the Slope is the extent of initial planning and tooling. The more extensive the preparation, the flatter the learning curve because more effort would have been made to optimize production before it actually began.

Cost-Quantity Relationships Applied to Shipbuilding

The author found that there is little published material on shipbuilding costs and cost estimating and that what *has* been written generally gives no more than token acknowledgement to the principle discussed here. What is more surprising is the absence of any published attempt (to the author's knowledge) to determine the relation between ship costs and the quantity built, prior to December, 1945.⁵ In July of that same year, in fact, H. M. Neuhaus,⁶ in discussing the effect on cost of a multiple ship contract and the information available on the subject, stated that, "Unfortunately no statistics or data were available, no reports, correspondence or literature having been published anywhere either in domestic or foreign papers."

Whether this absence of published material on the subject was due to a lack of knowledge of an applicable cost-quantity relationship or due to the industry's reluctance to release its cost data is difficult to determine. There is evidence to support either possibility. Since the early papers on shipbuilding costs attributed these costs savings sing-

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ularly to the resulting greater distribution of overhead⁷ or just to the non-recurring costs,⁸ there apparently was some lack of understanding of the problem and it is significant that the first application of a particular cost-quantity relationship to shipbuilding⁵ appeared in a government publication, *Monthly Labor Review*.

In this particular publication, A. D. Searle applied the learning curve concept to man-hour cost information obtained from several shipbuilders (where man-hour cost is that of direct labor plus overhead in proportion to direct labor and exclusive of non-recurring costs). Searle shows that for each yard's wartime Liberty, Victory, and tanker shipbuilding programs, the unit man-hour costs (man-hour cost for each additional, similar vessel) are log-linear. Specifically he shows that the Slope of these unit cost curves for each type of vessel built in the various yards is close to 80 per cent.

Three years after Searle's work appeared, W. B. Ferguson and B. V. Tornborgh⁹ presented another cost-quantity relationship for shipbuilding man-hours. This was

$$Z = C \sqrt{X - 1} \quad (7)$$

where

Z = Per cent savings of recurring direct labor costs

X = Number of units produced

C = A constant for any given yard, varying from 8 for an old and experienced yard to 20 for a "green" yard.

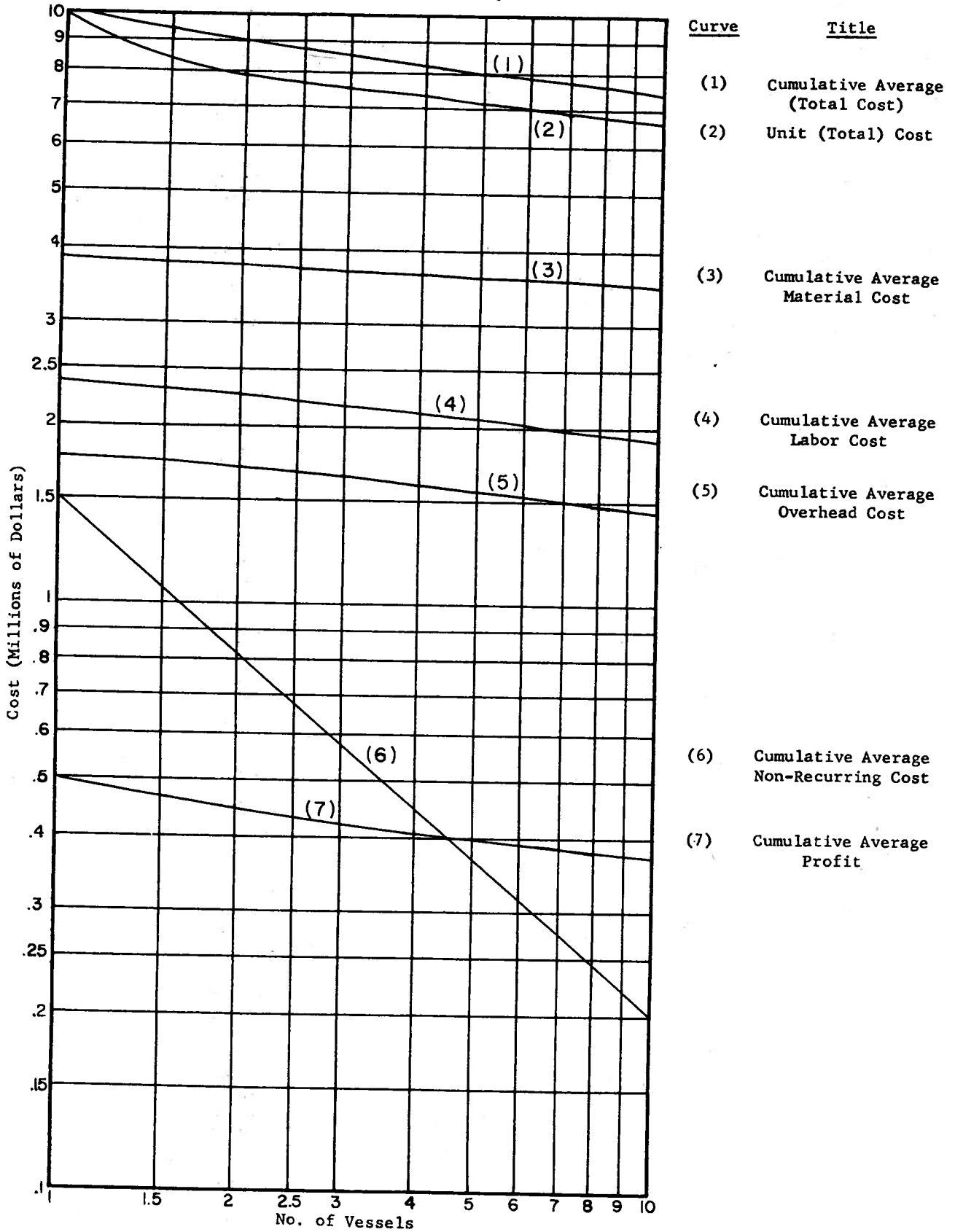
Although Ferguson and Tornborgh give an excellent comprehensive discussion of the whole problem, they present a mathematical formulation, Equation 7, only for direct labor costs, exclusive of non-recurring costs.

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Number of Ships	Number of Bidders	Type of Ship	y_1		\bar{Y}		Choice
			Slope	Accur.	Slope	Accur.	
3	11	T-AKA	0.825	E	0.91	A	\bar{Y}
5	1	Gen'l Cargo	0.890	C	0.945	A	"
10	1	Container		E	0.956	C	"
5	1	(Marad Data)		E	0.940	B	"
5	1	C-1		E	0.920	B	"
6	1	Farrel		E	0.910	A	"
10	Aver.	Cargo		E	0.930	A	"
6	1	Cargo		E	0.920	B	"
6	4	Cargo		E	0.937	B	"
3		Ferry	08.40	E	0.940	E	"
3	2	Ferry	0.887	A	0.940	A	TIE
8	1	Roll-On	0.950	B	0.965	A	\bar{Y}
10	1	Roll-On		C	0.970	A	"
5	Many		0.940	C	0.960	B	"
3	5	Cargo	0.930	A	0.960	A	TIE
3	5	Ferry	0.880	E	0.930	B	\bar{Y}
	1	Cargo	0.877	E	0.917	C	"
	1		0.930	E	0.970	B	"
3	1	Ferry	0.840	C	0.910	A	"
6	5	Cargo	0.92	C	0.950	A	"
7	Many	Cargo	0.83	E	0.940	C	"
			0.96	A	0.980	A	"
3	2	Container	0.93	B	0.960	A	"
3	7	Tanker	0.81	E	0.91	C	"
3	7	Bulk		E	0.982	B	"

*Bids rated A, B, C, D, or E, depending on how accurately they conformed to each type of Learning Curve.

Fig. 5
Component Cost Synthesis



(Continued from page 53)

Apparently no further information on shipbuilding cost-quantity relationships appeared until July, 1962, when Professor Harry Benford¹⁰ proposed a cumulative average log-linear relationship for total shipbuilding costs, with an average Slope of 93.3 per cent for general cargo ships. This proposal was based on the evaluation of limited data and was put forth without knowledge of work done in other industries.

In view of this dearth of conclusive information pertinent to shipbuilding cost-quantity relationships and with the knowledge that other industries have found strong evidence of log-linear cost quantity relationships, a group of students* at the University of Michigan undertook the following analysis to determine if these relationships would, in fact, hold true for total shipbuilding costs.

Total Shipbuilding Cost Analysis

The analysis consisted of the evaluation of recent bids, from many yards, for 23 different designs. In each case the bids applied to alternative numbers of ships. It was assumed that the estimators, in preparing the bids, were unfamiliar with the cumulative average and unit cost log-linear concepts, or at least they ignored them, and therefore their bids were influenced by neither school of thought. The various bids were converted, where necessary, to both cumulative average (total) costs and unit (total) cost and then both were plotted (versus the number of ships to be built) on logarithmic grid paper. The overwhelming conclusion of the analysis, which is summarized in Table I, is that cumulative average ship costs approach log-linearity; unit costs do not. It was found that cumulative average cost of recent general cargo ships, in particular, could be closely represented by a 93.5 per cent learning curve.

Table 2 shows the close agreement between the prices, determined by the apparent low bidder, for C4-S-65a vessels,¹² and those predicted by a 93.5 per cent learning curve, where $\bar{Y} = \$15,090,000 X^{-0.097}$. Note that the 93.5 per cent Slope is an average for all the yards bidding on the eleven general cargo vessels included in the study. Since the individual Slopes ranged from 91 to 96 per cent this average must be used with caution. Also,

*Naval Architecture and Marine Engineering students in Professor Benford's Fall, 1962, Shipbuilding Contracts and Cost Estimating class.

individual circumstances may cause alternating fluctuations above and below the theoretical line. Table 3 gives ratios of average cost per ship to the cost of the first ship for multiple general cargo vessels where cumulative average costs are log-linear with a 93.5 per cent Slope.

Because some of the bid data used for this analysis were the same as those used by Benford, the close agreement of the two separate conclusions does not mean one definitely confirms the other; however, the class study does strengthen Benford's original conclusion. This conclusion can be rationally substantiated only by evaluating separately each of the factors that contribute to the downward trend of cost with succeeding vessels. In fact, the reliability of estimating techniques, based on the cumulative average learning curve theory, will depend, to a large extent, on the individual yard's ability to evaluate the effect of the various component costs on the total. Because an analysis of the component shipbuilding costs would provide a better understanding of their inter-relationships and could serve as a check on the conclusion reached by Benford and the class study, the author prepared the following synthesis.

Component Cost Synthesis

Imaginary contracts of from one to ten vessels were assumed, with the cost of the first vessel set at \$10,000,000. The total ship cost was divided, to the best of the author's ability, into five broad components, the effect of multiple production on each was estimated, and they were then synthesized to provide total costs. Total costs were then plotted versus the number of ships, on logarithmic grid paper.

The five cost components used in the study were the recurring direct and miscellaneous production costs for *labor* and *materials*, the *non-recurring costs*, *overhead* and *profit*. The following assumptions were made in regard to the distribution of these component costs for the first vessel (See Table 4).

Miscellaneous Assumptions:

1. Labor and material costs were \$2,370,000 and \$3,860,000 respectively (i.e., they were assumed to be 38 and 62 per cent of a \$6,230,000 direct building cost).
2. Overhead charges (75 per cent of labor) were \$1,770,000.

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3. Labor, materials, and overhead amounted to 80 per cent of the ship's total cost to the customer.
4. Non-recurring costs amounted to 15 per cent of the total.
5. Profit was 5 per cent of the total.

The following reasoning determined how each component of this synthesis would be affected by multiple production.

Labor: Of the recurring costs in many industries, that of labor decreases most with succeeding units. Extensive analysis of labor learning in aircraft manufacturing³, as well as Searle's analysis of shipbuilding labor efficiency during World War II, indicated that labor learning curves are unit cost log-linear with approximately 80 per cent Slopes. In view of this evidence and with the additional consideration that peacetime shipyard labor forces are experienced and would show less

labor learning than the wartime industry as a whole, a 90 per cent unit cost, log-linear, labor curve was chosen.

Material: The cost savings resulting from buying materials in larger quantities and in more efficient shapes and sizes, together with the savings from reduced scrappage are usually relatively small. This is especially true in well established industries like shipbuilding and in industries that buy from manufacturers who have already produced their product in larger numbers. In 1936, Wright² proposed cumulative average, log-linear material cost curves with Slopes ranging from 90 to 95 per cent. He showed that the Slope depended on the amount of labor involved in processing the material before it was purchased from the vendor, the more complex, hardware items, having steeper Slopes than raw materials. Because large quantities of "raw" materials are used in shipbuilding and because of the many standard

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Table 2

**Cumulative Average Costs (\$1,000)
For C4-S-65a Vessels**

Cumulative Average Cost for Each of	1	2	3	4	5	6
Low Bidder	\$15,090	\$14,094	\$13,595	\$13,150	\$12,650	\$12,600
93.5% Learning Curve	15,090	14,109	13,566	13,189	12,905	12,679
Per cent Difference	—	0.1%	-0.2%	.03%	2.0%	0.6%

Table 3

**Multiple Ship Cost Reduction Factors
For 93.5 Per Cent Cumulative Average**

Number of Ships In Contract	Ratio of Average Cost per Ship to Cost of Single Ship	Ratio of Cost of Each Additional Ship to Cost of Single Ship
1	1.000	1.000
2	0.935	0.870
3	0.897	0.830
4	0.874	0.796
5	0.856	0.784
6	0.840	0.760
7	0.828	0.750
8	0.816	0.745
9	0.808	0.735
10	0.800	0.730

sizes and types of these materials that are purchased in quantity by today's yards, the author felt that a curve for shipbuilding material costs would have a somewhat flatter Slope than those predicted by Wright. Therefore a 97 per cent cumulative average, log-linear curve was chosen for material costs.

Overhead: Overhead charges were assumed to be a constant percentage (in this case 75 per cent) of direct labor costs, primarily because those factors causing a decrease in direct labor costs would also be at work in overhead costs.

Non-Recurring Costs: In shipbuilding the costs that, for the most part, are not repeated once production begins include:

1. Engineering and Drafting
2. Production Planning
3. Purchasing
4. Mold Loft Work
5. Jigs and Forms

The increase in these costs with increasing planned output is a recognized phenomenon in other industries³ and some mention of it in regard to shipbuilding is contained in Ref. 9. The lack of empirical data, however, precludes its accurate formulation. A study of the above components suggests that the increase would be slight and that it would be less as the planned number of vessels increases. A similar conclusion is reached in Ref. 11 with regard to total engineering man-hours in aircraft production. This assumed relationship is shown in Fig. 4. If plotted on a loga-

rithmic grid the curve would be linear. For this synthesis, therefore, total non-recurring costs versus the number of planned vessels were assumed log-linear, increasing from 15 per cent of the total for a single-vessel contract, to 20 per cent of the first vessel's cost for a ten-vessel contract.

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Table 4

1. Determination of Costs for First Ship

Recurring Material Cost.....	\$ 3,860,000
Recurring Labor Cost.....	2,370,000
Recurring Overhead Cost.....	1,770,000
<hr/>	
Recurring Direct Building Cost.....	\$ 8,000,000
Non-Recurring Cost.....	1,500,000
Profit	500,000
<hr/>	
Total Cost to Customer.....	\$10,000,000

2. Component, Cost-Quantity Relationships

- Recurring Material—Cumulative Average Cost Log-Linear (97% Slope)
- Recurring Labor—Unit Cost Log-Linear (90% Slope)
- Recurring Overhead—75% of Labor
- Total Non-Recurring Costs—Log-Linear
- Profit—5% of Total

3. Calculation of Unit Costs (Millions of Dollars)

No. Planned	1	2	3	4	5	6	7	8	9	10
Labor.....	2.37	2.13	2.01	1.92	1.86	1.81	1.76	1.73	1.70	1.67
Materials.....	3.86	3.63	3.54	3.49	3.46	3.43	3.40	3.36	3.34	3.32
O.H.....	1.77	1.60	1.51	1.44	1.40	1.36	1.32	1.30	1.28	1.25
Non-Recur.....	1.50	0.07	0.03	0.02	0.01	0.04
Profit.....	0.50	0.39	0.37	0.36	0.35	0.35	0.34	0.34	0.33	0.33
Total.....	10.00	7.82	7.46	7.23	7.08	6.96	6.82	6.73	6.65	6.57

4. Calculation of Cumulative Average Costs (Millions of Dollars)

No. Planned	1	2	3	4	5	6	7	8	9	10
Labor.....	2.37	2.25	2.17	2.11	2.06	2.02	1.98	1.95	1.92	1.90
Materials.....	3.86	3.74	3.68	3.63	3.59	3.56	3.54	3.52	3.51	3.49
O.H.....	1.77	1.69	1.63	1.58	1.55	1.52	1.49	1.46	1.44	1.43
Non-Recur.....	1.50	0.82	0.57	0.45	0.37	0.31	0.27	0.24	0.22	0.20
Profit.....	0.50	0.45	0.42	0.41	0.39	0.39	0.38	0.37	0.37	0.37
Total.....	10.00	8.95	8.47	8.18	7.96	7.80	7.66	7.54	7.46	7.39

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Profit: Profit was always assumed to be 5 per cent of the total cost to the customer.

The synthesis of the above cumulative average component costs yielded a cumulative average (total) cost curve, Fig. 5, which was very nearly log-linear and again the unit (total) cost curve was not. Since the cumulative curve has a mean Slope of 90.5 per cent, the cumulative average costs of multiples of the hypothetical ship could be expressed by the formula $\bar{Y} = \$10,000,000 X^{-0.144}$.

Because the final result of this synthesis is realistic, the author believes that the method is sound and that it suggests how careful evaluation of component cost-quantity relationships could predict reliable estimates for multiple contract ship costs. Cost estimators should have better inputs than used here, of course, and to use this technique, their accounting procedures should be modified to separate out the non-recurring costs and, where necessary, to group costs in similar categories so the data could be readily evaluated and synthesized.

Conclusion

The foregoing investigations indicate that reliable predictions of the cost savings of multiple ship production can be made by using the learning curve technique to estimate cumulative average ship costs for any number of vessels. Furthermore, individual yards, through careful analysis of cost records and the incorporation of accounting procedures that adequately evaluate component cost behavior, could determine accurate curves for different types of vessels and, in so doing, provide a valuable tool for easing the task of analyzing multiple cost savings.

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ACE BULLETIN KEYWORD CONCEPTS

— Link A —

Active (8) Estimating Calculating	Passive (9) Multiple Ship Costs Component Costs Non-Recurring Costs Unit Costs	Means or Methods (10) Cost-Quantity Relationships Learning Curve Techniques Accounting Procedures Log-Linearity
Used In (4) Shipbuilding	Dependent Variables (7) Unit Cost	Independent Variables (6) Number of Ships

Abstract:

This paper discusses the application of cost-quantity relationships used in other industries to shipbuilding. Specifically, the author shows how reliable estimates of the cost savings of multiple ship production can be made by careful accounting procedures which separate the component shipbuilding costs and by the application of learning curve techniques. The mathematical formulation of applicable cost-quantity relationships is included, as well as ship cost data. The author believes that these techniques provide a valuable tool for easing the task of cost estimating for multiple ship contracts.