No. 148
February 1974

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The preparation of this paper was supported by a grant of the National Science Foundation (Grant GK-36848)

THE DEPARTMENT OF NAVAL ARCHITECTURE AND MARINE ENGINEERING

THE UNIVERSITY OF MICHIGAN
COLLEGE OF ENGINEERING

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Paper prepared for
International Symposium on the Dynamics
of Marine Vehicles and Structures in Waves
London, April 1974

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ABSTRACT

Some of the fundamental assumptions of ship-motion theory are examined for the purposes of elucidating the success of the heuristically derived strip theory of Korvin-Kroukovsky and of recognizing some inadequacies of that theory. The formal approach employed is a systematic slender-body expansion. In the zero-speed problem, both far-field and near-field views can be used generally to recommend the assumption that frequency of oscillation be taken as "large," in the sense that the corresponding waves have wavelength that is comparable to ship beam. This assumption leads to considerable trouble in the head-sea case, however, and this special case has not been fully solved yet. Force and moment on a ship can be computed, even in the head-sea case, through use of the Khaskind formula, but computation of the load distribution necessitates solving the diffraction problem, or, possibly, solving two-dimensional near-field problems involving the Helmholtz equation. Rationalization of the short-wave assumption is not really successful in the forward-speed case, except in terms of the observed accuracy of the motion predictions. The most thorough analyses to date of the forced-motion and head-sea diffraction problems are based on disparate assumptions about the orders of magnitude of the characteristic wavelengths, although both require the product of speed and wave frequency to be large. Some discussion is presented on the interaction between ship oscillations and the steady-motion perturbation of the incident stream.

INTRODUCTION

There are three general approaches to the formulation of a theory of ship motions:

* heuristic stripwise analysis, in which two-dimensional boundary-value problems are formulated at the start;
* linearized analysis based on the complete three-dimensional boundary-value problem, stripwise approximations being introduced ultimately into the final formulas;
* systematic perturbation analysis, which may lead to any of several final forms, depending on what assumptions are made initially.

There have been enough reviews of this subject in recent years, and so I shall not describe these various approaches in a systematic or thorough fashion. (For a recent review, see Ogilvie & Beck (1973).) Instead, I hope to show how different initial assumptions lead to certain conclusions and why some of these assumptions are more reasonable than others. Thus, in a sense, I shall be emphasizing the third approach, and this paper can be considered perhaps as a sequel to the article by Newman (1970), which deals specifically with the systematic application of the slender-body idealization to ship problems.

There is one basic property of a ship which makes possible many hydrodynamic calculations, namely, its shape. Except in the regions near bow and stern, the cross-section changes slowly in both size and shape along the longitudinal axis. Of course, this property has been utilized in all of the above-mentioned approaches to predicting ship motions, for it is the key to reducing the intractable three-dimensional (3-D) boundary-value problems to manageable two-dimensional (2-D) problems. However, there are several possible ways of introducing the simplification, even within the context of a careful perturbation analysis, with results that differ profoundly. Moreover, the exception noted (concerning the ship ends) leads to many difficulties in any approach, and this situation has not been thoroughly studied yet.

One aspect of particular importance is the relationship assumed (or implied) between characteristic wavelengths and ship dimensions . Whenever a

3-D problem involving a free surface is reduced to a set of 2-D problems, there is some choice to be made as to how the wavelengths enter into the problem: Are they intimately connected with the problems in the transverse planes, or do they have their effect primarily on the 3-D aspects, in which interactions between sections arise and forward-speed effects modify predictions? This will be a major focus of attention in this paper.

It will be assumed (unless specifically stated otherwise) that the hydrodynamic problems of interest can be linearized, either heuristically or through the use of a systematic perturbation technique. In the latter case, there may be more than one small parameter, and "linearization" may have various meanings, but we shall always linearize with respect to a motion parameter, which means that superposition of our motions problems is permissible. This allows us to separate each ship-motion problem into a problem of wave excitation acting on a restrained body and a separate problem concerning the actual motions that result from the excitation. The latter is the easier problem to handle, and so we shall be examining it first.

It is also much easier to treat problems in which the ship has no forward speed. Several aspects of the zero-speed problem can be examined rather precisely, and we shall do this in some detail before even considering the forward-speed problem. Major difficulties arise when we try to extend the same considerations to the forward-speed problem, but obviously we must attempt it nevertheless.

It has long been recognized that the slender-body theory of ship motions is similar to the Korvin-Kroukovsky (1955) strip theory if one assumes that the frequency of oscillation is high enough. A disturbance at radian frequency ω produces gravity waves of length $\lambda = 2\pi/\nu = 2\pi g/\omega^2$, and so "high frequency" means "short waves." Similarity between the theories is achieved, specifically, if $\lambda = O(\varepsilon)$ or $\nu = O(\varepsilon^{-1})$, where ε is the slenderness parameter, which is assumed to be very small. Mathematically, of course, this means only that λ/ε remains bounded as $\varepsilon \to 0$ asymptotically. Physically, however, one is tempted to imply more, namely, that λ is comparable with ship beam*. This physical approach is justified by the fact that we intend ultimately to apply the theory to problems in which the ship beam is quite finite, and so the mathematical arguments about what happens as $\varepsilon \to 0$ are not very useful in the practical problem. However, the physical approach can be misleading, as we shall show.

Suppose that a ship is heaving with a sinusoidal motion at radian frequency ω. At points not too near the ship, we may expect that a fluid motion equivalent to that produced by the heaving ship can be produced by a line of pulsating sources located along the axis of the ship. Let the velocity potential of the source-induced motion be represented by

$$Re\,\{\phi(x,y,z)\, e^{i\omega t}\} ,$$

in which case $\phi(x,y,z)$ is given by

$$\phi(x,y,z) = -\frac{1}{2\pi}\int_{-L/2}^{L/2} d\xi\, \sigma(\xi) \int_{\underset{\frown\!\!\to}{0}}^{\infty} \frac{k\, e^{kz}}{k-\nu} J_0(kR)\, dk , \qquad (1)$$

where $R = \sqrt{(x-\xi)^2 + y^2}$, $\sigma(x)$ is the source density along the axis, $J_0(kR)$ is the Bessel function which is usually denoted this way, and the coordinate axes are taken with the x axis along the longitudinal axis of the ship, the z axis upwards, and the y axis laterally in the plane of the undisturbed free surface. The arrow under the inner integral sign indicates that the

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*We shall generally imply that ship length, L, remains fixed as $\varepsilon \to 0$, and so ε is a small parameter somehow representing the size of the beam of the ship.

integral is to be computed along a contour indented above the simple pole at $k = \nu$. If the complex conjugate of this potential function is multiplied by -4π , the result is identical to Equation (13.17") of Wehausen & Laitone (1960); the difference is caused by slightly different conventions adopted by those authors. For large $r = \sqrt{x^2+y^2}$, the above potential can be approximated by the following:

$$\phi(x,y,z) \sim \frac{i\nu}{2} e^{\nu z} \int_{-L/2}^{L/2} d\xi \ \sigma(\xi) \ H_0^{(2)}(\nu R) \quad ,$$

where $H_0^{(2)}$ denotes a Hankel function. From the addition theorem for Bessel functions, Equation (9.1.79) in Abramowitz & Stegun (1964), we rewrite this:

$$\phi(x,y,z) \sim \frac{i\nu}{2} e^{\nu z} \sum_0^\infty \varepsilon_n H_n^{(2)}(\nu r) \cos n\theta \int_{-L/2}^{L/2} d\xi \ \sigma(\xi) \ J_n(\nu\xi) \quad ,$$

where

$$\varepsilon_n = \begin{cases} 1 \ , & n = 0 \ , \\ 2 \ , & n \geq 1 \ . \end{cases}$$

For $\sigma(x)$ equal to a constant, say σ_0 , we can evaluate this easily. Only the terms for even n will be nonvanishing, and the integrals of the J_n functions can be computed from the series expressions:

$$\int J_p(x) \ dx = 2 \sum_{k=0}^\infty J_{p+2k+1}(x) \ .$$

At the level of the free surface, we have $z = 0$ approximately. Thus we obtain $\phi(x,y,0)$ as a function of r and θ . The amplitude of ϕ , that is, $|\phi|$, is shown in a polar diagram in Figure 1. This quantity is proportional to the wave amplitude. Thus, the radial distance to the curve at any angle represents the amplitude of the wave that propagates outwards in that direction. For $\nu L = 2$ (that is, for $L/\lambda = 1/\pi$), the waves propagate outwards with almost the same amplitude in every direction. On the other hand, for $\nu L = 20$ ($L/\lambda = 10/\pi$), the waves propagate almost entirely in the broadside direction. Two intermediate cases are also shown, and it is seen that even in the case $\nu L = 10$ ($L/\lambda = 5/\pi$) the waves moving laterally have an amplitude five times that of the waves moving away longitudinally.

The situation represented in Figure 1 is valid only at large distance from the disturbance. In fact, the Hankel function was even evaluated by its

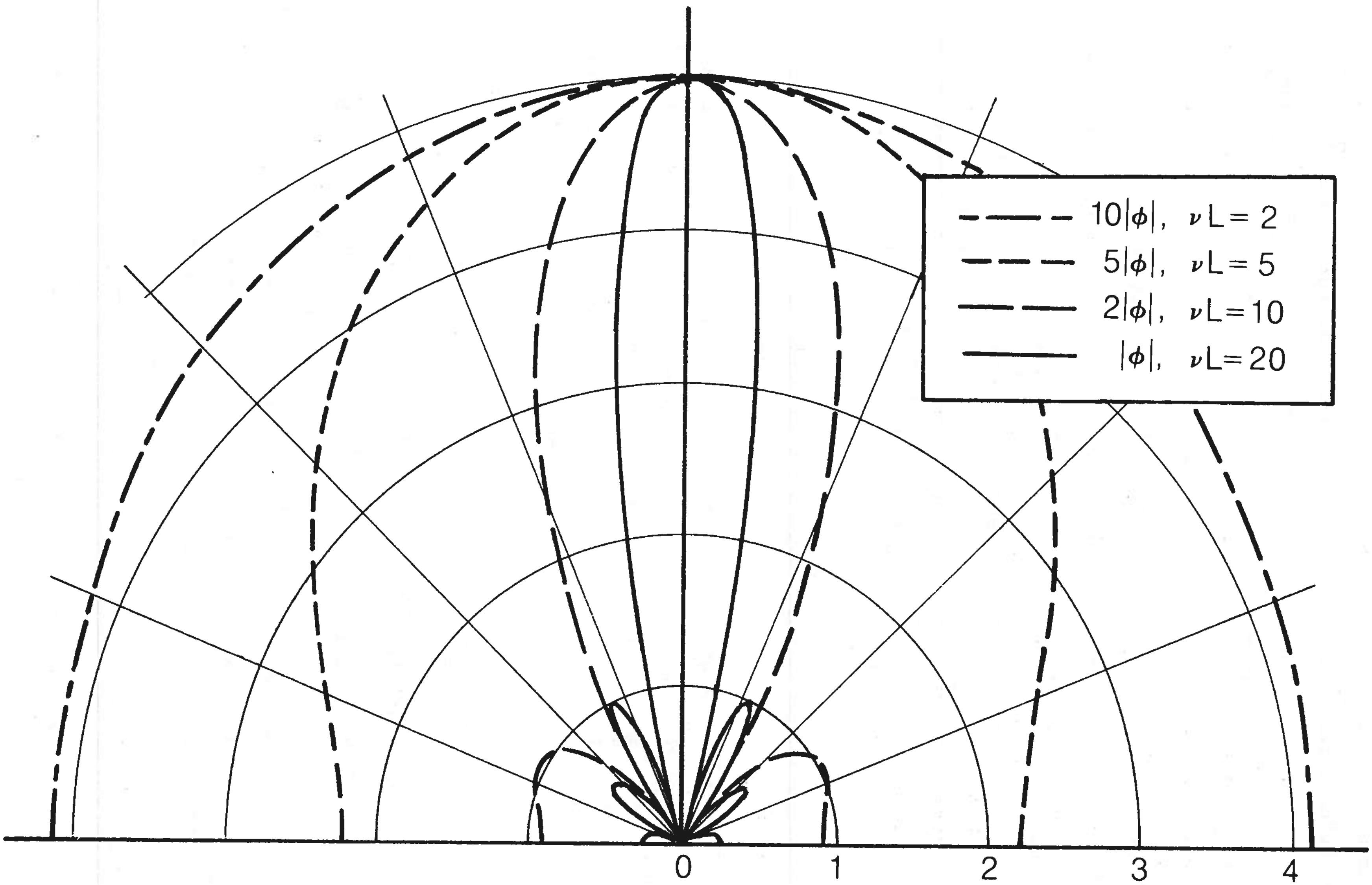

Figure 1. Polar diagram of $|\phi|$ in Equation (1), for $r = \sqrt{x^2+y^2} >> L$, with $z = 0$, $\sigma(x) = 1/2$.

large-argument approximation. However, the implication is clear: If the length of the disturbance is only slightly longer than the length of the waves that it generates, there is a strong focusing effect. In a qualitative way, this is not surprising, for such behavior is well-known in all branches of wave theory. However, the intensity of the focusing action is not well-recognized in ship hydrodynamics. "Short" waves do not have to be very short at all; they may be almost as long as the ship that generates them, at least when the ship oscillates in the heave mode.

In formulating a theory for predicting ship motions, we must make some explicit assumption relating wave length and ship dimensions. In Figure 1, we show a range of conditions, from L/λ approximately equal to $1/3$ up to L/λ equal to about 3, but we cannot very well encompass such a range in a single theory. We must assume a relationship between L/λ and the slenderness parameter, ε. There are two obvious possibilities:

i) Moderate-length waves, i.e., $\lambda = O(1)$ and $\nu = O(1)$;

ii) Short waves, i.e., $\lambda = O(\varepsilon)$ and $\nu = O(\varepsilon^{-1})$.

From a completely systematic point of view, we might add other possibilities, but no other seems to offer useful results. Newman (1970) discusses the literature on the above two possibilities, and so there is no need to cover that ground again. Ursell (1962) studied these two problems very thoroughly from the far-field point of view, although unfortunately his inner expansion of the far-field solution is not useful in the short-wave case since, in his near-field approximation, he requires always that lateral distances be small compared with wavelength, rather than small compared simply with ship length. The emphasis here will be on the physical interpretation of these results.

The velocity potential in (1) can be taken as the starting point for a far-field description in any case, although it is not "consistent" from the point of perturbation theory. In the moderate-wavelength case, the inner expansion of (1) is the sum of two terms, one being a function of x only, the other being proportional to $\log\sqrt{y^2+z^2}$. On the other hand, for $\lambda = O(\varepsilon)$, Ogilvie & Tuck (1969) showed that (1) can be simplified, even for $y = O(1)$, to the following:

$$\phi(x,y,z) \sim -\sqrt{\frac{\nu}{2\pi}}\, e^{\nu z} e^{-i\pi/4} \int_{-L/2}^{L/2} d\xi\, \frac{\sigma(\xi)\, e^{-i\nu R}}{R^{1/2}} \quad . \tag{2}$$

That is, (2) is a valid approximation even at distances from the ship that are comparable with ship length. The integrand in (2) has a point of stationary phase at $\xi = x$ if $|x| < L/2$, and so the integral can be approximated:

$$\phi(x,y,z) \sim i\,\sigma(x)\, e^{\nu z} e^{-i\nu|y|} , \quad |x| < L/2 , \tag{3}$$

which, when combined with the time-dependence factor, $e^{i\omega t}$, obviously represents an outgoing 2-D wave. There is no point of stationary phase if $|x| > L/2$, and so the solution in that region is smaller by an order of magnitude. The result expressed in (3) remains valid even if $y = O(\varepsilon)$, and so (3) represents the inner expansion of (1) for the short-wave case.

In the near field of the body, it is assumed in either case that all field variables change at a much higher rate in the transverse directions than in the longitudinal direction. Specifically, it is assumed that $\partial\phi/\partial y$, $\partial\phi/\partial z$, $\partial\phi/\partial r$, etc., are all $O(\phi\varepsilon^{-1})$, whereas $\partial\phi/\partial x$ is simply $O(\phi)$ as $\varepsilon \to 0$. The 3-D Laplace equation reduces to a 2-D Laplace equation to be solved in the planes of the cross-sections. The body boundary condition becomes a condition on just the transverse component of velocity. The free-surface condition degenerates to

$$\phi_z = 0 \quad \text{on} \quad z = 0 , \ \lambda = O(1) \quad (\nu = O(1)); \tag{4a}$$

$$\phi_z - \nu\phi = 0 \quad \text{on} \quad z = 0 , \ \lambda = O(\varepsilon) \quad (\nu = O(\varepsilon^{-1})). \tag{4b}$$

Condition (4a) represents a rigid-wall in place of the free-surface condition; of course, it is not possible for the corresponding solution to represent waves, but that solution can be made to match with the inner expansion of the outer solution, as shown by Newman & Tuck (1964). Condition (4b) is the usual free-surface condition in linearized gravity-wave problems, and solutions satisfying (4b) and the other conditions of the near-field problem generally represent gravity waves — standing waves or progressive waves. The solution is made unique by matching it with the inner expansion of the far-field solution, as given by (3), that is, the solution must represent outgoing waves, even in the near field. This seems to be quite obvious physically, and Korvin-Kroukovsky assumed that from the beginning. However, the situation is not so obvious when forward speed is introduced into the problem.

Mathematically, either of the wavelength assumptions leads to a tractable problem, and it appears that the choice between them depends entirely on what wavelength range is of interest in a particular problem. However, further consideration suggests that perhaps this is not really true. There are two reasons for this statement:

1) The situations represented in Figure 1 indicate that the short-wave solution is the proper one to use even when the waves are not extremely short.

2) The near-field solution satisfying the rigid-wall condition is nonuniform as $|y| \to \infty$. This is evident from the fact that, far away from the ship, the fluid motion is dominated by the wavelike motion, which cannot be represented in the near-field solution if one assumes that $\lambda = O(1)$. In fact, the long-wave assumption is an extremely strong assumption, for it implies not only that the beam is small compared with wavelength but that points at "infinity" in the near field are still not very far away compared with a wavelength. There is a vast difference between assuming that ship beam is small compared with wavelength and assuming that the entire near field is small compared with wavelength! The mathematical nonuniformity can be fixed up through the matching, but that does not relieve the severity of the basic assumption.

Furthermore, it may be noted that one probably never obtains an incorrect solution by solving the problem involving condition (4b). If ν is in fact not large, the solution satisfying (4b) will have an appropriate behavior *numerically* consistent with that fact. The boundary-value problem will be inconsistent itself, in the sense that some higher-order quantities are retained, but that should not seriously affect the accuracy of the solution. In other words, if the quantity $\nu\phi$ is everywhere much smaller than ϕ_z , then the solution satisfying (4b) will exhibit this property, and, to leading order, ϕ_z will vanish on $z = 0$.

SHIP AT ZERO SPEED IN INCIDENT WAVES

Korvin-Kroukovsky (1955) hypothesized that the wave-induced force on a ship could be computed in the same way as the motion-induced hydrodynamic force if one defined an effective "relative velocity" between the hull and the water surface. The precise way in which the relative velocity was defined was somewhat arbitrary, but the concept has proven to be very useful. In effect, he supposed that the local fluid motion around the hull did not depend on whether the hull was moving down into the water or the water was moving up around the hull. Thus, when computations such as those discussed in the previous section have been completed, it is not necessary to solve any more hydrodynamics problems in order to handle the incident-wave problem.

In recent years, this approach has largely been replaced by the use of the formulas of Khaskind (1957), Hanaoka (1959), and Newman (1965). They avoid having to solve the diffraction problem too, but they do not introduce new and perhaps artificial concepts such as the "relative velocity." Their formulas all require just the appropriate application of Green's theorem. For reference later, we state here the essential steps in their derivation.

Let the incident waves be described by the potential function,

$$Re\ \{\phi_0(x,y,z)\ e^{i\omega t}\}$$

where

$$\phi_0(x,y,z) \quad = \quad \frac{ga}{\omega}\ e^{\nu z} e^{-ikx} e^{-i\sqrt{\nu^2-k^2}\,y} .$$

The amplitude of the wave itself is a , and, as before, $\nu = \omega^2/g$. The new parameter here, k , is the apparent wave number along a track parallel to the x axis; it must be smaller than ν . It is sometimes useful to write:

$$k \quad = \quad \frac{2\pi}{\lambda} \cos\beta \quad = \quad \nu\cos\beta \quad ,$$

where β is the angle between the direction of propagation of the incident waves and the positive x axis.

We shall assume that the ship is situated on the x axis between $-L/2$

and $+L/2$. Let H denote the surface of the hull. The force* on the ship, first of all, according to the Froude-Krylov hypothesis is:

$$F_{0_j} = -i\rho\omega \int_H dS\, n_j\, \phi_0 \quad ,$$

where ρ is the water density and n_j is the direction cosine appropriate to the j-th** component of force. The force due to the diffracted wave can be computed similarly:

$$F_j = -i\rho\omega \int_H dS\, n_j\, \phi \quad , \tag{5}$$

where ϕ is the velocity potential for the diffracted wave. It satisfies the conditions:

[L] $$\phi_{xx} + \phi_{yy} + \phi_{zz} = 0 \quad \text{in the fluid region;}$$

[H] $$\frac{\partial\phi}{\partial n} = -\frac{\partial\phi_0}{\partial n} = -\frac{ga}{\omega}\{n_1(-ik) + n_2(-i\sqrt{\nu^2-k^2}) + n_3\nu\}e^{\nu z}e^{-ikx}e^{-i\sqrt{\nu^2-k^2}\,y}$$

on the body surface, H ;

[F] $$\phi_z - \nu\phi = 0 \quad \text{on} \quad z = 0 \; ;$$

[R] $$\phi(x,y,z) \sim \frac{A}{\sqrt{r}} e^{\nu z} e^{-i\nu r} \quad \text{as} \quad r = \sqrt{x^2+y^2} \to \infty \quad ;$$

$\mathbf{n} = (n_1, n_2, n_3)$ is the unit vector normal to H , directed into the hull, and A is some constant or possibly a function of θ , generally complex.

The point of the Khaskind method is to define a new potential function, ϕ_j , such that $\partial\phi_j/\partial n = n_j$ on H and then use Green's theorem on the integral in (5), so that the roles of ϕ_j and ϕ are interchanged:

$$F_j = -i\rho\omega \int_H dS \frac{\partial\phi_j}{\partial n}\phi = -i\rho\omega \int_H dS \frac{\partial\phi}{\partial n}\phi_j \quad . \tag{6}$$

If the use of Green's theorem is legitimate, this integral gives the force caused by the diffracted wave, and it can be computed if the corresponding ϕ_j problem has been solved, since the condition [H] gives the value of $\partial\phi/\partial n$.

*The actual expression should be multiplied by $\exp\{i\omega t\}$ and just the real part used.

** j can vary from 1 to 6 if suitable generalized direction cosines are used.

The use of Green's theorem is indeed proper if ϕ_j satisfies the Laplace equation and the same free-surface and radiation conditions as ϕ . The function satisfying all of these conditions is just the potential for the forced-motion problem, which has to be found in any case if the entire ship-motion problem is to be solved, and so Equation (6) permits the determination of F_j without the need to solve any further boundary-value problems. Furthermore, the solution for ϕ_j in the immediate vicinity of the ship satisfies approximately the 2-D Laplace equation, and one can use the strip-theory values for ϕ_j in the computation of F_j . This in no way implies that ϕ_j satisfies the 2-D Laplace equation everywhere — just in the neighborhood of the hull, where the strip-theory solution is actually used.

Newman (1970) points out that the last integrand in (6) cannot be identified in any way with the pressure on the hull, although the result can be interpreted loosely as a verification of Korvin-Kroukovsky's relative-motion hypothesis. The application of Green's theorem leads to the consequence that only the complete integral can be interpreted physically. If one needs to know the load *distribution* on the hull, an alternative procedure has to be found. One such alternative was described by Ogilvie (1971), in which the Khaskind idea is just slightly modified so that any particular load, e.g., bending moment at midship, can be computed in the same way as F_j above. However, that method is not of practical usefulness if it really is the pressure distribution that is needed or if a load (such as bending moment) must be found at many places. One must generally solve the diffraction problem in such cases.

However, the force per unit length acting on the hull can be computed without one's having to solve the diffraction problem. Suppose that we want to know the vertical force per unit length at $x = x_1$. It could be written:

$$\frac{dF_3}{dx} = -\, i\rho\omega \int_{C_H(x_1)} d\ell \; n_3 \, \phi \quad ,$$

where C_H is a contour around the hull H at $x = x_1$. Now we multiply this quantity by Δx and apply Green's theorem in a conventional 3-D manner. We construct the surface bounding the fluid region as shown in Figure 2: There is a cross-section plane at $x = x_1$ that we denote D , and a second such plane

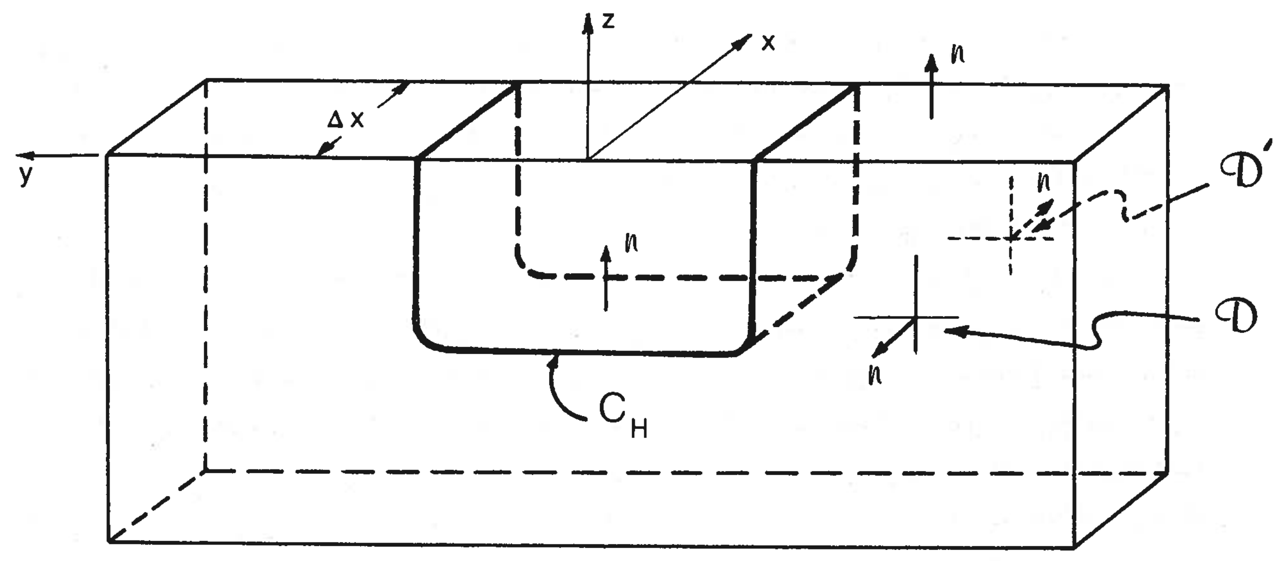

Figure 2. Surface Bounding Region in Which Green's Theorem Is Applied in Calculation of Force Per Unit Length

at $x = x_1 + \Delta x$ is called D' ; then there are strips of width Δx on the hull, the undisturbed free surface ($z = 0$), at $y = \pm R$, and at $z = -\infty$ As before, we assume that there is a function ϕ_3 which is the solution of the forced-heave problem, for which $\partial\phi_3/\partial n = n_3$. We also assume that ϕ_3 satisfies the usual linearized free-surface condition [F], and so the integral

$$\int \left(\frac{\partial\phi_3}{\partial n}\phi - \frac{\partial\phi}{\partial n}\phi_3 \right) ds$$

vanishes when computed over the free surface strip. It presumably also vanishes trivially on the horizontal strip at great depth. But the integral does not generally vanish over D and D' , nor does it necessarily vanish over the two vertical strips at $y = \pm R$ — unless ϕ_3 has certain special properties. First compute the above integral over the two cross-sections:

$$\int_{D'+D} \left(\frac{\partial\phi_3}{\partial n}\phi - \frac{\partial\phi}{\partial n}\phi_3\right) ds = \int_{D'} \left(\frac{\partial\phi_3}{\partial x}\phi - \frac{\partial\phi}{\partial x}\phi_3\right) ds - \int_{D} \left(\frac{\partial\phi_3}{\partial x}\phi - \frac{\partial\phi}{\partial x}\phi_3\right) ds$$

$$\approx \Delta x \int_{D} \frac{\partial}{\partial x}\left(\frac{\partial\phi_3}{\partial x}\phi - \frac{\partial\phi}{\partial x}\phi_3\right) ds = \Delta x \int_{D} \left(\frac{\partial^2\phi_3}{\partial x^2}\phi - \frac{\partial^2\phi}{\partial x^2}\phi_3\right) ds .$$

In a region near the ship hull, that is, in the near field, the diffraction wave potential presumably behaves like $\exp\{-ikx\}$ times a function that varies slowly in the x direction, and so, to a first approximation, ϕ

satisfies a Helmholtz equation:

$$\phi_{yy} + \phi_{zz} - k^2\phi = 0 \quad ;$$

Thus, in the last integral above, $\partial^2\phi/\partial x^2$ is replaced approximately by $-k^2\phi$, and the integral vanishes over this near-field region if ϕ_3 has approximately the same behavior. Furthermore, the potential ϕ behaves at large $|y|$ like $\exp\{\nu z - ikx - i\sqrt{\nu^2-k^2}|y|\}$, and, if we require that ϕ_3 also behave this way, the integral over the vertical strips in the application of Green's theorem vanishes. This leaves us finally with the result that the force per unit length on the ship is:

$$\frac{dF_3}{dx} = -\, i\rho\omega \int_{C_H} d\ell\; \phi_3 \frac{\partial\phi}{\partial n} = i\rho\omega \int_{C_H} d\ell\; \phi_3 \frac{\partial\phi_0}{\partial n} \quad . \tag{7}$$

This is equivalent to the result obtained by Newman (1970), although he simply used Green's theorem in two dimensions to obtain it. When the quantity in (7) is integrated along the length of the ship, the resulting expression for the total force is identical in appearance to (6), but now ϕ_3 has an entirely different meaning: Recall that in (6) ϕ_3 was supposed to satisfy the 2-D Laplace equation in the near field and to satisfy a radiation condition appropriate to a strictly 2-D wave problem. Newman pointed out too that, for a ship with pointed ends, the solution of the Helmholtz equation appropriate to the end cross-sections ought to be identical to the solution of the 2-D Laplace equation for the same sections, and so the two formulas are asymptotically equivalent for computing total force on the ship. But only the solution of the Helmholtz problem can be used for computing load distribution, unless the alternate procedure of Ogilvie (1971) is applied.

"Asymptotic equivalence" of two formulas is not an entirely satisfactory concept for the person who wants to use the formulas for practical computations. It is one thing to derive a formula on the assumption that a small parameter such as ε is approaching zero and another thing to use the formula in a problem in which the parameter is a finite — albeit small — quantity. There *may* be a serious difficulty in taking advantage of the equivalence discussed above, for the following reason: In deriving (7), we assumed that the two potential functions both behaved like the product of $\exp\{-ikx\}$ and a function varying slowly in x. This seems to be quite acceptable in studying the local behavior of the fluid motion, but, when these functions are integrated over the entire length of the hull, the "slowly varying" parts may produce cumulative contributions that cannot be neglected. I do not know of any investigations that can settle this question at present.

The above discussion has all been based on an implicit assumption that one should treat the waves as "short," in the sense of the preceding section. Even if we accept the conclusion of that section, i.e., that we should always take $\lambda = O(\varepsilon)$, there is a further uncertainty in that we have not specified from which direction the waves come. There are really three wave numbers in the problem, ν, k, and $\sqrt{\nu^2-k^2}$, and it may happen that only two of these are large. We should distinguish three cases:

a) *Waves from the beam (or nearly so),* for which case $k = O(1)$ and $\sqrt{\nu^2-k^2} = O(\varepsilon^{-1})$. The velocity potential, ϕ, for the diffraction wave satisfies the 2-D Laplace equation (approximately) in the near field, and the body boundary condition reduces to

[Ha] $$\frac{\partial\phi}{\partial n} \sim \omega a\,\{in_2 - n_3\}\, e^{\nu z} e^{-ikx} e^{-i\nu y} .$$

The variable x appears here as a parameter only. On the free surface, ϕ satisfies the same [F] condition given previously. This case gives no real difficulty of any kind; it leads to the simplest form of strip theory.

b) *Quartering seas,* for which both $k = O(\varepsilon^{-1})$ and $\sqrt{\nu^2-k^2} = O(\varepsilon^{-1})$. This is really the case discussed in detail above; the waves are neither from the beam nor from the bow. The potential for the diffraction wave satisfies the Helmholtz equation, the free-surface condition [F], the body boundary condition,

[Hb] $$\frac{\partial\phi}{\partial n} \sim \omega a\,\{in_2\frac{\sqrt{\nu^2-k^2}}{\nu} - n_3\}\, e^{\nu z} e^{-ikx} e^{-i\sqrt{\nu^2-k^2}\,y} ,$$

and the radiation condition,

[Rb] $$\phi \sim A\, e^{\nu z} e^{-ikx} e^{-i\sqrt{\nu^2-k^2}|y|} \quad \text{as} \quad |y| \to \infty ,$$

where A is some constant (generally different according to whether $y \to \pm\infty$).

The properties of the solutions of such problems have been studied by Ursell (1968a, 1968b) and actual solutions have been produced by Bolton & Ursell (1973) for the case of bodies of semicircular cross-section. For noncircular bodies, it will be necessary to use an integral equation technique involving Green's functions, since the techniques of conformal mapping are not useful with the Helmholtz equation.

c) *Waves from nearly end-on,* for which $k = O(\varepsilon^{-1})$ and $\sqrt{\nu^2-k^2} = O(1)$. The potential satisfies the Helmholtz equation again, the free-surface condition [F], the body condition,

[Hc] $$\frac{\partial\phi}{\partial n} \sim - \omega a\, n_3\, e^{\nu z} e^{-ikx} ,$$

and a radiation condition formally equivalent to [Rb] — provided that k is not precisely equal to ν , for which special difficulties arise. There appears to be no reason why this case cannot be solved just as well as case b) if the waves are really short enough compared with body length, but the solution may not be very useful in practical problems. This qualification is made because of the special problem that arises when $k \to \nu$, the limit representing precisely head waves. One can anticipate trouble by noting that solutions of problems in this case become undefined in this limit.

The special problem of head waves. Ursell (1968a) showed that a potential function satisfying the conditions:

$$\phi_{yy} + \phi_{zz} - \nu^2\phi = 0 \quad \text{in the fluid region,}$$

$$\phi_z - \nu\phi = 0 \quad \text{on } z = 0 ,$$

has no possible form that is symmetric in y and which remains bounded as $y \to \pm\infty$, except for some possible forms that vanish exponentially and which are thus not general enough to be useful. This means that the problem of head waves cannot be formulated in the manner just described for the oblique-wave case.

We can get some hint of the cause of the trouble by considering the oblique-wave problem again, but now from a far-field point of view. At a considerable distance from the ship, we may expect that the diffracted wave will appear to have been produced by a line distribution of singularities along the axis of the ship, just as in the problem of forced motion that was discussed in the preceding section. There is no reason to expect the diffracted wave to be symmetrical with respect to y , and so we should include both sources and lateral dipoles in the distribution. However, our point can be made quite adequately with just a distribution of sources.

We start with a potential identical to that given in (1), but we suppose that the source density varies rapidly along the axis. To be precise, let $\sigma(x) = \Sigma(x) \exp\{-iKx\}$ in the interval $|x| < L/2$, where $K < \nu = \omega^2/g$. (We use K instead of k in order to avoid confusion with the variable of integration in (1).) The function $\Sigma(x)$ is assumed to vary slowly with x , that is, it does not change much over a distance $2\pi/K$. The simplification represented in (2) is still possible, and we can again apply the method of stationary phase to simplify further. We find that there exists a point of stationary phase of the integrand provided that

$$\left| x - \frac{K|y|}{\sqrt{\nu^2-K^2}} \right| < \frac{L}{2} ,$$

and, when this condition is satisfied, the potential is given approximately by:

$$\phi(x,y,z) = i \frac{\nu}{\sqrt{\nu^2-K^2}} \Sigma\left(x - K|y|/\sqrt{\nu^2-K^2}\right) e^{\nu z} e^{-i\left(Kx + \sqrt{\nu^2-K^2}|y|\right)} , \qquad (8)$$

which represents a wave outgoing from the x axis at an angle $\beta = \cos^{-1}K/\nu$. What is more important is that the wave amplitude at any point (x,y) depends on the source density at $(x - K|y|/\sqrt{\nu^2-K^2}) = (x - |y|\cot\beta)$. This is most easily understood from the sketch of Figure 3. Starting at (x,y) , one follows a wave ray back to the axis from which the disturbance emanates; the intersection of that ray with the axis gives the point from which the wave at (x,y) seems to arise. In the corresponding forced-heave problem, (2) represents the outgoing wave, which, of course, appears to move directly out to the sides, and the wave amplitude at any (x,y) depends on the source strength at the same x .

In (8), the amplitude of the wave is multiplied by a factor $\nu/\sqrt{\nu^2-K^2}$, which was missing in (2). From Figure 3, it is apparent that this amplification arises because the length of the wave front is shorter than the length of the source distribution by a factor $\sqrt{\nu^2-K^2}/\nu$, and so the wave energy is concentrated into a smaller and smaller region as $K \to \nu$. In fact, as the limit is approached, the amplitude of the waves (according to this analysis) approaches infinity, and the waves move practically parallel to the x axis.

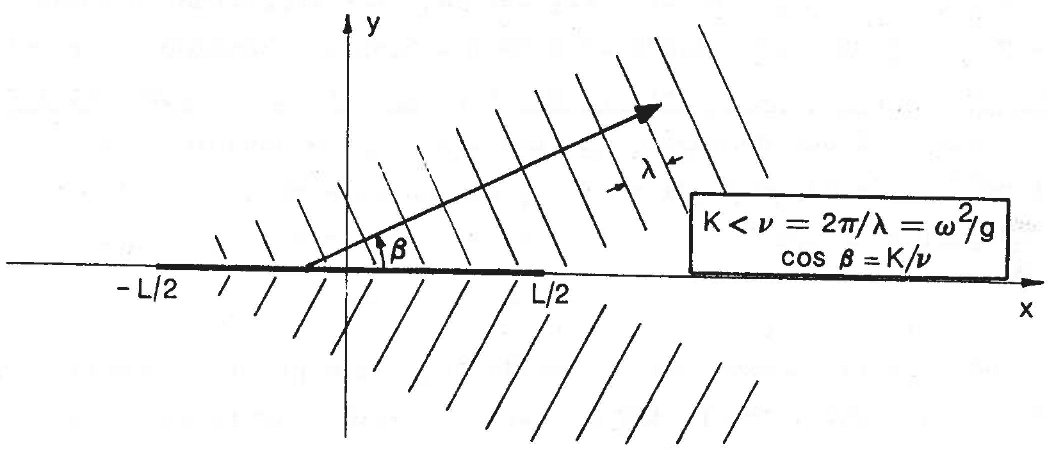

Figure 3. Waves radiated from a line of sources of density $\Sigma(x)\cdot\exp i\{\omega t - Kx\}$ on $|x| < L/2$.

Ursell (1968a) produced the general solution of the limit problem in which it is assumed that all field variables behave like $\exp\{-i\nu x\}$ for all x , the solution satisfying the free-surface condition $\phi_z - \nu\phi = 0$. He required only that the solution be bounded algebraically as $|y| \to \infty$, and he found that the solution generally increased linearly in y for large $|y|$. However, it should be noted that this solution is not in any sense a limit of the solutions for the oblique-wave case; one should note the unusual contour of integration in Ursell's solution of this special case. His analysis, taken along with the above discussion, suggests strongly that the case of *nearly* head waves should be treated as a perturbation of the head-wave case and not as an extreme case of the oblique-wave case.

It appears that Grim (1962) understood this problem many years ago, but unfortunately his analysis has been neglected. In the actual problem of wave diffraction by a ship, one may take the following point of view: If the waves come directly from ahead, diffraction waves will be generated at the bow portion of the ship, and these waves will radiate in all directions according to the usual laws of such fluid motion. However, those waves that move along the ship axis will have been generated with just the proper phase relationship so that they move right along with the incident wave that they serve to cancel in the bow region. Furthermore, the amplitude of these particular waves will drop off slowly *in this particular direction*. Since the amplitude does drop off, the portions of the ship aft of the bow will have to produce more diffraction waves, but the size of the additional diffraction wave will be very small, depending partly on the rate of change of hull cross-section and partly on the rate at which the bow-generated waves disperse laterally.

Faltinsen (1971) showed mathematically what Grim predicted heuristically. In Faltinsen's analysis, the problem is divided into near-field and far-field problems, in the usual way with the method of matched asymptotic expansions. In the near field, the first approximation to the diffraction wave is just the negative of the incident wave. This wave does not satisfy an ordinary radiation condition, of course, but Faltinsen did show that it matches with the inner expansion of the far-field solution provided the latter represents the fluid disturbance caused by a line of sources with density varying as $x^{-1/2}\exp i\{\omega t-\nu x\}$, x here being measured positive from the ship bow. This solution is singular at the bow itself and thus not valid in the immediate neighborhood of the bow, but it shows the expected general behavior: There is a high-density source distribution near the bow, creating waves that cancel the incident wave in the appropriate way on the hull surface, the density dropping off rapidly toward the stern because the diffraction wave travels along the hull with the incident wave, continuing partly to cancel it.

Faltinsen showed that his predictions agreed fairly well with experimental data (except in the immediate neighborhood of the bow) for the case $\lambda/L = 0.5$, but the agreement was not good for $\lambda/L = 0.75$. Since his analysis is based on the assumption that wavelength is very small compared with ship length, the agreement for the former case seems remarkable enough. It certainly strengthens my conjecture that one ought *always* to adopt a short-wave hypothesis. On the other hand, the failure of his analysis for the case $\lambda/L = 0.75$ appears to put a limit on the validity of that hypothesis. This is not clear at present, for the failure of his analysis in this case may be due to another cause: The singularity in the diffraction wave at the bow in Faltinsen's analysis may have important effects all along the ship if the waves are not very short compared with ship length; he has assumed, effectively, that the incident wave is canceled in the entire near field, starting abruptly at the bow, which is quite unrealistic, and the elimination of this idealization may improve his overall results. In order to improve his solution in this respect, it will be necessary to consider that the source density can be described in terms of a spectrum, which will undoubtedly be peaked at $k = \nu$ (instead of being concentrated there); in this way, it may be possible to obtain a smooth rise in the density of the source distribution. However, it will still be a short-wave analysis. The problem is a difficult one and has not been solved yet.

In concluding this section, we present a table comparing the orders of magnitude of the various terms in the body boundary condition according to three different systematic perturbation schemes: a) slender-body theory with the short-wave assumption ($\nu = O(\varepsilon^{-1})$); b) slender-body theory with the long-wave assumption ($\nu = O(1)$) ; c) thin-ship theory with the long-wave assumption . In the table, the exact body boundary condition is written, and, under it, the orders of magnitude of the terms are given, according to each of the three theories. These estimates all apply in the near field, of course. Several conclusions can be drawn:

1) In case a), the diffraction-wave potential, ϕ , is of the same order of magnitude as the incident-wave potential, and the value of $\partial h/\partial x$ affects only higher-order approximations of ϕ . (Here we represent the body surface by the equation, $y \mp h(x,z) = 0$, in order to be able to include the thin-ship case readily.)

2) In case b), the diffraction-wave potential, ϕ , is of the order of

TABLE 1

ORDER OF MAGNITUDE OF TERMS
IN BODY BOUNDARY CONDITION

Exact Condition:	ϕ_y	$\mp h_z\phi_z$	$\mp h_x\phi_x$	$= i\sqrt{\nu^2-k^2}\,\phi_0$	$\mp \nu h_z\phi_0$	$\mp ikh_x\phi_0$
a) Slender body, short waves	$[\phi/\varepsilon]$	$[\phi/\varepsilon]$	$[\phi]$	$[\phi_0/\varepsilon]$	$[\phi_0/\varepsilon]$	$[\phi_0]$
b) Slender body, long waves	$[\phi/\varepsilon]$	$[\phi/\varepsilon]$	$[\varepsilon\phi]$	$[\phi_0]$	$[\phi_0]$	$[\varepsilon\phi_0]$
c) Thin body, long waves	$[\phi]$	$[\varepsilon\phi]$	$[\varepsilon\phi]$	$[\phi_0]$	$[\varepsilon\phi_0]$	$[\varepsilon\phi_0]$

Notes:

1) The incident wave is given by

$$\phi_0(x,y,z) = \frac{ga}{\omega} e^{\nu z} e^{-ikx} e^{-i\sqrt{\nu^2-k^2}\,y} .$$

2) The body surface is given by:

$$y \mp h(x,z) = 0 .$$

$\varepsilon\phi_0$, and, as in case a), the value of $\partial h/\partial x$ does not affect the lowest-order approximation of ϕ .

3) For the thin ship, case c), two cases must be distinguished: i) If $k \neq \nu$ (waves not from directly ahead), then $\phi = O(\phi_0)$, and neither h_x nor h_z affects the lowest-order solution; all that matters is the size of the projection of the hull onto the center plane. ii) If the waves are from directly ahead, so that $k = \nu$, then $\phi = O(\varepsilon\phi_0)$. Furthermore, in such a case, h_z is likely to be negligible near the free surface, and so the longitudinal slope of the waterplanes, given by $\partial h/\partial x$, has a dominant effect on the diffraction potential.

SHIP OSCILLATIONS AT FORWARD SPEED

Ship motions can be predicted analytically with impressive accuracy, even in the case of a ship with forward speed, but the theory has many gaps and uncertainties. In keeping with the attitude expressed in the *Introduction*, I shall here be concerned more with the gaps and uncertainties than with the accomplishments of present-day ship-motion theory. However, my emphasis on the negative aspects of the situation should not be considered in any way as a denigration of the extant theory — which is one of the brightest facets of ship hydrodynamics.

The first difficulty to arise is the inadequacy of slender-ship theory for the steady-forward-motion problem. This inadequacy is perhaps not so bad as was once thought, some of the poor predictions having been the consequence of restrictions implied in early formulations of the theory. For the present purpose, I shall start with the steady-motion slender-ship theory of Tuck (1964). This theory is definitely deficient in some ways, and, were the present purpose to develop the steady-motion problem as fully as possible by means of slender-body theory, I would try to use an alternative to the Tuck theory. But shortcomings in the steady-motion theory clearly do not have catastrophic consequences in the ship-motion theory, and so I shall avoid digressing into the matter of possible improvements of Tuck's theory. Nevertheless, when the agreement between motion predictions and observations is imperfect, one should always recall that the fault could lie in the use of an inadequate steady-motion description.

In the discussion of the zero-speed problem, I concentrated rather heavily on the assumptions connecting wavelength with body dimensions. The wavelength of interest was easy to define, namely, $\lambda = 2\pi g/\omega^2$. In the forward-speed problem, there is no single characteristic wavelength. In the simple case of a pulsating, traveling source, there is a very complicated system of waves, comprising several fairly distinct wave systems in addition to the familiar Kelvin wave pattern. See Becker (1958) or Wehausen & Laitone (1960) (p. 494) for figures showing some of these wave systems. The entire nature of the wave pattern changes with speed and frequency, the most marked change occurring when $\tau \equiv \omega U/g = 1/4$; below this speed (or, for fixed speed, at a lower frequency), waves propagate ahead of the pulsating source, whereas at higher speed the wave motion is entirely confined to a region behind the source. Presumably the waves generated by an oscillating ship show the same kinds of behavior.

Led by the obvious practical success of Korvin-Kroukovsky's strip theory for predicting ship motions, Ogilvie & Tuck (1969) showed that similar results could be obtained in a first approximation by assuming that the frequency of oscillation is "high," again in the sense that $\omega = O(\varepsilon^{-1/2})$ — just as in the zero-speed problem. They carried out a systematic perturbation analysis of the forward-speed problem on this basis. They made no explicit assumption about the order of magnitude of the steady-motion wave number, $\kappa = 2\pi g/U^2$, which is equivalent to saying that they assumed that $\kappa = O(1)$ as $\varepsilon \to 0$. It is this fact that reduces their theory to Tuck's (1964) steady-motion theory when there is no oscillation.

Actually, Ogilvie & Tuck used two small parameters in their perturbation analysis, viz., the usual slenderness parameter, ε , and a "motion-amplitude" parameter, δ . Use of the latter allows the motion amplitude to be varied independently of ship dimensions, and, in particular, it is the means of linearizing the problem with respect to the motion amplitude without simultaneously introducing artificial restrictions on ship slenderness. Formally, they assumed the existence of a double asymptotic expansion, as follows:

$$\begin{aligned}\phi(x,y,z,t) \sim &\{\phi_{11} + \varepsilon^2\phi_{12} + \ldots\}\,\delta^0 \\ &+ \{\varepsilon^{3/2}\phi_{21} + \varepsilon^2\phi_{22} + \ldots\}\,\delta^1 \\ &+ \ldots^{**}. \qquad (9)\end{aligned}$$

The first term in the first line is really $\phi_{11} = Ux$, representing the uniform

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**I am following the possibly controversial practice of treating quantities that are $O(\log \varepsilon)$ as being $O(1)$. In any case, terms that are $O(\varepsilon^2)$ and $O(\varepsilon^2 \log \varepsilon)$ must certainly be carried along together, and so they are both included in the term denoted simply by the factor ε^2 .

incident stream. The orders of magnitude of the various terms were found systematically; they are certainly not known or even assumed in advance, as one might imply from the way in which the above series was written down.

The first line in (9) represents the steady-motion potential. It should be noted that the first term in the second line is lower order in terms of ε than the steady perturbation of the uniform stream. As a consequence, the lowest-order term in the formula for the time-dependent hydrodynamic force on an oscillating ship comes from ϕ_{21} , and it is precisely the same as the corresponding force formula in the zero-speed problem. It is the simplest kind of a strip-theory result: There are no forward-speed effects and no interactions between sections of the ship. ϕ_{21} satisfies Equation (4b) on $z = 0$, and, of course, it satisfies the 2-D Laplace equation in the plane of the cross-section. If one does not make the high-frequency assumption, the first term in the velocity potential for the motion problem satisfies a rigid-wall condition on $z = 0$, just as it does in the zero-speed problem.

In earlier attempts to derive a ship-motion theory on the basis of the thin-ship idealization, as in Stoker (1957), the free-surface condition for the first approximation was always taken as:

$$U^2\phi_{xx} + 2i\omega U\phi_x - \omega^2\phi + g\phi_z = 0 \quad \text{on} \quad z = 0 . \tag{10}$$

Here one could also have assumed that frequency is high, and it is then clear that the first two terms would be much smaller than the third term. Slender-body theory requires us to assume that $\partial\phi/\partial z$ is much larger than ϕ itself, and so the fourth term is "large" too. In introducing the high-frequency assumption into slender-body theory, one must realize that one — and only one — of the linear terms in the free-surface condition is reintroduced; the first and second terms in (10) remain of higher order. Of course, those two terms contain the explicit effect of forward speed in the boundary condition, and so we may expect them to appear in the conditions for higher-order approximations.

Ogilvie & Tuck completely formulated the boundary-value problem for ϕ_{22} . They found that it was not necessary to solve for this function in order to compute the corresponding term in the force expansion, but some of the properties of ϕ_{22} had to be used in that computation. Since this problem has not been correctly interpreted by some researchers, it appears desirable here to discuss it briefly.

Both ϕ_{21} and ϕ_{22} satisfy the 2-D Laplace equation in the cross-section planes. On the mean position of the hull, which we express:

$$S_0(x,y,z) = h_0(x,y) - z = 0 \quad ,$$

they satisfy the following condition:

$$\frac{\partial(\phi_{21} + \phi_{22})}{\partial n} = i\omega(n_3\xi_3 + n_5\xi_5) + U\,[(n_3\xi_3 + n_5\xi_5)(\nabla S_0 \cdot \nabla\phi_{12}) - n_3\xi_5] + \ldots , \qquad (11)$$

where we have now suppressed factors $\exp\{i\omega t\}$, and the heave and pitch displacements are given, respectively, by ξ_3 and ξ_5 , each multiplied by $\exp\{i\omega t\}$. The first line on the right-hand side of (11) would appear in the same form in the zero-speed problem. The last term in the second line represents an angle of attack (or cross-flow). The rest of the terms on the second line arise from the fact that the boundary condition, which ought to be applied on the exact, instantaneous hull surface, has been transferred to the mean hull position by means of a Taylor expansion of the field variables. Actually, the angle-of-attack term is obtained formally in the same way; however, the terms preceding it involve the gradient of the steady-motion potential, ϕ_{12} . (These terms are missing in all strip theories that are not based on a systematic perturbation procedure, but formally they are of the same order of magnitude as the angle-of-attack term.) Condition (11) can be obtained directly from the formula given by Timman & Newman (1962).

Because of the Ogilvie-Tuck assumption that $\omega = O(\varepsilon^{-1/2})$, the first line on the right-hand side of (11) is lower order than the second line by a factor $\varepsilon^{1/2}$. Thus, separate conditions can be written down for ϕ_{21} and ϕ_{22} :

$$\frac{\partial\phi_{21}}{\partial n} = i\omega \sum_j n_j\xi_j \qquad (j = 3,5) \quad ; \qquad (11a)$$

$$\frac{\partial\phi_{22}}{\partial n} = U[(\nabla S_0 \cdot \nabla\phi_{12}) \sum_j n_j\xi_j - n_3\xi_5] \qquad (j = 3,5) . \qquad (11b)$$

These potentials satisfy the following free-surface condition:

$$- \omega^2\phi_2 + g\phi_{2_z} = - 2i\omega U\phi_{2_x} - 2i\omega\phi_{12_y}\phi_{2_y} - i\omega\phi_{12_{yy}}\phi_2 + \ldots \quad \text{on } z = 0 , \qquad (12)$$

where we have written ϕ_2 to denote $\phi_{21} + \phi_{22}$. The terms on the left-hand side are the same as the third and fourth terms in (10), the usual linearized free-surface condition. Now note that the first term on the right-hand side is identical to another term, the second, in the linearized condition, (10). However, the entire right-hand side of (12) is higher order than the left-hand side by a factor $\varepsilon^{1/2}$, and so we sort out similar orders of magnitude, as follows:

$$- \omega^2\phi_{21} + g\phi_{21_z} = 0 ; \qquad (12a)$$

$$- \omega^2\phi_{22} + g\phi_{22_z} = - 2i\omega U\phi_{21_x} - 2i\omega\phi_{12_y}\phi_{21_y} - i\omega\phi_{12_{yy}}\phi_{21} . \qquad (12b)$$

Condition (12a) is, of course, the same as (4b), derived for the zero-speed problem.

Condition (12b) is the interesting condition. It is a nonhomogeneous condition on ϕ_{22}, the right-hand side depending on the lower-order solution, ϕ_{21}, which will presumably have been determined first. The terms on the right-hand side have two physical origins: a) The kinematic condition for ϕ_{21} was applied on $z = 0$, which introduced an error that must be accounted for in the next-higher-order problem; one can make a direct analogy to the origin of the right-hand side of (11b). b) The dynamic condition for ϕ_{21} was also satisfied on $z = 0$, and there was also an error of higher order in evaluating the pressure in the dynamic condition; these factors have no direct analogy in the body boundary condition, but their effect is similar — except for one thing: The first term on the right-hand side of (12b) is a solution of the homogeneous equation, and so the solution satisfying (12b) will have to be determined with great care. One can make a direct analogy with the elementary problem of applying a sinusoidal force to an undamped spring-mass system at the natural frequency: there is no periodic solution.

Ogilvie & Tuck showed that the solution of the ϕ_{22} problem represents an outgoing wave with amplitude increasing linearly with $|y|$; such a solution matches perfectly with the far-field solution representing the disturbance generated by a line distribution of pulsating, translating sources. It is not the purpose here to review that solution. What should be noted is that this perhaps unexpected kind of solution arises from a strictly linear problem. The difficulty in the ϕ_{22} problem starts with the first term on the right-hand side of (12b), and that term is equivalent to the second term in the old, familiar, linearized free-surface condition, (10). The other terms on the right-hand side of (12b) might be called nonlinear, in the sense that they represent interactions between the oscillatory fluid motion and the steady perturbation of the incident stream; they would be eliminated at the beginning in any analysis in which such interactions are considered to be "nonlinear." But these terms does not give rise to the unusual nature of ϕ_{22} . In fact, when the hydrodynamic force is computed, these terms play an essential role in allowing the force to be computed without the necessity for solving the ϕ_{22} problem explicitly. To some extent, they alleviate the trouble caused by the linear term on the right-hand side of (12b).

The analysis of Ogilvie & Tuck is based on the simple assumption that $\omega = O(\varepsilon^{-1/2})$, or the equivalent, $\nu = O(\varepsilon^{-1})$. The frequency is the actual frequency of oscillation of the ship, which will be equivalent to the frequency of encounter in an incident-wave problem. The steady-motion problem is treated on the basis that $\kappa = g/U^2 = O(1)$ as $\varepsilon \to 0$. The primary reason for accepting such assumptions is just that they give rather good results in comparison with experimental observations. I have not been able to develop any rationalizations comparable to those in the preceding section on the zero-speed problem that are really convincing in themselves. Considering the complicated nature of the wave system generated by an oscillating ship which is moving with a steady forward speed, I doubt that simple arguments are likely to be developed for this purpose.

One further implication of the Ogilvie-Tuck assumption should be noted, however: The critical parameter, $\tau = \omega U/g$, must be large, in fact, $O(\varepsilon^{-1/2})$, in the above analysis. Therefore the Ogilvie-Tuck theory applies to speeds and/or frequencies above the critical value. In particular, one cannot let the forward speed, U , approach zero in their results, and so the well-known singular behavior of the steady-motion problem for asymptotically low speed does not upset the results. On the other hand, it is difficult to reconcile this observation with the fact that their lowest-order results are precisely the same as what is obtained in the zero-speed problem itself. Furthermore, their forward-speed effects approach zero continuously as $U \to 0$. It is rather obvious that the forward-speed effects are not very sensitive to the manner of treating the steady-motion problem, but this does not explain much!

All of the discussion up to this point has dealt with the heave/pitch problem. As far as I know, no one has worked out the details of the analysis for motions in the lateral directions. I do not expect any great surprises to occur when this is done, but it is surprising that it has apparently not been done yet.

SHIP ADVANCING THROUGH INCIDENT WAVES

All of the questions that arose in connection with a restrained ship at zero speed now arise again, but with the expected further complications. This is the most difficult problem considered in this paper, and for precisely that reason I have the least to say about it.

For a ship moving into head waves, let the frequency of encounter be ω, and let the frequency of the waves be ω_0 when measured in a reference frame fixed to the fluid at infinity. There there is the following relationship connecting ω, ω_0, and U :

$$\omega \;=\; \omega_0 + \omega_0^2 U/g \quad . \tag{13}$$

The basic assumption of Ogilvie & Tuck (1969) was that $\omega = O(\varepsilon^{-1/2})$, and so (13) requires then that $\omega_0 = O(\varepsilon^{-1/4})$. Thus, the actual wave frequency has to be a different order of magnitude from that of the frequency of encounter. At zero speed, this is not required, of course.

Physically, this consequence of the Ogilvie-Tuck assumption has an interesting implication. At zero speed, the waves ought to have a wavelength which is $O(\varepsilon)$ in order for the strip theory to be valid, since $\omega = \omega_0$ and $\lambda = O(g/\omega_0^2) = O(\varepsilon)$. However, for the ship with forward speed, it is sufficient for the waves to be only short enough that $\lambda = O(\varepsilon^{1/2})$, since the wavelength is related to ω_0 and not to ω. Such a wavelength is long compared with ship beam and short compared with ship length — in a strict asymptotic sense. What this means practically remains to be determined.

Faltinsen's (1971) analysis, which was discussed previously, included the forward-speed case, but his assumptions were somewhat different from those of Ogilvie & Tuck. He assumed *always* that the waves are so short that $\omega_0 = O(\varepsilon^{-1/2})$, and he further assumed that U is asymptotically small, but in a very weak way, namely,

$$U \;=\; O(\varepsilon^{1/2 - a}) \;, \quad 0 < a \leq 1/2 \;.$$

This assumption implies that the characteristic steady-motion wavelength,

$2\pi U^2/g$, is short compared with ship length but long compared with ship beam. (Ogilvie & Tuck assumed that this wavelength was $O(1)$, or comparable with ship length.) The frequency of encounter under Faltinsen's assumptions has the estimate, $\omega = O(\varepsilon^{-1/2 - a})$, and so it is even larger than in the Ogilvie-Tuck analysis. Finally, we note that Faltinsen's theory is still a "large-τ " theory, since $\tau = \omega U/g = O(\varepsilon^{-2a})$, and so the forward-speed results are separated from the zero-speed results by the singular behavior that accompanies $\tau = 1/4$.

It is difficult to rationalize Faltinsen's assumptions except to note that he obtained rather good agreement with experimental data. His unique assumption about the order of magnitude of the forward speed has an important analytical consequence: There is no coupling between the steady-motion perturbation of the incident uniform stream and the time-dependent diffraction-wave motion. It was a comparable coupling that caused considerable complication in the Ogilvie-Tuck theory for added mass and damping.

Faltinsen undertook his analysis in order to be able to predict actual pressure distributions on ships in waves. He was particularly interested in the case of large ships, for which the net integrated force might be quite small and the motions themselves of little interest, but in which the distributed loads could cause design limitations. In calculating the forces and moments that cause ship motions, one is probably best off using the Khaskind-Newman formulas, as derived by Newman (1965) for the forward-speed case. As disucssed in the section above on the zero-speed problem, these formulas do not provide much of an insight into how and where the force is applied by the incident waves, but they do avoid the many questions about how to treat the various length scales of the problem — questions which we cannot answer. The derivation of the Khaskind-Newman formulas recently by McCreight (1973) under the general conditions assumed by Ogilvie & Tuck makes those formulas more generally useful and dependable than ever.

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