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Higher-Order Thin-Ship Theory
by Means of the Method of Matched Asymptotic Expansions

Hiroyuki Adachi

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Department of Naval Architecture
and Marine Engineering
College of Engineering
The University of Michigan
Ann Arbor, Michigan 48104

ABSTRACT

A new approach to the higher-order thin-ship theory is developed by means of the method of matched asymptotic expansions. The result which is obtained here is not new, but the approach is entirely different from others. The source distributions are determined by the process of matching of the far-field solution to the near-field solution. So the expression for the higher-order thin-ship theory is valid in the far field.

The near field is a combination of thin-ship and slender-ship near field. The slender-ship near field is necessary to take care of the free-surface condition, which is satisfied on the free surface instead of the $z = 0$ plane. A combination thin-ship and slender-ship approach makes it possible to construct higher-order potentials.

In the far field, the potential is composed of three types of potentials. They are the thin-ship potential, the slender-ship potential, and the pressure-distribution potential, all of which satisfy linearized free-surface conditions at the $z = 0$ plane. Among them, the slender-ship potential has the most interesting features.

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I. INTRODUCTION

It was shown by Ashley and Landahl (1965) how we can treat a thin-body problem by the method of matched asymptotic expansions. Ogilvie (1970) suggested that the symmetrical thin-ship problem may be treated by the method of matched asymptotic expansions if a combination thin-body/slender-body approach were taken. The introduction of such a combination approach will make it easy to handle the boundary conditions which must be satisfied on the free surface.

However, the problem which results from introducing such an approach is how one should define the two near fields and how one can match the solutions which will be obtained in each near field. Once one could get an appropriate definition of the near fields and a set of solutions there, one would have also a solution in the far field which will coincide with the well-known thin-ship potential.

By the method of matched asymptotic expansions, we may construct the higher-order thin-ship theory. This approach seems to be different from others which start from the Green's theorem in construction of the velocity potential which satisfies the higher-order boundary conditions on both the hull surface and the free surface. But, sometimes, the usual Green's-theorem approach leads to different expressions of the velocity potential according to the different treatments of the free-surface condition near (or at) the hull surface (See Wehausen (1963), Eggers (1966), and Yim (1968)). The approach which I employ also seems to be different from that of Maruo (1966), which originated in Sisov's method.

The difference in the expressions of the higher-order thin-ship potential arises mainly in the line integral part in the velocity potential. Although the importance of the line integral part in the thin-ship potential has been stressed, we do not have a generally accepted expression for the line integral part. Of course, Wehausen (1963) gave the complete expression for the higher-order thin-ship theory. But his result involves a lengthy and complicated expression which seems to make further discussion difficult.

It was shown by Wehausen (1963) that at the intersection of the free surface and the hull surface the solution is singular, and the singular part is represented by a line integral taken along the line of intersection. This suggests that, from the point of view of the method of matched asymptotic expansions, it will be possible to represent the far-field effects of this singular part by a line of singularities on the centerline and to determine the density of the singularity distribution by matching the solution to the near-field expansion. So the interference between the hull and the waves appears from the far-field point of view as a line of singularities on the centerline. This leads to the introduction of the slender-ship near field, in addition to the thin-ship near field. Thus, a combination of thin-ship and slender-ship near fields can be introduced to construct the higher-order thin-ship theory by means of the method of matched asymptotic expansions.

II. *FORMULATION OF THE MATHEMATICAL PROBLEM*

Consider a symmetrical ship at rest on the surface of a uniform stream of velocity U . The coordinate system is fixed to the ship, as shown in Figure 1. The ship is assumed not to be allowed to move. Also, the fluid is assumed to have infinite extent in the fluid domain $z < 0$. The origin of the Cartesian coordinates is on the undisturbed free surface. The x axis is in the direction of the stream, the y axis horizontal, and the z axis vertical, positive upwards.

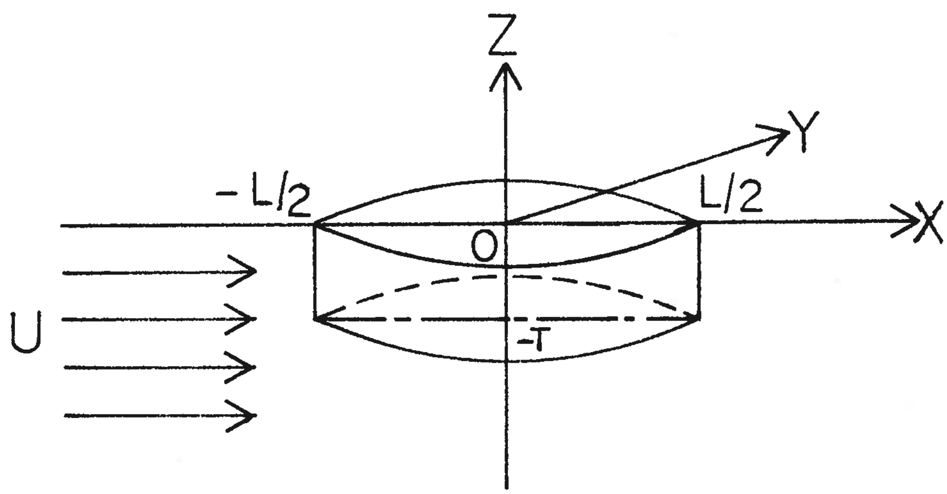

FIGURE 1. Coordinate System

The surface of the ship is expressed by an equation

$$y = \pm h(x,z) \; . \tag{1}$$

It is assumed that the fluid is inviscid and incompressible and that the fluid motion is irrotational, so that the velocity potential of the fluid motion due to the ship can be written

$$\phi = Ux + \Phi \; ,$$

where Φ is the perturbation velocity potential, which satisfies the three-dimensional Laplace equation:

[L] $$\Phi_{xx} + \Phi_{yy} + \Phi_{zz} = 0 \tag{2}$$

in the fluid domain. The velocity components of the fluid motion are:

$$u = U + \Phi_x = \phi_x \ , \quad v = \Phi_y = \phi_y \ , \quad w = \Phi_z = \phi_z \ . \tag{3}$$

The boundary condition satisfied by ϕ on the surface of the ship is:

[H] $$\pm \phi_x h_x - \phi_y \pm \phi_z h_z = 0 \quad \text{on} \quad y = \pm h(x,z) \ . \tag{4}$$

The boundary conditions on the free surface are, assuming the free surface is expressed by $z = \zeta(x,y)$ and neglecting surface tension, the dynamic free-surface condition,

[A] $$g\zeta + \frac{1}{2}[\phi_x^2 + \phi_y^2 + \phi_z^2] = \frac{1}{2} U^2 \quad \text{on} \quad z = \zeta(x,y) \ , \tag{5}$$

the kinematic free-surface condition,

[B] $$\phi_x \zeta_x + \phi_y \zeta_y - \phi_z = 0 \quad \text{on} \quad z = \zeta(x,y) \ . \tag{6}$$

These boundary conditions are nonlinear.

Moreover, the velocity potential ϕ must satisfy a radiation condition:

[R] There are water waves only on the downstream side.

It is assumed that the ship is thin, with beam/length ratio, $B/L \equiv \varepsilon$, much smaller than unity. So $h(x,z)$ is $O(\varepsilon)$, that is, $h(x,z) = \varepsilon H(x,z)$, where $H(x,z) = O(1)$, say.

Now let us define the far field. From the far-field point of view, a ship seems to be represented by a singularity distribution on the x-z plane as $\varepsilon \to 0$. We define the centerplane on which singularities are distributed:

$$\mathsf{H} : \quad -\frac{L}{2} \leq x \leq \frac{L}{2} \ , \quad -T \leq z \leq 0 \ , \quad y = 0 \ . \tag{7}$$

Moreover, the interaction between the ship hull and the waves generated by the ship may be seen from the far-field point of view as a disturbance due to a singularity distribution on the centerline, L, where L is defined:

$$\mathsf{L} : \quad -\frac{L}{2} \leq x \leq \frac{L}{2} \ , \quad y = 0 \ , \quad z = 0 \ . \tag{8}$$

So the far field is defined to be the entire space, $z \leq 0$, except the $y = 0$ plane. One may expect the far-field solution to be singular on the centerplane and on the centerline.

The near field is defined and shown in Figure 2. It is implied from the definitions in Figure 2 that

$$\frac{\partial}{\partial y} = O(\varepsilon^{-1}) , \quad \frac{\partial}{\partial x} = \frac{\partial}{\partial z} = O(1) \qquad \text{in Domain I,} \tag{9}$$

and

$$\frac{\partial}{\partial y} = \frac{\partial}{\partial z} = O(\varepsilon^{-1}) , \quad \frac{\partial}{\partial x} = O(1) \qquad \text{in Domain III.} \tag{10}$$

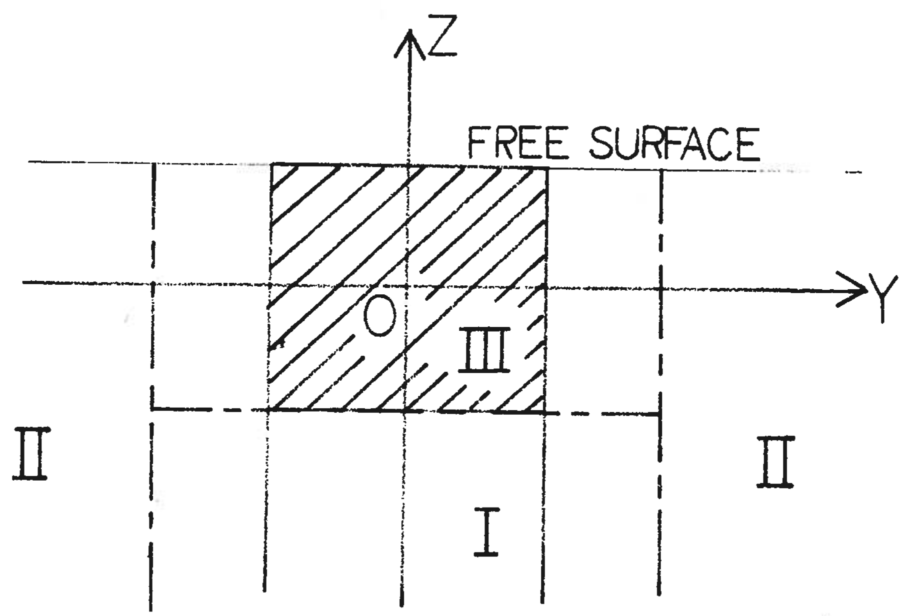

FIGURE 2. Near-Field Geometry

Domain I: Thin-ship near field, in which $x = O(1)$, $y = O(\varepsilon)$, $z = O(1)$ as $\varepsilon \to 0$.

Domain II: Far field.

Domain III: Slender-ship near field, in which $y = O(\varepsilon)$ and $z = O(\varepsilon)$ as $\varepsilon \to 0$.

III. FAR-FIELD EXPANSION

In the far field, in which $y = O(1)$, we assume the existence of the expansions,

$$\phi(x,y,z) \sim U x + \sum_{n=1}^{N} \phi_n(x,y,z) \quad ,$$

$$\xi(x,y) \sim \sum_{n=1}^{N} \xi_n(x,y) \quad ,$$

where

$$\phi_{n+1} = o(\phi_n) \qquad n \geq 1$$

as $\varepsilon \to 0$ for fixed (x,y,z).

$$\xi_{n+1} = o(\xi_n) \qquad n \geq 1$$

Moreover, we assume ϕ_n is composed of three parts,

$$\phi_n = \phi_{T_n} + \phi_{S_n} + \phi_{P_n} \quad , \tag{11}$$

ϕ_{T_n} being the potential due to a singularity distribution on the center-plane H , ϕ_{S_n} the potential due to a singularity distribution on the center line L , and ϕ_{P_n} the potential which will be introduced so as to satisfy the higher-order free surface condition.

On substituting the expansions into (2), we have

[L] $$\phi_{n_{xx}} + \phi_{n_{yy}} + \phi_{n_{zz}} = 0 \ . \tag{12}$$

A systematic treatment of the free surface conditions leads to:

[A] $U\phi_{1x} + g\zeta_1$ $\qquad [O(\varepsilon)]$

$+ U\phi_{2x} + U\phi_{1xz}\zeta_1 + \frac{1}{2}(\phi_{1x}^2 + \phi_{1y}^2 + \phi_{1z}^2) + g\zeta_2$ $\qquad [O(\varepsilon^2)]$

$+$ higher order terms $= 0$, at $z = 0$; (13)

[B] $$U\zeta_{1_x} - \phi_{1_z} \qquad [O(\varepsilon)]$$

$$+ U\zeta_{2_x} + \phi_{1_x}\zeta_{1_x} + \phi_{1_y}\zeta_{1_y} - \phi_{1_z}\zeta_1 - \phi_{2_z} \qquad [O(\varepsilon^2)]$$

$$+ \text{higher order terms} = 0 , \qquad \text{at} \quad z = 0 . \tag{14}$$

Combining (13) and (14), the lowest-order condition on $z = 0$ is

$$\phi_{1_{xx}} + \frac{g}{U^2}\phi_{1_z} = 0 . \tag{15}$$

The second order condition, from (13) and (14), is

$$\phi_{2_{xx}} + \frac{g}{U^2}\phi_{2_z} = -\frac{1}{U}\{\phi_{1_x}^2 + \phi_{1_y}^2 + \phi_{1_z}^2\}_x + \frac{1}{U}\phi_{1_x}\{\frac{U^2}{g}\phi_{1_{xx}} + \phi_{1_z}\}_z$$

$$\text{on} \quad z = 0 . \tag{16}$$

The free surface condition may also be written:

$$\phi_{n_{xx}} + \frac{g}{U^2}\phi_{n_z} = g_n(x,y) \qquad \text{on} \quad z = 0 , \tag{17}$$

where $g_n(x,y)$ can be expressed in terms of the lower order potentials, ϕ_m, with $m < n$. Moreover, ϕ_n must satisfy a radiation condition.

ϕ_n was assumed to be composed of three components of potential in (11). We are now more specific about these potentials. We assume that ϕ_{T_n} and ϕ_{S_n} are potentials that satisfy, respectively, the conditions

$$\phi_{T_{n_{xx}}} + \frac{g}{U^2}\phi_{T_{n_z}} = 0 \qquad \text{on} \quad z = 0 , \tag{18}$$

and

$$\phi_{S_{n_{xx}}} + \frac{g}{U^2}\phi_{S_{n_z}} = 0 \qquad \text{on} \quad z = 0 , \tag{19}$$

and ϕ_{P_n} is a potential that satisfies the condition

$$\phi_{P_{n_{xx}}} + \frac{g}{U^2}\phi_{P_{n_z}} = g_n(x,y) \qquad \text{on} \quad z = 0 . \tag{20}$$

Thus, the free surface boundary conditions satisfied by the velocity potentials are defined on the plane $z = 0$, and they are linear equations. It is well known that (20) is identical with the linearized free surface condition when a pressure distribution is applied to the water surface. The solution for such a problem is now well studied. So the velocity potentials, ϕ_{P_n}, can be known if ϕ_{T_n} and ϕ_{S_n} are known.

It is easily seen that ϕ_{T_n} and ϕ_{S_n} are of the forms

$$\phi_{T_n}(x,y,z) = -\frac{1}{4\pi}\iint_H \sigma_n(\xi,\zeta)G(x,y,z;\xi,0,\zeta)d\xi d\zeta , \qquad (21)$$

and

$$\phi_{S_n}(x,y,z) = -\frac{1}{4\pi}\int_L \gamma_n(\xi)G(x,y,z;\xi,0,\zeta)d\xi . \qquad (22)$$

Here, we assume that the distributed singularities with densities $\sigma_n(\xi,\zeta)$ and $\gamma_n(\xi)$, are sources. This assumption is valid for σ_n, $n = 1,2,3,\ldots$, and it is valid for at least the lowest-order γ_n.

Let us consider the inner expansion of the far field potentials. Since we have two different inner regions, we must make the inner expansions for each inner region, I and III. Here, we consider the inner expansion in the domain I, where $y = O(\varepsilon)$ and $z = O(1)$. Ogilvie (1970) gave the inner expansion for ϕ_{T_n} in the domain I:

$$\phi_{T_n}(x,y,z) \sim \phi_{T_n}(x,0,z) + \frac{1}{2}|y|\sigma_n(x,z;\varepsilon) + \text{higher order terms.} \qquad (23)$$

ϕ_{S_n}, the slender-ship potential, and ϕ_{P_n}, the pressure potential, are expanded into power series in y *:

$$\underset{[O(\varepsilon^n)]}{\phi_{S_n}(x,y,z) \sim \phi_{S_n}(x,0,z)} + \underset{[O(\varepsilon^{n+2})]}{\frac{1}{2}|y^2|\phi_{S_{n_{yy}}}(x,0,z)} + \text{higher order terms,} \qquad (24)$$

and

*This is permissible only because region I is really in the far field of the line distribution of singularities.

$$\phi_{P_n}(x,y,z) \sim \underset{[O(\varepsilon^n)]}{\phi_{P_n}(x,0,z)} + \underset{[O(\varepsilon^{n+2})]}{\frac{1}{2}|y^2|\phi_{P_{n_{yy}}}(x,0,z)} + \text{higher order terms.} \tag{25}$$

Here, we used the facts that $\phi_{S_{ny}}(x,0,z) = 0$ and $\phi_{P_{ny}}(x,0,z) = 0$, which are true because of the symmetry of the potentials.

Since it turns out $\phi_{S_1}(x,y,z) = 0$, the two-term inner expansion of the two-term outer expansion is

$$\phi(x,y,z) \sim \underset{[O(1)]}{Ux} + \underset{[O(\varepsilon)]}{\phi_{T_1}(x,0,z)} \quad . \tag{26}$$

The three-term inner expansion of the two-term outer expansion is

$$\phi(x,y,z) \sim \underset{[O(1)]}{Ux} + \underset{[O(\varepsilon)]}{\phi_{T_1}(x,0,z)} + \underset{[O(\varepsilon^2)]}{\frac{1}{2}|y|\sigma_1(x,z;\varepsilon)} \quad . \tag{27}$$

The three-term inner expansion of the three-term outer expansion is

$$\begin{aligned}\phi(x,y,z) \sim &\underset{[O(1)]}{Ux} + \underset{[O(\varepsilon)]}{\phi_{T_1}(x,0,z)} + \underset{[O(\varepsilon^2)]}{\frac{1}{2}|y|\sigma_1(x,z;\varepsilon)} + \underset{[O(\varepsilon^2)]}{\phi_{T_2}(x,0,z)} \\ &+ \underset{[O(\varepsilon^2)]}{\phi_{S_2}(x,0,z)} + \underset{[O(\varepsilon^2)]}{\phi_{P_2}(x,0,z)} \quad .\end{aligned} \tag{28}$$

The four-term inner expansion of the three-term outer expansion is

$$\begin{aligned}\phi(x,y,z) \sim &\underset{[O(1)]}{Ux} + \underset{[O(\varepsilon)]}{\phi_{T_1}(x,0,z)} + \underset{[O(\varepsilon^2)]}{\frac{1}{2}|y|\sigma_1(x,z;\varepsilon)} + \underset{[O(\varepsilon^2)]}{\phi_{T_2}(x,0,z)} \\ &+ \underset{[O(\varepsilon^2)]}{\phi_{S_2}(x,0,z)} + \underset{[O(\varepsilon^2)]}{\phi_{P_2}(x,0,z)} + \underset{[O(\varepsilon^3)]}{\frac{1}{2}|y|\sigma_2(x,z;\varepsilon)} \quad .\end{aligned} \tag{29}$$

We must here consider that $y = O(\varepsilon)$ in order to recognize the orders of magnitude as indicated above.

IV. *THIN-SHIP NEAR-FIELD EXPANSION*

Now, consider the near-field problem in the domain I. We use new coordinates in this near field, $y = \varepsilon Y$, $x = X$, and $z = Z$, where X , Y , Z are all $O(1)$ as $\varepsilon \to 0$. Substitute these into the formulation of the problem. Using the assumption,

$$\frac{\partial}{\partial y} = \frac{1}{\varepsilon}\frac{\partial}{\partial Y} = O(\varepsilon^{-1}) \ , \qquad \frac{\partial}{\partial X} = \frac{\partial}{\partial Z} = O(1) \ , \tag{9}$$

The Laplace equation, (2), becomes

[L]
$$\phi_{YY} + \varepsilon^2(\phi_{XX} + \phi_{ZZ}) = 0 \tag{30}$$

in the fluid domain, and the boundary condition, (4), becomes

[H]
$$\phi_Y = \pm\varepsilon^2[\phi_X H_X(X,Z) + \phi_Z H_Z(X,Z)] \qquad \text{on} \quad Y = \pm H(X,Z) \ . \tag{31}$$

Assume that there is an asymptotic expansion for the velocity potential ϕ :

$$\phi \sim UX + \sum_{n=1}^{N} \Phi_n(X,Y,Z;\varepsilon)$$

where $\Phi_{n+1} = o(\Phi_n)$ as $\varepsilon \to 0$ for fixed (X,Y,Z) . We then can express the conditions on the near-field expansion as follows:

[L]
$$\Phi_{1YY} + \Phi_{2YY} + \Phi_{3YY} + \cdots$$
$$\sim -\varepsilon^2[\Phi_{1XX} + \Phi_{1ZZ} + \Phi_{2XX} + \Phi_{2ZZ} + \cdots\cdots] \tag{32}$$

in the fluid domain;

[H]
$$\Phi_{1Y} + \Phi_{2Y} + \Phi_{3Y} + \cdots\cdots$$
$$\sim \pm\varepsilon^2[UH_X + \Phi_{1X}H_X + \Phi_{1Z}H_Z + \cdots\cdots]$$
$$\text{on} \quad Y = \pm H(X,Z) \ . \tag{33}$$

There is no free surface condition in Region I.

<u>Solution of the Φ_1 problem.</u> From (32) and (33), we have the conditions for Φ_1 :

[L] $$\Phi_{1YY} = 0 \qquad \text{in the fluid domain,} \tag{34}$$

[H] $$\Phi_{1Y}(X,\pm H(X,Z),Z;\varepsilon) = 0 \qquad \text{on } Y = \pm H(X,Z) \;. \tag{35}$$

Since the potential is symmetric with respect to Y , we can set

$$\Phi_1(X,Y,Z;\varepsilon) = A_1(X,Z;\varepsilon) + B_1(X,Z;\varepsilon)|Y| \quad \text{for } |Y| > H(X,Z) \;. \tag{36}$$

From (35), we have

$$B_1(X,Z;\varepsilon) = 0 \;.$$

The reason why we set $\Phi_{1Y} = 0$ in (35) is this: If we set $\Phi_{1Y} = \pm\varepsilon^2 UH_X = O(\varepsilon^2)$, we cannot match the inner solution with the outer solution. Then we have two-term inner solution,

$$\phi(x,y,z;\varepsilon) \sim UX + A_1(X,Z;\varepsilon) \;. \tag{37}$$

Its outer expansion is the same as (37), and it should match with the two-term inner expansion of the two-term outer expansion, (26):

$$\phi(x,y,z;\ ) \sim Ux + \phi_{T_1}(x,0,z) \;. \tag{26}$$

This matching determines $A_1(X,Z;\varepsilon)$:

$$A_1(X,Z;\varepsilon) = \phi_{T1}(x,0,z) = O(\varepsilon) \;. \tag{38}$$

Now the inner solution to two terms is

$$\phi(x,y,z;\varepsilon) \sim \underset{[O(1)]}{Ux} + \underset{[O(\varepsilon)]}{\phi_{T_1}(X,0,Z;\varepsilon)} \;. \tag{39}$$

<u>Solution of the Φ_2 problem.</u> In this case, we have

[L] $$\Phi_{2YY} = 0 \qquad \text{in the fluid domain,} \tag{40}$$

and

$$[H] \qquad \Phi_{2Y}(X,Y,Z;\varepsilon) = \pm\varepsilon^2 UH_X(x,z) \qquad \text{on} \quad Y = \pm H(X,Z) \; . \tag{41}$$

Then the solution is straightforwardly given by

$$\Phi_2(X,Y,Z;\varepsilon) = A_2(X,Z;\varepsilon) + B_2(X,Z;\varepsilon)|Y| \; , \tag{42}$$

where $$B_2(X,Z;\varepsilon) = \varepsilon^2 UH_X(X,Z) \; . \tag{43}$$

The three-term inner expansion is

$$\phi(x,y,z;\varepsilon) \sim \underset{[O(1)]}{Ux} + \underset{[O(\varepsilon)]}{\phi_{T1}(X,0,Z)} + \underset{[O(\varepsilon^2)]}{A_2(X,Z;\varepsilon)} + \underset{[O(\varepsilon^2)]}{\varepsilon^2 UH_X(X,Z)|Y|} \; . \tag{44}$$

Its two-term outer expansion is obtained by setting $Y = y/\varepsilon$ and re-ordering terms:

$$\phi(x,y,z;\varepsilon) \sim \underset{[O(1)]}{Ux} + \underset{[O(\varepsilon)]}{\phi_{T1}(x,0,z)} + \underset{[O(\varepsilon)]}{\varepsilon UH_x(x,z)|y|} \; . \tag{45}$$

The three-term inner expansion of the two-term outer expansion is, from (27)

$$\phi(x,y,z;\varepsilon) \sim Ux + \phi_{T1}(x,0,z) + \frac{1}{2}|y|\sigma_1(x,z;\varepsilon) \; .$$

These two match if:

$$\sigma_1(x,z;\varepsilon) = 2\varepsilon UH_X(X,Z) = 2Uh_x(x,z) = O(\varepsilon) \; . \tag{46}$$

Thus, $\sigma_1(x,z;\varepsilon)$ is determined by this matching; it is given as a function of the ship geometry. So the first-order outer potential coincides with the well-known Michell potential.

The source density, $\sigma_1(x,z;\varepsilon)$, is determined in the domain I, without regard to a free-surface condition. This means that the first-order velocity potential, which satisfies the free surface condition

$$\phi_{xx} + \frac{g}{U^2}\phi_z = 0 \qquad \text{at} \quad z = 0 , \tag{15}$$

can be determined without considering the free-surface condition near the body. But, in fact, the first-order inner solution can satisfy the free-surface condition (15), even near the body, because (37) is valid in $z = O(\varepsilon)$. But this will not be true when Φ_2 is included in the expansion.

The matching of the three-term outer expansion of the three-term inner expansion and the three-term inner expansion of the three-term outer potential will give $A_2(X,Z;\varepsilon)$, as follows:

$$A_2(X,Z;\varepsilon) = \phi_{T_2}(X,0,Z) + \phi_{S_2}(X,0,Z) + \phi_{P_2}(X,0,Z) = O(\varepsilon^2) . \tag{47}$$

So at this stage the inner potential in the domain I shows an influence of the potentials ϕ_{S2} and ϕ_{P2} . Then,

$$\Phi_2(X,Y,Z;\varepsilon) = \phi_{T_2}(X,0,Z) + \phi_{S_2}(X,0,Z) + \phi_{P_2}(X,0,Z) + \varepsilon^2 U H_X(X,Z)\,|Y| . \tag{48}$$

Φ_2 also seems to satisfy the free surface condition (17)

$$\Phi_{2xx} + \frac{g}{U^2}\Phi_{2z} = g_2(x,0) \qquad \text{at} \quad z = 0 \tag{17}$$

near the body when $H_X(X,Z) = 0$, at parallel-middle-body part, say. But Φ_2 is not now valid in $z = O(\varepsilon)$. The free surface conditions cannot be satisfied at $z = 0$ near the body. The first-order inner solution in the domain III turns out to be given by the same first-order thin-ship potential as in the first-order inner solution in the domain I. This indicates that, as far as the first-order potential is concerned, we do not need to introduce the domain III. Or, in other words, the problem is not yet a singular perturbation problem. To the contrary, the second-order problem will be a genuine singular perturbation problem.

Solution of the Φ_3 problem. In order to know the second-order thin-ship potential, ϕ_{T2} , we have to solve the problem of Φ_3 . In this case, the problem is slightly different from the previous problems because the Laplace equation degenerates into a nonhomogeneous differential equation. So Φ_3 is not linear in Y. The conditions on Φ_3 are:

$$[L] \qquad \Phi_{3YY}(X,Y,Z;\varepsilon) = -\varepsilon^2\{\Phi_{1XX}(X,Y,Z;\varepsilon) + \Phi_{1ZZ}(X,Y,Z;\varepsilon)\} \quad \text{in the fluid domain,} \tag{49}$$

$$[H] \qquad \Phi_{3Y}(X,Y,Z;\varepsilon) = \pm\varepsilon^2\{\Phi_{1X}(X,Y,Z;\varepsilon)H_X(X,Z) + \Phi_{1Z}(X,Y,Z;\varepsilon)H_Z(X,Z)\} \quad \text{on} \quad Y = \pm H(X,Z)\ . \tag{50}$$

Since Φ_{1XX} and ϕ_{1ZZ} do not depend on Y, the solution for Φ_3 is:

$$\Phi_3(X,Y,Z;\varepsilon) = \underset{[O(\varepsilon^3)]}{A_3(X,Z;\varepsilon)} + \underset{[O(\varepsilon^3)]}{B_3(X,Z;\varepsilon)|Y|} + \underset{[O(\varepsilon^3)]}{C_3(X,Z;\varepsilon)|Y|^2}, \tag{51}$$

where $C_3(X,Z;\varepsilon) = -\frac{\varepsilon^2}{2}\{\Phi_{1XX}(X,Y,Z;\varepsilon) + \Phi_{1ZZ}(X,Y,Z;\varepsilon)\}$, from (49).

Immediately from the boundary condition (50), $B_3(X,Z;\varepsilon)$ is given by

$$B_3(X,Z;\varepsilon) = \varepsilon^2\{(\Phi_{1X}H)_X + (\Phi_{1Z}H)_Z\}$$

So far, the potential Φ_3 is

$$\Phi_3(X,Y,Z;\varepsilon) = A_3(X,Z;\varepsilon) + \varepsilon^2\{(\Phi_{1X}H)_X + (\Phi_{1Z}H)_Z\}|Y| - \frac{\varepsilon^2}{2}\{\Phi_{1XX} + \Phi_{1ZZ}\}|Y|^2\ .$$

Then, the four-term inner solution is

$$\phi(x,y,z;\varepsilon) \sim \underset{[O(1)]}{UX} + \underset{[O(\varepsilon)]}{\phi_{T_1}(X,0,Z)}$$

$$+ \underset{[O(\varepsilon^2)]}{\varepsilon^2 UH_X(X,Z)|Y|} + \underset{[O(\varepsilon^2)]}{\phi_{T_2}(Z,0,Z)} + \underset{[O(\varepsilon^2)]}{\phi_{S_2}(X,0,Z)} + \underset{[O(\varepsilon^2)]}{\phi_{P_2}(X,0,Z)}$$

$$+ \underset{[O(\varepsilon^3)]}{A_3(X,Z;\varepsilon)} + \underset{[O(\varepsilon^3)]}{\varepsilon^2\{(\Phi_{1X}H)_X + (\Phi_{1Z}H)_Z\}|Y|} - \underset{[O(\varepsilon^3)]}{\frac{\varepsilon^2}{2}\{\Phi_{1XX} + \Phi_{1ZZ}\}|Y|^2}\ .$$

Its three-term outer expansion is obtained by setting $Y = y/\varepsilon$:

$$\phi(x,y,z;\varepsilon) \sim Ux + \phi_{T1}(x,0,z) + \varepsilon UH_x(x,z)|y| + \phi_{T2}(x,0,z)$$
$$+ \phi_{S_2}(x,0,z) + \phi_{P_2}(x,o,z) + \varepsilon\{(\Phi_{1_x}H)_x + (\Phi_{1_z}H)_z\}|y|$$

and this should match with the four-term inner expansion of the three-term outer potential, (29). This matching gives the source distribution, $\sigma_2(x,z;\varepsilon)$:

$$\sigma_2(x,z;\varepsilon) = 2\{\phi_{T1_x}(x,0,z)h(x,z)\}_x + 2\{\phi_{T1_z}h\}_z \ .$$

Now, we will stop at this stage of solving the problem in the domain I. The source distribution $\sigma_2(x,z)$ gives the second-order outer thin-ship potential. As in the first-order outer thin-ship potential, the second-order potential can be determined without considering the free-surface condition near the ship body. So far, in solving the inner problem, there is no difference between the infinite symmetric thin-body problem and the thin-body problem with the free surface, except for the introduction of both the slender-body potential and the pressure potential in Φ_n , $n \geq 2$.

In addition, it may be noted that *both* source distributions, σ_1 and σ_2 , represent well-known results in thin-ship theory.

V. *SLENDER-BODY NEAR-FIELD EXPANSION*

Next, we consider the near-field problem in the domain III, where $y = O(\varepsilon)$ and $z = O(\varepsilon)$. Now, since the free surface must be considered in this region, we must be careful in treating the conditions at the free surface.

Let us look for the inner expansions in the domain III of the outer potentials. As for the thin-ship potential, Ogilvie (1970) gave the expression of the two-term expansions:

$$\phi_{T_n}(x,y,z) \sim \underset{[O(\varepsilon^n)]}{\phi_{T_n}(x,0,0)} + \underset{[O(\varepsilon^{n+1})]}{\frac{1}{2}\sigma_n(x,0)|y|} - \underset{[O(\varepsilon^{n+1})]}{\frac{1}{\nu}\phi_{T_{n_{xx}}}(x,0,0)z} + \cdots, \tag{54}$$

with $\nu = \frac{g}{U^2}$. This expression is valid in the domain III. Tuck (1965) gave the inner expansions of the slender-ship potentials in his slender-ship theory. The one-term inner expansion of the outer slender-body potential, (22), is:

$$\phi_{S_n}(x,y,z) \sim \underset{[O(\varepsilon^n)]}{\frac{1}{\pi}\gamma_n(x)\log r} - \underset{[O(\varepsilon^n)]}{\frac{1}{2\pi}f_n(x)} - \underset{[O(\varepsilon^n)]}{g_n(x)} + \cdots, \tag{55}$$

where

$$f_n(x) = \int_{-\infty}^{\infty} d\xi\gamma_n'(\xi)\log 2|x-\xi|\,\text{sgn}(x-\xi) \tag{56}$$

$$= -\frac{1}{\pi}\int_{-\infty}^{\infty} dk e^{ikx}\gamma^*(k)\log\frac{C|k|}{2},$$

$$g_n(x) = \lim_{\mu\to 0}\frac{U^2}{4\pi^2}\int_{-\infty}^{\infty} dk e^{ikx}k^2\gamma^*(k)\int_{-\infty}^{\infty}\frac{d\ell}{\sqrt{(k^2+\ell^2)}\,[g\sqrt{(k^2+\ell^2)} - (Uk-i\mu/2)^2]}$$

and (57)

$$\gamma^*(k) = \int_{-\infty}^{\infty} dk e^{-ikx}\gamma(x).$$

The pressure potential, ϕ_{P_n} , can be expanded into a Taylor series in the domain III, and so, as in (25), its two-term expansion is:

$$\phi_{P_n}(x,y,z) = \underset{[O(\varepsilon^n)]}{\phi_{P_n}(x,0,0)} + \underset{[O(\varepsilon^{n+1})]}{\phi_{P_{nz}}(x,0,0)z} + \cdots\cdots , \tag{58}$$

Using the expansions (54), (55), and (58), the inner expansions of the outer potentials become as follows:

(1) The two-term inner expansion of the two-term outer expansion:

$$\phi(x,y,z) \sim Ux + \phi_{T_1}(x,0,0) . \tag{59}$$

(2) The three-term inner expansion of the two-term outer expansion:

$$\phi(x,y,z) \sim Ux + \phi_{T_1}(x,0,0) + \frac{1}{2}\sigma_1(x,0)|y| - \frac{1}{\nu}\phi_{T_{1xx}}(x,0,0)z . \tag{60}$$

(3) The three-term inner expansion of the three-term outer expansion:

$$\phi(x,y,z) \sim Ux + \phi_{T_1}(x,0,0) + \frac{1}{2}\sigma_1(x,0)|y| - \frac{1}{\nu}\phi_{T_{1xx}}(x,0,0)z + \phi_{T_2}(x,0,0) + \phi_{P_2}(x,0,0) + \frac{1}{\pi}\gamma_2(x)\log r - \frac{1}{2\pi}f_2(x) - g_2(x) . \tag{61}$$

Now consider the near-field problem in the domain III. We use new coordinates in the near-field:

$$y = \varepsilon Y , \quad z = \varepsilon Z , \text{ and } \quad x = X ,$$

where we treat X,Y,Z each as being $O(1)$ as $\varepsilon \to 0$. We substitute these into the equations (2), (4), (5), and (6). Using the assumptions

$$\frac{\partial}{\partial y} = \frac{1}{\varepsilon}\frac{\partial}{\partial Y} = O(\frac{1}{\varepsilon}) , \quad \frac{\partial}{\partial z} = \frac{1}{\varepsilon}\frac{\partial}{\partial Z} = O(\frac{1}{\varepsilon}) \quad \text{and} \quad \frac{\partial}{\partial x} = \frac{\partial}{\partial X} = O(1) ,$$

except that $\frac{\partial h}{\partial z} = O(h) = O(\varepsilon)$, the Laplace equation, (2), becomes

[L] $$\phi_{YY} + \phi_{ZZ} + \varepsilon^2\phi_{XX} = 0 \qquad (63)$$

in the fluid domain, and the boundary conditions are

[H] $$\varepsilon^2\phi_X H_X \mp \phi_Y + \phi_Z H_Z = 0 \qquad \text{on} \quad Y = \pm H(x,z) \;*\;, \qquad (64)$$

[A] $$g\zeta + \frac{1}{2}\{\phi_X^2 + \frac{1}{\varepsilon^2}(\phi_Y^2 + \phi_Z^2)\} = \frac{1}{2}U^2 \qquad \text{on the free surface,} \qquad (65)$$

[B] $$\phi_X\zeta_X + \frac{1}{\varepsilon^2}\phi_Y\zeta_Y - \frac{1}{\varepsilon}\phi_Z = 0 \qquad \text{on the free surface.} \qquad (66)$$

We assume that there are asymptotic expansions for the velocity potential ϕ and the wave elevation $\zeta(x,y)$:

$$\phi(x,y,z) \sim UX + \sum_{n=1}^{N} \Phi_n(X,Y,Z;\varepsilon) \;, \qquad (67)$$

where $\Phi_{n+1} = o(\Phi_n)$ as $\varepsilon \to 0$ for fixed (X,Y,Z) ;

$$\zeta(x,y) \sim \sum_{n=1}^{N} Z_n(X,Y;\varepsilon) \;, \qquad (68)$$

where

$$Z_{n+1} = o(Z_n) \quad \text{as} \quad \varepsilon \to 0 \quad \text{for fixed} \quad (X,Y) \;.$$

Substitution of these asymptotic expansions gives the following:

[L] $$\Phi_{1YY} + \Phi_{1ZZ} + \Phi_{2YY} + \Phi_{2ZZ} + \cdots = -\varepsilon^2(\Phi_{1XX} + \Phi_{2XX} + \cdots)$$

in the fluid domain. Thus, if $\Phi_1 = o(1)$, then

$$\Phi_{1YY} + \Phi_{1ZZ} = 0 \qquad (69)$$

and if $\varepsilon^2\Phi_1 = o(\Phi_2)$, then

$$\Phi_{2YY} + \Phi_{2ZZ} = 0 \;, \qquad (70)$$

$$\Phi_{3YY} + \Phi_{3ZZ} = -\varepsilon^2\Phi_{1XX} \;, \qquad (71)$$

and so on.

---

* Note that, with $h(x,z) = \varepsilon H(X,Z)$, we have: $h_x = \varepsilon H_X = O(\varepsilon)$, $h_z = H_Z + O(\varepsilon)$.

The body boundary condition becomes

$$[H] \quad \Phi_{1_Y} + \Phi_{2_Y} + \Phi_{3_Y} + \cdots = \pm\varepsilon^2[U + \Phi_{1_X} + \Phi_{2_X} + \cdots]H_X(X,Z)$$

$$\pm [\Phi_{1_Z} + \Phi_{2_Z} + \cdots]H_Z(X,Z) \quad \text{on} \quad Y = \pm H(X,Z) \; . \tag{72}$$

Thus, if $\Phi_1 = o(1)$, it follows that

$$\Phi_{1_Y} = 0 \; , \tag{73}$$

$$\Phi_{2_Y} = \pm\varepsilon^2 UH_X \pm \Phi_{1_Z}H_Z \; , \tag{74}$$

and so on. In this domain III, we assumed, as in (62), that if $\Phi_1 = O(\varepsilon)$, then $\frac{\partial\Phi_1}{\partial z} = O(1)$. However, it turns out that Φ_1 is just the first-order thin-ship potential, evaluated at $y = z = 0$. And so $\frac{\partial\Phi_1}{\partial z} = O(\varepsilon)$, which implies that in (72) we should consider the order of magnitude of $\Phi_{1_Z}H_Z$ to be $O(\varepsilon^2)$. Then (74) becomes

$$\Phi_{2_Y} = \pm\varepsilon^2 UH_X \; . \tag{75}$$

Conditions are satisfied on $Y = H(X,Z)$, but we make only a higher-order error if we satisfy them instead on $Y = H(X,0)$, since

$$H(X,Z) = H(X,0)[1 + O(\varepsilon)] \; .$$

On the free surface, the dynamic boundary condition is:

$$[A] \qquad gZ_1 + gZ_2 + \cdots + U[\Phi_{1_X} + \Phi_{2_X} + \cdots]$$

$$+ \frac{1}{2}[\Phi_{1_X}+\Phi_{2_X}+ \cdots]^2 + \frac{1}{2\varepsilon^2}\left([\Phi_{1_Y}+\Phi_{2_Y}+ \cdots]^2 + [\Phi_{1_Z}+\Phi_{2_Z}+ \cdots]^2\right) = 0 \; .$$

We assume that $Z_1 = O(\varepsilon)$. Thus, the lowest order condition on the free surface is

$$\frac{1}{2\varepsilon^2}[\Phi_{1_Y}^2 + \Phi_{1_Z}^2] = 0 \; ,$$

or

$$\Phi_{1_Y} = \Phi_{1_Z} = 0 \; . \tag{76}$$

The second-order condition is:

$$gZ_1 + U\Phi_{1_X} = 0 \; . \tag{77}$$

These conditions are satisfied on the free surface.

The kinematic free-surface condition is

[B]

$$(U + \Phi_{1_X} + \Phi_{2_X} + \cdots)(Z_{1_X} + Z_{2_X} + \cdots) + \frac{1}{\varepsilon^2}(\Phi_{1_Y} + \Phi_{2_Y} + \cdots)(Z_{1_Y} + Z_{2_Y} + \cdots) - \frac{1}{\varepsilon}(\Phi_{1_Z} + \Phi_{2_Z} + \cdots) = 0 \tag{78}$$

on the free surface. The lowest order relation, by using (76), is

$$UZ_{1_X} - \frac{1}{\varepsilon}\Phi_{2_Z} = 0 \; . \tag{79}$$

The second-order relation is

$$UZ_{2_X} + \Phi_{1_X}Z_{1_X} + \frac{1}{\varepsilon^2}\Phi_{2_Y}Z_{2_Y} - \frac{1}{\varepsilon}\Phi_{3_Z} = 0 \; . \tag{80}$$

These conditions are satisfied on the free surface.

In this near field, the free surface conditions cannot be satisfied on the surface $z = 0$. But at each stage of the inner problem, the position of the free surface should be specified. To this purpose, we try to evaluate potentials and their derivatives on $z = \varepsilon Z = Z_1$, which will turn out to be the wave elevation due to the first-order thin-ship potential. This procedure is valid because the free surface is $\zeta(x,y) = Z_1(x,y) + o(\varepsilon)$ in the domain III. Then we can expand some functions around $z = Z_1(X,Y)$ in the domain III, where $z = O(\varepsilon)$. So the conditions appearing in [A], [B], are to be satisfied on $z = Z_1(X,Y)$. Now we have conditions by which the inner problem will be solved.

<u>Problem for Φ_1</u> : From (69), (73), and (76), we have conditions on Φ_1 :

[L]

$$\Phi_{1_{YY}} + \Phi_{1_{ZZ}} = 0 \; , \tag{69}$$

[H] $$\Phi_{1_Y} = 0 \qquad \text{on} \quad Y = H(X,0) \ , \tag{73}$$

and

[A] $$\Phi_{1_Z} = \Phi_{1_Y} = 0 \qquad \text{on} \quad Z = \frac{Z_1(X,Y)}{\varepsilon} \ . \tag{76}$$

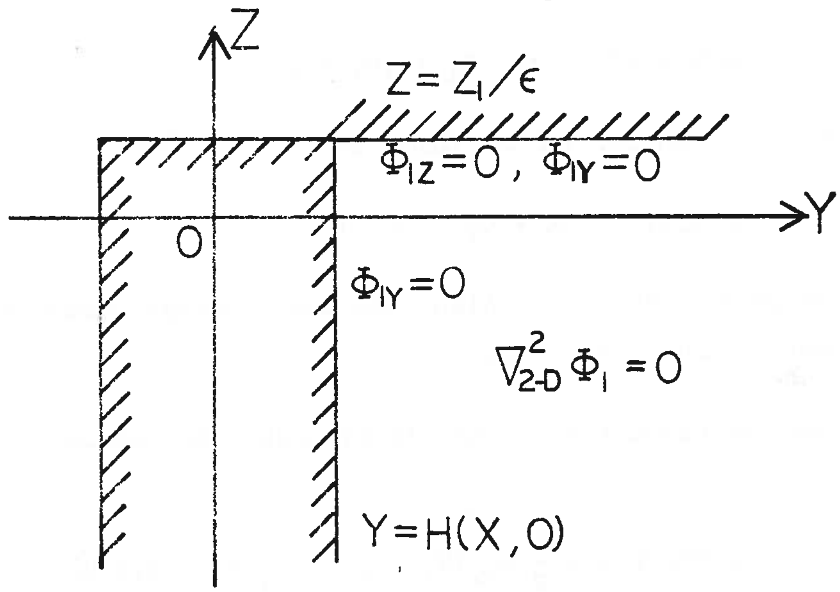

FIGURE 3. Domain III for Φ_1

From (69) and (76), in fact, Φ_1 cannot depend on Y and Z, and so we set:

$$\Phi_1(X,Y,Z) = \tilde{A}_1(X) \ . \tag{81}$$

This satisfies all the conditions on Φ_1. Moreover, this solution should match with the outer solution. With Φ_1 given in (81), the two-term inner solution is

$$\phi(x,y,z) \sim UX + \tilde{A}_1(X) \ . \tag{82}$$

This should match with the two-term inner expansion of the two-term outer potential, (59). They match if:

$$\tilde{A}_1(X) = \phi_{T_1}(x,0,0) = O(\varepsilon) \ .$$

So the two-term inner solution is

$$\phi(x,y,z) \sim \underset{[O(1)]}{Ux} + \underset{[O(\varepsilon)]}{\phi_{T1}(x,0,0)} \; . \qquad (83)$$

This solution also should match with the potential in the domain I, where $z = O(1)$. That potential is

$$\phi(x,y,z) \sim Ux + \phi_{T_1}(x,0,z) \; . \qquad (39)$$

Its inner expansion to two terms is

$$\phi(x,y,z) \sim Ux + \phi_{T_1}(x,0,0) \; .$$

This matches with (83). Also, the body boundary conditions are continuous between domains I and III.

Now we can determine the first-order free-surface elevation, Z_1 , from (77):

$$Z_1(X,Y) = -\frac{U}{g}\,\Phi_{1X}(X,Y,Z) = -\frac{U}{g}\,\phi_{T1_X}(x,0,0) \; . \qquad (84)$$

This represents just the waves along the centerplane due to the first-order thin-ship potential. Hereafter, the free surface conditions in the domain III are evaluated on $Z_1(X)$.

Problem for Φ_2 : From (70), (75) and (79), we have conditions on Φ_2 :

[L] $$\Phi_{2YY} + \Phi_{2ZZ} = 0 \quad \text{in the fluid domain,} \qquad (70)$$

[H] $$\Phi_{2Y} = \pm\varepsilon^2 UH_X(X,0) = \frac{1}{2}\,\varepsilon\sigma_1(x,0) \quad \text{on } Y = H(X,0) \; , \qquad (75)$$

[B] $$\Phi_{2Z} = \varepsilon UZ_{1X} = -\frac{U^2}{g}\,\phi_{T1_{xx}}(x,0,0) \quad \text{on } Z = \frac{Z_1(X)}{\varepsilon} \; . \qquad (79)$$

And the outer expansion of Φ_2 must have the form given by (61), or,

$$\Phi_2 \sim \frac{1}{2}\sigma_1(x,0)\,|y| - \frac{1}{\nu}\phi_{T1_{xx}}(x,0,0)z$$
$$+ \phi_{T_2}(x,0,0) + \frac{1}{\pi}\gamma_2(x)\log r - \frac{1}{2\pi}f_2(x) - g_2(x) + \phi_{P_2}(x,0,0) \; . \qquad (85)$$

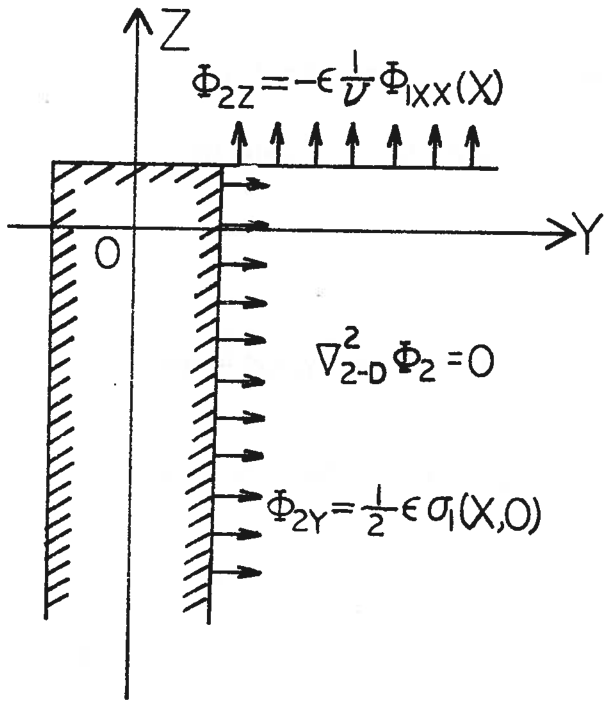

FIGURE 4a. Domain III for Φ_2

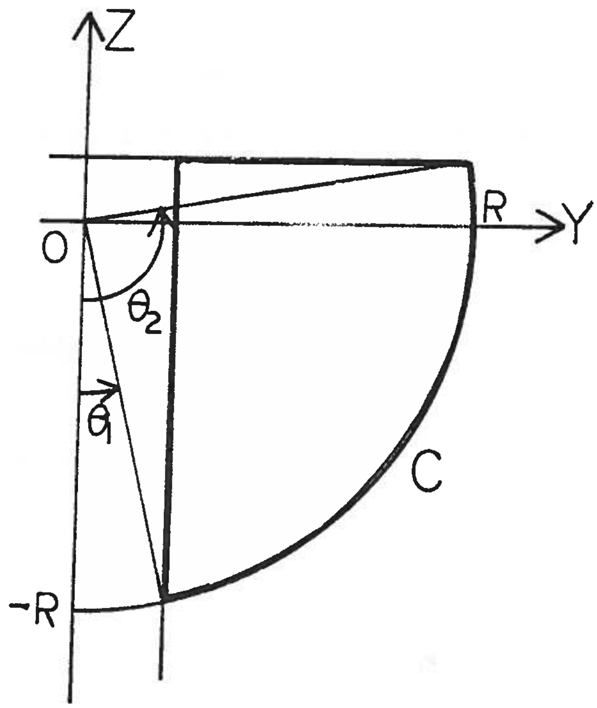

FIGURE 4b. Contour in Domain III

As shown in Fig. 4(a), there is inflow through the ship body and outflow through the free surface.

We may be able to solve the problem for Φ_2 explicitly, but we will not try to do so. We just determine the unknown source density, $\gamma_2(x)$, by the same method used by Tuck (1965). Using the conservation of mass, the flux through the contour C in Fig. 4(b) is given by:

$$Q = -\int_H^R \Phi_{2Z} dY + \int_{-R}^{\frac{Z_1}{\varepsilon}} \phi_{2Y} dZ$$

$$= = -\varepsilon U Z_{1X}[R-H] + \varepsilon^2 U H_X[\frac{Z_1}{\varepsilon} + R]$$

$$= \varepsilon R[-UZ_{1X}+\varepsilon U H_X] + \varepsilon U[HZ_{1X}+H_X Z_1] \qquad (86)$$

as $R \to \infty$.

Rewriting in the outer variables:

$$Q = r[-U\zeta_{1_x}(x,0) + Uh_x(x,z)] + U[h(x,z)\zeta_1(x,0)]_x \tag{87}$$

In the far-field, the flux from the contour C is given by:

$$Q = \int_{\theta_1}^{\theta_2} \Phi_{2\gamma} r d\theta$$

$$= \int_{\theta_1}^{\theta_2} [\frac{1}{2}\sigma_1(x,0)\sin\theta + \frac{1}{\nu}\phi_{T1_{xx}}(x,0,0)\cos\theta + \frac{1}{\pi}\gamma_2(x)\frac{1}{r}] r d\theta$$

$$= -\frac{r}{2}\sigma_1(x,0)[\cos\theta_2 - \cos\theta_1] + \frac{r}{\nu}\phi_{T1_{xx}}(x,0,0)[\sin\theta_2 - \sin\theta_1]$$

$$+ \frac{1}{\pi}\gamma_2(x)[\theta_2 - \theta_1] .$$

Since $\theta_1 \simeq \frac{h}{r} = O(\varepsilon)$ and $\theta_2 - \pi/2 = \frac{\zeta_1}{r} = O(\varepsilon)$, then, neglecting the higher-order terms, the flux Q is

$$Q = r[\frac{1}{2}\sigma_1(x,0) - U\zeta_{1_x}(x,0)] + \frac{1}{2}\gamma_2(x) + \text{higher order terms} \tag{88}$$

This matches with (87) if:

$$\frac{1}{2}\gamma_2(x) = U[h(x,0)\zeta_1(x,0)]_x . \tag{89}$$

The density of the line source distribution, γ_2 , is proportional to the rate of change of the volume in the x-direction caused by the waves due to the first-order thin-ship potential.

We will not step further into the Φ_3 problem in the domain III. So far, it becomes clear that the interaction of the ship hull and the ship waves is a higher-order phenomenon, and this effect can appear in the far field as the flow due to a line source distribution on the center-line.

VI. *HIGHER-ORDER THIN-SHIP POTENTIAL*

Now it remains to solve the problem for the pressure potential, ϕ_{P_2}, in order to get the full expression for the second-order thin-ship theory. From (12) and (16), we have

$$[L] \qquad \phi_{P_{2xx}} + \phi_{P_{2yy}} + \phi_{P_{2zz}} = 0 , \tag{90}$$

$$[A][B] \qquad \phi_{P_{2xx}} + \frac{g}{U^2}\phi_{P_{2z}} = -\frac{1}{U}\{\phi^2_{T_{1x}}+\phi^2_{T_{1y}}+\phi^2_{T_{1z}}\}_x + \frac{1}{U}\phi_{T_{1x}}\left(\frac{U^2}{g}\phi_{T_{1xx}}+\phi_{T_{1z}}\right)_z$$

$$= g_2(x,y) \qquad \text{on} \quad z = 0 . \tag{91}$$

From the previous result, we now know $\phi_{T_1}(x,y,z)$, so the problem ϕ_{P_2} can be solved. From the result for the potential due to a pressure distribution in Wehausen and Laitone (1960) (p. 598), we have for ϕ_{P_2}:

$$\phi_{P_2}(x,y,z) = -\frac{1}{4\pi U\nu}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left((\phi^2_{T_{1\xi}}+\phi^2_{T_{1\eta}}+\phi^2_{T_{1\zeta}})_\xi - \phi_{T_{1\xi}}\left(\frac{U^2}{g}\phi_{T_{1\xi\xi}}+\phi_{T_{1\zeta}}\right)_\zeta\right)$$

$$G(x,y,z;\xi,\eta,0)\,d\xi d\eta . \tag{92}$$

Then the velocity potential in the far field to three terms is now

$$\phi(x,y,z) \sim Ux + \phi_{T_1}(x,y,z;\varepsilon) + \phi_{T_2}(x,y,z;\varepsilon)$$

$$+ \phi_{S_2}(x,y,z;\varepsilon) + \phi_{P_2}(x,y,z;\varepsilon) , \tag{93}$$

where

$$\phi_{T_1}(x,y,z;\varepsilon) = -\frac{U}{2\pi}\iint_H h_\xi(\xi,\zeta)\; G(x,y,z;\xi,0,\zeta)\,d\xi d\zeta ,$$

$$\phi_{T_2}(x,y,z;\varepsilon) = -\frac{1}{2\pi}\iint_H [\phi_{T_{1\xi}}h_\xi+\phi_{T_{1\zeta}}h_\zeta-\phi_{T_{1\eta\eta}}h]G(x,y,z;\xi,0,\zeta)\,d\xi d\zeta ,$$

$$\phi_{S_2}(x,y,z;\varepsilon) = -\frac{U}{2\pi}\int_L \{h(\xi,0)\left(\zeta_1(\xi,0)\right)\}_\xi G(x,y,z;\xi,0,0)\,d\xi ,$$

and

$$\phi_{P_2}(x,y,z;\varepsilon) = -\frac{1}{4\pi U\nu}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(\{\phi^2_{T1\xi}+\phi^2_{T1\eta}+\phi^2_{T1\zeta}\}_\xi - \phi_{T1\xi}\left(\frac{U^2}{g}\phi_{T1\xi\xi}+\phi_{T1\zeta}\right)_\zeta\right)$$

$$G(x,y,z;\xi,\eta,0)d\xi d\zeta \ .$$

The wave heights are found to be:

$$\zeta_1(x,y) = -\frac{U}{g}\phi_{T1_x}(x,y,0) \tag{94}$$

$$\zeta_2(x,y) = -\frac{U}{g}[\phi_{T2_x}(x,y,0) + \phi_{S2_x}(x,y,0) + \phi_{P2_x}(x,y,0)]$$

$$-\frac{U}{g}\phi_{T1_{xz}}\zeta_1 - \frac{1}{2g}[\phi^2_{T1_x}+\phi^2_{T1_y}+\phi^2_{T1_z}] \tag{95}$$

Now we have completed the second-order thin-ship theory by making use of the method of matched asymptotic expansion. The results are not new, but the process to the result is clear. In the far field, the velocity potential is composed of the three parts. They are the thin-ship potential, the slender-ship potential, and the pressure potential. The thin-ship part of the potential is interpreted in the usual sense. This part is the disturbance due to the main hull under the free surface.

The slender-ship part of the potential is interpreted as that part which provides the correction which is necessary because of the existence of ship waves along the ship hull. Its source density is, in fact, the rate of change in the x-direction of the sectional area of the rectangle which is the product of the beam and wave height, as shown in Fig. 5. In this sense, this slender-ship potential part has the same property as the potential in the slender-ship theory. In the near-field, only the slender ship potential part becomes singular. The properties of the slender-ship potential are to some extent known by the investigation of the line integral which appeared in the thin-ship theory by Yim (1964). But we do not know so much about the higher order effect of the line integral part in the thin-ship theory. From the viewpoint of the inner region, especially in the domain III, where the free surface is included, the first-order thin-ship potential gives the entire description for the problem within the limit of the first-order theory. But this first-order solution in the domain III does not have any Y or Z dependent terms. In order to know the velocities in both Y and Z -direction near the free surface, we have to solve the second-order inner problem. In that boundary value problem, the free-surface condition must be satisfied on the first-order thin-ship waves. In order to make clear the behavior of the flow near the body, we have to take the effect of the free surface near the body into account. It suggests that the effects of the slender-ship potential part on the wave making theory may be more important than the other higher-order terms.

In the problem of a submerged body, the importance of the second-order pressure potential has been stressed. Maybe in the problem of a surface ship, this pressure potential will also play an important role, just as in the case of the submerged body.

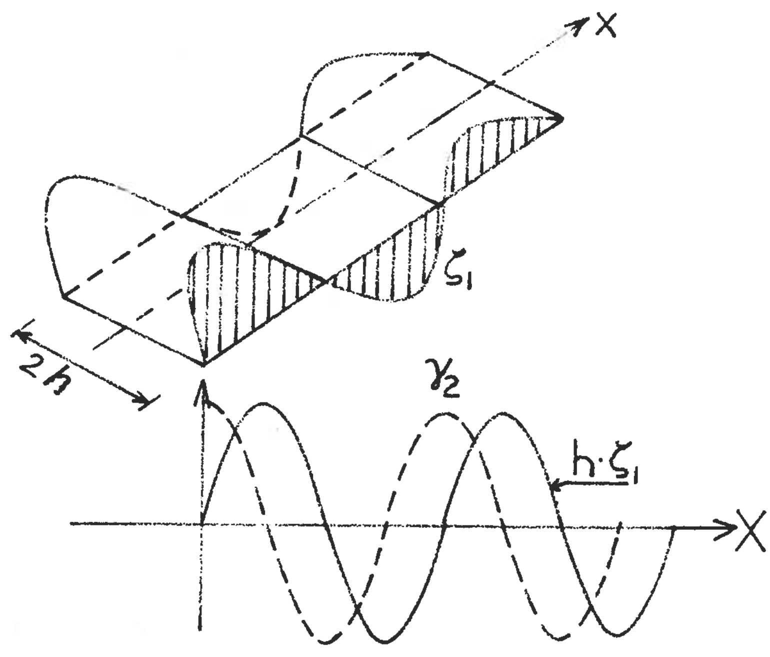

FIGURE 5. Line Source Density

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Hiroyuki Adachi

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13. ABSTRACT

A new approach to the higher-order thin-ship theory is developed by means of the method of matched asymptotic expansions. The result which is obtained here is not new, but the approach is entirely different from others. The source distributions are determined by the process of matching of the far-field solution to the near-field solution. So the expression for the higher-order thin-ship theory is valid in the far field.

The near field is a combination of thin-ship and slender-ship near field. The slender-ship near field is necessary to take care of the free-surface condition, which is satisfied on the free surface instead of the $z = 0$ plane. A combination thin-ship and slender-ship approach makes it possible to construct higher-order potentials.

In the far field, the potential is composed of three types of potentials. They are the thin-ship potential, the slender-ship potential, and the pressure-distribution potential, all of which satisfy linearized free-surface conditions at the $z = 0$ plane. Among them, the slender-ship potential has the most interesting features.