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ESTIMATION OF GREAT LAKES BULK CARRIER RESISTANCE BASED ON MODEL TEST DATA REGRESSION

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ERRATA in "Estimation of Great Lakes Bulk Carrier Resistance Based on Model Test Data Regression".

Page ii: C should read "C_∇"

Page 10, line 17: multiple correlative coefficient should read "multiple correlation coefficient"

Page 12, line 7 of table should read:

$$(C_B)^2(L/B) \quad (C_B)^2(B/T) \quad (C_B)^2(C_{WS}) \quad (C_B)^2(C_\nabla)$$

Page 36: add to term F_3:

$$+ A_9(B/T)^2(L/B) + A_{11}(C_\nabla)^2(L/B)$$

Page I-1, line 14: add after C_∇: x 10^3

Page I-5: line 3: add after C_∇: x 10^3

Fig. 27: C_V should read "C_∇=3.5"

Fig. 28: insert B/T=3.0

SYNOPSIS

Tank data have been collected, analyzed and standardized for 50 tests of Great Lakes Bulk Carriers. Regression analysis has been applied in order to estimate the coefficient of residuary resistance of such vessels in terms of their nondimensional form parameters. The results are presented for eight Froude numbers from 0.11 to 0.18 in the form of coefficients obtained by different regressions, and in the form of charts at Froude numbers 0.14 and 0.16. Examples illustrate the use of the regression formulas in estimating the full scale resistance.

NOMENCLATURE

B	beam (ft.)
C_A	model-ship correlation allowance
C_B	block coefficient, $\nabla/(LB\ T)$
C_F	frictional resistance coefficient, $R_F/(0.5\rho V^2 S)$
C_R	residuary resistance coefficient, $R_R/(0.5\rho V^2 S)$
C_T	total resistance coefficient, $R_T/(0.5\rho V^2 S)$
C	volumetric coefficient, ∇/L^3
C_{WS}	wetted surface coefficient, $S/\nabla^{2/3}$
EHP	effective horsepower (without appendages)
F_n	Froude number, $V/\sqrt{gL}$
g	acceleration of gravity (ft./sec.^{2})
i_E	half angle of entrance
L	length of the waterline (ft.)
LCB	longitudinal center of buoyancy
R_F	frictional resistance (lbs.)
R_R	residuary resistance (lbs.)

R_T total resistance (lbs.)

S wetted surface area (ft.^{2}), without appendages

T draft (ft.)

V speed (ft./sec.)

∇ displacement volume (ft.^{3})

ρ water density (lb. sec.^{2} ft.$^{-4}$)

The quantities defined in the foregoing are used in the paper only in such dimensionless context that, instead of the British units suggested here, any other consistent set of units may be used.

1. Introduction

This paper describes a series of methods to estimate the residuary resistance coefficient of Great Lakes bulk carriers based on model test data regression. The results of this work are intended to assist the designer in estimating the effective horsepower of such ships in the initial design stage.

A sample of 70 model tests of such vessels was collected from various sources. After some initial screening 50 test data sets, which formed a sufficiently coherent sample, were selected for further analysis. All data sets were converted to the ITTC line test evaluation as a uniform reference base. Then, a series of regression analyses was performed for different sets of independent variables at each of eight Froude numbers. The results of two of these analyses are presented in this paper, one aiming at a high level of accuracy, and another representing a simplified, design - oriented version.

The format of the paper is as follows: Section 2 describes the sample of model tests; Section 3 explains the regression analysis techniques, Section 4 presents the results of the regressions, and Appendix I contains some examples of the procedure for estimating Great Lakes bulk carrier resistance from the regression formulas.

2. The Sample

Model test data were assembled for 70 tank tests of Great Lakes bulk carriers. These tests were conducted during the last 30 years at the following establishments:

Naval Ship Research and Development Center, Carderock (formerly David Taylor Model Basin)

Netherlands Ship Model Basin, Wageningen

Ship Hydrodynamics Laboratory, University of Michigan, Ann Arbor

Ship Laboratory, National Research Council, Ottawa

The data were made available by the tanks, Ref. 1, the sponsors, from published reports and SNAME Data Sheets, Ref. 2.

Since some of the organizations that released their data did not want the identity of their ships to be revealed, we have to refrain from a detailed reference to the characteristics of each model in the sample.

For each model, the length of the waterline, L, beam, B, draft, T, wetted surface (without appendages), S, block coefficient, C_B, and load condition were noted together with the resistance measurements at each speed tested. Whenever available, the longitudinal center of buoyancy, LCB, and half - angle of entrance, i_E, were also recorded.

The data obtained from the various sources originally were not presented in accordance with any uniform standard but differed in the friction line, the tank water reference temperature, the roughness allowance when full scale data were given, and other minor details of the test evaluation procedure. It was, therefore, necessary to standardize all input data to the regression analysis: The residuary resistance coefficient C_R, at each speed was converted to the ITTC friction line for the model as a reference base, assuming a fresh water temperature of 59° F. The dimensionless hull form parameters were corrected to conform with the standard set given in the nomenclature of the paper.

The Froude number F_n, was calculated at each speed and C_R values were obtained by interpolation at F_n values, 0.11, 0.12, 0.13, 0.14, 0.15, 0.16, 0.17 and 0.18. The regression variables, C_B, L/B, B/T, C_{WS}, C_∇, i_E and LCB were determined for each model.

Values of i_E and LCB were only available for about 60% of the sample and initially it was decided that the mean value of the distributions of i_E and LCB should be used when data were lacking in the sample. However, early regression analyses indicated that, because of the consequent large clusters at the mean values, artificial emphasis was placed on the deviation from the mean. It was decided, therefore, to remove i_E and LCB from the list of regression variables.

These early regression analyses also suggested that tests in the ballast condition in 11 cases were not suitable for consideration with those in the full load and intermediate load (cubic load) condition, probably due to the nature of the trim effect. The ballast tests were, therefore, removed from the sample. Regrettably, this ballast sample was not of sufficient magntude to permit separate study.

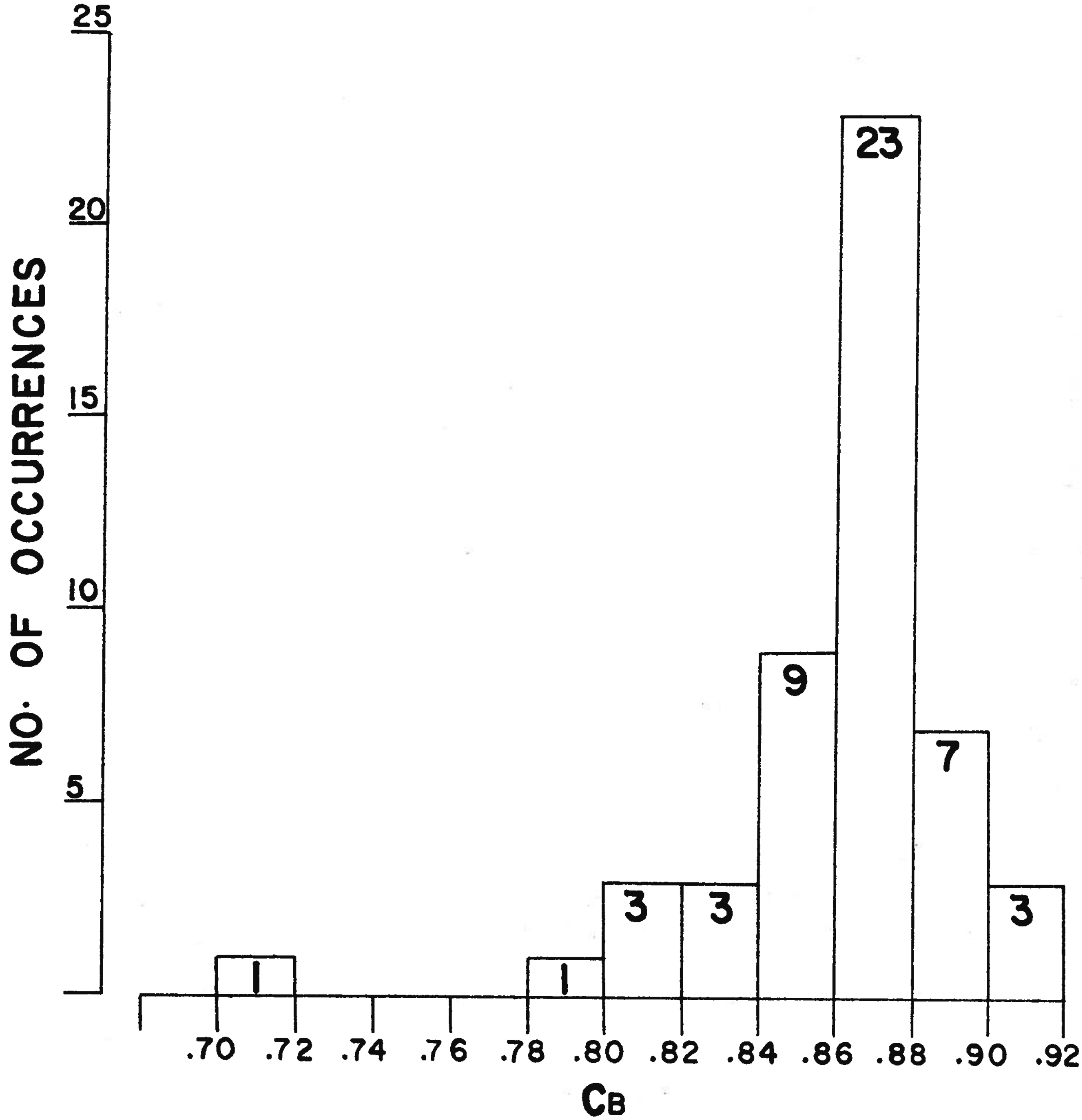
In three cases, the accuracy of the original input data was severely questioned and it was decided to discard these three sets.

In six of the tests, the data were only available over a small speed range, and rather than risk the error of extrapolation, these tests were also removed, thus providing a uniform size of sample over the whole speed range.

This left 50 sets of data, in the full load and intermediate load (near even keel) condition, in the sample.

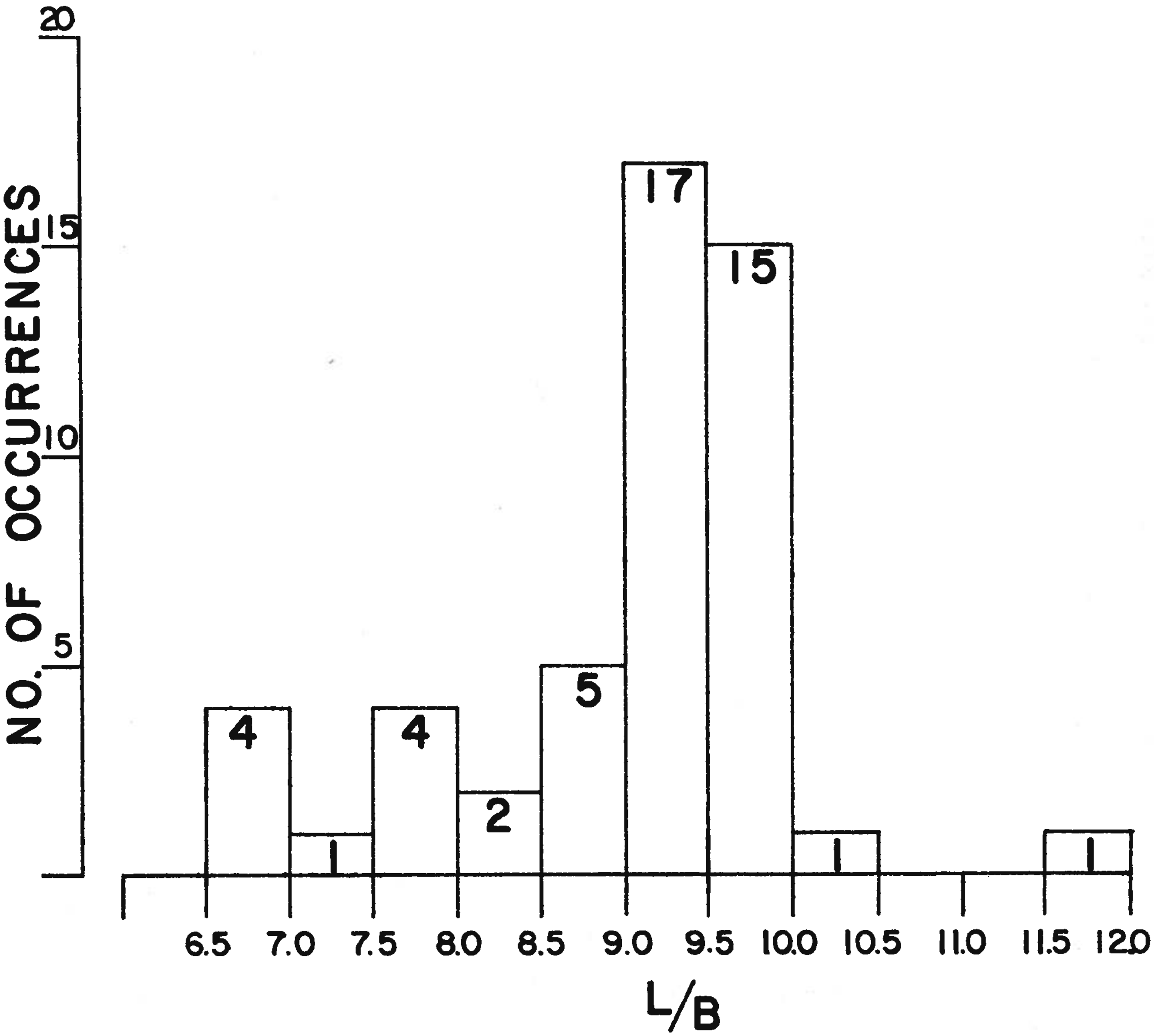
Figs. 1 - 5 show the distribution of each of the five regression variables, C_B, L/B, B/T, C_{WS}, C_∇, within the sample of these 50 tests. The great majority of the ships in the sample vary within only small limits in accordance with standard Great Lakes ship design practice. In a few cases where larger deviations do occur, these can for the most part be traced to intermediate draft conditions or jumboized designs.

Fig.1



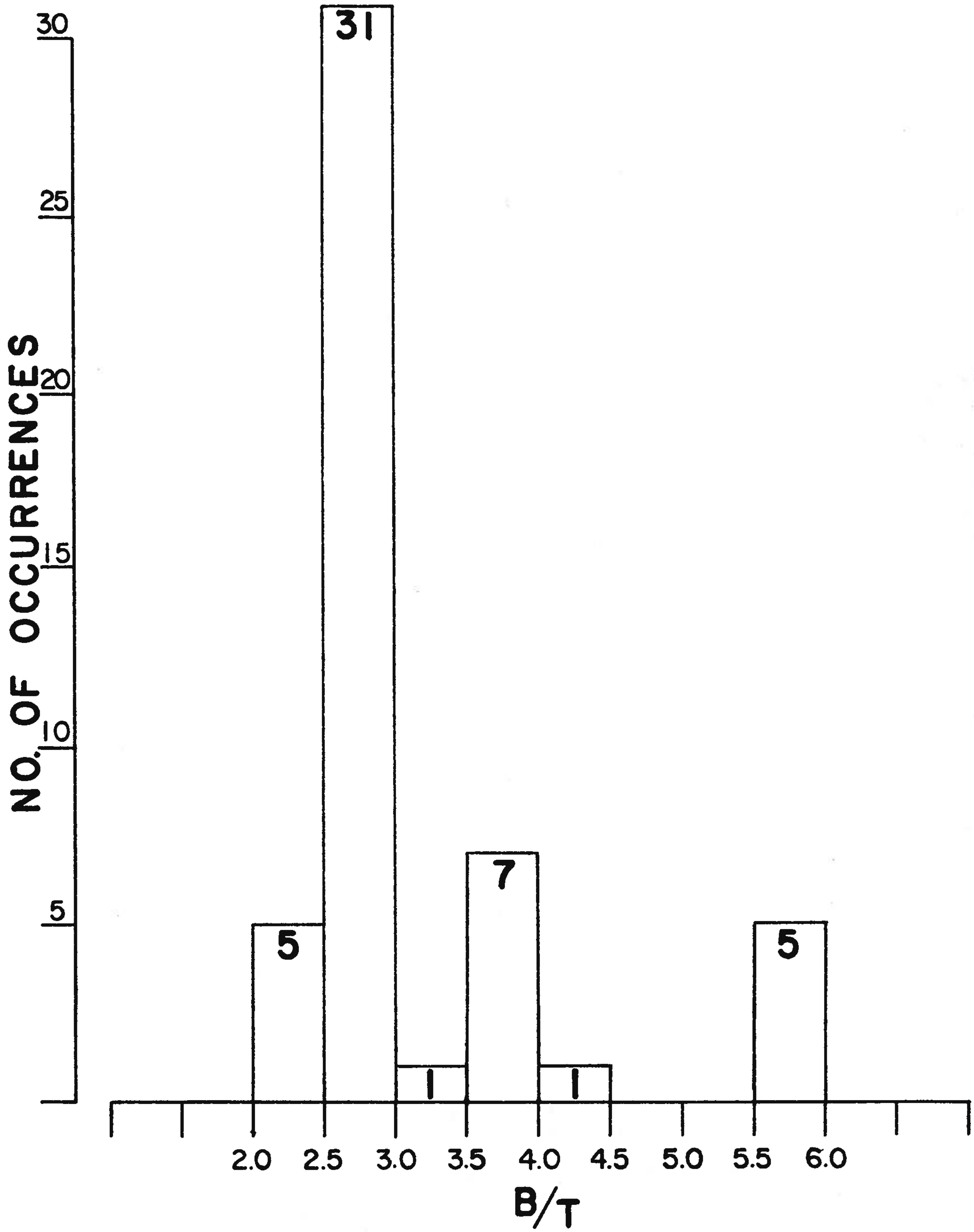
DISTRIBUTION OF C_B
IN REGRESSION SAMPLE

Fig.2



DISTRIBUTION OF L/B
IN REGRESSION SAMPLE

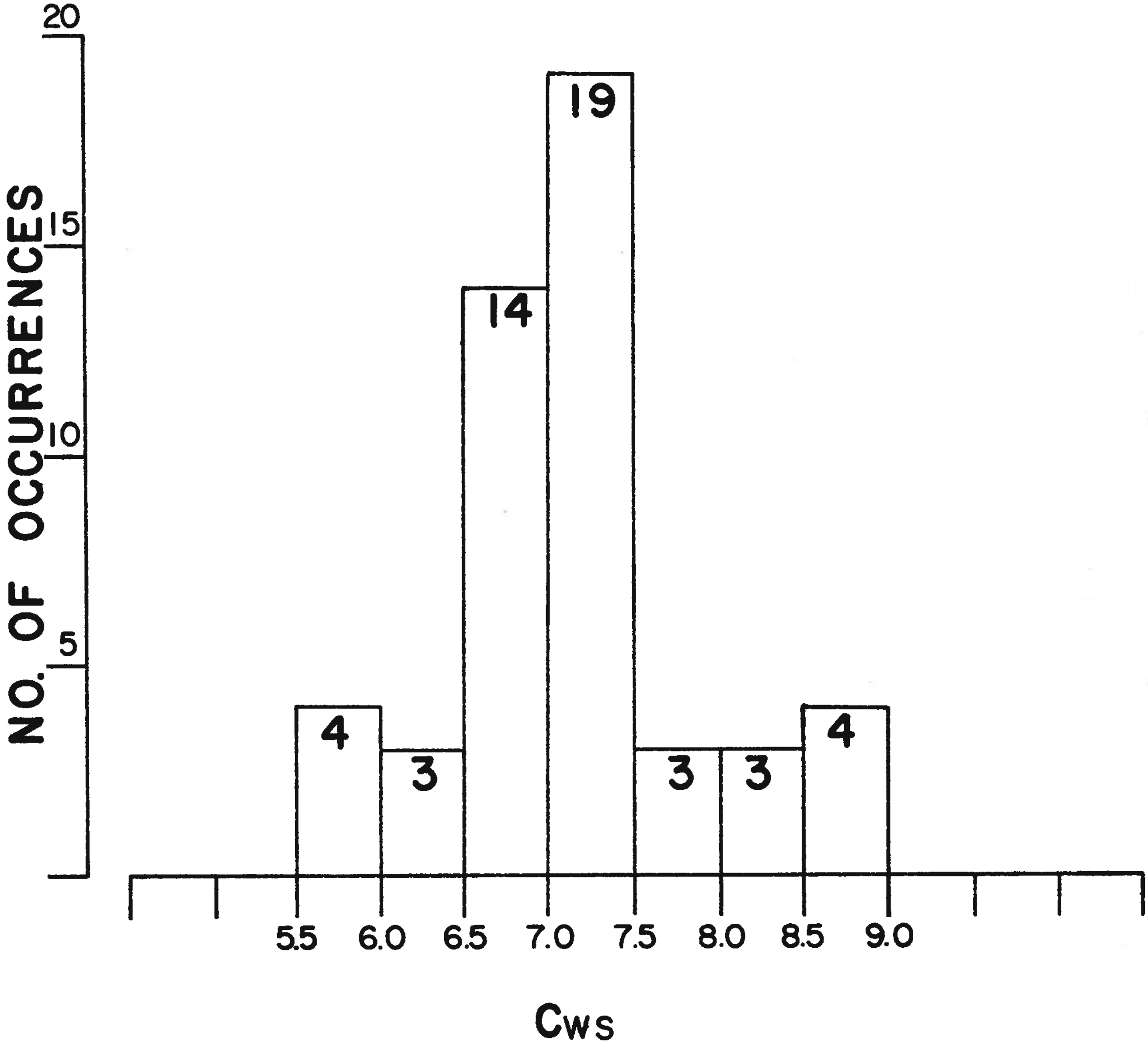
Fig.3



DISTRIBUTION OF B/T

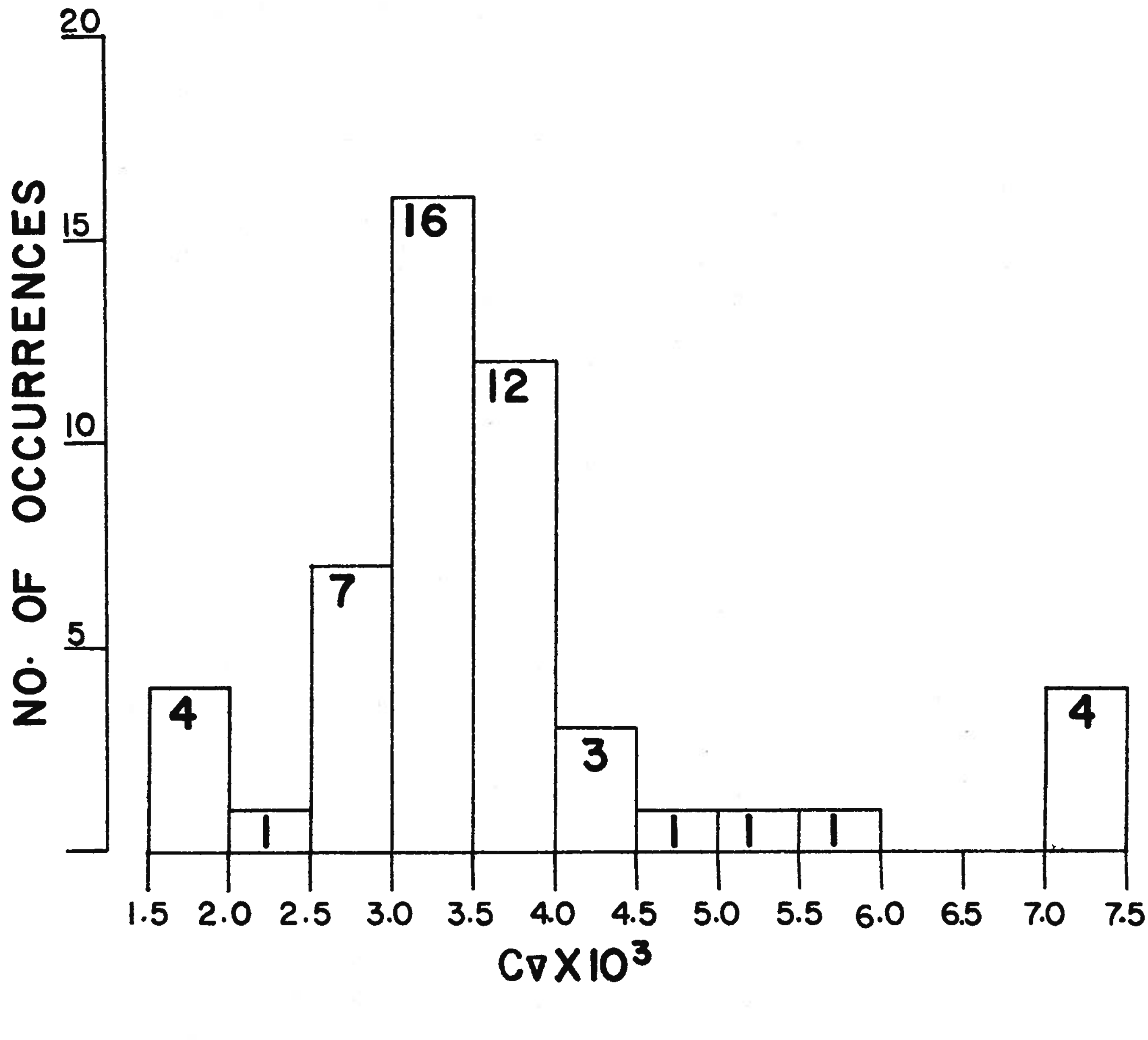
IN REGRESSION SAMPLE

Fig.4



DISTRIBUTION OF C_{WS}
IN REGRESSION SAMPLE

Fig.5



DISTRIBUTION OF C∇
IN REGRESSION SAMPLE

3. Regression Analysis

3.1 Terminology

Polynomial regression is the fitting of a dependent variable by a polynomial function of one independent variable with the degree of the polynomial specified.

Linear regression is the fitting of a dependent variable by a linear function of a specified independent variable. When more than one independent variable are specified, this is known as multiple linear regression.

In stepwise multiple linear regression a dependent variable is fitted in terms of specified independent variables in a stepwise manner.

In this procedure the regression equation is gradually built up, adding in each step one further term - out of a set initially specified - which is selected so that it improves the fit more than any other term still in contention for being included. (The quality of the fit is measured in terms of the multiple correlative coefficient.) This stepwise procedure continues until any further improvement to be expected by including another term does not exceed a specified tolerance. The stepwise regression offers the advantage that it establishes the relative significance of the terms within a preselected set.

3.2 Regression Programs

The regression programs used in these analyses were made available by The Statistical Research Laboratory of The University of Michigan and were run on the University of Michigan's IBM 360/67 computer.

In all programs, the option of zero intercept was available.

4. Results

4.1 Regression with 45 Variables at Each Speed

At Froude numbers 0.11, 0.12, 0.13, 0.14, 0.15, 0.16, 0.17, 0.18, a stepwise multiple linear regression with zero intercept was performed using as the independent variables the five original regression variables, the same regression variables squared and cross-products of these ten terms. This results in a total of 45 independent variables, Table I. From this set the regression at each speed extracted the 12-16 most significant variables for inclusion. (In accordance with the F-values and tolerance level specified.)

The results of the regressions are given in Tables II - IX.

Table I: List of independent variables in 45 term regression

C_B	L/B	B/T	C_{WS}	C_∇
$(C_B)^2$	$(L/B)^2$	$(B/T)^2$	$(C_{WS})^2$	$(C_\nabla)^2$

$(C_B)(L/B)$	$(C_B)(B/T)$	$(C_B)(C_{WS})$	$(C_B)(C_\nabla)$
$(L/B)(B/T)$	$(L/B)(C_{WS})$	$(L/B)(C_\nabla)$	$(B/T)(C_{WS})$
$(B/T)(C_\nabla)$	$(C_{WS})(C_\nabla)$		

$(C_B)^3$	$(L/B)^3$	$(B/T)^3$	$(C_{WS})^3$	$(C_\nabla)^3$

$(C_B)^2(L/B)$	$(C/B)^2(B/T)$	$(C/B)^2(C_{WS})$	$(C_B)^2(C_\nabla)$
$(L/B)^2(C_B)$	$(L/B)^2(B/T)$	$(L/B)^2(C_{WS})$	$(L/B)^2(C_\nabla)$
$(B/T)^2(C_B)$	$(B/T)^2(L/B)$	$(B/T)^2(C_{WS})$	$(B/T)^2(C_\nabla)$
$(C_{WS})^2(C_B)$	$(C_{WS})^2(L/B)$	$(C_{WS})^2(B/T)$	$(C_{WS})^2(C_\nabla)$
$(C_\nabla)^2(C_B)$	$(C_\nabla)^2(L/B)$	$(C_\nabla)^2(B/T)$	$(C_\nabla)^2(C_{WS})$

Table II: Regression equation with 45 independent variables at Froude number = 0.11

$$
\begin{aligned}
C_R = \; & \\
& \; 0.39093\text{E}{-03} \quad (B/T) \\
& + 0.44806\text{E } 01 \quad (C_\nabla) \\
& + 0.83000\text{E } 03 \quad (C_\nabla)^2 \\
& + 0.72338\text{E}{-05} \quad (B/T)(C_{WS}) \\
& - 0.26627\text{E } 01 \quad (B/T)(C_\nabla) \\
& - 0.70351\text{E}{-02} \quad (C_B)^3 \\
& + 0.12980\text{E}{-04} \quad (C_{WS})^3 \\
& - 0.53032\text{E } 05 \quad (C_\nabla)^3 \\
& + 0.51622\text{E}{-02} \quad (C_B)^2(B/T) \\
& + 0.17866\text{E } 01 \quad (C_B)^2(C_\nabla) \\
& - 0.70676\text{E}{-03} \quad (B/T)^2(C_B) \\
& + 0.30530\text{E } 00 \quad (B/T)^2(C_\nabla) \\
& - 0.11269\text{E}{-03} \quad (C_{WS})^2(C_B) \\
& - 0.19202\text{E}{-01} \quad (C_{WS})^2(C_\nabla) \\
& - 0.97511\text{E } 02 \quad (C_\nabla)^2(L/B) \\
& + 0.59050\text{E } 02 \quad (C_\nabla)^2(B/T)
\end{aligned}
$$

Statistical measures:

Multiple correlation coefficient = 0.9857

Standard error of estimate = 0.5757 E-4

F-values for inclusion and deletion = 0.00001

Tolerance level = 0.00001

Table III: Regression equation with 45 independent variables at Froude number = 0.12

$$
\begin{aligned}
C_R = \; & 0.19077\text{E}{-02} \quad (B/T) \\
& + 0.20311\text{E}\ 03 \quad (C_\nabla)^2 \\
& - 0.89316\text{E}{-03} \quad (B/T)(C_{WS}) \\
& - 0.60310\text{E}\ 00 \quad (B/T)(C_\nabla) \\
& - 0.13943\text{E}{-01} \quad (C_B)^3 \\
& + 0.11483\text{E}{-03} \quad (B/T)^3 \\
& - 0.26071\text{E}\ 05 \quad (C_\nabla)^3 \\
& + 0.11915\text{E}{-01} \quad (C_B)^2(B/T) \\
& - 0.21997\text{E}{-02} \quad (B/T)^2(C_B) \\
& - 0.45059\text{E}{-05} \quad (B/T)^2(C_{WS}) \\
& + 0.17105\text{E}\ 00 \quad (B/T)^2(C_\nabla) \\
& - 0.21553\text{E}{-03} \quad (C_{WS})^2(C_B) \\
& + 0.12299\text{E}{-04} \quad (C_{WS})^2(L/B) \\
& + 0.80595\text{E}{-04} \quad (C_{WS})^2(B/T) \\
& + 0.13902\text{E}\ 02 \quad (C_\nabla)^2(L/B)
\end{aligned}
$$

Statistical measures:

Multiple correlation coefficient = 0.9905

Standard error of estimate = 0.4848 E-4

F-values for inclusion and deletion = 0.00001

Tolerance level = 0.00001

Table IV: Regression equation with 45 independent variables at Froude number = 0.13

$$
\begin{aligned}
C_R = \; & 0.79997E-02 \; (B/T) \\
& -0.72573E\ 03 \; (C_\nabla)^2 \\
& -0.24164E-01 \; (C_B)(B/T) \\
& +\ 0.34961E\ 00 \; (B/T)(C_\nabla) \\
& -0.10479E-01 \; (C_B)^3 \\
& +\ 0.20239E-03 \; (B/T)^3 \\
& -0.25212E\ 05 \; (C_\nabla)^3 \\
& +\ 0.28514E-01 \; (C_B)^2(B/T) \\
& -0.59229E\ 01 \; (C_B)^2(C_\nabla) \\
& -0.33756E-02 \; (B/T)^2(C_B) \\
& +\ 0.41254E-04 \; (B/T)^2(L/B) \\
& +\ 0.11644E\ 00 \; (B/T)^2(C_\nabla) \\
& +\ 0.10623E\ 04 \; (C_\nabla)^2(C_B) \\
& +\ 0.74931E\ 02 \; (C_\nabla)^2(L/B) \\
& -0.22780E\ 02 \; (C_\nabla)^2(C_{WS})
\end{aligned}
$$

Statistical measures:

Multiple correlation coefficient = 0.9926

Standard error of estimate = 0.4513 E-4

F-values for inclusion and deletion = 0.00001

Tolerance level = 0.00001

Table V: Regression equation with 45 independent variables at Froude number = 0.14

$C_R =$

$$
\begin{aligned}
& 0.27304E-03 \quad (C_{WS}) \\
& + 0.22876E\ 03 \quad (C_{\nabla})^2 \\
& -0.11519E-01 \quad (C_B)(C_{WS}) \\
& + 0.56176E-03 \quad (L/B)(C_{WS}) \\
& + 0.85505E\ 00 \quad (B/T)(C_{\nabla}) \\
& + 0.40181E-05 \quad (L/B)^3 \\
& -0.12906E\ 05 \quad (C_{\nabla})^3 \\
& + 0.82250E-02 \quad (C_B)^2(C_{WS}) \\
& -0.49417E\ 00 \quad (C_B)^2(C_{\nabla}) \\
& -0.20037E-03 \quad (L/B)^2(C_B) \\
& -0.13293E-04 \quad (C_{WS})^2(L/B) \\
& -0.83626E\ 02 \quad (C_{\nabla})^2(B/T)
\end{aligned}
$$

Statistical measures:

Multiple correlation coefficient = 0.9943

Standard error of estimate = 0.4529 E-4

F-values for inclusion and deletion = 0.00001

Tolerance level = 0.00001

Table VI: Regression equation with 45 independent variables at Froude number = 0.15

$C_R =$

0.27252E-01	(C_B)
-0.57810E-03	$(C_B)(L/B)$
-0.18397E-02	$(L/B)(B/T)$
-0.65332E-05	$(L/B)^3$
-0.53638E-04	$(B/T)^3$
+ 0.10299E-04	$(C_{WS})^3$
+ 0.17691E 04	$(C_\nabla)^3$
-0.22049E-02	$(C_B)^2(C_{WS})$
+ 0.15197E-03	$(L/B)^2(B/T)$
-0.25054E-05	$(L/B)^2(C_{WS})$
+ 0.19035E-02	$(L/B)^2(C_\nabla)$
+ 0.68141E-03	$(B/T)^2(C_B)$
-0.24471E-05	$(B/T)^2(L/B)$
+ 0.18555E 00	$(B/T)^2(C_\nabla)$
+ 0.61236E-02	$(C_{WS})^2(C_\nabla)$
-0.71073E 02	$(C_\nabla)^2(B/T)$

Statistical measures:

Multiple correlation coefficient = 0.9954

Standard error of estimate = 0.4688 E-4

F-values for inclusion and deletion = 0.00001

Tolerance level = 0.00001

Table VII: Regression equation with 45 independent variables at Froude number = 0.16

$C_R =$

$$-0.34769E-03 \quad (L/B)$$
$$+\ 0.36576E-03 \quad (L/B)^2$$
$$-0.15297E-01 \quad (C_B)(B/T)$$
$$-0.33813E\ 01 \quad (C_B)(C_\nabla)$$
$$-0.18196E-04 \quad (L/B)^3$$
$$+\ 0.26462E-05 \quad (B/T)^3$$
$$-0.45552E\ 05 \quad (C_\nabla)^3$$
$$+\ 0.14342E-01 \quad (C_B)^2(B/T)$$
$$-0.84322E-03 \quad (C_B)^2(C_{WS})$$
$$-0.24405E-01 \quad (L/B)^2(C_\nabla)$$
$$-0.91883E-03 \quad (B/T)^2(C_B)$$
$$+\ 0.74922E-04 \quad (B/T)^2(L/B)$$
$$+\ 0.27256E\ 00 \quad (B/T)^2(C_\nabla)$$
$$+\ 0.58213E\ 02 \quad (C_\nabla)^2(L/B)$$
$$+\ 0.52824E\ 02 \quad (C_\nabla)^2(C_{WS})$$

Statistical measures:

Multiple correlation coefficient = 0.9964

Standard error of estimate = 0.4953 E-4

F-values for inclusion and deletion = 0.00001

Tolerance level =0.00001

Table VIII: Regression equation with 45 independent variables at Froude number = 0.17

$$
\begin{aligned}
C_R = \; & -0.70672E-03 \quad (L/B) \\
& + 0.11261E-03 \quad (C_{WS}) \\
& -0.59554E-03 \quad (L/B)(B/T) \\
& -0.49819E-05 \quad (L/B)^3 \\
& -0.92531E-04 \quad (B/T)^3 \\
& -0.15310E\ 05 \quad (C_\nabla)^3 \\
& + 0.16688E-02 \quad (C_B)^2 (L/B) \\
& -0.35594E-02 \quad (C_B)^2 (C_{WS}) \\
& + 0.94751E-04 \quad (L/B)^2 (B/T) \\
& + 0.15280E-02 \quad (B/T)^2 (C_B) \\
& -0.12069E-03 \quad (B/T)^2 (L/B) \\
& + 0.38688E-04 \quad (B/T)^2 (C_{WS}) \\
& + 0.48500E-01 \quad (C_{WS})^2 (C_\nabla) \\
& + 0.84527E\ 01 \quad (C_\nabla)^2 (L/B)
\end{aligned}
$$

Statistical measures:

Multiple correlation coefficient = 0.9954

Standard error of estimate = 0.6882 E-4

F-values for inclusion and deletion = 0.00001

Tolerance level = 0.00001

Table IX: Regression equation 45 independent variables at Froude number = 0.18

$C_R =$

−0.93931E−03	(C_{WS})
−0.16741E−03	$(L/B)^2$
+ 0.37866E−02	$(C_B)(C_{WS})$
−0.14179E−02	$(L/B)(B/T)$
−0.11011E−03	$(B/T)^3$
+ 0.17133E−02	$(C_B)^2(L/B)$
−0.73467E−02	$(C_B)^2(C_{WS})$
+ 0.20478E 01	$(C_B)^2(C_\nabla)$
+ 0.17675E−03	$(L/B)^2(B/T)$
+ 0.19050E−02	$(B/T)^2(C_B)$
−0.10805E−03	$(B/T)^2(L/B)$
+ 0.15162E 00	$(B/T)^2(C_\nabla)$
+ 0.72594E−01	$(C_{WS})^2(C_\nabla)$
−0.13581E 03	$(C_\nabla)^2(B/T)$

Statistical measures:

Multiple correlation coefficient = 0.9952

Standard error of estimate = 0.9100 E-4

F-values for inclusion and deletion = 0.00001

Tolerance level = 0.00001

4.2 Regression with 11 Variables at Each Speed

Although this stepwise multiple linear regression produces a very accurate fit of the sample by virtue of the great number of terms involved it also has some drawbacks in the practical application of its results. There are too many terms in the regression equation to arrive at a design-oriented, physical interpretation of the trends in the sample and, moreover, the set of independent variables producing this "most accurate" fit is a different one at each speed.

In the interest of greater simplicity and uniformity it was therefore desirable to attempt to reduce the number of independent variables to a reasonable minimum still ensuring adequate accuracy, and to use a standard set of these variables throughout the speed range.

It was logical to first eliminate those "independent" variables that showed a strong dependence on others in the set. This suggested itself especially for the wetted surface coefficient whose strong dependence on other hull form parameters is well known. This was also attractive since the wetted surface in the initial design stage generally cannot be treated as an independent design variable.

It was therefore attempted to express the wetted surface in the manner of the well known Denny formula, which reads (Ref. 3):

$$S = 1.7\ (L \cdot T) + \frac{\nabla}{T}$$

The coefficients in this formula were rederived by multiple linear regression of the 50 Great Lakes bulk carriers in our sample to yield:

$$S = 1.667\ (L \cdot T) + 1.047\ \frac{\nabla}{T}$$

with extremely small statistical error (multiple correlation coefficient = 0.9997, standard error estimate = $1.068 \cdot 10^3$).

To further test the significance of the wetted surface coefficient in the regressions, a stepwise multiple linear regression was then run at each Froude number with C_{WS} terms removed which resulted in only a small loss of accuracy. This further supported eliminating C_{WS} from the set of independent variables.

At the next stage we looked for any other terms in the regression equation whose role was relatively insignificant. This could be judged by the significance tests carried out during the stepwise regression, which identified the statistically less influential terms by their partial correlation values, as well as by judging the relative magnitude of the contributions from each term throughout the speed range.

On this basis it was possible to select 11 variables as appearing to be the most significant.

These were:

$(C_B)^3$ $\quad (L/B)^3$ $\quad (B/T)^3$ $\quad (C_\nabla)^3$

$(C_B)^2 (B/T)$ $\quad (L/B)^2 (B/T)$

$(L/B)^2 (C_\nabla)$ $\quad (B/T)^2 (C_B)$

$(B/T)^2 (L/B)$ $\quad (B/T)^2 (C_\nabla)$

$(C_\nabla)^2 (L/B)$

At each Froude number, a multiple linear regression was run using these independent variables. The coefficients obtained, A(i,j), are defined in Table X according to:

$$C_R = \sum_{i=1}^{11} A(i,j) \cdot x_i$$

for a given Froude number F_{nj} where x_i = independent variable, and their numerical values are given in Table XI.

Variable		Froude Number, F_{nj}							
i	x_i	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18
1	$(C_B)^3$	A(1,1)	A(1,2)	A(1,3)	A(1,4)	A(1,5)	A(1,6)	A(1,7)	A(1,8)
2	$(L/B)^3$	A(2,1)	A(2,2)	A(2,3)	A(2,4)	A(2,5)	A(2,6)	A(2,7)	A(2,8)
3	$(B/T)^3$	A(3,1)	A(3,2)	A(3,3)	A(3,4)	A(3,5)	A(3,6)	A(3,7)	A(3,8)
4	$(C_\nabla)^3$	A(4,1)	A(4,2)	A(4,3)	A(4,4)	A(4,5)	A(4,6)	A(4,7)	A(4,8)
5	$(C_B)^2(B/T)$	A(5,1)	A(5,2)	A(5,3)	A(5,4)	A(5,5)	A(5,6)	A(5,7)	A(5,8)
6	$(L/B)^2(B/T)$	A(6,1)	A(6,2)	A(6,3)	A(6,4)	A(6,5)	A(6,6)	A(6,7)	A(6,8)
7	$(L/B)^2(C_\nabla)$	A(7,1)	A(7,2)	A(7,3)	A(7,4)	A(7,5)	A(7,6)	A(7,7)	A(7,8)
8	$(B/T)^2(C_B)$	A(8,1)	A(8,2)	A(8,3)	A(8,4)	A(8,5)	A(8,6)	A(8,7)	A(8,8)
9	$(B/T)^2(L/B)$	A(9,1)	A(9,2)	A(9,3)	A(9,4)	A(9,5)	A(9,6)	A(9,7)	A(9,8)
10	$(B/T)^2(C_\nabla)$	A(10,1)	A(10,2)	A(10,3)	A(10,4)	A(10,5)	A(10,6)	A(10,7)	A(10,8)
11	$(C_\nabla)^2(L/B)$	A(11,1)	A(11,2)	A(11,3)	A(11,4)	A(11,5)	A(11,6)	A(11,7)	A(11,8)

Table X : Definition of coefficients in 11 term regression

Table XI: Coefficients A(i,j) as defined in Table X

A(1,1) =	-0.12243E-02
A(2,1) =	0.13516E-05
A(3,1) =	0.47534E-05
A(4,1) =	-0.16806E 05
A(5,1) =	0.61366E-03
A(6,1) =	0.74317E-05
A(7,1) =	-0.14642E-01
A(8,1) =	0.14582E-03
A(9,1) =	-0.30946E-04
A(10,1) =	-0.64174E-02
A(11,1) =	0.31273E 02

A(1,2) =	-0.19263E-03
A(2,2) =	0.11463E-05
A(3,2) =	0.71868E-05
A(4,2) =	-0.16419E 05
A(5,2) =	0.34805E-03
A(6,2) =	0.85753E-05
A(7,2) =	-0.15075E-01
A(8,2) =	0.14667E-03
A(9,2) =	-0.32269E-04
A(10,2) =	-0.38360E-02
A(11,2) =	0.30368E 02

A(1,3) =	0.10961E-02
A(2,3) =	0.10469E-05
A(3,3) =	-0.79792E-05
A(4,3) =	-0.15861E 05
A(5,3) =	-0.56531E-03
A(6,3) =	0.90785E-05
A(7,3) =	-0.14827E-01
A(8,3) =	0.34990E-03
A(9,3) =	-0.32326E-04
A(10,3) =	-0.27713E-02
A(11,3) =	0.29411E 02

A(1,4) =	0.71450E-03
A(2,4) =	0.12976E-05
A(3,4) =	-0.32405E-04
A(4,4) =	-0.12475E 05
A(5,4) =	-0.84064E-03
A(6,4) =	0.36554E-05
A(7,4) =	-0.10744E-01
A(8,4) =	0.55433E-03
A(9,4) =	-0.23176E-04
A(10,4) =	-0.11548E-01
A(11,4) =	0.23668E 02

A(1,5) =	-0.30748E-03
A(2,5) =	0.17779E-05
A(3,5) =	-0.53796E-04
A(4,5) =	-0.10457E 05
A(5,5) =	-0.70193E-03
A(6,5) =	-0.23687E-05
A(7,5) =	-0.76409E-02
A(8,5) =	0.70342E-03
A(9,5) =	-0.14043E-04
A(10,5) =	-0.22484E-01
A(11,5) =	0.20298E 02

A(1,6) =	0.91320E-06
A(2,6) =	0.21410E-05
A(3,6) =	-0.69529E-04
A(4,6) =	-0.11438E 05
A(5,6) =	-0.11691E-02
A(6,6) =	-0.12937E-05
A(7,6) =	-0.91107E-02
A(8,6) =	0.94282E-03
A(9,6) =	-0.21395E-04
A(10,6) =	-0.27928E-01
A(11,6) =	0.22709E 02

A(1,7) =	0.30289E-02
A(2,7) =	0.26432E-05
A(3,7) =	-0.10311E-03
A(4,7) =	-0.15742E 05
A(5,7) =	-0.32572E-02
A(6,7) =	0.44082E-05
A(7,7) =	-0.14989E-01
A(8,7) =	0.15325E-02
A(9,7) =	-0.40382E-04
A(10,7) =	-0.29265E-01
A(11,7) =	0.31250E 02

A(1,8) =	0.92385E-02
A(2,8) =	0.29208E-05
A(3,8) =	-0.14861E-03
A(4,8) =	-0.22253E 05
A(5,8) =	-0.69833E-02
A(6,8) =	0.15358E-04
A(7,8) =	-0.24153E-01
A(8,8) =	0.24127E-02
A(9,8) =	-0.70519E-04
A(10,8) =	-0.24494E-01
A(11,8) =	0.43452E 02

4.3 Cross-fairing of Regression Coefficients Against Speed

In order to further simplify the use of the regression equation, at the expense of some loss of accuracy, the coefficients for each of the 11 independent variables, $A(i,j)$, were cross-faired against Froude number, F_{nj}, in the range of the eight given speeds, $1 \leq j \leq 8$, in terms of cubic polynomials of F_n. This results in one speed-dependent function for the coefficient, $\bar{A}(i)$, for each term in the regression equation:

$$\bar{A}(i) = \sum_{k=o}^{3} B(i,k) F_n^{\,k}, \quad 1 \leq i \leq 11$$

which will serve to estimate the residual resistance coefficient by

$$C_R = \sum_{i=1}^{11} \bar{A}(i)\ x_i$$

The coefficients $B(i,k)$ were obtained by polynomial regression, to the third degree, of the $A(i,j)$ against the independent variables F_n, and are presented in Table XII. The functions $\bar{A}(i) = f(F_n)$ are shown in Figs. 6 - 16, where they are compared to the coefficients $A(i,j)$ prior to cross-fairing. The agreement is reasonable despite obvious discrepancies in some terms at some speeds. A more exacting test of the simplified, cross-faired regression formula is performed in Appendix I, where full scale total resistance coefficients are estimated using the exact 11 term regression at each individual speed versus the cross-faired regression formula. The agreement is fairly close in these instances, yet from a cautious viewpoint we must recommend the exact regression to be used for greater accuracy, the cross-faired version only when simplicity is at a premium.

It may be mentioned in passing that we did not want to include speed-dependent terms as independent variables, together with the set of 11 others we chose to start with, in order to conserve any differences in the physical trends at differing speeds, instead of smoothing them out from the beginning. Further, we could clearly have obtained a slightly more accurate speed-dependent regression in a single analysis with speed included, but our goal in this context was simplicity and transparency, and not primarily accuracy.

Table XII: Coefficients B(i,k)

B(1,0) = -0.44719E 00
B(1,1) = 0.98267E 01
B(1,2) = -0.71357E 02
B(1,3) = 0.17137E 03

B(2,0) = 0.49849E-04
B(2,1) = -0.10035E-02
B(2,2) = 0.66723E-02
B(2,3) = -0.14141E-01

B(3,0) = -0.71518E-04
B(3,1) = 0.11613E-02
B(3,2) = 0.35475E-02
B(3,3) = -0.68280E-01

B(4,0) = 0.43311E 06
B(4,1) = -0.10466E 08
B(4,2) = 0.79774E 08
B(4,3) = -0.19831E 09

B(5,0) = 0.19949E 00
B(5,1) = -0.44088E 01
B(5,2) = 0.32370E 02
B(5,3) = -0.79147E 02

B(6,0) = -0.10489E-02
B(6,1) = 0.23743E-01
B(6,2) = -0.17495E 00
B(6,3) = 0.42171E 00

B(7,0) = 0.68194E 00
B(7,1) = -0.15932E 02
B(7,2) = 0.11958E 03
B(7,3) = -0.29376E 03

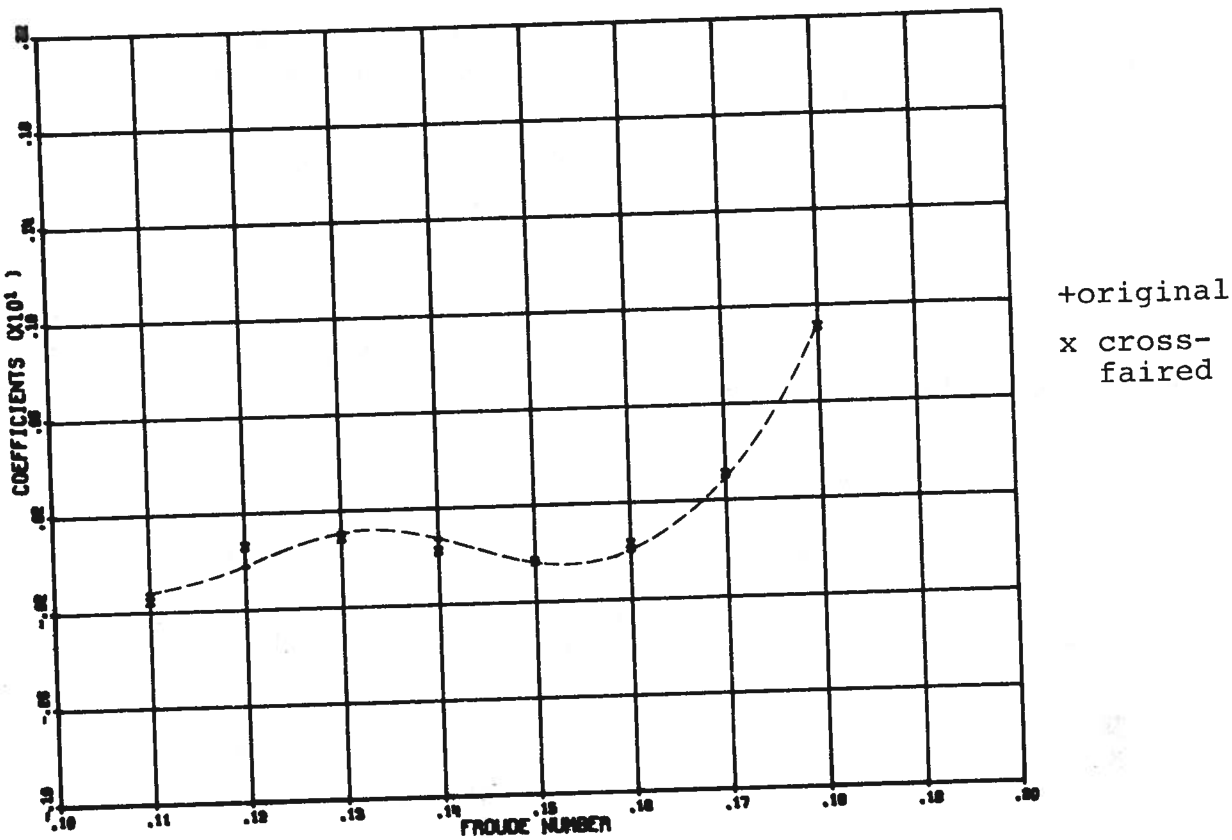
B(8,0) = -0.23874E-01
B(8,1) = 0.54814E 00
B(8,2) = -0.42080E 01
B(8,3) = 0.10963E 02

B(9,0) = 0.19879E-02
B(9,1) = -0.46226E-01
B(9,2) = 0.34795E 00
B(9,3) = -0.85950E 00

B(10,0) = -0.14838E 01
B(10,1) = 0.32257E 02
B(10,2) = -0.22939E 03
B(10,3) = 0.52907E 03

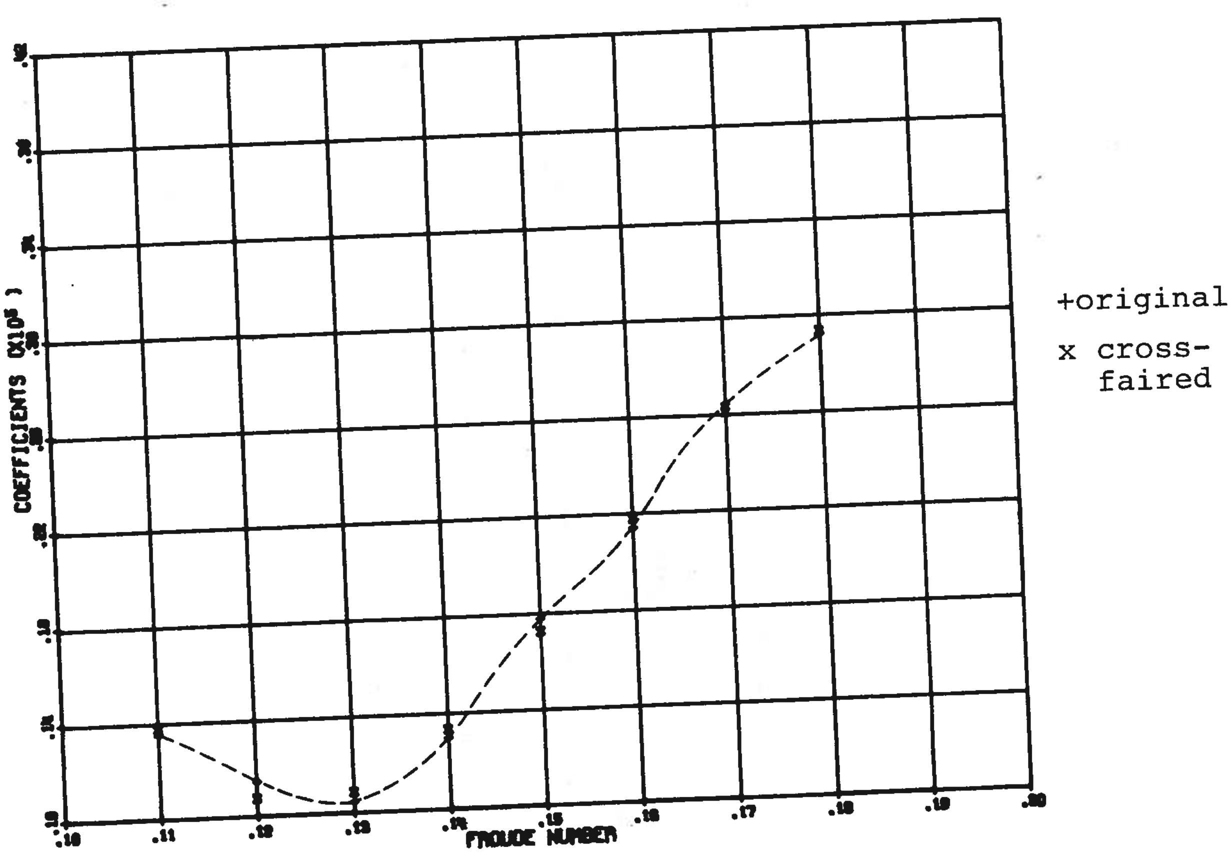
B(11,0) = -0.74263E 03
B(11,1) = 0.18117E 05
B(11,2) = -0.13903E 06
B(11,3) = 0.34815E 06

Fig.6



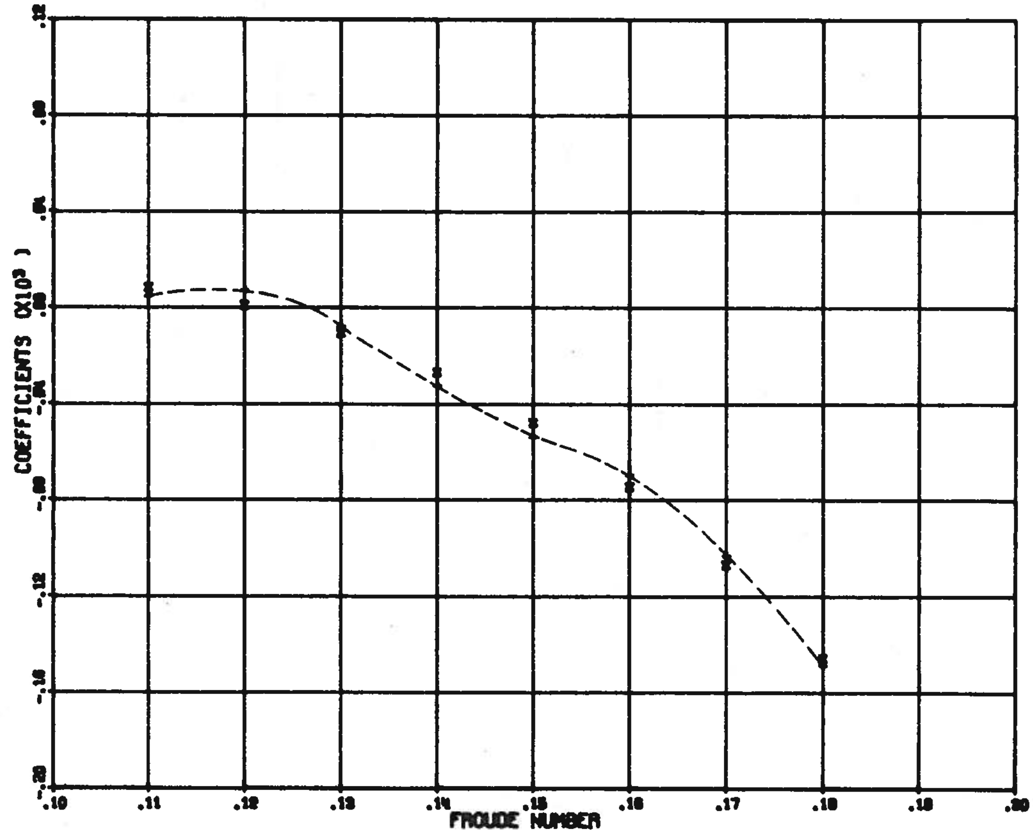
Coefficients A(i,j) and $\overline{A}$(i), i=1

Fig.7



Coefficients A(i,j) and $\overline{A}$(i), i=2

Fig.8

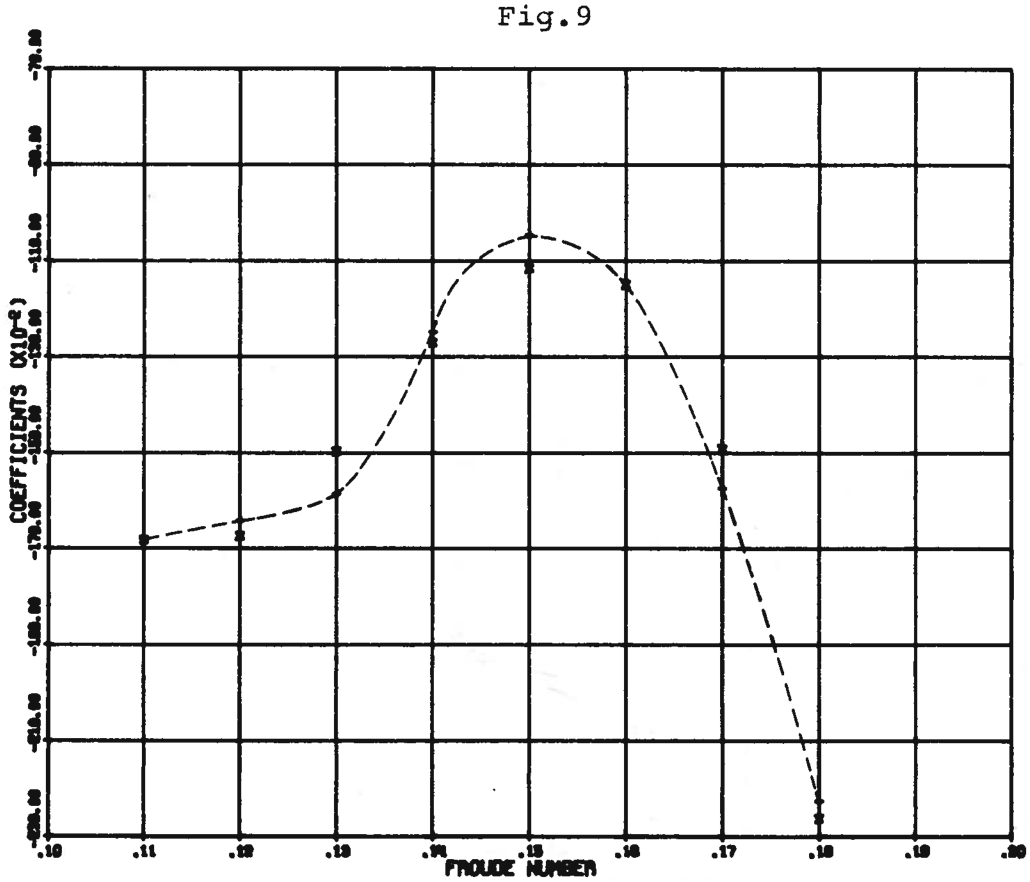


+original

x cross-
faired

Coefficients A(i,j) and $\overline{A}$(i), i=3

Fig.9

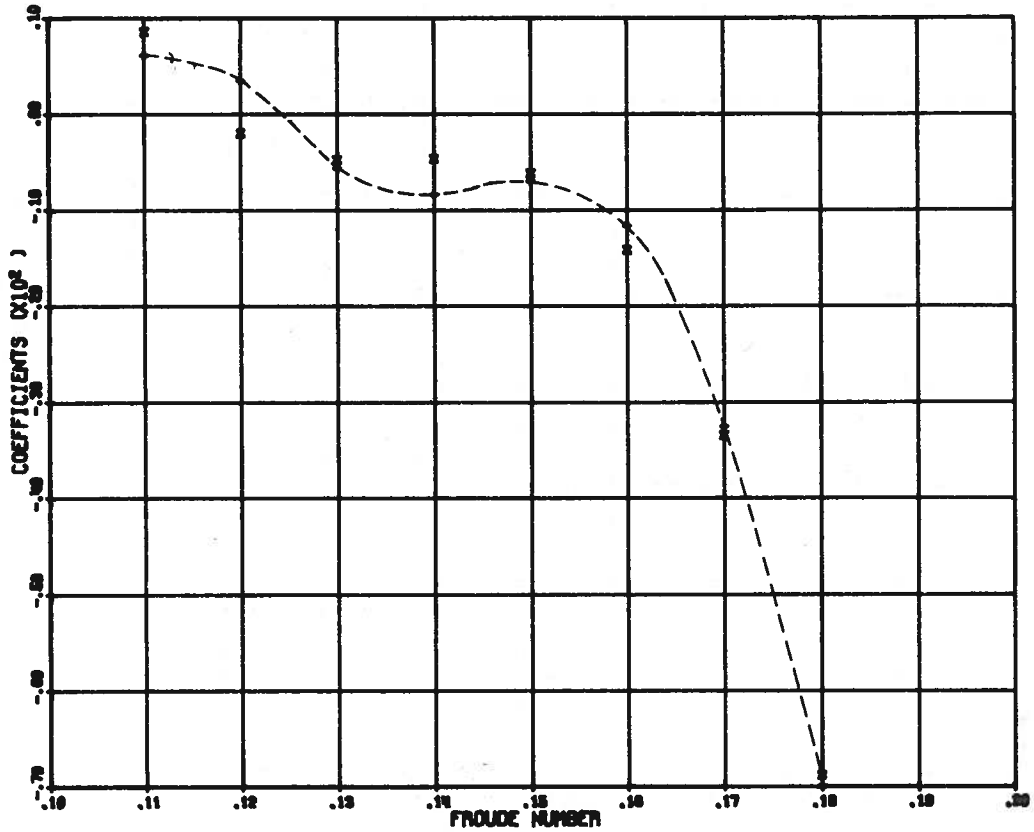


+original

x cross-
faired

Coefficients A(i,j) and $\overline{A}$(i), i=4

Fig.10

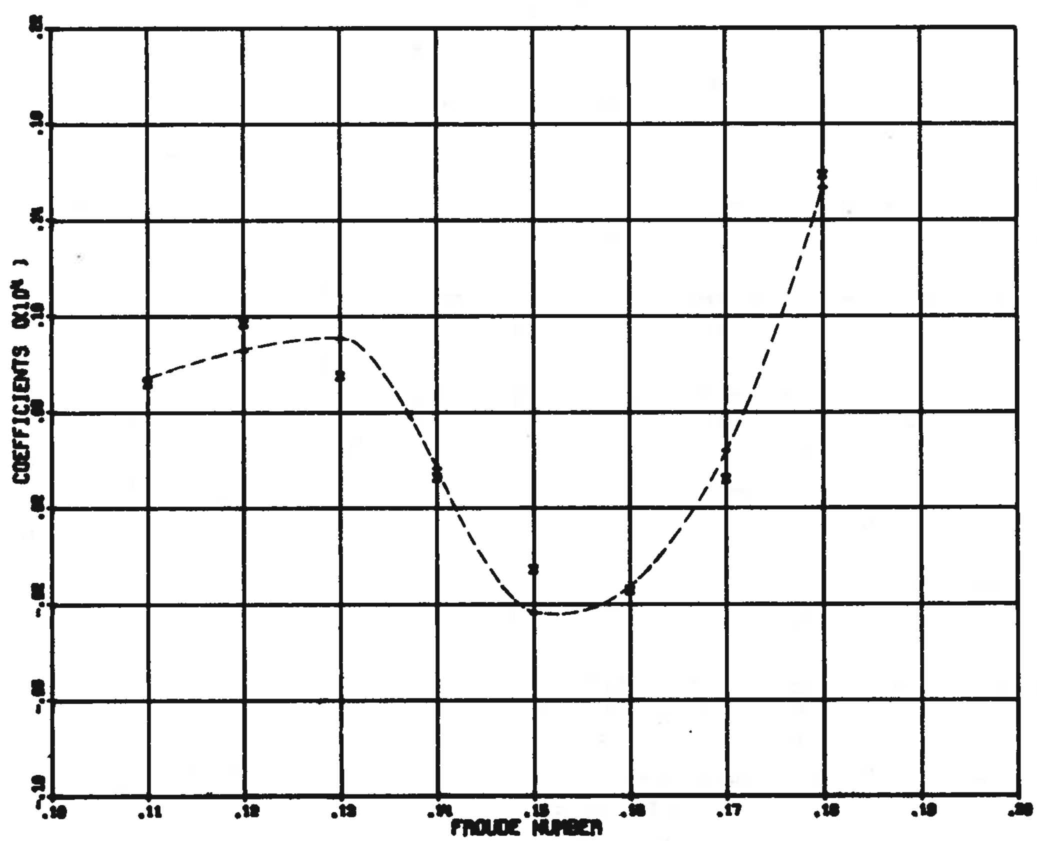


+original

x cross-
faired

Coefficients A(i,j) and $\overline{A}$(i), i=5

Fig.11

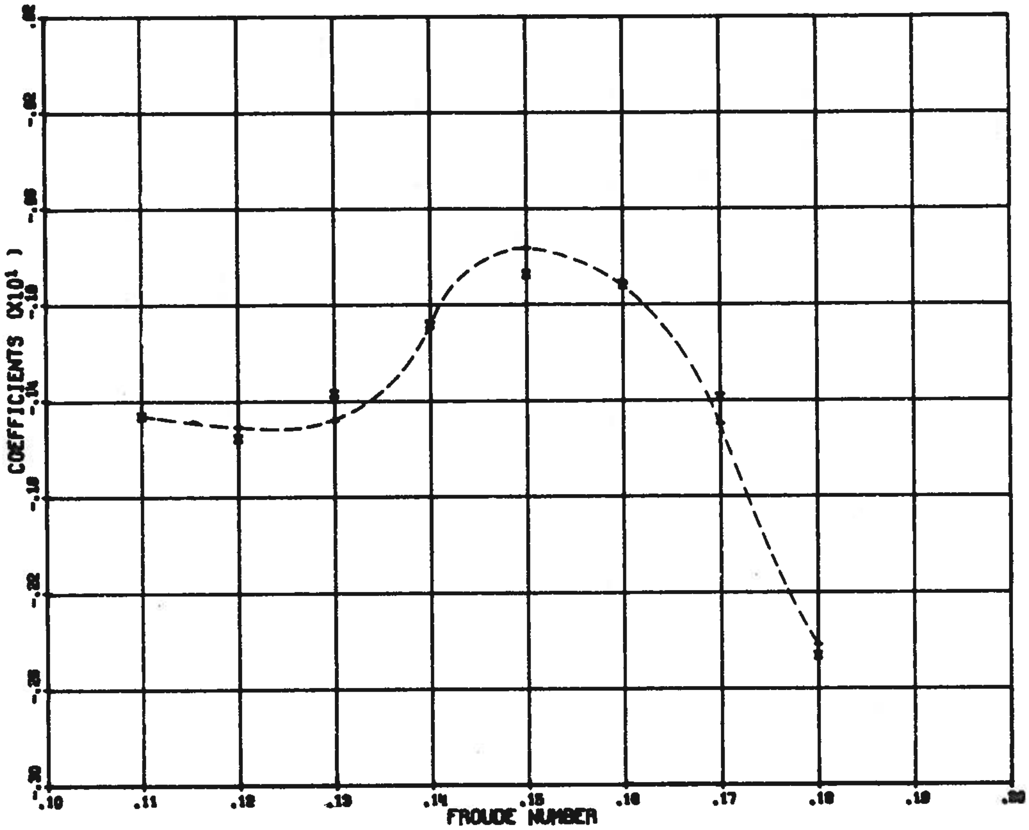


+ original

x cross-
faired

Coefficients A(i,j) and $\overline{A}$(i), i=6

Fig.12

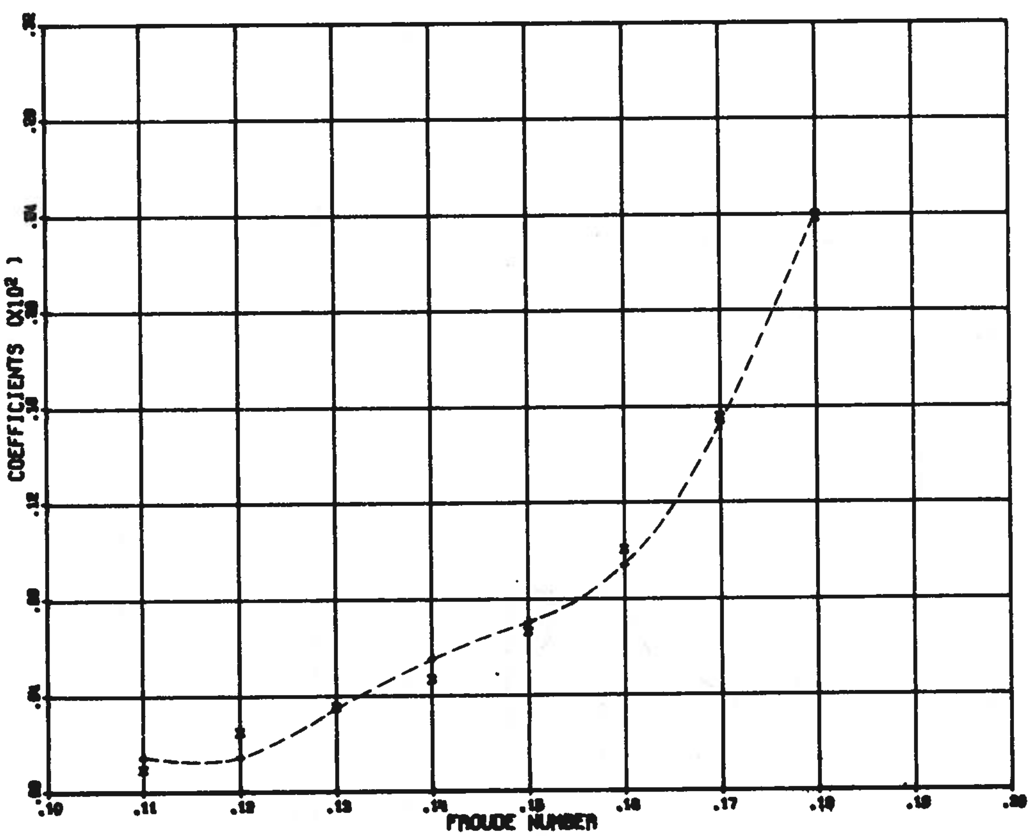


+original

x cross-faired

Coefficients A(i,j) and $\overline{A}$(i), i=7

Fig.13

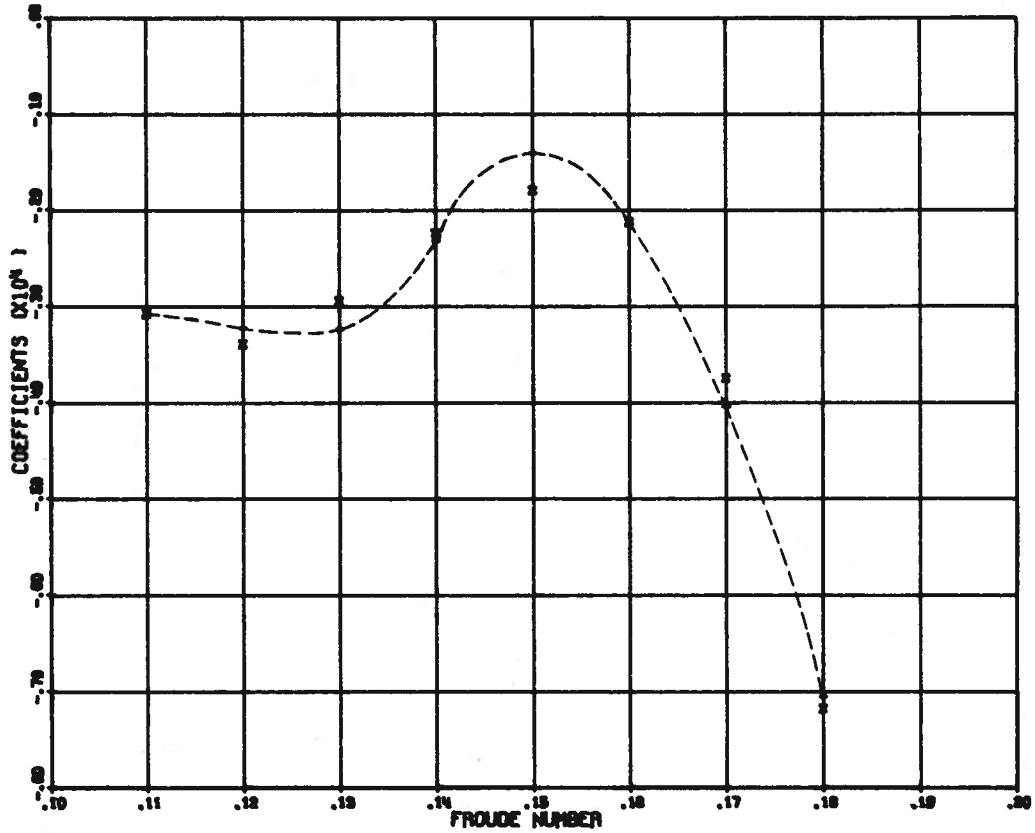


+original

x cross-faired

Coefficients A(i,j) and $\overline{A}$(i), i=8

Fig.14

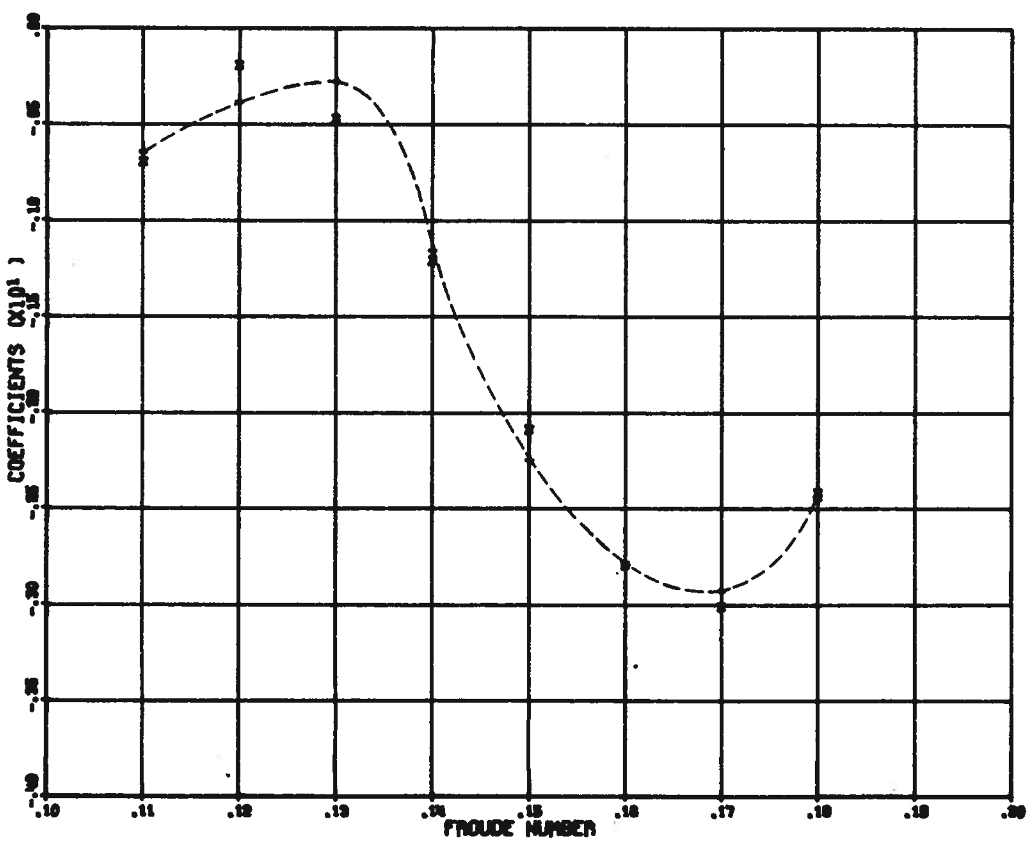


+original

x cross-
faired

Coefficients A(i,j) and $\bar{A}$(i), i=9

Fig.15

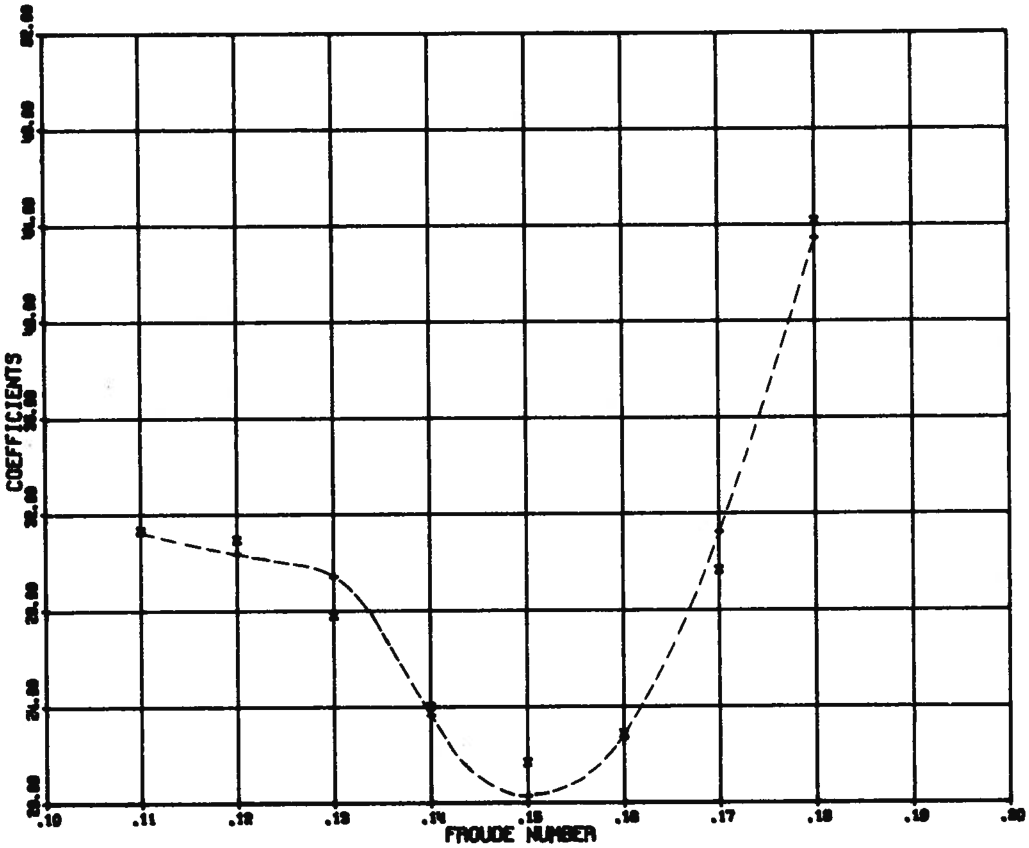


+original

x cross-
faired

Coefficients A(i,j) and $\bar{A}$(i), i=10

Fig.16



+original

x cross-
  faired

Coefficients A(i,j) and $\overline{A}(i)$, i=11

4.4 Discussion of Trends in the Sample

A physical, design-oriented interpretation of a regression formula is always difficult because of the multiplicity of terms and, moreover, cross-coupled terms involved. Further there is always the risk that the set of independent regression variables picked may not include the physically most relevant quantities. This leads the pure statistician to refrain from any physical interpretation, and to simply state that the regression formula represents the best possible fit, at the level attempted, of the population of physical observations in the sample.

Despite some reservations on our part that we might have missed some relevant physical parameters in the analysis, for instance some parameter of local afterbody shape influencing the extent of separation, we nonetheless wanted to discuss the trends exhibited by the variables in our regression in a design-oriented fashion without attempting a physical explanation.

For this purpose it was necessary to collect terms in the regression equation (11 term version at discrete speeds) forming a small set of functions each of which preferably should express the influence of some dominant variable modified by one or two weaker variables. The dominant variables should appear in only one function if possible. The basic scheme resulting from this is in full analogy to the approach taken by Doust and O'Brien in their trawler regression, Ref. 4.

After some experimentation the following set of functions appeared to be best suited to discuss the trends in the sample:

$$F_1 = f(C_B, B/T) =$$
$$= A_1(C_B)^3 + A_5(C_B)^2(B/T) + A_8(C_B)(B/T)^2$$

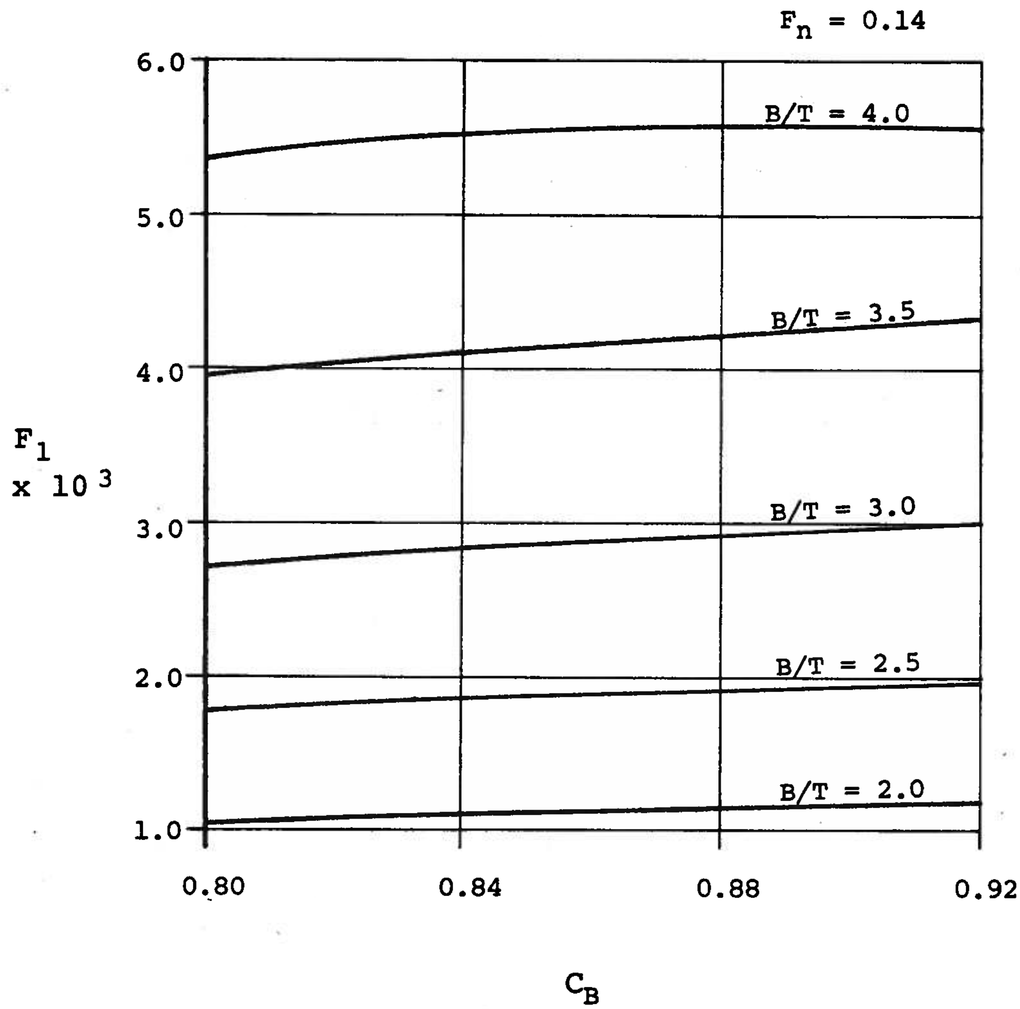
$$F_2 = f(B/T, C_\nabla) =$$
$$= A_3(B/T)^3 + A_4(C_\nabla)^3 + A_{10}(C_\nabla)(B/T)^2$$

$$F_3 = f(L/B, B/T, C_\nabla) =$$
$$= A_2(L/B)^3 + A_6(B/T)(L/B)^2 + A_7(C_\nabla)(L/B)^2$$

where $A_1, \ldots, A_{11}$ correspond to the coefficients A(1,j), ..., A(11,j) at a particular speed, F_{nj}, as defined earlier.

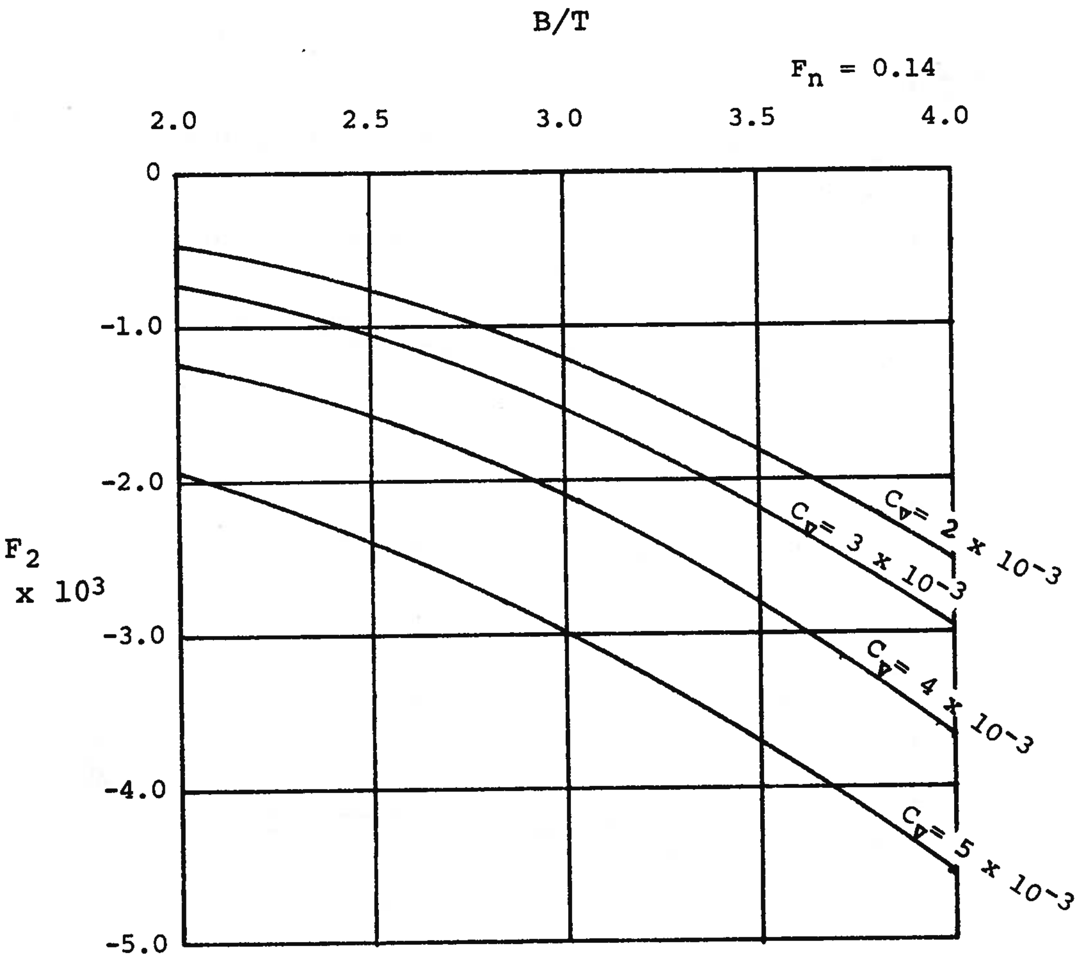
The trends in these functions against the basic regression variables are displayed in Figs. 17 to 30 for $F_n = 0.14$ and 0.16. Appendix I illustrates the use of the graphs in a sample calculation.

Fig.17



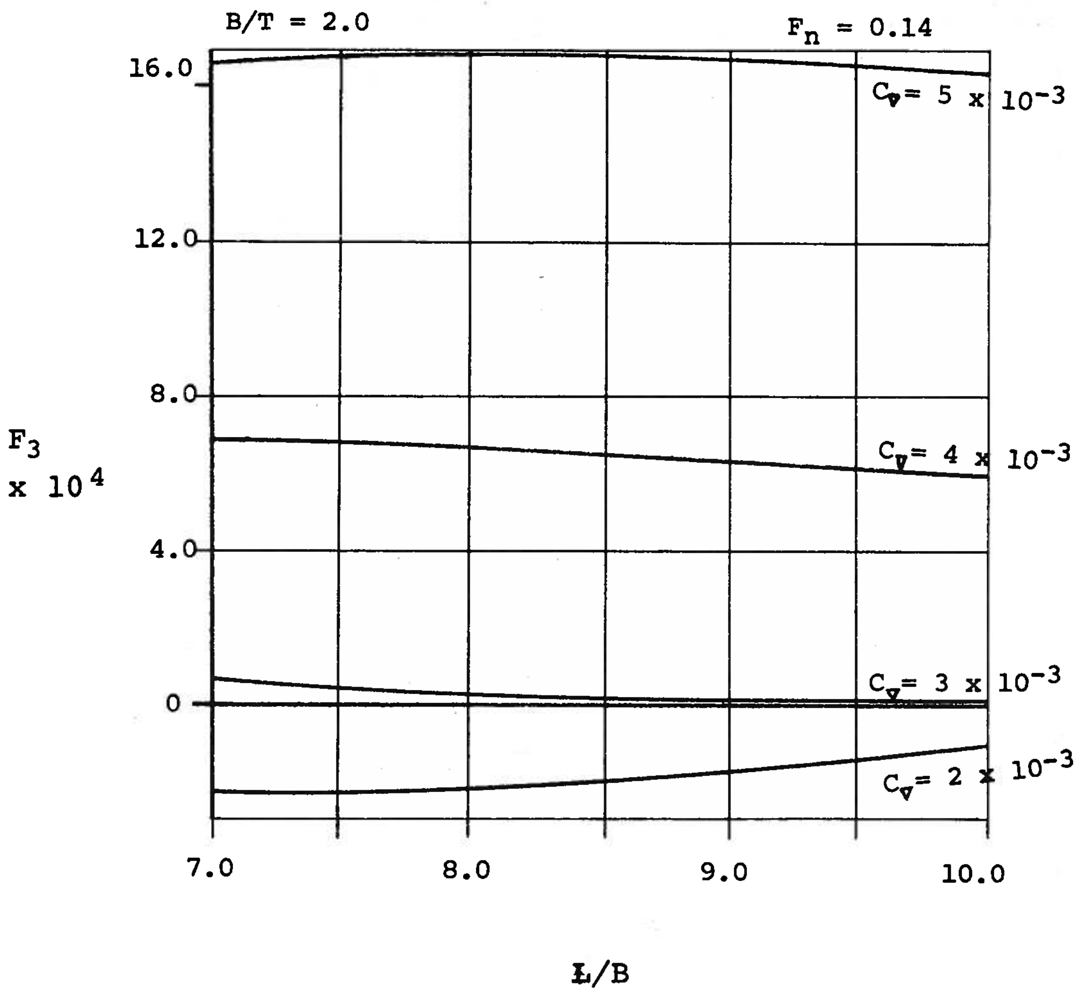
F_n = 0.14
6.0
5.0
4.0
3.0
2.0
1.0
F_1
x 10^3
B/T = 4.0
B/T = 3.5
B/T = 3.0
B/T = 2.5
B/T = 2.0
0.80
0.84
0.88
0.92
C_B

Fig.18



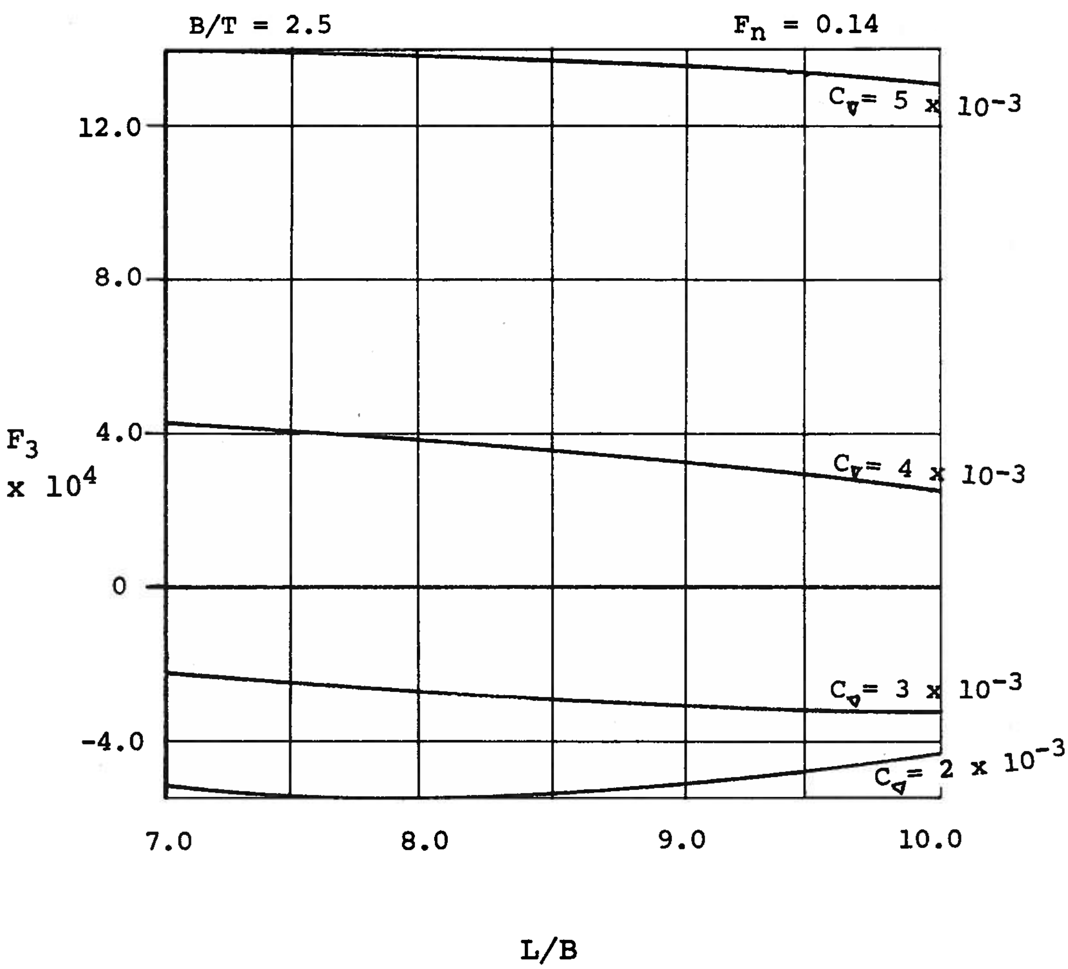
B/T
F_n = 0.14
2.0
2.5
3.0
3.5
4.0
0
-1.0
-2.0
-3.0
-4.0
-5.0
F_2 x 10^3
$C_\nabla = 2 \times 10^{-3}$
$C_\nabla = 3 \times 10^{-3}$
$C_\nabla = 4 \times 10^{-3}$
$C_\nabla = 5 \times 10^{-3}$

Fig.19



B/T = 2.0
Fn = 0.14
16.0
12.0
8.0
4.0
0
F3
x 10 4
Cv= 5 x 10-3
Cv= 4 x 10-3
Cv= 3 x 10-3
Cv= 2 x 10-3
7.0
8.0
9.0
10.0
L/B

Fig.20



B/T = 2.5
Fn = 0.14
12.0
8.0
4.0
0
-4.0
F3
x 10^4
Cv= 5 x 10^-3
Cv= 4 x 10^-3
Cv= 3 x 10^-3
Cv= 2 x 10^-3
7.0
8.0
9.0
10.0
L/B

Fig.21

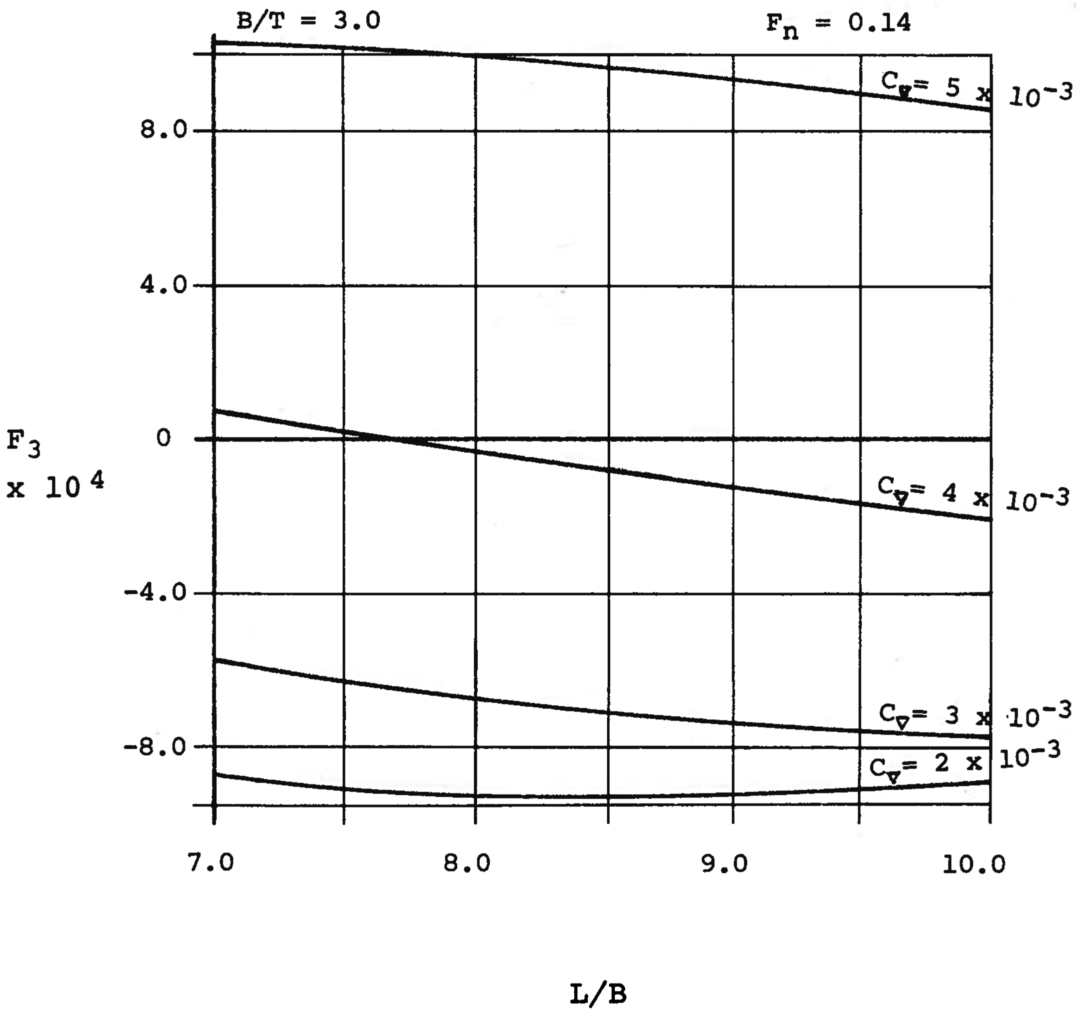
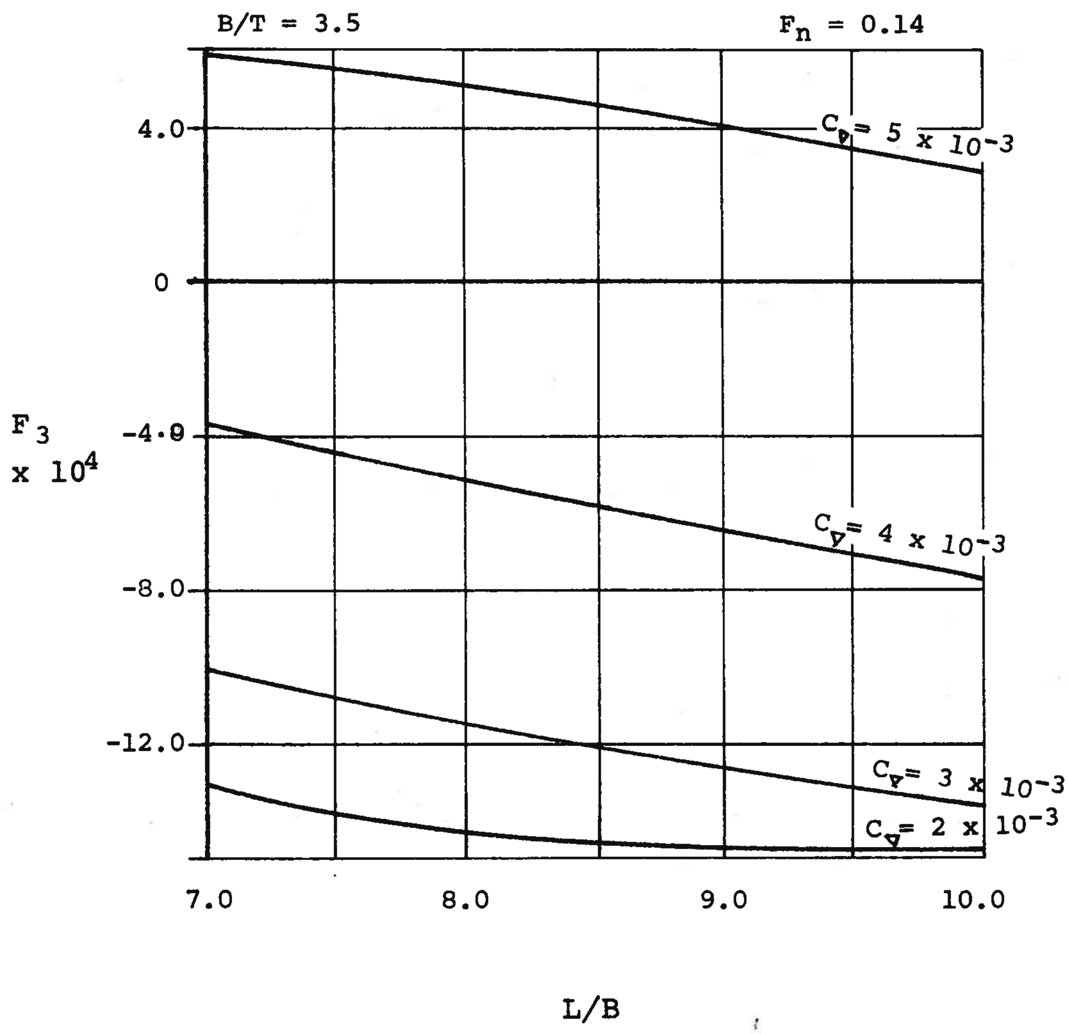
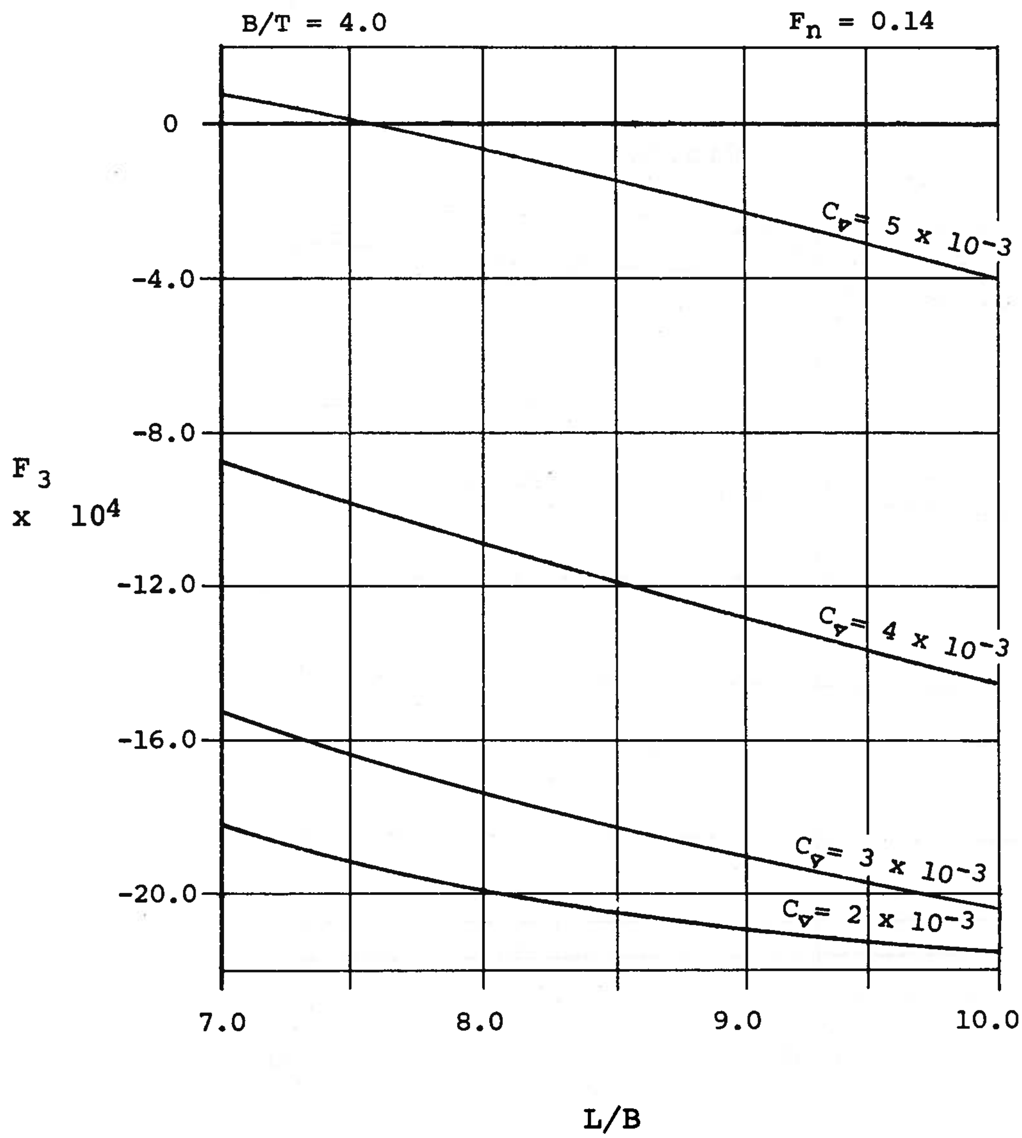

B/T = 3.0
Fn = 0.14
Cv= 5 x 10-3
Cv= 4 x 10-3
Cv= 3 x 10-3
Cv= 2 x 10-3
8.0
4.0
0
-4.0
-8.0
F3
x 10 4
7.0
8.0
9.0
10.0
L/B

Fig.22



B/T = 3.5
F_n = 0.14
4.0
0
-4.0
-8.0
-12.0
F_3 x 10^4
C_∇ = 5 x 10^{-3}
C_∇ = 4 x 10^{-3}
C_∇ = 3 x 10^{-3}
C_∇ = 2 x 10^{-3}
7.0
8.0
9.0
10.0
L/B

Fig.23



B/T = 4.0
F_n = 0.14
0
-4.0
-8.0
-12.0
-16.0
-20.0
F_3 x 10^4
$C_\nabla = 5 \times 10^{-3}$
$C_\nabla = 4 \times 10^{-3}$
$C_\nabla = 3 \times 10^{-3}$
$C_\nabla = 2 \times 10^{-3}$
7.0
8.0
9.0
10.0
L/B

Fig.24

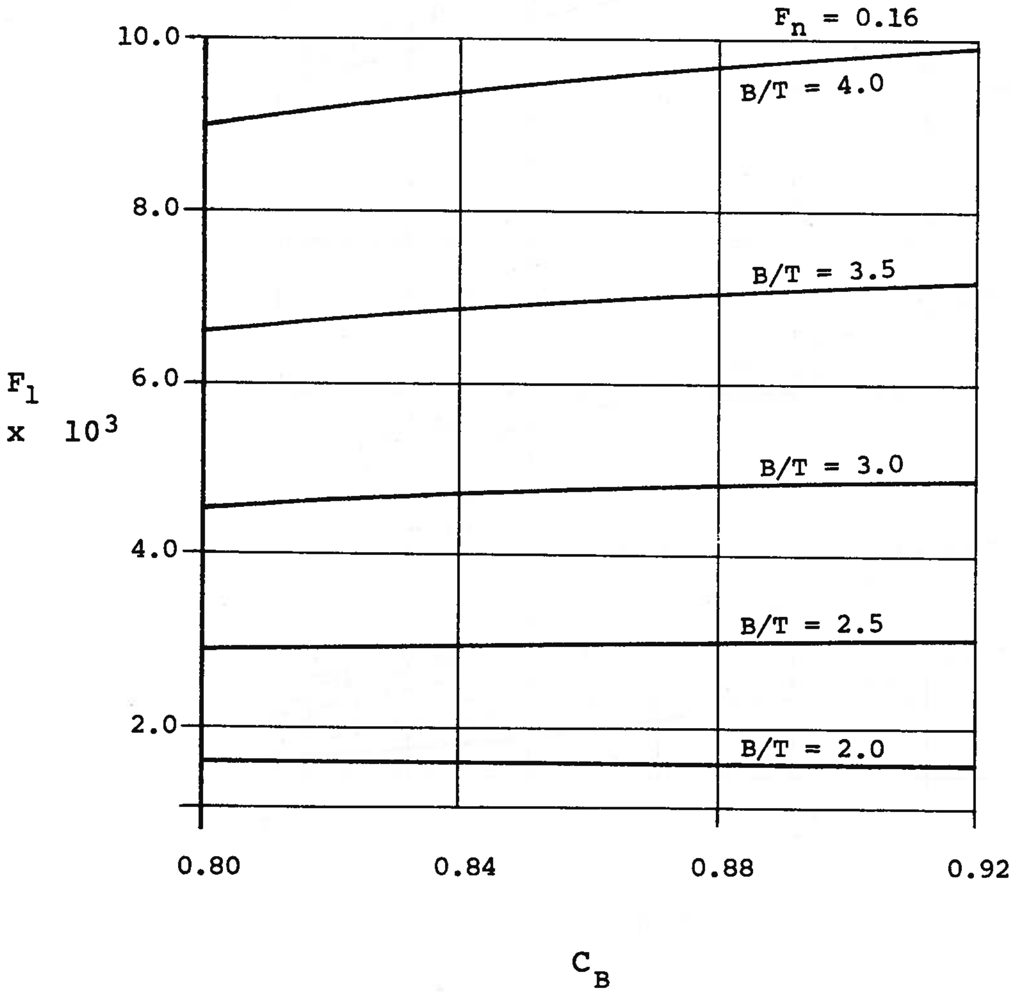
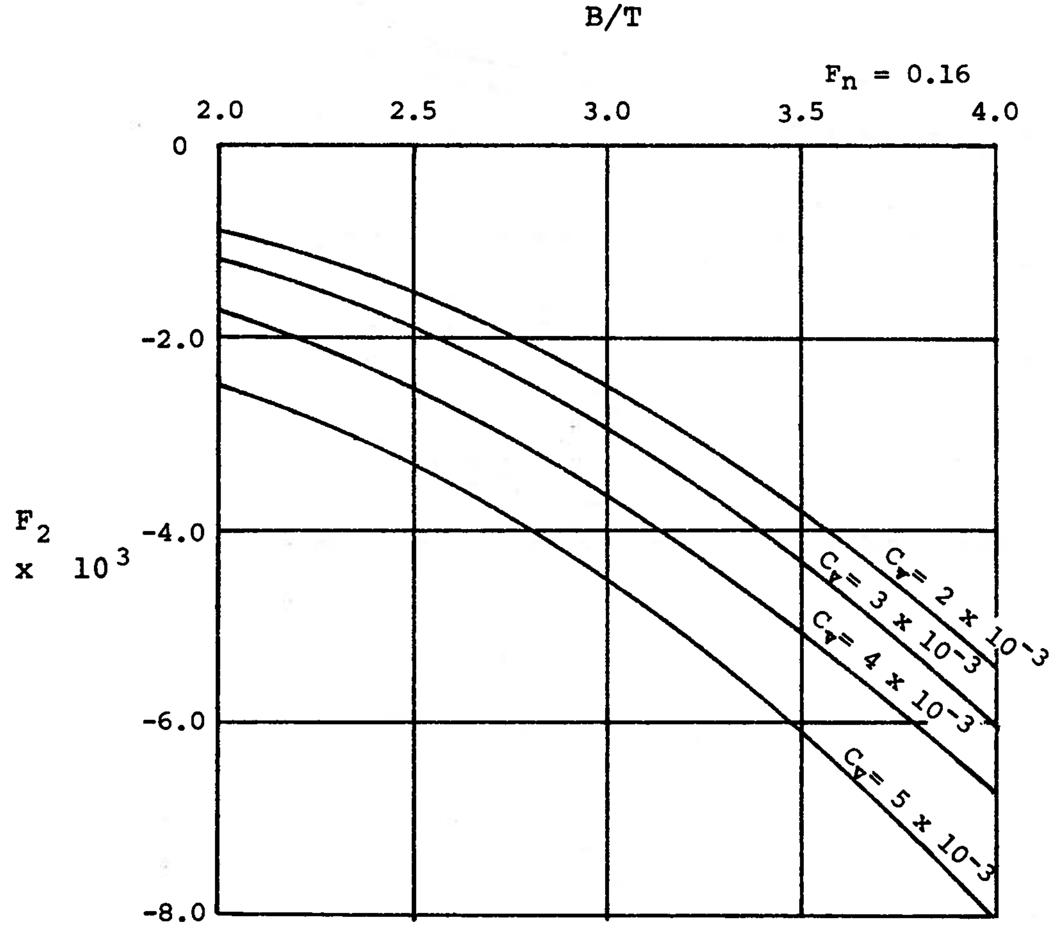
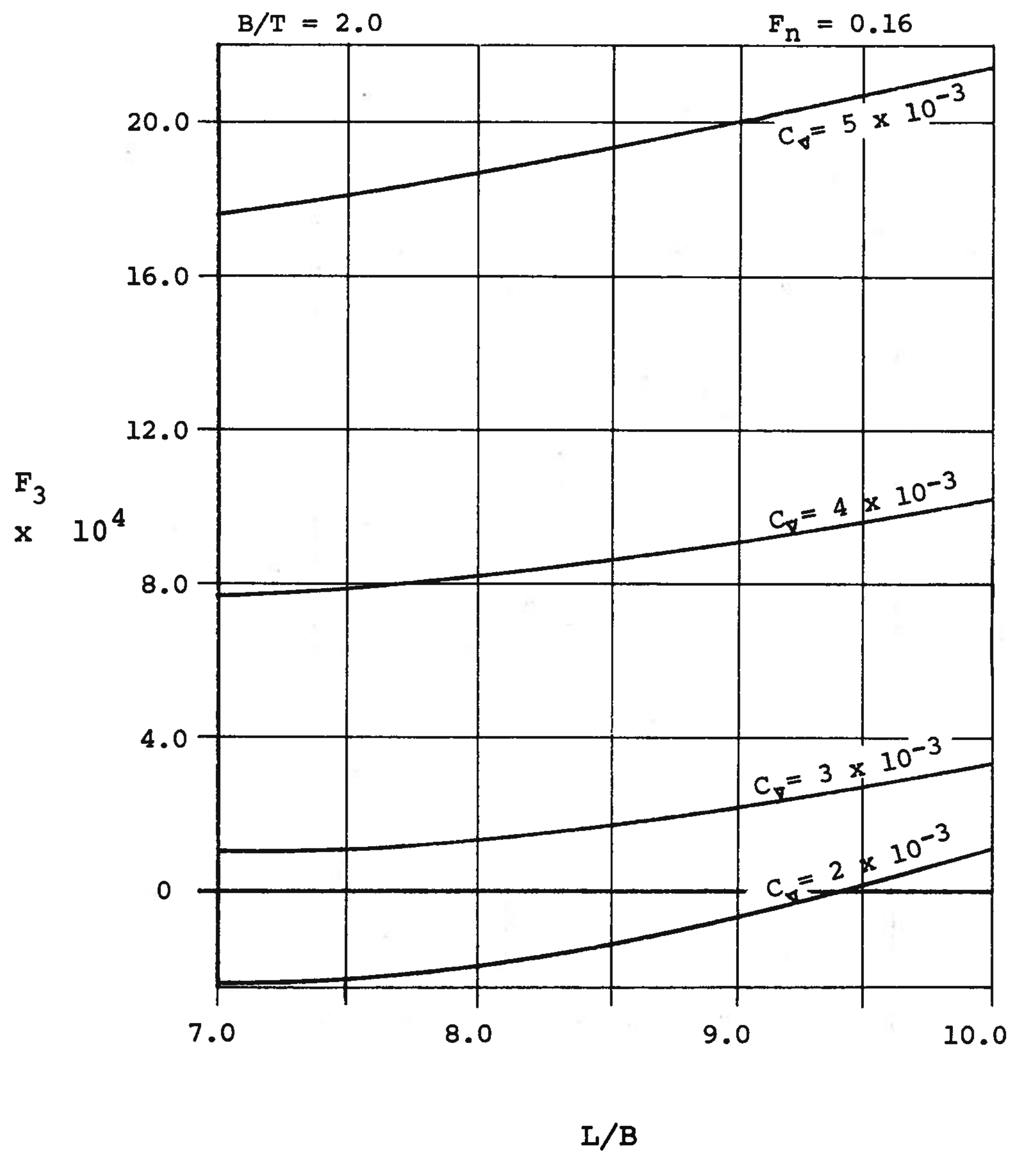

$F_n = 0.16$
B/T = 4.0
B/T = 3.5
B/T = 3.0
B/T = 2.5
B/T = 2.0
10.0
8.0
6.0
4.0
2.0
F_l
x 10^3
0.80
0.84
0.88
0.92
C_B

Fig.25



B/T
F_n = 0.16
2.0
2.5
3.0
3.5
4.0
0
-2.0
-4.0
-6.0
-8.0
F_2
x 10^3
C_v= 2 x 10^{-3}
C_v= 3 x 10^{-3}
C_v= 4 x 10^{-3}
C_v= 5 x 10^{-3}

Fig.26



B/T = 2.0
F_n = 0.16
20.0
16.0
12.0
8.0
4.0
0
F_3
x 10^4
$C_\nabla = 5 \times 10^{-3}$
$C_\nabla = 4 \times 10^{-3}$
$C_\nabla = 3 \times 10^{-3}$
$C_\nabla = 2 \times 10^{-3}$
7.0
8.0
9.0
10.0
L/B

Fig.27

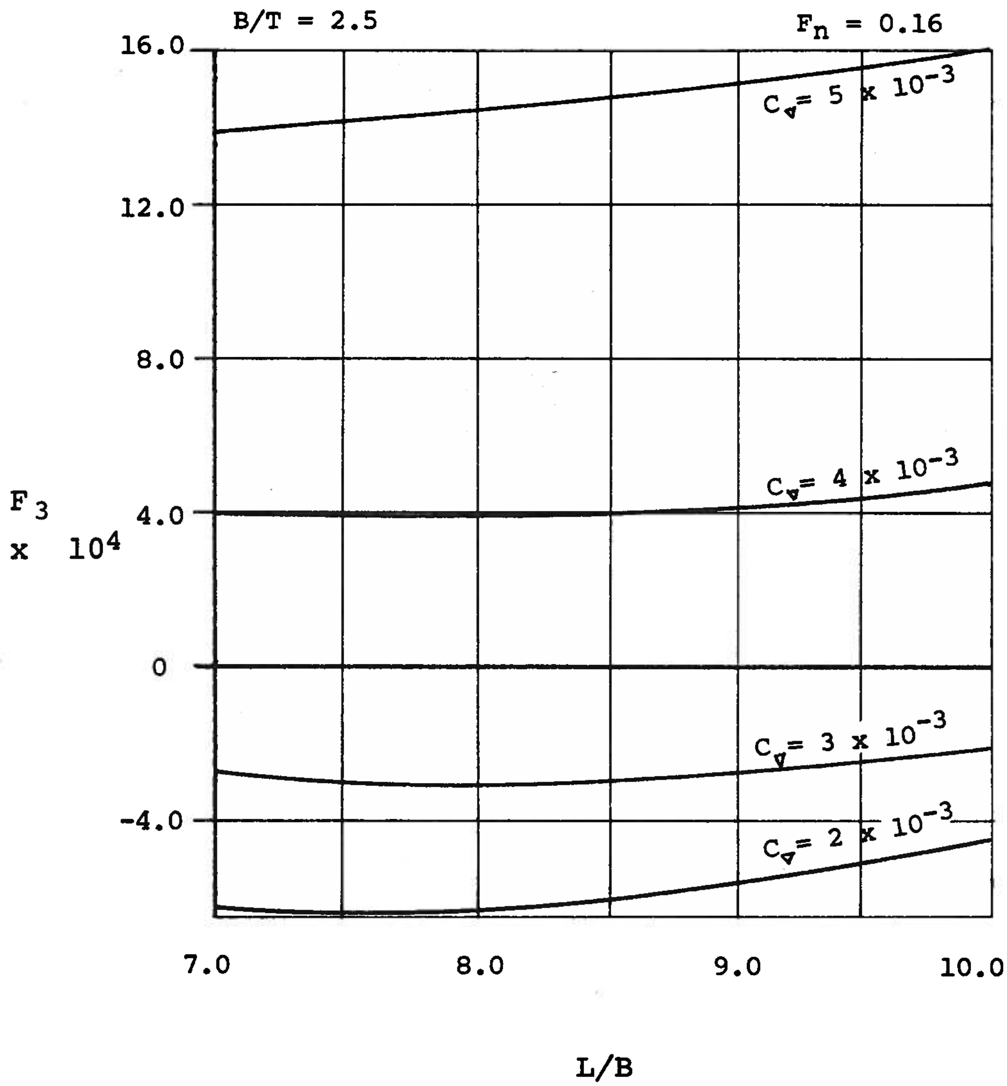

B/T = 2.5
F_n = 0.16
16.0
12.0
8.0
4.0
0
-4.0
F_3
x 10^4
C_∇ = 5 x 10^{-3}
C_∇ = 4 x 10^{-3}
C_∇ = 3 x 10^{-3}
C_∇ = 2 x 10^{-3}
7.0
8.0
9.0
10.0
L/B

Fig.28

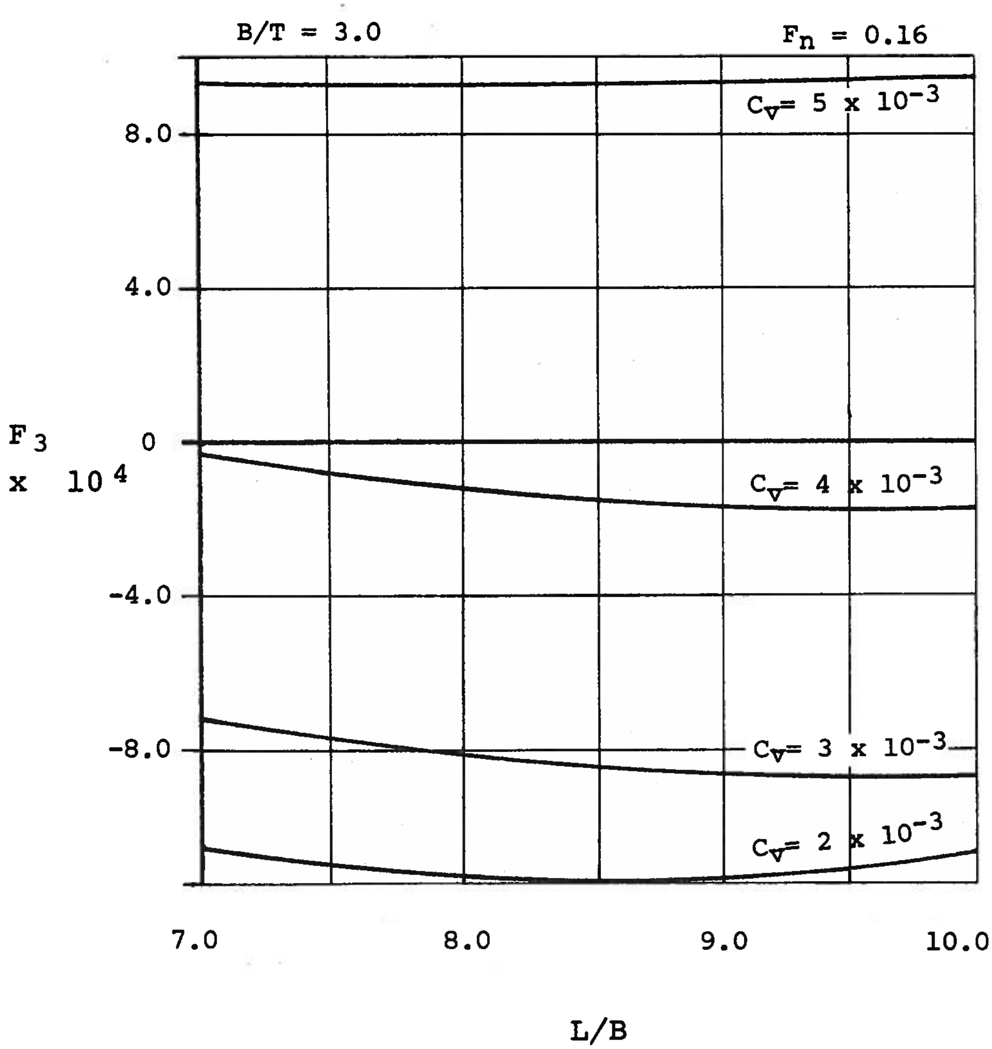

B/T = 3.0
F_n = 0.16
C_v= 5 x 10^{-3}
C_v= 4 x 10^{-3}
C_v= 3 x 10^{-3}
C_v= 2 x 10^{-3}
8.0
4.0
0
-4.0
-8.0
F_3 x 10^4
7.0
8.0
9.0
10.0
L/B

Fig.29

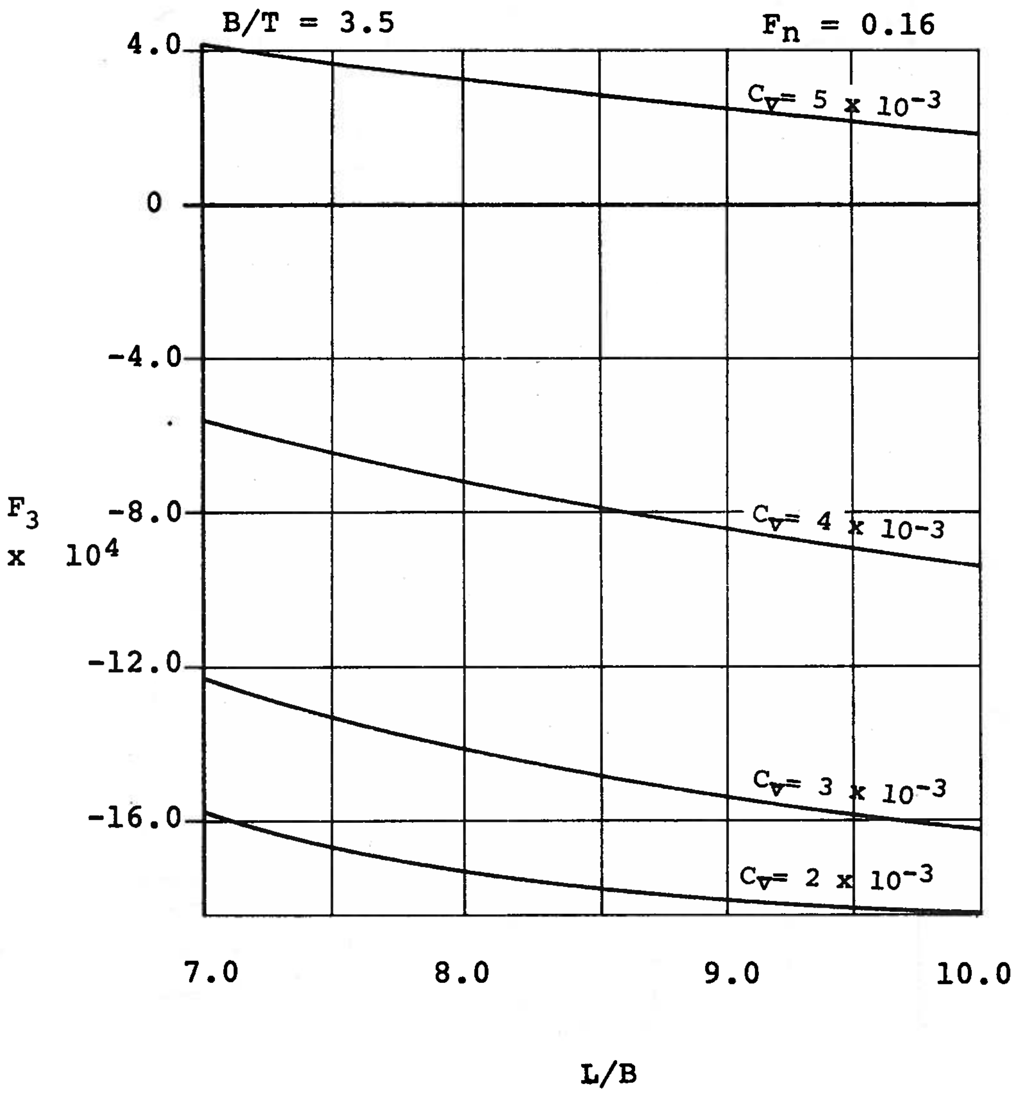
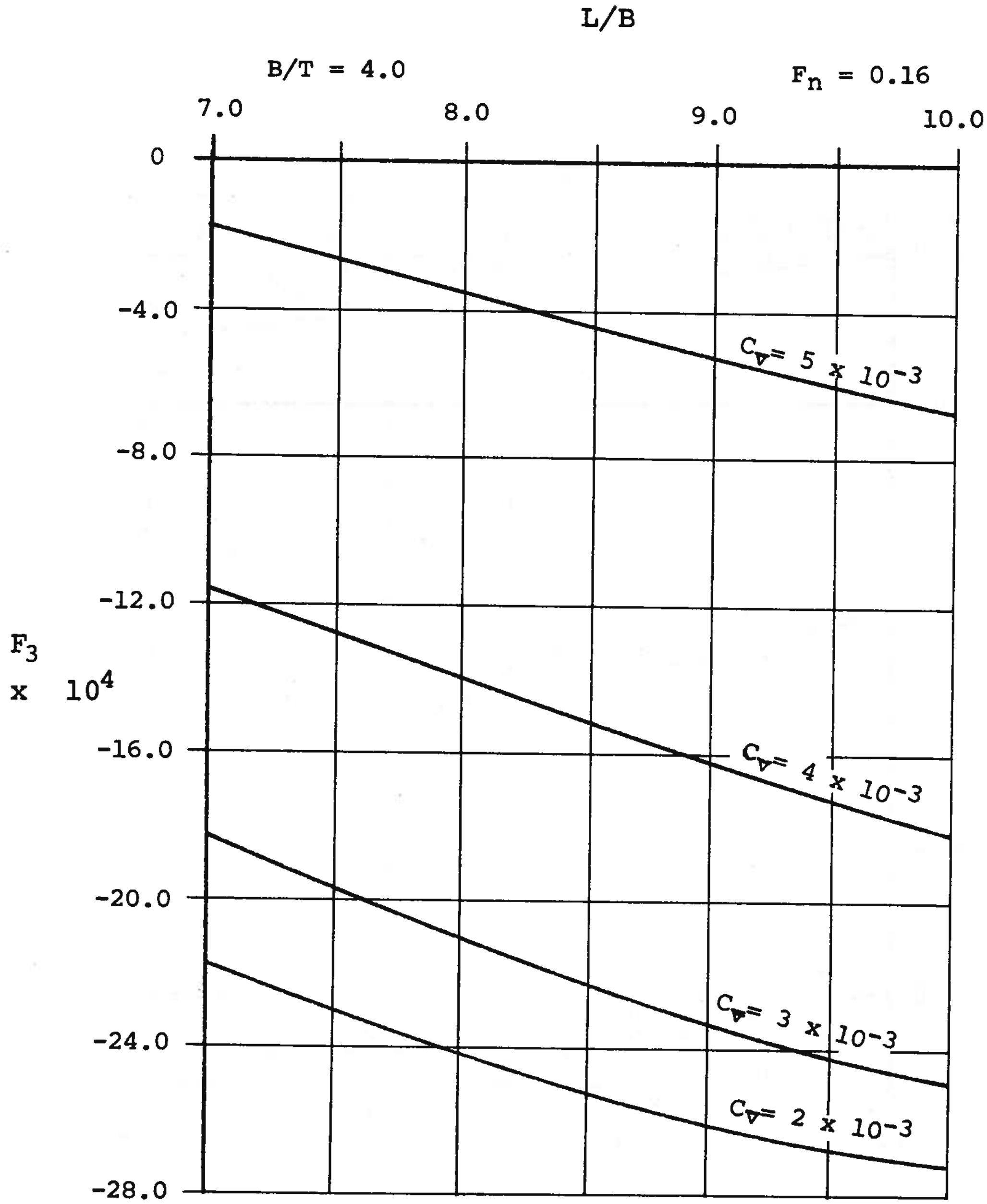

B/T = 3.5
Fn = 0.16
4.0
0
-4.0
-8.0
-12.0
-16.0
F3
x 10^4
Cv= 5 x 10-3
Cv= 4 x 10-3
Cv= 3 x 10-3
Cv= 2 x 10-3
7.0
8.0
9.0
10.0
L/B

Fig.30



L/B
B/T = 4.0
Fn = 0.16
7.0
8.0
9.0
10.0
0
-4.0
-8.0
-12.0
-16.0
-20.0
-24.0
-28.0
F3 x 10^4
Cv= 5 x 10-3
Cv= 4 x 10-3
Cv= 3 x 10-3
Cv= 2 x 10-3

5. Conclusion

A sample of model test data for Great Lakes bulk carriers was compiled from various sources which includes the great majority of all experiments conducted for this type of ship during the last thirty years.

The residuary resistance coefficient of these models was then regressed against a set of standard hull form parameters at three different levels of approximation:

1. A regression equation with 45 independent variables at each given speed ($0.11 \leq F_n \leq 0.18$), the most accurate fit attempted.
2. A regression equation with 11 independent variables at each given speed, a simplified version for initial design use.
3. A modified version of the 11 term regression with the coefficients cross-faired against speed, a further simplification for the designer's convenience.

Sample power estimates based on these formulas are presented in the Appendix.

The relative significance of the hull form parameters in the 11 term regression is discussed in terms of a graphical representation of three standard functions, of which the residuary resistance is composed, at two different speeds.

We hope that the results presented in this paper will offer the design community a choice of power estimating formulas for Great Lakes bulk carriers suitable for initial design work by manual or computer-aided methods.

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American Ship Building Company, Lorain, Ohio
Bethlehem Steel Corporation, Sparrows Point, Maryland
Columbia Transportation, Cleveland, Ohio
Hanna Mining Company, Cleveland, Ohio
Inland Steel Company, Chicago, Illinois
Marine Consultants and Designers, Cleveland, Ohio
U. S. Steel Corporation, Duluth, Minnesota

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Appendix I

Examples illustrating the use of the regression formulas in estimating the full scale resistance.

Examples:

	Vessel A	Vessel B
Length, L	998.0 ft.	661.2 ft.
Beam, B	104.6 ft.	70.0 ft.
Draft, T	25.75 ft.	24.5 ft.
Block coefficient, C_B	0.915	0.864
Wetted surface, S	150,450 ft.^{2}	69,700 ft.^{2}
Wetted surface, from regression formula	142,850 ft.^{2}	68,900 ft.^{2}
L/B	9.54	9.45
B/T	4.06	2.86
C_{WS}	8.26	7.06
C_{∇}	2.48	3.39

Calculation procedure for full scale resistance prediction

$$C_T = C_R + C_F + C_A$$

$$C_F = 0.075/(\log_{10}R_n - 2)^2 \quad \text{(ITTC line)}$$

where $R_n = \frac{VL}{\nu}$ = ship Reynold's number

C_A = appropriate model-ship correlation allowance

$$R_T = \frac{\rho V^2 S}{2} C_T$$

EHP

$$P_E = \frac{R_T V}{550}$$, R_T in lbs., V in ft./sec.

Vessel A

Full scale resistance prediction from test observation and three different regression techniques

		$C_R \times 10^3$						$C_T \times 10^3$				EHP			
F_n	MPH	Observed	45 variables	11 variables	11 variables cross-faired	C_F $\times 10^3$	C_A $\times 10^3$	Observed	45 variables	11 variables	11 variables cross-faired	Observed	45 variables	11 variables	11 variables cross-faired
0.11	13.44	0.911	0.907	0.831	0.834	1.445	0.200	2.556	2.552	2.476	2.479	5195	5187	5032	5038
0.12	14.67	0.987	0.967	0.892	0.886	1.439	0.200	2.617	2.597	2.522	2.516	6905	6853	6655	6639
0.13	15.89	1.051	1.074	0.967	0.971	1.416	0.200	2.667	2.690	2.583	2.587	8949	9026	8667	8680
0.14	17.11	1.219	1.193	1.118	1.114	1.403	0.200	2.822	2.796	2.721	2.717	11829	11720	11405	11388
0.15	18.33	1.462	1.385	1.338	1.341	1.392	0.200	3.054	2.977	2.930	2.933	15742	15345	15103	15118
0.16	19.56	1.785	1.720	1.673	1.677	1.381	0.200	3.366	3.301	3.254	3.258	21060	20653	20359	20384
0.17	20.78	2.189	2.122	2.053	2.044	1.376	0.200	3.761	3.694	3.625	3.616	28218	27715	27198	27130
0.18	22.00	2.674	2.587	2.666	2.669	1.367	0.200	4.236	4.149	4.228	4.231	37735	36960	37663	37690

Vessel B

Full scale resistance prediction from test observation
and three different regression techniques

		$C_R \times 10^3$						$C_T \times 10^3$				EHP			
F_n	MPH	Observed	45 variables	11 variables	11 variables cross-faired	C_F $\times 10^3$	C_A $\times 10^3$	Observed	45 variables	11 variables	11 variables cross-faired	Observed	45 variables	11 variables	11 variables cross-faired
0.11	10.94	0.393	0.418	0.440	0.441	1.558	0.200	2.151	2.176	2.198	2.199	1093	1105	1116	1117
0.12	11.94	0.438	0.464	0.475	0.469	1.542	0.200	2.180	2.206	2.217	2.211	1437	1454	1461	1457
0.13	12.93	0.504	0.502	0.500	0.508	1.528	0.200	2.230	2.228	2.226	2.234	1870	1868	1866	1873
0.14	13.93	0.589	0.566	0.574	0.573	1.512	0.200	2.301	2.278	2.286	2.285	2410	2385	2394	2393
0.15	14.92	0.701	0.684	0.685	0.681	1.500	0.200	2.401	2.384	2.385	2.381	3091	3070	3071	3066
0.16	15.92	0.887	0.852	0.850	0.847	1.488	0.200	2.575	2.540	2.538	2.535	4024	3969	3966	3962
0.17	16.91	1.080	1.065	1.082	1.088	1.477	0.200	2.757	2.742	2.759	2.765	5168	5140	5172	5183
0.18	17.91	1.424	1.397	1.419	1.417	1.467	0.200	3.091	3.064	3.086	3.084	6877	6817	6866	6862

Example:

Use of function charts.

$L/B = 9.5 \quad C_B = 0.86 \quad B/T = 2.75 \quad C_\nabla = 3.5$

$F_n = 0.14$

Fig.17 $C_B=0.86, B/T=2.75 \rightarrow F_1 = 2.38 \times 10^{-3}$

Fig.18 $B/T=2.75, C_\nabla=3.5 \rightarrow F_2 = -1.53 \times 10^{-3}$

$$\left.\begin{array}{ll} \text{Fig.20} & B/T=2.5 \\ & L/B=9.5, C_\nabla=3.5 \rightarrow F_3 = -0.8 \times 10^{-4} \\ & \qquad B/T=2.75, L/B=9.5, C_\nabla=3.5 \\ \text{Fig.21} & B/T=3.0 \\ & L/B=9.5, C_\nabla=3.5 \rightarrow F_3 = -5.2 \times 10^{-4} \end{array}\right\} F_3 = -3.0 \times 10^{-4}$$

$$\underline{C_R = F_1 + F_2 + F_3 = 0.55 \times 10^{-3}}$$

$F_n = 0.16$

Fig.24 $C_B=0.86, B/T=2.75 \rightarrow F_1 = 3.84 \times 10^{-3}$

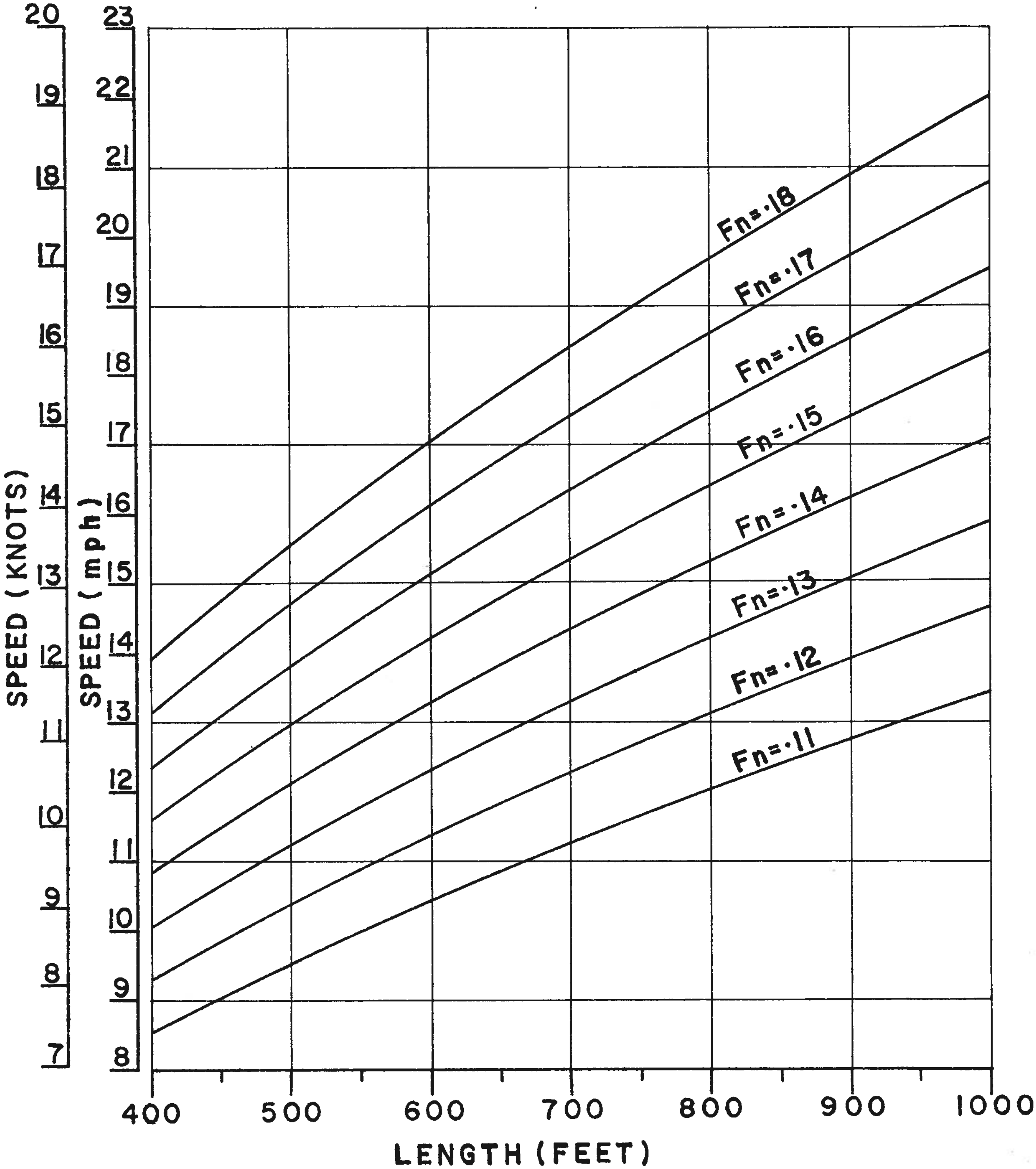
Fig.25 $B/T=2.75, C_\nabla=3.5 \rightarrow F_2 = -2.68 \times 10^{-3}$

$$\left.\begin{array}{ll} \text{Fig.27} & B/T=2.5 \\ & L/B=9.5, C_\nabla=3.5 \rightarrow F_3 = -0.9 \times 10^{-4} \\ & \qquad B/T=2.75, L/B=9.5, C_\nabla=3.5 \\ \text{Fig.28} & L/B=9.5, C_\nabla=3.5 \rightarrow F_3 = -5.3 \times 10^{-4} \end{array}\right\} F_3 = -3.1 \times 10^{-4}$$

$$\underline{C_R = F_1 + F_2 + F_3 = 0.85 \times 10^{-3}}$$

Appendix II

Speed-length plot of Froude numbers


SPEED (KNOTS)
SPEED (mph)
LENGTH (FEET)
Fn=·18
Fn=·17
Fn=·16
Fn=·15
Fn=·14
Fn=·13
Fn=·12
Fn=·11
20
19
18
17
16
15
14
13
12
11
10
9
8
7
23
22
21
20
19
18
17
16
15
14
13
12
11
10
9
8
400
500
600
700
800
900
1000