MODELING AND ANALYSIS OF REMANUFACTURING SYSTEMS WITH STOCHASTIC RETURN AND QUALITY VARIATION

by

Xiaoning Jin

A dissertation submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
(Industrial and Operations Engineering)
in The University Of Michigan
2012

Doctoral Committee:

Professor Jun Ni, Co-Chair
Professor S. Jack Hu, Co-Chair
Professor Yoram Koren
Associate Professor Amy Cohn
Guoxian Xiao, General Motors Company

To my family

ACKNOWLEDGEMENTS

I have learned many things since I began my journey toward my Doctoral degree; some things I realized early while other things I have only recently come to understand. Among the most important things I have learned along my journey are persistence, humbleness and diligence. I would not have been able to make any of these advancements without the help of many individuals, to whom I am deeply indebted and with whom I am very grateful.

I would like to express my most sincere gratitude to my co-advisor, Professor Jun Ni, for his continuous support, guidance and encouragement throughout my graduate study. Without him, I would never have had the chance to begin, continue, or complete my research. My foremost gratitude also goes to my co-advisor Prof. S. Jack Hu. I thank him not only for his insights and valuable feedback that contributed greatly to this dissertation, but for his support and guidance through the years. I would also like to thank Prof. Yoram Koren, who taught me how to be a better researcher and gave me valuable guidance and assistance on my research.

I also appreciate the invaluable guidance and advices provided by Professor Xiuli Chao, who devotes his precious time discussing research problems and providing me with insightful directions when I encountered difficulties during my research. I am very grateful for Professor Amy Cohn for serving on my doctoral committee and providing me with helpful suggestions on ways to improve my dissertation. I would also like to thank

Dr. Guoxian Xiao from GM R&D for offering me the opportunities to work with him on the battery research project. H is expert advices and experiences in automotive manufacturing systems inspired new ideas in my dissertation.

I am very grateful for my friends from the S. M. Wu Manufacturing Research Center, especially Adam John Brzezinski, Ahmad Almuhtady, Dr. Saumil Ambani, Kuen-Ren (Roland) Chen, Dr. Shiming Duan, Dr. Yi Liao, Dr. Li Jiang, Dr. Hsin-Yu (Cindy) Kuo, Dr. Seungchul Lee, Dr. Lin Li, Hao Yu, Jae-Wook Oh, Chaoye Pan, Xinran Liang, Xun Liu, Xi Gu, Yi Liu, Dr. Li-Jung (Bruce) Tai, as well as my other friends, Juil-Yum, Jin Yan, Dianyun Zhang and Dangdang Shao.

I greatly appreciate my friends in Prof. S. Jack Hu's lab, especially Dr. Jingjing Li, Dr. Liang Zhou, Dr. Hui Wang and Dr. Hui (John) Wang, Weihong (Grace) Guo, Chenhui Shao, Saumuy Suriano, Vernnaliz Carrasquillo, Robert Riggs, Taylor Tappe. They have inspired me with their intelligence and insight.

Finally, I would like to thank my family, especially my parents and my husband, Jie (Jay) Zhu. T heir unconditional support and encouragement has been the bedrock upon which my life has been built. It is them who have given me the confidence to face challenges and fight through difficulties over the years.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF SYMBOLS

μ	Aggregate product return rate in a Poisson process
α_i	Probability that module i is remaufactuable
p_{ij}	Probability that the retrieved module is type i with quality grade j
M	Number of types of modules in a single product
N	Number of quality grades for returned products
$x_{ij}(t)$	On-hand inventory level of module i with quality grade j at time t
D_j	Random demand variable for grade-j product returns
λ_j.	Demand rate for grade-j products in a Poisson process
$V_t\ (\boldsymbol{x})$	Relative value function of being in state $\boldsymbol{x}$
β	Uniformization factor for the discrete time Markov chain
c_j	Cost of using a unit with quality grade j
c_s	Unit penalty cost of substitution
s_j	Unit penalty cost of using new products to fulfill a demand
h_{ij}	Unit holding cost of module i with quality-j
$T^{(0)}$	Operator associated with random arrival of returned products
$T^{(j)}$	Operator associated with action regarding quality-j demand
G_i	Cumulative probability function of inter-arrival time of module i

T_i	Random variable of transportation delay of module i return
μ_{i}	Aggregate return rate of module i return
r_i	Admission threshold of the inventory of module i
θ_i	Capacity for shipment of products from customer returns
$Y_i(t)$	Outstanding orders for module i
$I_i(t)$	On-hand inventory of module i at time t, $0 \leq I_i \leq r_i$
$U_i(t)$	Stock out levels of module i at time t
F_i	Fill rate of module i
$\bar{F}_i$	Probability of stock out for module i inventory, i.e., $1 - F_i$
PCR	Partially covered by remanufacturing units
TCR	Totally covered by remanufacturing units
WCR	Warranty covered by remanufacturing units
W_i	Random variable of waiting time of module i
ρ	Traffic intensity
B	Total cost constraint
δ_i	Fill rate target (reuse level constraint) of module i
ε	Continuous-time value decay parameter
TC	Total cost
DV	Decayed value
EOL	Abbreviation of *End-of-life*
ATO	Abbreviation of *Assemble-to-order*
RATO	Abbreviation of *Reassemble-to-order*

Note: All the symbols with single subscript i are adapted to a multiple quality grade problem with ij as subscript.

LIST OF APPENDICES

CHAPTER 1

INTRODUCTION

1.1 Motivation

Energy and sustainability are two main societal challenges today. Since the energy consumed by transportation is around one third of the total energy consumption, electrification of transportation has attracted significant attention worldwide as electricity is one converging means for all forms of alternative, renewable energy. To store electricity for portable applications, lithium-ion batteries have attracted significant attention from vehicle manufacturers and are regarded as potentially the best technology because of their high cell voltage, high energy density and excellent cyclability (DOE Report, 2011). Furthermore, it is believed that the use of Lithium-ion batteries in hybrid and electric vehicles will be environmentally friendly by reducing exhaust emissions and improving fuel economy.

However, both lithium-ion battery cell manufacturers and automotive companies face the problem of battery warranty service and management of returns. Batteries returned from sold EVs for three main reasons. First, there may be cell damage or premature cell failure due to various reasons such as thermal runaway or electrical interconnections during the vehicle usage. For new battery-powered vehicles such as

Toyota Prius, GM Chevy Volt, and Nissan LEAF, whose batteries are warranted for 8-10 years or 100,000-150,000 miles, a premature end of life may be coming in the next few years during the warranty period. Second, batteries unavoidably lose capacity due to the charge-and-discharge cycling. When the charging capacity of a battery pack degrades to the level of 75-80% of its original capacity, the pack is no longer adequate for vehicle applications based on the current industrial specifications and should be "retired" from vehicle use. This will result in massive battery returns during the 8-10 years of warranty period. The third type of return is end-of-life battery when its warranty expires. These used batteries usually have much less residual value for recovery, remanufacturing and reuse, and are usually recycled for raw material. For the first two types of returns, EV manufacturers must fulfill the warranty obligation to service the EVs with failed batteries with functioning ones back to their customers. This obligation could potentially cost hundreds of million dollars to the vehicle manufacturers. Therefore, a big challenge arises in how to remanufacture and reuse the "retired" batteries and, at the same time, fulfill the demand from vehicle owners.

The value chain of EV batteries, as shown in Figure 1.1, consists of seven steps – component production (raw materials), cell production, assembly of cells into module and then modules into battery pack, integration of the battery pack into vehicle, use during the life of the vehicle, and reuse and recycling. In this research, we focus on the last four steps, which make up the end-of-life battery packs for reuse by automotive manufacturers. The goal is to close the loop using a value-added remanufacturing program (Dinger et al., 2010).

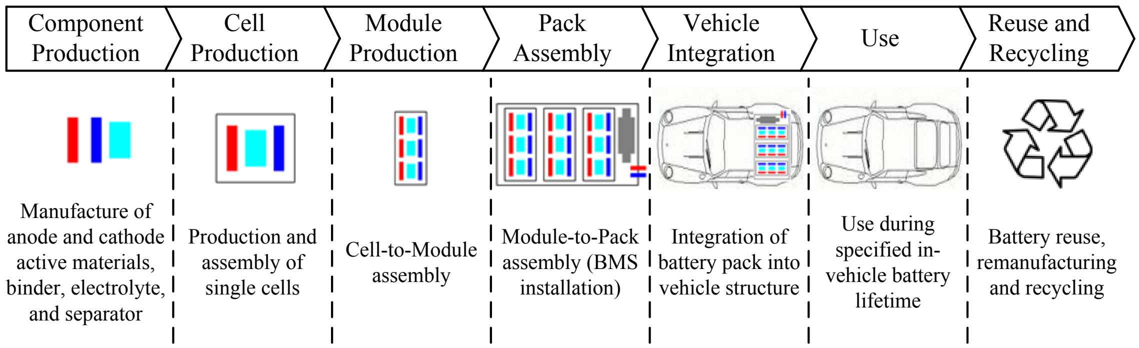


Figure 1.1 Value chain for EV batteries comprising of seven steps
[Source: BCG Focus]

The cost of Li-ion battery manufacturing is so high that it remains a hurdle to the industry development. The cells represent about 65% of the total cost of the battery pack. It is estimated that the current cost of an automotive Li-ion battery pack, as sold to OEMs, is between $1,000 and $1,200 per kW-h. The United States Advanced Battery Consortium (USABC) has set a cost target of $250 per kW-h.

Given the high cost of Li-ion battery manufacturing, it is too expensive to simply discard returned cells to landfill (Kalhammer et al., 2007). Generally, end-of-life battery packs may still have significant residual value for remanufacturing and secondary use. Therefore, remanufacturing system design and management for EV batteries are required in order to cut down the cost and enable EVs to successfully scale to the tens of millions (Fleichmann et al., 1997).

1.2 Significance of Remanufacturing

Balancing economic development with environmental protection is a key challenge to sustain U.S. manufacturing. The U.S. Environmental Protection Agency (EPA) recommends a 3R's (Recycle-Reduce-Reuse) strategy of sustainability on the End-

Of-Life (EOL) products and encourages manufactures to reduce raw material consumption and waste, recycle and reuse used material, components and products (Geyer et al., 1997). Conventional manufacturing is unsustainable because of its significant adverse environmental impacts. Remanufacturing can help companies achieve sustainable manufacturing by to saving costs via reductions in consumption of natural resources. R emanufacturing can also help reduce environment burden by decreasing landfill wastes and reclaim resources and energy already consumed in the original manufacturing of the products. Besides the environmental benefits, remanufacturing also provides economic incentives to firms by selling the remanufactured products and extending the life cycles of products. Successful examples from industry, such as BMW, IBM, and Xerox, show that remanufacturing can be profitable and there is a big market for the secondary use of remanufactured products. According to the EPA, the estimated total annual sales of 73,000 remanufacturing firms in the United States were $53 b illion in 1997. The fact that remanufacturing can be profitable has also been well documented by Lund (1983) and Ayres (1997).

Remanufacturing was defined as “an industrial process to recover value from the used and degraded products to ‘like-new’ condition by replacing components or reprocessing used component parts” (Lund, 1984). By remanufacturing, residual value of the durable components can be recaptured and some fraction of the original manufactured value is preserved. The significance of remanufacturing is that it would allow manufacturers to respond to environmental and legislative pressure by enabling them to meet waste legislation while maintaining high productivity for high-quality, lower-cost products with less landing filling and consumption of raw materials and energy.

A typical remanufacturing system includes disassembly, sorting and cleaning, refurbishing, and reassembly. The process of product remanufacturing is usually less expensive than producing a brand new unit because modules and components can be reused, thus avoiding the need to procure new components from suppliers (Ferguson, 2009). Therefore, a good strategy of battery remanufacturing for effective use on EVs will contribute to the sustainability of auto industry.

Remanufacturing and reuse of failed EV batteries are very different from the simple waste recycling. It requires a systematic method to manage the used vehicle battery subsystems by carefully evaluating their remanufacturability in terms of functionality, quality, cost, benefit, supply, and demand. The importance of sustainably managing large numbers of failed EV batteries with proper end-of-life treatment has been recognized, but the fundamental research issues are still not well-understood and systematic methodologies and challenges to solve the problems are lacking.

1.3 Research Issues

Logistics and remanufacturing systems for EOL products in a lifecycle management context differ from traditional forward logistics and manufacturing systems in terms of supply, production, inventory, and distribution. The major difference between the two lies in the supply side (Fleichmann et al., 2000; Trebilcock, 2002). In a remanufacturing system, supply is largely exogenous, i.e., timing, quantity, and quality of returns are much more uncertain than those of a traditional production system are. A significant consequence of these uncertainties is the inclusion of an inspection stage and a corresponding system with variable qualities of supply in a reverse logistics network

(Zikopoulos and Tagaras, 2008). With rare exceptions, traditional forward supply chains do not include such a focus on supply quality.

Furthermore, EV battery remanufacturing and reuse strategy is a new problem. It creates much more challenging research problems than other traditional products, such as cameras, machine tools, cartridge toners, etc., because of its unique ageing mechanism and failure modes. For battery packs that fail within their warranty period, some modules may still have sufficient capacity for EV usage and can be restored and reassembled to another battery pack again for customers who need warranty services. Battery remanufacturing, however, has stringent requirements on the balance of quality condition of subassemblies (cells and modules) than other products, because any mismatching among cells or modules could cause detrimental degradation to the entire battery system (Altemose, 2004; Barsukov, 2009). Normally, end-of-life EV batteries may still have significant residual values, even though they no longer meet automotive grade standards. Secondary use of retired EV batteries may include grid-integrated energy storage, off-grid backup power, storage for solar or wind power generation, etc.

1.3.1 Battery Degradation and Failure

Over the life of the battery, the battery may be charged and discharged for hundreds or even thousands of cycles. As this occurs, the individual cells may age differently. Some cells may become slightly (or more than slightly) mismatched with respect to the others. If this phenomenon is not corrected, one or more cells may become undercharged or overcharged, either of which can lead to failure of the battery.

Battery test, taken at cell level, module level and even complete pack level, is a critical step to determine the degradation in electrical performance as a function of life. (See, United States Advanced Battery Consortium, Electric Vehicle Battery Test Procedures Manual, 1996) EVs are designed to sense module-level degradation and to subsequently identify the failure module that affects the pack-level performance. The USABC established battery EOL standard for EVs as the stage at which specific failure criteria are met, e.g., capacity and/or power degradation. Specifically, when either (1) the net delivered capacity of a battery cell, module, or battery is less than 80% of its rated capacity when measured on the Reference Performance Test (RPT), or (2) the peak power capability, which is determined using the Peak Power Test, is less than 80% of the rated power at 80% depth of discharge. Generally, Battery Management Systems (BMS) in the battery packs are used to feed information to the controllers. A good BMS provides voltage (V), current (I), temperature (T), state of charge, cell balancing, and control of power-save mode, hence improving battery life and assuring safety. Advancements in BMS systems have important ramifications for how modern PHEVs and EVs manage their batteries as the batteries degrade over the vehicle's lifetime. Given these industry standards and rules of battery degradation testing, an accurate and complete health assessment of failed batteries is still challenging because the overall pack-level capability is dominated by module-level capability and individual module may degrade and age differently.

1.3.2 Variable Quality of EV Battery Returns

Quality variation in returned batteries is one of the sources of uncertainty in a remanufacturing system, and mainly caused by different "customer-use" stages and thus leads to different levels of degradation of individual battery modules or components. Quality variation adds to the complexity and challenge for EOL decision-making including disposal, recycling, remanufacturing, and reuse in secondary applications. More specifically, it complicates the EOL decisions and management of the remanufacturing system by increasing inventory variability, and reduces the precision, with which firms can control remanufactured product quality, balance component inventories, and make decisions on reassembly. The imbalance of inventories of modules at different quality levels could lead to shortage of certain modules while excessive inventories of other modules. The remanufacturers may have to resort to assembly with new modules to fulfill customer demands. The quality variation of the returned products also increases complexity to reassembly operations because the final remanufactured products could be assemblies of various combinations of the modules with different quality. As a result, remanufactured products may have different costs so that it may be more beneficial to satisfy one class of demand than the others.

Remanufacturing systems with the consideration of quality variation and uncertainties in returned products have been studied by several authors. Zikopoulos and Tagaras (2008) examined the economic attractiveness of a sorting procedure before disassembly and remanufacturing of used products. Their focus, however, was on how the sorting procedure could benefit the remanufacturing operations compared with the system without sorting. Galbreth and Blackburn (2006) investigated the optimal acquisition and sorting polices in the presence of used product condition variability for a remanufacturing system with both deterministic and uncertain demand . Their model and analysis followed a "make-to-stock" model. Souza et al. (2002) utilized a multi-class queueing network to model the remanufacturing system, where classes were determined by quality levels of product returns. Aras et al. (2004) analyzed quality-dependent remanufacturing and disposal decisions, and studied the conditions under which quality based categorization was most effective. Westkämper (2003) considered stochastic quality of cores with its associated capacity and cost implications for a multi-period remanufacturing planning problem. The problem was formulated as a stochastic program. Under certain probabilistic scenario, decisions were made on the number of cores to grade and remanufacture and the amount of inventory to carry over future periods .

These solution methods considered the variable quality levels for single product systems. The module-level quality variation was ignored, and the internal module-to-product assembly hierarchy was not taken into account in decision-making. In fact, for "cradle-to-grave' product design, the modularization of products is exceptionally important for disassembly and reassembly in remanufacturing, because they can facilitate customization and repeated utilization of single modules from different quality classes (See, e.g., Seliger et al 2004, Cunha et al., 2007, Wood et al., 2011).

1.3.3 End-of-Life Battery Decision-Making

One challenging issue for end-of-life battery management is to determine the optimal treatment for end-of-life batteries. Given the present state of EV battery technology, battery costs, and energy prices, it is highly unlikely that the majority of failed EV battery packs will be replaced by new ones. Cost analysis for EV battery lifecycle cost analysis is needed to provide insights and predictive information for both the vehicle manufacturers and EV consumers to make battery EOL decisions. The scientific issue is to develop an analytical model that can accommodate various qualitative and quantitative factors such as quality, residual life, legislation factors, costs related to the option, secondary use value, etc.

The literature on the EOL product recovery decision-making can be divided into two groups: (1) design of disassembly strategy, e.g., disassembly sequences and processes, and (2) end-of-life option models, e.g. recovery, recycling and disposal strategies. In Krikke et al. (1998), a stochastic Dynamic Program (DP) was used to determine a product recovery and disposal strategy for one product type based on the maximization of net profit considering relevant technical, ecological and commercial feasibility criteria at the product level. The same group of researchers further applied the methodology to real life cases on the recycling of copiers and monitors (Krikke et al., 1999). The work was further extended by taking into account multiple disassembly processes and partial disassembly (Das and Yedlarajiah, 2005). Some researchers proposed a mixed integer program to determine the optimal part disposal strategy based on the maximization of the net profit, while others developed a piecewise linear concave program to determine the optimal allocation of disassembled parts to five disposal options (refurbish, resell, reuse, recycle, and landfill) based on the maximization of the overall return (Jorjani et al., 2004). A piecewise linear concave program was developed to evaluate the economic viability of remanufacturing options under a government mandated take-back program [Richey et al. 2005]. A linear programming model was studied by Tan and Kumar (2008) to evaluate three EOL options for each part, namely, repair, repackage, or scrap. The underlying assumption of these studies on the EOL strategy was that non-destructive disassembly and reassembly techniques were available as the prominent steps for remanufacturing. In addition, these models considered disassembly and other EOL options, yet did not consider the quality variability and uncertainty of components at different assembly level.

A battery pack for an EV contains several hundred individual cells connected in series and parallel to achieve the total voltage and current requirements of the pack. To assist in manufacturing and assembly, the large stack of cells is typically grouped into smaller stacks called modules. Several of these modules are placed into a single pack. Within each module, the cells are welded together to complete the electrical path for current flow. A typical EV battery pack is a multi-level assembly structure as shown in Figure 1.2. For each level of the structure, the quality of each subassembly is evaluated in terms of remaining useful life. The EOL options are defined as whether to conduct remanufacturing by disassembly, or resale the entire module or components to the secondary market without recovery processes. Usually, a complete disassembly during an EOL battery treatment may or may not be necessary and cost-effective for all subsystems and components. Disassembly planning should be evaluated together with EOL options so that unnecessary disassembly costs could be avoided and optimal EOL profits achieved. Remanufacturing, as an important option in the EOL decisions, is not a simple recovery process. It includes further operations such as disassembly, cleaning, testing, and reassembly that are highly affected by the quality of the returns.

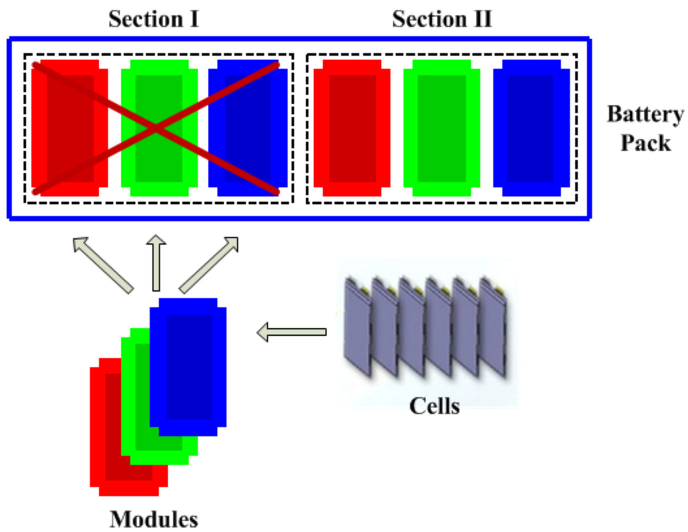


Figure 1.2 Basic structure of an EV battery (with a failure in Section I).

1.3.4 Module-to-Pack Re-matching and Reassembly Issues

Although new EV battery pack testing in a manufacturing environment has already been used in automotive industry, there are still emerging test and validation challenges to the EV industry as electronic and software contents of the vehicles grow. Testing for an EOL EV battery pack involves breaking the process down into manageable pieces, i.e., pack level testing and module level testing. Currently, modules in EV battery packs can incorporate cooling mechanisms, temperature monitors, and other devices. In most cases, modules also allow for monitoring the voltage produced by each battery cell in the stack by a BMS. Module-level performance degradation dominates over pack level effects such as module imbalance in capacity.

Strict constraints on the compatibility of modules and cells require decision-making models to produce the best re-matching and reassembly policies for remanufacturing. With the development of new generation EV batteries, EV manufacturers will be able to provide better battery performance characteristics. Therefore, remanufacturing strategy should also incorporate re-matching, reassembling, and packaging used cells from different primary usage sources and even different generations of batteries. Newer cells will need to be compatible with older cells to enable the continuation of battery technology development and keep up with the EV requirements. One aspect of re-matching and reassembly is the safety issue when assembly processes deal with all used battery modules with various physical conditions. Another very important aspect of re-matching includes determining a dynamic solution to the reassembly problem that accounts for the control of battery inventory and multiple classes of demand of different battery types/generations. Mismatching during reassembly may result in system instability such as inventory accumulation or low reuse level. Reassembly policy must be designed to cope with the compatibility of cells and modules of different usage ages as well as different generations.

1.3.5 Managing Inventory in Remanufacturing Systems

Production planning and inventory management problems with a deterministic supply have been successfully addressed for forward supply chains in manufacturing systems. These forward supply chain polices, however, may no longer perform well in the remanufacturing environment. In reverse logistics, the supply of used product returns has significant heterogeneity in timing, quantity, and quality. Therefore, we need to consider the quality variations in inputs, resulting in different EOL decisions as well as different routes of remanufacturing processes with different remanufacturing costs.

Remanufacturing systems may carry a variety of manufacturing inventories: cores (products not yet remanufactured), new parts, spare parts, finished goods, and Work-In-Process (WIP) inventories. In particular, cores are usually disassembled upon their arrivals, thus generating core parts to be stored in the inventories. In a typical remanufacturing facility, core inventories occupy the most resources in terms of space, maintenance, labor and logistics. Core inventories can be modules, components, or durable parts of used products. They differ greatly from other manufacturing inventories in several ways. First, the functions are different. New parts, spare parts have deterministic supply and serve as inputs of the production and assembly line, while finished goods inventory and WIP inventory can be considered the outputs of the system. The core inventories, however, exist as an intermediate to hold acceptable EOL product disassemblies and are ready to be used in remanufacturing processes. Uncertainties and variations in the quantity and timing of product returns could cause high core inventories in the remanufacturing facility. Although large core inventories are held to buffer against variations in the supply of cores and the variability in demand, holding cost, maintenance cost, and degrading core value overtime can still be a big burden on the total operation costs for the firm. Second, the policy for governing core inventories differs from those for regular inventories. WIP and final products inventories in conventional manufacturing system can be increased or decreased by changing production rates. Core inventories, however, largely depend on the compatibility (re-matching) between modules/components and how they are reassembled. In this research, core parts inventories are used to hold battery modules or cells and serve as an important buffer generated from disassembling returned EV batteries, and consumed by a part recovery process. The cores together with some new spare parts are reassembled to create final remanufactured products. Since reassembly requires the compatibility and balance between modules, quality grading will have a major impact on module inventories and will require a jointly management of module inventories of different quality levels.

The generation and consumption of the core inventories greatly rely on the strategy that the remanufacturing system adopts. The majority of remanufacturers use Make-to-Stock (MTS), Make-to-Order (MTO), or Assemble-to-Order (ATO) strategies or a mix of them from conventional manufacturing systems within a single remanufacturing facility (see, e.g., Rajagopalan, 2002; Blecker et al., 2005; Zhao and Simchi-Levi, 2006; Chod et al., 2010,). In order to develop optimal strategic policies for EV battery remanufacturing, system performance and stability in terms of levels of customer service, inventory level, and reusability of returned batteries will be modeled. To this end, we plan to focus on the Reassemble-to-Order (RATO) strategy which is commonly implemented by manufacturers with remanufacturing operations for their warranty services, especially because of the issues of highly uncertain product returns and demand and inventory fluctuation.

Although there is a growing body of literature addressing production planning and control for remanufacturing (see, e.g., Van Der Laan and Salomon, 1997; Van Der Laan et al., 1999; Toktay et al., 2000; Golany et al., 2001; Van Der Laan and Teunter , 2006;), most of them assume a single quality grade for all returns. No existing method is able to solve the production and inventory control problems in EV battery remanufacturing which suffers from high uncertainty and variance in the quality of returns. Only a few research papers considered different quality of returns with uncertainty issue. Souza et al. (2002) modeled the remanufacturing system as a multi-class open queueing network where they classified returned products according to quality levels and dispatch them to different remanufacturing stations based on dispatching rules. Ferguson et al. (2006)

investigated a remanufacturing planning problem when returned products had different quality levels and demand had a network-like structure. They formulated the problem as a linear program and provided optimal decisions on the quantity to remanufacture or to salvage for each quality level during each period. Extended study was found in "multi-period remanufacturing planning with uncertain quality of inputs" and a stochastic program was formulated to account for the fact that the quality levels of returned cores may take on any configuration of bad to good quality levels based on pre-established probabilities (Denizel et al., 2010). These methods consider the variable quality levels at the single product level. However, they ignored the module-level quality variation because the internal module-to-product assembly hierarchy was not taken into account in decision-making. In fact, for "cradle-to-grave' product design, structuring, and modularization of products is exceptionally important for disassembly and reassembly in remanufacturing because they can facilitate customization and repeated utilization of single modules from different quality classes (Westkämper 2003, Seliger et al. 2004).

1.4 Research Objectives

This thesis has three main objectives:

First, we want to develop a reassemble-to-order model to provide optimal reassembly-inventory decisions in a remanufacturing system. This model should be able to take into account the uncertainties of EOL product returns in terms of quality, quantity, and arriving times. We want to characterize the structural properties of the optimal policy for reassembly and inventory, and develop robust and computationally efficient heuristic approaches to support reassembly decisions for more complex remanufacturing systems.

Second, we want to model and analyze several key performance measures of the remanufacturing system with return admission. The modeling technique and solution method should provide important managerial insights on system performance under varying admission levels, thus providing better decision guidance on managing random EOL product returns.

Third, we want to design optimal admission policies for the remanufacturing system with different performance targets and constraints. These modeling methodologies should be applicable to manufacturing or service systems with uncertain supply in quality and delivery time, and multiple classes of demand. F igure 1.3 illustrates decisions for reusable EV lithium-ion batteries at different stages of their life cycles (indicated by % capacity). This thesis focuses on the batteries with more than 80% capacity and the module-to-pack reassembly strategy in a remanufacturing system.

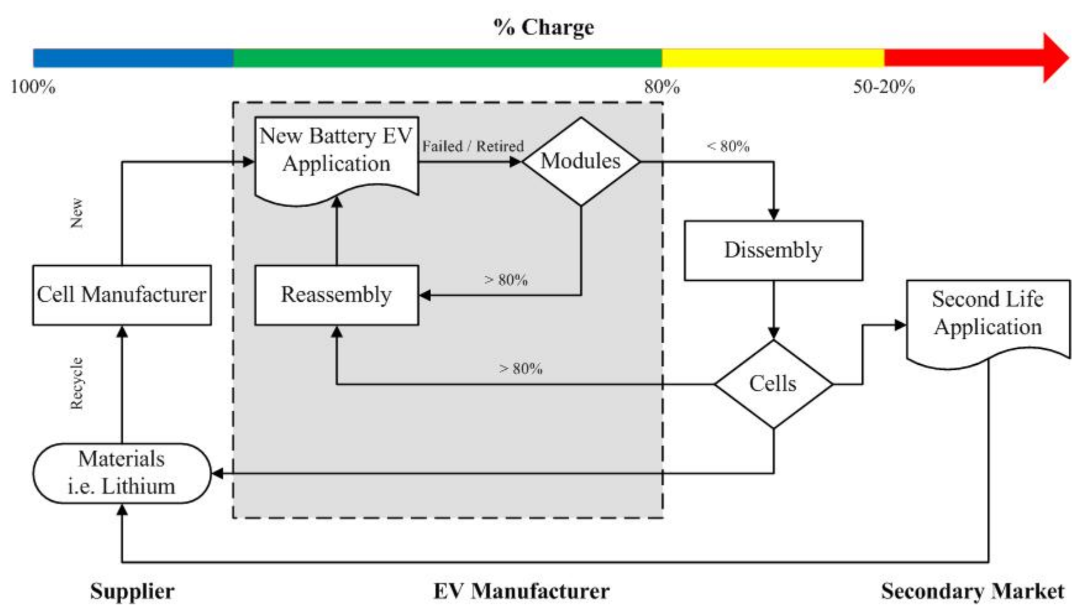


Figure 1.3 Different stages of an EV battery life cycle.

1.5 Outline of the Dissertation

This rest of this dissertation is organized as follows.

In Chapter 2, a stochastic optimal control model is developed for modular product reassembly and inventory decisions within a remanufacturing setting where a firm receives product returns with variable quality and reassembles products of multiple classes to customer orders for remanufactured products. This proposed model allows one to obtain the optimal policy for remanufacturing and reassembling end-of-life products as well as controlling the inventory, so as to minimize the overall system cost. Uncertainty in the form of end-of-life quality, quantity and arriving times adds to the problem complexity. Following Song et al. (1999), the remanufacturing-inventory strategy is referred to as the assemble-to-order strategy (ATO), but in a remanufacturing environment. This chapter is based on paper "Assembly Strategies for Remanufacturing Systems with Variable Quality Returns", by X. Jin, S. J. Hu, J. Ni and G. Xiao, conditionally accepted by *IEEE Trans. on Automation Science and Engineering*.

In Chapter 3, the ATO model with variable quality returns and multi-class customers introduced in Chapter 2 is revisited, with additional admission control on the arrivals of end-of-life product returns. Multiple classes of customer orders are generated by warranty claims which at the same time trigger product returns at random times and quality. The modeling has an emphasis on exactly expressing the impact of admission thresholds on the system performance at multiple levels: module level, order level and system level. These results are essential to understanding how the variation and uncertainty in the end-of-life quality impact the system behavior and provide insights into the design of an effective admission policy. This chapter is based on the paper

“Admission Policy and Performance Analysis in a Remanufacturing System with Warranty Returns”, by X. Jin, J. Ni, S.J. Hu, G. Xiao and S. Biller. This paper is submitted to the IIE Transactions.

Chapter 4 extends the work in Chapter 3 by designing optimal admission policies for the RATO remanufacturing systems. A discrete-event simulation model is developed by accommodating complex assumptions and constraints. The simulation model is used for what-if analysis and validation of the analytical model in Chapter 3. The results provide insights for implementing admission control policy and its potential benefits to system performance. Two admission threshold policies are proposed based on different objectives and constraints. The first policy is driven by minimizing the expected total cost of inventory and shortage penalty with the reuse level constraint. A heuristic decomposition algorithm is developed and used to obtain near-optimal solutions. The second policy is defined from the time value perspective such that the value decay due to the time delay in transportation of EOL products and their waiting time in inventory is minimized.

In Chapter 5, the contributions of this dissertation are summarized and possible future work is proposed.

CHAPTER 2

ASSEMBLY STRATEGIES FOR REMANUFACTURING SYSTEMS WITH VARIABLE QUALITY RETURNS

2.1 Introduction

Economic and environmental balance is one of the key challenges in sustainable manufacturing. Among various solutions to sustainable manufacturing, remanufacturing helps to significantly reduce carbon emissions and of energy required in manufacturing by diverting materials from landfill. In addition, remanufacturing also provides profits to a firm by selling the remanufactured products and extending product life cycle (Oakdene Hollins Ltd., 2004).

Over the past decade, literature and research has focused on remanufacturing as a set of industrial processes to recapturing the value added to the material when a product was first manufactured by replacing components or reprocessing used component parts (Lund, 2004, Charter and Gray, 2008). General processes for remanufacturing include disassembly, sorting and cleaning, refurbishing, and reassembly. The actual process of remanufacturing is usually less expensive than producing a brand new unit of the same product because many parts and components can be reused, thus avoiding the need to procure them from suppliers (Ferguson, 2010). Design and analysis of remanufacturing systems has been an important area of research as the future direction in much of the world is moving toward the green economy, energy efficiency and renewable energy.

In this chapter, we consider multiple grades of quality of remanufactured products. Conventionally, remanufactured products were assumed to have the same quality condition as the original product. In practice, however, some of used components cannot always be restored to a like-new condition. Usually, there exists significantly high variation in quality from both supply and demand perspectives. High quality variation implies extensive inventories of parts/modules that must be sourced, sorted and maintained, generating additional costs over the price of standard cores and components. Therefore, the supply of core is so vital to remanufacturers such as Caterpillar, Cummins that they initially pay for cores in order to secure a good flow of high quality cores.

We model remanufacturing as a value recovery process by reassembling products from modules/components with variable quality condition. We study the remanufacturing systems for modular products with the following characteristics:

(1) Customer orders for remanufactured products are 100% fulfilled. No lost sale or backorder is considered,

(2) Reassemble-to-order (RATO) strategy is adopted in the remanufacturing system,

(3) Quality condition of product returns varies,

(4) A higher quality module can be used to substitute for a lower quality module during reassembly, and

(5) There are multiple quality grades in remanufactured products for multiple classes of customer needs.

The above problem characteristics add more uncertainties and complexity to the system, hence making firm's decision making more difficult regarding how to utilize returned modules with variable quality and quantity. In this chapter, we consider the problem of jointly determining optimal inventory and reassembly policies for the above mentioned systems. Research issues of interest include: (a) what the optimal reassembly decision is to satisfy demands for remanufactured products, (b) how quality variation affects the decisions and system performance, and (c) how the system parameters impact the system performance. To address these questions, we formulate the problem as a Markov Decision Process (MDP) to derive the optimal policy. MDP enables us to characterize the optimal reassembly policy for the remanufacturing system, evaluate the benefits of module substitution, and analyze the effects of jointly managing the module inventories of different quality levels.

The rest of the chapter is organized as follows. A review of literature closely related to our study summarized in Section 2.2. Section 2.3 introduces a product reassembly and inventory decision making problem and notations. An MDP-based formulation is developed to seek the optimal reassembly decisions for two classes of demand respectively. This MDP enables us to indentify the structure of the optimal reassembly policy. The policy is characterized to be multiple state-dependent. In Section 2.4, we also show the monotonic properties of the thresholds that govern the optimal decisions. In Section 2.5, the performance of our derived optimal policy is compared with that of an exhaustive reassembly policy. We also illustrate how changes in parameters of the stochastic processes affect the optimal decisions and the average cost. Using these results we propose some computationally efficient heuristic algorithms for larger scale problems. Section 2.6 concludes the chapter.

2.2 Literature Review

Literature that is mostly relevant to our work include: variable quality of returns, assemble-to-order system and substitution. The state-of-the-art of these research areas are reviewed below.

2.2.1 Quality Variation of Product Returns

Quality condition variation of returned products is one main source of uncertainty in a remanufacturing system. Quality variation is mainly due to the different "customer-use" stage of products and thus leads to different levels of degradation of individual modules or components. Quality variation of the uncertain product returns complicates the management of the remanufacturing system by increasing inventory variability, thus reducing the precision with which firms can control remanufactured product quality, balance component inventories and make decisions on reassembly. Firstly, the imbalance of module inventories at different quality levels could lead to shortage of certain modules when demand arrives while others have excessive inventories. The remanufacturer may have to resort to assembly with new modules to fulfill customer demands just in time. Secondly, the quality variation of the returned products adds complexity to reassembly operations because the final remanufactured products could be assemblies of various combinations of the modules with different quality.

The literature dealing with remanufacturing systems with quality variation and uncertainties in returned products is relatively limited. Zikopoulos and Tagaras (2008) examined the economic attractiveness of a sorting procedure before disassembly and remanufacturing of used products. But their focus is on how the sorting procedure could benefit the remanufacturing operations compared with the system without sorting. Galbreth and Blackburn (2006) investigated the optimal acquisition and sorting polices in the presence of used product condition variability for a remanufacturing system with both deterministic and uncertain demand. Souza et al. (2002) considered a multi-class queueing network to model the remanufacturing system where classes were determined by the quality levels of product returns. Aras et al. (2004) analyzed quality-dependent remanufacturing and disposal decisions and studied the conditions under which quality based categorization was most effective. Ferguson et al. (2010) is among the few papers considering stochastic quality of cores with its associated capacity and cost implications for a multi-period remanufacturing planning problem. The problem was formulated as a stochastic program where decisions were made under certain probabilistic scenario that how many units of cores to grade and how many to remanufacture for each quality grade, and what the amount of inventory to carry over future periods.

These solution methods consider the variable quality levels at the single product level. However, they ignore the module-level quality variation and the internal module-to-product assembly hierarchy is not taken into account in decision-making. In fact, for "cradle-to-grave" product design, the modularization of products is exceptionally important for disassembly and reassembly in remanufacturing because they can facilitate customization and repeated utilization of single modules from different quality classes (Westkämper, 2003).

2.2.2 Assemble-to-Order (ATO) Systems

Assemble-to-orders are multi-item inventory systems with dependent demands across items. The literature on ATO systems in operations management includes inventory/replenishment strategy and performance analysis. (See, e.g., Song et al., 1999; ElHafsi, 2009) ATO strategy considers a trade-off between the product portfolio and the assembly lead time. Usually, modular product design is used in support of the ATO strategy because the assembly, the storage and logistics are affected by the product modular structure (Cunha et al., 2007). Inventory rationing/admission control for ATO systems has been studied extensively to solve the problem of dynamically allocating inventory to different demand classes upon demand arrivals (Zhao and Simchi-Levi, 2006; Benjaafar and ElHafsi, 2006; Deshpande et al., 2007,). These models considered inventory control problem in a dynamic sense, where an inventory may be reserved in anticipation of demand from higher margin customer. In particular, Zhao and Simchi-Levi (2006) evaluated the performance of base-stock policies for a multiproduct ATO system, particularly with stochastic sequential lead time. In Benjaafar and ElHafsi (2006), the authors considered the optimal production-inventory control of an ATO system with multiple customer classes. Their decisions deal with (1) when to produce each component and (2) whether or not to satisfy an order from on-hand inventory. They finally presented a multi-level rationing policy that was optimal for the inventory allocation.

In this chapter, we consider the optimal assembly decisions with unreliable supply in a remanufacturing environment, particularly with quality variation consideration. Our work interconnects the two research areas of stochastic modeling of remanufacturing systems and production/inventory control in ATO systems.

2.2.3 Substitution

Due to the quality variation of product returns, we adopt the concept of module substitution into the ATO remanufacturing system. In existing production and operations management literature, product substitution provides the assembly system with flexibility such that the shortage of one type of modules can be covered by a nother available inventory. In this way, the inventory level of products may be reduced due to the risk pooling effect. Only a few studies investigated substitution via simple models. Xu et al. (2010) studied an inventory system where a supplier supplied the customer using two mutually substitutable products and may choose to offer substitution or may choose not to; where as the customer may or may not accept the substitution. They provided the solution to the optimal replenishment quantities. In Iravani et al. (2012), the authors studied an inventory system in which a supplier fulfilled demand using two mutually substitutable products. In their model, substitution was on the product level such that when demand for a certain type of product exceeded its supply, the inventory of another product type could be used as a substitute to fulfill the demand.

The substitution decisions in our remanufacturing-reassembly model considers module substitution during reassembly process and these decisions are integrated with the module inventory control. A module substitution occurs whenever a demand from one is filled using module inventories of another class. In Jin et al. (2011), a reassemble-to-order system is studied to seek an optimal policy for modular product reassembly within a remanufacturing setting. The system receives end-of-life product returns with quality variance and faces multiple classes of demands for remanufactured products. This chapter studies the benefits of module substitution in reassembly decision-making as well as providing sufficient conditions for optimality both with mathematical and intuitive verification. The optimal inventory and reassembly policy developed in this study will be beneficial in broad manufacturing practices because one can jointly manage inventories across variable quality (in a way of pooling inventory) in order to achieve timely customer order fulfillment with minimal cost. The optimization modeling will provide solutions for minimal average cost by trading off the penalty cost using new modules and holding cost for excessive returns (Chen et al. 2001, Frank et al. 2003).

2.3 Problem Formulation

We consider a reassemble-to-order (RATO) system with multiple demand classes. The problem is based on the steady-state analysis of an MDP model rather than short-term transient behavior and effects (Puterman, 1994; Bertsekas, 1995). The MDP model seeks to balance the penalty cost of fulfilling demand with new products and the holding cost of having excessive cores in stock. By analyzing the structure of the model, we determine an optimal control inventory and reassembly policy to support practical decision making for remanufacturing systems.

To model arrivals of the returned products, we assume that product returns follow a Poisson process with a total rate μ. Each product consists of M non-interchangeable modules, indexed by $i = 1, 2, \ldots, M$. Here we assume that the failure of the product is due to failure in one of the modules and the remaining one still functional and retains value to be reused. The probability that module i is remanufacturable is α_i, $i =$

$1, 2, \ldots, M$. The remanufacturable modules are further categorized by its quality, i.e., each module of type i has a quality grade q indexed by $j = 1, 2, \ldots, N$ ($j = 1$ represent the lowest quality grade and j=1, the highest.) with probability p_{ij}. All the probabilities are given constants. Therefore, the actual arrival rate of module i with quality j is $\mu \cdot \alpha_i \cdot p_{ij}$. Let $x_{ij}(t) \in \boldsymbol{X}$ denote the on-hand inventory level of module i with quality grade j at time t, where $\boldsymbol{X}$ represents the discrete state space.

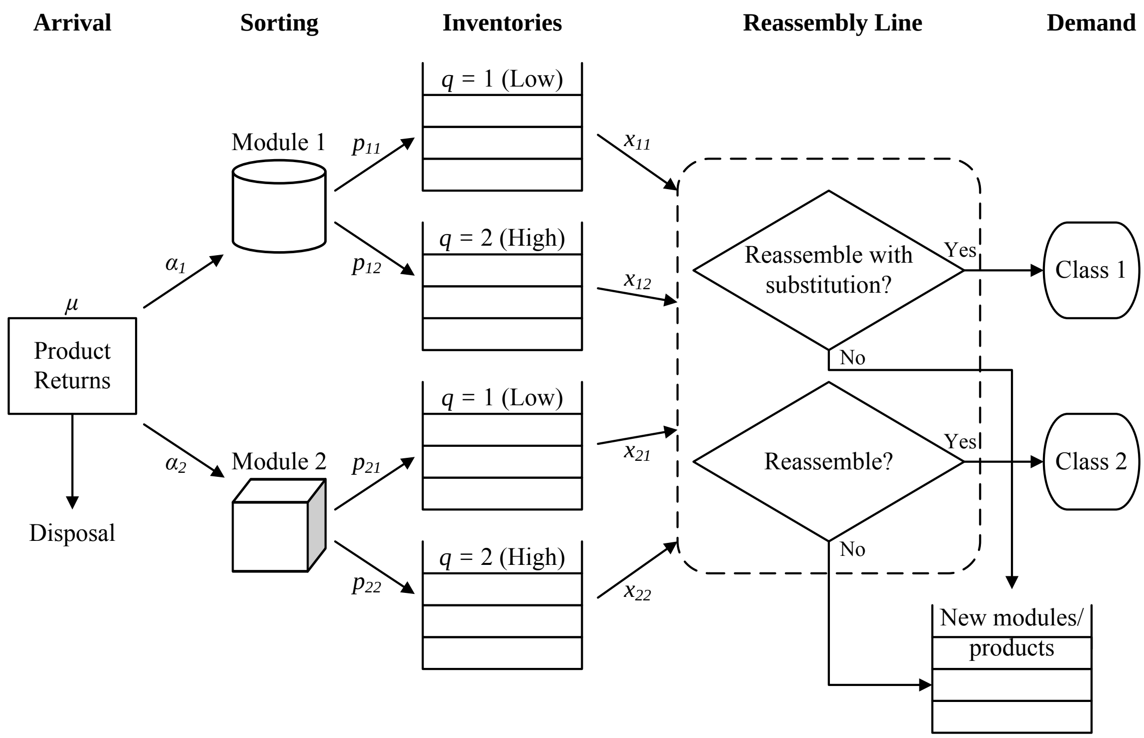


Figure 2.1 The RATO remanufacturing system.

There are N classes of random demand for the remanufactured products, denoted as D_j, $j = 1, 2, \ldots, N$. Classes 1 through N represent N quality grades of product from low to high. Arrivals of class j demand forms a Poisson process with rate λ_j. The demands are distinguished by the customer preference for the residual life and price of the remanufactured products. Figure 2.1 shows an example of a two module – two quality grade ATO system.

Each time an order is placed, the system manager must decide on the way of assembly to satisfy the order from on-hand inventory, if any is available. If a decision is made to satisfy the incoming order from available inventory, the products are immediately assembled and delivered to the customer. The assumption of instantaneous assembly is consistent with most of the literature on ATO systems. (See, Song et al. 1999, Benjaafar and ElHafsi 2006). We assume that the product assembly is subject to the weakest link principle (Baiman et al. 2005) such that the demand of quality class j can only be assembled from all modules with quality j or higher modules with at most one grade higher, i.e., $(j+1)^{st}$ quality grade.

The decision epochs considered in this model consist of all demand arrivals. At each decision epoch, a policy specifies how the order is fulfilled at the reassembly system. The goal is to find a policy, which minimizes the *average cost per unit time* through an infinite horizon. The cost considered in this model consists of holding costs of returned modules, costs of usage of modules during assembly, penalty cost of module substitution, and penalty cost of using new modules in place of returned modules.

The reassembly and inventory system consists of $M \times N$ inventories where the quality-specific modules are stocked before demand is realized. For the ease of exposition, we start with a system of two modules – and two quality grades, i.e., $M = 2$, $N = 2$. Let $\boldsymbol{x}(t)$ denote the state of the system at time t:

$$\boldsymbol{x}(t) = \begin{bmatrix} x_{11}(t) & x_{12}(t) \\ x_{21}(t) & x_{22}(t) \end{bmatrix} \tag{2.1}$$

There is a reassembly station at the downstream of the inventories, where the reassembly operation is triggered by an order from any of the two classes of demand. During reassembly, a quality-specific module is turned into a final secondary product by joining of modules with different quality grades shared with both demand classes. Inventory costs are incurred at a constant h_{ij} for each unit of returned module i with quality j kept in stock.

Let $V_t\ (\boldsymbol{x})$ denote the relative value function of being in state $\boldsymbol{x}$ and g be the average cost per transition, where transitions occur with rate $\beta = \sum_{i=1}^{M} \lambda_i + \mu$, resulting an average cost per unit time of $g\beta$. The decision epochs correspond to all demand arrivals of two classes of remanufactured products. The reassembly policy governs the binary decision-making options in the following way.

We first consider the reassembly decision for the demand class of low quality products (class 1). When an order for the low quality product arrives:

(1) if both modules of low quality have positive inventories, the order is fulfilled immediately from reassembly,

(2) if one of the low quality modules, however, has zero inventory, the firm has the option of either reassembling using a substitutable high quality module or using a new product to fill the order.

Each reassembly option incurs a cost of using a unit with quality grade j, c_j, $j = 1,2$, and penalty cost of substitution c_s. If the "reassemble with substitution" option is chosen, the order is filled with reassembly with cost c_j and an additional substitution penalty cost c_s. If the order is supplied with a new product, a penalty cost of s_j occurs. To keep the notation simple, we introduce a set of operators, $T^{(0)}$, $T^{(1)}$ and $T^{(2)}$, to represent system state changes due to return arrivals, and demand arrivals for two quality classes of remanufactured products, respectively. These operators are extremely useful in decomposing the problem and establishing the structure of the optimal policy individually.

First, let operator $T^{(1)}$ represent the reassembly decision regarding the low quality demand. As shown in Equation 2.1, there are four possible cases when a class 1 demand arrives: (1) if both modules of low quality are available in stock; the order is met by immediate assemble from inventories resulting in $x_{11}-1$,and $x_{21}-1$; (2) if low quality module 1 is not available but substitutable module is available; (3) if low quality module 2 is not available but substitutable module is available, and (4) if no substitutable module is available when needed.

$$T^{(1)}V_t(\boldsymbol{x}) \tag{2.2}$$

$$= \begin{cases} V_t(\boldsymbol{x}-e_{11}-e_{21})+2c_1 & if\ x_{11}\neq 0, x_{21}\neq 0 \quad (i) \\ \min\,[\underbrace{V_t(\boldsymbol{x}-e_{21}-e_{12})+c_1+c_s}_{\text{Reassmeble with substitution}}, \underbrace{V_t(\boldsymbol{x})+s_1}_{\text{Use new}}] & if\ x_{11}=0, x_{21}\neq 0, x_{12}\neq 0 \quad (ii) \\ \min\,[\underbrace{V_t(\boldsymbol{x}-e_{11}-e_{22})+c_1+c_s}_{\text{Reassmeble with substitution}}, \underbrace{V_t(\boldsymbol{x})+s_1}_{\text{Use new}}] & if\ x_{21}=0, x_{11}\neq 0, x_{22}\neq 0 \quad (iii) \\ V_t(\boldsymbol{x})+s_1 & if\ x_{i1}+x_{i2}=0, i=1,2 \quad (iv) \end{cases}$$

Second, when an order for the higher quality product arrives, we check the inventory levels of the high quality modules.

- If both of them are positive, i.e., $x_{12} > 0$, $x_{22} > 0$, then the firm has the option to reassemble from on-hand inventories or use a new product to meet the order.
- If one of the high quality modules is not available in inventory, the firm has the

options (a) to use a new module for substitution, or (b) use a new product to fulfill the order.

The decision regarding the high quality demand is represented by the operator $T^{(2)}$. Mathematically, we define $\boldsymbol{e}_{ij} \equiv [b_{j-1} \quad e_j \quad b_{m-j}] \in \mathbb{R}^{n \times m}$, where b_k is an n by k zero matrix and e_j is the i^{th} unit vector. We assume the initial condition $V_0(\boldsymbol{x}) \equiv 0$ so that the initial condition has no effect on the results for the infinite-horizon problem.

$$T^{(2)}V_t(\boldsymbol{x}) \tag{2.3}$$

$$= \begin{cases} \min [\underbrace{V_t(\boldsymbol{x} - e_{12} - e_{22}) + 2c_2}_{\text{Reassmeble}}, \ \underbrace{V_t(\boldsymbol{x}) + s_2}_{\text{Use new}}] & if\ x_{12} \cdot x_{22} \neq 0 \\ \min [\underbrace{V_t\big(\boldsymbol{x} - e_{j2}\big) + c_2 + c_s}_{\text{Substitution with new}}, \ \underbrace{V_t(\boldsymbol{x}) + s_2}_{\text{Use new pack}}] & if\ x_{i2} = 0, x_{j2} \neq 0, i \neq j \\ V_t(\boldsymbol{x}) + s_2 & if\ x_{12} \cdot x_{22} = 0 \end{cases}$$

Finally, $T^{(0)}$ is associated with the state transitions due to random arrivals of product returns of the system and there is no decision triggered by this type of event.

$$T^{(0)}V_t(x) = V_t\big(x + \boldsymbol{e}_{ij}\big), \mathrm{i}, \mathrm{j} = 1,2 \tag{2.4}$$

We now present the average cost infinite horizon dynamic programming formulation by the following recursive optimality equation. The uniformization technique is applied to discretize the original continuous-time Markov chain to a discrete-time Markov chain (Lippman, 1979).

$$V_{t+1}(\boldsymbol{x}') = V_t(\boldsymbol{x}) + g$$

$$= \frac{1}{\beta}[h(\boldsymbol{x}) + \sum_{i=1}^{2}\sum_{j=1}^{2} \mu \cdot \alpha_i \cdot p_{ij}\, T^{(0)}V_t(\boldsymbol{x}) + \lambda_1 T^{(1)}V_t(\boldsymbol{x}) + \lambda_2 T^{(2)}V_t(\boldsymbol{x})] \tag{2.5}$$

Here, β is the uniformization factor for the discrete time Markov chain (DTMC). As mentioned earlier, the state transitions occur with rate $\beta = \sum_{i=1}^{2} \lambda_i + \mu$. The holding cost is a linear combination of the cost of inventories of all types and all quality levels per decision epoch:

$$h(\boldsymbol{x}) = \sum_{i=1}^{2} \sum_{j=1}^{2} h_{ij} x_{ij} \tag{2.6}$$

The terms multiplied by μ in (2.5) correspond to state transitions due to the arrivals of the returned products. Finally, the terms multiplied by λ_j correspond to the transition due to action upon the arrivals of a class j demand for remanufactured products.

We use a value iteration algorithm to solve the dynamic programming for the optimal control problem given in (2.5).

2.4 Characterization of the Optimal Policy Structure

In this section, we answer the question of optimal decisions on reassembly to fulfill quality-specific demand for remanufactured products. More specifically, (1) if a demand of lower quality product arrives but the required modules are not available in the inventories, should the firm reassemble using substitutable modules from the high quality module inventory, or fulfill it with new products, and (2) if a demand of high quality product arrives and all required modules are available, should the firm reassemble from inventories or use new product to meet the order hence keeping the inventory for future demand?

2.4.1 Sufficient Conditions for Optimality

In order to characterize the structure of the optimal control policy for reassembly, we introduce the following difference operators for the lower quality and higher quality classes respectively to facilitate the characterization of the optimal policy structure.

DEFINITION 2.1 For any relative cost function V on the state space X, we define

$$\Delta_{low}V(\boldsymbol{x}) \tag{2.7}$$
$$= \begin{cases} [V(\boldsymbol{x}-\boldsymbol{e}_{12}-\boldsymbol{e}_{21})+c_1+c_s]-[V(\boldsymbol{x})+s_1] & if\ x_{11}=0, x_{12}>0, x_{21}>0 \\ [V(\boldsymbol{x}-\boldsymbol{e}_{11}-\boldsymbol{e}_{22})+c_1+c_s]-[V(\boldsymbol{x})+s_1] & if\ x_{21}=0, x_{11}>0, x_{22}>0 \end{cases}$$

The operator $\Delta_{low}V(\cdot)$ represents the additional cost of reassembling by using on-hand inventories relative to the cost of keeping inventories level by using new product. The lower the value of $\Delta_{low}V(\cdot)$, the more the willingness one has to use substitutable modules.

Let Ω be the set of functions defined on $\mathbb{Z}^+$ such that if $V \in \Omega$, then the following conditions hold for the two demand classes, respectively.

(1) Reassembly policy for class 1 demand (low quality)

Suppose the current state is $\boldsymbol{x}|x_{11}=1, x_{12}>0, x_{21}>0, x_{22}>0$, then the following conditions characterize the optimal reassembly and substitution policies for class 1 demand (which is derived from (2.5) by using relative iteration algorithm).

- *Condition A1:* $\Delta_{low}V(\boldsymbol{x}) \geq \Delta_{low}V(\boldsymbol{x}+\boldsymbol{e}_{22})$
- *Condition A2:* $\Delta_{low}V(\boldsymbol{x}) \leq \Delta_{low}V(\boldsymbol{x}+\boldsymbol{e}_{21}+\boldsymbol{e}_{12})$

- *Condition A3:* $\Delta_{low}V(\boldsymbol{x}) \geq \Delta_{low}V(\boldsymbol{x}+\boldsymbol{e}_{21}+\boldsymbol{e}_{12}+\boldsymbol{e}_{22})$

The above conditions imply that the structure of the optimal reassembly policy is a state-dependent threshold policy. Difference operator $\Delta_{low}V(\cdot)$ represents the marginal cost of choosing the "reassemble with substitution" option. The lower $\Delta_{low}V(\cdot)$ is, the more likely for the firm to reassemble with substitution. Condition A1 implies that the marginal cost (willingness) of choosing "reassembly" option decreases when there is an additional unit of non-substitutable module in the inventory, i.e. $\boldsymbol{x}+\boldsymbol{e}_{22}$. Equivalently, if it is optimal to reassemble in state $\boldsymbol{x}$, it remains optimal to reassemble in state $\boldsymbol{x}+\boldsymbol{e}_{22}$. Condition A2 implies that the marginal cost of choosing "reassembly with substitution" option increases when we have both an additional unit of available lower quality module and a substitutable higher quality module in the inventory, i.e., $\boldsymbol{x}+\boldsymbol{e}_{21}+\boldsymbol{e}_{12}$. In brief, conditions A1 and A2 present two opposite effects of having additional unit of inventory, i.e., $\Delta_{low}V(\cdot)$ decreases as x_{22} increases while it increases in the direction of $(\boldsymbol{e}_{12},\boldsymbol{e}_{21})$. Condition A3 reveals the total impact of having additional units of inventory x_{12}, x_{21} and x_{22} on the firm's willingness to reassemble with substitution. Consequently, we derive the following sufficient conditions for optimal decisions.

LEMMA 2.1 If $V \in \Omega$ satisfies conditions A1-A3, then there exist conditions B1-B3 that are equivalent to conditions A1-A3, respectively:

- *Condition B1:* $x_{22} \geq \gamma_{22}(\boldsymbol{x})$
- *Condition B2:* $x_{21}+x_{12} \leq \kappa(\boldsymbol{x})$
- *Condition B3:* $x_{21}+x_{12}+x_{12} \geq \zeta(\boldsymbol{x})$

Proof LEMMA 2.1 *See Appendix A.* □

A similar conclusion holds when a demand occurs in state $\boldsymbol{x}|x_{21} = 0, x_{11} > 0, x_{12} > 0, x_{22} > 0$.

(2) Reassembly policy for class 2 demand (higher quality)

DEFINITION 2.2 For any relative cost function V on the state space $\boldsymbol{X}$, we define

$$\Delta_{high}V(\boldsymbol{x}) = [V(\boldsymbol{x} - \boldsymbol{e}_{12} - \boldsymbol{e}_{21}) + 2c_2] - [V(\boldsymbol{x}) + s_2] \tag{2.8}$$

where $\Delta_{high}V(\cdot)$ is a difference operator representing the cost difference associated with the reassembly decision for a higher quality demand.

The smaller the value of $\Delta_{low}V(\cdot)$, the more the willingness one has to reassemble from on-hand inventory. The optimal reassembly policy for class 2 demand is characterized by the following sufficient condition:

- *Condition A4:* $\Delta_{high}V(\boldsymbol{x}) \leq V(\boldsymbol{x} + \boldsymbol{e}_{12} + \boldsymbol{e}_{22})$

Condition A4 says that the marginal cost of assembly increases when there is an additional unit of x_{12} and an additional unit of x_{22}. Equivalently, the willingness to assemble with substitution decreases in the direction of (x_{12}, x_{21}). Similar to the argument of conditions B1-B3, we obtain B4 shown in the following lemma:

LEMMA 2.2 If $V \in \Omega$ satisfies Condition A4, then the following Condition B4 that are equivalent to Condition A4.

- *Condition B4:* $x_{12} + x_{22} \leq \eta(x_{11}, x_{21})$

Proof LEMMA2.2 *See Appendix A.* □

There exists a function of low quality inventory levels, $\eta(\boldsymbol{x})$, such that when the sum of inventories $x_{12},$ and x_{22} exceeds $\eta(\boldsymbol{x})$, it is no longer optimal to reassemble from inventories to satisfy the demand of higher class. This result provides important managerial insights that when the inventory of high quality modules are relatively low, fulfilling the demand of higher class is prior to the fulfillment of a lower class demand. However, if there are relatively sufficient inventories of higher quality modules, then it is more beneficial to use them as substitutable units for lower quality demand.

Now we can present Lemma 2.3 which shows that conditions B1-B4 are preserved under operator T.

LEMMA 2.3 *If* $V \in \Omega$, *then* $T^{(1)}V \in \Omega$ *and* $T^{(2)}V \in \Omega$.

Proof LEMMA2.3 *See Appendix A.* □

Now we present the main result of the optimal policy. The policies defined in the following theorem reflect the structure of the optimal reassembly-inventory policy for our model.

THEOREM 2.1 *The optimal reassembly policy for an order from demand class j, consists of a state-dependent threshold policy with parameters* $(\gamma, \kappa, \zeta, \eta)$, *such that (1) if Conditions B1-B3 are all satisfied, it is optimal to fulfill an incoming class* 1 *demand by reassembling with substitution. Otherwise, use new modules; (2) if Condition B4 is satisfied, it is optimal to fulfill a class* 2 *demand by reassembly. Otherwise, use new ones to fill the order. In addition, the optimal policy has the following properties:*

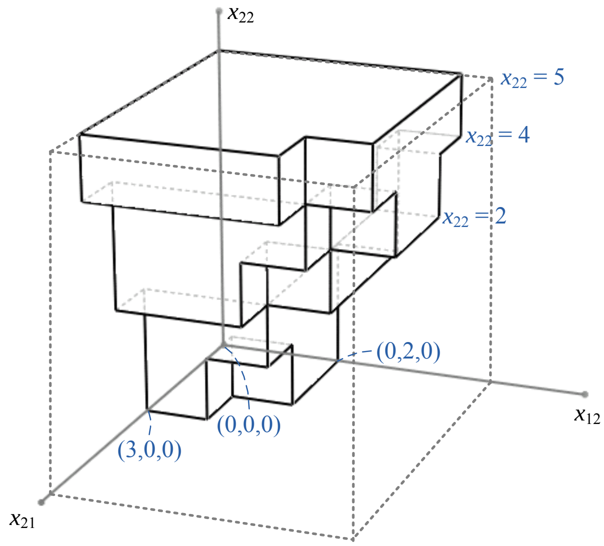
(1) *The thresholds (inventory rationing levels and rejection levels) are monotone.*

(2) *It is always optimal to satisfy class* 1 *(low) demand if all modules of class* 1 *have positive on-hand inventory.*

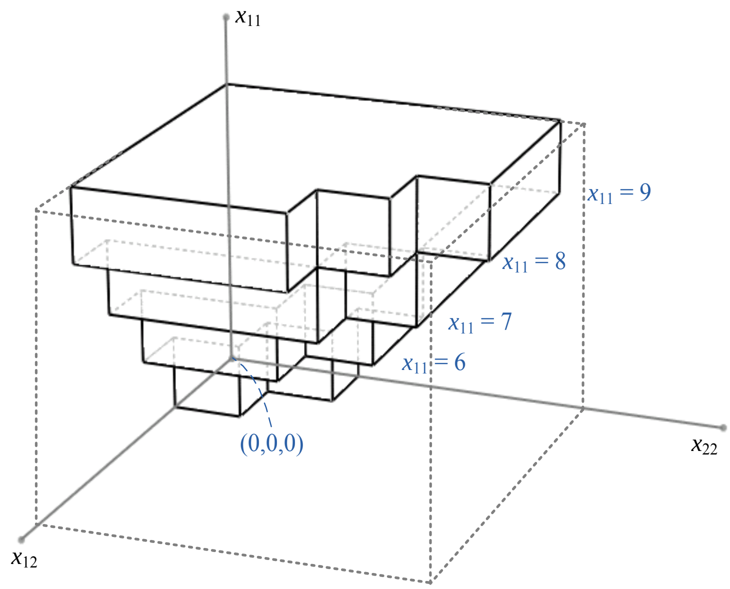
Proof Theorem 2.1 *See Appendix A.* . □

Theorem 2.1 states that the assembly decision of each demand is governed by a state-dependent threshold policy. The theorem states that the reassembly with substitution is essentially an inventory allocation problem where state-dependent thresholds determine whether a demand from a particular class should be fulfilled by assembling from on-hand inventories (with substitution or not) or new ones. In particular, substitution is not optimal if all module inventories of the lowest quality grade ($j = 1$) are positive.

For a system with two modules - two quality levels and zero x_{11}, the structure of the optimal reassembly policy derived from (2.5) is graphically illustrated as 3D plots in Figure 2.2. The 3D plot shows the subspace containing all the possible states with $x_{11} = 0$. In Figure 2.2 (a), several critical switching surfaces divide the whole state space into two regions, i.e., the highlighted part of the space as the region of "reassemble with substitution" and the remaining region as "do not reassemble". For the high quality demand, if we fix x_{21} at a certain value, the optimal policy determined by the all other three inventories is also visualized as Figure 2.2 (b). The highlighted region stands for "optimal to reassemble" and the remaining region stands for "optimal not to reassemble".



(a)



(b)

Figure 2.2 The structure of the optimal reassembly policies: (a) low quality class demand; (b) high quality class demand.

$$\mu = 2, \lambda_1 = \lambda_2 = 0.6, \alpha_1 = \alpha_2 = 0.5, p_{11} = p_{12} = 0.5, p_{21} = p_{22} = 0.5,$$
$$c_1 = 2, c_2 = 3, c_s = 5, s_1 = 10, s_2 = 8, h_{11} = h_{21} = 4, h_{12} = h_{22} = 6$$

In order to see the structure of the optimal policy more clearly, we use a plane to cut the state space along a direction orthogonal to x_{21} at different x_{21} values, e.g. $x_{21} = 3$ 4, 5 and 6. The resulting cross section of each cut is shown in Figure 2.3. For each cut, there is a "step-shaped" switching curve dividing the state space into two regions. This stresses the fact that the thresholds described in Theorem 2.1 are state-dependent and sensitive to the changes in the system state, e.g., an increase in x_{12} from 3 to 4 leads to an increase in γ_{22} from 2 to 4.

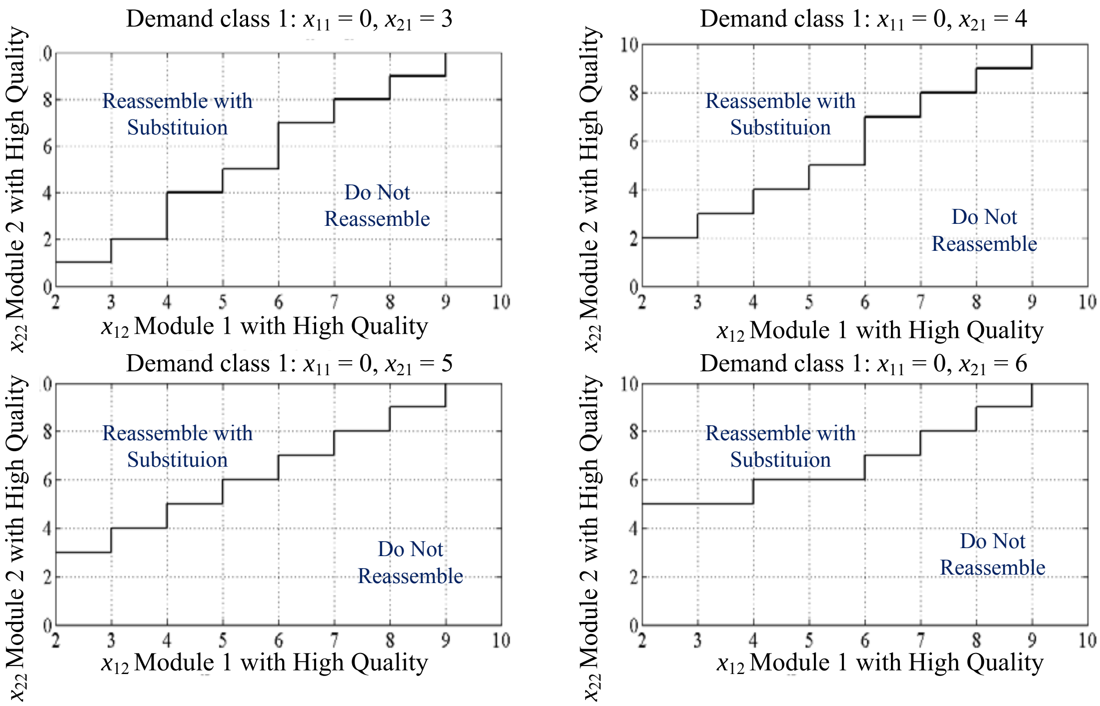


Figure 2.3 Projection of optimal policy structure on the (x_{12}, x_{21}) plane: reassembly decisions for the class 1 (lower quality) demand.

$$\mu = 2, \lambda_1 = \lambda_2 = 0.6, \alpha_1 = \alpha_2 = 0.5, p_{11} = p_{12} = 0.5, p_{21} = p_{22} = 0.5$$

2.4.3 Monotonicity of the Optimal Policy

As we observed the step shape of the optimal policy in optimal policy structure, we need to show the monotonicity of the policy parameters $(\gamma_{ij}, \kappa, \zeta, \eta)$ under the optimal policy claimed in Theorem 2.1.

DEFINITION 2.3 Let $V \in \Omega$ and define the region-switching levels $\gamma_{ij}(\boldsymbol{x}), \kappa(\boldsymbol{x})$, and $\zeta(\boldsymbol{x})$ as follows.

$$\gamma_{ij}(\boldsymbol{x}) = min\{x_{22} | \Delta_{low}V(\boldsymbol{x}) \leq 0\} \quad (2.9)$$

$$\kappa(\boldsymbol{x}) = min\{x_{21} + x_{12} | \Delta_{low}V(\boldsymbol{x}) \geq 0\} \quad (2.10)$$

$$\zeta(\boldsymbol{x}) = min\{x_{21} + x_{12} + x_{22} | \Delta_{low}V(\boldsymbol{x}) \leq 0\} \quad (2.11)$$

Then the above definitions imply:

$$\begin{cases} \Delta_{low}V(\boldsymbol{x}) \leq 0 & if\ x_{22} \geq \gamma_{22}(\boldsymbol{x}) \\ \Delta_{low}V(\boldsymbol{x}) > 0 & otherwise \end{cases} \quad (2.12)$$

$$\begin{cases} \Delta_{low}V(\boldsymbol{x}) \leq 0 & if\ x_{21} + x_{12} \leq \kappa(\boldsymbol{x}) \\ \Delta_{low}V(\boldsymbol{x}) > 0 & otherwise \end{cases} \quad (2.13)$$

$$\text{and} \begin{cases} \Delta_{low}V(\boldsymbol{x}) \leq 0 & if\ x_{21} + x_{12} + x_{22} \geq \zeta(\boldsymbol{x}) \\ \Delta_{low}V(\boldsymbol{x}) > 0 & otherwise \end{cases} \quad (2.14)$$

Combining conditions A1, A2 and A3 with the above definitions leads to the following lemma.

LEMMA 2.4 Let $V \in \Omega$, if $x_{11} = 0$ then

(L2.4.1) $\gamma_{22}(\boldsymbol{x}) \leq \gamma_{22}(\boldsymbol{x} + \boldsymbol{e}_{21} + \boldsymbol{e}_{12})$

(L2.4.2) $\kappa(\boldsymbol{x}) \leq \kappa(\boldsymbol{x} + \boldsymbol{e}_{22})$

Lemma 2.4 states the monotonic properties of the switching curve of the optimal policy. In particular, L2.4.1 states that for demand class 1, the optimal threshold $\gamma_{i2}(\boldsymbol{x})$ for a module i with high quality (non-substitutable module) is non-decreasing in the inventory level of other modules (see Figure 2.3). L2.4.2 indicates that increasing the non-substitutable module inventory increase the likelihood of choosing the "reassemble with substitution" option. We also use Figure 2.3 to explain two structural properties. When there are more inventories x_{21} and x_{12}, we need more inventory x_{22} to enter the reassembly with substitution region. On the other hand, when there are more x_{21} inventories, we have more chance to enter the reassembly with substitution region (the region expands as x_{21} increases). Similarly, we define the following threshold for the higher quality module inventories.

DEFINITION 2.4 Let $V \in \Omega$ and define the rationing levels and rejection levels as following:

$$\eta(\boldsymbol{x}) = min\{x_{i2} + x_{j2} | \Delta_{high}V(\boldsymbol{x}) \geq 0, i \neq j\} \tag{2.15}$$

LEMMA 2.5 Let $V \in \Omega$, then $\eta(\boldsymbol{x}) \leq \eta(\boldsymbol{x} + \boldsymbol{e}_{11} + \boldsymbol{e}_{21})$

Lemma 2.5 states that for high quality demand, the region for "reassemble" is bounded by a state-dependent value $\eta(\boldsymbol{x})$ which is non-decreasing in the direction of $(\boldsymbol{e}_{11}, \boldsymbol{e}_{21})$, which implies that with an additional pair of low quality modules in the inventory, the "reassembly" region would expand.

2.5 Numerical Study

2.5.1 Value of Inventory Rationing

In this section, we present our numerical experiments and results. We use the same basic parameters as the example in Section 2.4. In order to investigate the benefits of state-dependent reassembly policy, we compare the cost benefits of this policy with that of an exhaustive policy in which reassembly with substitution is allowed as long as modules are available upon demand arrivals. The exhaustive policy attempts to utilize the inventory in a more "greedy" manner instead of having any control on the balance of the penalty costs and holding costs.

Let V^* and $\hat{V}^*$ be the optimal average costs using the state-dependent policy and the exhaustive reassembly policy, respectively. We define the relative cost reduction, percentage gap (PG), as follows:

$$PG = \frac{(\hat{V}^* - V^*)}{\hat{V}^*} / \times 100\% \tag{2.16}$$

We use the PG value to evaluate the effectiveness of the optimal reassembly policy under different operating conditions of the system. The larger the PG value is, the more beneficial it is to employ a st ate-dependent optimal reassembly policy. We investigate the following dimensionless parameters: (1) holding cost ratio: h_1/h_2. and (2) penalty cost ratio s_1/s_2. These ratios reflect the relative value of two quality classes of modules.

Noting that PG is a f unction of these dimensionless parameters, we want to investigate the cross-sectional relationship of PG with each of them. The numerical experiment is composed of several representative cases to test the performance of the state-dependent threshold policy in a wide range of practice. In this numerical experiment, the total demand rate is assumed to be 1.2 units a week ($\lambda_1 = \lambda_2 = 0.6$). The performance comparisons are made under three different return (supply) rates $\mu_1 = \mu_2$ =0.9, 1.0 and 1.1. The results are plotted in Figure 2.4 (a) to (d). Figure 2.4 (a) demonstrate that the optimal policy results in over 10% performance improvement in all three cases. The PG value is not sensitive to the unit holding cost ratio. Figure 2.4 (b) shows that the more the return rate is similar to the demand rate (i.e., $\lambda_1 + \lambda_2 \approx \mu_1 + \mu_2$), the smaller the optimal average cost is. This is not surprising, because there is better balance between demand and supply and hence reduced shortage penalty. In addition, the average cost increases as the holding cost ratio h_1/h_2 increases.

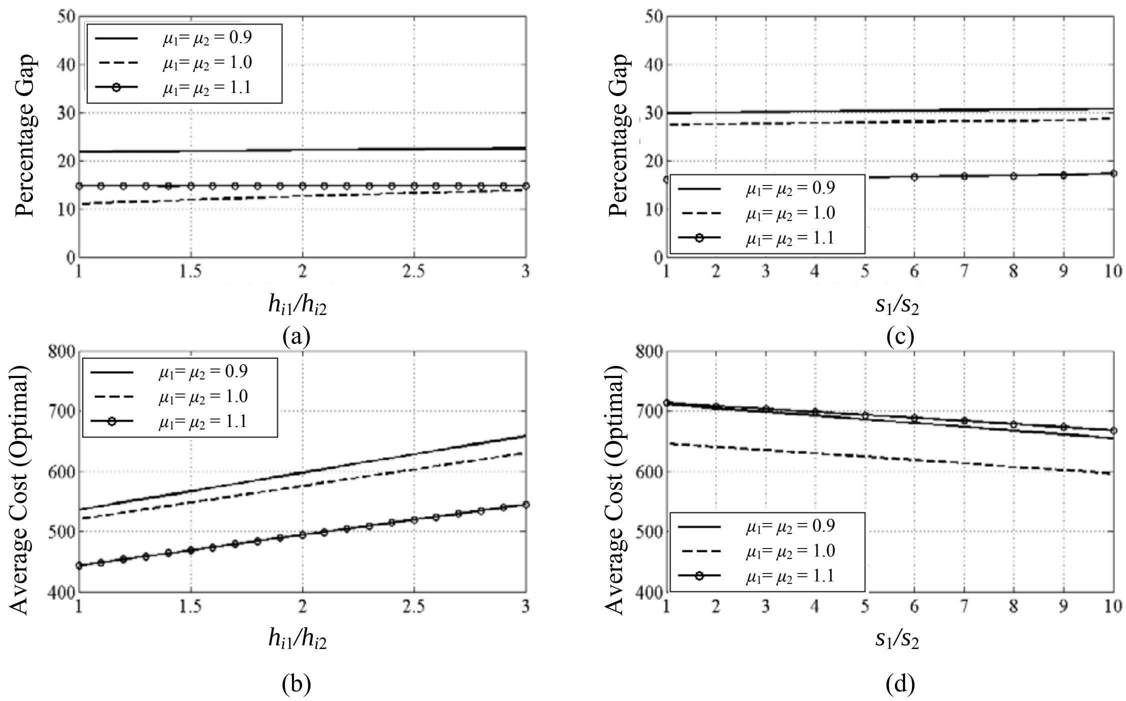


Figure.2.4 Effects of Parameters: (a) and (b): effects of holding cost variation; (c) and (d): effects of penalty cost variation.

Figure 2.4 (c) and (d) demonstrate how the penalty cost ratio s_1/s_2 affects the average cost and show the performance comparison between the optimal policy and the exhaustive policy. When s_1/s_2 equals to one, the penalty costs by utilizing new modules/product for lower class of demand and higher class of demand are the same. In practice, we always have s_1/s_2 >1 because modules with different level of quality condition are usually distinguished by their salvage value. We note that the PG value monotonically increases as t he ratio value s_1/s_2 increases. This behavior can be explained as follows: a larger ratio implies that the penalty of using a new product to fulfill a lower quality order is much higher than that of a high quality order, thus the module substitution is more desirable in order to reduce the chances of using new modules and associated penalty costs. Indeed, the higher penalty cost ratio, the more beneficial it is to implement the optimal policy with substitution option (see Figure 2.4(d)).

2.5.2 Sensitivity Analysis

In this section, we investigate the influence of the system parameters on the system behavior under optimal control. We present results of varying some particular parameter while keeping all other parameters unchanged.

From Table 2.1, when the demand rates for both classes decrease (λ_1, λ_2) simultaneously, the number of new products used to fulfill the orders decreases while the number of substitution does not necessary decrease. The reuse level (WCR) increases. Therefore, the system performance is improved indicating by the reduced average cost. Table 2.2 demonstrates that when the imbalance of the two types of module returns gets larger (i.e., difference between α_1 and α_2), the usage of new products increases significantly due to less opportunities for matching and substitution. Accordingly, the reuse rate WCR decreases and the average inventory level increases. Table 2.3 shows that when the proportion of higher quality module returns ($p_{i2}, i = 1,2$) increases, there are fewer substitution but more use of new products. This observation implies that it is more desirable to take the substitution option when return rates of low quality and high quality module are closer.

Table 2.1 Results for Various Demand Rates ($\mu = 2$).

$\alpha_1\mu$	$\alpha_2\mu$	λ_1	λ_2	p_{i1}	p_{i2}	h_{i1}	h_{i2}	New Products	Substitution	Optimal Avg. Cost	Inventory	Reuse Level(%)
1.0	1.0	1.0	1.0	0.5	0.5	0.2	0.4	568	95	177.11	51	47.21
1.0	1.0	0.9	0.9	0.5	0.5	0.2	0.4	518	99	158.80	36	52.71
1.0	1.0	0.8	0.8	0.5	0.5	0.2	0.4	482	121	140.87	16	54.79
1.0	1.0	0.7	0.7	0.5	0.5	0.2	0.4	469	92	122.99	12	57.08
1.0	1.0	0.6	0.6	0.5	0.5	0.2	0.4	402	76	105.63	29	63.33

Table 2.2 Results for Various Return Rates($\lambda = 2$).

$\alpha_1\mu$	$\alpha_2\mu$	λ_1	λ_2	p_{i1}	p_{i2}	h_{i1}	h_{i2}	New Products	Substitution	Optimal Avg. Cost	Inventory	Reuse Level(%)
1.0	1.0	1.0	1.0	0.5	0.5	0.2	0.4	568	95	177.11	51	48.71
0.9	1.1	1.0	1.0	0.5	0.5	0.2	0.4	589	70	166.81	151	48.00
0.8	1.2	1.0	1.0	0.5	0.5	0.2	0.4	678	64	157.10	220	40.83
0.7	1.3	1.0	1.0	0.5	0.5	0.2	0.4	736	56	148.17	299	36.33
0.6	1.4	1.0	1.0	0.5	0.5	0.2	0.4	798	54	146.24	360	31.25

Table 2.3 Results for Various Probabilistic Quality Parameters($\lambda = 2, \mu = 2$).

$\alpha_1\mu$	$\alpha_2\mu$	λ_1	λ_2	p_{i1}	p_{i2}	h_{i1}	h_{i2}	New Products	Substitution	Optimal Avg. Cost	Inventory	Reuse Level(%)
1.0	1.0	1.0	1.0	0.6	0.4	0.2	0.4	533	105	185.55	36	51.21
1.0	1.0	1.0	1.0	0.5	0.5	0.2	0.4	568	95	177.11	51	48.71
1.0	1.0	1.0	1.0	0.4	0.6	0.2	0.4	613	61	168.68	177	46.38
1.0	1.0	1.0	1.0	0.3	0.7	0.2	0.4	667	58	149.50	441	42.00
1.0	1.0	1.0	1.0	0.2	0.8	0.2	0.4	844	47	132.54	630	27.72

2.5.3 Heuristic Policies

In this section, we propose two simple heuristic policies which are constructed based on the structure of the optimal policy in Section 2.4.

Heuristic Policy I: The first heuristic consists of controlling the assembling from module inventories independently of each other via a vector $\boldsymbol{r} = (r_{11}, r_{12}, \ldots, r_{MN})$ of fixed threshold levels. We refer to this policy as the independent threshold policy. The advantage of this heuristic policy is that it is much simpler to implement than the optimal policy because it does not require the storage of the state-dependent threshold levels. This is important when the number of modules or components is large and it becomes difficult to compute the optimal policy because the size of the state space grows exponentially. However, this policy is clearly suboptimal and lacks the coordination in managing the production of different components carried out under the optimal policy.

Heuristic Policy II: The second heuristic is similar to the first heuristic policy, except that additional coordination parameters are introduced as a vector $\boldsymbol{z} = (z_{11}, z_{12}, \ldots, z_{MN})$ such that the difference between the inventory level of non-substitutable module and the substitutable module is at least $\boldsymbol{z}$. We refer to this policy as the coordinated threshold policy. The policy is motivated by the fact that decisions on assembly from various modules are state-dependent and mutually coordinated under the optimal policy. This policy attempts to mimic the structural properties of the optimal policy by c onsidering the interdependency between different module inventories (See, Figure 2.5).

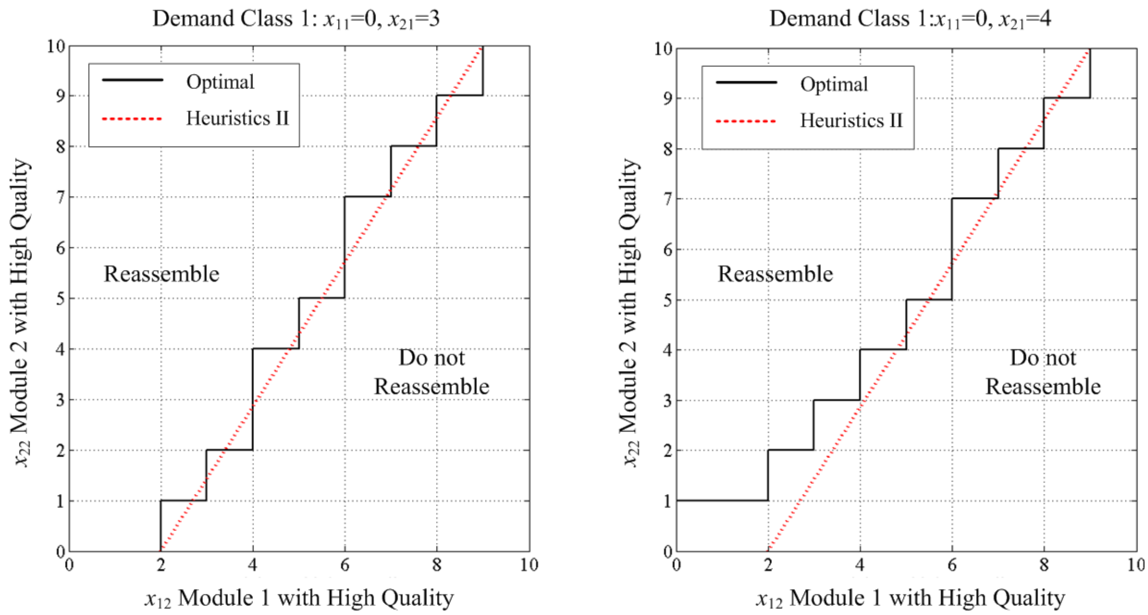


Figure 2.5 The structure of the heuristic policy II.

2.6 Conclusions

In this chapter, we studied a reassemble-to-order system in a remanufacturing setting, which receives random product returns with variable quality, reassembles modules to remanufactured products for multiple classes of demand. Quality variation of returned modules adds complexity to the reassembly decisions due to the imbalance of module inventories. We formulate the problem as a Markov decision process and characterized the structure of the optimal control policy. It is shown that module substitution provides the remanufacturing system with flexibility such that shortage in lower quality module can be smoothed by higher quality module inventories. In particular, we prove that the optimal reassembly policy is a state-dependent threshold policy, where module substitution and reassembly decisions are made based on inventory level of other modules. Monotonicity properties for the threshold polices were also provided. Using numerical experiments, we investigated the effects of holding costs, penalty costs on the optimal average cost rate and showed the performance percentage difference between the optimal policy and the exhaustive reassembly policy. Two heuristic policies are proposed for the problem with large number of modules and quality grades. Finally, both analytical results and numerical examples are presented to illustrate some interesting and unexpected behaviors of the system and provide useful insights for managers of this type of systems.

These insights include:

- Substitution may not be the optimal decision when you have more substitutable modules in stock; the optimal decision is jointly determined by both substitutable and non-substitutable module inventories.
- Optimal decisions for reassembly are very different from currently implemented practices. Basic but simple policies such as FCFS based exhaustive reassembly policy may cause the system to be less cost-effective.
- Using high quality modules for the high quality class product assembly may not be optimal, it is sometimes more beneficial to save them for future module substitution and avoid high penalty from using new ones for low class demands.
- Large variation in module quality leads to significant imbalance during reassembly, hence reducing the total utilization of returned modules.

CHAPTER 3

PERFORMANCE ANALYSIS OF A REMANUFACTURING SYSTEM WITH RETURN ADMISSION

3.1 Introduction

Among various approaches to sustainable manufacturing, remanufacturing serves as a solution to reduce both the consumption of natural resources and the environmental burden by recovering materials and reducing landfill. Remanufacturing recovers the residual value added to cores through the processes of disassembly, cleaning, refurbishing, reassembly and testing products at a cost which is typically 40-50% of the original new product price (Atasu et al., 2008). Remanufacturing is now common in the machinery and other capital goods industries.

The work reported here is motivated by warranty return issues faced by an electric vehicle (EV) battery remanufacturing facility. Currently, EV manufacturers must fulfill warranty obligations to service the electric vehicles and return functional vehicles back to their owners. Remanufacturing of returned batteries from customers for warranty service will save cost by recapturing the residual value of the original products (Kalhammer et al., 2007). In remanufacturing, used batteries are taken back from the customer and disassembled into modules, then the battery modules are sorted and assigned different quality grades, and sent to the module inventories for remanufacturing. On the customer side, warranty claims and requests for replacement need to be immediately satisfied by the vehicle manufacturer. Remanufactured modules are reassembled to customer orders for specific warranty replacement in a speedy and reliable manner. Zhou et al. (2011) presented optimal manufacturing/remanufacturing policies for systems in which returns can be actively controlled by the firm through acquisition effort. However, their model considered inventories for serviceable products and cores but did not look at the multi-item reassemble-to-order operational strategies. Evidently, new models and methods are needed for consider both multi-item assemble-to-order operations and admission control on product returns, so as to help industries to better balance supply and demand for warranty replacement.

In a typical remanufacturing system, there is quality variation in both product returns and remanufactured products demand. Demand for warranty replacement must be fulfilled by matching the quality of the return with a similar quality in the replacement. Since each customer order requires the simultaneous availability of several modules, the issues remanufacturing and inventory managers are faced with are the following: (1) What is the probability a warranty claim can be satisfied immediately? (2) What is the probability that the order is fulfilled by used modules (reuse level)? (3) How long on average does a returned module wait in the inventory before being reused (old module waiting time)?

In this chapter, we address the problem of evaluating and analyzing a reassemble-to-order system where warranty claims are the major reason for returns. In this situation, remanufacturers must adapt quickly to uncertainties and changes in after-sale markets and move toward "assemble-to-order" inventory systems as opposed to the traditional "assemble-to-stock" systems. Order fulfillment and reuse level of returned products become important metrics for such manufacturers.

In the remainder of this chapter, we briefly review the literature in Section 3.2. In Section 3.3, we describe the model and introduce the performance measures to be analyzed. Section 3.4 contains performance model in detail for the single-quality class system and a solution method by geometric-matrix analysis. In Section 3.5, we extend the model to the multi-quality class system by developing a donor-beneficiary queueing model, and derive analytical results of a wide range of system performance measures. In Section 3.6, numerical studies are conducted to illustrate the results presented in Sections 3.4 and 3.5. Interesting behavior of the system and managerial insights for effective assembly planning and admission control are also provided. Finally, conclusions are provided in Section 3.7.

3.2 Literature Review

Literature on assemble-to-order (ATO) strategy for traditional manufacturing systems can be broadly classified into two categories: systems with periodic review and systems with continuous review; see Song and Zipkin (2003) for a comprehensive literature review. Assemble-to-order strategy emerges in conventional manufacturing systems where commonality of components or modules exists among customer orders and assembly times are relatively small (Gerchak and Henig, 1989). This work focuses on *re*assemble-to-order systems within the *continuous review* category.

Standard models for ATO systems with multiple demand classes usually assume that demands are classified by different combination/assembly of a specific kit of modules and components, for example, mixed model assemble-to-order systems (Song et al. 1999, Iravani et al. 2003, Gunasekaran and Ngai 2005). Our work is different in the sense that, customer classes are defined using product quality grades such that customers of a specific class request warranty replacements satisfying a specific quality standard. Generally, companies would choose remanufactured products to match the quality requirement of the warranty replacement. For example, if a warranty claim occurs at the time point of 5 remaining warranty years, the company would fill the warranty with a product which must last for at least another 5 years rather than replace with a brand new one.

When the quality uncertainty and variation is considered, our system is similar to an ATO system with multi-item inventory and stochastic supply. There have been several research efforts in studying multi-item inventory systems with stochastic supply. (See, e.g., Forsberg 1995, Huausman et al. 1998, Zhao and Simchi-Levi 2006, Zhou and Yu 2011) These studies focus on trade-off between inventory level, the order service level and fill rate. Backorders or lost sales are allowed and backorder levels are investigated as a system performance factor. These models also assume systems with i.i.d. lead times and Poisson demand. The common performance measures in their studies include: fill rate, expected inventory level and expected waiting times of each order type.

Our model adapts to the remanufacturing systems with additional features and challenges in deriving exact expressions for system performance measures. First, we incorporate *dependency across modules* and *allow module substitution* during assembly. The shortage of lower quality modules could be met by the higher quality modules. This effect is called risk pooling and allows the manufacturer more flexibility to avoid understock (Hillier 2000). However, using the higher quality items is more expensive. Second, our analysis also contributes to the knowledge base of performance measures that are unique and important for remanufacturing systems: (1) waiting times for a returned item in the inventory before being reused; and (2) reuse level, which reflects the utilization of returned items in the system. We develop models and solution methods to address such important issues in reassemble-to-order systems for OEM with remanufacturing operations.

The purpose of this chapter is to conduct an exact analysis on a wide range of performance measures in the reassemble-to-order system with stochastic return and demand processes and the effects of a return admission threshold on these performance measures. In particular, we model the warranty returns process as a multivariate Poisson process. That is, the overall order arrives according to a Poisson process, but there is a probability that a demand falls in a certain quality grade and requests different modules of that particular quality condition. Each module's inventory has a separate admission control policy, which is a threshold policy saying that returns are accepted only when inventory level is below the threshold. Orders from warranty claims are filled on FCFS basis. Demands for a module that is not available in the inventory are fulfilled by a newer module which costs much higher than a used one. For any given admission threshold policy, we present a procedure to evaluate the module-level and order-level performance measures, such as order fill rate, reuse level, inventory level, inventory holding times (waiting times), etc. Conducting performance measures analysis can enhance our understanding of the system's inner working, hence providing insights for effective remanufacturing system design and control.

3.3 Model Description

This section describes the model assumptions and introduces the basic notation. We consider a multi-item reassemble-to-order system (RATO) within a remanufacturing environment. Throughout this chapter, we refer modules as the atomic level for assembly. Inventories stocked with M non-interchangeable modules. Module substitution described in Chapter 2 is also considered in this model. Let $\Lambda = \{1,2,\dots,M\}$ be the set of all module indices. Each order for remanufactured product requires one of each module type to be assembled. After sorting, each module is assigned to a quality grade q, $q = 1,2,\dots,N$. We consider an infinite planning horizon and assume that the times to reassemble the modules into the end products are negligible compared to the transportation time for the product returns. Figure 3.1 illustrates the RATO system.

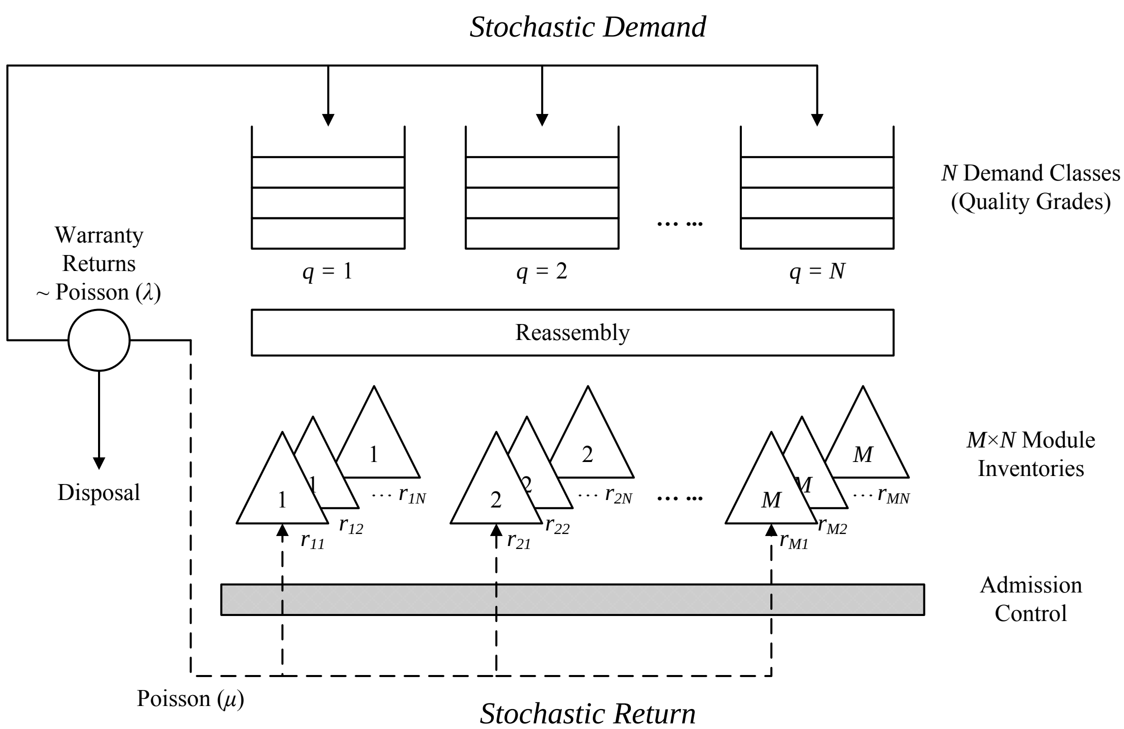


Figure 3.1 Reassemble-to-order system with admission control.

3.3.1 The Demand Process

Consider a continuous review inventory system where M modules are assembled into a single product. Warranty claims (demand for warranty repair or replacement) arrive following a Poisson process $D(t), t \geq 0$ with rate λ. The overall demand process is stationary in time and forms a Poisson process. Each order requests one unit of each module simultaneously. Since no backorder or lost sale is allowed, if module i is missing, a new module will be used as a substitution during reassembly, hence a stock out penalty cost (cost of using new module) s_i will incur.

3.3.2 Cores Return and the Arrival Process

We assume that the warranty claims occur if one of the M modules of a product is defective or malfunctioning while remaining ones are still remanufacturable. The probability that the warranty return is caused by the failure or defect of module i is denoted by α_i. Non-remanufacturable modules are disposed or recycled.

Demands for customer warranty replacement are fulfilled on a first-come first-serve (FCFS) basis. Upon a w arranty claim from customers, if there are positive inventories for all modules, then fulfill the demand by immediate assembly. Let G_i be the cumulative probability function of inter-arrival time of module i return and let T_i denote the generic random variable with distribution G_i and mean $E[T_i] = t_i$. In particular, we assume that G_i for module i; $i = 1,2, \ldots, M$ are i.i.d. *exponentially* distributed (see Figure 3.2). We need to emphasize that the assumption of Markovian arrival process on the returns does affect the module-level performance analysis in this study because all the state variables depend on the return process T_i only through its mean $E[T_i] = t_i$. However, the assumption is important in conducting the exact analysis for deriving joint distribution and order-level performance measures.

The return rate for module i is proportional to the demand rate since each return is triggered simultaneously by a customer warranty claim. The return process for module i is a P oisson process with aggregate rate $\mu_{\mathrm{i}} = \sum_{j=i}^{\Lambda} \alpha_j \lambda$, $i = 1,2, \ldots, M$ (random split property of Poisson process). Thus the module returns--serving as the supply system--can be viewed as M parallel stochastic production facilities, where processing times at facility i are i.i.d exponentially distributed random variables T_{i} with rate μ_{i}, $i = 1,2, \ldots, M$.

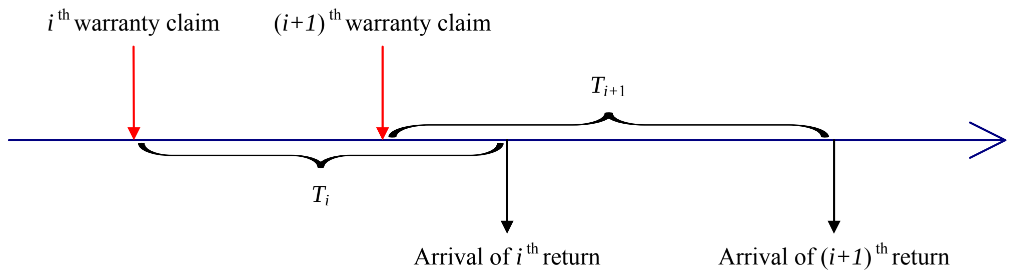


Figure 3.2 Delay of product returns shipment and arrival.

3.3.3 Admission Control Policy for Product Returns

We want to control the inventory of each module by an admission policy with admission level r_i which is a threshold for module i inventory (see Figure 3.1). The admission policy allows a module return from a warranty claim whenever the module i inventory position is below r_i. Mathematically, the admission policy is as follows:

- If inventory position of module $i < r_i$, then admit the return to the remanufacturing system,
- If inventory position of module $i \geq r_i$, then reject the return to the remanufacturing system.

We define Y_i to be the outstanding orders for module i, that is, the number of warranty claims that have been placed but not arrived at the remanufacturing system. The admission threshold r_i, can be viewed as the maximum inventory position we allow for module i.

Since the final remanufactured product consists of a unit of each type of module, each customer order triggers the consumption of a unit of each module. Meanwhile, admission policy allows a shipment of remanufacturable modules and replenishment at the module inventories. In addition, we assume that the capability for shipment of products collected from customer warranty returns is bounded by a finite capacity θ_i for module i. Therefore, the outstanding orders for each module i at time t, $Y_i(t)$, is equal to the number of jobs in service in an $M/M/1/r_i{+}\theta_i$ queue, for $i = 1,2, \dots, M$ (see Figure 3.3). Queue i contains at most $r_i + \theta_i$ outstanding orders at any time. The M queues are not independent because they are driven by a common Poisson arrival process $\{D(t)\}$.

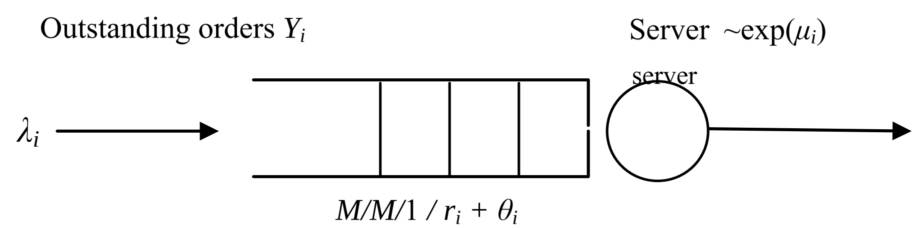


Figure 3.3 Queueing representation of a RATO system with admission control.

3.3.4 Performance Measures

For the single quality grade problem, we are interested in two levels of performance measures: *module-level* and *order-level* performance measures. For the multiple quality grades problem, we further consider the *system-level* performance measures when there are interactions between different quality classes.

1) Module-Level Performance Measures

Module-level performance measures reflect the availability of a module when requested. We use the following measures usually used in standard inventory models. For the single quality module i, $i \in \Lambda$, we define:

- $I_i(t) = [r_i - Y_i(t)]^+$ as on-hand inventory of module i at time t, $0 \le I_i \le r_i$;
- $U_i(t) = [Y_i(t) - r_i]^+$ as stock out levels of module i at time t;

- $Y_i(t)$ as inventory on order of module i at time t, $0 \le Y_i(t) \le r_i+\theta_i$

Expectations of the above measures are: $E[I_i(t)], E[U_i(t)]$ and $E[Y_i(t)]$. We also define the fill rate of module i as $F_i = P(I_i > 0) = P(Y_i < r_i), i = 1,2, \dots, M$.

The fill rate F_i is the probability of immediately fulfilling a warranty claim by using one unit of module i during reassembly. It can be computed through the stationary probability that the number of outstanding order is less than the admission threshold of module i inventory r_i.

2) Order-Level Performance Measures (Single Quality)

In this section, we focus on some order-level performance measures for the single quality system. A demand for a remanufactured product is fulfilled by assembling using either all used but remanufacturable modules, or partial used modules (at least one). Since the on-hand inventory level of module i is given by $r_i - Y_i$, the order-level performance measures are determined by the joint distribution of $(Y_1, Y_2, \dots, Y_M)$. We will develop an approach to obtain the joint distribution of $(Y_1, Y_2, \dots, Y_M)$ and then derive order-level performance measures using this distribution in Section 3.3.

One of the standard order-level performance measures is the proportion of old modules reutilized in order fulfillment. This measure can be divided to two more detailed measures: one is warranty partially covered by remanufactured units (PCR), the other is warranty totally covered by remanufactured units (TCR). PCR is the percentage of class-j orders that are assembled by partial used modules and partial new modules. TCR is the percentage of class-j orders that are assembled totally by used modules.

An overall performance of the reutilization rate is a linear combination of these two, referred as warranty covered by remanufactured units (WCR), e.g., $WCR = TCR + 0.5 \times PCR$ for a two-module product example. As we assume all warranty claims are satisfied and there are no lost sales, then the percentage of order fulfilled by a totally new product is $(1 - TCR - 2 \times PCR)$. The order-level performance measures represents the reuse level of returned products.

3) System-Level Performance Measures (Multiple Quality Grades)

If we consider the system with multiple quality classes, then we will need system-level performance measures. System performance combines the order-based measures of different quality classes of orders into one measure which reflects the overall performance of the remanufacturing system. I n particular, we study the system performance from the remanufacturer's perspective. If we specify each class order-level performance with subscript j, $j = 1,2, \dots, N$, then the aggregate system performance measures are: $PCR = \sum_j^N PCR_j, TCR = \sum_j^N TCR_j,\ and\ WCR = \sum_j^N WCR_j$. As is clear, the system-level performance measures are the summation of order-level performance measures of all quality classes.

3.4 Analysis for Single-Quality Remanufacturing

3.4.1 Module-Level Performance Analysis

Let $N(\alpha)$ denote a P oisson random variable with mean α, and define the following notation (Zipkin, 2000) :

$$p(n|\alpha) \coloneqq P[N(\alpha) = n] = \frac{\alpha^n}{n!} e^{-\alpha} \tag{3.1}$$

$$P(n|\alpha) \coloneqq P[N(\alpha) \leq n] = \sum\nolimits_{k=0}^{n} p(k|\alpha) \tag{3.2}$$

$$\bar{P}(n|\alpha) \coloneqq P[N(\alpha) > n] = 1 - P(n|\alpha) \tag{3.3}$$

If λ is the total warranty claim rate (demand rate), then the Poisson arrival rate of module i at the system is $\mu_i = \sum_{i \in \Lambda} \alpha_i \lambda$. Let λ_i represent the demand rate of module i, we can express the marginal distribution of Y_i as a Poisson distribution with mean $E[Y_i] = \lambda_i E[t_i] = \lambda_i / \mu_i$, which depends on the return transportation time distribution Poisson(μ_i) only through its mean. Since $Y_i \triangleq N(\lambda_i t_i)$, we can write the module-level performance measures as follows.

- The loss function of Y_i (expected stock-out level), $E[U_i]$:

$$E[U_i] = E[(Y_i - r_i)^+] = \sum\nolimits_{y_i \geq r_i} (Y_i - r_i) p(n|\lambda_i) = \sum\nolimits_{y_i \geq r_i} \bar{P}(n|\lambda_i) \tag{3.4}$$

  For non-negative Y_i, $\bar{P}(0|\lambda_i) = E[Y_i]$. Hence, we have

$$E[U_i] = \bar{P}(0|\lambda_i) - \sum\nolimits_{0 \leq n \leq r_{i-1}} \bar{P}(n|\lambda_i) = E[Y_i] - \sum_{n=0}^{r_i - 1} \bar{P}(n|\lambda_i) \tag{3.5}$$

  The loss function is non-negative, non-increasing, and convex in r_i.

- The expected on-hand inventory level E[I_i]:

$$E[I_i] = r_i - E[Y_i] + E[U_i] \tag{3.6}$$

- Fill rate of module i (probability that module i is available when a demand for it occurs), F_i:

$$F_i = P(I_i > 0) = P(Y_i < r_i) = P(r_i - 1|\lambda_i) \tag{3.7}$$

Figure 3.4 and Figure 3.5 graph $E[I_i]$ and $E[U_i]$ as functions of r_i for three different values of λ and delay t_i, respectively. We obtain the family of curves if we fix λ at 1 but increase $1/\mu_i = E[t_i]$ to 1, 3 and 5. Then, we obtain the similar family of curves if we fix $E[t_i]$ but increase λ to 10, 30 and 50.

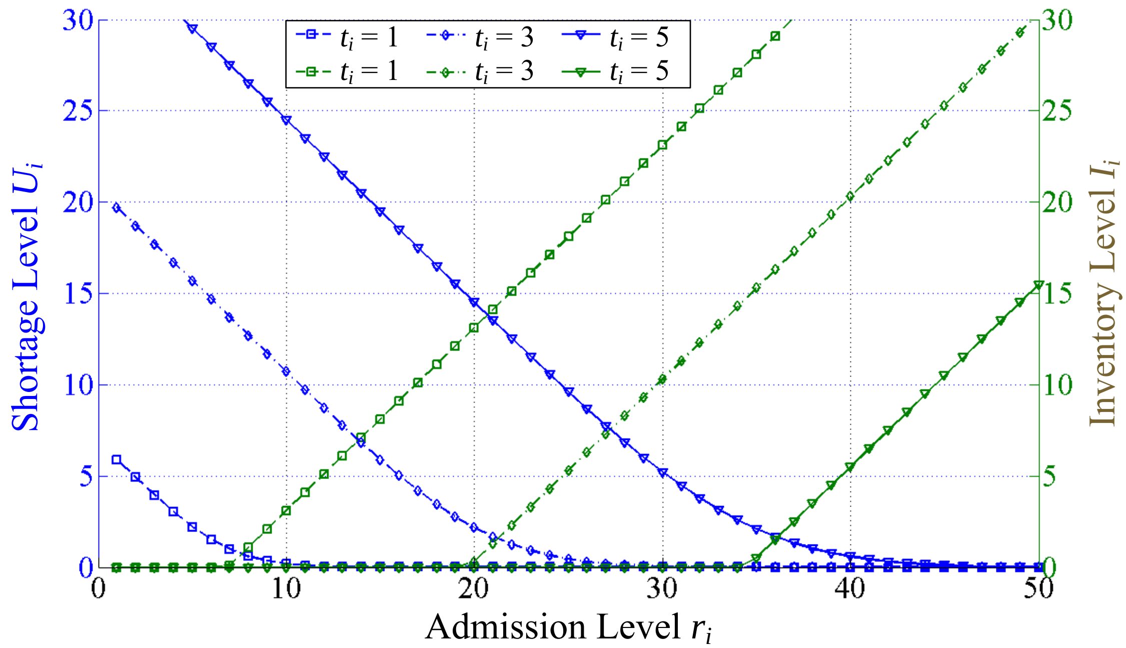


Figure 3.4 Effects of t_i on inventory and stock out level.

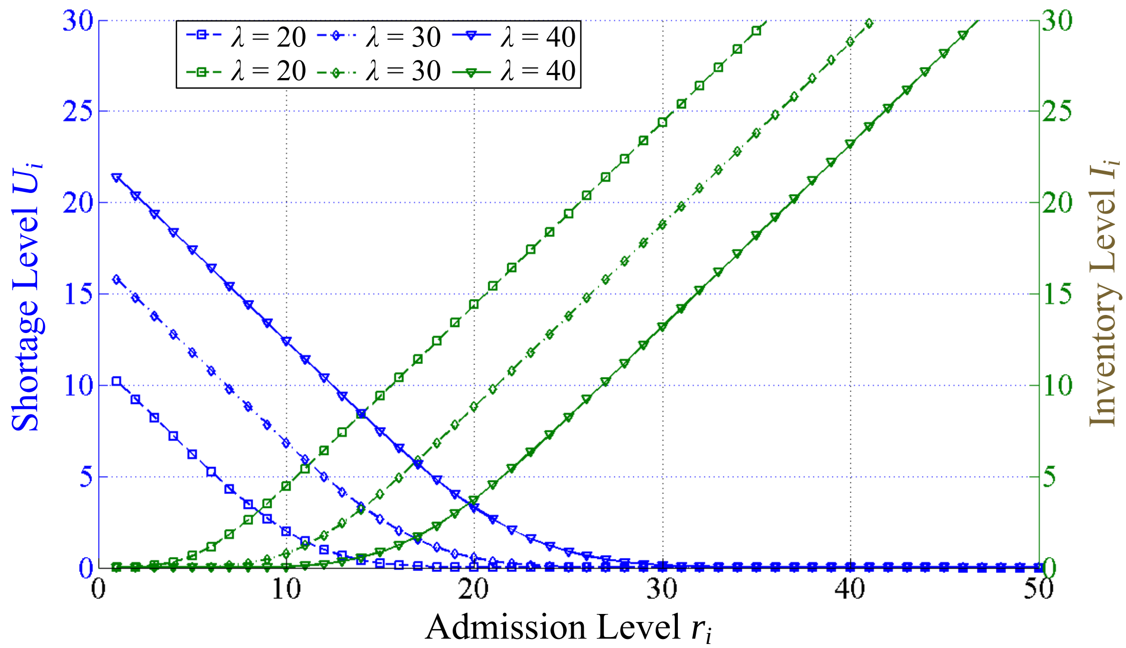


Figure 3.5 Effects of λ on inventory and stock out level.

The performance measures depend on the system parameters λ and μ_i only through their ratio λ/μ_i, which is equivalent to $\lambda \cdot t_i$, the mean demand on-order. All systems with the same $\lambda \cdot t_i$ behave the same and their performance characteristics are identical. From the above analysis, we have the following propositions.

PROPOSITION 3.1 Given a fixed overall demand rate λ, the system with a longer delay time t_i of return needs a larger admission threshold.

PROPOSITION 3.2 Given a fixed delay of return t_i, $i = 1, 2.$ a system with a larger demand rate λ needs a larger admission threshold.

Selecting a policy to meet a desired reuse level involves the trading off between customer service level and costs of operations that guarantee that service level. Given an upper limit of average stock out level $E[U_i]$, we want to set r_i large enough to meet this constrain. Conversely, given an upper limit on $E[I_i]$, set r_i as small as possible to minimize inventory cost, while maintaining certain service level.

3.4.2 The Joint Distribution of Outstanding Orders

At any given time t, the outstanding orders of module i, $Y_i(t)$, is a continuous-time Markov chain (CTMC) in a f inite system state space $\boldsymbol{Y} \coloneqq \{(\vec{Y}_1(t), \vec{Y}_2(t), \ldots, \vec{Y}_M(t)) | 0 \leq \vec{Y}_i(t) \leq b_i, i = 1,2, \ldots, M\}$. In steady states, each system state is associated with a nonnegative probability number, $p(Y_1, Y_1, \ldots, Y_M)$, which is between zero and one representing its probability at that specific state. The state space $\boldsymbol{Y}$ is composed of two subsets: (1) the boundary state set $\boldsymbol{Y_b}$, when $\vec{Y}_i = 0$, for any $i = 1,2, \ldots, M$, and (2) the repetitive state set $\boldsymbol{Y}_r$, when $0 < \vec{Y}_i(t) \leq b_i$, $i = 1,2, \ldots, M$. In the two-module reassembly example, the area shaded in gray is the boundary state set and the area in blue is repetitive state set.

The Poisson demand arrivals and Poisson return process form an *embedded* Markov Chain in which we only focus on t he moments when the state of the system changes. A transition of the system state only occurs when a customer warranty claim happens which triggers a new order with rate λ or when product returns arrive at remanufacturing facility with rate μ_i. Mat hematically, with transition rate λ, state y enters state y' where

$$y' = \begin{cases} y_j + 1 & if\ y_i < b_i, for\ all\ i \in S \\ y_j & otherwise. \end{cases} \tag{3.8}$$

and with transition rate $\mu_{i,}$ state y enters state y'' where

$$y'' = \begin{cases} y_j - 1 & if\ j = i\ and\ y_i > 0 \\ y_j & otherwise. \end{cases} \tag{3.9}$$

The relation between outstanding orders of two modules is depicted in Figure 3.6, which clearly demonstrates that the system state can be partitioned in two dimensions: the outstanding inventory of module 1, Y_1 on the vertical axis, and the outstanding inventory of module 2, Y_2 on the horizontal axis, respectively.

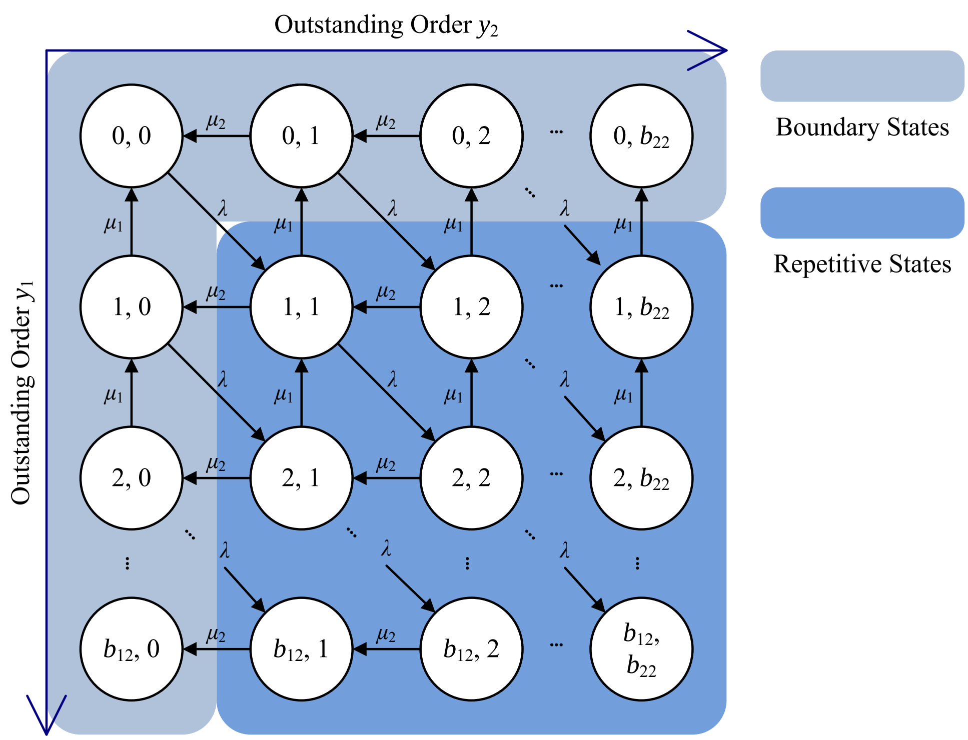


Figure 3.6 Two-module assembly system: state transition in a 2D Markov chain.

From Figure 3.6, we see that the transitions in the diagonal direction represent that an order arrival triggers an increment of both Y_1 and Y_2 by 1 simultaneously. Module return events for two modules could occur independently at different rates, μ_1 and μ_2.

The stationary joint distribution of $\boldsymbol{Y}$ can be solved by establishing balance equation for each state (there are $\prod_{i=1}^{M}(b_i+1)$ states in total) plus a normalization equation.

3.4.3 Matrix-Geometric Methods

The Markov chain in (3.8) and (3.9) has $\prod_{i=1}^{M}(b_i+1)$ states. Therefore, the steady-state probability distribution can be obtained by solving a system of $\prod_{i=1}^{M}(b_i+1)$ balance equations. However, when M and N becomes large, the Markov chain also becomes a large-scale one. This motivates us to consider numerical solution for the computation of stationary distributions (Stewart 2009, Chapter 10). By rearranging the system state space it can be shown that the transition matrix represents a quasi-birth-and death (QBD) process (Latouche and Ramaswami, 1999). This is because a state transition can only occur if a demand (warranty claim) arrives or a module return arrives at the system. In general, a QBD process is a Markov chain on the states $\{1,2,\ldots\}$, where transitions are allowed only to the neighboring states. That is, from state l, there are transitions only to state $l-1$ and state $l+1$ for $l \geq 1$, and from state 0, there is a transition only to state 1. Thus, a QBD process has a generator matrix of the form:

$$Q=\begin{Bmatrix} A^{(0)} & C^{(0)} & & & \\ B^{(1)} & A^{(1)} & C^{(1)} & & \\ & B^{(2)} & A^{(2)} & C^{(2)} & \\ & & \ddots & \ddots & \ddots \end{Bmatrix} \tag{3.10}$$

where $A^{(k)}, B^{(k)}$, and $C^{(k)}$, $k=0,1,2,\ldots$ are block matrices and form the tri-diagonal matrix Q.

Liu et al. (2012) discussed that the advantage of working with QBD models on generic Markov processes is that there exist efficient algorithms and methods to quickly solve the MDP. They used the Matrix-Analytic method for solving the generic QBD problem to analyze the throughput of a two-stage manufacturing systems with independent unreliable machines. The system discussed in this chapter can also be transformed into a matrix form fitting the properties of the QBD process. This allows us to obtain the stationary distribution efficiently by using *Matrix-Geometric* methods which was first presented by Neuts (1981).

By definition, $\boldsymbol{Y}$ is solvable because the second dimension of process is finite and bounded by b_2. Hence, our approach can be applied to obtain the exact expression for the system. We rearrange the system state in order of $\boldsymbol{Y} = \{\bar{Y}_0, \bar{Y}_1, \dots, \bar{Y}_{b_1}\}$, thus

$$\begin{gathered}\vec{Y}_j = \{(k, 0, \dots, 0,0), \dots, (k, 0, \dots, 0, b_m), \dots, (k, 0, \dots, 1,0), \dots, \\ (k, 0, \dots, 0, b_M), \dots, (k, 0, \dots, b_{M-1}, 0), \dots (k, 0, \dots, b_{M-1}, b_M), \dots, \\ (k, b_2, \dots, b_{M-1}, 0), \dots, (k, b_2, \dots, b_{M-1}, b_M)\}\end{gathered} \tag{3.11}$$

where $b_i = r_i + c_i$ for $i = 1,2, \dots M$.

Let $\boldsymbol{\pi}_j$be the stationary probabilities of states in $\vec{Y}_j$. Thus we can rewrite the stationary distribution of $\boldsymbol{Y}$ as $\boldsymbol{\pi} = (\boldsymbol{\pi}_0, \boldsymbol{\pi}_1, \dots, \boldsymbol{\pi}_{b_1})$. It is obvious that the above generator matrix is in a block-tridiagonal form which can be written as the following *infinitesimal generator matrix*.

$$Q' = \left\{\begin{matrix} A & A_0 & 0 & \cdots & 0 & 0 \\ \mu_1 I & A - \mu_1 I & A_0 & \cdots & 0 & 0 \\ 0 & \mu_1 I & A - \mu_1 I & \ddots & \vdots & 0 \\ \vdots & \vdots & \mu_1 I & \ddots & A_0 & \vdots \\ 0 & 0 & \cdots & \cdots & A - \mu_1 I & A_0 \\ 0 & 0 & \cdots & 0 & \mu_1 I & A - \mu_1 I \end{matrix}\right\} \tag{3.12}$$

Since the Markov chain is irreducible, there exists a unique matrix $\boldsymbol{\pi}$. Then from (3.11), we can write the balance equations in the matrix form:

$$\boldsymbol{\pi}_0 A + \mu_1 \boldsymbol{\pi}_1 = 0$$

$$\boldsymbol{\pi}_{j-1} A_0 + \boldsymbol{\pi}_j (A - \mu_1 I) + \mu_1 \boldsymbol{\pi}_{j+1} = 0, 1 \leq j \leq b_1 \tag{3.13}$$

$$\boldsymbol{\pi}_{b_1 - 1} A_0 + \boldsymbol{\pi}_{b_1} (A - \mu_1 I) = 0.$$

In the QBD process, there exists a matrix $\boldsymbol{R}^{(j)}$ such that $\boldsymbol{\pi}_j = \boldsymbol{\pi}_{j-1} \boldsymbol{R}^{(j)}$ for each $j \geq 1$. Specifically, the stationary probability vector in the QBD process is given recursively by

$$\boldsymbol{\pi}_j = \boldsymbol{\pi}_{j-1} \boldsymbol{R}^{(j)} \tag{3.14}$$

where $\boldsymbol{R}^{(j)}$ is given recursively via:

$$\boldsymbol{A_0} + \boldsymbol{R}^{(j)} (\boldsymbol{A} - \mu_1 \boldsymbol{I}) + \boldsymbol{R}^{(j)} \boldsymbol{R}^{(j+1)} \mu_1 \boldsymbol{I} = \boldsymbol{0} \tag{3.15}$$

Once $\boldsymbol{R}^{(j)}$ for all j are obtained, the stationary probability vector $\boldsymbol{\pi}_j$ can be calculated recursively from $\boldsymbol{\pi}_j$ via (3.14) for all $j \geq 1$. Thus, all that remains is to calculate $\boldsymbol{\pi}_0$, which can be found by a positive solution of $\boldsymbol{\pi}_0 \left(A + \boldsymbol{R}^{(1)} \mu_1 \boldsymbol{I} \right) = \boldsymbol{0}$, and the normalization equation

$$\boldsymbol{\pi}_0 \sum_{j=0}^{b_1} \prod_{i=1}^{j} R^{(i)} \vec{1} = 1 \tag{3.16}$$

where A and A_0 are square matrices of order b_2. The above equation can be used to find the probability vector $\boldsymbol{\pi}_0$ and then $\boldsymbol{\pi}_k$, $k = 1,2, \dots, b_1$ as positive solutions to (3.14) by using (3.13).

3.4.4 Translating Stationary Probabilities into Performance Measures

- Mean number of modules in the system: $E[Y] = \sum_{k=0}^{b_1} k\boldsymbol{\pi}_k \vec{\mathbf{1}}$.
- Mean inventory level: $E[I] = \sum_{i=0}^{\infty}(r_i - Y_i)\vec{\mathbf{1}}$.
- Mean waiting time: $E[W] = E[Y_i]/\lambda$.
- Order fill rates with all remanufactured module:

$$F = P(I_i > 0, \dots, I_M > 0) = P(Y_i < r_i, \dots, Y_M < r_M)$$

- Warranty Covered by Remanufacturing (WCR):

$$\frac{Z(i{:}i\in\Omega)}{Z(i+k{:}i\in\Omega,k\in\bar{\Omega})}\, P(I_i > 0, I_k = 0{:}\, i \in \Omega, k \in \bar{\Omega})$$

where Ω is the subset of module inventories which has positive on-hand inventory, $\bar{\Omega}$ is the complementary set, and Z is a function that counts the number of items in a specific subset. Hence, the above expression means that the WCR is the product of fraction of modules with positive on-hand inventory and the probability that the system is exactly in that state.

3.5 Analysis for Multiple Quality Grade Remanufacturing

3.5.1 Embedded Donor-Beneficiary Model

Conventionally, remanufactured products were assumed to have the same quality condition as the original product. In practice, however, some of used components cannot always be restored to a like-new condition but are still highly durable and good for reuse. Distinguishing them by different quality and corresponding different price can lead to multiple customer classes. Decision making on the admission of product returns is challenging when additional quality variation is considered, because it complicates the problem of compatibility in reassembly by increasing inventory variability. Jin et al. (2011) studied optimal assembly-inventory strategy in a multi-quality remanufacturing system. In this section, we extend our single-quality model in Section 3.4 to a model with multiple quality grades of returns along with quality-specified customer orders of warranty replacement. Usually, returned products are inspected, sorted and categorized in q different quality grades j, ($j = 1,2, \ldots N$) before heading into the remanufacturable inventories. All the notations are modified from single subscript, i, to two subscripts, i and j. For example, let r_{ij} denote the admission threshold value for the inventory of module type i with quality grade j. Now, the Markov chain becomes more complicated due to the inventory queues are extended from one dimension (module type) to two dimensions (with a second dimension in quality grade).

For the ease of exposition, we use a case of two (quality) classes of remanufactured products: class 2 products have higher quality than class 1 products. Each remanufactured product is assembled from two non-interchangeable modules or components. The assembly strategy can be described as a *donor-beneficiary* model as follows: when an order of low quality grade (class 1) arrives, we first use available modules from low grade module inventories for reassembly to fulfill the order; if there is one module not available, we resort to the same type of module with high quality grade and use them as a substitutable module in assembly. The *substitution* is one-way such that only high grade one can be used to substitute low grade and not vice versa. Figure 3.7 illustrates a queueing network of a two-quality grade *donor-beneficiary* network that specifically appears in a multi-server analysis (Nozaki and Ross 1978, Kao and Wilson, 1999). Here, the low quality class of module i arrives according to a Poisson process with rate λ_{i1} and high class of jobs of module i arrives according to a Poisson process with rate λ_{i2}, $i = 1,2$. The queues are not independent but linked under the condition that the low grade queue y_{i1} exceeds r_{i1}.

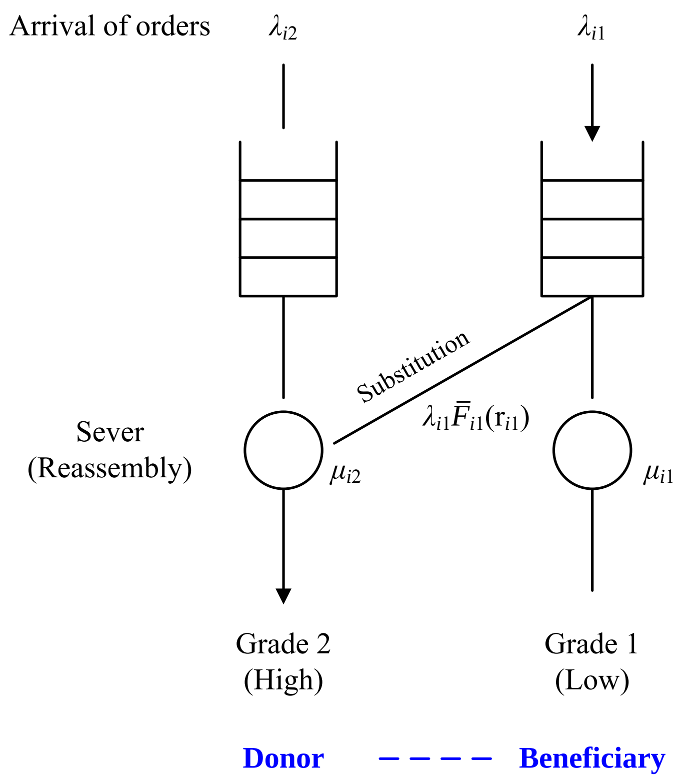


Figure 3.7 Multi-server model with substitution.

Beforehand, we investigate the interaction between queues of outstanding orders with different quality of the same module type. This will help us understand how module substitution affects the behavior of the inventory system. From Figure 3.7, as we observe the dependency exists between the high grade queues and its counterpart low grade queues. A 2D Markov chain cannot simply be decomposed into two 1D Markov chains (Harchol-Balter et al., 2005). When the inventories of low quality module are empty ($I_{i1} = 0$), there is an impact on the transition rates of the counterpart high quality module queue due to *substitution*. Therefore, there is very loose dependency between the two quality classes of modules, that is, the high quality modules can be consumed by an additional rate $\lambda_{i1}\bar{F}_{i1}(r_{i1})$ from low class demand. As a result, the performance analysis of the multidimensional Markov chain can be decomposed by solving high class queue conditioning on the low class queue status.

For the *low* quality modules, the queues of returned items before being assembled can be modeled as an $M/M/1/\ b_{i1}$ queue with infinite capacity b_{i1}, where $b_{i1} = r_{i1} + c_{i1}$. Let $P_{i1}(n)$ be the stationary probability that the queue length of module i with quality grade 1, y_{i1} is *n*:

$$P_{i1}(n) = \frac{(1-\rho_{i1})\rho_{i1}^{n}}{1-(\rho_{i1})^{b_{i1}+1}}, i = 1,2 \tag{3.17}$$

where $\rho_{i1} = \lambda_{i1}/\mu_{i1}$ is the traffic density of queue i_1.

For the *high* quality queues, the demand rates are conditional on the state of the counterpart module inventories with low quality. Define the stock out rate as $\bar{F}_{i1}(r_{i1}) \triangleq P(I_{i1} = 0) = P(y_{i1} > r_{i1})$. The actual demand rate for module i with high quality, $\tilde{\lambda}_{i2}$, is the summation of original return rate λ_{i2} and $\lambda_{i1}\bar{F}_{i1}(r_{i1})$. Thus we have

$$\begin{aligned}\tilde{\lambda}_{i2} &= \lambda_{i2} + \lambda_{i1}\bar{F}_{i1}(r_{i1}) \\ &= \lambda_{i2} + \lambda_{i1}[1 - \frac{(1-\rho_{i1})}{1-(\rho_{i1})^{b_{i1}+1}}(1+\rho_{i1}+\rho_{i1}^{2}+\cdots+\rho_{i1}^{r_{i1}})]\end{aligned} \tag{3.18}$$

The model can be easily extended for multi-server with substitution for multiple quality grades ($\geq$ 3 quality grades). Without loss of generality, we assume $q_1 > q_2 > \cdots > q_N$ (i.e., higher grade number indicates better quality). The dependency among queues of on-order inventories Y_{ij} are shown in Figure 3.8.

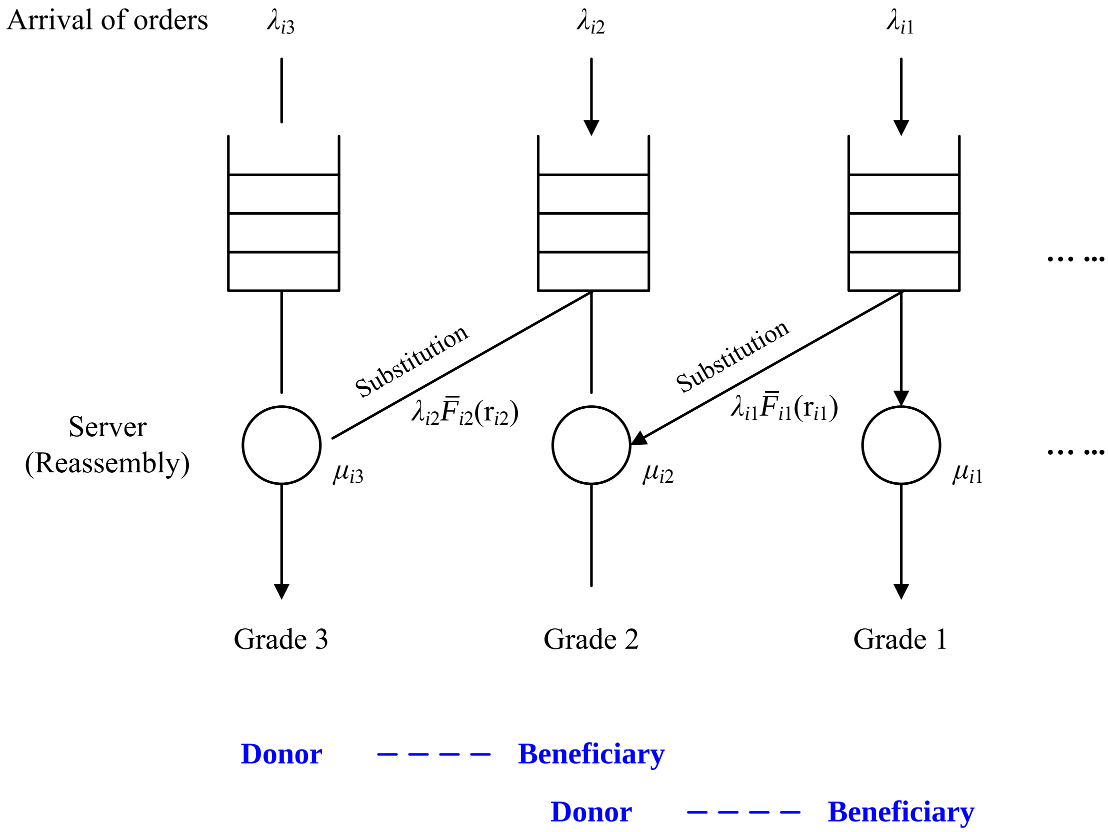


Figure 3.8 Donor-Beneficiary models for multi-servers (>2).

The demand rate for all queues can be derived using the following iterations:

$$\tilde{\lambda}_{i(j+1)} = \lambda_{i,(j+1)} + \lambda_{ij}\bar{F}_{ij}(r_{ij}), i = 1,2, \dots, M, j = 1,2, \dots N \tag{3.19}$$

3.5.2 Joint Distribution of Multi-Class Outstanding Orders

In this section, we solve for the joint stationary distribution of Y_{ij} in the case of multiple demand classes. The matrix-geometric approach for the single quality model is adapted for the multi-class problem. Given the revised demand rates for the substitutable module queues, we need to revise the Markov chain for the substitutable (high quality) modules (Y_{12}, Y_{22}) as depicted in Figure 3.9. The red dotted arrows represent the state transitions when substitutions occur in the "*donor-beneficiary*" queueing model. The remaining arrowed arcs are of the same structure as the one in the single-class model (see Figure 3.6 in Section 3.4.2).

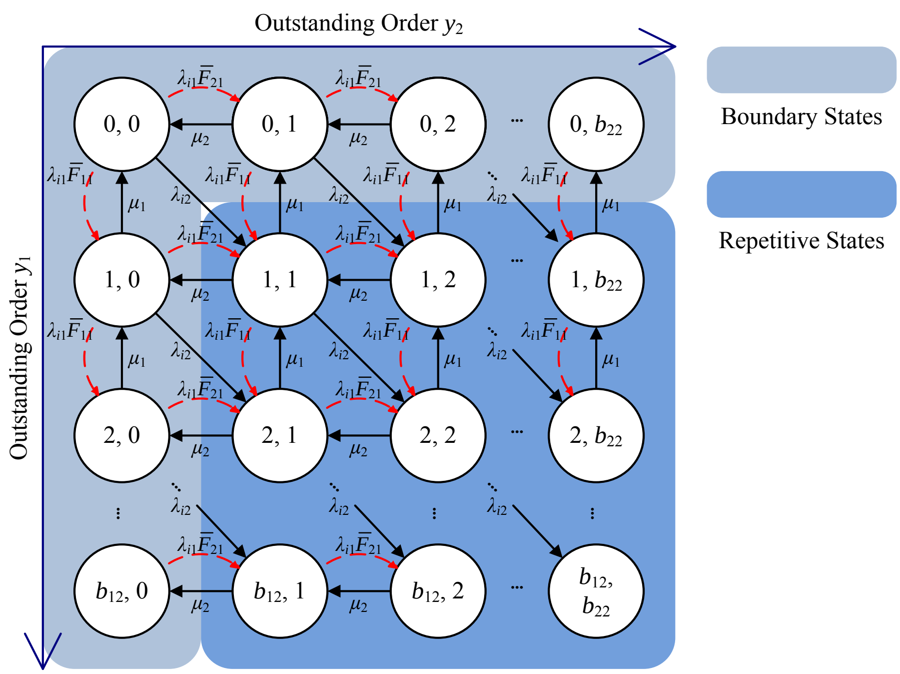


Figure 3.9 Two-module two quality grade system: state transitions for class 2

The transition rate matrix for the low quality modules is the same as in Section 3.4 while the one for the high quality modules is slightly different such that additional transitions (red dotted arcs) need to be added to the original graph. Accordingly the new matrix $\tilde{A}$ and $\tilde{A}_0$ need to be adapted from the original matrix A and A_0 with additional off-diagonal components, respectively:

$$\tilde{A} = \begin{bmatrix} -(\lambda_2 + \lambda_1 \bar{F}_{11}) & \lambda_1 \bar{F}_{11} & 0 & \cdots & 0 & 0 \\ \mu_1 & -(\mu_2 + \lambda_2) & \lambda_1 \bar{F}_{11} & \cdots & 0 & 0 \\ 0 & \mu_2 & -(\mu_2 + \lambda_2) & \ddots & \vdots & \vdots \\ 0 & 0 & \mu_2 & \ddots & \lambda_1 \bar{F}_{11} & 0 \\ \vdots & \vdots & \vdots & \ddots & -(\mu_2 + \lambda_2) & \lambda_1 \bar{F}_{11} \\ 0 & 0 & 0 & \cdots & \mu_2 & -\mu_2 \end{bmatrix} \tag{3.20}$$

$$\tilde{A}_0 = A_0 + \begin{bmatrix} \lambda_1 \bar{F}_{21} & & & & \\ & \lambda_1 \bar{F}_{21} & & & \\ & & \lambda_1 \bar{F}_{21} & & \\ & & & \ddots & \\ & & & & \lambda_1 \bar{F}_{21} \end{bmatrix}$$

$$= \begin{bmatrix} \lambda_1 \bar{F}_{21} & \lambda_2 & & & \\ & \lambda_1 \bar{F}_{21} & \lambda_2 & & \\ & & \lambda_1 \bar{F}_{21} & \ddots & \\ & & & \ddots & \lambda_2 \\ & & & & \lambda_1 \bar{F}_{21} \end{bmatrix} \tag{3.21}$$

Now, the procedure of computing the stationary distribution for the multi-quality multi-type module system is exactly the same as described in Section 3.4. The translation and analysis of the order-level and system-level performance follow the similar procedures. We describe the analysis and results in detail in the numerical study in the following section.

3.6 Numerical Study

In this section we present the results of the numerical experiments and discuss our observations. Our main purposes are two-fold:

(1) To investigate the effects of various system parameters on module-level performances. These parameters include the policy parameters (r_{ij}) and system parameters λ_{ij} and μ_{ij}. This kind of information will help us understand the behavior of the system with multiple modules and supply with uncertain quality. We also gain insight on how to better manage these systems, e.g., how to select admission thresholds to improve system performance and reduce cost.

(2) To understand the benefits from module substitution in the presence of quality variation. This will shed light on further development of threshold-type heuristics for problems with a modestly large number of components, i.e., more than two types of modules and more than two quality grades.

We present our findings under different scenarios. First, we show module-level performance for *single quality - two module* cases under both *symmetric* arrival rate and *asymmetric* arrival rate scenarios, respectively. Second, we show order-level and system-level performance for a *two module – two quality level* case when two classes of demands have equal arrival rate ($\mu_1 = \mu_2$) and non-equal arrival rate ($\mu_1 \neq \mu_2$).

3.6.1 The Single Quality Grade System

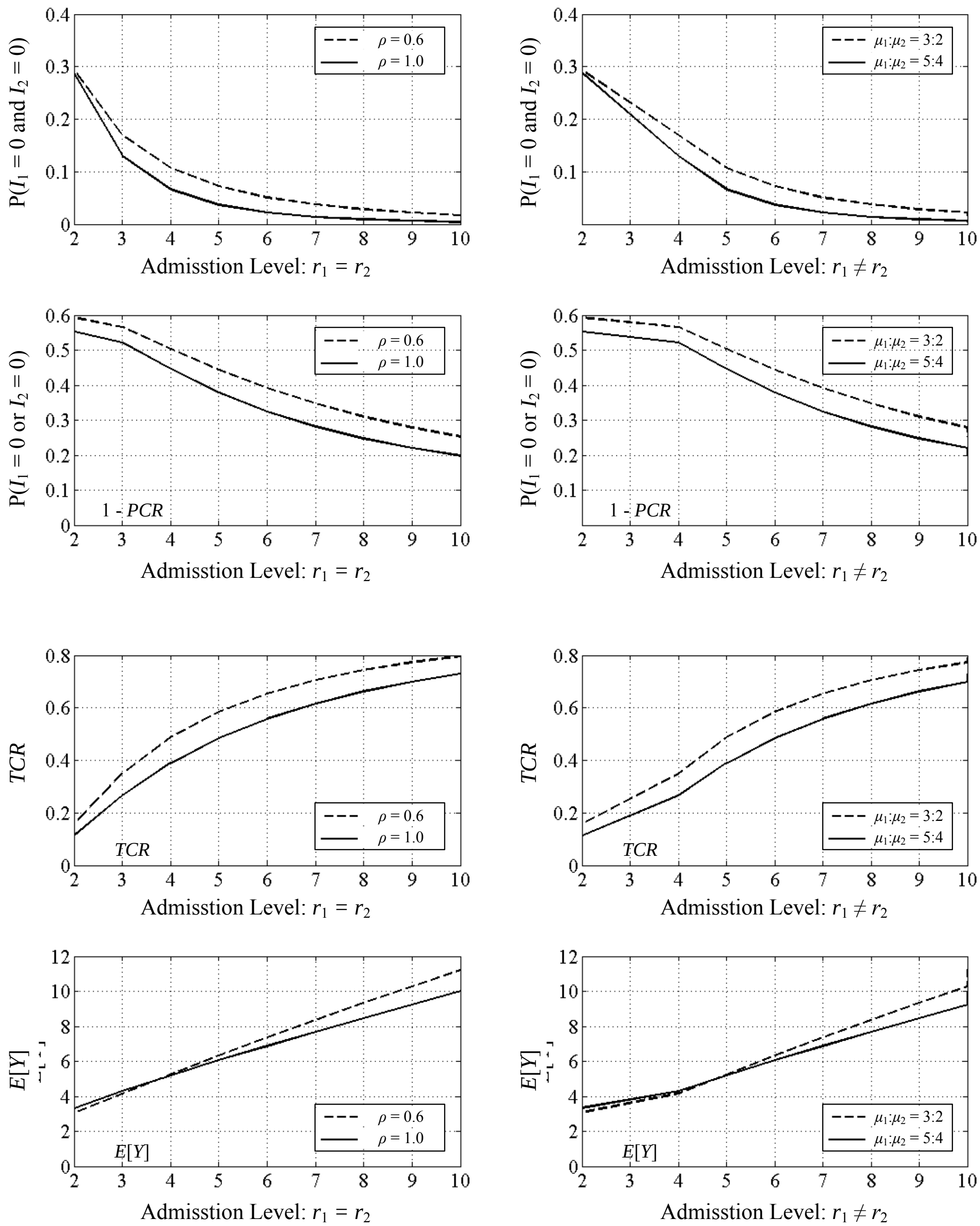
For each performance measure, we divide the graphs into two subgroups: symmetric cases (i.e., groups of graphs in Figure 3.10 (a)) and asymmetric cases (i.e., groups of graphs in Figure 3.10 (b)). Each horizontal pairs compare the order-level performances of the different scenarios under the same parameter configuration except for the module return rates μ_i, $i = 1, 2$. Other parameters are fixed: demand rate for remanufactured product λ is 4, admission threshold takes the value from 3 to 10. The following summarizes the key observations and their implications.

(1) For the symmetric cases, $\mu_1 = \mu_2$, we choose the traffic indensity of two queues such that $\rho_1 = \rho_2 (= \rho)$ is 1 and 0.6 corresponding to the systems with heavy load and moderate load, respectively. Since the traffic indensities of two queues are equal in the symmetric cases, we select a symmetric configuration of admission threshold for the module inventories (r_1, r_2) such that $r_1 = r_2$. We change the admission threshold from 2

to 10 and show the total stock out probability, partial stock out probabiliy, total warranty covered by remanufactured products, and expected outstanding orders in Figure 3.10 (a).

(2) For the asymmetic cases, $\mu_1 \neq \mu_2$, non-equal return rates for two modules reflect a realistic situation when one type of module is more likely to fail than the other. We select two configurations of Poisson return parameters: $\mu_1/\mu_2 = 5/4$ and $3/2$ corresponding to the systems with lower imbalance and higher imbalance, respectively. We explore the effects of increasing the degree of imbalance between returned modules on the inventory and stock-out level at the module level.

(3) We found that for both cases, the traffic intensity $\rho(= \lambda/\mu)$ has a pronounced adverse effect on each of performance. As the traffic intensity ρ increases, the module-level performance gets poorer and also provides influence on the order-level performance (Figure 3.10 (a)). For example, higher ρ leads to higher probability of totally stock out level (equivalent to the probability of using a new product to fill the order), and decreased reuse level and higher outstanding orders.



(a) (b)

Figure 3.10 Comparison of order-level performance measures: (a)Symmetric cases; (b) Asymmetric cases.

(4) For comparison, we also put the results of highly asymmetric case ($\mu_1/\mu_2 = 4/1$) and a sy mmetric case ($\mu_1 = \mu_2$) together in a single graph (see Figure 3.11). The

imbalance between two types of module arrivals has a negative effect on system performance such that higher imbalance leads to higher average stock out and the lower fill rate. Therefore, the managers of the system may take possible remedy to take-back program so that these negative effects from return imbalance is mitigated.

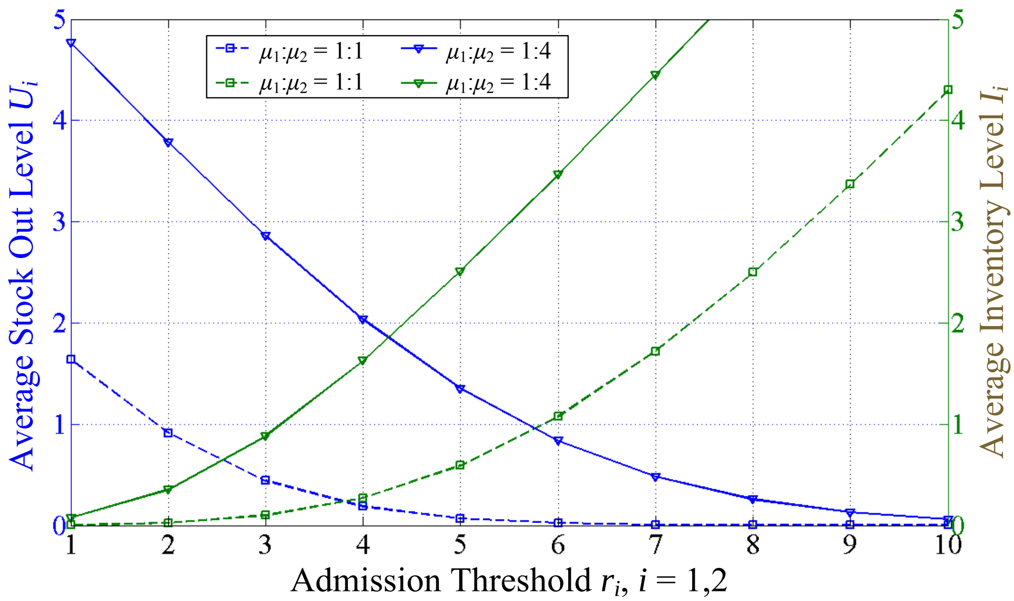


Figure 3.11 Effects of module return imbalance.

The results also reveal the qualitative effects on the performance of changes in admission thresholds r_i: (1) the average stock out level U_i is decreasing and convex as a function of r_i as the threshold-type admission policy is similar to a base-stock level (Song 1998, Hausman 1998, Song and Yao 2002, H uh and Janakiraman 2010); (2) as r_i increases, the stock out level declines with an ever slower rate; (3) the average inventory level $E[I_i]$ is increasing and convex in r_i; and (4) the more imbalance among the modules return is, the larger admission threshold is needed to achieve optimality (see Figure 3.11).

We further study the effects of return rate imbalance on an important performance measuring the reuse rate of return modules, i.e., warranty covered by remanufacturing rate (WCR). In general, we found that, in both symmetric and asymmetric cases, the order-level performance measure, WCR and system inventory responds to the admission threshold changes in an increasing manner. Figure 3.12 (a) and Figure 3.12 (b) show the similar trend of WCR versus admission thresholod under two cases (increasing and concave). Also, the traffic density ρ has a positive effect on WCR, that is higher ρ will result in higher WCR value. For the comparison of the two cases, one observes that there is performance deterioration in the asymmetric cases because of the imbalanced return rates.

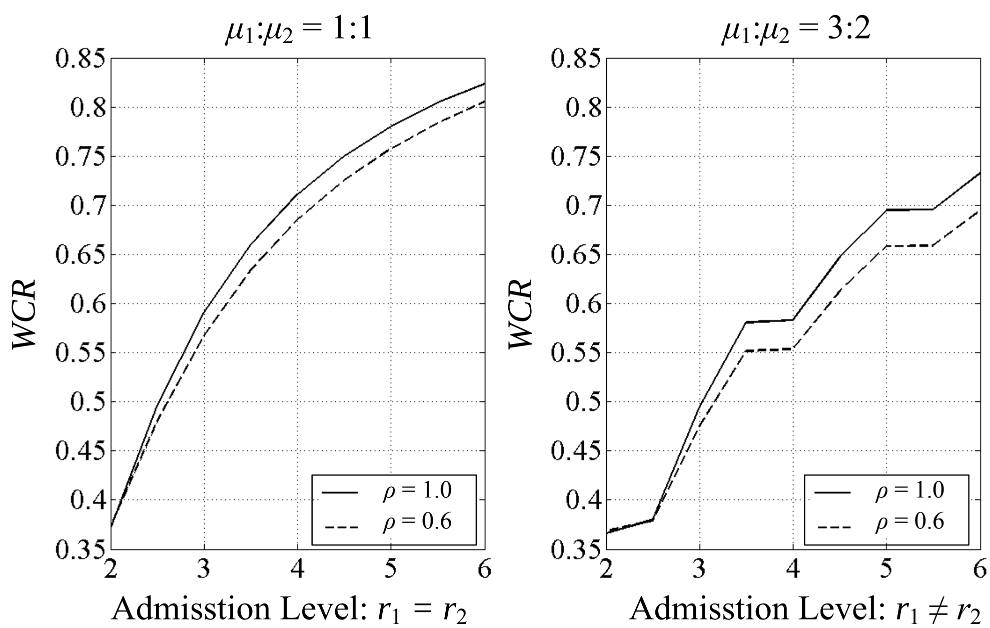


Figure 3.12 Comparison of reuse level: (a) WCR when $\mu_1 = \mu_2$; (b) WCR when $\mu_1 \neq \mu_2$.

3.6.2 The Two-Quality Grade System

For all the experiments, we still use a *two module type – two quality grade* problem and fix the following parameters: total demand rate λ is 4. For the symmetric scenario, demand rate for two classes (low and high quality classes) are equal, e.g., $\lambda_1 + \lambda_2 = 2$, return rates for four modules yield $\mu_{11} + \mu_{21} = \lambda_1/\rho$ and $\mu_{12} + \mu_{22} = \lambda_2/\rho$, $\rho = 1$ and 0.8. For the asymmetric cases, the total return rate and total demand rate keep the same while the low-to-high ratio becomes $3:7$ which corresponds to the system with high quality variable and imbalance between low class of return/demand and higher class of return/demand. Besides the performance measures discussed above, we consider the total system cost with unit inventory cost $c_{ij} = 2$ and unit penalty cost of using new modules (stock out penalty*)* $s_{ij} = 30$ for $i = 1,2$ and $j = 1,2$. Hence, the total cost of the system is $TC = c_{ij}\sum_{j=1}^{N} E\left[I_{ij}\right] + s_{ij}\sum_{j=1}^{N}\sum_{i=1}^{N} E[U_{ij}]$. The results for this example are illustrated in Figure 3.13 and Figure 3.14. In general, we found that, in both cases, the system-level performances respond to the admission threshold changes in a similar manner as the single-quality systems. We found interesting observations and gained useful insights from the numerical cases as follows.

(1) In both cases, WCR increases as the admission levels for module inventories increase. However, it does not guarantee an increase in overal system cost-effectiveness. In fact, inventory levels also increase so that the combinational effect on system total cost is not monotonic in admission thresholds. Specifically, the total cost (TC_1) of fulfullment of class 1 customers is convex in admission threshold for low quality module inventories. Howerver, TC_2 is not convex for the higher class customers. The inflection point in the

left bottom graph of Figure 3.13 is associated with the system behavior when substititions for assembly occur.

(2) Although substitution provides the chances that higher quality modules are used for fulfillment of low quality class of demand, the WCR value for the higher quality class of demand is still greater than that for the low quality class in both cases. This implies that the "donor" does not necessarily sacrifies its own performenace when it is used as a substitutable item for the "beneficiary". In fact, taking advantage of substitution between multiple classes benefits the overall system performance.

(3) From Figure 3.14, we found that an increase in the imbalance between quality classes of returns lead to poorer system performance in terms of higher inventory level and higher total cost, althougth WCR values are not necessarily affected.

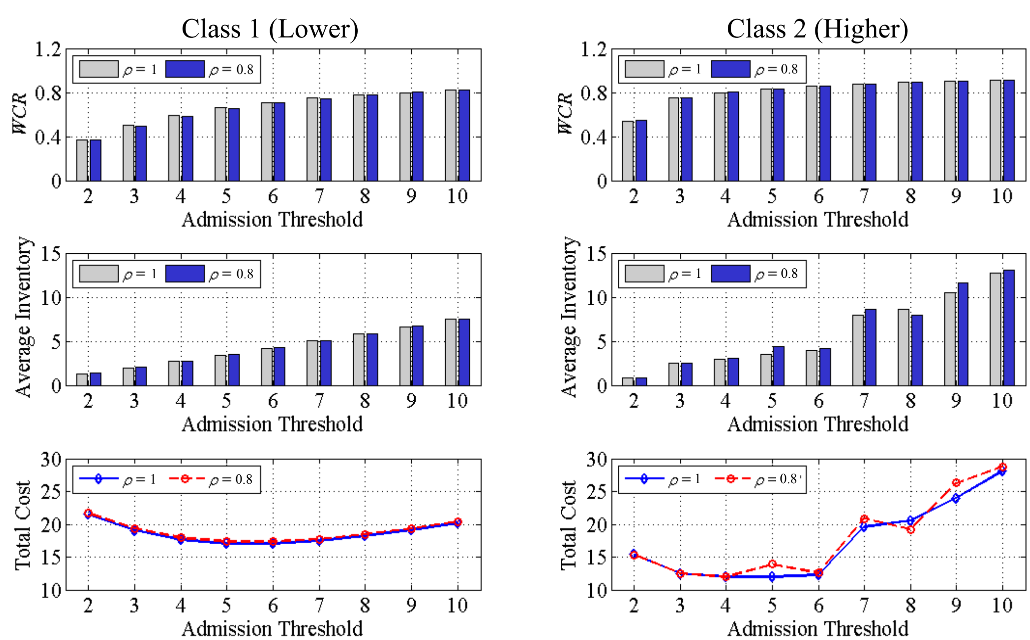


Figure 3.13 Symmetric cases for two-quality-grade RATO systems.

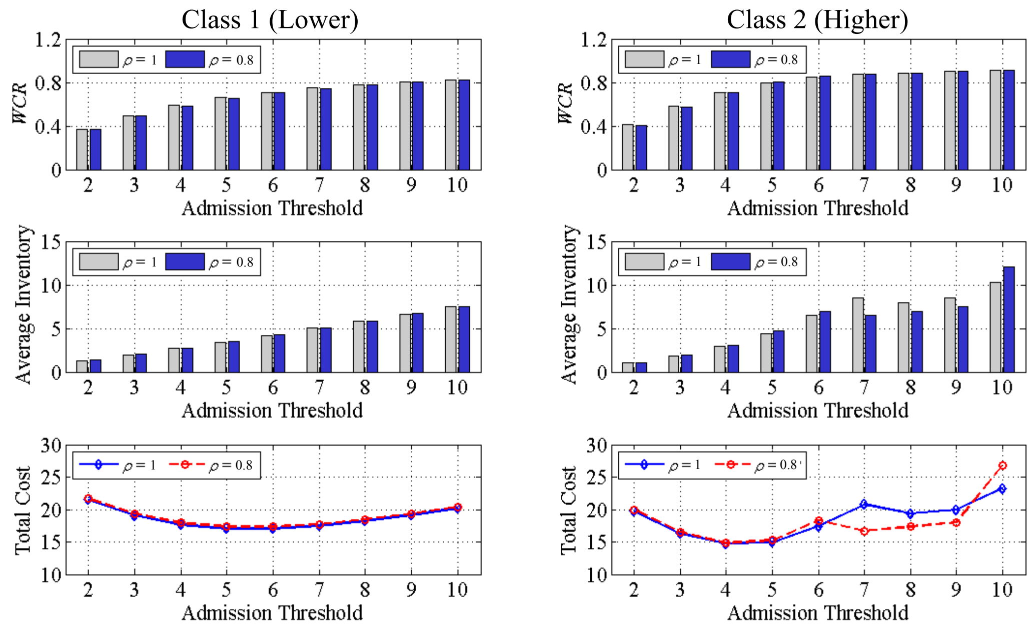


Figure 3.14 Asymmetric cases for two-quality-grade RATO systems.

3.7 Conclusions

This chapter studied the performance of a reassemble-to-order remanufacturing system receiving variable quality product returns with admission control. We developed analytical models and numerical solutions to evaluate how admission thresholds affect the performances and discussed their implications.

The contribution of this study includes the following: (1) a multi-dimensional Markov model transformed from parallel and dependent $M/M/1/C$ queues is presented to characterize the Poisson arrivals of module returns and quality-specified demand; (2) the dynamic process of the system is shown to be a QBD process by rearranging the original transition matrix; (3) the Matrix-Geometric method for solving generic QBD process is applied to compute system performance (i.e., inventory level, stock out level, waiting time, reuse level); (4) numerical studies show how admission levels affect these performance measures and provide valuable implications; and (5) different system parameter setting are tested to understand the inner trade-off between service level and operation costs.

CHAPTER 4

DESIGNING OPTIMAL ADMISSION POLICIES FOR REMANUFACTURING SYSTEMS

4.1 Introduction

In the remanufacturing industry, worn or used parts (cores) are rebuilt when received from customers and other sources. For cores that have been or will be remanufactured, the proper inventory value ordinarily will be cost, since these cores are not offered for sale or salvage at scrap prices (IRS, 1991). Effective management of core inventory can be a key issue not only for the remanufacturers, but also for the entire remanufacturing distribution chain including distributors and retailers. T he core inventory management in remanufacturing business requires a more comprehensive view of inventory behavior in terms of quantity, quality and timing.

The work in this chapter is motivated by a real battery remanufacturing case study in which there are not only uncertainties in returns and demand but also quality compatibility issues. Several elements make the end-of-use product return management and remanufacturing decisions challenging. First, product life cycle constrains the remanufacturing for warranty replacement. Firms introduce the next-generation product every 12 – 18 months. Since product usually have a finite residence time (duration of one use of the product by a customer) that is shorter in expectation than the life cycle of the product, product returns from the older generation will could happen at different points of the newer generation products' life cycle. For example, the 1st generation (Gen I) products often return from the market during the life cycle of the 2nd generation (Gen II) products. There is a compatibility issue such that well-functioning modules or components of the Gen I products cannot be reused in Gen II warranty replacement, while used modules/components from a Gen II product can be reused in a remanufactured Gen I product from warranty replacement. Second, the random quality of product returns constrains the matching and reassembly in remanufacturing. Third, the component (module) commonality leads to different inventory behavior and hence different remanufacturing strategies.

We analyze the behavior of the remanufacturing-reassembly-inventory system with random product returns through a simulation model with consideration of module compatibility during reassembly.. Different return processes and different system settings are investigated to understand the system behavior. Analysis based on the simulation results includes comparison of inventory level and variances, waiting times of core modules, reuse level and usage of new modules. The simulation program will be used to conduct what-if scenario analysis and provide a foundation for an optimization model with objectives specified by the remanufacturer. The framework of the simulation and optimization models is presented in Figure 4.1.

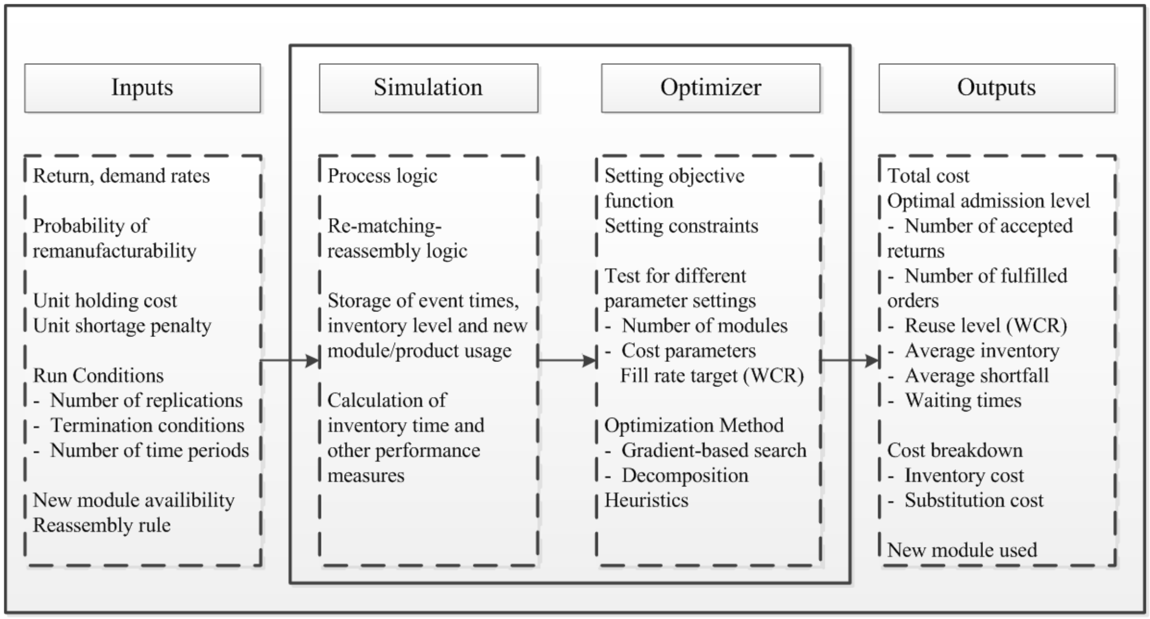


Figure 4.1 Overall simulation and optimization framework

The objective of this chapter is twofold: (1) develop a simulation model to analyze a real and complex remanufacturing-inventory system under different scenarios; and (2) integrate the implication from the simulation with an optimization tool for optimal acquisition control on returned products (as an extension from Chapter 3).

The reminder of Chapter 4 is organized as following:

We first develop a simulation model in ProModel® and Matlab® for a complex multiple-module reassemble-to-order system in Section 4.2. In Section 4.3, m odel assumptions, random generation of returns as model input and a basic model are discussed. In Section 4.4, extensive scenario modeling and analysis allow us to confirm the significance of inventory control under complex scenarios when there are multiple product generations and backward compatibility issues in reassembly. The purpose of the simulation tool is to analyze remanufacturing and inventory system behavior and performance under different model settings, hence providing a better understanding of what and how remanufacturing strategy may be integrated into a firm's business. In Section 4.5, we address that based on t he results of inventory analysis from the simulation program, product returns can be controlled by an optimal admission policy described in Chapter 3. Sections 4.6 and 4.7 present two optimal admission policies that optimally trade off aggregate inventory level and stock out level and reduce the value decay of core in inventory. Section 4.8 concludes this chapter.

4.2 Problem Description

4.2.1 Battery Quality and Capacity Degradation

All battery packs from any lithium ion battery maker gradually lose capacity. This is the nature of batteries in general, that they lose electrical storage capacity upon charge and discharge cycles. The quality of a used battery module is indicated by the battery capacity or the state of health (SOH). State of health (SOH) is a figure of merit of the condition of a battery (or a cell, or a battery pack), compared to its ideal conditions. The units of SOH are percent points (100% = the battery's conditions match the battery's specifications). Current engineering specification for electric vehicle (EV) requires that a battery with a SOH of 80% or above is sufficient for EV use, while a standard battery warranty period, its expected useful life-time, is 8 years. For example, when the new Nissan Leaf battery pack is rated to hold 24 kilowatt-hours of electricity, Nissan expects the capacity to gradually decline after a 10 year period to hold only 19.2 kilowatt-hours (80% of 24 kilowatt-hours) (HybridCars, 2010). Experimental data by ba ttery manufacturerss reveals a relatively graceful battery degradation trend, which indicates a relatively linear loss of capacity over time. Spotnitz (2003) studied a typical shape for calendar life (capacity versus time) data. As time proceeds, the capacity loss appears to be linear with time. Therefore, we can approximately correlate residence time with the capacity fading process as illustrated in Figure 4.2. The battery quality is divided into four grades with each grade covering a two-year age range. For example, a battery with age in [0,2] year is a grade *A* battery.

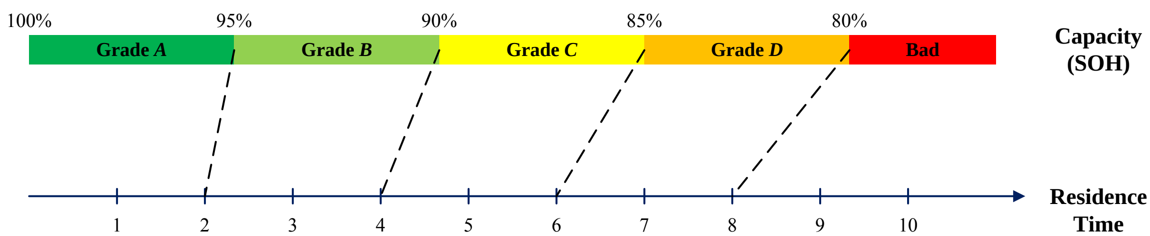


Figure 4.2 Sorting and grading of EV Li-ion batteries

4.2.2 Core Inventory Behavior Analysis

In this study, **core inventory** is the inventory of remanufacturable modules disassembled from returned products. It differs from other manufacturing inventories in several ways. First, the functions are different. In a conventional manufacturing system, there are work-in-process (WIP) inventories, finished product inventory, and spare parts inventory. These inventories have deterministic supply. For the case of remanufacturing, core inventories exist as an intermediate to hold acceptable end-of-life product disassemblies and are ready to be used in the reassembly processes.

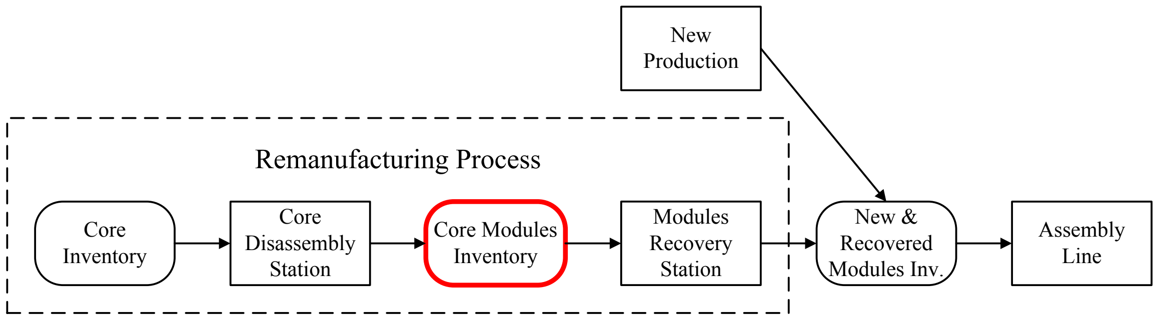


Figure 4.3 Core inventory in a remanufacturing system

Second, the policy for governing core inventory differs from those for regular inventories. WIP and final product inventories in conventional manufacturing system can be increased or decreased by changing production rates. Core inventory, however, largely depends on the return rate and the compatibility between parts and how it is reassembled. Core inventories serve as an important buffer generated from disassembling accepted returned products, and followed and consumed by a part recovery process (see Figure 4.3). The recovered parts together with some new spare parts are reassembled to get a final remanufactured product. Since reassembly requires the compatibility of the parts, the quality grade can have a major impact upon the recoverable parts inventories. The compatibility is highly determined by the quality level of each part. Manufacturers may have different rules for re-assembly and definition of compatibility. In Section 4.3.2, the definition of subassembly's compatibility in our study is given.

4.3 The Simulation Model

Simulation is the most commonly used technique to study the effect of different policies on the long term behavior of a remanufacturing system. Biehl et al. (2007) presented a discrete event simulation (DES) model for a carpet reverse logistics and analyzed the impact of the system design factors and environmental factors on the performance of the reverse supply chain system. Kara et al. (2007) developed a reverse logistics network of EOL white goods using DES model to determine the most important factors in the reverse logistics system. The use of simulation in modeling the core inventory management problem represents a popular alternative to mathematical programming since simulation has the ability of describing multivariate non-linear relationships which can be difficult to put in an explicit analytical form. However, simulation modeling is not an optimization technique. It is, therefore, necessary to integrate the simulation model with an optimization tool in the later part of this chapter.

In our simulation model, the remanufacturing system is first modeled with a basic setting and then extended to various module assumptions and parameter settings such as the number of quality grades and other probabilistic parameters. We will carry out what-if scenario analysis and experimental design study to find the most important factors in the long term performance of the remanufacturing system.

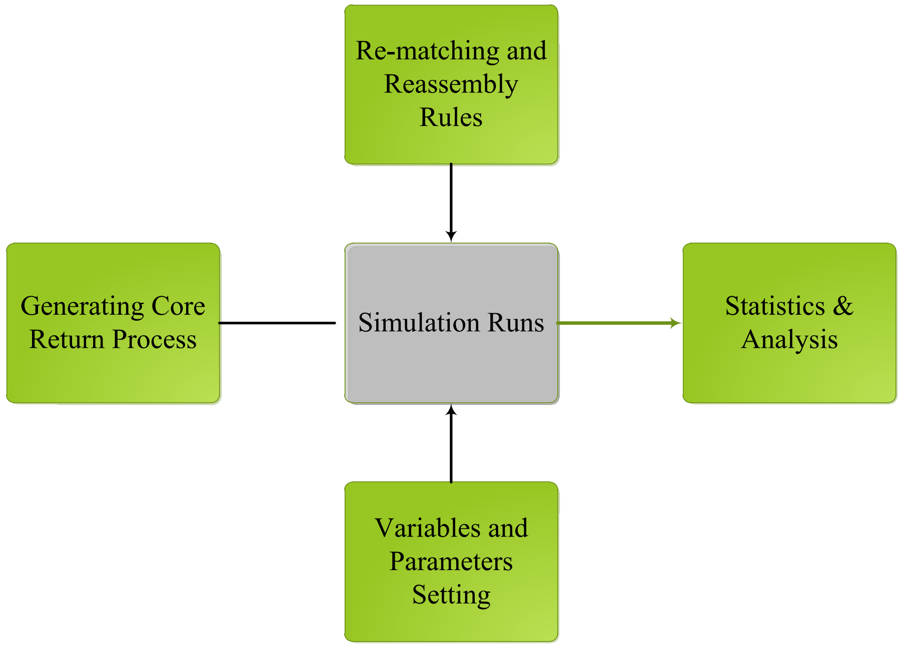


Figure 4.4 Overview of the simulation model

We want to simulate the preceding model, keeping track of the amounts of time spent in the system by each returned module, and the number of modules in each inventory. To model and analyze the system we will use the following variables:

Table 4.1 Variables and Parameters for Simulation Model

Time variable	
t	Time
Parameters	
λ	Aggregate return rate
α_i:	Probability that module i is defective and not remanufacturable
p_j:	Probability that the module is assigned as a grade-j module
Counter Variables	
D_j	Number of orders for quality-j product by time t

R_{ij}	Number of returns of module i with quality grade j by time t
N_i	Number of new module i being used by time t
Input Variables	
$A_{ij}(n)$:	The arrival time of n^{th} returned module i with quality grade j, $n \geq 1$
$D_{ij}(n)$:	The departure time of n^{th} i with quality grade j, $n \geq 1$
Output Variables	
I_{ij}	Inventory level of module i with quality grade j
U_{ij}	Stock out level of module i with quality grade j
W_{ij}	Waiting time of old module i with quality grade j
WCR	Warranty covered by remanufacturing, as an indication of reuse level.

Note: $i = 1,2,3;\ j = 1,2,3,4$

4.3.1 Generating the Core Return Process $\{A_{ij}\}$

The simulation model is able to accommodate discrete events for multiple generation products, i.e., arrivals of returns, departure of modules. The arrival of returned product is predicted to be a fixed fraction of the sales volume. For each product generation, product returns are predicted to follow a Poisson arrival process. Figure 4.5 represents an example of a EV sales volume for multiple product generation (green bars) and the time period for new production modules/parts for warranty repair (gray bars). Then, the compound arriving process of returns can be illustrated by Figure. 4.6 (a two generation product case).

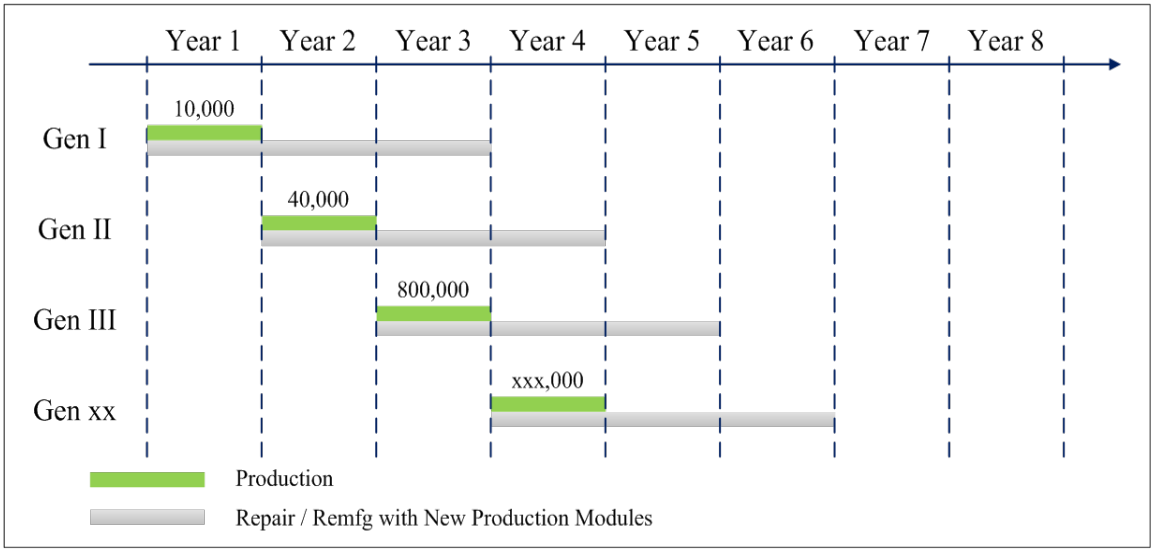


Figure 4.5 Production volume and new repair parts for batteries.

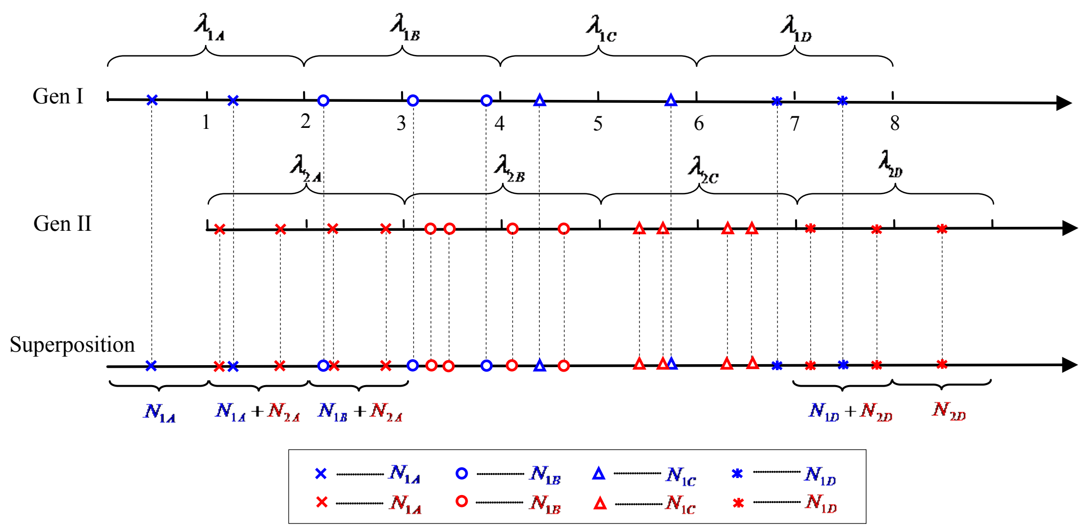


Figure 4.6 Random arrivals of product return of two generations.

For each generation of module i with quality grade j, ($i = 1,2,3$, and $j = A, B, C, D$), there is a counting process $\{N_{ijt}, t \geq 0\}$ which is a stochastic process defined on a sample space Θ such that for each $\omega \in \Theta$, the function $N_{ijt}(\omega)$ is a realization of the number of "return events" happening in the interval $(0, t]$, with $N_{ij0}(\omega) = 0$. The arrival process of the returns of one generation within eight-year warranty period consists of four consecutive homogeneous Poisson processes associated with four different quality grades. For example, Gen I arrivals occurring in calendar years 1 and 2 form a Poisson process $\{N_{1At}, t \geq 0\}$ with rate λ_{1A}, and arrivals form a Poisson process $\{N_{1Bt}, t \geq 0\}$with rate λ_{1B} in calendar years 3 and 4, $\{N_{1Ct}, t \geq 0\}$ with rate λ_{1C} in calendar years 5 and 6, and $\{N_{1D,t}, t \geq 0\}$ with rate λ_{1D} in years 7 and 8. Gen II arrivals in calendar years 2 and 3 form another Poisson process $(N_{2At}, t \geq 0\}$ with rate λ_{2A}, and so on so forth. Without loss of generality, we assume $\lambda_{1A} < \lambda_{2A} < \lambda_{3A} < \lambda_{4A}$, due to the increasing volume of sold vehicles with generation upgrading. We also assume $\lambda_{iA} = \lambda_{iB} = \lambda_{iC} = \lambda_{iD}$, so that the returns of different quality grade within same product generation have equal probabilities.

One should note that the degradation of products are assumed to be all identical and only associated with their ages. In the later part of this study, we will extend the models with additional degradation variation and uncertainty, i.e., a two years old battery can be in the *C* grade quality category with certain probability.

For each individual Poisson process $\{N_{ijt}, t \geq 0\}$, we need to generate the core arrival times in different time intervals $(0,2T), (2T, 4T), \dots, (6T, 8T)$ for Gen I products, and $(T, 3T), (3T, 5T), \dots, (7T, 9T)$ for Gen II, and so on so forth.

4.3.2 Backward Compatibility for Reassembly

The concept of backward or downward compatibility first appeared in the context of telecommunications and computing, such that a device or technology is said to be **backward** or **downward compatible** if it can work with input generated by an older device. More generally, a new component is said to be backward compatible if it provides all of the functionality of the old component. For lithium ion battery systems, the backward compatibility of the compact lithium ion battery enables the manufacturer to continue to install newer generation modules in the older generation battery platform.

Backward compatibility is important as a part of the engineering decisions for product design and development. Generally, newer-generation technology will reach its fullest potential only when all the parameters are optimized for its characteristics. However, in order to be backward compatible, the new arrays are designed to confirm to older-generation parameters so that part of their benefits is scarified.

4.3.3 Model Assumptions

Assumptions concerning the remanufacturing systems and assemble-to order operations are the following:

- Customer demands are classified to different quality grades. E ach order is confined to a unit of remanufactured product which is reassembled from M non-interchangeable modules.
- Each customer demand generated from a warranty claim is fulfilled from either remanufactured products or new products or a product reassembled by a mix of new and returned modules.
- New products are always available when needed to fulfill customer demand. No backorder or lost sales is allowed.
- The re-assemble process is instantaneous and the time is negligible.
- The remanufacturing facilities have unlimited capacities (e.g., inventory, assembly, etc.)

4.3.4 Battery Assembly with Weakest-link Principle

Most batteries are assembled in serial and parallel configurations, from cell-to-module and from module-to-pack. For assembling new battery packs, it is important to use the same battery type with equal capacity throughout and never mix different makes and sizes. A weaker cell causes an imbalance. This is especially critical in a serial configuration and a battery is only as strong as the weakest link. If the battery consists of serially connected modules, the whole battery is considered discharged when the weakest module is discharged. If all the modules have identical characteristics then they should all reach the discharged state about the same time. They all get charged equally and the charger monitors the total voltage to determine if they are fully charged. Imagine a chain with strong links and weak links. When the tension rises by a small weight pull applied to the chain, the weakest link will break first. The same happens when cells with different capacities are connected in serial in a battery. The weakest cell (module) will get exhausted more quickly than the stronger ones when in continued charge and discharge cycling. During charge, the weak ones get fully charged before the strong ones and get hot; during discharge the weak ones become empty before the strong ones and they are getting stressed.

Battery capacity inconformity is unavoidable in used batteries, it can hardly find the battery modules with exactly “identical” unused capacity (quality) – it is always desired to assemble the modules with closest capacity. The overall capacity of the reassembled battery will diminish to weakest link and the unused capacity of all the other modules is wasted.

We first show the simulation results of a basic model with out admission control. In Figure 4.7, the sample paths of on-hand inventory level for Module 1 of different quality grades and generations show the significant fluctuation in core inventory and large number of end inventory. The distribution of waiting time of core inventory is shown in Figure 4.8. Waiting times distribution can be fitted as a normal distribution with a mean of 82 days and a standard deviation of 27 days. These simulation-based results imply that with additional constraints on module re-matching in reassembly (issues of quality compatibility and backward compatibility), system performance is worsen by increased inventory level, fluctuation, and waiting time. Therefore, we add threshold-type admission control to the basic simulation model and discuss the system behavior.

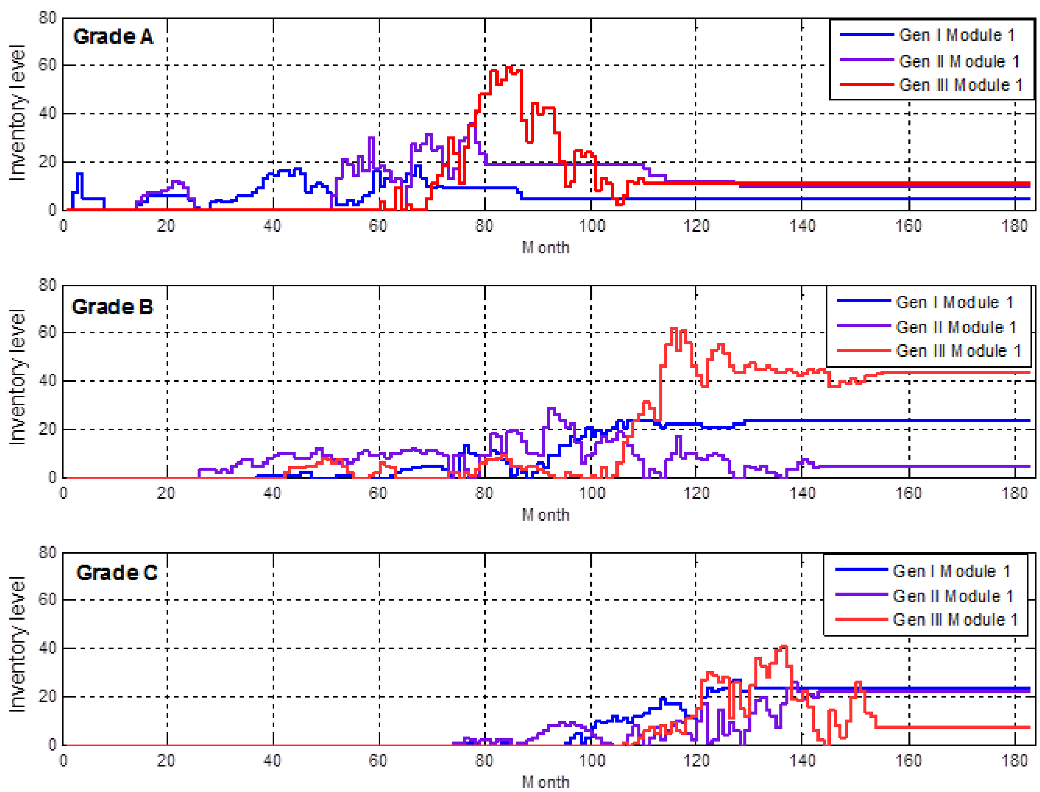


Figure 4.7 Sample paths of on-hand inventory without admission control.

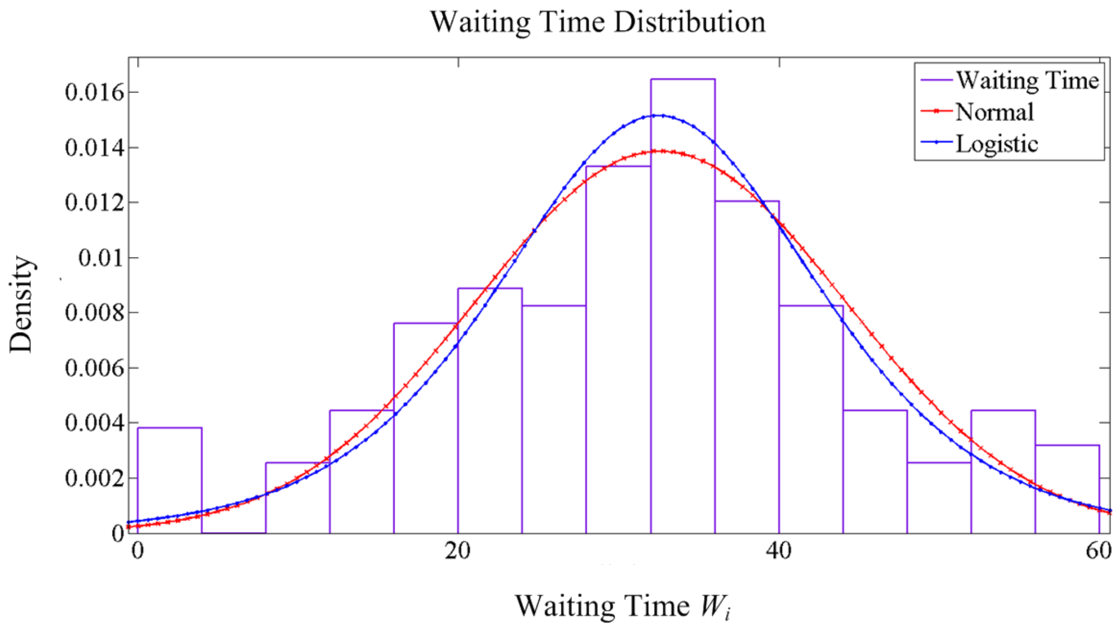


Figure 4.8 Waiting time distribution.

4.3.5 Simulation Results of a Basic Case

Despite the simplified assumptions of the analytical model, the results from the simulation model generally support the analytical results, suggesting the validity of admission control effects on system performance. This can be explained as follows. Analytical approaches have to do with simplifying reality so we can deal with the systems more effectively. The system dynamics in the analytical model doesn't change the realistic settings in the simulation model structurally.

The analytical result in the previous chapters shows that there is a dilemma in choosing admission - an increase in admission level improves the system reuse ratio while causes higher inventory level, which is also observed in the simulation. In other words, we observe a trade-off between WCR and inventory level (Table 4.2). When the admission level r_i increases, the WCR increases, indicating an increasing reuse level. However, the inventory cost increases due to increased inventory level.

Table 4.2 Simulation Results with Admission Control (Expected Values)

Admission level r_i			# New Module	# New Pack	*WCR*	End Inventory
r_1	r_2	r_3				
5	5	5	297	605	68.06%	30
10	10	10	297	579	68.83%	60
15	15	15	297	572	69.36%	90
20	20	20	297	569	69.59%	120
25	25	25	297	567	69.73%	150
30	30	30	297	565	69.89%	180
35	35	35	297	563	70.03%	210
40	40	40	297	562	70.12%	240

Figure 4.9 illustrates the relationship between the admission level r_i and the reuse ratio indicated by WCR. The experiment is simulated for 30 replications. Confirming analytical results, it shows that the WCR value increases with r_i.

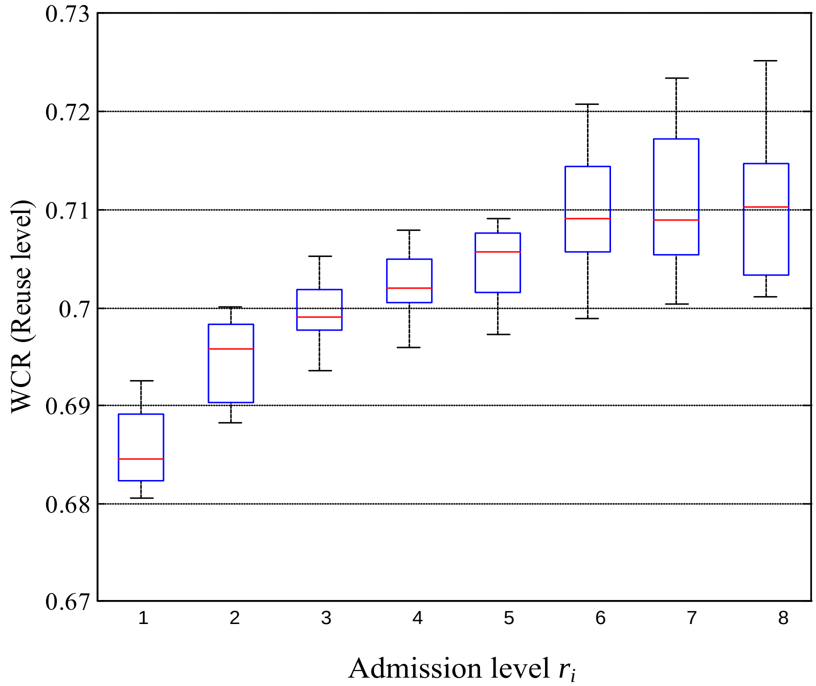


Figure 4.9 Simulation result – effect of admission level on reuse level.

4.4 Scenario Analysis

Using the simulation model, we also investigate different scenarios and discuss the impact of balance in module return, impact of quality variation, and disassembly degree for remanufacturing. The experimental setting is shown in Table 4.3.

Table 4.3 Experimental Design for Simulation Study

	Setting		
Return rates	Equal $\alpha_1 = \alpha_2 = \alpha_3$		Proportional $\alpha_1 \neq \alpha_2 \neq \alpha_3$
Matching Rule	Fixed grade		2-year window
Quality variance	Low		High
Disassembly level	1	2	3

We start the simulation with a basic model setting which ignores many possible variations and uncertainties in the real remanufacturing systems. We then expand the model gradually by including more complex rules and relaxing some assumptions, e.g. from a static matching rule to a dynamic matching rule with a moving window, relaxing the assumption of degradation conformity to models with high quality variation, etc. Then, the model is further extended from equal return probability to a unequal scenario in which the return rate of different type of module is proportional to the number of cells in that module. In order to examine the effects of the length of new module availability, the model is developed by assuming that new-generation module is available and backward compatible with old modules for a certain period.

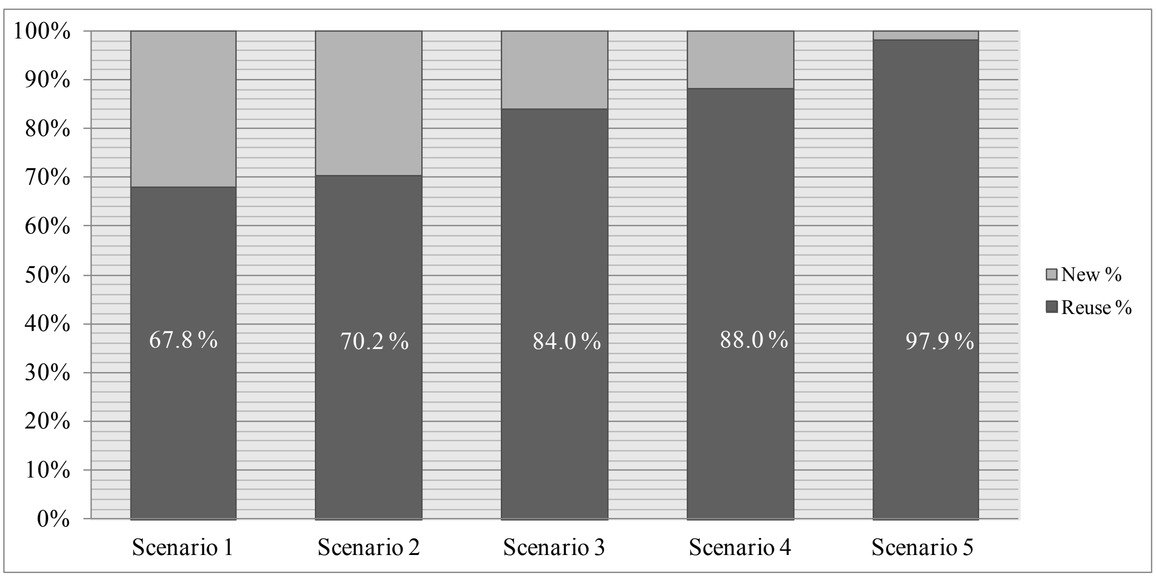


Figure 4.10 Comparison of *WCR* for different scenarios.

We illustrate some interesting insights from the comparison in Figure 4.10. (1) A moving window for re-matching improves the reuse rate. The explanation for this is that a two-year window increases the opportunities for re-matching which could be ruled out by a fixed same grade matching rule. (2) More balance in module returns, higher reuse ratio by reducing the risk of shortage of one type of module. (3) Techniques in transforming non-interchangeable modules to interchangeable ones have significant benefits in improving reusability. (4) Further disassembly techniques increase the reuse ratio of functioning subassemblies/components. For example, if module-to-cell disassembly can be done in a non-destructive way, more remanufacturable and reusable cells can be recovered.

4.5 Optimization of RATO Systems with Admission Control

In Chapter 3, we have derived exact expressions for different performance measures for the RATO system under admission control and analyzed how the system performance is affected with admission level variation for different parameter settings. In the previous part of this chapter, a simulation model was developed for a more complicated real production case. In this chapter, we will further develop the optimization models to determine the optimal or near-optimal admission thresholds for a given RATO system.

The goal of setting admission control on the collection site of end-of-life products is to meet certain performance target without violating budget constraints on the total cost. In a typical remanufacturing system, the performance target is the proportion of warranty replacement demand fulfilled by returned products and the total cost is usually a linear combination of inventory holding costs of returned products and cost of using new products (penalty due to shortage of remanufacturable inventory). High admission level leads to higher inventory level and hence higher fill rate. Low admission level reduces inventory cost but leads to great chances of shortfall. In addition to determining an effective admission level approach, managers seek to identify which parameter will be best to improve system performance. The work studied in this chapter is also motivated by end-of-life products management issues faced by a major Li-ion battery vehicle company in one of its remanufacturing facilities.

Our problem is most similar to the base-stock policy for an assemble-to-order (ATO) system with component supply uncertainty. Extensive work has been done in such type of systems; see, e.g., Song et al (1999, 2000), Song and Yao (2002), Gallien and Wein (2001). In those models for traditional ATO strategy in manufacturing systems, supply uncertainty is typically modeled as random lead times of components. Gurnani et al (1996, 2000) assumed a different type uncertainty that the yields at component suppliers were random. The reassemble-to-order (RATO) system studied in this work considers both supply uncertainties due to random quality and arriving times of end-of-life products.

4.6 Admission Policy I: Minimizing Cost under a Reuse Level Constraint

The system operates under a continuous-review threshold-based admission policy. Under this policy, when the inventory position of a module falls below a specific admission level r_i, an incoming module return is admitted to the remanufacturing facility; otherwise, rejected. If a customer demand for warranty replacement cannot be fulfilled by on-hand reusable modules immediately, there is a shortfall in other words, then initiate using new modules or product to satisfy the demand.

In Chapter 3, system state $Y_i\ (t)$ represents the number of outstanding order of module i in period t in the presence of transportation delay of returns; and r_i represents the admission level of module i. Now, let c_i be the corresponding cost per unit of node i.

Let $I_i(r_1, \dots, r_M)$ denote the steady-state on-hand inventory level of module i at the beginning of a period before demand for warranty replacement is fulfilled. Let α be the desired TCR level for the single quality demand. (see Section 3.4) That is, the chance of fulfilling demand by reassembly using all returned module inventories must be at least δ. Then the problem can be formulated as follows:

$$\text{Total Cost} = \min_{r_i} > 0 \sum_{i=1}^{M} h_i I_i + s_i U_i$$
$$s.t. P[I_1 \geq 0, \dots, I_M \geq 0] \geq \delta \tag{4.1}$$

The objective in (4.1) is a summation of inventory holding cost and the penalty cost due to shortfall. The reuse-level (i.e.,TCR) constraint is similar to a service-level constraints in conventional ATO system. The service-level constraint is commonly used in traditional ATO systems where δ is desired or target service level (Baker et al., 1986). In our problem δ is a d esired reuse level that reflects the fraction of orders totally fulfilled with remanufactured modules. The objective is to minimize the expected cost as a summation of inventory holding cost and penalty cost of shortage (substitution or use new), it reflects holding least inventories while maintaining a certain level of reusing remanufactured items.

Note that both $I_i(t) = [r_i - Y_i(t)]^+$ and $U_i(t) = [Y_i(t) - r_i]^+$ are convex functions because the $[\cdot]^+$ is a maximum operator and the maxima of linear functions is convex. Therefore, we can obtain the results as follows. On-hand inventory level I_i is increasing and convex in admission level r_i, and stock out level U_i is decreasing and convex in admission level r_i for all i. Here, decreasing is in the sense of stochastic ordering and convex is in the sense of strong convexity as defined in Shanthikumar and Yao (1991).

The increasing (decreasing) and convexity properties also apply to $E[I_i]$ and $E[U_i]$. The properties of on-hand inventories implies an optimization problem on t rading off inventory and fill rate.

4.6.1 Relation between Inventory Levels and Admission Thresholds

Note that the on-hand inventory level of module i is $I_i(t) = [r_i - Y_i(t)]^+$. Hence, the probability of *totally covered by remanufacturing* (TCR) is $P[r_i - Y_i \geq 0, \dots, r_M - Y_M \geq 0]$. Then the problem in (4.1) can be rewritten as follows:

$$\begin{aligned} &\min_{r_i} > 0 \sum_{i=1}^{M} h_i I_i + s_i U_i \\ &s.t. P[r_i - Y_i \geq 0, \dots, r_M - Y_M \geq 0] \geq \delta \end{aligned} \tag{4.2}$$

Since the joint distribution of $Y_i(t), i = 1, \dots, M$ is *quasi-concave*, and the feasible region, $\varphi(r_1, \dots, r_M)$, is a closed convex set. Hence formulation (4.2) is a convex programming problem.

Note that in the performance analysis in Chapter 3, admission levels for a single quality case are set independently and they ignore the inherent relationships. We now characterize the relationship between the admission levels r_1 and r_2 (for the two module case). Propositions 4.1 and 4.2 describe the relationship between feasible admission levels of different modules.

PROPOSITION 4.1 *r_j is decreasing in r_k $(k \neq j)$ with $(k, j) \in \{1,2, \dots, M\}$*

The intuition behind Proposition 4.1 is as follows: An additional inventory of a module allows the inventory of the other module to be reduced due to the decreased coordination pressures.

PROPOSITION 4.2 *If the return rate of Module 1 is stochastically greater than that of Module 2 (i.e., $\alpha_2 \geq \alpha_1$), then the optimal admission threshold satisfy $r_1{}^* \leq r_2{}^*$.*

Proof Proposition 4.2 *See Appendix B.*

COROLLARY 4.1 *If $\alpha_1 = \alpha_2$, then in the optimal solution, $r_1{}^* = r_2{}^*$*

PROPOSITION 4.3 The optimal admission threshold, $r_i{}^*$, decreases with h_i, for $i = 1, \dots, M$.

Proposition 4.3 is intuitively true because module inventory should be kept at a lower level as the unit inventory cost of this module increases.

4.6.2 Simulation-Based Optimization

The RATO optimization problem in (4.1) is a convex programming problem. Therefore, we can obtain the optimal admission threshold levels using a gradient-based search. Since our problem has a nonlinear fill rate δ constraint that add "noise" to the problems, it requires an estimation of the fill rate measure of δ by simulation (Bashyam and Fu, 1998). We propose the following related problem:

$$\begin{aligned} \delta(B) &= \max F \\ s.t. \sum_{i=1}^{M} &(h_i I_i + s_i U_i) = B \end{aligned} \tag{4.3}$$

Here, F is adapted from the fill rate defined in Chapter 3, that is, $F = P(I_1 > 0, \dots, I_M > 0)$. The solution to this problem provides the highest fill rate $\delta(B)$ for a given total cost constraint B. If we approximate the problem by considering the module-wise fill rate and rewrite it in terms of the decision variable r_i, then we have:

$$\delta(B) = max_{r_i} P[r_i - Y_i > 0]$$
$$s.t. \sum_{i=1}^{M} (h_i[r_i - Y_i]^+ + s_i[Y_i - r_i]^+) = B, \quad i = 1,2,\dots,M \tag{4.4}$$

Noting that $\delta(B)$ is increasing in B, the original optimization problem (4.1) can be solved by searching for the minimal B that satisfies $\delta(B) \geq \delta$. This problem can be solved using gradient-based search but it can be computationally intensive, especially when the number of modules increases. These difficulties motivate us to develop a more computationally efficient heuristic approach.

4.6.3 A Heuristic Decomposition Algorithm

Let F_i be the probability that on-hand inventory of module i is no less than zero, that is, $F_i = P[r_i - Y_i \geq 0]$. In Chapter 3, this probability is defined as the fill rate of individual module i, which depends only on r_i and Y_i. Hence, the determination of admission level r_i is independent of other modules once F_i is specified.

PROPOSITION 4.4 *$f(F_i)$ is unimodal in F_i.*[1]

From the numerical experiments in Chapter 3, we found that the overall inventory level and associated cost are insensitive to the individual module fill rate. This motivated us to use the F_i as the module-specific parameters in developing a heuristic approach for near-optimal solutions. Then we solve the following related problem:

1. A function $f(x)$ is unimodal on [a,b] if for some point $\bar{x}$ on [a,b], f(x) is strictly increasing on [a, $\bar{x}$] and strictly decreasing on [$\bar{x}$,b].

$$\min_{r_i} > 0 \sum_{i=1}^{M} h_i I_i + s_i U_i$$
$$s.t. \mathrm{P}[r_i - Y_i \geq 0] = \delta_i, \qquad \text{for } i = 1, \dots, M \tag{4.5}$$

where δ_i is a given positive number between 0 and 1 representing the specified target fill rate. Despite its simple form, the problem in (4.5) is computationally intensive. We develop heuristic approaches which are computationally-tractable as su rrogates in the optimization problem. Set δ_i to be a function of single module-specific problem parameters. That is, we set $\delta_i = f^i(r_i, \lambda, \mu_i, c_i, s_i)$. Then we solve the following M module-wise problems:

$$\min_{0 \leq \delta_i \leq 1} f(\delta_i) = \min_{r_i > 0}\{h_i I_i + s_i U_i\}$$
$$s.t. P[r_i - Y_i \geq 0] = \delta_i, i = 1, \dots, M \tag{4.6}$$

Analysis in Chapter 3 demonstrates that cost function $f(F_i)$ is quasi-convex in F_i. We only need to search over F_i and find the optimal F^* that minimizes$f(F_i)$. In order to test the heuristic algorithm, we set $F_i = \delta$, for all i, and compare the resulting solution with the optimal solution. The setting of identical fill rate among component inventories is commonly implemented in industrial practice (Agrawal and Cohen, 2001). The module-based fill rate provides a simple but effective mechanism to coordinate inventory of different modules.

The *golden section search* is a technique for finding the minimum or maximum of a unimodal function by successively narrowing the range of values inside which the extremum is known to exist. The sequence of probing the function is by selecting triples of points whose distances form a golden ratio (Kiefer 1953, Avriel and Wilde 1966). Fibonacci search and binary search are two closely related algorithms. Diagram in Figure

4.11 illustrates the basic idea of a golden section search method. Details of the heuristic decomposition algorithm are presented in Figure 4.12.

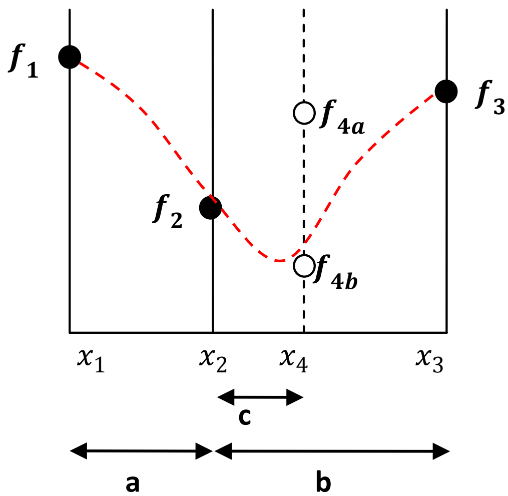


Figure 4.11 Diagram of a golden section search.

Procedure of the heuristic decomposition algorithm

Step 0.	$\varphi = (\sqrt{5} - 1)/2$
Step 1.	$x_1 = 0, x_3 = 1$
Step 2.	$x_2 = \varphi x_1 + (1-\varphi)x_3, x_4 = (1-\varphi)x_1 + \varphi x_3$

Step 3.	for i from 1 to $N-2$
	{
	$r_i = \min\{r_i \mid P(r_i - Y_i > 0) > x_2\}$,
	$f_2 = f(x_2) = (c_i I_i + s_i U_i)\vert_{F_i > x_2}$,
	$f_4 = f(x_4) = (c_i I_i + s_i U_i)\vert_{F_i > x_4}$.
	If $f_2 < f_4$, then $x_1 = x_2,\ x_2 = x_4$
	Else, then $x_3 = x_4,\ x_4 = x_2$
	If $\vert x_1, x_3\vert < \varepsilon$, then go to Step 4.
	}

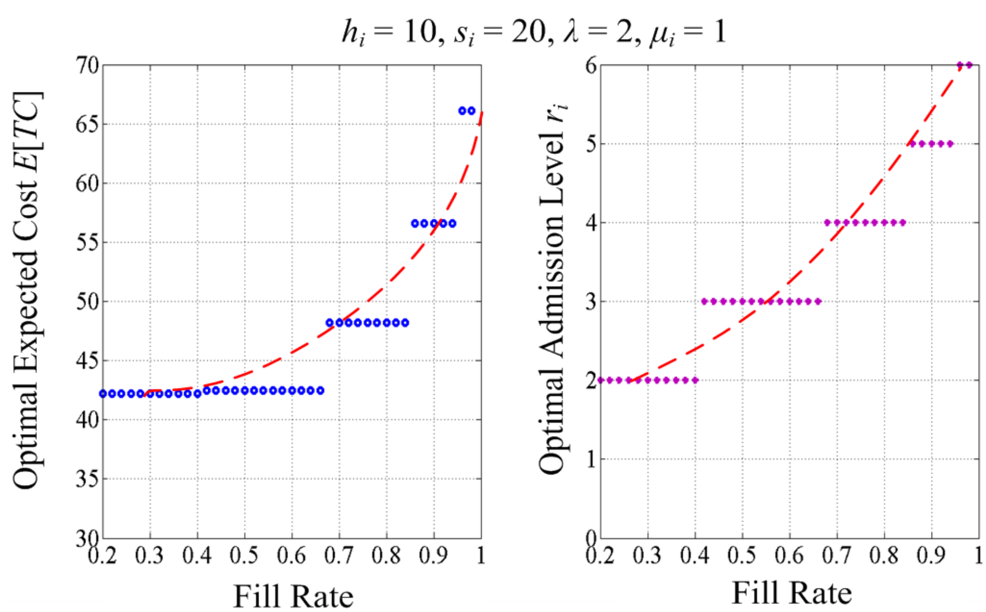
Step 4.	if $f_2 < f_4$, then the desired F_i is in interval $[x_2, x_3]$ else F_i lies in the interval $[x_1, x_4]$ end. □

Figure 4.12 Heuristic approach using golden section search.

The independent fill rate provides a simple yet efficient way for jointly managing inventories of different modules. In this case, the central planner may specify how much the target reuse level, then different module inventories will be controlled independently and will carry the desired module inventories.

4.6.4 Computational Results

The test problem instances were generated for small instances in order to test the robustness of the decomposition heuristic, i.e., the number of modules was chosen from [2,3,4,5]. Larger instances cannot be efficiently computed by the optimization algorithm in Section 4.6.2. We set the unit inventory cost of module i as c_i = 10, and unit shortage penalty $c_i = \{20, 80, 100, 200\}$. The demand rate for warranty replacement is $\lambda =$ 1, 2, 3, and exponentially-distributed return transportation time parameter μ_i= 0.8, 1, 2, 3. For all these cases, the decomposition heuristic algorithm provides the same optimal solution as the simulation-based optimization method. In the first experiment, we fixed h_i = 10, $\lambda = 2$, and μ_i=1 and studied the optimal expected cost $E[TC]$ and optimal admission level r_i with three different levels of unit penalty cost, i.e., $s_i = 20, 80$ and 200. Figure 4.13 shows the case when c_i and s_i values are relatively close indicating similar weights on inventory and shortage in the objective function. In this case, the optimal expected cost increases as the fill rate target increases. Clearly, the optimal admission level also increases as F_i increases. In Figure 4.13, the $E[TC]$ value jumps because the admission level can only take integer numbers. Figure 4.14 represents a case when there is an intermediate difference between unit penalty cost and unit inventory cost (c_i<h_i). The results of the heuristic algorithms show that the optimal expected cost is a convex function of the fill rate target and exist an optimal fill rate target F_i^* that minimize the objective function. In this case, the optimal admission level is 4 and the corresponding $E[TC]$ is 104. The last case shown in Figure 4.15 represents the situation when the penalty of shortage has a much higher weight than the unit inventory cost - which is common for battery manufacturing industry. Since the shortage penalty dominates the overall cost, low fill rate target could lead to high $E[TC]$, see Figure 4.15(a). Therefore, optimal admission level should be high (in this case r_i^*=5) to avoid risks of shortage with relatively small effect of increased inventory.



(a) (b)

Figure 4.13 Optimal inventory-fill rate trade-off: $s_i = 20$.

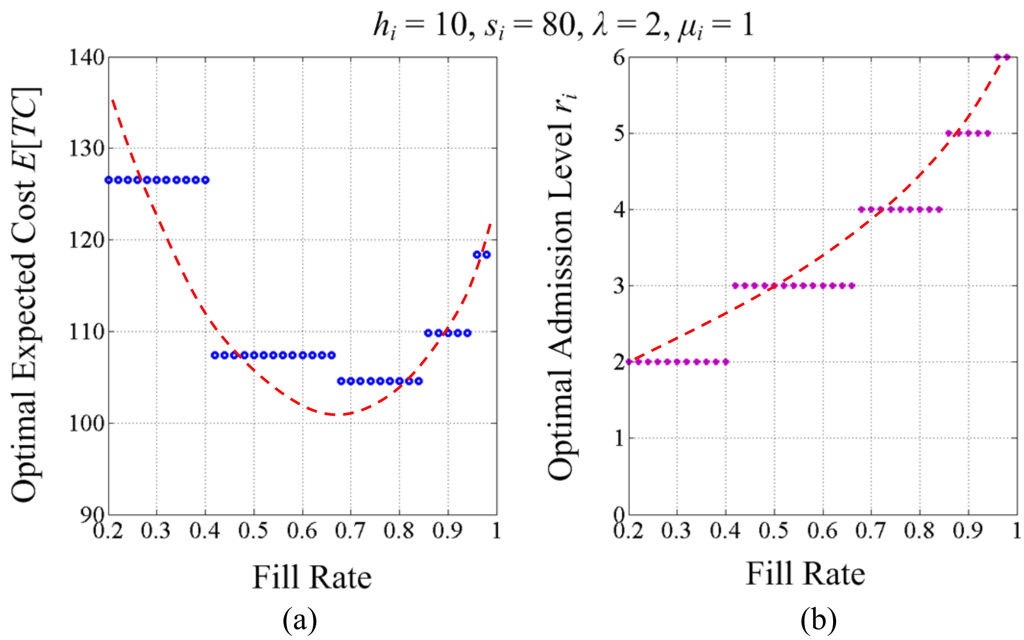


Figure 4.14 Optimal inventory-fill rate trade-off: $s_i = 80$.

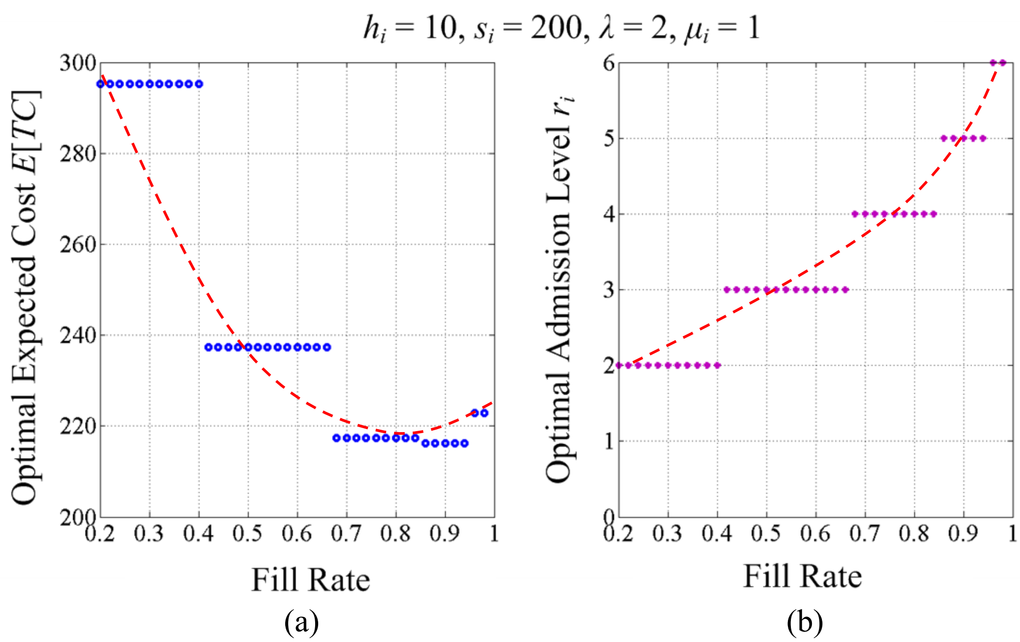
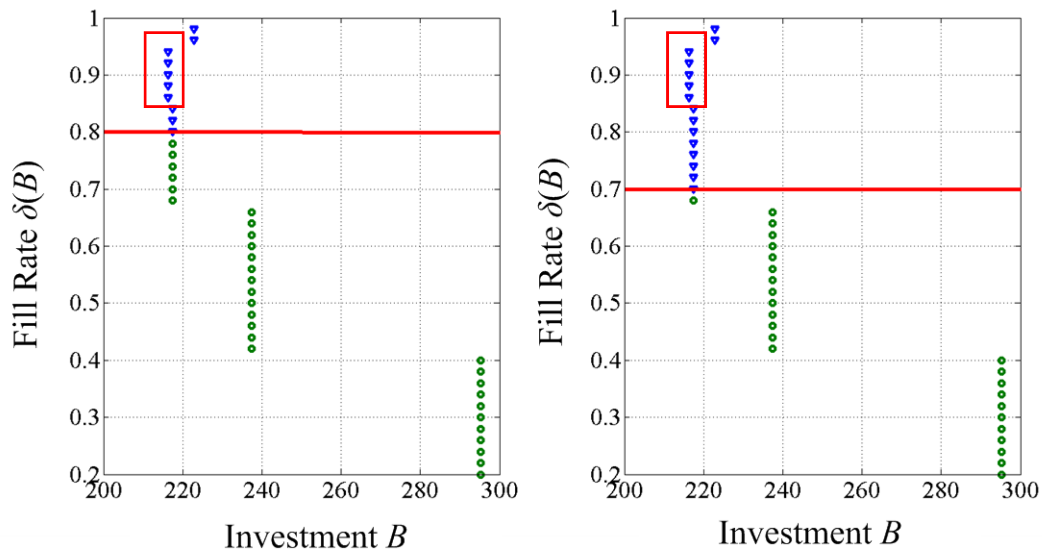
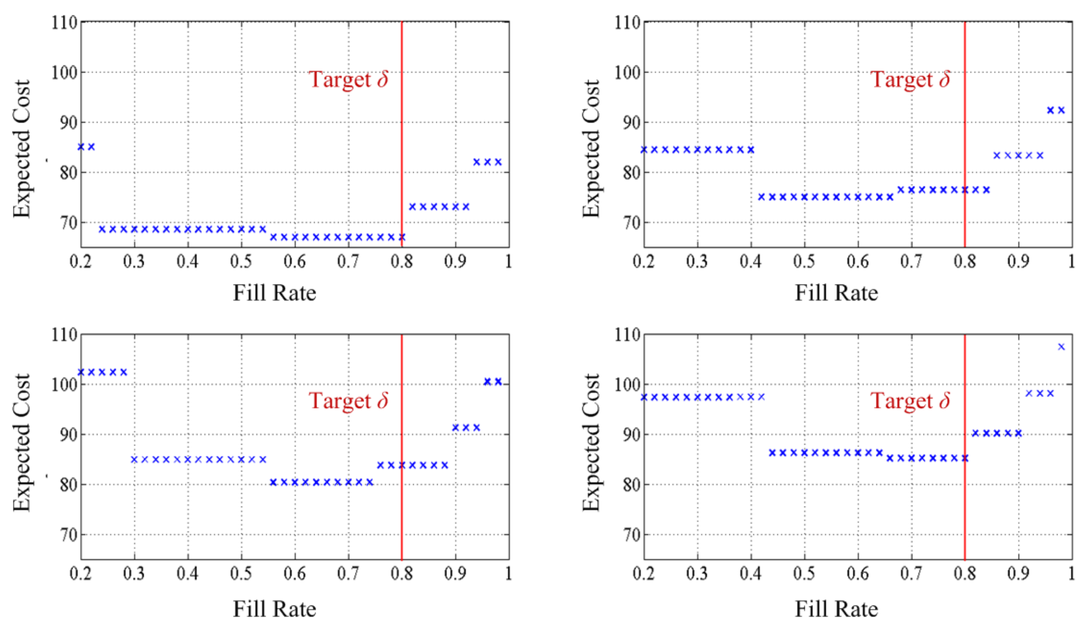


Figure 4.15 Optimal inventory-fill rate trade-off: $s_i = 200$.



(a) $\delta = 0.8$ (b) $\delta = 0.7$

Figure 4.16 $\delta(B)$ versus total constraint$B$.



(a) (b)

Figure 4.17 Comparison of optimal cost achieved for different λ.

Recall the optimization problem (4.4), we solve the problem by searching for the minimal B that satisfies $\delta(B) \geq \delta$. For different fill rate constraint, $\delta = 0.7$ or 0.8, we find that the minimal B values for both cases (highlighted in red boxes) are the same (see Figure 4.16). In fact, when the fill rate target falls in [0.85, 0.95], the expected cost achieves the minimum value.

Figure 4.17 compares four instances with different demand rate λ for the objective function value. It is observed that with certain reuse level constraint (fill rate target), the overall expected cost doesn't necessarily achieve global minimum. For example, when fill rate target is 0.8 with $\lambda = 1.5$ and 2, the expected cost has the second best value. This implies that if the firm seeks to achieve minimal expected cost, the fill rate will be reduced. For the instances with $\lambda = 1$ and 2.5, the objective function achieves the minimum with the fill rate target $\delta = 0.8$.

Table 4.4 Minimum Cost with Fill Rate Constraint $\delta = 0.9$

Instance	c_i	s_i	λ	μ_i	$E[TC]$	$r_1{}^*$	$r_2{}^*$	$E[I_i]$	$E[U_i]$	δ_i
1	10	200	1	1	157.64	3	3	2.65	0.065	0.9161
2	10	200	3	1	240.19	6	6	4.00	1.001	0.9161
3	10	200	5	1	259.26	9	9	5.04	1.047	0.9319
4	10	200	1	0.8	207.65	3	3	2.65	0.905	0.9197
5	10	200	2	0.8	316.30	5	5	3.89	1.387	0.9474
6	10	200	3	0.8	391.15	7	7	4.97	1.717	0.9665
7	10	200	3	1	240.19	6	6	4.00	1.001	0.9161
8	10	200	3	2	40.00	6	6	4.00	0	0.9161
9	10	200	3	3	40.00	6	6	4.00	0	0.9161

10	10	20	1	1	39.66	3	3	2.65	0.655	0.9197
11	10	20	3	1	56.61	5	5	3.89	0.887	0.9473
12	10	20	5	1	60.03	6	6	4.00	1.001	0.9161
13	10	20	1	0.8	44.66	3	3	2.65	0.905	0.9197
14	10	20	2	0.8	66.61	5	5	3.89	1.387	0.9473
15	10	20	3	0.8	75.03	6	6	4.00	1.751	0.9161
16	10	20	3	1	56.61	5	5	3.89	0.887	0.9473
17	10	20	3	2	20.03	6	6	4.00	0	0.9161
18	10	20	3	3	20.03	6	6	4.00	0	0.9161

In Table 4.4, we show the results for a subset of tested (λ, μ_i) values with target fill rate $\delta = 0.9$. Specifically, we divide our test into two groups: the first half set with expensive unit penalty cost and the second half instances with relatively low unit penalty cost. From instances {1,2,3}, {4,5,6}, {10,11,12} and {13,14,15}, we see that (i) as λ increases, the total inventory investment increases; (ii) the individual module admission level r_i should increase as demand rate λ increases. From instances {7,8,9} and {16,17,18}, we observe that higher return rate could improve the system performance in terms of the expected total cost for both groups of instances while the optimal admission levels and fill rates are stable. An explanation for the observed "stable" performance of these cases is as follows. When return rate increases, the probability of shortfall decreases while inventory level increases. The advantage of deploying admission policy is strengthened because the admission policy can effectively control the incoming returns to achieve minimal cost with fill rate constraints. This suggests that the admission control might be more effective and critical for high return rate (or intermediate traffic intensity, e.g., $\frac{\lambda}{\mu_i} = \frac{3}{1}, \frac{3}{2}, \frac{3}{3}$).

4.7 Admission Policy II: Time Value Driven Policy

In the section, we consider an admission policy driven by an objective of minimizing the waiting time of returned modules. This is motivated by t he goal of achieving high time value of EOL products by reducing the non-value added time and value decay in the remanufacturing system. In the existing literatures on the production planning problem with remanufacturing, there is typically no consideration of perishable items or items whose quality deteriorates over time. T herefore, there is usually no limitation on how long a used item can be carried in inventory (Golany et al., 2001).

In conventional manufacturing systems, the idea of "lean manufacturing" emphasizes the significance of reducing inventory level and inventory time hence reduced costs. For a typical remanufacturing system, the costs are not only incurred by holding inventory but also incurred by m aintaining and keeping the returned products from condition degradation and being perished. Common practice in some manufacturing and inventory systems where the maintenance and storage of products or components incurs additional costs, e.g., regular maintenance cost, large spaces for inventory, temperature control, discounting effect on some short-life cycle products, etc.

The value of the recoverable assets from product returns obsoletes or erodes away in the return process and during their stay in the remanufacturing system. Most return processes and EOL products acquisition process in place today were designed for an earlier stage of the product life cycle when return rates were relatively low and the effects of time value decay is insignificant. Typically, return processes were typically designed for cost efficiency where collection networks minimized logistics costs and the need for managerial oversight. Current design of return processes driven by a narrow operational cost focus (e.g., only inventory costs, logistics cost) can lead to time delays and waiting times that limit the options available for remanufacturing and reuse. The limited product remanufacturing opportunities can lead to substantial losses in product value recovery, especially for short life- cycle, time-sensitive products (Guide et al. 2006). Therefore, there is a need for design admission strategies that achieve efficient asset recovery by remanufacturing and reduce time value decay.

4.7.1 The Time Value for Product Returns

In this study, we explicitly consider and evaluate the decayed product value because of time delays at each stage after the product return is created. Blackburn (1991) has studied a time-based competition model which showed that faster response in business processes could add to the company's competitiveness. Some other studies have developed models to quantify the negative impact of time delays in conventional manufacturing supply chains (Blackburn 2002). Guide et al.(2006) presented a network flow with delay models to identify the drivers of reverse supply chain design. They examined how industry clockspeed generally affect the choice between an efficient and a responsive returns process.

According to the theory of time preference (Frederick 2002), investors would rather have cash immediately than having to wait and must therefore be compensated by paying for the delay. Returned products, as an asset of a company, also have the value discounted because they there is an increase of risk that they might not be valuable for remanufacturing in the future.

We design and evaluate admission policies for the multiple quality RATO model with the goal of maximizing the time value of a returned product. First, we derive an exact expression of the probability distribution of module's waiting times in inventory. Second, we model the optimization problem such that the optimal admission level is determined to maximize time value of products. Here, waiting time refers to the total time a returned module spent in the inventory before being reused.

4.7.2 Waiting Times of Cores in a RATO System with Admission Control

In this section, we derive the probability distribution for the waiting times of each module i following the analysis in Section 3.5. The basic idea of finding the distribution of waiting times of module i, W_i, is to condition on the state (number of outstanding orders of module i) that a demand observes upon its arrival, $Y_i = y_i$, and inventory on hand is $r_i - y_i$, $i \in S$. Conditioning on $Y_i = y_i$, we obtain

$$P(W_i \le t) = \sum_{y_i=0}^{\infty} P(W_i \le t | Y_i = y_i) P(Y_i = y_i) \quad (4.8)$$

Observe that, if a warranty return observes state $Y_i = y_i$, then there will be $r_i - y_i$ orders in front of it in the queue i ($r_i \ge y_i$ in this case). Hence the new warranty return will be the $(r_i - y_i + 1)^{st}$ item in the inventory queue whose waiting time is the sum of $(r_i - y_i + 1)$ exponential random variables with rate μ_i, which follows an $Erlang - (i, \lambda_i)$ distribution. Letting $\varphi_i(t)$ be the c.d.f. of the *Erlang* distribution, we now have

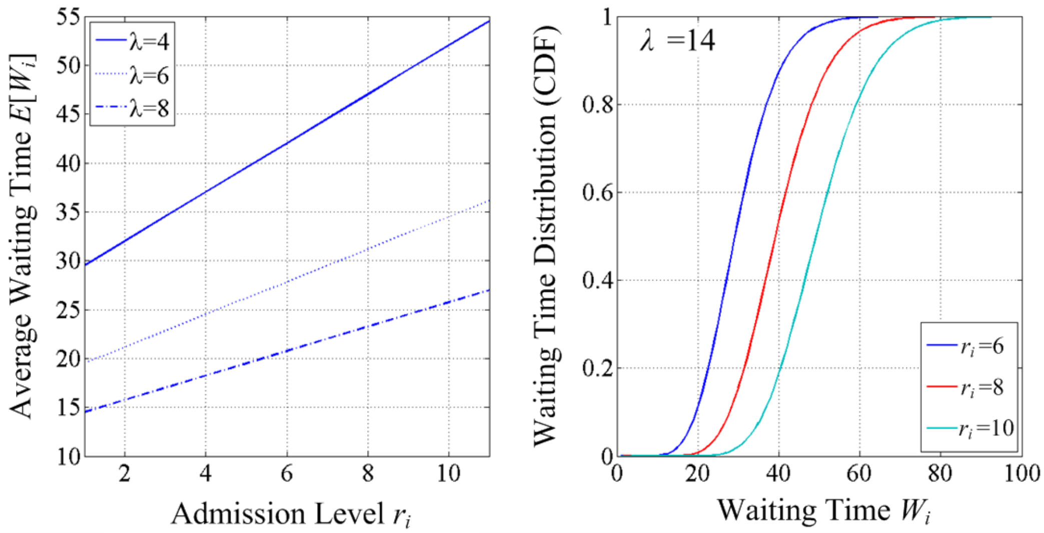
$$\sum_{y_i=0}^{\infty} P(W_i \le t | Y_i = y_i) P(Y_i = y_i) = \sum_{y_i=r_i}^{\infty} \varphi_i(t) P(Y_i = y_i) \tag{4.9}$$

where $\varphi_i(t) = \begin{cases} 1 - \sum_{k=0}^{r_i - y_i + 1} \frac{(\lambda_i t)^k e^{-\lambda_i t}}{k!}, & y_i \le r_i \\ 0, & otherwise. \end{cases}$ and $P(Y_i = y_i) = \frac{(\lambda_i t)^{y_i}}{y_i!} e^{-\lambda_i t}$.

Substituting (4.9) into (4.8), we get

$$P(W_i \le t) = \sum_{y_i=0}^{r_i} [1 - \sum_{k=0}^{r_i - y_i + 1} \frac{(\lambda_i t)^k e^{-\lambda_i t}}{k!}] \frac{(\lambda_i t)^{y_i}}{k!} e^{-\lambda_i t} + \sum_{y_i = r_i + 1}^{\infty} \frac{(\lambda_i t)^{y_i}}{k!} e^{-\lambda_i t} \tag{4.10}$$

The resulting curves show the stocking time distribution for different admission threshold r_i. It is clear from the figure that, the probability that the module i is reused by time t declines as r_i increases. That is, the modules are more likely to have longer waiting time with a large admission threshold (see Figure 4.18 (a) and Figure 4.18 (b)). This observation is not surprising because larger admission thresholds increase the inventory queue length and thus longer waiting time for incoming returns according to the FCFS policy.



(a) (b)

Figure 4.18 (a) Average waiting times; (b) Waiting time cumulative density function.

4.7.3 Time Value Decay of Returned Products

We define time $t = 0$ as the beginning of the steady state period for product returns, and $t = \Gamma$ as the end of steady state period for returns. We assume returned and remanufacturable products (modules) have exponential value decay functions, i.e.,

$$V[t] = V[0]e^{-\varepsilon t} \tag{4.11}$$

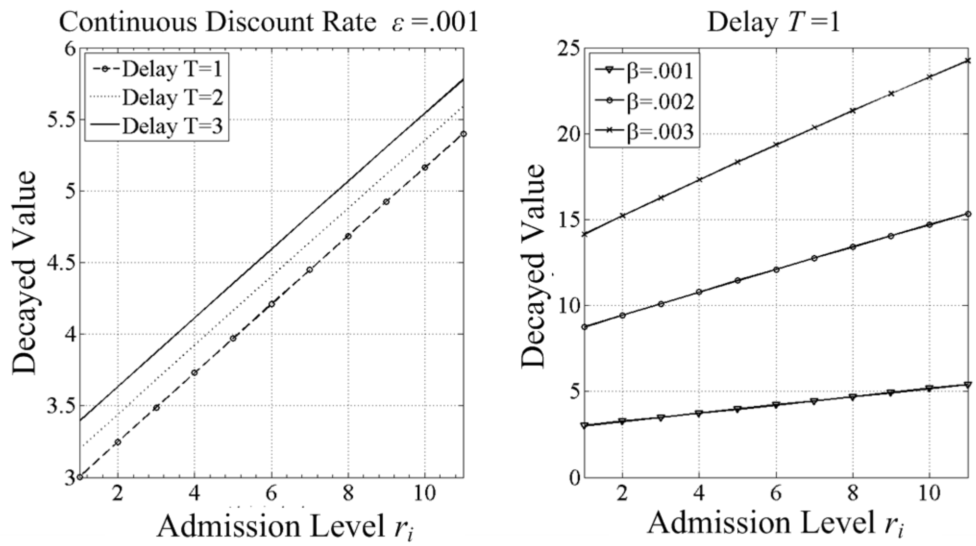
where ε is continuous-time value decay parameter (i.e., % value decay per unit of time). The decay parameter can be viewed as a m easure of industry clockspeed (see., e.g., Williams 1992, Mendelson and Pillail 1999). We make an assumption on the end-of-use product returns as follows:

ASSUMPTION 4.1 *Products and modules are only remanufacturable once. That is, a module goes through the remanufacturing system only once.*

At time t, the product failure occurs and it is shipped to the remanufactured facilities at a unit value $V[t]$. At time $t + T_i$, the product arrives at the remanufacturing facility and remanufacturable or reusable modules are sent to inventory after disassembly and sorting. Here T_i is the transportation delay for the returned products (modules), the value of remanufacturable modules decays to $V[t + T_i]$. At time $t + T_i + W_i$, the returned module is used for reassembly for a warranty replacement. W_i is the sojourn time of Module i in the inventory, where the module waits until they are reused. The residual value of the product becomes $V[t + T_i + W_i]$. Noticing that W_i is determined by the admission threshold r_i, more precisely, we can rewrite it as $W_i[r_i]$. Let DV denote the decayed value during the period of transportation delay and waiting time in inventory. Then,

$$DV = V[t + T_i + W_i(r_i)] - V[t]$$
$$= 1 - e^{-\varepsilon[T_i + W_i(r_i)]} \qquad (4.12)$$

Formula (4.12) represents, discounted over $T_i + W_i(r_i)$, the residual value of returned products decays exponentially with discount rate β. For tractability, we assume deterministic identical transportation delay for all types of module, i.e., $T_i = T$ for all i. Figure 4.19 (a) shows that, given discount rate β = 0.001, how the module residual value decays through time $T_i + W_i(r_i)$ with admission level r_i for three different T $(= 1,2,3)$. It is observed that the residual value decays almost linearly as the admission threshold increases. In Figure 4.19 (b), we show that given a fixed transportation delay, T, how the value decays for different admission thresholds with three levels of discount rate ε = 0.001, 0.002 and 0.003, respectively. The larger discount rate is, the steeper the decay curve turns out to be. Most important insights from Figure 4.19 are that time value decay always increases as the admission threshold r_i increases. This can be explained by (4.10) and Figure 4.18 that W_i increases as r_i increases.



(a) (b)

Figure 4.19 Time value decay (a) Effects of T; (b) Effects of β.

Further analysis could consider the trade-off between time value decay and the penalty due to shortfall. Optimal admission threshold can be determined by minimizing the time-value decay function with some fill rate target as constraints. This optimization problem can be formulated and solved in a similar way as the total cost optimization model in Section 4.6.

4.8 Conclusions

In this chapter, we presented a first attempt to find optimal admission threshold levels under both return and demand uncertainty in a RATO system with a reuse level constraint. We showed that total cost composed of inventory holding cost and penalty cost due to shortfall could be minimized using convex programming (with quasi-concave constraint). To solve this problem with sizable module types, we proposed a heuristic approach where admission levels could be determined independently based on decomposed module-based fill rate targets. We also demonstrated that heuristic algorithm was very effective in attaining near-optimal solutions by comparing with a simulation-based optimization solutions and current practice.

The second admission level decisions are provided by examining the time value decay of returned products. By analyzing waiting time probability distribution, we derived a value decay function for returned products and determine the admission levels that are within the value decay limit.

CHAPTER 5

CONCLUSIONS AND FUTURE WORK

5.1 Conclusions and Contributions

This dissertation focuses on decision-making for returned product remanufacturing and inventory management with variable quality grades. The chapters within the dissertation studied three problems in the domain of remanufacturing systems and inventory management, managing exogenous demand for remanufactured products for warranty replacement, and allocation of uncertain end-of-life products/module returns in a reassemble-to-order environment.

The major achievements of this dissertation can be summarized as follows:

(1) *Development of an optimal reassembly-inventory model for stochastic remanufacturing systems*: A reassemble-to-order system in a remanufacturing setting has been considered, in which there are uncertainties in terms of product returns quality, quantity and arriving times, as well as multiple quality classes of demand for remanufactured products. A full characterization of the joint reassembly and inventory decisions is provided by modeling and solving a multi-dimensional Markov Decision Process. We showed that the optimal reassembly policy could be characterized by a multi-level state-dependent threshold policy that exhibited distinct forms across two regions of the entire

state-space. For each demand class (indicated by quality grade), we showed how the state-dependent thresholds depended on the inventory levels of other modules.

(2) *Managerial insights for optimal reassembly-inventory decisions for remanufacturing systems*: We explored the monotonicity properties of the optimal policy under various problem parameters. Extensive numerical study provided interesting and important insights to the remanufacturing and reassembly decisions such as (1) substitution may not be the optimal decision when you have more substitutable modules in stock; the optimal decision is jointly determined by both substitutable and non-substitutable module inventories; (2) large variation in module quality leads to significant imbalance during reassembly, hence reducing the total utilization of returned modules; and (3) optimal decisions for reassembly are very different from currently implemented practices. Simple rules such as FCFS based exhaustive reassembly policy may cause the system to be less cost-effectively. Based on the structural properties, we also introduced two novel heuristic policies for large-scale problems motivated by the insights we gained from the optimal policy structure. The performance of the two heuristic policies is tested against the optimal solution as well as a commonly implemented “exhaustive” policy.

(3) *Investigation of system performance with admission policy in product returns*: A reassemble-to-order system with certain admission rules has been proposed to analyze the system performance under various end-of-life return admission

policies. Our first contribution was to derive exact expressions of performance measures (e.g., module-level inventory, stock out level, reuse level, waiting time, etc.) when both warranty returns and demand are uncertain in time and quality. An efficient computational solution method for the multi-dimensional queueing problem is proposed. I t provides computationally efficient and stable solutions especially when the problem is scaled to many module types and many quality classes. We developed analytical solutions to evaluate how admission thresholds affect the performances and discuss their implications. The results shed light on ho w system parameters affect performance, and also lead to inner trade-off between reuse level and operation costs.

(4) *Optimization modeling for admission thresholds*: The work has been extended to an optimization problem of selecting optimal admission policy. Two different objectives for attaining optimal admission level for each module inventory were set: The first model was to minimize the summation of inventory holding cost and penalty cost due to shortfall with a reuse level constraint. This model was essentially to seek an optimal trade-off between excessive inventory and risk of understock. A decomposition heuristic algorithm was proposed in order to solve for the convex program. The main results showed that the admission threshold level for module i, r_i, uniquely determined by the module-based fill rate target. The second decision model presented a novel approach from the time value perspective. The probability distribution of waiting times of returned modules was derived under variant

admission levels. A time value decay function was formulated to examine the impact of time delays in inventory on the residual value of returned products. Hence, optimal admission levels can be determined for managing inventory and system performance in the remanufacturing system.

The original contributions of this research can be summarized as follows:

(1) An initial work has been done in incorporating product remanufacturing into assemble-to-order and inventory strategy for a cost-effective system operations dealing with uncertain end-of-life product returns. A formal relationship between product returns process and demand process, and a systematic methodology for reassembly decisions and inventory management has been proposed. The proposed model is novel because it accounts for the quality variation and uncertainty of returns in an assemble-to-order system which has not been investigated before. This research provides a step forward in modeling modular products remanufacturing in a stochastic environment, as well as in exploring assembly strategies considering multiple component quality.

(2) A formal framework is developed to obtain a wide range of performance of reassemble-to-order systems and admission policies for EOL products. Exact analysis for performance measures such as inventory level, shortfall, waiting times in inventory, reuse level are conducted in Chapter 3; and provides managerial insights of the system behavior under admission rules.

(3) A discrete-event simulation framework is developed for a real battery remanufacturing case study. This simulation model frees one from restrictive assumptions on system characteristics and enables one to run extensive what-if analysis to understand practical system behavior. Furthermore, it provides a validation for the stylized model and the advantages of incorporating admission policies on product returns.

There are still some limitations for the proposed modeling and analysis methodology to be implemented in more comprehensive remanufacturing applications. First, the reassembly operation that is subject to the *weakest link principle* is valid for battery assembly and many other electronic components. However, this principle may not apply to mechanical assemblies. The structural of the optimal reassembly policy will change and may be more difficult to identify if this assumption is relaxed. Second, the modeling and solution methods in this dissertation are based on steady-state analysis and stationary probability distribution of system state. However, the transient behavior of the stochastic remanufacturing systems remains practically unexplored. Transient characteristics may have significant operational and managerial implications when the factors of product life cycle and product upgrading are considered. New analytical and numerical investigation in this direction should be one of the possible future works. Third, for complex products that have numerous components and modules, the computational efficiency of the proposed algorithms and solution methods may not be sufficient because of the increasing computational complexity.

5.2 Proposed Future Work

Future work can be conducted in each area addressed by this dissertation. A short summary of some possible directions for future research is provided below.

For the reassembly strategy in the stochastic remanufacturing system, the heuristic policies should be tested on large scale problems and compared with other types of commonly implemented policies, e.g., first-come first-serve policy (FCFS), last-come first serve policy (LCFS), exhaustive policies, etc. The optimal selection of coordination vector in the heuristic policy II must be investigated. In addition, the assumption of "weakest link property" for reassembly could be relaxed to provide more comprehensive strategies for general remanufacturing and reassembly systems. Future work should investigate the effect of the product life cycle on the return-demand process and the according reassembly-inventory decision-making.

Due to time and data constraints, the first optimal admission policy proposed in Chapter 4 only considered minimizing the expected sum of total inventory cost and shortage penalty cost as the objective function. Future research should also investigate the opportunity cost of rejecting some of returns. The rejected products could lose the opportunity to be restored to a better condition and higher reuse value for the manufacturer. For the optimal admission policy II, additional investigation for time value decay effects on both reassembly strategy and admission strategy is required.

Finally, decision-making models of other end-of-life options (material recycling, secondary use or disposal) during different stages of product's life cycle, as shown in Figure 5.1, are also a subject of future work. The quality condition of returned products could be modeled as a state variable added to the proposed optimal control model.

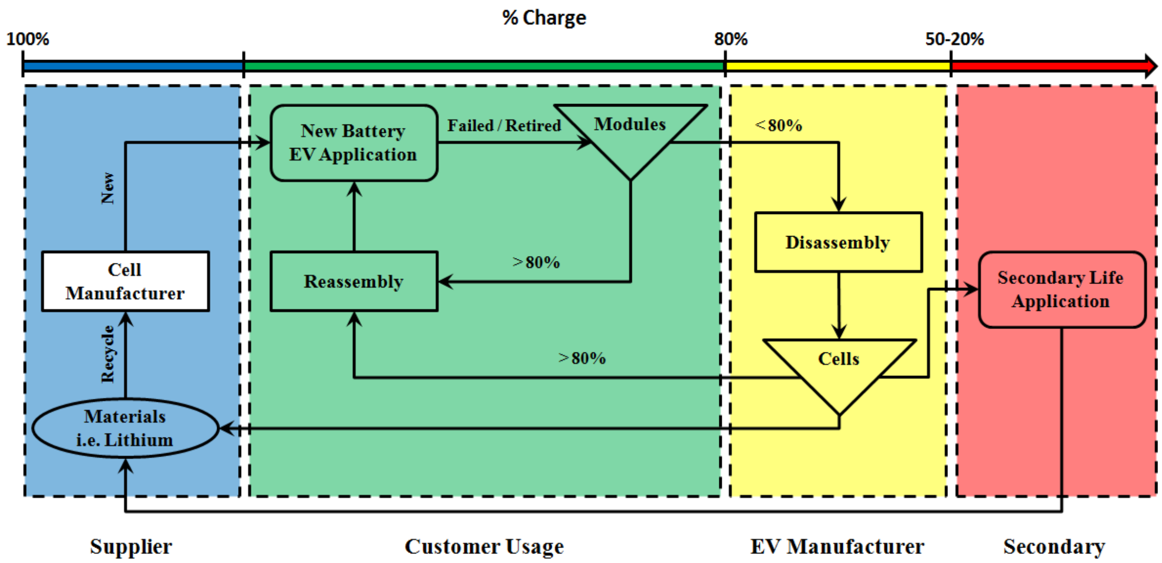


Figure 5.1 Multi-level battery life cycle decision-making.

APPENDICES

Appendix A

In this section, proofs of sufficient conditions for optimality, proofs of optimal policy structure and proofs of monotonicity of the properties in Chapter 2 are presented.

Proof Lemma 2.1

Condition (A1) implies that if it is optimal to satisfy a demand using substitutable inventory x_{12} in state $V(0, x_{21}, x_{12}, x_{22})$, i.e.,

$V(x_{11}, x_{21}-1, x_{12}-1, x_{22}) + c_1 + c_s < V(x_{11}, x_{21}, x_{12}, x_{22}) + s_1$, it is also optimal to satisfy a demand by assembly with substitution in state $V(0, x_{21}, x_{12}, x_{22}+1)$, i.e.,

$V(x_{11}, x_{21}-1, x_{12}-1, x_{22}+1) + c_1 + c_s < V(x_{11}, x_{21}, x_{12}, x_{22}+1) + s_1$. Thus reassembly with substitution decision policy follows a threshold policy where the threshold for switching decision is determined as $\gamma_{22}(\mathbf{x}) \triangleq \min\{x_{22} \mid \Delta_{low} V(\mathbf{x}) \le 0\}$.

Condition (A2) indicates that if

$V(x_{11}, x_{21}-1, x_{12}-1, x_{22}) + c_1 + c_s > V(x_{11}, x_{21}, x_{12}, x_{22}) + s_1$ (optimal not to fulfill demand with substitutable module assembly in state $V(0, x_{21}, x_{12}, x_{22})$), then it is also optimal not to fulfill demand with substitutable module assembly in state $V(0, x_{21}+1, x_{12}+1, x_{22})$, i.e.,

$V(x_{11}, x_{21}, x_{12}, x_{22}) + c_1 + c_s > V(x_{11}, x_{21} + 1, x_{12} + 1, x_{22}) + s_1$, thus implying a threshold for the assembly with substitution control.

Similarly, Condition A3 implies the assembly with substitution optimal decision is also constrained by a combinational effect of x_{21}, x_{12} and x_{22} such that if is optimal to assembly with substitution in state $V(0, x_{21}, x_{12}, x_{22})$, then it is also optimal to assemble with substitution in state $V(0, x_{21} + 1, x_{12} + 1, x_{22} + 1)$.

Proof of Lemma 2.2

The proof is similar and hence omitted.

Proof of Lemma 2.3

In our model, we formulate the controlled queueing systems using dynamic programming framework which encounters a set of properties and operations. Recent work by Benjaafar et al (2008) establishes preservation results for an assembly system with multiple stages. Our model requires similar sub/super-modularity conditions, but our analysis extends to a multi-dimensional case due to the arbitrary number of quality levels and types of modules which prevent us from using their results directly.

Given Lemma 2.1 and 2.2 and Definitions 2.1 and 2.2, we know that the thresholds conditions B1-B4 are implied by conditions A1-A4, respectively. It only remains to show that Conditions A1 through A4 are preserved under the operators $T^{(1)}$ and $T^{(2)}$.

Condition A1

(B1) $\Delta_{low}V(\mathbf{x}) \geq \Delta_{low}V(\mathbf{x}+\mathbf{e}_{22})$

The difference operator remains the same: $\Delta_{low}V(\mathbf{x}) = V(\mathbf{x}-\mathbf{e}_{12}-\mathbf{e}_{21}) - V(\mathbf{x})$

For the ease of exposition, we use the up/down arrows $\uparrow$ and $\downarrow$ to represent how $\Delta_{low}V$ changes with system state. For example, $\Delta_{low}V(\mathbf{x}) \downarrow x_{22}$ represents that the marginal cost to assemble with substitution in state **x**, $\Delta_{low}V$, decreases when there is an additional inventory x_{22}.

Now from condition A1, we have $\Delta_{low}V(\mathbf{x}) \downarrow x_{22}$ and we need to show that $\Delta_{low}T^{(1)}V(\mathbf{x}) \downarrow x_{22}$ to prove that property A1 preserves under operator $T^{(1)}$

That is equivalent to show:

$$\Delta_{low}T^{(1)}V(\mathbf{x}+\mathbf{e}_{22}) - \Delta_{low}T^{(1)}V(\mathbf{x}) \leq 0$$

$$\begin{aligned}\Delta_{low}T^{(1)}v(\mathbf{x}+\mathbf{e}_{22}) = \min\{&V(x_{11},x_{21}-1,x_{12}-1,x_{22}+1)+c_1+c_s, V(x_{11},x_{21},x_{12},x_{22}+1)+s_1\} \\ &-\min\{V(x_{11},x_{21},x_{12},x_{22}+1)+c_1+c_s, V(x_{11},x_{21}+1,x_{12}+1,x_{22}+1)+s_1\}\end{aligned} \quad ((1)$$

$$\begin{aligned}\Delta_{low}T^{(1)}v(\mathbf{x}) = \min\{&V(x_{11},x_{21}-1,x_{12}-1,x_{22})+c_1+c_s, V(x_{11},x_{21},x_{12},x_{22})+s_1\} \\ &-\min\{V(x_{11},x_{21},x_{12},x_{22})+c_1+c_s, V(x_{11},x_{21}+1,x_{12}+1,x_{22})+s_1\}\end{aligned} \quad \ldots \quad (2)$$

Now we need to show (1)-(2) ≤ 0, there are four outcomes of (1):

Case1: $V(x_{11}, x_{21}-1, x_{12}-1, x_{22}+1)-V(x_{11}, x_{21}, x_{12}, x_{22}+1)$

Case2: $V(x_{11}, x_{21}-1, x_{12}-1, x_{22}+1)-V(x_{11}, x_{21}+1, x_{12}+1, x_{22}+1)+c_1+c_s-s_1$

Case3: $V(x_{11}, x_{21}, x_{12}, x_{22}+1)-V(x_{11}, x_{21}, x_{12}, x_{22}+1)-(c_1+c_s-s_1)$

Case4: $V(x_{11}, x_{21}, x_{12}, x_{22}+1)-V(x_{11}, x_{21}+1, x_{12}+1, x_{22}+1)$

Let $\tilde{c}=c_1+c_s-s_1$

h) Suppose case1 is the outcome of (1), and there are 3 possible outcomes of (2):

- If $V(x_{11}, x_{21}-1, x_{12}-1, x_{22})-V(x_{11}, x_{21}, x_{12}, x_{22})$ is the outcome of (1), then (1)-(2) becomes $\Delta_{low}V(\mathbf{x}+\mathbf{e}_{22})-\Delta_{low}V(\mathbf{x})\leq 0$ by condition A1.
- If $V(x_{11}, x_{21}-1, x_{12}-1, x_{22})-V(x_{11}, x_{21}+1, x_{12}+1, x_{22})+\tilde{c}$ is the outcome of (2), then (1)-(2) $\leq \Delta_{low}V(\mathbf{x}+\mathbf{e}_{22})-\Delta_{low}V(\mathbf{x})\leq 0$ by condition A1.
- If $V(x_{11}, x_{21}, x_{12}, x_{22})-V(x_{11}, x_{21}+1, x_{12}+1, x_{22})+\tilde{c}$ is the outcome of (2), then (1)-(2) = $=\Delta_{low}V(\mathbf{x}+\mathbf{e}_{22})-\Delta_{low}V(\mathbf{x}+\mathbf{e}_{21}+\mathbf{e}_{12})\leq 0$ by condition A2.

i) Suppose case2 is the outcome of (1), and there are 3 possible outcomes of (2):

- If $V(x_{11}, x_{21}-1, x_{12}-1, x_{22})-V(x_{11}, x_{21}, x_{12}, x_{22})+\tilde{c}$ is the outcome of (2), then (1)-(2) $\leq \Delta_{low}V(\mathbf{x}+\mathbf{e}_{22})-\Delta_{low}V(\mathbf{x})\leq 0$ because of property A1.
- If $V(x_{11}, x_{21}-1, x_{12}-1, x_{22})-V(x_{11}, x_{21}+1, x_{12}+1, x_{22})+\tilde{c}$ is the outcome of (2), then (1)-(2) $=\Delta_{low}V(\mathbf{x}+\mathbf{e}_{22})-\Delta_{low}V(\mathbf{x})+\Delta_{low}V(\mathbf{x}+\mathbf{e}_{21}+\mathbf{e}_{12}+\mathbf{e}_{22})-\Delta_{low}V(\mathbf{x}+\mathbf{e}_{21}+\mathbf{e}_{12})\leq 0$

- If $V(x_{11},x_{21},x_{12},x_{22})-V(x_{11},x_{21}+1,x_{12}+1,x_{22})+\tilde{c}$ is the outcome of (2), then (1)-(2) because of $\le \Delta_{low}V(\mathbf{x}+\mathbf{e}_{21}+\mathbf{e}_{12}+\mathbf{e}_{22})-\Delta_{low}V(\mathbf{x}+\mathbf{e}_{21}+\mathbf{e}_{12})\le 0$ condition A1.

j) Suppose case 4 is the outcome of (1), and there are three possible outcomes of (2):

- If $V(x_{11},x_{21}-1,x_{12}-1,x_{22})-V(x_{11},x_{21},x_{12},x_{22})+\tilde{c}$ is the outcome of (2), then (1)-(2)= $\le \Delta_{low}v(\mathbf{x}+\mathbf{e}_{21}+\mathbf{e}_{12}+\mathbf{e}_{22})-\Delta_{low}v(\mathbf{x})\le 0$ because of condition A3.
- If $V(x_{11},x_{21}-1,x_{12}-1,x_{22})-V(x_{11},x_{21}+1,x_{12}+1,x_{22})+\tilde{c}$ is the outcome of (2), then (1)-(2) $=\Delta_{low}(\mathbf{x}+\mathbf{e}_{21}+\mathbf{e}_{12}+\mathbf{e}_{22})-\Delta_{low}(\mathbf{x})\le 0$ because of condition A3.
- If $V(x_{11},x_{21},x_{12},x_{22})-V(x_{11},x_{21}+1,x_{12}+1,x_{22})+\tilde{c}$ is the outcome of (2), then (1)-(2) $=\Delta_{low}(\mathbf{x}+\mathbf{e}_{21}+\mathbf{e}_{12}+\mathbf{e}_{22})-\Delta_{low}(\mathbf{x}+\mathbf{e}_{21}+\mathbf{e}_{12})\le 0$ because of condition A1.

Now we have shown that condition A1 is preserved under operator $T^{(1)}$.

Condition A2

(B2) $\Delta_{low}V(\mathbf{x})\le \Delta_{low}V(\mathbf{x}+\mathbf{e}_{21}+\mathbf{e}_{12})$

The difference operator remains the same: $\Delta_{low}V(\mathbf{x})=V(\mathbf{x}-e_{12}-e_{21})-V(\mathbf{x})$

We have A2 $\Delta_{low}V(\mathbf{x})\uparrow(x_{21},x_{12})$ and we need to show that $\Delta_{low}T^{(1)}V(\mathbf{x})\uparrow(x_{21},x_{12})$ to prove that property A2 preserves under operator $T^{(1)}$

That is equivalent to show: $\Delta_{low}T^{(1)}V(\mathbf{x}+\mathbf{e}_{21}+\mathbf{e}_{12})-\Delta_{low}T^{(1)}V(\mathbf{x})\le 0$

$$\Delta_{low}T^{(1)}V(\mathbf{x}+\mathbf{e}_{21}+\mathbf{e}_{12})$$

$$=\min\{V(x_{11},x_{21},x_{12},x_{22})+c_1+c_s,V(x_{11},x_{21}+1,x_{12}+1,x_{22})+s_1\} \qquad (3)$$

$$-\min\{V(x_{11},x_{21}+1,x_{12}+1,x_{22})+c_1+c_s,V(x_{11},x_{21}+2,x_{12}+2,x_{22})+s_1\}$$

$$\Delta_{low}T^{(1)}V(\mathbf{x})=\min\{V(x_{11},x_{21}-1,x_{12}-1,x_{22})+c_1+c_s,V(x_{11},x_{21},x_{12},x_{22})+s_1\}$$

$$-\min\{V(x_{11},x_{21},x_{12},x_{22})+c_1+c_s,V(x_{11},x_{21}+1,x_{12}+1,x_{22})+s_1\} \quad \ldots\ldots\ \ldots \quad (4)$$

Now we need to show (3)-(4) ≥ 0, there are four outcomes of (3):

Case1: $V(x_{11},x_{21},x_{12},x_{22})-V(x_{11},x_{21}+1,x_{12}+1,x_{22})+c_1+c_s-s_1$

Case2: $V(x_{11},x_{21},x_{12},x_{22})-V(x_{11},x_{21}+2,x_{12}+2,x_{22})+c_1+c_s-s_1$

Case3: $V(x_{11},x_{21}+1,x_{12}+1,x_{22})-V(x_{11},x_{21}+2,x_{12}+2,x_{22})+c_1+c_s-s_1$

Let $\tilde{c}=c_r+c_s-s_1$

a) Suppose case1 is the outcome of (3), and there are 3 possible outcomes of (4):

- If $V(x_{11},x_{21}-1,x_{12}-1,x_{22})-V(x_{11},x_{21},x_{12},x_{22})+\tilde{c}$ is the outcome of (4), then (3)-(4)

  $=\Delta_{low}V(\mathbf{x}+\mathbf{e}_{21}+\mathbf{e}_{12})-\Delta_{low}V(\mathbf{x})\geq 0$ because of condition A2.

- If $V(x_{11},x_{21}-1,x_{12}-1,x_{22})-V(x_{11},x_{21}+1,x_{12}+1,x_{22})+\tilde{c}$ is the outcome of (4), then

  (3)-(4)

  $=V(x_{11},x_{21},x_{12},x_{22})-V(x_{11},x_{21}+1,x_{12}+1,x_{22})$

  $-V(x_{11},x_{21}-1,x_{12}-1,x_{22})+V(x_{11},x_{21}+1,x_{12}+1,x_{22})$

- $\geq V(x_{11},x_{21},x_{12},x_{22})-V(x_{11},x_{21}+1,x_{12}+1,x_{22})$

  $-V(x_{11},x_{21},x_{12},x_{22})+V(x_{11},x_{21}+1,x_{12}+1,x_{22})$

  $=0$

- If $V(x_{11},x_{21},x_{12},x_{22})-V(x_{11},x_{21}+1,x_{12}+1,x_{22})+\tilde{c}$ is the outcome of (4), then (3)-(4)=0.

b) Suppose case2 is the outcome of (3), and there are 3 possible outcomes of (4):

- If $V(x_{11},x_{21}-1,x_{12}-1,x_{22})-V(x_{11},x_{21},x_{12},x_{22})+\tilde{c}$ is the outcome of (4), then (3)-(4) $\geq \Delta_{low}V(\mathbf{x}+\mathbf{e}_{21}+\mathbf{e}_{12})-\Delta_{low}V(\mathbf{x})\geq 0$ because of condition A2.

- If $V(x_{11},x_{21}-1,x_{12}-1,x_{22})-V(x_{11},x_{21}+1,x_{12}+1,x_{22})+\tilde{c}$ is the outcome of (4), then

  (3)-(4)

$$
\begin{aligned}
&=V(x_{11},x_{21},x_{12},x_{22})-V(x_{11},x_{21}+2,x_{12}+2,x_{22})\\
&-V(x_{11},x_{21}-1,x_{12}-1,x_{22})+V(x_{11},x_{21}+2,x_{12}+2,x_{22})\\
&\geq V(x_{11},x_{21},x_{12},x_{22})-V(x_{11},x_{21},x_{12},x_{22})\\
&=0
\end{aligned}
$$

- If $V(x_{11},x_{21},x_{12},x_{22})-V(x_{11},x_{21}+1,x_{12}+1,x_{22})+\tilde{c}$ is the outcome of (4), then (3)-(4)

$$
\begin{aligned}
&=V(x_{11},x_{21},x_{12},x_{22})-V(x_{11},x_{21}+2,x_{12}+2,x_{22})\\
&-V(x_{11},x_{21},x_{12},x_{22})+V(x_{11},x_{21}+1,x_{12}+1,x_{22})\\
&\geq V(x_{11},x_{21},x_{12},x_{22})-V(x_{11},x_{21}+1,x_{12}+1,x_{22})\\
&-V(x_{11},x_{21},x_{12},x_{22})+V(x_{11},x_{21}+1,x_{12}+1,x_{22})\\
&=0
\end{aligned}
$$

c) Suppose case3 is the outcome of (3), and there are 3 possible outcomes of (4):

- If $v(x_{11}, x_{21}-1, x_{12}-1, x_{22}) - v(x_{11}, x_{21}, x_{12}, x_{22}) + \tilde{c}$ is the outcome of (4), then (3)-(4)

  $$= V(x_{11}, x_{21}+1, x_{12}+1, x_{22}) - V(x_{11}, x_{21}+2, x_{12}+2, x_{22})$$
  $$-V(x_{11}, x_{21}-1, x_{12}-1, x_{22}) + V(x_{11}, x_{21}, x_{12}, x_{22})$$
  $$\geq \Delta_{low}V(\mathbf{x}+2\mathbf{e}_{21}+2\mathbf{e}_{12}) - \Delta_{low}V(\mathbf{x})$$
  $$= \Delta_{low}V(\mathbf{x}+2\mathbf{e}_{21}+2\mathbf{e}_{12}) - \Delta_{low}V(\mathbf{x}+\mathbf{e}_{21}+\mathbf{e}_{12})$$
  $$+\Delta_{low}V(\mathbf{x}+\mathbf{e}_{21}+\mathbf{e}_{12}) - \Delta_{low}V(\mathbf{x}) \geq 0$$

  because of condition A2.

- If $V(x_{11}, x_{21}-1, x_{12}-1, x_{22}) - V(x_{11}, x_{21}+1, x_{12}+1, x_{22}) + \tilde{c}$ is the outcome of (4), then

  (3)-(4)

$$= V(x_{11}, x_{21}+1, x_{12}+1, x_{22}) - V(x_{11}, x_{21}+2, x_{12}+2, x_{22}) - V(x_{11}, x_{21}-1, x_{12}-1, x_{22})$$
$$+V(x_{11}, x_{21}+1, x_{12}+1, x_{22})$$
$$\geq \Delta_{low}V(\mathbf{x}+2\mathbf{e}_{21}+2\mathbf{e}_{12}) - \Delta_{low}V(\mathbf{x}+\mathbf{e}_{21}+\mathbf{e}_{12}) \geq 0$$

because of condition A2.

- If $V(x_{11}, x_{21}, x_{12}, x_{22}) - V(x_{11}, x_{21}+1, x_{12}+1, x_{22}) + \tilde{c}$ is the outcome of (4), then (3)-(4)

  $$= V(x_{11}, x_{21}+1, x_{12}+1, x_{22}) - V(x_{11}, x_{21}+2, x_{12}+2, x_{22})$$
  $$-V(x_{11}, x_{21}-1, x_{12}-1, x_{22}) + V(x_{11}, x_{21}+1, x_{12}+1, x_{22})$$
  $$= \Delta_{low}V(\mathbf{x}+2\mathbf{e}_{21}+2\mathbf{e}_{12}) - \Delta_{low}V(\mathbf{x}+\mathbf{e}_{21}+\mathbf{e}_{12}) \geq 0$$

because of condition A2.

Condition A3

(A3) $\Delta_{low}V(\mathbf{x}) \geq \Delta_{low}V(\mathbf{x}+\mathbf{e}_{21}+\mathbf{e}_{12}+\mathbf{e}_{22})$

The difference operator remains the same: $\Delta_{low}V(\mathbf{x}) = [V(\mathbf{x}-e_{12}-e_{21})+c_r]-[V(\mathbf{x})+s_1]$

We have A3 $\Delta_{low}V(\mathbf{x}) \downarrow (x_{21}, x_{12}, x_{22})$ and we need to show that

$\Delta_{low}T^{(1)}V(\mathbf{x}) \downarrow (x_{21}, x_{12}, x_{22})$ to prove that property A3 preserves under operator $\boldsymbol{T}^{(1)}$

That is equivalent to show:

$$\Delta_{low}T^{(1)}V(\mathbf{x}+\mathbf{e}_{21}+\mathbf{e}_{12}+\mathbf{e}_{22}) - \Delta_{low}T^{(1)}V(\mathbf{x}) \leq 0$$

$$\begin{aligned}&\Delta_{low}T^{(1)}V(\mathbf{x}+\mathbf{e}_{21}+\mathbf{e}_{12}+\mathbf{e}_{22}) \\ &= \min\{V(x_{11}, x_{21}, x_{12}, x_{22}+1)+c_1+c_s, V(x_{11}, x_{21}+1, x_{12}+1, x_{22}+1)+s_1\} \\ &-\min\{V(x_{11}, x_{21}+1, x_{12}+1, x_{22}+1)+c_1+c_s, V(x_{11}, x_{21}+2, x_{12}+2, x_{22}+1)+s_1\}\end{aligned} \tag{5}$$

$$\begin{aligned}\Delta_{low}T^{(1)}V(\mathbf{x}) = &\min\{V(x_{11}, x_{21}-1, x_{12}-1, x_{22})+c_1+c_s, V(x_{11}, x_{21}, x_{12}, x_{22})+s_1\} \\ &-\min\{V(x_{11}, x_{21}, x_{12}, x_{22})+c_1+c_s, V(x_{11}, x_{21}+1, x_{12}+1, x_{22})+s_1\}\end{aligned} \quad \ldots\ldots \tag{6}$$

Now we need to show (5)-(6) ≥ 0, there are four outcomes of (5):

Case1: $V(x_{11}, x_{21}, x_{12}, x_{22}+1) - V(x_{11}, x_{21}+1, x_{12}+1, x_{22}+1)$

Case2: $V(x_{11}, x_{21}, x_{12}, x_{22}+1) - V(x_{11}, x_{21}+2, x_{12}+2, x_{22}+1) + c_1 + c_s - s_1$

Case3: $V(x_{11}, x_{21}+1, x_{12}+1, x_{22}+1) - V(x_{11}, x_{21}+2, x_{12}+2, x_{22}+1)$

Let $\tilde{c} = c_1 + c_s - s_1$

a) Suppose case1 is the outcome of (5), and there are three possible outcomes of (6):

- If $V(x_{11}, x_{21}-1, x_{12}-1, x_{22}) - V(x_{11}, x_{21}, x_{12}, x_{22}) + \tilde{c}$ is the outcome of (6), then (5)-(6) $= \Delta_{low}V(\mathbf{x}+\mathbf{e}_{21}+\mathbf{e}_{12}+\mathbf{e}_{22}) - \Delta_{low}V(\mathbf{x}) \leq 0$ because of condition A3.

- If $V(x_{11},x_{21}-1,x_{12}-1,x_{22})-V(x_{11},x_{21}+1,x_{12}+1,x_{22})+\tilde{c}$ is the outcome of (6), then (5)-(6)

$$=V(x_{11},x_{21},x_{12},x_{22}+1)-V(x_{11},x_{21}+1,x_{12}+1,x_{22}+1)-V(x_{11},x_{21}-1,x_{12}-1,x_{22})$$
$$+V(x_{11},x_{21}+1,x_{12}+1,x_{22})$$
$$\leq V(x_{11},x_{21},x_{12},x_{22}+1)-V(x_{11},x_{21}+1,x_{12}+1,x_{22}+1)$$
$$-V(x_{11},x_{21},x_{12},x_{22})+V(x_{11},x_{21}+1,x_{12}+1,x_{22})$$
$$=\Delta_{low}V(\mathbf{x}+\mathbf{e_{21}}+\mathbf{e_{12}}+\mathbf{e_{22}})-\Delta_{low}V(\mathbf{x}+\mathbf{e_{21}}+\mathbf{e_{12}})\leq 0$$ because of condition A1.

- If $V(x_{11},x_{21},x_{12},x_{22})-V(x_{11},x_{21}+1,x_{12}+1,x_{22})+\tilde{c}$ is the outcome of (6), then (5)-(6)

$$=V(x_{11},x_{21},x_{12},x_{22}+1)-V(x_{11},x_{21}+1,x_{12}+1,x_{22}+1)-V(x_{11},x_{21},x_{12},x_{22})$$
$$+V(x_{11},x_{21}+1,x_{12}+1,x_{22})$$
$$=\Delta_{low}V(\mathbf{x}+\mathbf{e_{21}}+\mathbf{e_{12}}+\mathbf{e_{22}})-\Delta_{low}V(\mathbf{x}+\mathbf{e_{21}}+\mathbf{e_{12}})\leq 0$$ because of condition A1.

b) Suppose case2 is the outcome of (5), and there are 3 possible outcomes of (6):

- If $V(x_{11},x_{21}-1,x_{12}-1,x_{22})-V(x_{11},x_{21},x_{12},x_{22})+\tilde{c}$ is the outcome of (6), then (5)-(6)

$$\leq \Delta_{low}V(\mathbf{x}+2\mathbf{e}_{21}+2\mathbf{e}_{12}+\mathbf{e}_{22})-\Delta_{low}V(\mathbf{x})\leq 0$$ because of condition A3.

- If $V(x_{11},x_{21}-1,x_{12}-1,x_{22})-V(x_{11},x_{21}+1,x_{12}+1,x_{22})+\tilde{c}$ is the outcome of (6), then (5)-(6)

$$\leq V(x_{11},x_{21}+1,x_{12}+1,x_{22}+1)-V(x_{11},x_{21}+2,x_{12}+2,x_{22}+1)$$
$$-V(x_{11},x_{21}-1,x_{12}-1,x_{22})+V(x_{11},x_{21}+1,x_{12}+1,x_{22})$$
$$\leq \Delta_{low}V(\mathbf{x}+2\mathbf{e}_{21}+2\mathbf{e}_{12}+\mathbf{e}_{22})-\Delta_{low}V(\mathbf{x})\leq 0$$

- If $V(x_{11},x_{21},x_{12},x_{22})-V(x_{11},x_{21}+1,x_{12}+1,x_{22})+\tilde{c}$ is the outcome of (6), then (5)-(6)

  $\leq \Delta_{low}V(\mathbf{x}+2\mathbf{e}_{21}+2\mathbf{e}_{12}+\mathbf{e}_{22})-\Delta_{low}V(\mathbf{x}+\mathbf{e}_{21}+\mathbf{e}_{12})\leq 0$

  because of condition A1 and A3.

c) Suppose case 3 is the outcome of (5), and there are 3 possible outcomes of (6):

- If $V(x_{11},x_{21}-1,x_{12}-1,x_{22})-V(x_{11},x_{21},x_{12},x_{22})+\tilde{c}$ is the outcome of (6), then (5)-(6)

  $=\Delta_{low}V(\mathbf{x}+2\mathbf{e}_{21}+2\mathbf{e}_{12}+\mathbf{e}_{22})-\Delta_{low}V(\mathbf{x})\leq 0$

- If $V(x_{11},x_{21}-1,x_{12}-1,x_{22})-V(x_{11},x_{21}+1,x_{12}+1,x_{22})+\tilde{c}$ is the outcome of (6), then (5)-(6)

  $\leq \Delta_{low}V(\mathbf{x}+2\mathbf{e}_{21}+2\mathbf{e}_{12}+\mathbf{e}_{22})-\Delta_{low}V(\mathbf{x})\leq 0$

- If $V(x_{11},x_{21},x_{12},x_{22})-V(x_{11},x_{21}+1,x_{12}+1,x_{22})+\tilde{c}$ is the outcome of (6), then (5)-(6)

  $=\Delta_{low}V(\mathbf{x}+2\mathbf{e}_{21}+2\mathbf{e}_{12}+\mathbf{e}_{22})-\Delta_{low}V(\mathbf{x}+\mathbf{e}_{21}+\mathbf{e}_{12})\leq 0$
  $=0$ because of condition A1 and A3.

Now we have shown that condition A1, A2 and A3 are preserved under operator $T^{(1)}$.

Condition A4

(A4) $\Delta_{high}V(\mathbf{x})\leq \Delta_{high}v(\mathbf{x}+\mathbf{e}_{12}+\mathbf{e}_{22})\Leftrightarrow \Delta_{high}V(\mathbf{x})\uparrow (x_{12},x_{22})$

For the higher demand class, the difference operator is $\Delta_{high}V(\mathbf{x})\triangleq[V(\mathbf{x}-\mathbf{e}_{12}-\mathbf{e}_{22})+2c_2]-[V(\mathbf{x})+s_2]$, and we need to show that conditions A4 are preserved under operator $T^{(2)}$.

We have $\Delta_{high}V(\mathbf{x})\uparrow(x_{12},x_{22})$ and we need to show that $\Delta_{high}T^{(2)}V(\mathbf{x})\uparrow(x_{12},x_{22})$ to prove that property A4 preserves under operator $T^{(2)}$.

That is equivalent to show: $\Delta_{high}T^{(2)}V(\mathbf{x}+\mathbf{e}_{12}+\mathbf{e}_{22})-\Delta_{high}T^{(2)}V(\mathbf{x})\geq 0$

$$\begin{aligned}&\Delta_{high}T^{(2)}V(\mathbf{x}+\mathbf{e}_{12}+\mathbf{e}_{22})\\&=\min\{V(x_{11},x_{21},x_{12},x_{22})+2c_2,V(x_{11},x_{21},x_{12}+1,x_{22}+1)+s_2\}\\&-\min\{V(x_{11},x_{21},x_{12}+1,x_{22}+1)+2c_2,V(x_{11},x_{21},x_{12}+2,x_{22}+2)+s_2\}\end{aligned}\tag{7}$$

$$\begin{aligned}\Delta_{high}T^{(2)}V(\mathbf{x})&=\min\{V(x_{11},x_{21},x_{12}-1,x_{22}-1)+2c_2,V(x_{11},x_{21},x_{12},x_{22})+s_2\}\\&-\min\{V(x_{11},x_{21},x_{12},x_{22})+2c_2,V(x_{11},x_{21},x_{12}+1,x_{22}+1)+s_2\}\end{aligned}\tag{8}$$

Now we need to show (7)-(8) ≥ 0, there are two feasible outcomes of (7):

Case1: $V(x_{11},x_{21},x_{12},x_{22})-V(x_{11},x_{21},x_{12}+1,x_{22}+1)$

Case2: $V(x_{11},x_{21},x_{12}+1,x_{22}+1)-V(x_{11},x_{21},x_{12}+2,x_{22}+2)$

Case3: $V(x_{11},x_{21},x_{12},x_{22})-V(x_{11},x_{21},x_{12}+2,x_{22}+2)+(2c_2-s_2)$

Let $\hat{c}=2c_2-s_2$

a) Suppose case1 is the outcome of (7), and there are 2 possible outcomes of (8):

- If $V(x_{11},x_{21},x_{12}-1,x_{22}-1)-V(x_{11},x_{21},x_{12},x_{22})$ is the outcome of (8), then (7)-(8)

$\geq\Delta_{high}V(\mathbf{x}+\mathbf{e}_{12}+\mathbf{e}_{22})-\Delta_{high}V(\mathbf{x})\geq 0$ because of condition A4.

- If $V(x_{11},x_{21},x_{12},x_{22})-V(x_{11},x_{21},x_{12},x_{22})-\hat{c}$ is the outcome of (8), then (7)-(8)

  $\geq\Delta_{high}V(\mathbf{x}+\mathbf{e}_{12}+\mathbf{e}_{22})-V(x_{11},x_{21},x_{12}-1,x_{22}-1)-2c_2$

  $+V(x_{11},x_{21},x_{12}+1,x_{22}+1)+2c_2$

  $=\Delta_{high}V(\mathbf{x}+\mathbf{e}_{12}+\mathbf{e}_{22})-\Delta_{high}V(\mathbf{x})=0$ by condition A4.

b) Suppose case 2 is the outcome of (7), and there are two possible outcomes of (8):

- If $V(x_{11}, x_{21}, x_{12}-1, x_{22}-1)+2c_2-V(x_{11}, x_{21}, x_{12}+1, x_{22}+1)-s_2$ is the outcome of (8), then (7)-(8)

$\geq \Delta_{high}V(\mathbf{x}+2\mathbf{e}_{12}+2\mathbf{e}_{22})-V(x_{11}, x_{21}, x_{12}, x_{22})-s_2+V(x_{11}, x_{21}, x_{12}+1, x_{22}+1)+s_2$

$=\Delta_{high}V(\mathbf{x}+2\mathbf{e}_{12}+2\mathbf{e}_{22})-\Delta_{high}V(\mathbf{x}+\mathbf{e}_{12}+\mathbf{e}_{22})\geq 0$ by condition A4.

- If $V(x_{11}, x_{21}, x_{12}, x_{22})+s_2-V(x_{11}, x_{21}, x_{12}+1, x_{22}+1)-s_2$ is the outcome of (8), then (7)-(8) $=\Delta_{high}V(\mathbf{x}+2\mathbf{e}_{12}+2\mathbf{e}_{22})-\Delta_{high}V(\mathbf{x}+\mathbf{e}_{12}+\mathbf{e}_{22})\geq 0$ by condition A4.

c) Suppose case 3 is the outcome of (7), and there are two possible outcomes of (8):

- If $V(x_{11}, x_{21}, x_{12}-1, x_{22}-1)+2c_2-V(x_{11}, x_{21}, x_{12}, x_{22})-2c_2$ is the outcome of (8), then (7)-(8)

$\geq V(x_{11}, x_{21}, x_{12}, x_{22})-V(x_{11}, x_{21}, x_{12}+1, x_{22}+1)$

$-V(x_{11}, x_{21}, x_{12}-1, x_{22}-1)-2c_2+V(x_{11}, x_{21}, x_{12}, x_{22})+2c_2$

$=\Delta_{high}V(\mathbf{x}+\mathbf{e}_{12}+\mathbf{e}_{22})-\Delta_{high}V(\mathbf{x})\geq 0$ by condition A4.

- If$V(x_{11}, x_{21}, x_{12}, x_{22})+s_2-V(x_{11}, x_{21}, x_{12}, x_{22})-2c_2$ is the outcome of (8), then (7)-(8)

$$=V(x_{11}, x_{21}, x_{12}, x_{22})-V(x_{11}, x_{21}, x_{12}+2, x_{22}+2)+(2c_2-s_2)$$
$$-V(x_{11}, x_{21}, x_{12}, x_{22})-s_2+V(x_{11}, x_{21}, x_{12}, x_{22})+2c_2$$
$$=V(x_{11}, x_{21}, x_{12}, x_{22})-V(x_{11}, x_{21}, x_{12}+2, x_{22}+2)+2\widehat{c}$$
$$=\Delta_{high}V(\mathbf{x}+\mathbf{e}_{12}+\mathbf{e}_{22})+\Delta_{high}V(\mathbf{x}+2\mathbf{e}_{12}+2\mathbf{e}_{22})\geq 0$$

Now we have shown that Condition A4 is preserved under operator $T^{(2)}$.

Proof of Lemma 2.4

(L2.4.1): From condition A2 and the definition of $\gamma_{22}(\mathbf{x})$, we have

$\Delta_{low}V(x_{11},x_{21},x_{12},\gamma_{22}(\mathbf{x}+\mathbf{e}_{21}+\mathbf{e}_{12})) \le \Delta_{low}V(x_{11},x_{21}+1,x_{12}+1,\gamma_{22}(\mathbf{x}+\mathbf{e}_{21}+\mathbf{e}_{12})) \le 0.$ Hence, we have $\Delta_{low}V(x_{11},x_{21},x_{12},\gamma_{22}(\mathbf{x}+\mathbf{e}_{21}+\mathbf{e}_{12})) \le 0$. Using the definition $\gamma_{22}(\mathbf{x})$, we can obtain $\gamma_{12}(\mathbf{x}) \le \gamma_{12}(\mathbf{x}+\mathbf{e}_{21}+\mathbf{e}_{12})$.

(L2.4.2): We know that $\Delta_{low}V(x_{11},x_{21},x_{12},x_{22}+1 \mid x_{21}+x_{12}=\kappa(\mathbf{x}+\mathbf{e}_{22})) \ge 0$ because of (8) in Definition 2.3.

From condition A1, we have that $\Delta_{low}V(x_{11},x_{21}+1,x_{12}+1,x_{22}+1 \mid x_{21}+x_{12}=\kappa(\mathbf{x}+\mathbf{e}_{22}))$ $\ge \Delta_{low}V(x_{11},x_{21},x_{12},x_{22}+1 \mid x_{21}+x_{12}=\kappa(\mathbf{x}+\mathbf{e}_{22})) \ge 0$. Using the definition of $\kappa(\mathbf{x})$, we obtain that $\kappa(\mathbf{x}) \le \kappa(\mathbf{x}+\mathbf{e}_{22})$.

The proof of **Lemma 2.5** is very similar, and so it is omitted. □

Proof of Theorem 2.1

Consider a value iteration algorithm to solve the dynamic programming for the optimal control problem given in (2.5) where initial values $V_0(\mathbf{x})=0$ are used for each state $\mathbf{x}$. Conditions B1-B4 are trivially satisfied by $V_0(\mathbf{x})$, hence $V_0(\mathbf{x})=0$. We apply $V_{t+1}(\mathbf{x})=TV_t(\mathbf{x})$ for $t=0,1,2,...$ to determine the relative value functions for sequential iterations. Suppose $V_t(\mathbf{x})\in\Omega$, then, Lemma 2.1 and Lemma 2.2 show that $V_{t+1}(\mathbf{x})$ also satisfy condition B1-B4, i.e., $V_{t+1}(\mathbf{x}) \in \Omega$. Furthermore, the problem given in (2.5) and assumes that the total rates of receiving returned modules are less than the total demand rates for modules, the inventories of various module types and quality grades are finite and bounded. The action set of the problem consists of finite number of actions, i.e., for lower class of demand: (a) reassemble with substitution, and (b) do not reassemble but use new ones; for higher class of demand: (c)reassemble and (d) do not reassemble but use new ones. Thus, the problem is a finite state, finite action set problem. In addition, the underlying Markov chain is also *unichain*. Therefore, the existence of a long-run average cost and the validity of the value iteration algorithm are ensured by Theorem 8.4.5 of Puterman (1994).

Appendix B

Proof of Proposition 4.2. By contradiction. Suppose ${r_1}^* > {r_2}^*$ instead, then there exists $\epsilon > 0$ such that ${r_1}^* - \epsilon > {r_2}^*$. Because the return volume of Module 1 is stochastically greater than that of Module 2, we have $Y_1 \leq_{st} Y_2$. Therefore, there exist $\widehat{Y_1}$ and $\widehat{Y_2}$ defined on the same probability space such that $\widehat{Y_1} =_{st} Y_1$, $\widehat{Y_2} =_{st} Y_2$ and $\widehat{Y_1} \leq_{st} \widehat{Y_2}$ a.s. Combining the above statements, we have

$$d - {r_1}^* + \widehat{Y_1} \leq d - ({r_1}^* - \epsilon) + \widehat{Y_1} \leq d - {r_2}^* + \widehat{Y_2} \text{ a.s.} \quad \text{(B.1)}$$

where d is the one-period demand.

It follows that

$$P\left[\left({r_1}^* - \max\left(0, Y_1 + d - ({r_1}^* - \epsilon)\right), {r_2}^* - \max(0, Y_2 + d - {r_2}^*)\right) \geq 0\right]$$

$$\xRightarrow{\widehat{Y_i} =_{st} Y_i} P\left[\left({r_1}^* - \max\left(0, \widehat{Y_1} + d - ({r_1}^* - \epsilon)\right), {r_2}^* - \max\left(0, \widehat{Y_2} + d - {r_2}^*\right)\right) \geq 0\right]$$

$$\xRightarrow{(B.1)} P\left[\left({r_1}^* - \max\left(0, \widehat{Y_1} + d - {r_1}^*\right), {r_2}^* - \max\left(0, \widehat{Y_2} + d - {r_2}^*\right)\right) \geq 0\right]$$

$$\xRightarrow{\widehat{Y_i} =_{st} Y_i} P[({r_1}^* - \max(0, Y_1 + d - {r_1}^*), {r_2}^* - \max(0, Y_2 + d - {r_2}^*)) \geq 0] \geq \alpha$$

Thus, we find a feasible solution $({r_1}^* - \epsilon, {r_2}^*)$ for the problem with a lower objective function value than the optimal, which yields the contradiction.

BIBLIOGRAPHY

Altemosel G, 2004. Achieving cell balancing for lithium-ion batteries, http://www.aeroflex.com, 2004.

Aras N, T Boyaci, and V Verter, 2004. The effect of categorizing returned products in remanufacturing. *IIE Transactions*. 26(4): p. 319-331.

Atasu A., Sarvary, M., and Van Wassenhove, L N, 2008 Remanufacturing as a marketing strategy. *Management Science*. 54(10):p. 1731-1746.

Agrawal N, and M A Cohen, 2001. Optimal material control in an assembly system with component commonality. *Naval Res. Logist.* 48:p. 409-429.

Avriel M, and J D Wilde, 1966. Optimality proof for the symmetric Fibonacci search technique. *Fibonacci Quarterly.* 4. p: 265–269.

Ayres R U, Ferrer G, and T Van Leynseele, 1997. Eco-efficiency, asset recovery and remanufacturing. *Eur. Manag. J.* 15(5): p. 557–574.

Barsukov Y, 2009. Battery Cell Balancing: What to Balance and How, Texas Instruments, http://focus.ti.com.

Blecker T and N Abdelkafi, 2005. Modularity and delayed product differentiation in assemble-to-order systems: analysis and extensions from a complexity perspective. *Proceedings of IMCM*'05: p. 29-46.

Bertsekas D, Dynamic Programming and Optimal Control. Volume2, Athena, MA, 1995.

Benjaafar S and M ElHafsi, Production and inventory control of a single product assemble-to-order system with multiple customer classes, 2006. *Management Science*, 52(12): p. 1896-1912.

Baiman S, S Netessine, and H Kunreuther, Procurement in supply chains when the end-product exhibits the “weakest link” property, The Wharton School, Philadelphi, 2004.

Batteries and electric vehicles, The Recovery Act: Transforming America’s Transportation Sector, *Department of Energy*. January 14, 2010.

Baker K R, M Magazine and H Nuttle, 1986. The effects of commonality on safety stocks in simple inventory model. *Management Science*. 32: p. 982-988.

Bashyam S, and M C Fu, 1998. Optimization of (s, S) inventory systems with random lead times and a service level constraint. *Management Science*. 44: p. 243-S256

Blackburn, J D, 1991. Time-based competition: The next battleground in American manufacturing. (Business One Irwin), Homewood, IL.

Blackburn J D, Limits of time-based competition: valuing time in make-to-stock manufacturing, *Quantitative Approaches to Distribution Logistics and Supply Chain Management*. Chapter 3. Springer-Verlag, 2002.

Biehl M, E Prater, and M J Realff, 2007. Assessing performance and uncertainty in developing carpet reverse logistics systems. *Computers & Operations Research*. 34: p. 443–463

Cunha C, B Agard, and A Kusiak, Deisgn for cost: module-based mass customization. 2007. *IEEE Transactions on Automation Science and Engineering*. 4(3): p. 350-359.

BCG Focus, *Batteries for Electric Cars, Challenges, Opportunities, and the Outlook to* 2020. The Boston Cousulting Group.

Charter M and C Gray, Remanufacturing and product design, 2008. *International Journal of Product Development*. 6(3/4) p: 375 – 392.

Chen J, D Yao and S Zheng, 2001. Optimal replenishment and rework with multiple unreliable supply sources, *Operations Research*. 49(3):p. 430-443.

Dinger A, R Martin, X Mosquet, M Rabl, D Rizoulis, M Russo and G Sticher, Batteries for electric cars-challenges: Opportunities and the outlook to 2020. *Energy & Environment Publications*. Janurary 2010.

Das S, and D Yedlarajiah, 2002. An integer programming model for prescribing material recovery strategies. *IEEE International Symposium on Electronics and the Environment*. p.118-122.

Denizel M, M Ferguson, G Souza, 2010. Multi-period remanufacturing planning with uncertain quality of inputs, *IEEE Transactions on Engineering Management*.57 (3): p.394-404.

Deshpande V, M A Cohen, and K Donohue, 2003. A threshold inventory rationing policy for service-differentiated demand classes, *Management Science*. vol.49(6): p. 683–703.

ElHafsi M, 2009. Optimal integrated production and inventory control of an assemble-to-order system with multiple non-unitary demand classes, *European Journal of Operational Research*. 2009, 194: p.127-142.

Fleichmann M, J M Bloemhof-Ruwaard, R Dekker, E A van der Laan, J A E E van Nunen, and L N Van Wassenhove, 1997. Quantitative models for reverse logistics: a review, *European Journal of Operations Research.* 103: p.1-17.

Frguson M, 2009. Strategic issues in closed-loop supply chains with remanufacturing. http://www.gsb.stanford.edu/facseminars/events/oit/documents/oit_03_09_ferguson_relatedpaper.pdf.

Fleichmann M, H R Krikke, R Dekker, and S D P Flapper, 2000. A characterization of logistics networks for product recovery, *Omega*. 28(6): p.653-666.

Ferguson M, V D R Guide, E Koca and G Souza, 2006. Remanufacturing planning with different quality levels for product returns. *Robert H. Smith School Research Paper*. No. RHS 06-050.

Frguson M., Strategic issues in closed-loop supply chains with remanufacturing, Closed-Loop Supply Chains: New Developments to Improve the Sustainability of Business Practices, Auerbach Publications, 2010: p. 9–22.

Frank K C, R Q Zhang and I Duenyas, 2003, Optimal policies for inventory systems with priority demand classes. *Operations Research.* 51(6): p. 993-1002.

Forsberg R, 1995. Optimization of order-up-to-S policies for two-level inventory systems with compound Poisson demand, *European Journal of Operational Research*. 81(1):p. 143–153.

Frederick S, G. Loewenstein, and T O'Donoghue, 2002. Time discounting and time preference: a critical review, *Journal of Economic Literature*. 40(2):p. 351–401.

Geyer R, L N Van Wassenhove, A Atasu, 2007. The economics of remanufacturing under limited component durability and finite product life cycles, *Management Science*. 53(1):p. 88–100.

Galbreth M R and J D Blackburn, 2006. Optimal acquisition and sorting policies for remanufacturing. *Production and Operations Management.* 15(3): p. 384-392.

Golany B, Yang J, and Yu G, 2001. Economic lot-sizing with remanufacturing options. *IIE Transactions.* 33: p.995–1003.

Gerchak Y and M Henig, 1989. Component commonality in assemble-to-order systems: models and properties, *Naval Research Logistics*. 3:p. 61-68.

Gunasekaran A. and E Ngai, 2005. Build-to-order supply chain management: A literature review and framework for development, *Journal of Operations Management*. 23: p. 423–451.

Gallien J and L Wein, 2001. A simple and effective component procurement policy for stochastic assembly systems. *Queueing Systems.* 38: p. 221-248.

Gurnani H, R Akella and J Lehoczky, 1996. Optimal ordering policies in assembly systems with random demand and random supply delivery. *IIE Transactions*. 28: p.865-878.

Gurnani H, R Akella and J Lehoczky, 2000. Supply management in assembly systems with random yield and random demand. *IIE Transactions*. 32: p. 701-714.

Guide V D R, G Souza, L N Van Wassenhove and J D Blackburn, 2006. Time value of commercial product returns, *Management Science.* 52(8): p. 1200-1214.

Guide V D R and V Jayaraman, 2000. Product acquisition management: Current industry practice and a proposed framework, *International Journal of Production Research*. 38(16): p. 3779–3800.

Hu S J, J Ko, L Wenyand, H A ElMaraghy, T K Lien, Y Koren, H Bley, C Chryssolouris, N Nasr and M. Shpitalni, 2011.Assembly system design and operations for product variety, *CIRP Annals – Manufacturing Technology,* 60(2): p. 715-733.

Harchol-Balter M., T Osogami, A Scheller-Wolf and A Wierman, 2005. Multi-server queueing systems with multiple priority classes. *Queueing Systems*: *Theory and Applications*.51: p. 331–360.

Hausman W H, H L Lee and A X Zhang, 1998. Joint demand fulfillment probability in a multi-item inventory system with independent order-up-to policies, *European Journal of Operations Research*. 109: p. 646-659.

Huh W T and G Janakiraman, 2010. Base-stock policies in capacitated assembly systems: Convexity properties, *Naval Research Logistics*. 57(2):p. 109-118.

Hillier M S, 2000. Component commonality in multiple-period, assemble-to-order systems, *IIE Transactions*. 32(8): p. 755-766.

Iravani S M R , B Kolfal and M.P. Van Oyen, 2012. Process flexibility and inventory flexibility via product substitution, To appear in *Flexible Services and Manufacturing*.

Iravani S M R, K L Luangkesorn and D Simchi-Levi, 2003. On assemble-to-order systems with flexible customers, *IIE Transactions.* 35: p. 389-403.

Coordinated issue motor vehicle industry remanufacturers' inventory of cores, October 31, 1991, IRS (http://www.irs.gov/businesses/article/0,,id=181533,00.html)

Jorjani S, J Leu and C Scott, 2004. Model for the allocation of electronics components to reuse options, *International Journal of Production Research*. 42: p. 1131–1145

Jin X, J Ni, S J Hu, G Xiao and S Biller, Performance analysis of a remanufacturing system with warranty return admission (*under review*).

Jin X, J Ni and Y Koren, Optimal control of reassembly with variable quality returns in a product remanufacturing system, *CIRP Annals - Manufacturing Technology*, 2011, 60(1): p. 25–28.

Kalhammer F R, B M Kopf, D H Swan, V P Roan and M P Walsh, 2007. Status and prospects for zero emissions vehicle technology, Final Report Prepared for State of California Air Resources Board, Sacramento, CA.

Krikke H R, A Van Harten and P C Schuur, 1998. On a medium term product recovery and disposal strategy for durable assembly products, *International Journal of Production Research*. 36(1): p. 111-140.

Krikke H R, A Van Harten and P C Schuur, 1999. Business Case: reverse logistic network re-design for copiers, *OR-Spektrum*. 21: p. 381–409.

Kao E P C and Wilson S D, 1999. Analysis of non-preemptive priority queues with multiple servers and two priority classes, *European Journal of Operational Research*. 118(1): p. 181-193.

Kiefer J, 1953. Sequential minimax search for a maximum, *Proceedings of the American Mathematical Society*. 4(3): p. 502–506.

Kara S, F Rugrungruang and H Kaebernick, 2007. Simulation modelling of reverse logistics networks. *International Journal of Production Economics*. 106: p. 61 – 69.

Lund R T, 1983. Remanufacturing: United States Experience and Implications for Developing Nations, Center for Policy Alternatives, Massachusetts Institute of Technology.

Lund R T, 1984. Remanufacturing. *Technology Review*. 87(2): p. 19-23,28-29.

Lippman S A, 1975. Applying a new device in the optimization of exponential queueing systems, *Operations Research*. 23: p. 687-710.

Latouche G and V Ramaswami, 1999. Introduction to matrix analytic methods in stochastic modeling, *ASA-SIAM Series on Statistics and Applied Probability* (Chapter 1).

Liu J, S Yang, A Wu and S J Hu, 2012. Multi-state throughput analysis of a two-stage manufacturing system with parallel unreliable machines and a finite buffer. *European Journal of Operational Research*.219(2): p.296-304.

Mendelson H and R R Pillai, 1999. Industry clockspeed: Measurement and operational implications. *Manufacturing & Service Operations Management* 1: p. 1–20.

Neuts M F, 1981. Matrix-Geometric Solutions in Stochastic Models: An Algorithmic Approach. Baltimore: The Johns Hopkins University Press.

Nozaki S and R Ross, 1978. Approximations infinite capacity multiserver queues with Poisson arrivals. *Journal of Applied Probability*. 15(4): p. 826-834.

Oakdene Hollins Ltd. Remanufacturing in the UK: a significant contributor to sustainable development, Skipton: *The Resource Recovery Forum*. 2004.

Puterman M L, Markov Decision Processes. Wiley, 1994.

Richey R G, S E Genchev and P J Daugherty, 2005. The role of resource commitment and innovation in reverse logistics performance, *International Journal of Physical Distribution & Logistics Management*. 35(4): p. 233-257.

Souza G C, M Ketzenberg and V D R Guide Jr., 2002. Capacitated remanufacturing with service level constraints, *Production and Operations Management*. 11(2): p. 231-248.

Seliger G, C Franke and M Ciupek, 2004. Process and facility planning for mobile phone remanufacturing. *CIRP Annals – Manufacturing Technology*. 53(1): p. 9-12.

Song J S, S Xu and B Liu, 1999. Order-fulfillment performancemeasure in an assemble-to-order system with stochastic leadtimes, *Operations Research*. 47: p.131–149.

Song J S, 1998. On the order fill rate in a multi-item, base-stock inventory system, *Operations Research*. 46: p. 831–845.

Song J S, and Yao D D, 2002. Performance analysis and optimization in assemble-to-order systems with random leadtimes, *Operations Research*. 50: p. 889-903.

Song J S and P Zipkin, 2003. Supply chain operations: assemble-to-order systems. *Handbooks in Operations Research and Management Science*, 11: *Supply Chain Management*. Elsevier, Amsterdam, The Netherlands, p. 561–596.

Song J S, Yano C and P Lerssrisuriya, 2000. Contract assembly: Dealing with combined supply lead time and demand quantity uncertainty, *Manufacturing Service Operations Management* 2: p. 287-296.

Shanthikumar J G and D D Yao, 1991. Strong stochastic convexity and its applications, J. Appl. Probab. 28 642-659.

Spotnitz R, 2003. Simulation of capacity fade in lithium-ion batteries, *Journal of Power Sources.* 113: p. 72-80.

Stewart W J, 2009. Probability, Markov Chains, Queues, and Simulation: The Mathematical Basis of Performance Modeling, Princeton University Press.

Trebilcock B, 2002. The seven deadly sins of reverse logistics, *Logistics Management*. 41(6): p. 31-34.

Teunter R H, 2006. Determining optimal disassembly and recovery strategies, *Omega*. 34: p.533-537.

Tan A and A Kumar, 2008. A decision making model to maximize the value of reverse logistics in the computer industry, *International Journal of Logistics Systems and Management*. 4: p.297–312.

Toktay B, L M Wein and S A Zenios, 2000. Inventory management of remanufacturable products, *Management Science*, 46(11): p. 1412–1426.

United States Advanced Battery Consortium, *Electric Vehicle Battery Test Procedures Manual*, Revision 2, Southfield, MI, 1996.

Van Der Laan E and R H Teunter, 2006. Simple heuristics for push and pull remanufacturing policies. *European Journal of Operational Research*. 175(2): p. 1084–1102.

Van Der Laan E and M Salomon, 1997. Production planning and inventory control with remanufacturing and disposal, *European Journal of Operational Research*. 102(2): p. 264–278.

Van Der Laan E, M Salomon, R Dekker and L N Van Wassenhove, 1999. Inventory control in hybrid systems with remanufacturing, *Management Science*. 45(5): p. 733–747.

Westkämper E, 2003. Assembly and disassembly processes in product life cycle perspectives. *CIRP Annals – Manufacturing Technology*. 52(2): p. 579–588.

Wood E, M Alexander and T H Bradley, 2011. Investigation of battery end-of-life conditions for plug-in hybrid electric vehicles, *Journal of Power Sources*. 196: p.5147–5154.

Williams J. 1992. How sustainable is your competitive advantage? *California Management Rev*. 34(3) p. 29–51.

Xu H, D D Yao and S. Zheng, 2010.Optimal control of replenishment and substitution in an inventory system with nonstationary batch demand, *Production and Operations Management* (early view), 2010.

Zikopoulos C and G Tagaras, 2008.On the attractiveness of sorting before disassembly in remanufacturing. *IIE Transactions*. 40(3): p. 313-323.

Zhao Y and D Simchi-Levi, 2006. Performance analysis and evaluation of assemble-to-order systems with stochastic sequential leadtimes, *Operations Research*. 54(4): p. 706–724.

Zipkin P, 2000. Foundations of Inventory Management, McGraw Hill, Boston, Mass, USA.

Zhou S X, Z Tao and X Chao, 2011. Optimal control of inventory systems with multiple types of remanufacturable products. *Manufacturing Service Operations Management*. 13(1) : p. 20–34.

Zhou S X and Yu Y, 2011. Optimal product acquisition, pricing, and inventory management for systems with remanufacturing, *Operations Research*. 59 (2): p. 514-521.

13 Key Questions and Answers about Nissan Leaf Battery Pack and Ordering. HybridCars.com. 2010-05-27.