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This paper studies capacity collaboration between two (potentially competing) firms. We explore the ways that the firms can collaborate by either building capacity together or sharing the existing capacity for production. We consider cases where the two firms' products are potential substitutes and also where the firms' products are independent. We find that a firm can benefit from collaboration even with its competitor. Moreover, the firms do not have to jointly make the production decisions to realize the benefits of collaboration. We consider a model where firms build capacity before demand is realized and make production decisions after they receive a demand signal. They can potentially collaborate in jointly building capacity and/or in exchanging capacity once they receive their demand signals. Interestingly, we find that having firms compete at the production stage can result in firms deciding to build less overall capacity than if they coordinated capacity investment and production. Also, we find that though collaboration in capacity investment is beneficial, collaboration in production using existing capacity is often more beneficial. The benefits of collaboration is largest when competition is more intense, demand is more variable and cost of investment is higher.

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1. Introduction

Most capacity decisions require major resource commitments, and can substantially change the firm's asset structure. Capacity decisions are often hard to reverse, and they are made when there is significant demand uncertainty. Several approaches are used to mitigate risks associated with capacity decisions. These can include real options (Trigeorgis, 1997), flexible capacity (Bish and Wang, 2004), quick response and delayed differentiation (Aviv and Federgruen, 2001). Another option that is becoming more common is for firms to jointly invest in capacity and/or coordinate the use of capacity. Interestingly, it is becoming common for even firms that are competing to undertake such coordination. One way to do this is to establish a joint venture (Harrigan, 1986). Firms that participate in a joint venture contribute equity to build capacity, agree on how to utilize the capacity, and share the resulting revenue. For instance, Sony and Samsung, two of the biggest players in the LCD-TV market, established S-LCD to manufacture flat panels (Lee, 2010). Note that, although Sony and Samsung produced in the same facility, they competed with each other in

the end-user market. Another example is the New United Motor Manufacturing, Inc. (NUMMI), which was jointly formed by Toyota and General Motors in 1984, which manufactured cars under both brands (Henry, 2010). Other examples include Inotera Memories, a joint venture by Nanya Technology and Micron Technologies (Clark, 2008), and Spansion by AMD and Fujitsu (Devine, 2003).

Of course, firms can collaborate and share capacity without forming a joint venture. For instance, in the automotive industry, Toyota and Fuji Heavy Industries agree to share their existing manufacturing facilities (Toyota, 2006). In the airline industry, the code-sharing practice allows different carriers to share flight capacity (Wassmer et al., 2010; Chun et al., 2012). The competing newspapers Detroit Free Press and Detroit News have a joint operating agreement to print in the same facility (Busterna and Picard, 1993), although they put out separate newspapers every weekday using the same capacity.

In this paper, we study capacity collaboration between two (competing) firms. We consider a two-stage model where capacity decisions are made when there are significant uncertainties about market conditions (*first stage*), and production decisions are made after most of these uncertainties are resolved (*second stage*). To capture different collaboration scenarios, we consider several models that differ in the extent that the firms collaborate in capacity and/or in production decisions.

Although there are many examples of capacity collaboration, it is not always clear whether a firm benefits from capacity collaboration, especially when the firm collaborates with a competitor. Capacity collaboration allows firms to reduce investment costs. But doing so with a competing firm may intensify competition, since it gives the competing firm easy access to more capacity. Furthermore, as previous examples show, there are different collaboration scenarios: Firms can build and operate capacity together, or firms can build capacity together and operate autonomously, or firms can just share the existing capacity. How much each of these collaboration scenarios improves firms' profits and when simpler collaboration scenarios (such as sharing existing capacities but not collaborating in building capacity) are as effective as more complicated collaboration scenarios are interesting questions.

Several research questions are central to this paper: (1) Can a firm benefit from collaborating with a competitor? (2) For a given collaboration scenario, what is the total capacity and how is it allocated to each firm? (3) How do firms gain from collaboration? Is most of the gain from deciding capacity together or from utilizing (allocating) capacity together to fulfill demands? (4) How do the outcome and the gains from collaboration change in business parameters such as variability, cost, etc.?

We find that a firm can considerably benefit from collaboration even with a competitor. If the firms collaborate both in capacity and in production decisions, we show that there is a mutually beneficial agreement under which the firms select the centrally optimal decisions. This supports the case for a joint venture. We also find that most of the benefit of collaboration can be captured even when the two firms are allowed to compete in the production stage as long as they build the capacity together and trade their allocations after they observe the demand signals. However, if the firms are not able to collaborate in either of the stages, some efficiency is lost. We find that, if the firms can collaborate either in the capacity investment stage or in the production stage alone, collaborating in the production stage alone provides more benefits, except when demands are extremely predictable.

We find that when the firms compete in the production stage, but are allowed to trade their capacity endowments after they observe the demand signals, they build smaller capacity together than the capacity of a centralized firm. We also find that the total equilibrium capacity when the firms compete in both stages can be smaller than the total equilibrium capacity when the firms compete in capacity investment and collaborate in production. These are surprising results, since competition typically leads to higher resource levels than centralization in most of the existing literature (Eppen, 1979; Yang and Schrage, 2009).

Several papers study capacity and production decisions for two products from the perspective of single or multiple firms. Wang and Yang (2012) consider the effect of knowledge spill over when two buyers share a supplier. Van Mieghem (1998) considers a single firm that produces two products and examines the value of flexible capacity. Bish and Wang (2004) extend this to a price-setting firm, and Chod and Rudi (2005) and Bish and Suwandechochai (2010) consider a model where the two products are substitutes. With a perspective of multiple firms, Anupindi and Jiang (2008) study a duopoly where the firms with homogeneous products compete in capacity and in quantity. Goyal and Netessine (2007), on the other hand, consider partially substitutable products and study a model where two firms, each capable of producing both products, compete in capacity and in quantity. They focus on symmetric equilibria in the capacity game when the firms are allowed to choose between flexible or dedicated technologies. All of these models assume that either one firm produces both products, or two firms compete without any collaboration. Our paper, on the other hand, focuses on collaboration between two (potentially) competing firms.

Collaborating in capacity and/or in production, is related to sharing inventory through transshipment and centralization. Anupindi et al. (2001) and Granot and Sosic (2003) consider a setting where the firms individually make quantity decisions, but they can trade inventory after observing

the demand signals. Rudi et al. (2001) study inventory transshipment and show that, in general, the firms' quantities in equilibrium are not centrally optimal. They propose a contract that induces the firms to choose the centrally optimal quantities. Van Mieghem (1999) studies a model with a manufacturer and a subcontractor. In this model, each firm separately decides on its capacity ex-ante but has an option to trade capacity ex-post. He shows that the firms can reach a centrally optimal solution only when the contract terms are contingent on the demand realizations. Slikker et al. (2005) use cooperative game theory to study inventory centralization with coordinated ex-ante orders and ex-post allocation among retailers. They show that the core of the cooperative game is nonempty. Other papers that consider inventory centralization include Hanany and Gerchak (2008), Ozen et al. (2008) and Chen and Zhang (2009). All of the aforementioned papers assume that the price is fixed and the demand is exogenous. Our paper, on the other hand, considers that demand is endogenous.

Only a few papers study capacity collaboration with endogenous demand. Chod and Rudi (2006) consider two firms that engage in capacity trade after revising their forecasts, and characterize the trading contracts that lead to the centrally optimal capacity levels. Plambeck and Taylor (2005) study a model of two OEMs that collaborate in capacity and decide investment levels in demand-stimulating innovations. They characterize the effects of collaboration structures on equilibrium innovation levels. Both papers assume that the demand of one product is independent from the other. In contrast, our paper allows the two firms to compete, and analyzes how competition incentivizes (or discourages) capacity or production collaboration.

We model the outcome of collaboration between two firms using a bargaining game. Bargaining has been extensively studied in economics literature and has been applied to model the outcomes of negotiations on wage settlement between unions and firms, price decisions between retailers and consumers, and terms of mergers and acquisitions. (See Muthoo, 1999; for an extensive review). To characterize the outcome of a bargaining game, we use the Nash Bargaining Solution (NBS). The NBS establishes that the equilibrium outcome maximizes the product of the firms' surpluses net of their disagreement payoffs (Nash, 1950). Although the NBS does not directly specify the bargaining process, it has been shown that the outcomes of several bargaining processes (or situations) can be modeled as variants of the NBS, including alternating offers Rubinstein (1982). Furthermore, a number of extensions of Rubinstein's model, such as the possibility of negotiation breakdown or presence of inside or outside options, lead to outcomes that are slight variations of the NBS outcome (Muthoo, 1999). There is also significant experimental evidence indicating that the NBS is successful in predicting the outcomes of various bargaining situations (Roth, 1995). As a

result, a number of papers in operations management literature uses the NBS to model bargaining between two parties: Van Mieghem (1999), Chod and Rudi (2006), Plambeck and Taylor (2005), Nagarajan and Bassok (2008), Kostamis and Duenyas (2009), Kuo et al. (2011), etc. Nagarajan and Sosic (2008) provide a comprehensive review of the use of cooperative game theory in operations management.

The remainder of this paper evolves as follows. In Section 2, we introduce the model, the notation, and the preliminaries. In Section 3, we present the analysis and the results, starting with the production subgame, followed by the capacity investment decision. We carry out a computational study to gain further insights, which we present in Section 4. Section 5 provides future research directions and concluding remarks.

2. Model, Notation and Preliminaries

We consider two firms, each producing a single product, engaging in competition / collaboration over two stages. In the first stage, firms build capacity before demand information is known. In the second stage, firms observe the demand signals, and then decide the production quantities. We assume that the two firms either compete or collaborate in either or both of the two stages. If they compete, each firm chooses its decision to maximize its own payoff. If they collaborate, the firms jointly make the decisions and negotiate over the division of the total payoff. Along with the benchmark scenario with a single centralized firm, four scenarios represent a varying degree of collaboration, as summarized in Table 1.

Scenario	Capacity Decision	Production Decision
Nn	no collaboration (N)	no collaboration (n)
Nc	no collaboration (N)	collaboration (c)
Cn	collaboration (C)	no collaboration (n)
Cc	collaboration (C)	collaboration (c)
Monopoly	centralized	centralized

Table 1 Capacity and production decisions under each scenario.

We will separately analyze each of the four collaboration scenarios, along with the benchmark centralized scenario. Our aim is to analyze the benefit that firms get from collaborating on joint capacity investments and/or using the capacity that they each have to jointly decide on production levels. Depending on the collaboration scenario, firms invest in capacity – together (C) or separately (N) – in the first stage. Let c be the capacity building cost per unit. We denote K_i as the capacity endowment of firm i, for which, firm i holds the ownership rights. If there is no collaboration in the subsequent production stage, K_i is the capacity level that firm i can use for production. If the

firms collaborate in the production stage, they can negotiate over the use of capacity so that a firm can produce beyond its initial endowment.

In the second stage, the firms observe the demand signals. Let (Θ_1, Θ_2) be random variables that represent the demand information and θ_i be the realization of Θ_i, $i = 1, 2$. We assume that Θ_i has mean μ_i, standard deviation σ_i, marginal density function $f_i(\cdot)$, $i = 1, 2$, and joint density function $f(\cdot, \cdot)$. The observed demand information, (θ_1, θ_2), and the firms' capacity endowments, (K_1, K_2), define the production subgame $\omega := (K_1, K_2, \theta_1, \theta_2)$. Let Ω denote the set of all subgames with given capacity endowments: $\Omega := \{(K_1, K_2, \Theta_1, \Theta_2)\}$.

In a given production subgame $\omega \in \Omega$, firms decide the production quantities $\Big(q_1(\omega), q_2(\omega)\Big)$. If the firms do not collaborate in the production stage (scenarios Cn or Nn), each firm can only produce up to its initial endowment K_i. On the other hand, if the firms collaborate during the production stage (scenarios Cc and Nc), they jointly choose the production quantities based on the demand signals. We allow the two products to be (partially) substitutable. Hence, the price for product i is given as a function of quantities as follows:

$$p_i\Big(q_1(\omega), q_2(\omega), \omega\Big) = \theta_i - b_i q_i(\omega) - \hat{b} q_j(\omega), \qquad i, j = 1, 2, \quad i \neq j$$

We assume that $b_i \geq \hat{b} \geq 0$. In other words, a product's own quantity is more influential in its price than the other product's quantity. In addition, note from the inverse demand function that, *ceteris paribus*, the price for product i increases in θ_i (i.e., the demand signal becomes more *favorable* for firm i). This inverse demand function delivers a reasonable representation of reality as it arises from a choice model where a consumer maximizes a quadratic and concave utility function (Singh and Vives, 1984). Essentially, we are modeling firms competing in quantity. This demand model has been used in several papers, including Chod and Rudi (2005), Goyal and Netessine (2007), Zhou and Zhu (2010) and Bish and Suwandechochai (2010). To avoid trivial outcomes, we assume that $b_i > 0$ and $\mu_i > \frac{\hat{b}}{2b_j}\mu_j + c$ for $i, j = 1, 2$ and $i \neq j$. The model captures many practical situations: for example, in the U.S. car industry, historically car companies produce quantities but then discount at the end of the model year to ensure all cars that were produced are sold. So in this case, quantities are decided first and the price depends on the quantities. Similarly, electronics components markets such as memory chips function similarly.

If the firms collaborate in the production stage, to allocate the revenues in a mutually agreeable way, an exchange of transfer payment can occur between the firms. Let $\Gamma(\omega)$ be the net transfer payment from firm 1 to firm 2 in a subgame ω: if $\Gamma(\omega) > 0$, firm 1 pays firm 2; if $\Gamma(\omega) < 0$, firm 2 pays firm 1.

For each of the four scenarios, we solve the problem using backward induction. We first determine the equilibrium production quantities and transfer payment for a production subgame ω. We use superscripts to denote the equilibrium decisions and outcomes. For instance, we let $\left(q_1^{\mathfrak{s}*}(\omega), q_2^{\mathfrak{s}*}(\omega)\right)$ be the equilibrium production quantities. Note that, these quantities are chosen together if the firms collaborate in the production stage ($\mathfrak{s} = \mathrm{c}$, for the Cc and Nc scenarios), or separately if the firms compete in the production stage ($\mathfrak{s} = \mathrm{n}$, for the Cn and Nn scenarios). Similarly, we let $\Gamma^{\mathrm{c}*}(\omega)$ denote the equilibrium transfer payment under the Cc and Nc scenarios. If the firms do not collaborate in production, there is no transfer payment in the second stage (Cn and Nn scenarios): $\Gamma^{\mathrm{n}*}(\omega) = 0$. Then, the equilibrium revenue for firm i in a subgame $\omega = (K_1, K_2, \theta_1, \theta_2)$, denoted by $R_i^{\mathfrak{s}*}(\omega)$, can be written as follows:

$$R_i^{\mathfrak{s}*}(\omega) = q_i^{\mathfrak{s}*}(\omega) p_i\Big(q_i^{\mathfrak{s}*}(\omega), q_j^{\mathfrak{s}*}(\omega), \omega\Big) + (-1)^i \Gamma^{\mathfrak{s}*}(\omega), \qquad i, j = 1, 2,\ i \neq j,\ \mathfrak{s} \in \{c, n\} \tag{1}$$

Once we determine the equilibrium revenues for each setting, we calculate the expected profits by taking the expectation with respect to the demand signals and subtracting the capacity costs. For given capacity endowments (K_1, K_2), we let $\pi_i^{\mathfrak{s}*}(K_1, K_2)$ denote the expected profit of firm i under setting $\mathfrak{s}$:

$$\pi_i^{\mathfrak{s}*}(K_1, K_2) = \mathbb{E}\Big[R_i^{\mathfrak{s}*}(K_1, K_2, \Theta_1, \Theta_2)\Big] - cK_i, \qquad i = 1, 2 \quad \mathfrak{s} \in \{c, n\} \tag{2}$$

If the firms do not collaborate in the first-stage investment game, each firm chooses the capacity that only maximizes its expected profit (equation (2)). On the other hand, if the firms collaborate in the investment stage, firms negotiate to jointly build the capacity and share the profit according to the Nash bargaining solution. Under the NBS, the capacity level that maximizes the sum of the profits, $\pi_1^{\mathfrak{s}*}(\cdot) + \pi_2^{\mathfrak{s}*}(\cdot)$ is chosen, and the profit is split in a manner that neither firm wants to deviate from the agreement: see Section 3.3.

Once we obtain the profit functions, we solve for the equilibrium capacities under each scenario. To distinguish the equilibrium capacities for each scenario, we use superscripts. For instance, $K_i^{\mathrm{Nn}*}$ is the equilibrium capacity of firm i under the Nn scenario, etc. To denote the outcomes and profit of a single centralized firm, we use a superscript 'm' (representing *monopoly*).

Remark: Without loss of generality, we assume that the second stage production cost is zero, since, any positive (and possibly asymmetric) cost can be accommodated in our model by shifting the demand variable Θ_i, $i = 1, 2$.

When a variable or a function represents a joint/total value, we use subscript 'T'. For instance, while K_1 denotes the capacity for firm 1, K_{T} denotes the joint/total capacity ($K_{\mathrm{T}} = K_1 + K_2$). We

use $\mathbf{1}_{\{\}}$ to denote the indicator function. We also use ∇ as the differentiation operator: $\nabla g(x) = \frac{d}{dx}g(x)$, $\nabla^2 g(x) = \frac{d^2}{dx^2}g(x)$ for a function with single variable, and $\nabla_1 g(x_1,x_2) = \frac{\partial}{\partial x_1}g(x_1,x_2)$, $\nabla_2 g(x_1,x_2) = \frac{\partial}{\partial x_2}g(x_1,x_2)$ and $\nabla^2_{12} g(x_1,x_2) = \frac{\partial^2}{\partial x_1 \partial x_2} f(x_1,x_2)$ for a function with multiple variables. Moreover, differentiation operator has precedence over assignment. Hence, for instance, if $g(x) = x^2$, then $\nabla g(y^2) = 2y^2$ rather than $\nabla g(y^2) = 4y^3$.

2.1. Preliminaries - A Centralized Firm

We first consider a single firm who decides the capacity, and the production quantities for both products. For given capacity K_{T} and demand signals (θ_1,θ_2), the centralized firm solves the following problem to determine the optimal production quantities:

$$\begin{aligned} \max_{q_1,q_2\geq 0} \; & q_1\big(\theta_1 - b_1 q_1 - \hat{b} q_2\big) + q_2\big(\theta_2 - b_2 q_2 - \hat{b} q_1\big) \\ \text{s.t.} \quad & q_1 + q_2 \;\leq\; K_{\mathrm{T}} \end{aligned} \tag{3}$$

Let $\Big(q_1^{\mathrm{m}^*}(K_{\mathrm{T}},\theta_1,\theta_2), q_2^{\mathrm{m}^*}(K_{\mathrm{T}},\theta_1,\theta_2)\Big)$ denote the optimal production quantities. Then, the resulting revenue is:

$$\begin{aligned} R^{\mathrm{m}^*}(K_{\mathrm{T}},\theta_1,\theta_2) \;=\; & q_1^{\mathrm{m}^*}(K_{\mathrm{T}},\theta_1,\theta_2)\Big(\theta_1 - b_1 q_1^{\mathrm{m}^*}(K_{\mathrm{T}},\theta_1,\theta_2) - \hat{b} q_2^{\mathrm{m}^*}(K_{\mathrm{T}},\theta_1,\theta_2)\Big) \\ & + q_2^{\mathrm{m}^*}(K_{\mathrm{T}},\theta_1,\theta_2)\Big(\theta_2 - b_2 q_1^{\mathrm{m}^*}(K_{\mathrm{T}},\theta_1,\theta_2) - \hat{b} q_1^{\mathrm{m}^*}(K_{\mathrm{T}},\theta_1,\theta_2)\Big) \end{aligned} \tag{4}$$

Utilizing this, the expected profit is given $\pi^{\mathrm{m}^*}(K_{\mathrm{T}}) = \mathbb{E}\Big[R^{\mathrm{m}^*}(K_{\mathrm{T}},\Theta_1,\Theta_2)\Big] - cK_{\mathrm{T}}$. In the capacity building stage, the centralized firm selects the capacity level $K_{\mathrm{T}}^{\mathrm{m}^*}$ to maximize the expected profit:

$$K_{\mathrm{T}}^{\mathrm{m}^*} = \operatorname*{arg\,max}_{K_{\mathrm{T}}\geq 0} \; \pi^{\mathrm{m}^*}(K_T) \tag{5}$$

The following proposition characterizes the optimal production and capacity decisions.

Proposition 1.

i. For given (K_T,θ_1,θ_2), define the two switching curves:

$$\tau_i^{m^*}(K_T,\theta_j) = \begin{cases} 2b_i K_T, & \text{if } \theta_j \leq 2\hat{b}K_T; \\ \frac{2(b_i b_j - \hat{b}^2)K_T - (b_i - \hat{b})\theta_j}{b_j - \hat{b}}, & \text{if } \theta_j > 2\hat{b}K_T. \end{cases} \qquad i,j=1,2, \quad i\neq j \tag{6}$$

Then, the optimal production quantities of a centralized firm – $q_1^{m^*}(K_T,\theta_1,\theta_2), q_2^{m^*}(K_T,\theta_1,\theta_2)$– are as follows:

$$\left[\frac{\Big(\theta_1 b_2 - \theta_2\hat{b}\mathbf{1}_{\{\theta_2 b_1 > \theta_1\hat{b}\}}\Big)^+}{2\Big(b_1 b_2 - \hat{b}^2\mathbf{1}_{\{\theta_2 b_1 > \theta_1\hat{b}\}}\Big)}, \frac{\Big(\theta_2 b_1 - \theta_1\hat{b}\mathbf{1}_{\{\theta_1 b_2 > \theta_2\hat{b}\}}\Big)^+}{2\Big(b_1 b_2 - \hat{b}^2\mathbf{1}_{\{\theta_1 b_2 > \theta_2\hat{b}\}}\Big)}\right] \quad \text{if } \theta_1 \leq \tau_1^{m^*}(K_T,\theta_2), \;\; \theta_2 \leq \tau_2^{m^*}(K_T,\theta_1);$$

$$\left[\frac{\Big(2(b_2-\hat{b})K_T + \min\Big((\theta_1-\theta_2), 2(b_1-\hat{b})K_T\Big)\Big)^+}{2(b_1+b_2-2\hat{b})}, \frac{\Big(2(b_1-\hat{b})K_T + \min\Big((\theta_2-\theta_1), 2(b_2-\hat{b})K_T\Big)\Big)^+}{2(b_1+b_2-2\hat{b})}\right], \quad \text{otherwise.} \tag{7}$$

ii. There exists a unique optimal capacity $K_T^{m^*}$.

Figure 1 illustrates the optimal production policy characterized in (7). In this figure, we can observe that it is optimal to fully utilize the capacity only when the firm gets favorable demand signals. (These areas are marked "binding" for binding capacity in Figure 1). The allocation of the capacity to production of products 1 and 2 depends on the relative values of the demand signals. (The gray areas in Figure 1 are areas where it is optimal to produce just one product while in the white areas, it is optimal to produce both products.) Finally Figure 1 also shows both thresholds τ_1, τ_2. The centralized case solution is interesting as it will serve as a benchmark against the decentralized cases with different levels of collaboration.

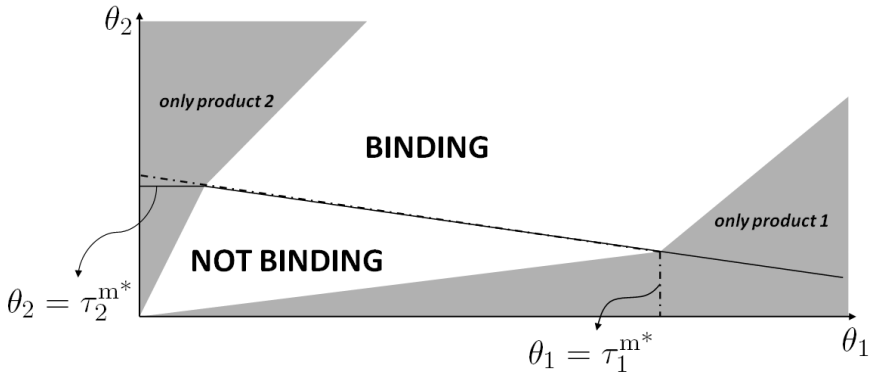

Figure 1 Optimal production strategy for a centralized firm with respect to demand signals.

3. Analysis of Decentralized Firms under Different Collaboration Scenarios

We now provide an analysis of the different scenarios of collaboration. We first start with the production subgames for each scenario and analyze the two different settings of the second stage production subgame - no collaboration (n) and collaboration (c) - for given capacity endowments and demand signals. We then roll back the outcome of the corresponding subgame to the first stage and determine the equilibrium strategies for each of the four different scenarios.

3.1. Noncollaborative Production

If the firms do not collaborate in the production stage (Nn and Cn scenarios), each firm individually chooses the quantity that maximizes its own revenue. Specifically, the equilibrium production quantities for a given subgame $\omega = (K_1, K_2, \theta_1, \theta_2)$, must satisfy the following system of equations:

$$q_1^{n^*}(\omega) = \underset{q_1 \in [0,K_1]}{\arg\max}\ q_1 p_1\Big(q_1, q_2^{n^*}(\omega), \omega\Big) \qquad \text{and} \qquad q_2^{n^*}(\omega) = \underset{q_2 \in [0,K_2]}{\arg\max}\ q_2 p_2\Big(q_1^{n^*}(\omega), q_2, \omega\Big) \tag{8}$$

Solving this, we obtain the following equilibrium outcomes:

PROPOSITION 2. *In a subgame* $\omega = (K_1, K_2, \theta_1, \theta_2)$*, define the two switching curves* $\tau_i^{n^*}(\omega)$*:*

$$\tau_i^{n^*}(\omega) = \begin{cases} 2b_i K_i, & \text{if } \theta_j \le \hat{b} K_i; \\ \frac{(4b_i b_j - \hat{b}^2)K_i + \theta_j \hat{b}}{2b_j}, & \text{if } \hat{b}K_i < \theta_j \le 2b_j K_j + \hat{b} K_i; \\ 2b_j K_j + \hat{b} K_i, & \text{if } 2b_j K_j + \hat{b} K_i < \theta_j. \end{cases} \qquad i,j = 1,2 \quad i \ne j \tag{9}$$

There exists a unique equilibrium production strategy $\left(q_1^{n^*}(\omega), q_2^{n^*}(\omega)\right)$ *such that:*

$$\left(q_1^{n^*}(\omega), q_2^{n^*}(\omega)\right) = \begin{cases} \left(\frac{\left(2\theta_1 b_2 - \theta_2 \hat{b} \mathbf{1}_{\{2\theta_2 b_1 > \theta_1 \hat{b}\}}\right)^+}{4b_1 b_2 - \hat{b}^2 \mathbf{1}_{\{2\theta_2 b_1 > \theta_1 \hat{b}\}}}, \frac{\left(2\theta_2 b_1 - \theta_1 \hat{b} \mathbf{1}_{\{2\theta_1 b_2 > \theta_2 \hat{b}\}}\right)^+}{4b_1 b_2 - \hat{b}^2 \mathbf{1}_{\{2\theta_1 b_2 > \theta_2 \hat{b}\}}} \right), & \text{if } \theta_1 \le \tau_1^{n^*}(\omega) \text{ and } \theta_2 \le \tau_2^{n^*}(\omega); \\ \left(K_1, \frac{\left(\theta_2 - \hat{b} K_1\right)^+}{2b_2} \right), & \text{if } \theta_1 > \tau_1^{n^*}(\omega) \text{ and } \theta_2 \le \tau_2^{n^*}(\omega); \\ \left(\frac{\left(\theta_1 - \hat{b} K_2\right)^+}{2b_1}, K_2 \right), & \text{if } \theta_1 \le \tau_1^{n^*}(\omega) \text{ and } \theta_2 > \tau_2^{n^*}(\omega); \\ (K_1, K_2), & \text{if } \theta_1 > \tau_1^{n^*}(\omega) \text{ and } \theta_2 > \tau_2^{n^*}(\omega). \end{cases} \tag{10}$$

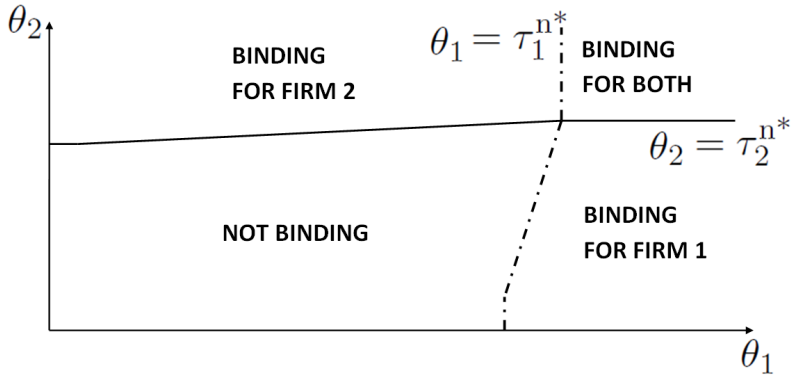

Figure 2 **Equilibrium production strategies when the firms compete in the production subgame, with respect to demand signals.**

Figure 2 illustrates the equilibrium quantities with respect to demand signals, θ_1 and θ_2. If both firms get poor demand signals, their production quantities are low and the initial capacity endowments do not play any role. If only one firm's demand signal is favorable, the firm with favorable demand signal produces at its capacity. If both firms get favorable demand signals, they both produce at their capacities.

We let $R_i^{n^*}(\omega)$ denote the revenue that firm i will earn under the sub game ω,

$$R_i^{n^*}(\omega) = q_i^{n^*}(\omega) p_i \left(q_1^{n^*}(\omega), q_2^{n^*}(\omega), \omega \right) \qquad i = 1,2 \tag{11}$$

3.2. Collaborative Production

If the firms collaborate in the production stage (Cc and Nc scenarios), they jointly set the production quantities and share the total revenue obtained from both products. We assume that firms will decide on the optimal production quantities, and then use Nash bargaining to split the revenues.

To characterize the Nash bargaining solution (NBS), we first need to specify the disagreement payoff for each firm (that is, the payoff that each firm earns if there is no deal). Note that if the firms fail to reach an agreement, each firm chooses the quantity that maximizes its own revenue within its capacity endowment. Hence, the disagreement payoff for firm i is the equilibrium revenue under the no collaboration setting, $R_i^{\mathrm{n}^*}(\omega)$, for a subgame $\omega = (K_1, K_2, \theta_1, \theta_2)$.

The equilibrium quantities and transfer payment of the NBS solve the problem below and the following proposition characterizes the equilibrium.

$$\max_{\Gamma, q_1, q_2} \quad \Big(q_1 p_1(q_1, q_2, \omega) - \Gamma - R_1^{\mathrm{n}^*}(\omega)\Big)\Big(q_2 p_2(q_1, q_2, \omega) + \Gamma - R_2^{\mathrm{n}^*}(\omega)\Big) \tag{12a}$$

$$\text{subject to} \quad q_1 + q_2 \leq K_1 + K_2 \tag{12b}$$

$$q_1 p_1(q_1, q_2, \omega) - \Gamma \geq R_1^{\mathrm{n}^*}(\omega) \tag{12c}$$

$$q_2 p_2(q_1, q_2, \omega) + \Gamma \geq R_2^{\mathrm{n}^*}(\omega) \tag{12d}$$

Proposition 3. *Suppose that firms with capacity endowments (K_1, K_2) collaborate in the production stage. Then, for given $\omega = (K_1, K_2, \theta_1, \theta_2)$, there exists a unique equilibrium $\big(q_1^{c^*}(\omega), q_2^{c^*}(\omega), \Gamma^{c^*}(\omega)\big)$ such that:*

i. The firms produce the same quantities as a centralized firm would: $q_i^{c^*}(K_1, K_2, \theta_1, \theta_2) = q_i^{m^*}(K_1 + K_2, \theta_1, \theta_2)$, $i = 1, 2,$.

ii. The transfer payment (from firm 1 to firm 2) is:

$$\Gamma^{c^*}(\omega) = \frac{q_1^{c^*}(\omega) p_1\Big(q_1^{c^*}(\omega), q_2^{c^*}(\omega), \omega\Big) - R_1^{n^*}(\omega)}{2} - \frac{q_2^{c^*}(\omega) p_2\Big(q_1^{c^*}(\omega), q_2^{c^*}(\omega), \omega\Big) - R_2^{n^*}(\omega)}{2} \tag{13}$$

Proposition 3 establishes that, under the NBS, the quantities produced by the two collaborating firms are equal to those of a single centralized firm. That is, for any demand signal, there exists a negotiation outcome where no efficiency is lost. In order to make the arrangement mutually beneficial for both firms (i.e., each firm's payoff is no less than its disagreement payoff), the transfer payment $\Gamma^{\mathrm{c}^*}(\omega)$ is used to allocate the gains from collaboration, and capacity may have to be reallocated between the firms. Let $\chi^{\mathrm{c}^*}(\omega)$ be the net capacity allocated from firm 2 to firm 1 in equilibrium, which is expressed as follows:

$$\chi^{\mathrm{c}^*}(\omega) = \begin{cases} \big(q_1^{\mathrm{c}^*}(\omega) - K_1\big), & \text{if } q_1^{\mathrm{c}^*}(\omega) > K_1; \\ -\big(q_2^{\mathrm{c}^*}(\omega) - K_2\big), & \text{if } q_2^{\mathrm{c}^*}(\omega) > K_2; \\ 0, & \text{otherwise.} \end{cases} \tag{14}$$

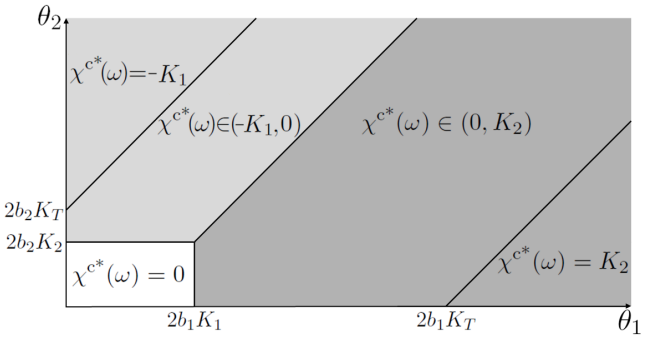

Figure 3 **Equilibrium capacity trade with respect to demand signals.**

Note that $\chi^{c*}(\omega)$ is positive if firm 1 borrows capacity from firm 2, and negative if firm 2 borrows capacity from firm 1. Figure 3 illustrates how capacity is shared with respect to the demand signals, (θ_1, θ_2).

If both firms get poor demand signals, then each firm can serve its demand with its endowed capacity, hence no capacity reallocation need to take place. Otherwise, the two firms readily trade the capacity to produce the quantities that maximize the total revenue. Note that there are regions under which the entire capacity of one firm is reallocated to the other. This happens when one firm is better off by selling its entire capacity and receiving the transfer payment than by producing and selling its own product.

Note that for given total capacity, the equilibrium quantities depend on the demand signals but not on the individual capacity endowments of each firm. Consequently, for a given total capacity, the revenue a firm earns directly from sales (before the transfer payment) depends only on the demand signals, (θ_1, θ_2). However, the transfer payment (which depends not only on the demand signals, but also on the firms' individual capacity endowments K_1, K_2) balances the firms' payoffs according to their initial contributions to the total capacity.

One may expect the transfer payment to be monotone in demand signals or in capacity endowments since capacity becomes more valuable with higher demand. Figure 4 presents the transfer payment from firm 1 to firm 2, $\Gamma^{c*}(\omega)$, and the amount of capacity that firm 1 acquires from firm 2, $\chi^{c*}(\omega)$, with respect to the demand signal for firm 2. When the demand signal is poor (when θ_2 is low), firm 2 transfers its whole capacity to firm 1. In this range, as the demand signal for firm 2 becomes more favorable, the disagreement payoff of firm 2 and hence, its opportunity cost for the capacity increases. Consequently, although the amount of capacity traded remains the same, the transfer payment from firm 1 to firm 2 increases. However, when θ_2 becomes moderate or high, firm 2 finds it optimal to keep some of its capacity to satisfy its own demand. In this range, the

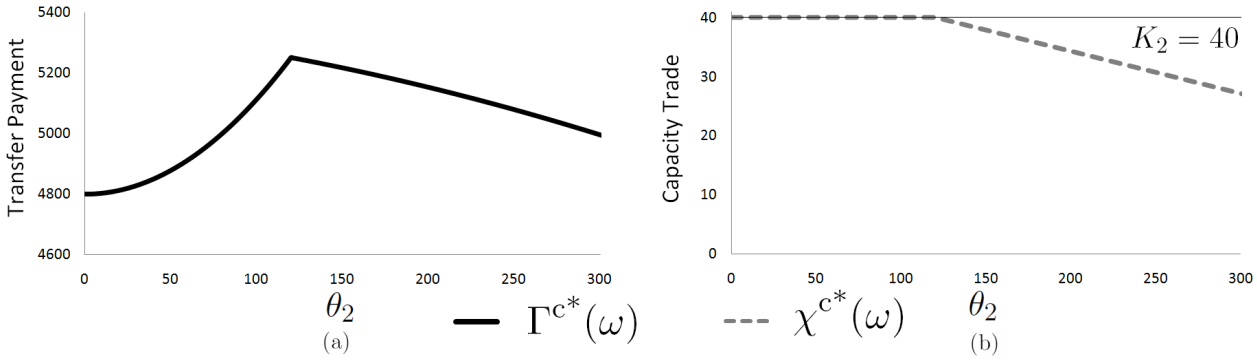

Figure 4 **(a) Transfer payment from firm 1 to firm 2, $\Gamma^{c*}(\omega)$, and, (b) amount of capacity traded from firm 2 to firm 1, $\chi^{c*}(\omega)$, with respect to the demand signal, θ_2. ($b_1 = 3$, $b_2 = 4$, $K_1 = K_2 = 40$, $\theta_1 = 400$)**

amount of capacity that firm 2 trades to firm 1 decreases in θ_2. Consequently, firm 1 receives less capacity in trade, and earns less with the traded capacity. Therefore, the transfer payment, $\Gamma^{c*}(\omega)$, decreases.

To examine how much a unit capacity is worth when it is reallocated, we define the price per unit of reallocated capacity: $\gamma^{c*}(\omega) = \Gamma^{c*}(\omega)/\chi^{c*}(\omega)$, for $\chi^{c*}(\omega) \neq 0$. The next result shows that although the total transfer payment is not monotone, the unit price of capacity is monotone in demand signals and in capacity endowments, when the two products are not substitutes.

PROPOSITION 4. *Suppose that the two products are not substitutes (i.e., $\hat{b} = 0$). Then,*

i. The price of capacity (per unit), $\gamma^{c*}(\omega)$, increases in θ_i, and decreases in K_i, $i = 1, 2$.

ii. Transfer payment is nonzero if and only if there is capacity trade: $\Gamma^{c*}(\omega) \neq 0 \Leftrightarrow \chi^{c*}(\omega) \neq 0$.

When a firm's demand signal becomes more favorable (i.e. θ_i increases),the market clearing price for its products will be higher. Thus, the firm's per unit profit margin will increase. This increases the price that it is willing to pay for a unit of capacity. Likewise, when the demand signal for the firm that sells capacity becomes more favorable, the firm's opportunity cost of the capacity increases, which increases the per unit capacity price he will charge to transfer capacity. Thus, the price of unit capacity increases when either demand signal gets more favorable. On the other hand, when either firm starts with larger capacity, capacity trade becomes less valuable, and hence the transfer price of unit capacity decreases. We also find that, when the products are not substitutes, transfer payment is made if and only if there is nonzero capacity trade. This is intuitive, as one would exchange the payments to balance the revenue only when physical capacity is traded.

However, none of these intuitive results hold when the products are substitutes. Firms pay non-zero transfer payment even when there is no capacity trade. In addition, even when capacity is traded, the unit capacity price is not necessarily increasing as either firms' demand increases. Figure

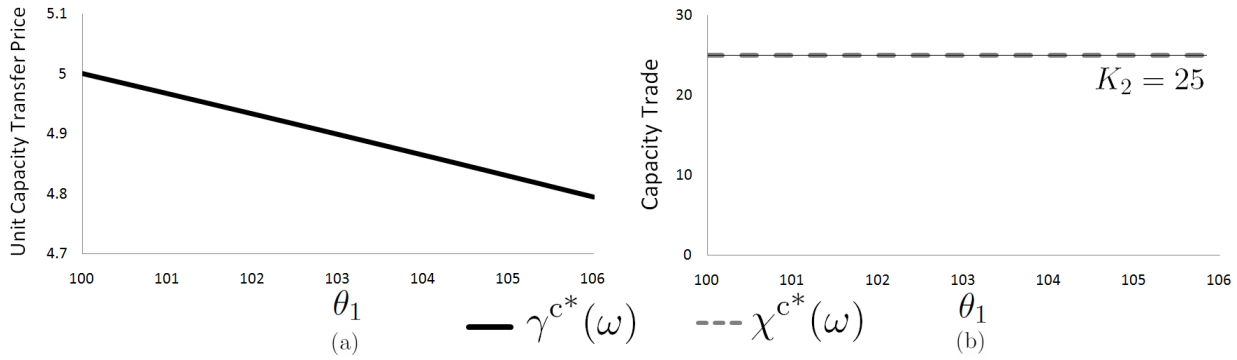

Figure 5 **(a) Unit capacity transfer price, $\gamma^{c*}(\omega)$, and, (b) the amount of capacity traded from firm 2 to firm 1, $\chi^{c*}(\omega)$, with respect to the demand realization of firm 1, θ_1. ($b_1 = b_2 = 2$, $\hat{b} = 1$, $K_1 = K_2 = 25$ and $\theta_2 = 40$)**

5 presents an example. In this example, the amount of capacity firm 1 buys from firm 2 is constant, but the price per unit capacity decreases in the demand signal of firm 1, θ_1. In other words, firm 1 earns more but pays less per unit of the capacity it acquires from firm 2. To understand why, first note that, firm 2 sells its whole capacity to firm 1 in this case. As θ_1 increases, the price of product 1 increases. Hence, the production quantity of firm 1 also increases as a response to a higher market price. But, as firm 1 increases the quantity available in the market, price for product 2 decreases. Consequently, the opportunity cost for firm 2 for its capacity decreases, and, as a result, firm 2 is willing to give up its capacity at a lower price. Note that, when the products are not substitutes, this outcome will never happen, because, the opportunity cost for firm 2 will be independent from the demand signal of firm 1.

3.3. Capacity Investment Stage

After solving the production subgame, we next study the capacity investment stage under the two different cases - No collaboration (N) and Collaboration (C).

No collaboration

If the firms do not collaborate in the first stage (Nc and Nn scenarios), each firm strategically decides its capacity level to maximize its own expected profit. Therefore, the equilibrium capacity levels $(K_1^{\mathrm{N}\mathfrak{s}*}, K_2^{\mathrm{N}\mathfrak{s}*})$ must satisfy the following system of equations:

$$K_1^{\mathrm{N}\mathfrak{s}*} = \underset{K_1 \geq 0}{\arg\max}\ \pi_1^{\mathfrak{s}*}(K_1, K_2^{\mathrm{N}\mathfrak{s}*}) \quad \text{and} \quad K_2^{\mathrm{N}\mathfrak{s}*} = \underset{K_2 \geq 0}{\arg\max}\ \pi_2^{\mathfrak{s}*}(K_1^{\mathrm{N}\mathfrak{s}*}, K_2) \quad \text{for } \mathfrak{s} \in \{n, c\} \tag{15}$$

Collaboration in capacity investment

If the firms collaborate in the first stage (Cc and Cn scenarios), they negotiate to build capacity jointly and share the capacity and its investment costs according to the Nash bargaining solution

(NBS). We assume that each firm pays the capital to obtain its initial endowment, that is, firm i pays cK_i to have an endowment of K_i, which is a part of $\pi_k(\cdot)$, defined in equation (2).

To determine the bargaining outcome, we first specify the disagreement payoff, i.e., what each firm earns if the negotiation fails. Let π_i^{d} denote the disagreement payoff for firm i. For both the Cc, and Cn scenarios, if the firms fail to reach an agreement to collaborate in the first stage, then they never collaborate in the subsequent stages. Notice that we are modeling a situation where firms agree to collaborate in a multi-stage partnership. Thus, in the Cc scenario, the firms are agreeing to collaborate first in capacity, then in production. If the agreement fails at the first stage, then we are assuming that this collaboration will no longer take place. Of course, we will also separately analyze the case where firms agree to collaborate in capacity but not in production Cn and vice versa Nc.

Thus, in a a multi-period collaboration agreement between parties under the Cc scenario, if the firms fail to reach an agreement at the first stage, each firm will decide its own capacity and production quantity separately to maximize its own profit. Therefore, each firm will then earn equilibrium profits under the Nn scenario, and we have $(\pi_1^{\mathrm{d}}, \pi_2^{\mathrm{d}}) = \left(\pi_1^{\mathrm{n}*}(K_1^{\mathrm{Nn}*}, K_2^{\mathrm{Nn}*}), \pi_2^{\mathrm{n}*}(K_1^{\mathrm{Nn}*}, K_2^{\mathrm{Nn}*})\right)$.

If there is a deal, the firms invest in the capacity and obtain the capacity endowments: K_1 and K_2. Then, demand signals are realized, and the firms play the production subgame and earn the revenue as illustrated in subsections 3.1 and 3.2. But, note that the firms agree to a deal if their (ex ante) profits are at least as large as their diagreement payoffs, $\pi_i^{\mathrm{n}*}(K_1^{\mathrm{Nn}*}, K_2^{\mathrm{Nn}*})$, $i = 1, 2$. To guarantee this, one firm may need to make a transfer payment to the other firm so that she earns at least its disagreement payoff. Let η be the first-stage transfer payment firm 1 makes to firm 2 to induce an agreement (negative if the actual payment is from firm 2 to firm 1). According to the NBS, the equilibrium outcome– $(K_1^{\mathrm{C}\mathfrak{s}*}, K_2^{\mathrm{C}\mathfrak{s}*}, \eta^{\mathrm{C}\mathfrak{s}*})$, for $\mathfrak{s} \in \{n, c\}$– is the solution to the following optimization problem:

$$\begin{aligned} \max_{\substack{K_1, K_2 \geq 0 \\ \eta}} \quad & \left(\pi_1^{\mathfrak{s}*}(K_1, K_2) - \eta - \pi_1^{\mathrm{d}}\right)\left(\pi_2^{\mathfrak{s}*}(K_1, K_2) + \eta - \pi_2^{\mathrm{d}}\right) \\ \text{s.t.} \quad & \pi_1^{\mathfrak{s}*}(K_1, K_2) - \eta \geq \pi_1^{\mathrm{d}} \\ & \pi_2^{\mathfrak{s}*}(K_1, K_2) + \eta \geq \pi_2^{\mathrm{d}} \end{aligned} \tag{16}$$

Notice that the transfer payment, η, plays a role of investment subsidy. Consequently, an equilibrium outcome in which $\eta = 0$ implies that each firm pays only for its own endowment as no firm gives or receives a payment in the first stage.

When the products are not substitutes ($\hat{b} = 0$), all of the results that follow will hold for any continuous demand distribution with positive support. This includes cases when the demand signals

are correlated. However, when the products are substitutes ($\hat{b} > 0$), obtaining analytical results is much more difficult. This is because the derivative of the firm's profit (following the subgame outcome) is discontinuous, and makes verifying the second-order condition formidable (see Goyal and Netessine, 2007 for a thoughtful discussion). To overcome this, we make the following additional assumptions for the case of substitutable products: (i) Θ_1 and Θ_2 are independent, and (ii) Θ_i follows either an uniform or exponential distribution, $i = 1, 2$. Although our results are proven for the two distributions, our computational study shows that the results are still valid with other distributions as well.

THEOREM 1.

i. In all four scenarios (Nn, Nc, Cn and Cc) a pure strategy equilibrium exists. Moreover, the equilibrium is unique under the Nn and the Nc scenarios.

ii. When the firms collaborate in capacity investment (Cn or Cc scenarios):

(a) *The difference between the firms' equilibrium profits is equal to the difference in their disagreement payoffs:*

$$\left(\pi_1^{\mathfrak{s}*}(K_1^{C\mathfrak{s}*}, K_2^{C\mathfrak{s}*}) - \eta^{C\mathfrak{s}*}\right) - \left(\pi_2^{\mathfrak{s}*}(K_1^{C\mathfrak{s}*}, K_2^{C\mathfrak{s}*}) + \eta^{C\mathfrak{s}*}\right) = \pi_1^d - \pi_2^d, \quad \text{for} \quad \mathfrak{s} \in \{n, c\} \tag{17}$$

(b) *Under the Cc scenario, the total capacity in equilibrium is equal to the optimal capacity of a centralized firm:* $K_1^{Cc*} + K_2^{Cc*} = K_T^{m*}$.

Theorem 1 implies that, while the equilibrium collaboration outcome makes both firms better off, the difference in their profits remains the same as the difference in their disagreement payoffs. In other words, the negotiation outcome only increases the total surplus without changing the difference in profits.

Theorem 1 also establishes that, if the firms are allowed to collaborate both in capacity and in production stages (Cc scenario), the total capacity is the same as a centralized firm's capacity. Under this scenario, the firms not only produce the centrally optimal quantities (Proposition 3) but also agree to build the optimal capacity of a centralized firm. Thus, no efficiency is lost in either stage.

An interesting question is how firms pay for the capacity they will purchase. Suppose the two firms agree to collaborate and build capacity K_1 and K_2. Do they each pay for the capacity they build or is it the case that one of the firms has to provide a subsidy to the other? One would expect that each firm's contribution should be proportional to its endowment and that it is subsidy-free: in other words, $\eta^{C\mathfrak{s}*} = 0$. The next result characterizes the condition under which this occurs.

PROPOSITION 5. *For the Cc and Cn scenarios:*

i. An investment equilibrium is subsidy-free (i.e., $\eta^{C\mathfrak{s}*}=0$), if and only if the equilibrium endowments $(K_1^{C\mathfrak{s}*}, K_2^{C\mathfrak{s}*})$ satisfy the following:

$$\pi_1^{n*}(K_1^{C\mathfrak{s}*}, K_2^{C\mathfrak{s}*}) - \pi_2^{n*}(K_1^{C\mathfrak{s}*}, K_2^{C\mathfrak{s}*}) = \pi_1^d - \pi_2^d \qquad for \quad \mathfrak{s} \in \{n, c\} \tag{18}$$

ii. If the products are not substitutes ($\hat{b}=0$), then there always exists a subsidy-free investment equilibrium. If the products are substitutes ($\hat{b}>0$), a subsidy-free investment equilibrium exists when the firms are allowed to collaborate in the second stage (Cc scenario) and $K_T^{Nn*} \geq K_T^{m*}$. Otherwise, a subsidy-free investment equilibrium may not exist in general.

The condition in (18) leads to a subsidy-free investment. Notice that the left hand side is the difference between the profits when the firms compete in the second stage with the endowments $(K_1^{C\mathfrak{s}*}, K_2^{C\mathfrak{s}*})$. The right hand side is the difference between the disagreement payoffs, same as in the condition in (17). This implies that, under the subsidy-free equilibrium, the difference between the firms' profits must be the same regardless of whether they collaborate in the subsequent production stage.

Proposition 5 also implies that, when the two products are not substitutes, a subsidy-free equilibrium exists regardless of whether the firms collaborate in the production stage or not (Cc and Cn scenarios). Under the Cn scenario, as no capacity sharing will occur in the second stage, there is no gain from joint capacity investment (because the products are not substitutes). Hence, each firm agrees to build the endowment that maximizes its own profit and there is no investment subsidy. On the other hand, under the Cc scenario, the firms share capacity in the second stage, and hence they gain from joint investment in capacity. The second stage negotiation allocates these gains so that the firms do not need the investment subsidy to select the centrally optimal joint capacity in the first stage.

On the other hand, when the products are substitutes, a subsidy-free equilibrium exists only when the firms collaborate in both stages (Cc scenario) and the total capacity of a centralized firm is smaller than the total capacity under the Nn scenario (i.e., $K_{\mathrm{T}}^{\mathrm{m}*} \leq K_{\mathrm{T}}^{\mathrm{Nn}*}$). Note that, when the products are substitutes, and the firms collaborate in both stages, the gains of collaboration come not only from pooling capacities but also from pooling demands (i.e., forgoing competitive behavior). The negotiation in the second stage allocates the gains from pooling demands. If the total capacity of a centralized firm is smaller than the total capacity under the Nn scenario, by jointly building the centrally optimal capacity in the first stage, the firms achieve the investment cost savings. Consequently, in the first stage the firms jointly build the centrally optimal capacity

and select the endowments at which the savings from capacity investment cost are split without an investment subsidy.

When the products are substitutes, and the total capacity of a centralized firm is larger than the total capacity under the Nn scenario (i.e., $K_{\mathrm{T}}^{\mathrm{m}*} > K_{\mathrm{T}}^{\mathrm{Nn}*}$), a subsidy-free investment equilibrium does not necessarily exist even when the firms collaborate in both stages. Without the investment subsidy, the firms' total capacity may be less than that of a centralized firm. Thus, even if they collaborate in the second stage, the firms can earn less than what a centralized firm can earn for the same realization of demand signals. Therefore, the firms lose efficiency. Hence, one firm finds it beneficial to pay a subsidy to the other firm and induce it to agree on building a larger capacity to guarantee that the revenue in the second stage is the same as the revenue of a centralized firm.

When the products are substitutes and the firms do not collaborate in the second stage (Cn scenario), a subsidy-free investment equilibrium does not exist in general. Even though there is no capacity sharing in the second stage, there still exists a gain from joint investment in capacity for substitutable products. However, because there is no ex-post recourse to resolve the inefficiencies (due to possible imbalance between endowments and demand signals), whether each firm realizes the gain or not depends on its initial endowment. Consequently, an upfront subsidy is generally needed so that the firms select the endowments that maximize the gains of collaboration.

When the firms collaborate in both stages, it is interesting to know how much each firm will invest in a subsidy-free equilibrium. First, note from Theorem 1 that the total capacity is equal to that of a centralized firm. This implies that $K_2^{\mathrm{Cc}*}$, can be replaced by $K_{\mathrm{T}}^{\mathrm{m}*} - K_1^{\mathrm{Cc}*}$, in (18). Moreover, note that (18) is defined by the disagreement payoffs and $\pi_i^{\mathrm{n}*}(\cdot,\cdot)$, $i = 1, 2$. The disagreement payoffs do not depend on the endowments. In addition, $\pi_i^{\mathrm{n}*}(\cdot,\cdot)$ is a well-defined continuous function. Therefore, one can simply solve (18) with a search in single variable in a bounded interval to determine the investment level under a subsidy-free equilibrium.

3.4. Collaboration in Capacity and Partial Collaboration in Production (Cp)

So far, we have analyzed the equilibrium outcomes for four scenarios and show that the firms gain the most if they can fully collaborate in both stages (Cc scenario). However, to achieve this outcome, the two firms must build capacity and set production quantities together. Furthermore, the two firms not only need to make decisions together but they also need to agree on how to split the profit for each contingency in detail. In the previous section, we show that, if the two products are substitutes, the firms may exchange the transfer payment even when there is no physical exchange of the capacity. One alternative arrangement is that the firms collaborate on strategic decisions (e.g., building capacity), but each firm individually sets its production quantity, while they trade

the capacity endowments if necessary. We call this arrangement *Cp scenario* (where 'p' stands for *partial* collaboration in production).

Under this scenario, the firms build a joint capacity in the first stage. In the second stage, after the firms observe the demand signals, they trade capacity to establish new endowments. Then, each firm individually decides its production quantity within its new endowment. An example of such collaboration can be found in an arrangement between AMD and Fujitsu for producing flash memory chips (Devine, 2003). Under this arrangement, the firms built a plant together (thus collaboratively chose the total capacity), but each firm individually decided how much to purchase from the plant's output (Plambeck and Taylor, 2005). Another such example is the limited joint operating agreement between two newspapers: Detroit Free Press and Detroit News. Under this arrangement, the newspapers operate separately, but are printed in the same, jointly built facility (Busterna and Picard, 1993).

Once again, we solve for the equilibrium outcome using backward induction. Let $\hat{K}_i$, $i=1,2$, be the new endowment of firm i after the capacity trade, and let $\hat{\omega}$ be the vector that represents the new capacity endowments and demand signals: $\hat{\omega} := (\hat{K}_1, \hat{K}_2, \theta_1, \theta_2)$. For each realization of $\hat{\omega}$, the equilibrium production quantities must satisfy the following set of equations:

$$q_1^{\mathrm{Cp}^*}(\hat{\omega}) = \underset{q_1 \in [0,\hat{K}_1]}{\arg\max}\ q_1 p_1\Big(q_1, q_2^{\mathrm{Cp}^*}(\hat{\omega}), \hat{\omega}\Big) \quad \text{and} \quad q_2^{\mathrm{Cp}^*}(\hat{\omega}) = \underset{q_2 \in [0,\hat{K}_2]}{\arg\max}\ q_2 p_2\Big(q_1^{\mathrm{Cp}^*}(\hat{\omega}), q_2, \hat{\omega}\Big) \tag{19}$$

Note that the equilibrium outcome of this quantity-setting game is identical to (8) except that $(\hat{K}_1, \hat{K}_2)$ replaces (K_1, K_2). In other words, the production quantities are the same as the ones under the Nn scenario with the endowments $(\hat{K}_1, \hat{K}_2)$. Therefore, by equation (11), firm i earns the revenue $R_i^{\mathrm{n}^*}(\hat{\omega})$ from sales. Given this, we can consider the preceding capacity trading game for given initial endowments (K_1, K_2) and demand signals (θ_1, θ_2). As before, let Γ be the transfer payment that firm 1 pays to firm 2 (negative if firm 2 pays to firm 1). Thus, if the firms agree on a deal, firm i earns:

$$R_i^{\mathrm{n}^*}(\hat{K}_1, \hat{K}_2, \theta_1, \theta_2) + (-1)^i \Gamma, \quad i = 1,2$$

If the firms fail to reach a deal, each firm individually decides its production quantity using its initial endowment, K_i, as the capacity constraint. Hence, the disagreement payoff of firm i is $R_i^{\mathrm{n}^*}(K_1, K_2, \theta_1, \theta_2)$, $i=1,2$, and the NBS is a solution to the following problem:

$$\begin{aligned} \max_{\Gamma, \hat{K}_1, \hat{K}_2} \ & \Big(R_1^{\mathrm{n}^*}(\hat{K}_1, \hat{K}_2, \theta_1, \theta_2) - \Gamma - R_1^{\mathrm{n}^*}(K_1, K_2, \theta_1, \theta_2)\Big)\Big(R_2^{\mathrm{n}^*}(\hat{K}_1, \hat{K}_2, \theta_1, \theta_2) + \Gamma - R_2^{\mathrm{n}^*}(K_1, K_2, \theta_1, \theta_2)\Big) \\ \text{subject to} \quad & \hat{K}_1 + \hat{K}_2 = K_1 + K_2 \\ & R_1^{\mathrm{n}^*}(\hat{K}_1, \hat{K}_2, \theta_1, \theta_2) - \Gamma \geq R_1^{\mathrm{n}^*}(K_1, K_2, \theta_1, \theta_2) \end{aligned}$$

$$R_2^{n*}(\hat{K}_1, \hat{K}_2, \theta_1, \theta_2) + \Gamma \geq R_2^{n*}(K_1, K_2, \theta_1, \theta_2)$$
$$K_1, K_2 \geq 0$$

The first constraint implies that the total capacity will remain the same before and after the trade. The remaining two constraints guarantee that both firms' payoffs must be improved after negotiation. The next result characterizes the properties of the equilibrium outcome.

PROPOSITION 6.

i. There exists an equilibrium in the capacity trading game. If $\hat{K}_i = K_i$, $i = 1, 2$ in equilibrium, then it must be $\Gamma^{Cp*}(\omega) = 0$ for any $\omega = (K_1, K_2, \theta_1, \theta_2)$.

ii. If the products are not substitutes (i.e., $\hat{b} = 0$) the equilibrium quantities in the subsequent production game are the same as those of a centralized firm with capacity $K_1 + K_2$.

Note that the equilibrium outcome described by Proposition 6 and equation (19) is different from the equilibrium outcome in a subgame under full collaboration (i.e., Cc scenario). For instance, unlike the Cc scenario, the firms exchange the transfer payment only if there is a physical trade of capacity. To see why, note that, under the Cp scenario, each firm chooses the quantity that maximizes its own revenue. Therefore, if there is no capacity trade, each firm's revenue would be equal to its disagreement payoff, and hence there would be no transfer payment.

Another interesting outcome is how the shared capacity is used. Under the Cp scenario, one firm may buy capacity from the other firm and idle the purchased capacity: In Figure 6, observe that firm 2 buys 10 units of capacity from firm 1, but uses only 3.75 units in production. Although such outcome is counterintuitive at first (why pay for capacity and waste?), firm 2 gains more by reducing the intensity of competition by limiting firm 1's production. At the same time, firm 1 also gains by selling a portion of capacity to firm 2 and limiting the competition.

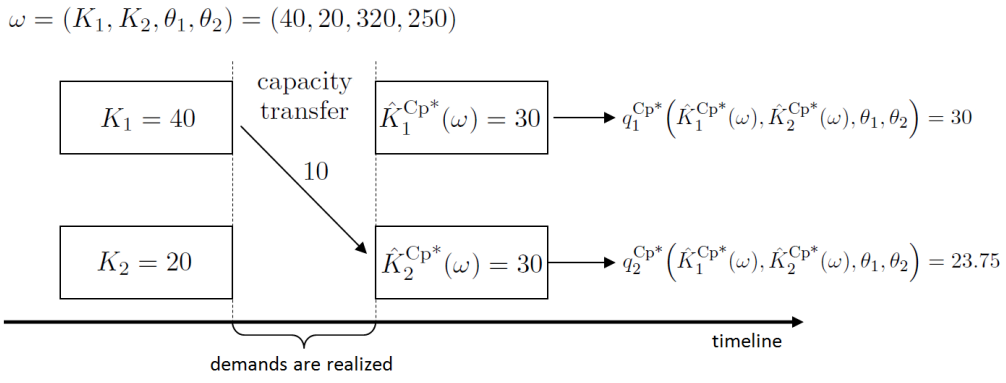

Figure 6 **Equilibrium capacity trade and production under the Cp scenario. ($b_1 = b_2 = 4$, $\hat{b} = 2$)**

After substituting the equilibrium outcome in the second stage, we write the equilibrium payoff of firm i, $R_i^{\mathrm{Cp}^*}(\omega)$ for a given $\omega = (K_1, K_2, \theta_1, \theta_2)$ as follows:

$$R_i^{\mathrm{Cp}^*}(\omega) = R_i^{\mathrm{n}^*}\left(\hat{K}_1^{\mathrm{Cp}^*}(\omega), \hat{K}_2^{\mathrm{Cp}^*}(\omega), \theta_1, \theta_2\right) + (-1)^i \Gamma^{\mathrm{Cp}^*}(\omega) \tag{20}$$

Now consider the first stage game. If the two firms choose the initial endowments (K_1, K_2), the expected profit of firm i is $\pi_i^{\mathrm{Cp}^*}(K_1, K_2) = \mathbb{E}\left[R_i^{\mathrm{Cp}^*}(K_1, K_2, \Theta_1, \Theta_2)\right] - cK_i$. As in the analysis of the Cc scenario, an (upfront) investment subsidy might be needed for the firms to agree on a deal. Putting them altogether, the equilibrium capacity endowments, $(K_1^{\mathrm{Cp}^*}, K_2^{\mathrm{Cp}^*})$, and investment subsidy, η^{Cp^*}, are given by an optimization problem analogous to the one in (16). The next proposition characterizes the properties of the bargaining outcome.

PROPOSITION 7. *Under the Cp scenario,*

i. There exists a pure strategy equilibrium in the capacity investment game such that the difference in the firms' equilibrium profits is equal to the difference between their disagreement payoffs.

ii. If the two products are not substitutes ($\hat{b} = 0$), the equilibrium outcome under the Cp scenario is identical to the equilibrium outcome under the Cc scenario.

iii. A subsidy-free investment equilibrium exists (i.e., $\eta^{Cp^*} = 0$), if and only if the equilibrium endowments $(K_1^{Cp^*}, K_2^{Cp^*})$ satisfy the following condition:

$$\pi_1^{n^*}(K_1^{Cp^*}, K_2^{Cp^*}) - \pi_2^{n^*}(K_1^{Cp^*}, K_2^{Cp^*}) = \pi_1^d - \pi_2^d \tag{21}$$

iv. Let $K_T^{Cp^*}$ be the total capacity in equilibrium under the Cp scenario. In addition to the conditions of Proposition 5(ii), assume that $K_T^{Cp^*} \geq K_i^{Nn^*}$, for $i = 1, 2$. Then, there exists a subsidy-free investment equilibrium under the Cp scenario.

Proposition 7 implies that, when the products are not substitutes, the level of equilibrium joint capacity is centrally optimal (equal to that of a centralized firm). However, this is not necessarily true when the products are substitutes. This is because, when the products are substitutes, even if the total capacity is the same, the production quantities under the Cp scenario is different from the quantities that a centralized firm would produce. For instance, when both firms get poor demand signals, there will be no capacity trade and the firms engage in quantity-setting game with their initial endowments. Since the second stage outcome is not always the same as the centrally optimal outcome, the firms' total expected profit is not the same as the profit of a centralized firm. As a result, under the Cp scenario, the firms build a joint capacity that is different from the capacity of a centralized firm.

Proposition 7 provides the condition that must be satisfied by the subsidy-free investment equilibrium. Note that this condition, given in equation (21), is analogous to (18), the equation that determines the subsidy-free investment equilibrium under the Cc scenario. Therefore, for the cases where the existence is guaranteed, similar to the Cc scenario, one can simply solve the condition in (21) with a search in single variable in a bounded interval to determine the investment level in a subsidy-free equilibrium.

4. Comparison of Equilibrium Capacities

In the previous section, we have solved for the equilibrium capacity under the five different scenarios. We show that, under some cases, collaboration leads to a centrally optimal outcome. For example, if the two firms can fully collaborate in both stages (Cc scenario), the equilibrium joint capacity level is equal to the optimal capacity of a centralized firm. Similarly, if the two products are not substitutes (i.e., $\hat{b}=0$), the equilibrium joint capacity level under the Cp scenario is also equal to the optimal capacity of a centralized firm. However, in all other scenarios, some efficiency is lost and the equilibrium capacity level deviates from the capacity of a centralized firm. The next proposition compares the total equilibrium capacity levels under different scenarios.

Proposition 8.

A. When the products are not substitutes (i.e., $\hat{b}=0$), the following results hold:

i) $K_T^{Cn^*} = K_T^{Nn^*}$,

ii) $\min(K_T^{Cc^*}, K_T^{Nn^*}) \le K_T^{Nc^*} \le \max(K_T^{Cc^*}, K_T^{Nn^*})$,

iii) $K_T^{Cp^*} = K_T^{Cc^*}$.

B. When the products are substitutes (i.e., $\hat{b}>0$), the following results hold:

i) $K_T^{Cn^*} \le K_T^{Nn^*}$,

ii) $\min(K_T^{Nn^*}, K_T^{Cc^*}) \le K_T^{Nc^*}$,

iii) $K_T^{Cp^*} < K_T^{Cc^*}$.

The parts A.i) and B.i) of Proposition 8 compare the total capacity under the two scenarios– Nn and Cn. If the two products are not substitutes (part A.i), the total capacity under the Cn scenario (i.e., collaborating in the capacity game, but not collaborating in the production subgame) is the same as that under the Nn scenario (i.e., not collaborating in both games). In other words, in terms of the total capacity, not collaborating in the production subgame is the same as not collaborating at all. On the other hand, if the products are substitutes, part B.i implies that the Cn scenario yields smaller capacity than the Nn scenario although the firms do not lend or borrow capacity from each other in the producton subgame: $K_{\mathrm{T}}^{\mathrm{Cn}^*} \le K_{\mathrm{T}}^{\mathrm{Nn}^*}$. This is because, although no

capacity is physically shared in the second stage, jointly deciding the total capacity together allows the firms to curb the downstream competition by making capacity more scarce. (Recall that in part B.i, products are substitutes whereas in part A.i, they are not, and it is interesting that just collaborating in capacity investments without any production (and therefore implicitly pricing) coordination results in decreasing competition when products are substitutes).

The parts A.ii) and B.ii) compare the total capacity under the Nc scenario with those under the Nn and Cc scenarios. If the products are not substitutes, the total capacity under the Nc scenario is always in between $K_{\mathrm{T}}^{\mathrm{Cc}^*}$ and $K_{\mathrm{T}}^{\mathrm{Nn}^*}$. To see why, recall that, firm i selects its capacity to maximize $\pi_i^{\mathrm{c}^*}(K_1, K_2)$ under the Nc scenario. Note that from Proposition 3),

$$\pi_i^{\mathrm{c}^*}(K_1, K_2) = \frac{1}{2}\Big(\pi^{\mathrm{m}^*}(K_1 + K_2) + \pi_i^{\mathrm{n}^*}(K_1, K_2) - \pi_j^{\mathrm{n}^*}(K_1, K_2)\Big) \qquad i, j = 1, 2 \quad i \neq j$$

Since the products are not substitutes, for given K_j, the disagreement payoff of firm j, $\pi_j^{\mathrm{n}^*}(K_1, K_2)$, is independent of firm i's capacity, K_i. Thus, firm i chooses the capacity that maximizes $0.5\pi^{\mathrm{m}^*}(\cdot) + 0.5\pi^{\mathrm{n}^*{}_i}(\cdot)$. Notice that if each firm wants to maximize $\pi^{\mathrm{m}^*}(\cdot)$, then this results in the total capacity of $K_{\mathrm{T}}^{\mathrm{Cc}^*}$. On the other hand, each firm maximizing $\pi^{\mathrm{n}^*}(\cdot)_i$ will result in the total of $K_{\mathrm{T}}^{\mathrm{Nn}^*}$. Consequently, $K_{\mathrm{T}}^{\mathrm{Nc}^*}$ falls between between $K_{\mathrm{T}}^{\mathrm{Cc}^*}$ and $K_{\mathrm{T}}^{\mathrm{Nn}^*}$.

However, this result is no longer true when the products are substitutes. Although we can establish $K_{\mathrm{T}}^{\mathrm{Nc}^*} \geq \min(K_{\mathrm{T}}^{\mathrm{Cc}^*}, K_{\mathrm{T}}^{\mathrm{Nn}^*})$, $K_{\mathrm{T}}^{\mathrm{Nc}^*}$ can exceed the maximum of $K_{\mathrm{T}}^{\mathrm{Cc}^*}$ and $K_{\mathrm{T}}^{\mathrm{Nn}^*}$. Figure 7 illustrates an example of this: $K_{\mathrm{T}}^{\mathrm{Nc}^*} > K_{\mathrm{T}}^{\mathrm{Nn}^*}$ when demand variability is low (the coefficient of variation is less than 0.35) and $K_{\mathrm{T}}^{\mathrm{Nc}^*} < K_{\mathrm{T}}^{\mathrm{Nn}^*}$ otherwise.

When the demand variability becomes smaller, the chances that one firm needs to borrow capacity from the other firm decrease. However, larger capacity can still be beneficial to the firm when products are substitutes. Note that the disagreement payoff of the firm i's competitor, $\pi_j^{\mathrm{n}^*}(K_1, K_2)$, decreases in K_i (see Lemma 2 in Appendix). Hence, increasing the firm's initial endowment can weaken the competitor's bargaining position and improve the firm's own bargaining position. Consequently, both firms try to get an edge for the negotiation that will occur in the production sub game and end up building larger capacity in the capacity game than they would otherwise and this is why the total capacity under the Nc scenario exceeds the total capacity under the Nn scenario. On the other hand, when demand becomes highly variable, the opposite effect prevails. When demands are highly variable, the value of pooling and sharing the capacity is high, thus, resulting in smaller total capacity.

Finally, parts A.iii) and B.iii) compare the equilibrium capacity under the Cp and the Cc scenarios. One would expect the total capacity to be higher with increased competition, i.e., it may

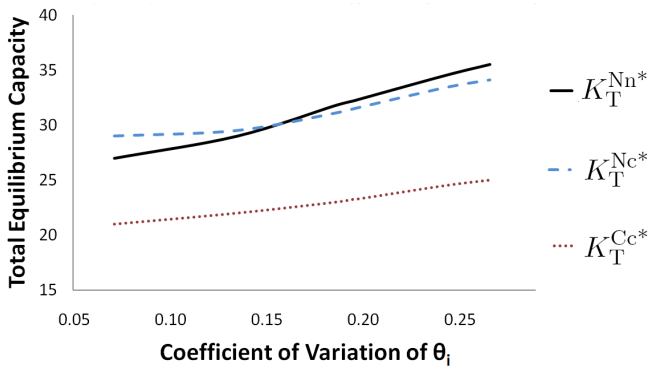

Figure 7 **Total capacity with respect to coefficient of variation. (**$\mu_1 = \mu_2 = 600$**,** $b_1 = b_2 \in \{5, \ldots, 35\}$**,** $\hat{b}/b_i \in \{0, \ldots, 0.99\}$**, and** $c \in \{20, \ldots, 160\}$**)**

be reasonable to expect that $K_T^{Cp*} \geq K_T^{Cc*}$. However, surprisingly, Proposition 8 shows that the opposite is true. To understand why, first consider the case where each firm has ample capacity. Then, under the Cp scenario, for most demand realizations, each firm can satisfy its demand with its own capacity. In this case, it is unlikely that the two firms will trade capacity , thus they will be competing in the production sub-game, deviating from producing the centrally optimal quantities. On the other hand, such adverse effect of competition can be prevented if the firms do not build too much capacity in the first place. Therefore, in the first stage, the firms anticipate this outcome, and build smaller joint capacity and reduce the intensity of competition.

5. Computational Study

We conduct a computational study to gain further managerial insights into the benefit of capacity collaboration. In particular, we aim to: (1) measure the gains the firms can achieve via collaboration, (2) find out how the gains change in the level and the form of collaboration, and (3) assess the impacts of business parameters (such as costs, demand variability, etc.) on the gains from collaboration.

For this, we compare the performances of the firms under the five scenarios: Cc, Cp, Cn, Nc, and Nn. To examine the effects of the business parameters on the gains from collaboration, we systematically vary cost and demand parameters. Specifically, the following combinations of parameters (in total of 2268 problem instances) are used in the computational study:

- $b_i \in \{5, 10, \ldots, 35\}$,
- $\hat{b}/b_i \in \{0, 0.25, 0.5, 0.75, 0.99\}$,
- $\text{CV}_i = \sigma_i/\mu_i \in [0.03, 0.50]$,
- $c \in \{20, 40, \ldots, 120\}$.

Throughout the experiments, we use several distributions – uniform, truncated normal, triangular, etc. – with $E[\theta_i] = 600$, $i = 1, 2$. To measure the gains from collaboration under different

scenarios, we compute the percentage improvement in total profit over the profit under the no collaboration scenario:

$$\mathcal{I}^{\mathrm{Cc}} = \frac{\Pi_{\mathrm{T}}^{\mathrm{Cc}^*} - \Pi_{\mathrm{T}}^{\mathrm{Nn}^*}}{\Pi_{\mathrm{T}}^{\mathrm{Nn}^*}} \times 100, \quad \mathcal{I}^{\mathrm{Cp}} = \frac{\Pi_{\mathrm{T}}^{\mathrm{Cp}^*} - \Pi_{\mathrm{T}}^{\mathrm{Nn}^*}}{\Pi_{\mathrm{T}}^{\mathrm{Nn}^*}} \times 100, \quad \mathcal{I}^{\mathrm{Cn}} = \frac{\Pi_{\mathrm{T}}^{\mathrm{Cn}^*} - \Pi_{\mathrm{T}}^{\mathrm{Nn}^*}}{\Pi_{\mathrm{T}}^{\mathrm{Nn}^*}} \times 100, \quad \mathcal{I}^{\mathrm{Nc}} = \frac{\Pi_{\mathrm{T}}^{\mathrm{Nc}^*} - \Pi_{\mathrm{T}}^{\mathrm{Nn}^*}}{\Pi_{\mathrm{T}}^{\mathrm{Nn}^*}} \times 100$$

The descriptive statistics are summarized in Table 2.

	$\mathcal{I}^{\mathrm{Cc}}$	$\mathcal{I}^{\mathrm{Cp}}$	$\mathcal{I}^{\mathrm{Cn}}$	$\mathcal{I}^{\mathrm{Nc}}$
Mean	20.45 %	20.34 %	4.50 %	14.36 %
Standard Deviation	17.78 %	17.80 %	5.35 %	14.50 %
Minimum	0.05 %	0.05 %	0.00 %	0.02 %
25^{th} percentile	5.41 %	5.24 %	0.17 %	2.87 %
Median	15.37 %	15.13 %	2.24 %	9.62 %
75^{th} percentile	31.31 %	31.29 %	7.47 %	21.99 %
Maximum	72.79 %	72.78 %	30.62 %	62.60 %

Table 2 Summary Statistics for improvements in the dataset of 2268 problem instances.

We observe that the overall gains from collaboration are significant except for the Cn scenario. In particular, the performance under the Cp scenario (where the firms invest in capacity together, and decide whether to trade capacity but determine the production quantities individually) is very close to the performance under the Cc (full collaboration) scenario. We also observe that the gain is significant when the firms are allowed to collaborate only in the production stage (Nc scenario): the total profit increases by 14.4% on average. On the other hand, if the firms are not allowed to collaborate in the production stage, they do not gain much even when they build the capacity together (4.5% on average). To understand why, notice that, the investment decisions are made before, but production decisions are made after observing the demand signals. Therefore, even if the firms build capacities close to the ideal level, they cannot capture most of the potential gain if they are not allowed to share the capacity to respond to demand variability.

5.1. The Impact of Business Parameters

For each scenario, we examine the change in the performance with respect to the changes in the problem parameters. These results are presented in Figures 8(a) (substitutability), 8(b) (capacity cost) and 9 (demand variability).

Figure 8(a) plots the percentage improvements with respect to $\hat{b}/b_i$. Note that b_i is the elasticity and $\hat{b}$ is the cross-elasticity of the inverse demand function. Therefore, the ratio of the two, $\hat{b}/b_i$ measures the degree of substitutability, or the intensity of competition between the two firms (Lus and Muriel, 2009). We observe that the percentage improvement increases in the ratio $\hat{b}/b_i$, which implies that the gains from collaboration increase as the competition becomes more intense under

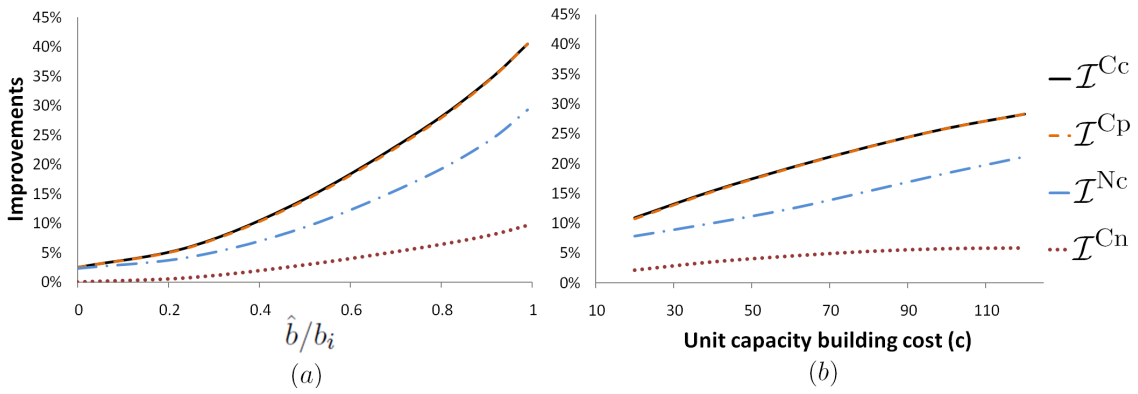

Figure 8 **Improvements vs. (a) $\hat{b}/b_i$ ratio, and (b) unit capacity building cost c.**

all scenarios. On the other hand, we observe that the performance gaps between the scenarios where the firms collaborate only in one stage – the Cn and Nc scenarios – and the scenarios where the firms collaborate in both stages – the Cc and Cp scenarios – also increase. To see why, first observe that, by collaborating in both stages, the firms mitigate the adverse effects of competition in both investment and resource allocation decisions. Under the Cc scenario, the firms completely coordinate the decisions. Under the Cp scenario, the firms compete in quantity, but they are allowed to trade the capacity, so that the degree of competition in the second stage is reduced as well. As the substitutability increases, the value of reduced competition enabled by collaborating in both stages (Cc or Cp scenarios) increases. On the other hand, under the Cn and Nc scenarios, the firms collaborate in one stage, but compete in the other stage. As a result, the competition is still a driver of each firm's (capacity or quantity) decision. Thus, although the gains under the Cn and Nc scenarios increase in substitutability, the growth is much slower than the growth under the Cc or Cp scenarios.

Figure 8(b) shows that the percentage gains increase in the unit capacity building cost. When capacity becomes more expensive, the firms build smaller capacity. When the firms collaborate in the production stage (Cc, Cp and Nc scenarios), they utilize the limited capacities more efficiently. Therefore, under these scenarios, the gains from collaboration increase significantly when the unit capacity building cost increases. On the other hand, under the Cn scenario, the gain slightly increases first, but the rate of increase diminishes as the cost increases. To see why, recall that the only source of gain under the Cn scenario is building smaller capacity to reduce competition in the production stage. Since the firms are already building smaller capacities under the Nn scenario when the capacity cost is high, the gain under the Cn scenario is small and the growth diminishes with higher capacity cost.

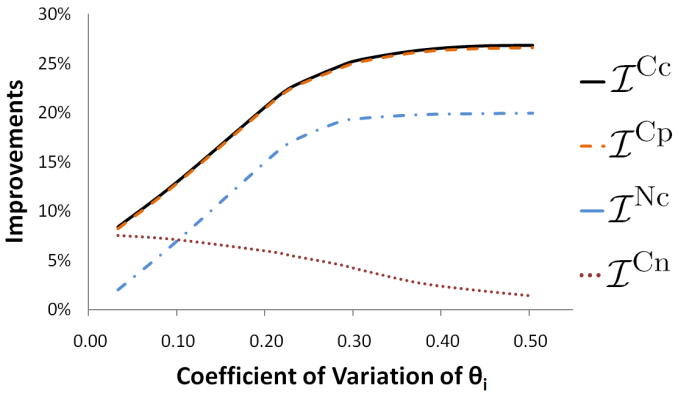

Figure 9 **Improvements vs. the coefficient of variation of demand.**

Figure 9 illustrates how the gains change in demand variability. Overall, the gains increase in demand variability when the firms collaborate in the production stage (Cc, Cp and Nc scenarios). If the firms do not collaborate in the production stage at all, however, the gain from collaboration in capacity decision alone (Cn scenario) decreases in demand variability. To see why, note that for given capacity endowments, as the demand variability increases, the probability that at least one firm is short of capacity increases as well. Thus, collaborating in production after observing the demand signals (reactive collaboration) becomes more valuable. On the other hand, when demand becomes more predictable (i.e., less variable), the opportunity to curb the competition by preventing overinvestment (proactive collaboration) becomes more valuable. That is, most of the benefit comes from the savings from the investment cost and reduced competition (thus, the performances under the Cc and the Cn scenarios are almost equal). As a result, when demand is extremely predictable, collaborating in capacity investment alone (Cn scenario) could be more beneficial than collaborating in production alone (Nc scenario).

5.2. Full vs. Partial Collaboration in Production and Implications on Price

Recall that the Cp scenario allows each firm to set its own production quantity only allowing the firms to exchange capacity once they realize demand signals. Therefore, one might expect that the increased competition compared to the full collaboration (Cc) scenario will force the firms to produce larger quantities so that the consumers pay lower prices. However, when we compare the equilibrium quantities and prices between the Cc and the Cp scenarios (see Figure 10), we see that the exact opposite case may happen. When demand signals for both products are unfavorable to moderately favorable, the price under the Cp scenario is strictly lower. When one firm observes favorable demand signal and the other firm gets poor demand signal, the price under the Cp scenario is equal to the price under the Cc scenario. In all the other cases, however, the price

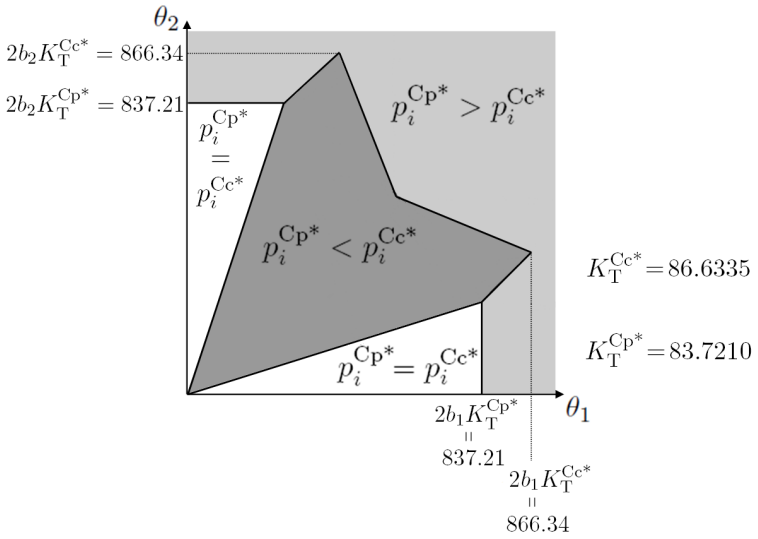

Figure 10 **Regions on $(\theta_1 \times \theta_2)$ space that determine whether the prices under the Cp scenario are larger than the prices under the Cc scenario in equilibrium. ($b_1 = b_2 = 10$, $\hat{b} = 2.5$, $c = 20$, $\mu_1 = \mu_2 = 600$)**

under the Cp scenario is strictly higher than the price under the Cc scenario. To understand why such behavior arises, first note that, although the firms eventually engage in quantity competition under the Cp scenario, they are allowed to trade the capacity when doing so is mutually beneficial. Thus, the firms engage in the full level of competition only when both firms have sufficient capacity (i.e., when they get demand signals that are unfavorable to moderately favorable). Otherwise, the capacity becomes binding. Recall that, to reduce the harmful effects of competition, the firms build smaller joint capacity under the Cp scenario (Proposition 8). Therefore, when the capacity is binding under both the Cp and the Cc scenarios, the total output is smaller under the Cp scenario. As a result, the consumers pay higher prices.

6. Discussion and Conclusions

In this paper, we considered collaboration between two competing firms. Specifically, we allowed firms to collaborate in both capacity building and production decisions. We considered several different arrangements of collaboration and examined the resulting equilibrium outcomes.

We find that, if the firms can fully collaborate both in capacity and in production decisions (Cc scenario), they can achieve the centrally optimal outcome in equilibrium. In other words, no efficiency would be lost. Moreover, compared to the scenario where the firms do not collaborate at all, the gains from collaboration with a competitor can be substantial. This supports the joint venture agreements between competing firms, in which, the firms jointly take the decisions under a separate economic entity. However, for a joint venture agreement to be sustainable, the firms may need to agree on investment subsidies and on a detailed transfer payment schedule. Interestingly,

we find that an investment scheme that is proportional to the capacity endowment structure (e.g., one firm owns 60 % of capacity and the other owns 40% of capacity in a 60-40 joint venture) may not always be an equilibrium, and subsidies from one firm to the other may be necessary to achieve mutually beneficial collaboration.

As an alternative to joint venture agreements, motivated by examples from different industries, we study an arrangement where the firms jointly build capacity but still compete in production after trading their capacity endowments (Cp scenario). We find that most of the benefits from full collaboration can be captured under the Cp scenario. Therefore, it is a viable alternative to forming a joint venture. We also find that, although the Cp scenario is a more competitive arrangement than a joint venture, equilibrium total capacity is lower than the total capacity of a joint venture. This interesting result is due to the firms' tendency in the capacity building stage to limit the intensity of competition in the operational (e.g., production) stage.

If the firms can collaborate only in capacity investment or in operational decisions, we find that collaboration only in operational decisions (Nc scenario) outperforms collaboration only in capacity investment (Cn scenario), as long as demand is not too predictable. This result may explain collaborative production agreements between competing firms such as Toyota and Fuji Heavy, and the code sharing agreements among airlines. However, we also observe that, even though the firms gain considerable benefits when they only collaborate in operational decisions (e.g., production), they cannot do as well as they would do under a joint venture. This is because of the competition in the capacity building stage that deteriorates the firms' earnings.

On the other hand, when demand is very predictable, the gains of collaborating only in capacity investment may exceed the gains of collaborating only in production. When the firms collaborate only in capacity investment, the only gain comes from preventing overinvestment to curb competition in the production stage. However, when the firms collaborate only in production, the gain comes from pooling capacity in the face of demand variability. Therefore, when demand is extremely predictable, collaborating only in capacity investment could be more valuable than collaborating only in production.

The sensitivity of the gains of collaboration with respect to different business parameters imply that overall gains increase when: (1) the products are more substitutable, (2) capacity is more costly to build, and (3) demand is more variable. This is because: (1) when the products become more substitutable, the value of reduced competition, which is enabled by collaboration, increases, (2) when capacity is more costly to build, the collaboration becomes more valuable as it allows the firms to utilize the limited resources more efficiently, and (3) with more variable demand, there is

higher probability of instances where a firm needs to use the other firm's capacity, under which capacity trade is more profitable.

For future research, it would be interesting to consider capacity collaboration structures other than the ones we consider in this paper. For instance, we assume that when the firms collaborate in the operational stage they negotiate to agree on contract terms such as the capacity trade and the transfer payment. This causes the contract terms to be contingent on the demand signals (these contracts are labeled as *incomplete contracts* by Van Mieghem, 1999). One extension is to consider a simpler capacity sharing contract that can be agreed upon before demand signals are observed. It will be interesting to see how much of the benefit can be captured through a rather simple mechanism.

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Appendix

Proof of Proposition 1: The proof immediately follows from the fact that $r^{\mathrm{m}}(q_1, q_2, \theta_1, \theta_2)$ is concanve in (q_1, q_2) (part i) and the fact that $\pi^{\mathrm{m}^*}(K_{\mathrm{T}})$ is concave in K_{T}.

Proof of Proposition 2: Notice that, for a given subgame ω, the second-stage payoff of firm i, $q_i(\theta_i - b_i q_i - \hat{b} q_j)$, is concave in q_i. Also, the strategy space for (q_1, q_2) is compact, a pure strategy equilibrium exists (Fudenberg and Tirole, 1991). To show uniqueness, for given q_j, the best response of firm i is $q_i^{\mathrm{n}}(q_j, \omega) = \min\left(\left(\frac{\theta_i - \hat{b} q_j}{2b_i}\right)^+, K_i\right)$. Taking its derivative with respect to q_j, we get:

$$\frac{d}{dq_j} q_i^{\mathrm{n}}(q_j, \omega) = \begin{cases} -\frac{\hat{b}}{2b_i}, & \text{if } 0 \le \frac{\theta_i - \hat{b} q_j}{2b_i} \le K_i; \\ 0, & \text{otherwise.} \end{cases}$$

Since $b_i > \hat{b}$, we have $-1 < \frac{d}{dq_j} q_i^{\mathrm{n}}(q_j, \omega) \le 0$ and the best response mapping is a contraction. Hence, the equilibrium (Fudenberg and Tirole, 1991) described in equation (10) is unique.

Proof of Proposition 3: To determine the NBS, we solve the optimization problem defined in (12). We first determine the optimal transfer payment, $\Gamma^{\mathrm{c}^*}(\omega)$, for given production quantities. Then, we solve for the optimal production quantities. For given (q_1, q_2), it can be shown that (12a) is strictly concave in Γ, and hence the optimal transfer payment $\Gamma^{\mathrm{c}^*}(\omega)$ is unique. To solve for $\Gamma^{\mathrm{c}^*}(\omega)$ (thereby proving part ii), we first write the KKT conditions. Let ν_1 and ν_2 be the Lagrangian multipliers. Then, the KKT conditions are as follows:

$$\Big(q_1 p_1(q_1, q_2, \omega) - q_2 p_2(q_1, q_2, \omega)\Big) - \Big(R_1^{\mathrm{n}^*}(\omega) - R_2^{\mathrm{n}^*}(\omega)\Big) - 2\Gamma - \nu_1 + \nu_2 = 0 \tag{22a}$$

$$\nu_1 \Big(q_1 p_1(q_1, q_2, \omega) - \Gamma - R_1^{\mathrm{n}^*}(\omega)\Big) = 0, \qquad \nu_1 \ge 0 \tag{22b}$$

$$\nu_2 \Big(q_2 p_2(q_1, q_2, \omega) + \Gamma - R_2^{\mathrm{n}^*}(\omega)\Big) = 0, \qquad \nu_2 \ge 0 \tag{22c}$$

From the KKT condition, we obtain

$$\Gamma^{\mathrm{c}^*}(\omega) = \frac{q_1 p_1(q_1, q_2, \omega) - R_1^{\mathrm{n}^*}(\omega)}{2} - \frac{q_2 p_2(q_1, q_2, \omega) - R_2^{\mathrm{n}^*}(\omega)}{2}, \tag{23}$$

To obtain the optimal production quantities, we rewrite (12) utlizing equation (23):

$$\left(\frac{q_1 p_1(q_1, q_2, \omega) + q_2 p_2(q_1, q_2, \omega)}{2} - \frac{R_1^{\mathrm{n}^*}(\omega) + R_2^{\mathrm{n}^*}(\omega)}{2}\right)^2 = \left(\frac{r^{\mathrm{m}}(q_1, q_2, \theta_1, \theta_2)}{2} - \frac{R_1^{\mathrm{n}^*}(\omega) + R_2^{\mathrm{n}^*}(\omega)}{2}\right)^2$$

Since the second part of the expression within the parentheses is independent of (q_1, q_2), the solution will maximize $r^{\mathrm{m}}(q_1, q_2, \theta_1, \theta_2)$ and this proves the result.

Proof of Proposition 4: i. We first determine the equilibrium transfer payment and capacity trade for given market signals. Then, we evaluate the unit price of capacity to establish the result. As illustrated in Figure 11, depending on the demand signals, the optimal capacity trade and the transfer payment can fall in one of 10 different regions. Table 3 presents the equilibrium capacity trade, $(\chi^{\mathrm{c}^*}(\omega))$, and unit price of capacity, $\gamma^{\mathrm{c}^*}(\omega)$, and condition for each of the 10 regions.

It can be shown that $\chi^{\mathrm{c}^*}(\omega)$ and $\Gamma^{\mathrm{c}^*}(\omega)$, are continuous functions in (θ_1, θ_2). Thus, $\gamma^{\mathrm{c}^*}(\omega)$ is also continuous in regions where $\chi^{\mathrm{c}^*}(\omega) \neq 0$. To show $\gamma^{\mathrm{c}^*}(\omega)$ is increasing in θ_i and decreasing in K_i, it suffices to show that

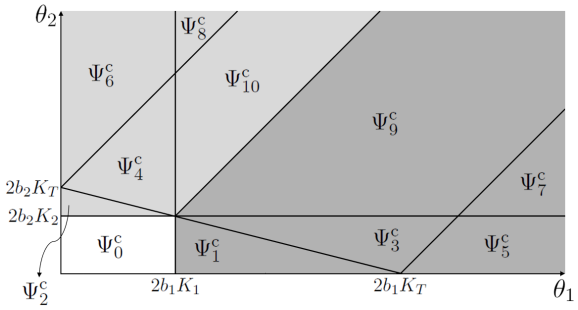

Figure 11 **The capacity trade and transfer payment when firms collaborate in production.**

Region	Definition	Capacity Trade, $\lvert\chi^{c*}(\omega)\rvert$	Unit Price of Capacity, $\gamma^{c*}(\omega)$
Ψ_0^c	$\theta_1 \le 2b_1K_1,\ \ \theta_2 \le 2b_2K_2$	0	Not defined
Ψ_1^c	$\theta_1 > 2b_1K_1,\ \ \theta_1b_2+\theta_2b_1 \le 2b_1b_2(K_1+K_2)$	$\dfrac{\theta_1}{2b_1}-K_1$	$\dfrac{\theta_1-2b_1K_1}{4}$
Ψ_2^c	$\theta_2 > 2b_2K_2,\ \ \theta_2b_1+\theta_1b_2 \le 2b_1b_2(K_1+K_2)$	$K_2-\dfrac{\theta_2}{2b_2}$	$\dfrac{\theta_2-2b_2K_2}{4}$
Ψ_3^c	$\theta_1b_2+\theta_2b_1 > 2b_1b_2(K_1+K_2),$ $\theta_2 \le 2b_2K_2, \theta_1-\theta_2 \le 2b_1(K_1+K_2)$	$\dfrac{\theta_1-\theta_2-2b_1K_1+2b_2K_2}{2(b_1+b_2)}$	$\dfrac{\left[\begin{array}{l}4b_2(b_1+b_2)^2K_1(b_1K_1-\theta_1)+(b_1+b_2)^2\theta_2^2\\+b_2(2b_1(K_1+K_2)-\theta_1+\theta_2)\left[-2b_1\theta_2-b_2(\theta_1+\theta_2)\right]\\+b_2(2b_2(K_1+K_2)+\theta_1-\theta_2)\left[2b_2\theta_1+b_1(\theta_1+\theta_2)\right]\\+4b_1b_2^2(K_1+K_2)\left[(b_1-b_2)(K_1+K_2)-\theta_1+\theta_2\right]2\end{array}\right]}{4b_2(b_1+b_2)(\theta_1-\theta_2-2b_1K_1+2b_2K_2)}$
Ψ_4^c	$\theta_2b_1+\theta_1b_2 > 2b_1b_2(K_1+K_2),$ $\theta_1 \le 2b_1K_1, \theta_2-\theta_1 \le 2b_2(K_1+K_2)$	$\dfrac{\theta_1-\theta_2-2b_1K_1+2b_2K_2}{2(b_1+b_2)}$	$\dfrac{\left[\begin{array}{l}4b_1(b_1+b_2)^2K_2(\theta_2-b_2K_2)-(b_1+b_2)^2\theta_1^2\\+b_1(2b_1(K_1+K_2)-\theta_1+\theta_2)\left[-2b_1\theta_2-b_2(\theta_1+\theta_2)\right]\\+b_2(2b_2(K_1+K_2)+\theta_1-\theta_2)\left[2b_2\theta_1+b_1(\theta_1+\theta_2)\right]\\+4b_1^2b_2(K_1+K_2)\left[(b_1-b_2)(K_1+K_2)-\theta_1+\theta_2\right]\end{array}\right]}{4b_1(b_1+b_2)(\theta_1-\theta_2-2b_1K_1+2b_2K_2)}$
Ψ_5^c	$\theta_2 \le 2b_2K_2,\ \ \theta_1-\theta_2 > 2b_1(K_1+K_2)$	K_2	$\dfrac{\theta_2^2+4b_2K_2\left[\theta_1-b_1(2K_1+K_2)\right]}{8b_2K_2}$
Ψ_6^c	$\theta_1 \le 2b_1K_1,\ \ \theta_2-\theta_1 > 2b_2(K_1+K_2)$	$-K_1$	$\dfrac{\theta_1^2+4b_1K_1\left[\theta_2-b_2(2K_2+K_1)\right]}{8b_1K_1}$
Ψ_7^c	$\theta_2 > 2b_2K_2,\ \ \theta_1-\theta_2 > 2b_1(K_1+K_2)$	K_2	$\dfrac{\theta_1+\theta_2-2b_1K_1-(b_1+b_2)K_2}{2}$
Ψ_8^c	$\theta_1 > 2b_1K_1,\ \ \theta_2-\theta_1 > 2b_2(K_1+K_2)$	$-K_1$	$\dfrac{\theta_1+\theta_2-2b_2K_2-(b_1+b_2)K_1}{2}$
Ψ_9^c	$\theta_1-\theta_2 > 2b_1K_1-2b_2K_2,$ $\theta_2 > 2b_2K_2,\ \theta_1-\theta_2 \le 2b_1(K_1+K_2)$	$\dfrac{\theta_1-\theta_2-2b_1K_1+2b_2K_2}{2(b_1+b_2)}$	$\dfrac{\left[\begin{array}{l}\theta_2(3b_1+b_2)+\theta_1(b_1+3b_2)\\-2\left[b_1(b_1+3b_2)K_1+b_2(3b_1+b_2)K_2\right]\end{array}\right]}{4(b_1+b_2)}$
Ψ_{10}^c	$\theta_2-\theta_1 > 2b_2K_2-2b_1K_1,$ $\theta_1 > 2b_1K_1, \theta_2-\theta_1 \le 2b_2(K_1+K_2)$	$\dfrac{\theta_1-\theta_2-2b_1K_1+2b_2K_2}{2(b_1+b_2)}$	$\dfrac{\left[\begin{array}{l}\theta_2(3b_1+b_2)+\theta_1(b_1+3b_2)\\-2\left[b_1(b_1+3b_2)K_1+b_2(3b_1+b_2)K_2\right]\end{array}\right]}{4(b_1+b_2)}$

Table 3 **The equilibrium capacity trade for the regions in Figure** 11.

$\gamma^{c*}(\omega)$ is monotone in each region. To illustrate this, we will show that $\gamma^{c*}(\omega)$ is increasing in θ_1 in Ψ_3^c. Other cases are similar, thus omitted. Taking the derivative of $\gamma^{c*}(\omega)$ in Ψ_3^c with respect to θ_1 and applying algebra, we get:

$$\frac{d}{d\theta_1}\gamma^{c*}(\omega)=\frac{\left[\begin{array}{l}(2b_2^2-b_1b_2-b_1^2)(\theta_1-\theta_2-2b_1K_1+2b_2K_2)^2-(b_1+b_2)^2(\theta_1-2b_1K_1)^2\\+2(b_1+b_2)^2(\theta_1-2b_1K_1)(\theta_1-\theta_2-2b_1K_1+2b_2K_2)\end{array}\right]}{4b_2(b_1+b_2)(\theta_1-\theta_2-2b_1K_1+2b_2K_2)^2}>0$$

ii. The proof follows from Table 3, theorefore omitted.

Proof of Theorem 1: i. We present the proof when Θ_1 and Θ_2 are exponentially distributed with rates λ_1 and λ_2. The proof utilizes Lemma 1 which is stated and proved below.

LEMMA 1. *Suppose that Θ_1 and Θ_2 are independent exponential random variables with rates λ_1 and λ_2, respectively. Let $K_i^{N\mathfrak{s}}(K_j)$, $i \neq j$, be the firm i's best response when firm j sets its capacity to K_j of firm j under the scenario $N\mathfrak{s}$, $\mathfrak{s} \in \{n, c\}$. Then, we have the following:*

i. $\pi_i^{n^*}(K_1, K_2)$ *is concave in* K_i, $i = 1, 2$.

ii. $\nabla K_i^{Nn}(K_j) \in (-1, 1)$, $i = 1, 2$, $i \neq j$.

iii. $\tilde{\pi}_i^{n^*}(K_1, K_2)$ *is concave in* K_i, $i = 1, 2$.

iv. $\nabla K_i^{Nc}(K_j) > -1$, $i = 1, 2$, $i \neq j$.

Proof of Lemma 1: We show that $\pi_1^{\mathrm{n}^*}(K_1, K_2)$ is concave in K_1. The proof for $\pi_2^{\mathrm{n}^*}(K_1, K_2)$ is symmetric. The second order derivative of $\pi_1^{\mathrm{n}^*}(K_1, K_2)$ with respect to K_1 is

$$\begin{aligned}\nabla_{11}^2 \pi_1^{\mathrm{n}^*}(K_1, K_2) \;=\;& \tfrac{\hat{b}^3 K_1}{2b_2}\left[\int_{2b_1K_1+\hat{b}K_2}^{\infty} f(t_1, 2b_2K_2+\hat{b}K_1)dt_1 - \int_{2b_1K_1}^{\infty} f(t_1, \hat{b}K_1)dt_1\right] \\ & - \tfrac{\hat{b}^2(4b_1b_2-\hat{b}^2)K_1}{4b_2^2}\int_{\hat{b}K_1}^{2b_2K_2+\hat{b}K_1} f(\tfrac{(4b_1b_2-\hat{b}^2)K_1+\hat{b}t_2}{2b_2}, t_2)dt_2 \\ & - 2b_1\left[\int_0^{\hat{b}K_1}\int_{2b_1K_1}^{\infty} f(t_1,t_2)dt_1dt_2 + \int_{2b_2K_2+\hat{b}K_1}^{\infty}\int_{2b_1K_1+\hat{b}K_2}^{\infty} f(t_1,t_2)dt_1dt_2\right] \\ & - \tfrac{2b_1b_2-\hat{b}^2}{b_2}\int_{\hat{b}K_1}^{2b_2K_2+\hat{b}K_1}\int_{\frac{(4b_1b_2-\hat{b}^2)K_1+\hat{b}t_2}{2b_2}}^{\infty} f(t_1,t_2)dt_1dt_2 \end{aligned} \tag{24}$$

Using the fact that Θ_i is exponentially distributed, the above equation can be simplified to

$$\begin{aligned}\nabla_{11}^2 \pi_1^{\mathrm{n}^*}(K_1, K_2) = \;& -2b_1 e^{-2b_1K_1\lambda_1} \\ & - \tfrac{\hat{b}(1-e^{\hat{b}K_2\lambda_1+2b_2K_2\lambda_2})(\hat{b}\lambda_2(2-\hat{b}K_1\lambda_2)+2b_1\lambda_1(1-\hat{b}K_1\lambda_2))}{\hat{b}\lambda_1+2b_2\lambda_2} e^{-(2b_1K_1+\hat{b}K_2)\lambda_1-(2b_2K_2+\hat{b}K_1)\lambda_2} \end{aligned} \tag{25}$$

Noting that $\hat{b}\lambda_1 + 2b_2\lambda_2 > 0$ and $e^{-(2b_1K_1+\hat{b}K_2)\lambda_1-(2b_2K_2+\hat{b}K_1)\lambda_2} > 0$, $\nabla_{11}^2\pi_1^{\mathrm{n}^*}(K_1, K_2)$ must have the same sign as $\frac{\hat{b}\lambda_1+2b_2\lambda_2}{e^{-(2b_1K_1+\hat{b}K_2)\lambda_1-(2b_2K_2+\hat{b}K_1)\lambda_2}}\pi_1^{\mathrm{n}^*}(K_1, K_2)$. After some algebra, we have:

$$\begin{aligned}&\frac{\hat{b}\lambda_1 + 2b_2\lambda_2}{e^{-(2b_1K_1+\hat{b}K_2)\lambda_1-(2b_2K_2+\hat{b}K_1)\lambda_2}}\pi_1^{\mathrm{n}^*}(K_1, K_2) \\ &= -2b_1\left(e^{\hat{b}K_1\lambda_2}\right)\left(e^{\hat{b}K_2\lambda_1+2b_2K_2\lambda_2}\right)(\hat{b}\lambda_1 + 2b_2\lambda_2) - \hat{b}\left(1 - e^{\hat{b}K_2\lambda_1+2b_2K_2\lambda_2}\right)\left(\hat{b}\lambda_2(2-\hat{b}K_1\lambda_2) + 2b_1\lambda_1(1-\hat{b}K_1\lambda_2)\right) \\ &\leq -\lambda_2\left(2(2b_1b_2 - \hat{b}^2) + \hat{b}^2(2b_1\lambda_1 + \hat{b}\lambda_2)K_1\right)e^{\hat{b}K_2\lambda_1+2b_2K_2\lambda_2} - \hat{b}\left(\hat{b}\lambda_2(2-\hat{b}K_1\lambda_2) + 2b_1\lambda_1(1-\hat{b}K_1\lambda_2)\right) \\ &\leq -2b_1\left(\hat{b}\lambda_1 + 2b_2\lambda_2\right) < 0. \end{aligned} \tag{26}$$

The first inequality comes from the fact that $-2b_1\left(e^{\hat{b}K_1\lambda_2}\right) \leq -2b_1$ and the second inequality comes from the fact $-\lambda_2 e^{\hat{b}K_2\lambda_1+2b_2K_2\lambda_2} \leq -\lambda_2$. The last inequality implies that $\nabla_{11}^2\pi^{\mathrm{n}^*}(K_1, K_2) < 0$, hence $\pi_1^{\mathrm{n}^*}(K_1, K_2)$ is concave in K_1. The proofs for remaining parts use similar logic, thus omitted.

Proof of part (i): Nn Scenario: The best response of firm i to the firm j's action, K_j is uniquely determined $\big($Lemma 1.$i\big)$, and the mapping is a contraction $\big($Lemma 1.$ii\big)$. Note that $\lim_{K_i\to 0}\pi_i^{\mathrm{Nn}^*}(K_1, K_2) = 0$ and $\lim_{K_i\to\infty}\pi_i^{\mathrm{Nn}^*}(K_1, K_2) \to -\infty$. Without loss of generality, it suffices to restrict the firm i's strategy space to be a compact interval, $[0, \bar{K}_i]$ for some $\bar{K}_i < \infty$. From Fudenberg and Tirole (1991), the equilibrium exists and it is unique.

Nc Scenario: Applying Proposition 3, the expected profit of firm i who has endowment K_i and will collaborate with firm j (with endowment K_j), in the second stage, $\pi_i^{c^*}(K_1, K_2)$, is :

$$\pi_i^{c^*}(K_1,K_2) = \frac{1}{2}\Big(\pi^{m^*}(K_1+K_2) + \pi_i^{n^*}(K_1,K_2) - \pi_j^{n^*}(K_1,K_2)\Big)$$

Define $\tilde{\pi}_i^{n^*}(K_1,K_2) = \pi_i^{n^*}(K_1,K_2) - \pi_j^{n^*}(K_1,K_2)$, $i \neq j$ and substitue this into the previous equation, we rewrite

$$\pi_i^{c^*}(K_1,K_2) = \frac{1}{2}\Big(\pi^{m^*}(K_1+K_2) + \tilde{\pi}_i^{n^*}(K_1,K_2)\Big) \qquad i,j=1,2 \quad i\neq j. \tag{27}$$

It should be noted that $\pi_i^{c^*}(K_1,K_2)$ is concave in K_i (this follows from the fact that $\pi^{m^*}(K_1+K_2)$ and $\tilde{\pi}_i^{n^*}(K_1,K_2)$ are concave: see Lemma 1.(iii). Similar to the Nn scenario, it suffices to restrict the firm i's strategy space to be a compact interval, which guarantees the existence (Fudenberg and Tirole, 1991).

For the uniqueness, first note that the best response mapping under the Nc scenario is not a contraction in general. We show the uniqueness by showing that the slopes of the best response functions are bounded in a way so that they will intersect exactly once. To start with, observe that the best response of firm i to the capacity K_j of firm j under the Nc scenario, $K_i^{Nc}(K_j)$, is

$$K_i^{Nc}(K_j) = \begin{cases} 0, & \text{if } \nabla_i \pi_i^{c^*}(0,K_j) \leq 0; \\ \tilde{K}_i^{Nc}(K_j), & \text{otherwise.} \end{cases} \qquad i,j=1,2 \quad i\neq j \tag{28}$$

where $\tilde{K}_i^{Nc}(K_j)$ is the solution to the following first order condition for given K_j:

$$\nabla_i \pi_i^{c^*}\big(\tilde{K}_i^{Nc}(K_j), K_j\big) = \frac{1}{2}\Big\{\nabla \pi^{m^*}(\tilde{K}_i^{Nc}(K_j)+K_j) + \nabla_i \tilde{\pi}_i^{n^*}(\tilde{K}_i^{Nc}(K_j), K_j)\Big\} = 0$$

Then, implicitly differentiating the first order condition, we get:

$$\nabla K_i^{Nc}(K_j) = -1 \cdot \frac{\nabla^2 \pi^{m^*}\big(K_i^{Nc}(K_j)+K_j\big) + \nabla^2_{ij}\tilde{\pi}_i^{n^*}\big(K_i^{Nc}(K_j), K_j\big)}{2\nabla^2_{ii}\pi_i^{c^*}\big(K_i^{Nc}(K_j), K_j\big)}.$$

To obstain the bound on $\nabla K_i^{Nc}(K_j)$, we first use Lemma 1(iv) and $\nabla K_i^{Nc}(K_j) > -1$. Furthermore, since $\pi_i^{c^*}(K_1,K_2)$ is strictly concave in K_i, the denominator has a negative sign. Hence, $\nabla K_i^{Nc}(K_j)$ has the same sign as the numerator. Note that the first term in the numerator is negative since $\pi^{m^*}(\cdot)$ is concave. Thus, in order for $\nabla K_i^{Nc}(K_j)$ to be positive, the sign of the second term in the numerator must be postive. Subsutitute $q_i^{n^*}(\omega)$ into $\tilde{\pi}_i^{n^*}\big(K_1,K_2\big)$ and taking cross-partial derivative in K_i and K_j yield

$$\nabla^2_{ij}\tilde{\pi}_i^{n^*}\big(K_i,K_j\big) = \hat{b}^2\Big\{K_i \int_{2b_iK_i+\hat{b}K_j}^{\infty} f(t_i, 2b_jK_j+\hat{b}K_i)dt_i - K_j \int_{2b_jK_j+\hat{b}K_i}^{\infty} f(2b_iK_i+\hat{b}K_j, t_j)dt_j\Big\}. \tag{29}$$

If Θ_is are exponentially distributed, $\nabla^2_{ij}\tilde{\pi}_i^{n^*}\big(K_i,K_j\big)$ becomes

$$\nabla^2_{ij}\tilde{\pi}_i^{n^*}\big(K_i,K_j\big) = \hat{b}^2 e^{-(2b_iK_i+\hat{b}K_j)\lambda_i - (2b_jK_j+\hat{b}K_i)\lambda_j}[K_i\lambda_j - K_j\lambda_i] \tag{30}$$

Note that this term is positive only if $K_i\lambda_j > K_j\lambda_i$. Therefore, $\nabla K_i^{Nc}(K_j)$ can be positive only if $K_i\lambda_j > K_j\lambda_i$. This and the fact that $\nabla K_i^{Nc}(K_j) > -1$ together implies that $\nabla K_i^{Nc}(K_j) \in [-1,0]$ for $K_i\lambda_j < K_j\lambda_i$ and $\nabla K_i^{Nc}(K_j) \geq -1$ for $K_i\lambda_j > K_j\lambda_i$: This is shwon in Figure 12(a). We now use this to show that the best response functions cannot cross multiple times. For this, suppose that the best responses cross twice or more and one of these intersections occurs at point A in Figure 12(b). Notice that from the fact that

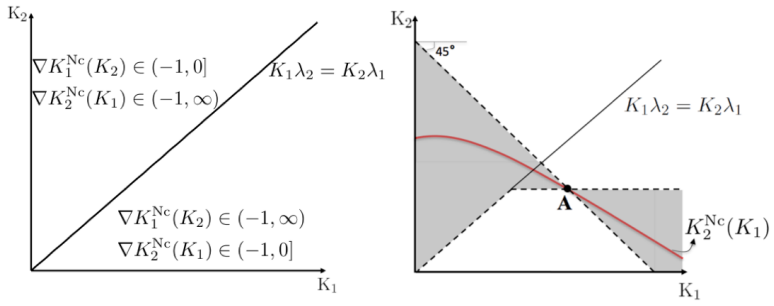

Figure 12 **(a) Signs of the best response functions' derivatives with respect to the regions on $K_1 \times K_2$ space, (b) Example for contradiction to prove that the intersection of the best response functions is unique.**

$\nabla K_2^{\mathrm{Nc}}(K_1) \in [-1,0]$ for $K_2\lambda_1 < K_1\lambda_2$ and $\nabla K_2^{\mathrm{Nc}}(K_1) \geq -1$ elsewhere, the additional intersection point must be in the shaded region in Figure 12(b). However, if $K_1^{\mathrm{Nc}}(K_2)$ passes through point A, the other intersection (which is in the shaded region) must have $\nabla K_1^{\mathrm{Nc}}(K_2) < -1$, contradicts the fact that $\nabla K_1^{\mathrm{Nc}}(K_2) \geq -1$ (see Figure 12(a)). Thus, the response functions cannot cross more than once.

Cc and Cn Scenarios: The existence comes from the fact that $(K_1, K_2, \eta) = (K_1^{\mathrm{Nn}*}, K_2^{\mathrm{Nn}*}, 0)$ and the fact that the total profit is bounded above by that of a centralized firm.

The proof of part ii.(a) is algebraic, thus omitted. The proof for uniform distribution is similar.

The proofs of Propositions 5, 7, and 8 use the following technical lemma.

LEMMA 2. *We have the following:*

i. $\pi_i^{n*}(K_i, K_j)$ *is decreasing in* K_j, $i = 1,2$, $i \neq j$.

ii. The optimal capacity of a centralized firm is larger than an individual capacity of a firm under the Nn scenario: $K_T^{m*} \geq K_i^{Nn*}$, $i = 1,2$.

iii. The joint capacity under the Cp scenario is smaller than the capacity of a centralized firm: $K_T^{Cp*} < K_T^{m*}$.

Proof of Lemma 2. We present the proof of Part **iii**. The proofs of parts (i) and (ii) are algebraic, therefore omitted. Under the Cp scenario, the firms negotiate to determine the reallocation of the total capacity in the second stage, before each firm individually sets its production quantity. Therefore, the total revenue, $R_{\mathrm{T}}^{\mathrm{Cp}*}(K_{\mathrm{T}}, \theta_1, \theta_2)$, for given demand signals (θ_1, θ_2), is a function of the total capacity K_{T}. By the NBS, $K_{\mathrm{T}}^{\mathrm{Cp}*}$ maximizes $E\big[R_{\mathrm{T}}^{\mathrm{Cp}*}(K_{\mathrm{T}}, \theta_1, \theta_2)\big] - cK_{\mathrm{T}}$. We can show that $E\big[R_{\mathrm{T}}^{\mathrm{Cp}*}(K_{\mathrm{T}}, \theta_1, \theta_2)\big] - cK_{\mathrm{T}}$ is increasing at $K_{\mathrm{T}} = 0$ and decreasing when $K_{\mathrm{T}} \to \infty$. Therefore, the first order condition is a necessary condition for $K_{\mathrm{T}}^{\mathrm{Cp}*}$ to be optimal:

$$\tfrac{d}{dK_{\mathrm{T}}}|_{K_{\mathrm{T}}=K_{\mathrm{T}}^{\mathrm{Cp}*}} \Big\{\mathbb{E}\big[R_{\mathrm{T}}^{\mathrm{Cp}*}(K_{\mathrm{T}}, \Theta_1, \Theta_2)\big] - cK_{\mathrm{T}}\Big\} = 0$$

Note that for any incremental capacity, a centralized firm optimally allocates it to production while the efficiency is not guaranteed in the Cp scenario. This implies that, for any marginal increase in capacity, the increase in the total profit of a centralized firm is always larger than the increase in the total profit under the Cp scenario. Hence,

$\tfrac{d}{dK_{\mathrm{T}}}\Big\{\mathbb{E}\big[R_{\mathrm{T}}^{\mathrm{m}*}(K_{\mathrm{T}}, \Theta_1, \Theta_2)\big] - cK_{\mathrm{T}}\Big\} > \tfrac{d}{dK_{\mathrm{T}}}\Big\{\mathbb{E}\big[R_{\mathrm{T}}^{\mathrm{Cp}*}(K_{\mathrm{T}}, \Theta_1, \Theta_2)\big] - cK_{\mathrm{T}}\Big\}$ for any K_{T}, and we have

$$\tfrac{d}{dK_{\mathrm{T}}}|_{K_{\mathrm{T}}=K_{\mathrm{T}}^{\mathrm{Cp}^*}}\Big\{\mathbb{E}\big[R_{\mathrm{T}}^{\mathrm{m}^*}(K_{\mathrm{T}},\Theta_1,\Theta_2)\big]-cK_{\mathrm{T}}\Big\}>\tfrac{d}{dK_{\mathrm{T}}}|_{K_{\mathrm{T}}=K_{\mathrm{T}}^{\mathrm{Cp}^*}}\Big\{\mathbb{E}\big[R_{\mathrm{T}}^{\mathrm{Cp}^*}(K_{\mathrm{T}},\Theta_1,\Theta_2)\big]-cK_{\mathrm{T}}\Big\}=0$$

As $\mathbb{E}\big[R_{\mathrm{T}}^{\mathrm{m}^*}(K_{\mathrm{T}},\Theta_1,\Theta_2)\big]-cK_{\mathrm{T}}$ is concave in K_{T}, this implies that $K_{\mathrm{T}}^{\mathrm{Cp}^*}<K_{\mathrm{T}}^{\mathrm{m}^*}$.

Proof of Proposition 5: i. We first show that a subsidy-free equilibrium, $(K_1^{\mathrm{C}\mathfrak{s}*},K_2^{\mathrm{C}\mathfrak{s}*})$ satisfies equation (18). Note that applying $\eta^{\mathrm{C}\mathfrak{s}*}=0$, the equation in (17) is simplified as follows:

$$\pi_1^{\mathfrak{s}*}(K_1^{\mathrm{C}\mathfrak{s}*},K_2^{\mathrm{C}\mathfrak{s}*})-\pi_2^{\mathfrak{s}*}(K_1^{\mathrm{C}\mathfrak{s}*},K_2^{\mathrm{C}\mathfrak{s}*})=\pi_1^{\mathrm{d}}-\pi_2^{\mathrm{d}}\qquad \mathfrak{s}\in\{n,c\} \tag{31}$$

For the Cn scenario (i.e., $\mathfrak{s}=n$), this directly implies equation (18). For the Cc scenario (i.e., $\mathfrak{s}=c$), from (27), we have

$$\pi_i^{\mathrm{c}^*}(K_1,K_2)=\pi_i^{\mathrm{n}^*}(K_1,K_2)+\frac{\pi^{\mathrm{m}^*}(K_1+K_2)-\pi_i^{\mathrm{n}^*}(K_1,K_2)-\pi_j^{\mathrm{n}^*}(K_1,K_2)}{2}\qquad i,j=1,2\quad i\neq j \tag{32}$$

Substituting $K_i=K_i^{\mathrm{Cc}*}$, $i=1,2$ and applying in (31), we get (18).

We now show that any equilibrium $(K_1^{\mathrm{C}\mathfrak{s}*},K_2^{\mathrm{C}\mathfrak{s}*})$ that satisfies equation (18) is subsidy-free. The result $\eta^{\mathrm{Cn}^*}=0$ follows from algebra for the Cn scenario. For the Cc scenario, we first rewrite (18) using the expression of $\pi_i^{\mathrm{c}^*}(K_1,K_2)$ in equation (32). Then, we have $\pi_1^{\mathrm{c}^*}(K_1^{\mathrm{Cc}^*},K_2^{\mathrm{Cc}^*})-\pi_1^{\mathrm{c}^*}(K_1^{\mathrm{Cc}^*},K_2^{\mathrm{Cc}^*})=\pi_1^{\mathrm{d}}-\pi_2^{\mathrm{d}}$. Substituting this in (17), we obtain $2\eta^{\mathrm{Cc}^*}=0$, which completes the proof.

ii. From the fact that $(\pi_1^{\mathrm{d}},\pi_2^{\mathrm{d}})=\Big(\pi_1^{\mathrm{n}^*}(K_1^{\mathrm{Nn}^*},K_2^{\mathrm{Nn}^*}),\pi_2^{\mathrm{n}^*}(K_1^{\mathrm{Nn}^*},K_2^{\mathrm{Nn}^*})\Big)$, equation (18) can be expressed as:

$$\pi_1^{\mathrm{n}^*}(K_1^{\mathrm{C}\mathfrak{s}*},K_2^{\mathrm{C}\mathfrak{s}*})-\pi_2^{\mathrm{n}^*}(K_1^{\mathrm{C}\mathfrak{s}*},K_2^{\mathrm{C}\mathfrak{s}*})=\pi_1^{\mathrm{n}^*}(K_1^{\mathrm{Nn}^*},K_2^{\mathrm{Nn}^*})-\pi_2^{\mathrm{n}^*}(K_1^{\mathrm{Nn}^*},K_2^{\mathrm{Nn}^*})\qquad \mathfrak{s}\in\{n,c\} \tag{33}$$

For $\hat{b}=0$, we will first consider the Cn scenario. Since $\hat{b}=0$, $q_i^{\mathrm{n}^*}(\omega)$ and the inverse demand function, $p_i(q_1,q_2,\omega)=\theta_i-b_iq_i$, are independent from K_j. Therefore, the profit $\pi_i^{\mathrm{n}^*}(K_i,K_j)$ is independent from K_j. Under the Cn scenario, the equilibrium, $(K_1^{\mathrm{Cn}^*},K_2^{\mathrm{Cn}^*})$ will maximize $\pi_1^{\mathrm{n}^*}(K_1,K_2)+\pi_2^{\mathrm{n}^*}(K_1,K_2)$. Since $\pi_i^{\mathrm{n}^*}(K_i,K_j)$ is independent from K_j, we have $K_i^{\mathrm{Cn}^*}=K_1^{\mathrm{Nn}^*}$. The proof for the Cc scenario is similar, thus omitted. Now, consider the Cc scenario when $\hat{b}>0$. First note that, by Theorem 1, $K_1^{\mathrm{Cc}^*}+K_2^{\mathrm{Cc}^*}=K_{\mathrm{T}}^{\mathrm{m}^*}$. For the proof, define the following function

$$h(K_1)=\Big(\pi_1^{\mathrm{n}^*}(K_1,K_{\mathrm{T}}^{\mathrm{m}^*}-K_1)-\pi_1^{\mathrm{n}^*}(K_1^{\mathrm{Nn}^*},K_2^{\mathrm{Nn}^*})\Big)-\Big(\pi_2^{\mathrm{n}^*}(K_1,K_{\mathrm{T}}^{\mathrm{m}^*}-K_1)-\pi_2^{\mathrm{n}^*}(K_1^{\mathrm{Nn}^*},K_2^{\mathrm{Nn}^*})\Big)$$

We next prove that there exists a K_1 such that $h(K_1)=0$. For this, observe that $h(\cdot)$ is continuous (since $\pi_1^{\mathrm{n}^*}(\cdot,\cdot)$ is continuous). In addition, we have:

$$h(K_1^{\mathrm{Nn}^*})=\Big(\pi_1^{\mathrm{n}^*}(K_1^{\mathrm{Nn}^*},K_{\mathrm{T}}^{\mathrm{m}^*}-K_1^{\mathrm{Nn}^*})-\pi_1^{\mathrm{n}^*}(K_1^{\mathrm{Nn}^*},K_2^{\mathrm{Nn}^*})\Big)-\Big(\pi_2^{\mathrm{n}^*}(K_1^{\mathrm{Nn}^*},K_{\mathrm{T}}^{\mathrm{m}^*}-K_1^{\mathrm{Nn}^*})-\pi_2^{\mathrm{n}^*}(K_1^{\mathrm{Nn}^*},K_2^{\mathrm{Nn}^*})\Big)$$

Since $K_2^{\mathrm{Nn}^*}\geq K_{\mathrm{T}}^{\mathrm{m}^*}-K_1^{\mathrm{Nn}^*}$ and $\pi_1^{\mathrm{n}^*}(K_1,K_2)$ is decreasing in K_2 (Lemma 2(i)), the terms in the first parenthesis is positive. Also, from the fact that $K_2^{\mathrm{Nn}^*}$ is the best response to $K_1^{\mathrm{Nn}^*}$, the terms in the second parantehesis is negative. Combining these, we have $h(K_1^{\mathrm{Nn}^*})\geq 0$.

A similar argument shows that $K_{\mathrm{T}}^{\mathrm{m}^*}-K_i^{\mathrm{Nn}^*}\geq 0$ (Lemma 2(ii)) and

$$h(K_{\mathrm{T}}^{\mathrm{m}^*}-K_2^{\mathrm{Nn}^*})=\Big(\pi_1^{\mathrm{n}^*}(K_{\mathrm{T}}^{\mathrm{m}^*}-K_2^{\mathrm{Nn}^*},K_2^{\mathrm{Nn}^*})-\pi_1^{\mathrm{n}^*}(K_1^{\mathrm{Nn}^*},K_2^{\mathrm{Nn}^*})\Big)-\Big(\pi_2^{\mathrm{n}^*}(K_{\mathrm{T}}^{\mathrm{m}^*}-K_2^{\mathrm{Nn}^*},K_2^{\mathrm{Nn}^*})-\pi_2^{\mathrm{n}^*}(K_1^{\mathrm{Nn}^*},K_2^{\mathrm{Nn}^*})\Big)\leq 0$$

These two inequalities imply that there exists K_1 between $\min(K_{\mathrm{T}}^{\mathrm{m}^*} - K_2^{\mathrm{Nn}^*}, K_1^{\mathrm{Nn}^*})$ and $\max(K_{\mathrm{T}}^{\mathrm{m}^*} - K_2^{\mathrm{Nn}^*}, K_1^{\mathrm{Nn}^*})$ that satisfies equation (33) (i.e., $h(K_1) = 0$).

Proof of Proposition 6: i. The proof is similar to that for the existence of the NBS solution for the Cc senario, thus omitted. **ii.** Under the Cp scenario, the firms negotiate to trade capacity before they make quantity decisions. Notice that, if $\hat{b} = 0$, the firm i's revenue is indepedent from the other firm's quantity. Hence, as long as the total endowment is the same, the quantities that firms will produce when they do not collaborate during the production are the same as the those chosen by a centralized firm.

Proof of Proposition 7: We omit the proof of part i. since it is similar to the proof of Theorem $1(i)$ (existence) and the proof of Theorem $1(ii)a$ (difference), respectively. For part **ii.**, from Proposition 6, observe that, the equilibrium production quantities are the same as the quanitites that a centralized firm with the same total capacity would produce. Since the firms fully collaborate in capacity investment in the first stage, the equilibrium outcome must coincide with the equilibrium outcome under the Cc scenario. The proof of part **iii.** is similar to the proof of Proposition $5(i)$, thus omitted. Finally, for part **iv.**, the case where $\hat{b} = 0$ immediately follows from part (ii) of Proposition 7. Now, consider the case where $\hat{b} > 0$. Note from Lemma $2(iii)$ that $K_{\mathrm{T}}^{\mathrm{m}^*} > K_{\mathrm{T}}^{\mathrm{Cp}^*}$. Furthermore, from the assumption, it must be $K_{\mathrm{T}}^{\mathrm{Cp}^*} \geq K_i^{\mathrm{Nn}^*}$ for $i = 1, 2$. Then, the result follows from a similar argument used in the proof of Proposition $5(ii)$ with $K_{\mathrm{T}}^{\mathrm{Cp}^*}$ replacing $K_{\mathrm{T}}^{\mathrm{m}^*}$.

Proof of Proposition 8.

(A) First note that from Proposition 2, $q_i^{\mathrm{n}^*}(\omega)$ and $\pi_i^{\mathrm{n}^*}(K_i, K_j)$ are independent from K_j when $\hat{b} = 0$. Thus, we will simplify the notation and drop K_j from the arguments of $\pi_i^{\mathrm{n}^*}(\cdot)$, $i, j = 1, 2$, $i \neq j$ in this proof.

(i.) Note that $(K_1^{\mathrm{Cn}^*}, K_2^{\mathrm{Cn}^*})$ maximizes $\pi_1^{\mathrm{n}^*}(K_1) + \pi_2^{\mathrm{n}^*}(K_2)$. Since $\pi_i^{\mathrm{n}^*}(\cdot)$ is independent of K_j, $i, j = 1, 2$, $i \neq j$, $K_i^{\mathrm{Cn}^*} = \arg\max \pi_i^{\mathrm{n}^*}(K_i) = K_i^{\mathrm{Nn}^*}$, $i = 1, 2$.

(ii.) From Theorem 1(ii)b, we have $K_{\mathrm{T}}^{\mathrm{Cc}^*} = K_{\mathrm{T}}^{\mathrm{m}^*}$. Therefore, it suffices to show that $\min(K_{\mathrm{T}}^{\mathrm{m}^*}, K_{\mathrm{T}}^{\mathrm{Nn}^*}) \leq K_{\mathrm{T}}^{\mathrm{Nc}^*} \leq \max(K_{\mathrm{T}}^{\mathrm{m}^*}, K_{\mathrm{T}}^{\mathrm{Nn}^*})$. We provide the proof for the case where the equilibrium under the Nc scenario, $(K_1^{\mathrm{Nc}^*}, K_2^{\mathrm{Nc}^*})$ is an interior solution where both capacity endowments satisfy the first order conditions. The treatment for the boundary solution as the analysis is similar. From equation (27), the first order conditions that determine the equilibrium for the Nc scenario is expressed as follows:

$$\nabla_i \pi_i^{\mathrm{c}^*}(K_i^{\mathrm{Nc}^*}, K_j^{\mathrm{Nc}^*}) = \nabla \pi^{\mathrm{m}^*}(K_1^{\mathrm{Nc}^*} + K_2^{\mathrm{Nc}^*}) + \nabla \pi_i^{\mathrm{n}^*}(K_i^{\mathrm{Nc}^*}) = 0 \qquad i, j = 1, 2 \;\; i \neq j \tag{34}$$

Thus, we must have $\nabla \pi_1^{\mathrm{n}^*}(K_1^{\mathrm{Nc}^*}) = \nabla \pi_2^{\mathrm{n}^*}(K_2^{\mathrm{Nc}^*})$.

Consider the case that $K_{\mathrm{T}}^{\mathrm{Nn}^*} > K_{\mathrm{T}}^{\mathrm{m}^*}$. Suppose that $K_1^{\mathrm{Nc}^*} + K_2^{\mathrm{Nc}^*} = K_{\mathrm{T}}^{\mathrm{Nc}^*} < K_{\mathrm{T}}^{\mathrm{m}^*}$. Since $\pi^{\mathrm{m}^*}(\cdot)$ is concave, we have $\nabla \pi^{\mathrm{m}^*}(K_{\mathrm{T}}^{\mathrm{Nc}^*}) > 0$. Then, from equation (34), it must be $\nabla \pi_1^{\mathrm{n}^*}(K_1^{\mathrm{Nc}^*}) < 0$. Since, $\pi_i^{\mathrm{n}^*}(\cdot)$ is also concave in K_i and it is independent of K_j, it must be the case that $K_i^{\mathrm{Nc}^*} > K_i^{\mathrm{Nn}^*}$, $i = 1, 2$. Hence, we have

$$K_{\mathrm{T}}^{\mathrm{m}^*} \;>\; K_1^{\mathrm{Nc}^*} + K_2^{\mathrm{Nc}^*} \;>\; K_1^{\mathrm{Nn}^*} + K_2^{\mathrm{Nn}^*} = K_{\mathrm{T}}^{\mathrm{Nn}^*}$$

which contradicts $K_{\mathrm{T}}^{\mathrm{Nn}^*} > K_{\mathrm{T}}^{\mathrm{m}^*}$. Hence, we must have $K_{\mathrm{T}}^{\mathrm{Nc}^*} \geq K_{\mathrm{T}}^{\mathrm{m}^*}$.

Now, to prove $K_{\mathrm{T}}^{\mathrm{Nc}*} \leq K_{\mathrm{T}}^{\mathrm{Nn}*}$, suppose that $K_1^{\mathrm{Nc}*} > K_1^{\mathrm{Nn}*}$. Since $\pi_1^{\mathrm{n}*}(\cdot)$ is concave, $\nabla\pi_1^{\mathrm{n}*}(K_1^{\mathrm{Nc}*}) < 0$. Then, from equation (34), it must be the case that $\nabla\pi^{\mathrm{m}*}(K_{\mathrm{T}}^{\mathrm{Nc}*}) > 0$. As $\pi^{\mathrm{m}*}(\cdot)$ is concave, this implies $K_{\mathrm{T}}^{\mathrm{Nc}*} < K_{\mathrm{T}}^{\mathrm{m}*}$. Note that $\nabla\pi_1^{\mathrm{n}*}(K_1^{\mathrm{Nc}*}) < 0$ also implies that $\nabla\pi_2^{\mathrm{n}*}(K_2^{\mathrm{Nc}*}) < 0$, and hence $K_2^{\mathrm{Nc}*} > K_2^{\mathrm{Nn}*}$. Therefore:

$$K_{\mathrm{T}}^{\mathrm{m}*} \;>\; K_1^{\mathrm{Nc}*} + K_2^{\mathrm{Nc}*} \;>\; K_1^{\mathrm{Nn}*} + K_2^{\mathrm{Nn}*} = K_{\mathrm{T}}^{\mathrm{Nn}*}$$

which contradicts $K_{\mathrm{T}}^{\mathrm{Nn}*} > K_{\mathrm{T}}^{\mathrm{m}*}$. Hence, we must have $K_1^{\mathrm{Nc}*} \leq K_1^{\mathrm{Nn}*}$ and hence $K_2^{\mathrm{Nc}*} \leq K_2^{\mathrm{Nn}*}$, establishing $K_{\mathrm{T}}^{\mathrm{Nc}*} \leq K_{\mathrm{T}}^{\mathrm{Nn}*}$. Therefore, we have $K_{\mathrm{T}}^{\mathrm{m}*} \leq K_{\mathrm{T}}^{\mathrm{Nc}*} \leq K_{\mathrm{T}}^{\mathrm{Nn}*}$. The proof when $K_{\mathrm{T}}^{\mathrm{Nn}*} \leq K_{\mathrm{T}}^{\mathrm{m}*}$ is similar.

(iii.) Directly follows Proposition 7(*ii*).

(B) The proof is similar to part (A) but uses Lemma 2(*i*). Hence, we only provide the sketches and highlight difference. For part **(i)**, notice that, when the firms collaborate in the capacity building stage (as in Cn scenario), the NBS stipluates that the firms select the capacities to maximize the total profit. Therefore $(K_1^{\mathrm{Cn}*}, K_2^{\mathrm{Cn}*})$ satisfies the following conditions:

$$\nabla_1\pi_1^{\mathrm{n}*}(K_1^{\mathrm{Cn}*}, K_2^{\mathrm{Cn}*}) + \nabla_1\pi_2^{\mathrm{n}*}(K_1^{\mathrm{Cn}*}, K_2^{\mathrm{Cn}*}) = 0 \quad \text{and} \quad \nabla_2\pi_1^{\mathrm{n}*}(K_1^{\mathrm{Cn}*}, K_2^{\mathrm{Cn}*}) + \nabla_2\pi_2^{\mathrm{n}*}(K_1^{\mathrm{Cn}*}, K_2^{\mathrm{Cn}*}) = 0$$

Lemma 2(*i*) establishes that $\nabla_j\pi_i^{\mathrm{n}*}(K_i, K_j) < 0$, for $i, j = 1, 2$, and $i \neq j$. Therefore, we have: $\nabla_1\pi_1^{\mathrm{n}*}(K_1^{\mathrm{Cn}*}, K_2^{\mathrm{Cn}*}) > 0$ and $\nabla_2\pi_2^{\mathrm{n}*}(K_1^{\mathrm{Cn}*}, K_2^{\mathrm{Cn}*}) > 0$. Recall that $K_i^{\mathrm{Nn}}(K_j)$ is the best response of firm i to the capacity K_j of firm j under the Nn scenario. Since $\pi_i^{\mathrm{n}*}(K_i, K_j)$ is concave in K_i, we have $\nabla_i\pi_i^{\mathrm{n}*}(K_i^{\mathrm{Nn}}(K_j), K_j) = 0$. Thus, $K_1^{\mathrm{Cn}*} < K_1^{\mathrm{Nn}}(K_2^{\mathrm{Cn}*})$ and $K_2^{\mathrm{Cn}*} < K_2^{\mathrm{Nn}}(K_1^{\mathrm{Cn}*})$. This is depicted in Figure 13, where $(K_1^{\mathrm{Cn}*}, K_2^{\mathrm{Cn}*})$ can only be in the lightly shaded region. In this figure, the dashed line represents the values where $K_1 + K_2 = K_{\mathrm{T}}^{\mathrm{Nn}*}$. The lightly shaded region is to the left of this line because $\nabla K_i^{\mathrm{Nn}}(\cdot) \in (-1, 1)$ by Lemma 1(*ii*). Therefore, $K_{\mathrm{T}}^{\mathrm{Cn}*} \leq K_{\mathrm{T}}^{\mathrm{Nn}*}$.

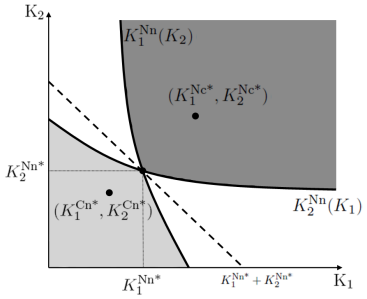

Figure 13 **The best response curves under the Nn scenario and the equilibrium capacity vectors under the Cn and Nc scenarios.**

(ii.) We use contradiction to prove the result that $\min(K_{\mathrm{T}}^{\mathrm{m}*}, K_{\mathrm{T}}^{\mathrm{Nn}*}) \leq K_{\mathrm{T}}^{\mathrm{Nc}*}$. The argument is similar to the proof of part (A.ii) in Proposition 8, thus omitted.

(iii.) The result immediately follows Lemma 2.(*iii*).