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THE DEVELOPMENT OF NEW WAKE SURVEY TECHNIQUES AT THE UNIVERSITY OF MICHIGAN

John C. Gebhardt
Finn C. Michelsen

ORA Project 07402

under contract with:

Maritime Administration
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Task VI, Part III
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OFFICE OF RESEARCH ADMINISTRATION · ANN ARBOR

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ABSTRACT

New instrumentation and data handling techniques developed at The University of Michigan for performing wake surveys behind ship models is described. Preliminary experimental results are presented. Doubts are raised as to the validity of the Betz-Tullin approximation for separating the viscous drag and the wave drag of ship models.

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List of Symbols

a	angle between tube axis and holes
CS, CP, CT, CB	pressure differences
g	acceleration of gravity
K, K'	constants
n	outward normal to control surface
p_o	static pressure far upstream
p_1	fictional pressure in wake (assumed)
S_o	area of transverse plane far upstream
S	area of transverse plane close behind model
U	model speed
u	x velocity component
V	uniform stream velocity
v	y velocity component
w	z velocity component
u_1, v_1, w_1	assumed potential flow velocites in wake
X	horizontal flow angle
x	distance behind the model
Y	vertical flow angle
y	distance to starboard of the model ℄ in inches
z	distance above the free surface in inches
ζ	free surface elevation
ρ	mass density of fluid
ϕ	velocity potential

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Introduction

Increased interest in two distinct but related problems concerning ship design prompted the design and implementation of a system utilizing a spherical 5-hole pitot tube and pressure transducers for measuring temporal and spatial velocity variations in the wake of a ship model. First, the problem of correctly calculating or measuring the drag of a three-dimensional body operating near or on a free surface has never been satisfactorily solved, although William Froude's hypothesis that the viscous drag of a ship is independent of the Froude number and is equal to the viscous resistance of a flat plate of the same length and wetted surface seems to give reasonable values. The subsequent development of wave resistance theories for surface ships has now shed serious doubts about the validity of Froude's hypothesis. It has been found that the discrepency between the calculated wave resistance and the measured value obtained by subtracting the flat plate friction from the tow rope pull is relatively large. Either the calculated wave resistance is in error or the three dimensional and free surface effects on the viscous resistance are larger than have been thought. The only logical way to resolve this dilemma is to formulate a rational method for either calculating or measuring directly the viscous drag, including free surface effects, of three dimensional bodies. In 1951, Tulin(1) suggested a method, based on work by Betz, for measuring the viscous drag of a ship by means of a wake survey. At the University of Iowa, Landweber(2) and Wu(3) developed and modified Tulin's theory and applied the method to a ship model with good results. While the work which has been done at Iowa shows some promise, the experimental techniques are slow and tedious and the nature of the measurements leave many important questions unanswered. The instrumentation which has been designed and built at The University of Michigan is capable of measuring the necessary quantities for determining the viscous drag according to Landweber's theory while at the same time measuring additional quantities which make possible an evaluation of the errors involved in the method.

The second problem which led to the design of the instrumentation to be described in detail later in this report is the problem of determining a true picture of the wake in which a ship propeller operates. It seems that at the present time the only factor preventing naval architects from designing larger and higher speed conventional single screw ships is the non-uniform flow in the area where the propeller must operate. This disturbed flow can cause severe vibration problems which if left uncorrected can reduce the usefulness of the ship to almost zero. If the wake could be rendered more uniform, the allowable shaft horsepower could be increased without concern. Since the flow in the wake contains both spatial and temporal variations, any instrument for measuring the wake velocities must be capable of responding to both types of fluctuation. The instrumentation described herein has sufficient frequency response so that the measurement of time and space fluctuations of the velocity are now possible.

At the outset of this project the new hardware and techniques were to be used almost exclusively to gain experimental data for viscous drag research and propeller vibration studies. However, because of the expense and complexity of the instrumentation, it was decided to try to design it such that it could be useful for other applications. This decision has proven to be extremely sound. For example, the light-weight trailing carriage (see Section II. C.), has been used as a platform for photography and for observing wave systems as well as for making ordinary propeller plane wake surveys. In addition to its intended duties as a platform for the 5-hole pitot tube (see Section II. A. for details of the 5-hole pitot tube) while making viscous drag measurements. In the future it may be used as a towing carriage for testing high speed vehicles and for other studies requiring a stable lightweight platform separate from the main carriage.

The pitot tube data analysis program (Section II. A.) which enables the data to be recorded and converted directly to a computer compatible form was written so that it will accept any analog data which can be recorded on magnetic tape. So far, this program has been used only for digitizing pressure data from the pitot tube, but there is no reason why it could not be used to advantage with other data.

In the following pages the wake survey instrumentation and associated hardware is described in detail. Following the description some theoretical considerations and results of several wake surveys are presented.

II. The 5-Hole Pitot Tube and Associated Instrumentation

A. The 5-Hole Pitot Tube

For a number of years the instrument used in many towing tanks for making wake measurements has been the 5-hole pitot tube which was first developed at the David Taylor Model Basin(4). The instrument in use at The University of Michigan is essentially a copy of the DTMB design and is shown in Figure 1. The location of the holes on the surface of the one half inch diameter spherical tip is shown in Figure 2. By utilizing the pressure differences measured between the four peripheral holes and the center hole as dictated by the theory presented below, the speed and direction of the flow impinging on the spherical tip of the tube can be determined.

Consider a 5-hole spherical pitot tube as shown in Figure 3 located in a uniform stream of velocity V and direction specified by the angles X and Y relative to the axis of the tube. If the fluid is inviscid and the flow is steady the following relationships can be written. (See Pien. 4)

$$\frac{CS-CP}{CS+CP} = K \tan 2X \qquad (1)$$

$$\frac{CT-CB}{CT+CB} = K \tan 2Y \qquad (2)$$

$$\frac{CS-CP}{V_h 2} = \frac{9}{8g} K' \sin 2X \qquad (3)$$

$$\frac{CT-CB}{V_v 2} = \frac{9}{8g} K' \sin 2Y \qquad (4)$$

where

CP, CS, CT, CB = Pressure at center hole - pressure at hole P,

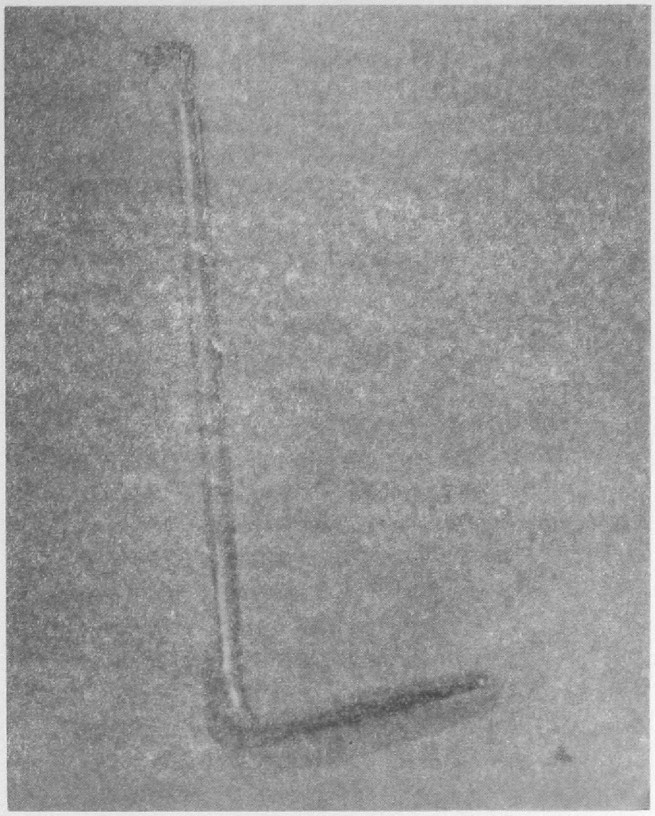


Fig. 1. The University of Michigan five-hole pitot tube.

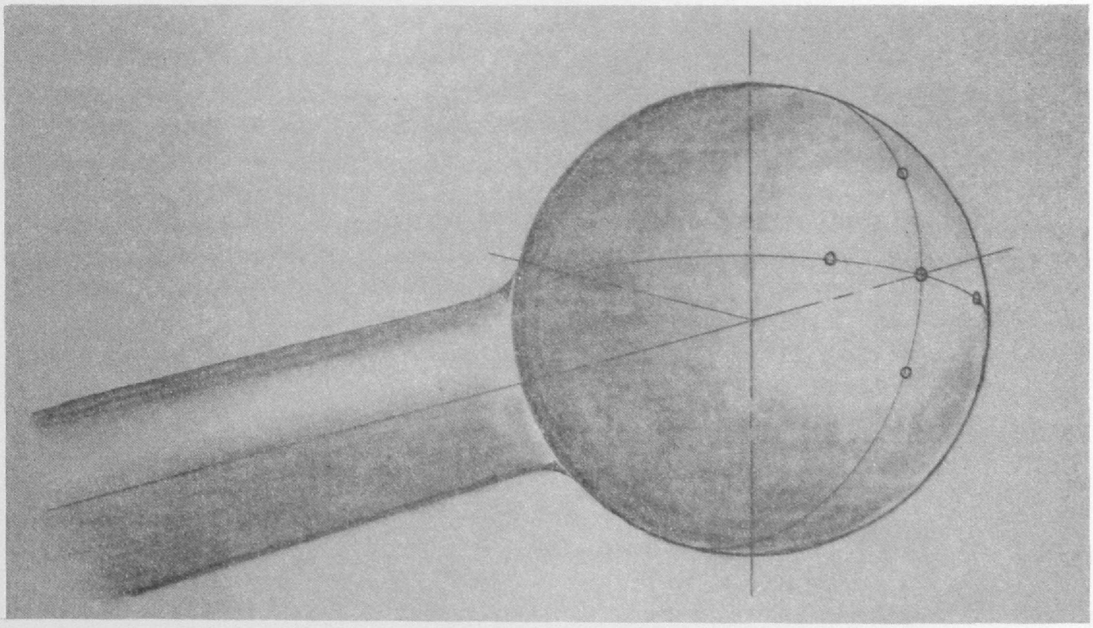


Fig. 2. Five-hole pitot tube illustrating hole configuration.

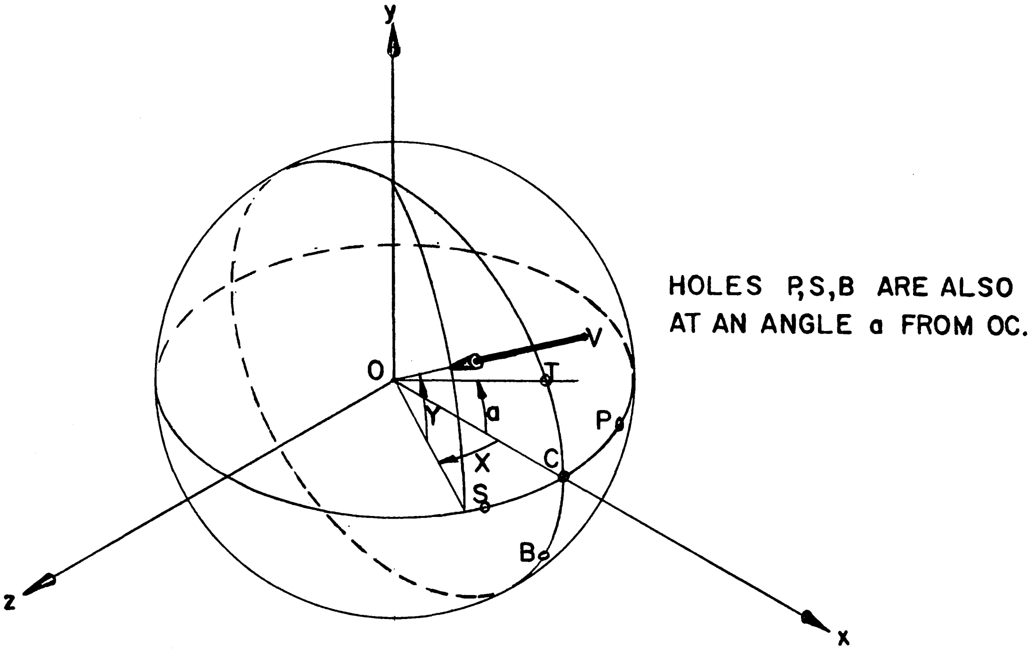


Fig. 3. Geometry of the five-hole pitot tube.

S, T, B respectively. and

$$K = \frac{\sin 2a}{1-\cos 2a}$$

$$K' = \sin 2a$$

g = acceleration of gravity

$$V_h = V \cos Y$$

$$V_v = V \cos X$$

It can be seen that X is a function of $\frac{CS-CP}{CS+CP}$ only whereas Y is similarly a function of $\frac{CT-CB}{CT+CB}$. Thus it appears that the angle X can be obtained entirely from pressure measurements obtained from the three holes in the horizontal plane and Y can likewise be determined from corresponding measurements taken at the three holes in the vertical plane. The calibration curve of X vs. $\frac{CS-CP}{CS+CP}$ should therefore be independent of Y, just as the curve of Y vs. $\frac{CT-CB}{CT+CB}$ should show no dependency upon X.

If these simple relationships were to hold it would be a relatively easy matter to determine the velocity components from measured data, i.e., CP, CS, CT and CB. With this information one could enter curves of $\frac{CS-CP}{CS+CP}$ and $\frac{CP-CS}{V_n 2}$ vs. X and $\frac{CT-CB}{CT+CB}$ and $\frac{CT-CB}{V_v 2}$ vs. Y which had been obtained from calibration runs of the tube to compensate from deviations of pressures from theoretical predictions. Such a procedure is referred to by Pien(4) and has been used at David Taylor Model Basin. Figure 4 shows typical simple calibration curves of this type. At The University of Michigan we have found, however, that all the quantities referred to above

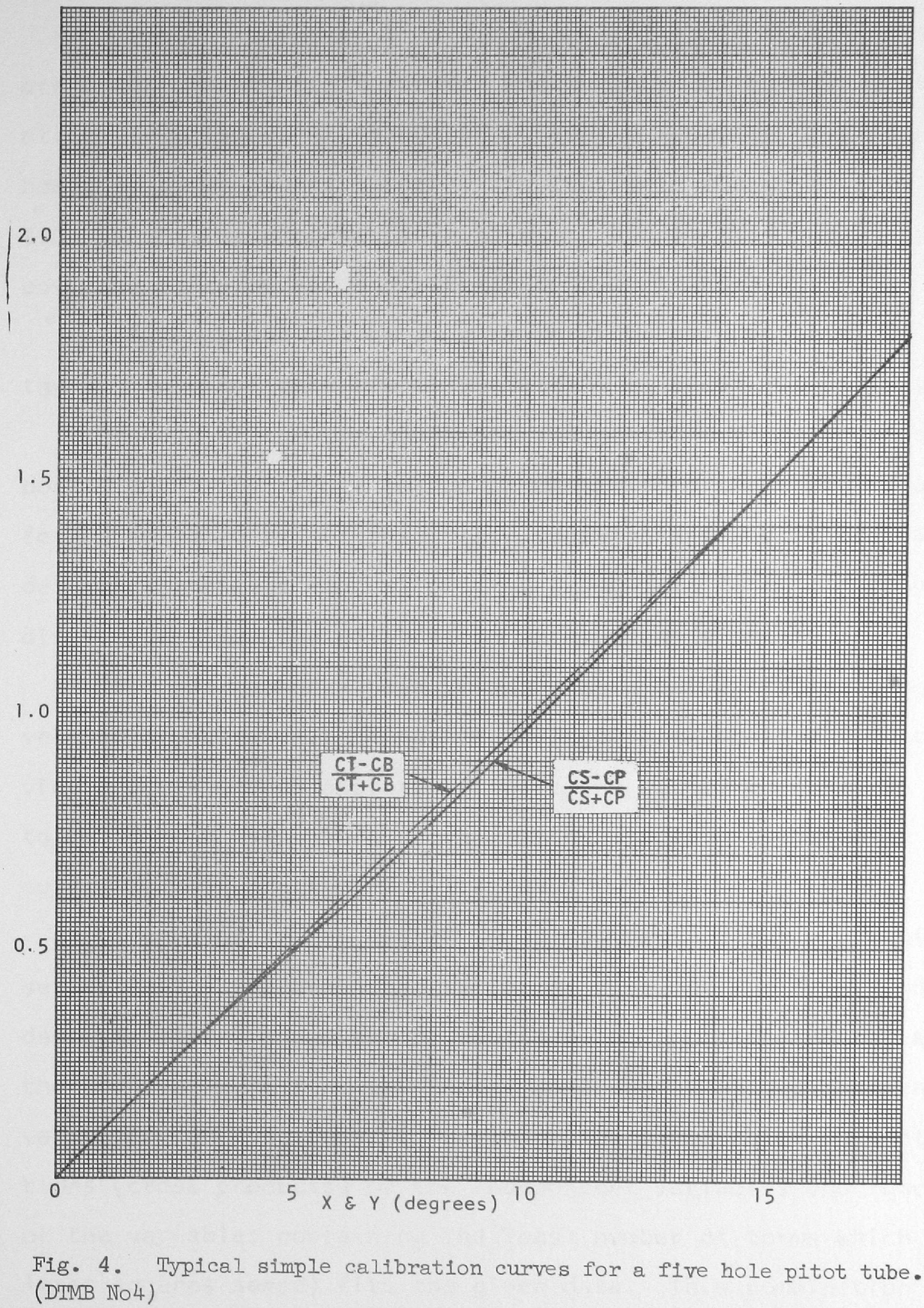


Fig. 4. Typical simple calibration curves for a five hole pitot tube. (DTMB No4)

are functions of both X and Y. Whether this is caused by deviations from theory or inaccuracies in the geometry of pitot tube head, or both, is not certain. The complete answer to this question is of little consequence however, since it is necessary in any case to calibrate the pitot tube experimentally.

Figures 5 and 6 show some calibration data as obtained from the 5-hole pitot tube now in use at The University of Michigan.

Because of the dependency of the pressure differences upon both X and Y the simple method of determining these angles as referred to above is not possible. Ideally it would be desirable to determine analytic expressions for X and Y in terms of the pressure differences from the calibration data of the pitot tube.

This approach has been tried using a curve fitting program developed by Westervelt(5) but the polynomials obtained did not provide a close enough approximation to the actual calibration curves to be of much use. This curve fitting approach seems to be better suited to less exacting applications.

Very briefly, the method is as follows: Given a set of observed data for a dependent variable, y, a maximum of 60 independent variables, and a set of functions of the independent variable, the computer will find the linear combination of the independent variables, or functions of the independent variables, or interactions (cross products) of the independent variables and functions of the variables containing the least number of terms which (in the least squares sense) fits the given data. This combination is called the predicting equation. The set of functions may be chosen

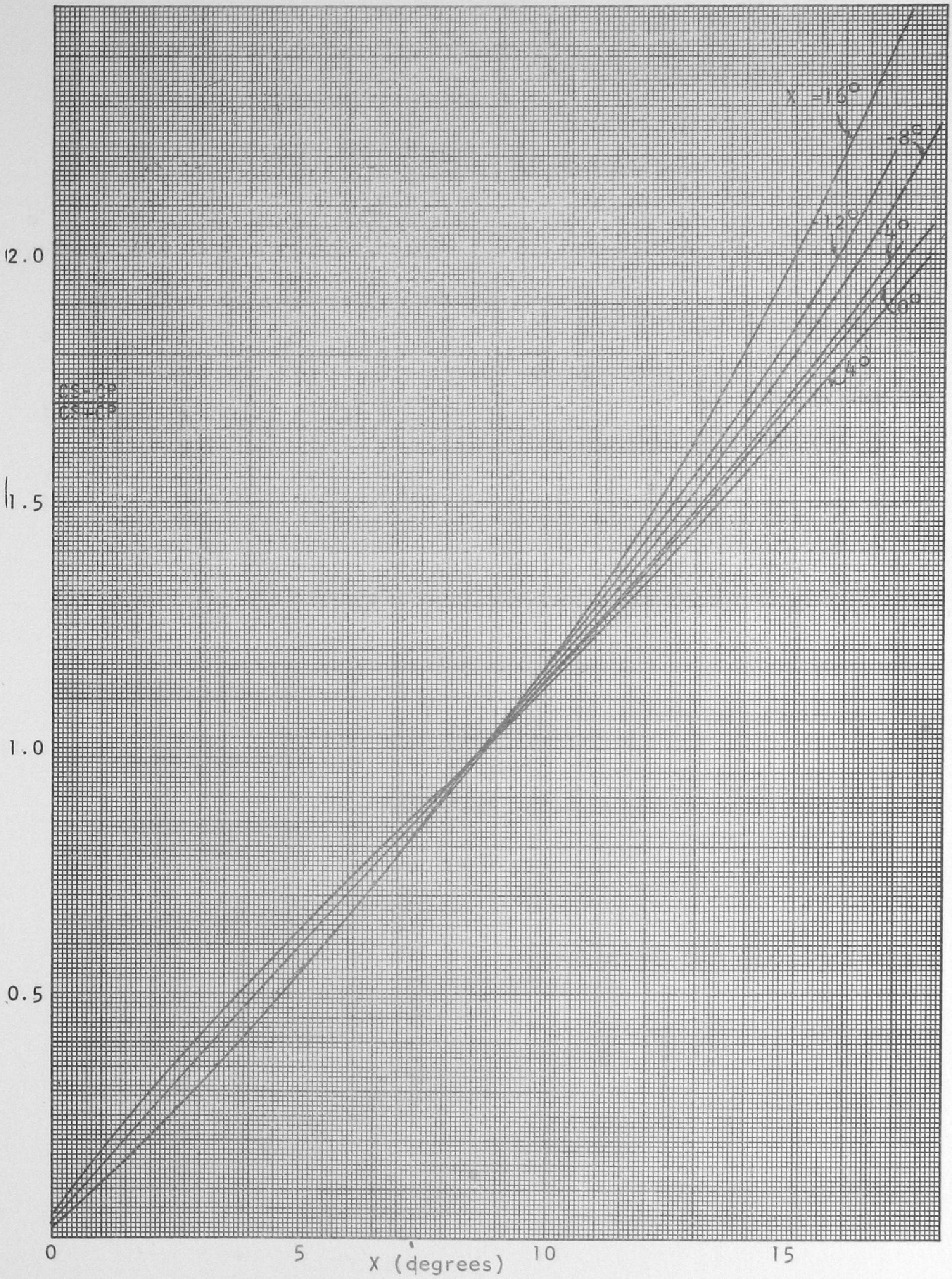


Fig. 5. X vs $\frac{CS-CP}{CS+CP}$ with Y as parameter. University of Michigan five hole pitot tube.

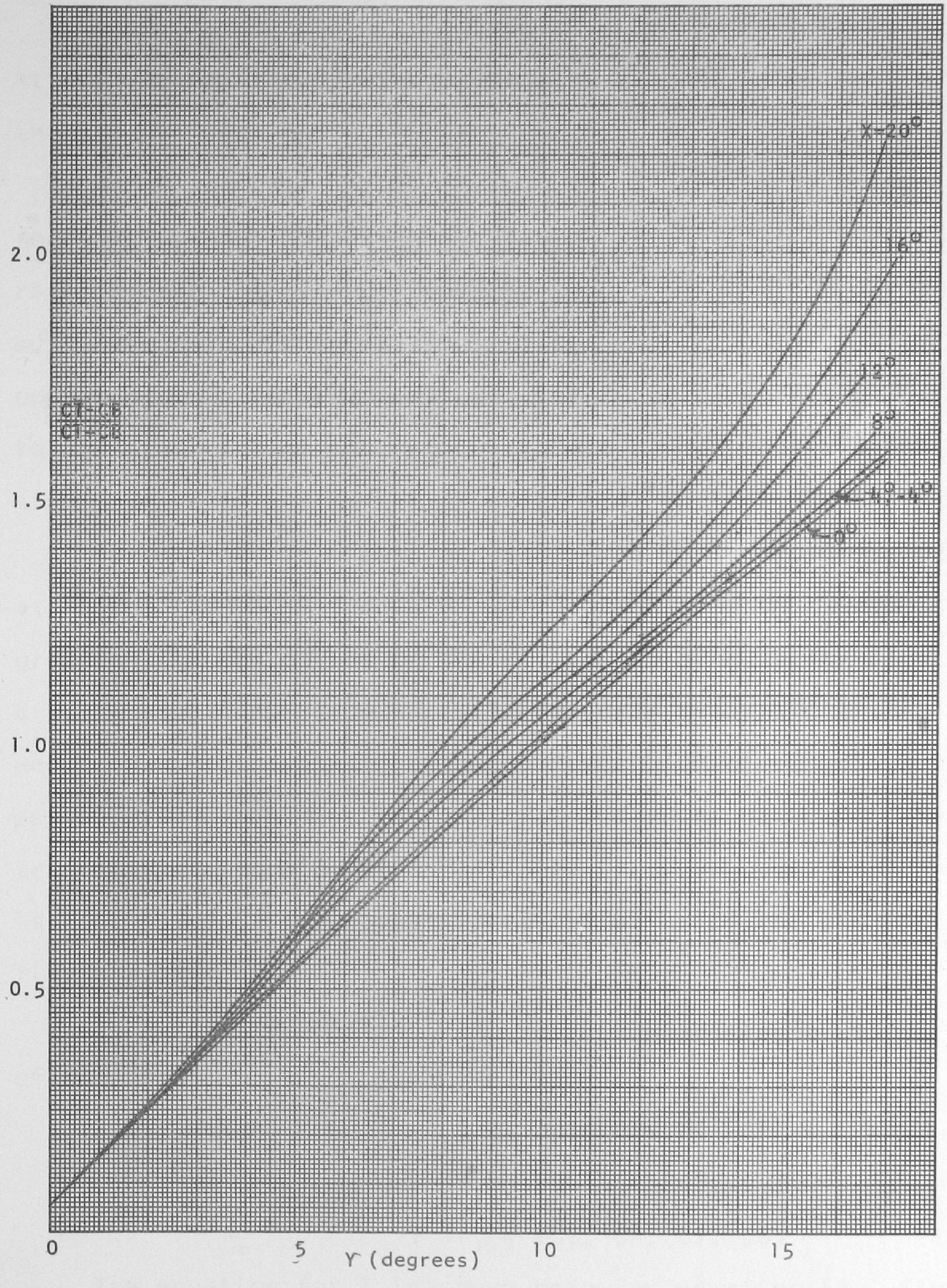


Fig. 6. Y vs $\frac{CT-CB}{CT+CB}$ with X as parameter. University of Michigan five hole pitot tube.

at will to fit the particular problem at hand. For this problem the set of functions

$$(X_m)^{i/j} \quad = 1,\ 2,\ 3 \quad j = 1;\ j = 1,\ 2,\ 3 \quad i = 1$$

where X_m is any independent variable will suffice. Even for this restricted class of functions the number of possible terms that must be considered is extremely large and it would be very time consuming to attempt to evaluate the importance of each of them to the final fit of the data. To help find the predicting equation more rapidly a method of artificial intelligence is employed.

During the first pass a small number of terms are selected at random from many thousands which are available. Stepwise regression is carried out on these terms to produce the first predicting equation. From this the computer "learns" certain things about the problem which influence its choice of terms for the next pass. After each pass the equation is modified and more terms are tried based on the experience of the previous ones. This process is continued until the postulated criteria for the predicting equation are met.

This program has been used successfully to generate functions of the form

$$X = P_1\ (CS,\ CP,\ CT,\ CB) \qquad (5)$$

$$Y = P_2\ (CS,\ CP,\ CT,\ CB) \qquad (6)$$

$$V^2 = P_3\ (CS,\ CP,\ CT,\ CB) \qquad (7)$$

The equation for X is shown below as equation 5a. Figure 7 shows the errors between this predicting equation and the experimental data.

```
                                   PREDICTED  RESULTS  VERSUS  DATA  POINTS

OBS. NO.                      PREDICTIONS                                     DATA                          DEVIATIONS
          Y - SIGMA                Y                   Y + SIGMA             POINTS                 (DATA - Y)       PERCENT
   1   -.79841582E 00       .30396879E-01         .85920958E 00        .00000000E 00          -.30396879E-01       -.000
   2   -.17296811E 02      -.16467998E 02        -.15639186E 02       -.15000000E 02           .14679985E 01       -9.787
   3   -.21105000E 02      -.20276187E 02        -.19447374E 02       -.20000000E 02           .27618718E 00       -1.381
   4   -.65742747E 01      -.57454621E 01        -.49166494E 01       -.50000000E 01           .74546212E 00      -14.909
   5   -.56003825E 00       .26877445E 00         .10975871E 01        .00000000E 00          -.26877445E 00       -.000
   6    .32670106E 01       .40958233E 01         .49246359E 01        .50000000E 01           .90417671E 00       18.084
   7    .85103401E 01       .93391528E 01         .10167965E 02        .10000000E 02           .66084719E 00        6.608
   8    .14097063E 02       .14925876E 02         .15754689E 02        .15000000E 02           .74123740E-01         .494
   9    .19314786E 02       .20143599E 02         .20972411E 02        .20000000E 02          -.14359856E 00        -.718
  10    .19410404E 02       .20239216E 02         .21068029E 02        .20000000E 02          -.23921657E 00       -1.196
  11    .13867203E 02       .14696016E 02         .15524829E 02        .15000000E 02           .30398381E 00        2.027
  12    .90781586E 01       .99069713E 01         .10735784E 02        .10000000E 02           .93028665E-01         .930
  13    .33916151E 01       .42204278E 01         .50492404E 01        .50000000E 01           .77957219E 00       15.591
  14   -.44496878E 00       .38384392E 00         .12126566E 01        .00000000E 00          -.38384392E 00       -.000
  15   -.49159645E 01      -.40871518E 01        -.32583391E 01       -.50000000E 01          -.91284817E 00       18.257
  16   -.10344537E 02      -.95157243E 01        -.86869115E 01       -.10000000E 02          -.48427570E 00        4.843
  17   -.16321220E 02      -.15492407E 02        -.14663594E 02       -.15000000E 02           .49240720E 00       -3.283
  18   -.21375022E 02      -.20546209E 02        -.19717397E 02       -.20000000E 02           .54620957E 00       -2.731
  19   -.21514229E 02      -.20685417E 02        -.19856604E 02       -.20000000E 02           .68541694E 00       -3.427
  20   -.15519417E 02      -.14690604E 02        -.13861791E 02       -.15000000E 02          -.30939579E 00        2.063
  21   -.11784163E 02      -.10955350E 02        -.10126538E 02       -.10000000E 02           .95535028E 00       -9.554
  22   -.51348676E 01      -.43060549E 01        -.34772422E 01       -.50000000E 01          -.69394505E 00       13.879
  23   -.34686147E 00       .48195124E 00         .13107639E 01        .00000000E 00          -.48195124E 00       -.000
  24    .36720560E 01       .45008688E 01         .53296814E 01        .50000000E 01           .49913120E 00        9.983
  25    .97027548E 01       .10531568E 02         .11360380E 02        .10000000E 02          -.53156757E 00       -5.316
  26    .14670238E 02       .15499051E 02         .16327864E 02        .15000000E 02          -.49905133E 00       -3.327
  27    .18650831E 02       .19479644E 02         .20308457E 02        .20000000E 02           .52035570E 00        2.602
  28    .14679909E 02       .15508722E 02         .16337534E 02        .15000000E 02          -.50872159E 00       -3.391
  29    .98591939E 01       .10688007E 02         .11516819E 02        .10000000E 02          -.68800664E 00       -6.880
  31   -.88166577E 00      -.52853073E-01         .77595963E 00        .00000000E 00           .52853073E-01         .000
  32   -.62888300E 01      -.54600173E 01        -.46312046E 01       -.50000000E 01           .46001738E 00       -9.200
  33   -.19617439E 02      -.18788626E 02        -.17959813E 02       -.20000000E 02          -.12113740E 01        6.057
  39   -.72864056E 01      -.64575929E 01        -.56287802E 01       -.50000000E 01           .14575930E 01      -29.152
  40   -.32955658E 00       .49925612E 00         .13280688E 01        .00000000E 00          -.49925612E 00       -.000
  41    .42620790E 01       .50908917E 01         .59197044E 01        .50000000E 01          -.90891778E-01       -1.818
  42    .99535680E 01       .10782381E 02         .11611193E 02        .10000000E 02          -.78238070E 00       -7.824
  43    .14249958E 02       .15078771E 02         .15907584E 02        .15000000E 02          -.78771353E-01        -.525
  44    .18620238E 02       .19449050E 02         .20277863E 02        .20000000E 02           .55094957E 00        2.755
  50   -.11082795E 02      -.10253983E 02        -.94251701E 01       -.10000000E 02           .25398290E 00       -2.540
  51   -.15740688E 02      -.14911876E 02        -.14083063E 02       -.15000000E 02          -.88124156E-01         .587
  52   -.20223242E 02      -.19394429E 02        -.18565616E 02       -.20000000E 02          -.60557079E 00        3.028
  53   -.19264523E 02      -.18435710E 02        -.17606897E 02       -.20000000E 02          -.15642896E 01        7.821
  54   -.15340470E 02      -.14511658E 02        -.13682845E 02       -.15000000E 02          -.48834217E 00        3.256
  55   -.10853508E 02      -.10024695E 02        -.91958823E 01       -.10000000E 02           .24695039E-01        -.247
  56   -.49576070E 01      -.41287944E 01        -.32999817E 01       -.50000000E 01          -.87120563E 00       17.424
  57   -.63612863E 00       .19268407E 00         .10214968E 01        .00000000E 00          -.19268407E 00       -.000
  58    .36568264E 01       .44856391E 01         .53144518E 01        .50000000E 01           .51436085E 00       10.287
  59    .88413416E 01       .96701543E 01         .10498967E 02        .10000000E 02           .32984567E 00        3.298
```

Fig. 7. Deviations of the predicting equation for X from the experimental data points.

$$X = 6.10-1.05(CS-CP)^2 + 0.01 \left(\frac{CS+CP}{CS-CP}\right)^3 - 0.07 \left(\frac{CS+CP}{CS-CP}\right)^{3/2} +$$
$$+ 0.43 (CS-CP)^3 + 0.03 (CS-CP)^2 \left(\frac{CT-CB}{CT+CB}\right)^3 - 1.99 \frac{1}{(CS-CP)^2}$$
$$\left(\frac{CT-CB}{CT+CB}\right) - 2.20 \frac{CS+CP}{(CS-CP)^{1/3}} -3.07 \qquad \frac{1}{(CS-CP)^2} \frac{(CT+CB)^{1/3}}{CT-CB}$$
$$+ 5.62 \frac{1}{(CS-CP)^2} - 0.62 (CS-CP)^{1/3} (CS+CP)^2 \qquad (5a)$$

A much more accurate method of putting the pitot tube calibration curves into the computer has been devised and implemented into the MAD computer program "PITSEAR". The complete grid of calibration data: $\frac{CT-CB}{CT+CB}$, $\frac{CS-CP}{CS+CP}$, $\frac{CS}{V_h^2}$, $\frac{CP}{V_h^2}$, $\frac{CT}{V_v^2}$ and $\frac{CB}{V_v^2}$ for the various values of X and Y are stored in the computer memory in table form. Then when give a set of pressures CT, CB, CS and CP, the program calculates the parameters $\frac{CT-CB}{CT+CB}$ and $\frac{CS-CP}{CS+CP}$, enters the first two calibration tables and interpolates to find a unique value of X and Y. This X and Y are then used to enter two of the last four tables, depending on whether X and Y are positive or negative, from which unique values of $\frac{CS}{V_h^2}$ or $\frac{CP}{V_h^2}$ and $\frac{CT}{V_v^2}$ or $\frac{CB}{V_v^2}$ are found. The values of these parameters are then used to calculate V_h and V_v and the problem is solved. A listing of the computer program "PITSEAR" can be found in Appendix 1.

It appears that the only reason for using five holes on a spherical pitot tube is to make Equations (1) - (4) valid. Since there are only three unknowns in the problem, X, Y and V it would appear that three independent pressure measurements would be enough. This could be achieved using a 4-hole pitot tube. The euqations for a 4-hole pitot tube are derived in Appendix II.

B. Pressure Transducers and Electronics

In search of a pressure transducer of sufficient sensitivity to measure very small pressure differences, and also of fast enough response so that when the tube moves at a fairly slow speed (1 ft/sec) in a steady state flow field it records with high accuracy the space variations of fluid velocity relative to the tube, we found that the Sandborn Model 268B Physiological Pressure Transducer would meet our requirements. Figure 8 is a picture of four of these transducers, mounted in an instrument package for use with the 5-hole pitot tube. Each transducer has two separate pressure chambers and is made so that the difference between the pressures in each chamber can be recorded directly. This instrument has proved to be very stable and with good linearity properties. Instrument characteristics are given in Figure 9. We have also found it to be fairly rugged, but one must be cautioned against exceeding maximum design pressures. The frequency response depends upon length of plastic tubing from the top of the pitot tube to the pressure transducer. With copper tubing connected with short lengths of plastic tubing it is possible to record at frequencies up to about 2 cycles/sec. Which is considered quite adequate for the intended purposes. Figure 10 shows the system response to pressure changes of varying frequency.

Each transducer output is amplified by a Sandborn Carrier Amplifier and the resulting signal is recorded on a 7-channel 1/4" magnetic tape along with the carriage speed and a voltage which corresponds to the position of the pitot tube.

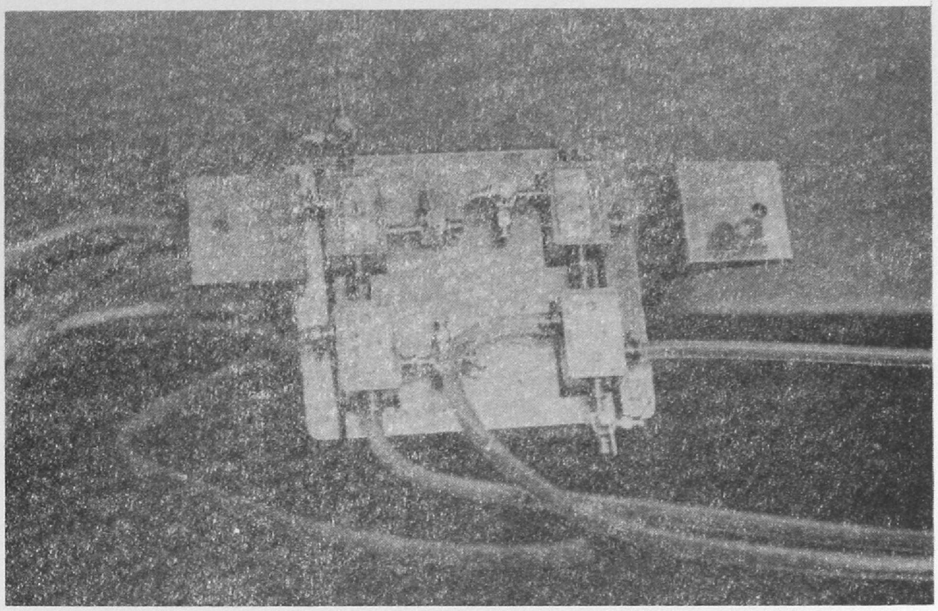


Fig. 8. Pressure transducers used with five-hole pitot tube.

Linear Range:	-21.4 in. H_2O to +21.4 in. H_2O pressure with respect to atmosphere
Output Voltage:	750 microvolts open circuit output per volt excitation per inch of water
Overload Pressure:	± 40 inches of water maximum permissible pressure
Internal Volume:	0.2 cc maximum internal volume. .187 mm^3/ 20 inches water nonimal volume displacement.
Hysteresis and Non-Linearity	less than ± 1.5% of full scale
Temperature Limitations	50°C (120°F) Maximum permissible

Fig. 9. Pressure transducer characteristics.

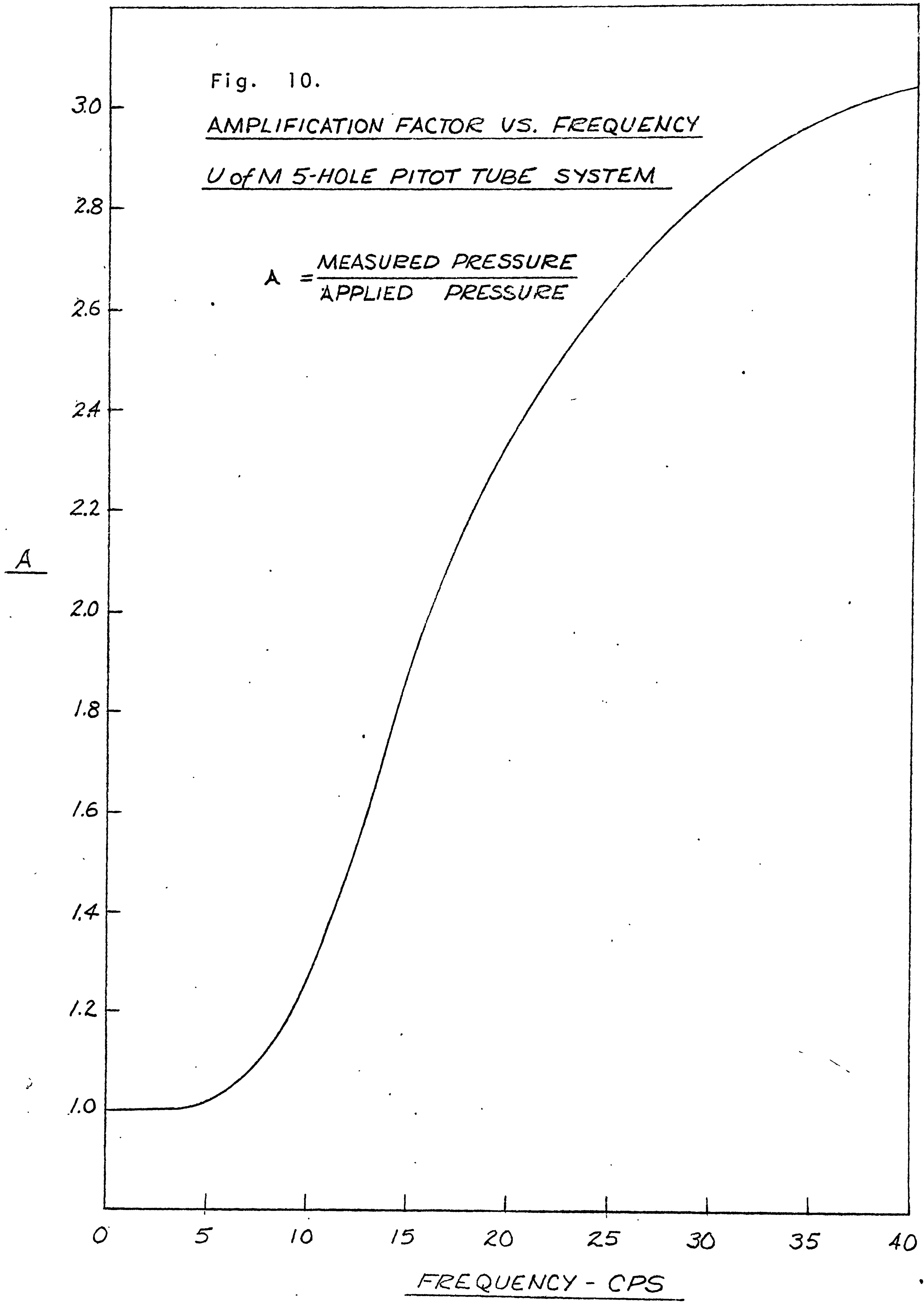


Fig. 10.

AMPLIFICATION FACTOR VS. FREQUENCY

U of M 5-HOLE PITOT TUBE SYSTEM

Figure 11 gives the specifications of the Sanborn carrier amplifiers and Figure 12 shows the performance characteristics of the tape recorder. This recorder is the new Ampex SP 300, a relatively low cost portable instrumentation recorder. It operates at four different tape speeds and each of the seven channels can record in either direct or FM mode. In the direct mode the recorder's frequency response is from 50 cps to 2,600 x N cps where N is the tape speed in inches per second. In FM mode the recorder's response is from D.C. to 167 x N cps. All pressure data is recorded in the FM mode while the carriage speed data, which is a series of square waves produced by a magnetic pulser mounted on a carriage wheel and whose frequency is proportional to the speed, is recorded in the direct mode.

The pressure transducers are calibrated by placing a known pressure difference, across each of them and adjusting the gains on the amplifiers so that the output voltage of all the transducers is identical (1 volt). The calibration is then recorded on tape at the beginning of each test, and upon playback, the data is compared with the calibration.

C. The Pitot Tube Drive Mechanisms

Since the 5-hole pitot tube, when connected to electronic pressure transducers described above, can measure the instantaneous fluid velocity at a point by measuring the pressure, it is possible to move the pitot tube through the fluid at a known velocity and calculate the velocity with respect to a stationary pitot tube. For this purpose two separate drive systems have been built, one which moves the tip of the pitot tube in concentric

Input:	Any external transducer having not less than 100 ohms resistance at excitation terminals and not more than 1000 ohms at the input terminals. These requirements are met by most resistance bridge transducers.
Carrier Frequency:	2400 cps ± 2%, internally supplied
Carrier Amplitude:	Approximately 5 volts
Sensitivity:	Depends on transducer. Nominal sensitivity of a 100 ohms four arm bridge is 100 microvolts of signal for one volt at the output.
Frequency Response:	0 to 480 cps within 3 db at 1 volt peak-to-peak output amplitude.
Linearity:	Error not to exceed ± .2% at the output
Noise:	10 millivolts maximum (peak-to-peak) with no output. Additional ripple present with input signal is less than 1% of the instantaneous d-c output.
Drift:	Less than 25 millivolts/10°C ambient temperature change to 50°C. 2 millivolts maximum for line voltage change from 103 to 127 volts
Power:	115 volts ± 10%, 60 cps

Fig.11. Performance data on Sanborn carrier preamplifer, model 350 1100 B.

TAPE TRANSPORT

Tape Speeds:	15, 7-1/2, 3 3/4, 1 7/8 ips, electrically switchable. The transport speed switch automatically selects the proper equalization or frequency determining unit for each speed.
Tape Speed Deviation:	± 0.2% at 15 and 7-1/2 ips, ± 0.4% at 3 3/4 and 1 7/8 ips
Tape and Reel Size:	1/4 inch tape width on either 10-1/2 inch NAB reels or 7 inch plastic reels
Controls:	Record, Stop, Fast Forward, Play, Rewind, Power, Reel Size, Erase, Speed, Calibration Voltage, Meter Function, FM/Direct, Amput Level.

TAPE HEADS

Track Dimensions:	Track width is 0.024 inches (0.061 cm); 7 channel track spacing is 0.035 inches (0.084 cm) on center
Interstacking Spacing:	1 7/32 inch (3.09 cm) nominal

DIRECT RECORD/REPRODUCE SYSTEM

Frequency Response:	50 to 266 x N cps where N is the tape speed in ips. RMS signal to noise ration is nominally 30 db.
Input Level:	1.0 volt RMS to produce normal recording level. Adjustable from 0.5 to 10 volts RMS by input potentiometer.
Input Impedance:	Minimum 10,000 ohms resistance, unbalanced to ground.
Output Level:	1 volt RMS nominal, DC coupled (capable of delivering 20 ma into short circuit load).
Output Impedance:	100 ohms ± 20%, unbalanced to ground

FM RECORD/REPRODUCE SYSTEM

Frequency Response:	D.C. to 167 x N cps. RMS signal to noise ratio is 38 db. Total harmonic distortion is 1.5%.
DC Drift:	Less than ± 1.0% of full deviation over a 8 hour period, ambient temperature and line voltage constant, after a 1 hour warmup.
Record/Reproduce Voltage Linearity:	± 1.0% of full band (best straight line)
Input Level:	Input of 1 volt RMS to produce ± 40% deviation: adjustable from 0.5 to 10 volts RMS by input potentiometer
Input Impedance:	Minimum 10,000 ohms, unbalanced to ground.
Output Level:	1 volt RMS nominal, DC coupled (capable of delivering 20ma into short-circuit load).
Output Impedance:	100 ohms 20%, unbalanced to ground.

GENERAL CHARACTERISTICS

Power Requirement:	105 to 125 volts single phase 60 cps AC, 250 watts
Temperature and Humidity Limits:	30° to 110° F (-1° to 40°C); 0 to 85% without condensation.
Weight:	Approximately 90 lbs. (40.99 kg).

Fig. 12. Performance data for ampex SP 300 instrumentation tape recorder.

circles in the propeller plane, the other across the width of the model propeller. Figure 13 shows pitot tube mounted to the drive mechanism only four test runs are required for the standard wake survey.

A small precision potentiometer is connected to a shaft on the drive mechanism and wired so that the output voltage is proportional to the angle through which the tip of the pitot tube has travelled from top dead center. This voltage is recorded on a tape channel simultaneously with the pressure and speed data.

The device used to move the pitot tube across the width of the towing tank consists of a small subcarriage running on tracks mounted on a specially built carriage spanning the width of the tank. This carriage is towed behind the main carriage at a distance which can be varied from 18 inches to 20 feet. Figure 14 shows the "trailing carriage" and the subcarriage. Figure 15 is an end view of the track showing the arrangement of the drive wheels on the subcarriage. The pitot tube head can be lowered to approximately 3 feet below the free surface on a precision screw mechanism. The subcarriage was originally driven across the track at a constant speed by a syncrogenerator and a rack and pinion whose mate was in turn driven by a variable speed electric motor. A precision potentiometer was geared to the syncrogenerator and wired such that the output voltage was proportional to the distance of travel of the carriage from a fixed point on its track. This configuration was reliable and convenient but because of the response characteristics of the slave synchrogenerator severe oscillations could be excited by small pieces of dirt or imperfections on the

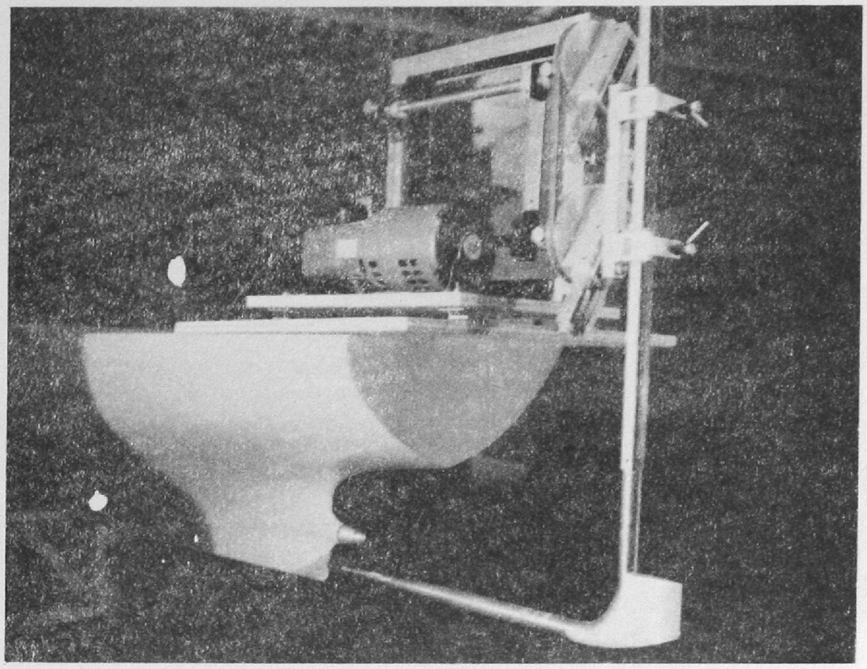


Fig. 13. Pitot tube drive mounted on a model ready for testing.

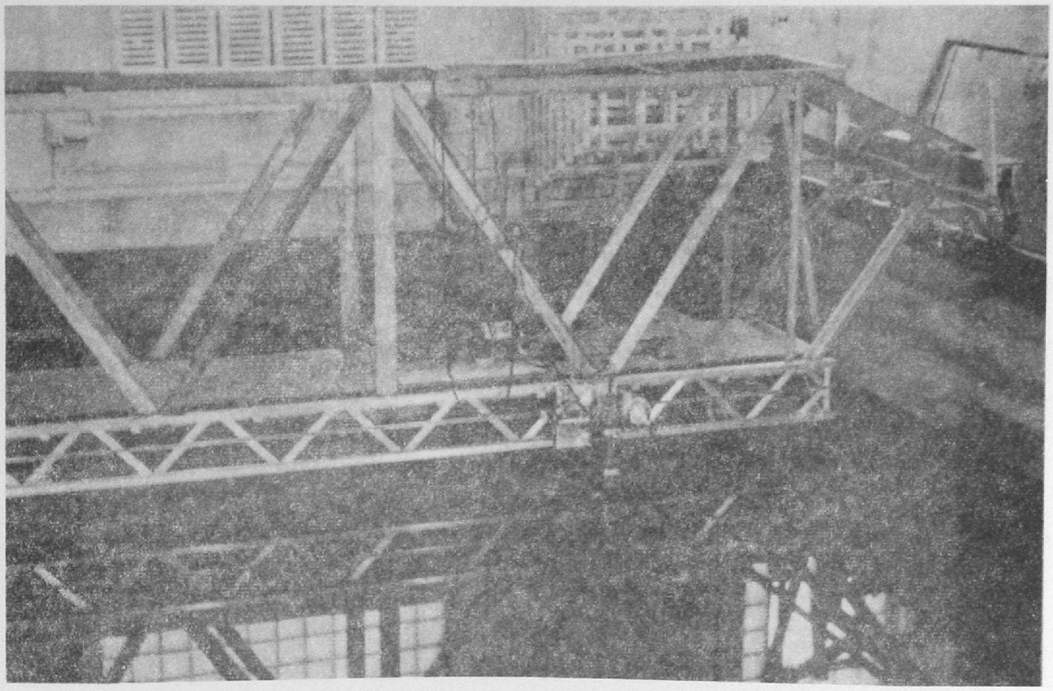


Fig. 14. The trailing carriage and the subcarriage.

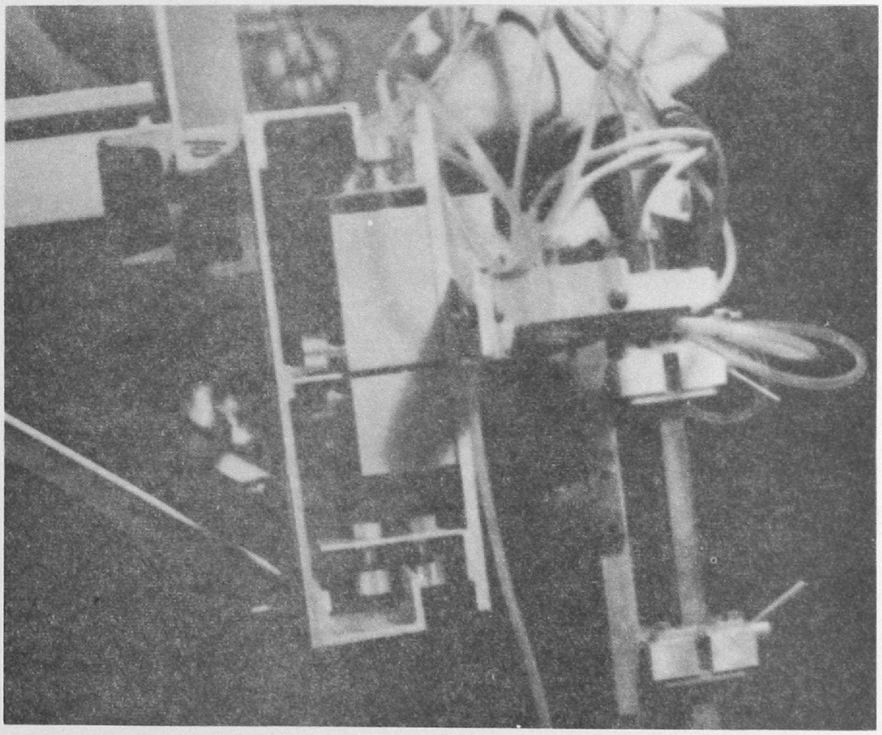


Fig. 15. End view of the sub-carriage.

subcarriage rails. Rather than devise a elaborate damping mechanism to cure the problem, the variable speed drive is used to drive the subcarriage directly by means of a simple cable and pulley arrangement. The resulting system is much smoother and much more room is available on the subcarriage for attaching equipment. Figure 16 shows the present system with the precision screw for vertical motion mounted on the subcarriage.

Figures 17 and 18 are drawings of the trailing carriage and the track with the subcarriage.

D. The Complete Wake Survey Instrumentation Package

The wake survey instrumentation package in use at The University of Michigan includes the following components, all of which have been described in some detail above:

1. 5-hole spherical pitot tube
2. pitot tube drive mechanisms
    a. horizontal and vertical drive
    b. rotary drive
3. electronic pressure transducers
4. carrier preamplifiers (one for each transducer)
5. signal conditioners (R-L-C filters)
6. seven channel 1/4" magnetic tape recorder

Figure 19 is a picture of the instrumentation package and Figure 20 is a schematic representation of the same system.

References 5 and 6 also describe the wake survey instrumentation.

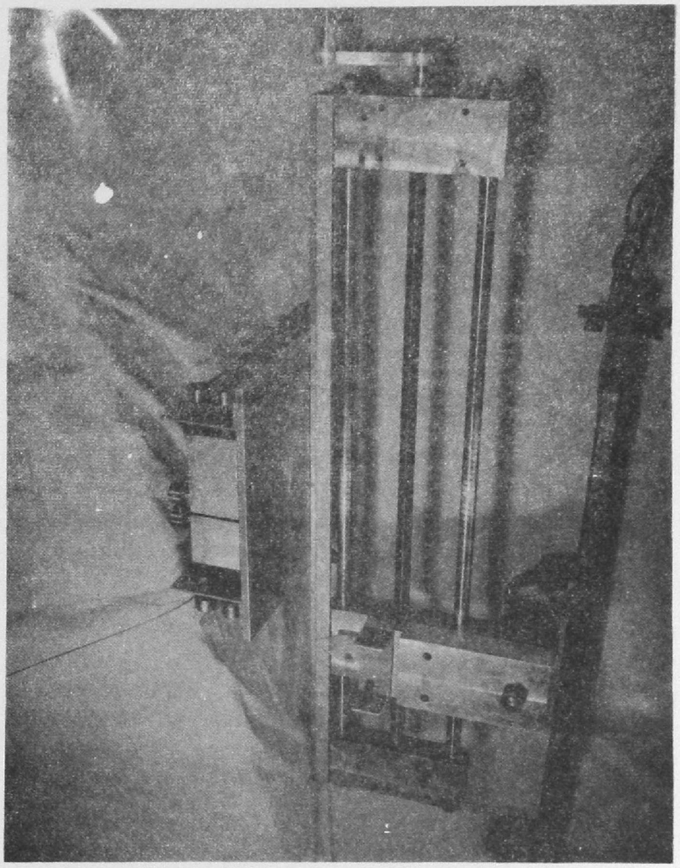
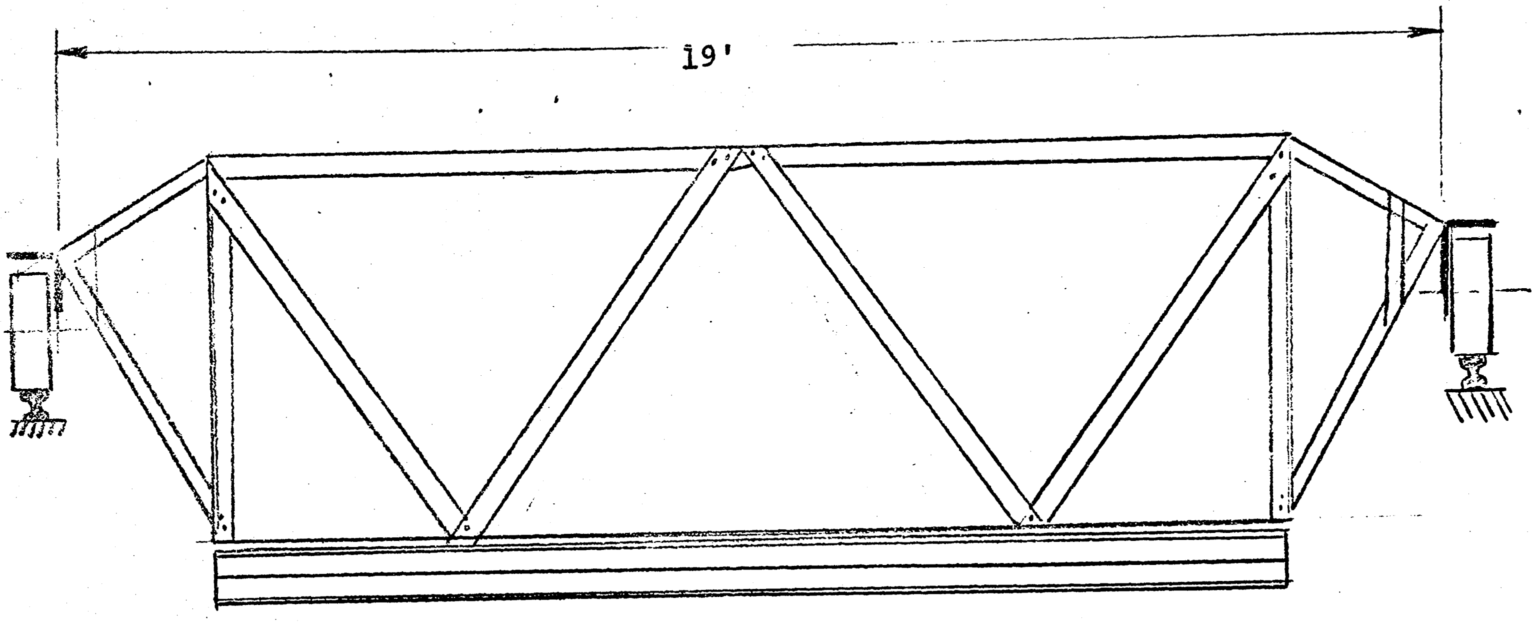
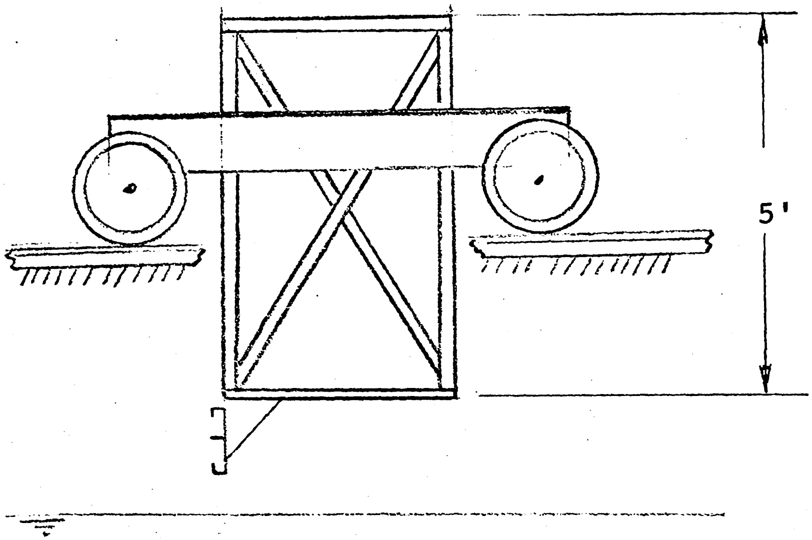


Fig. 16. Subcarriage with Vertical Screw Attached



Front View



Right Side View

Fig. 17. Trailing Carriage Drawing

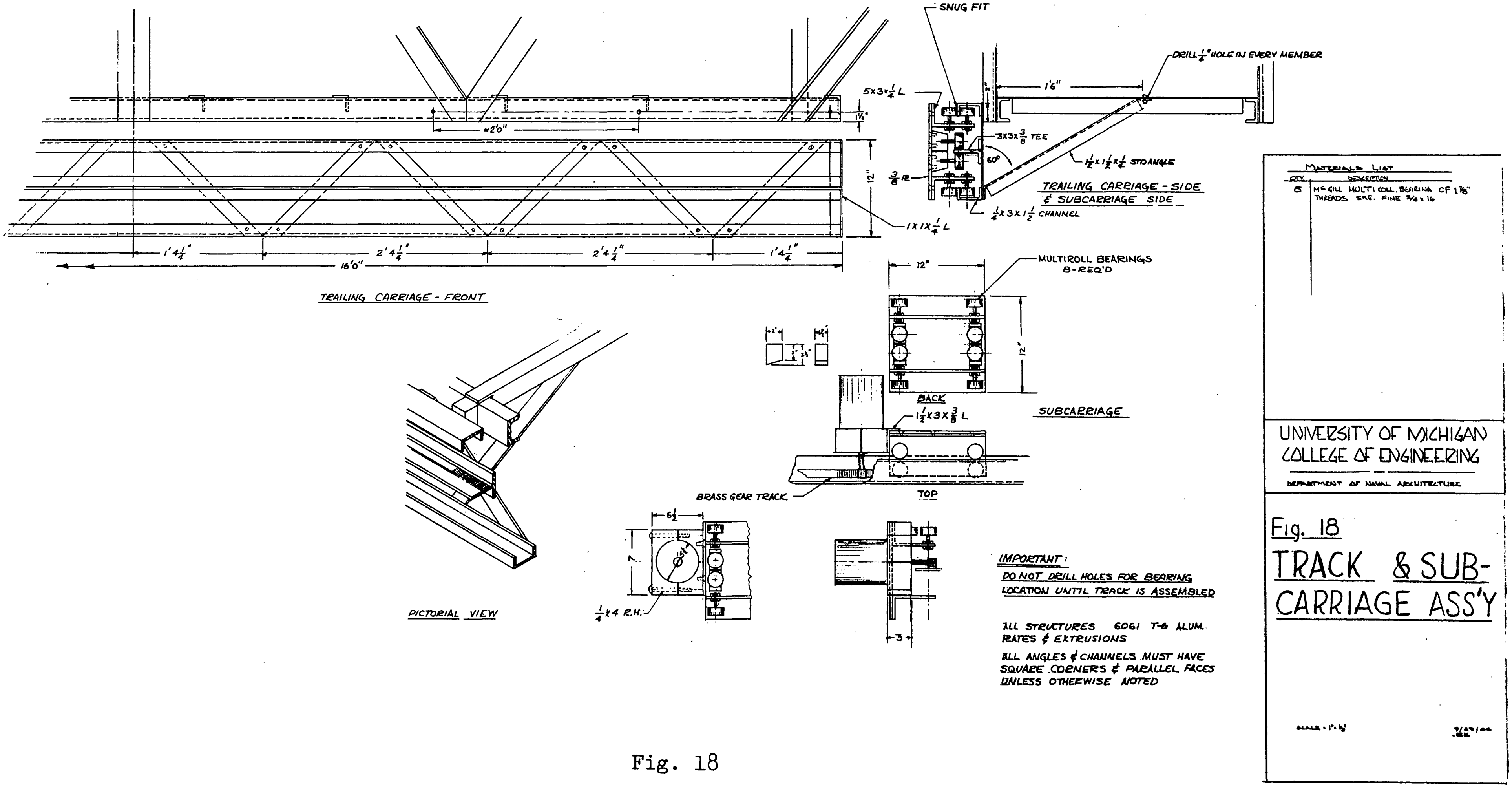

SNUG FIT
DRILL 1/4" HOLE IN EVERY MEMBER
1'6"
5x3x1/4 L
1 1/2"
3x3x3/8 TEE
60°
1 1/2 x 1 1/2 x 1/4 STD ANGLE
3/8 R
1/4 x 3 x 1 1/2 CHANNEL
TRAILING CARRIAGE - SIDE
& SUBCARRIAGE SIDE
≈2'0"
12"
1 x 1 x 1/4 L
1'4 1/4"
2'4 1/4"
2'4 1/4"
1'4 1/4"
16'0"
TRAILING CARRIAGE - FRONT
MULTIROLL BEARINGS
8-REQ'D
12"
12"
BACK
1 1/2 x 3 x 3/8 L
SUBCARRIAGE
TOP
BRASS GEAR TRACK
6 1/2
7
1/4 x 4 R.H.
3
PICTORIAL VIEW
IMPORTANT:
DO NOT DRILL HOLES FOR BEARING
LOCATION UNTIL TRACK IS ASSEMBLED
ALL STRUCTURES 6061 T-6 ALUM.
PLATES & EXTRUSIONS
ALL ANGLES & CHANNELS MUST HAVE
SQUARE CORNERS & PARALLEL FACES
UNLESS OTHERWISE NOTED
MATERIALS LIST
QTY.
DESCRIPTION
8
McGILL MULTI ROLL BEARING CF 1 7/8"
THREADS SAE. FINE 3/4 x 16
UNIVERSITY OF MICHIGAN
COLLEGE OF ENGINEERING
DEPARTMENT OF NAVAL ARCHITECTURE
Fig. 18
TRACK & SUB-
CARRIAGE ASS'Y

Fig. 18

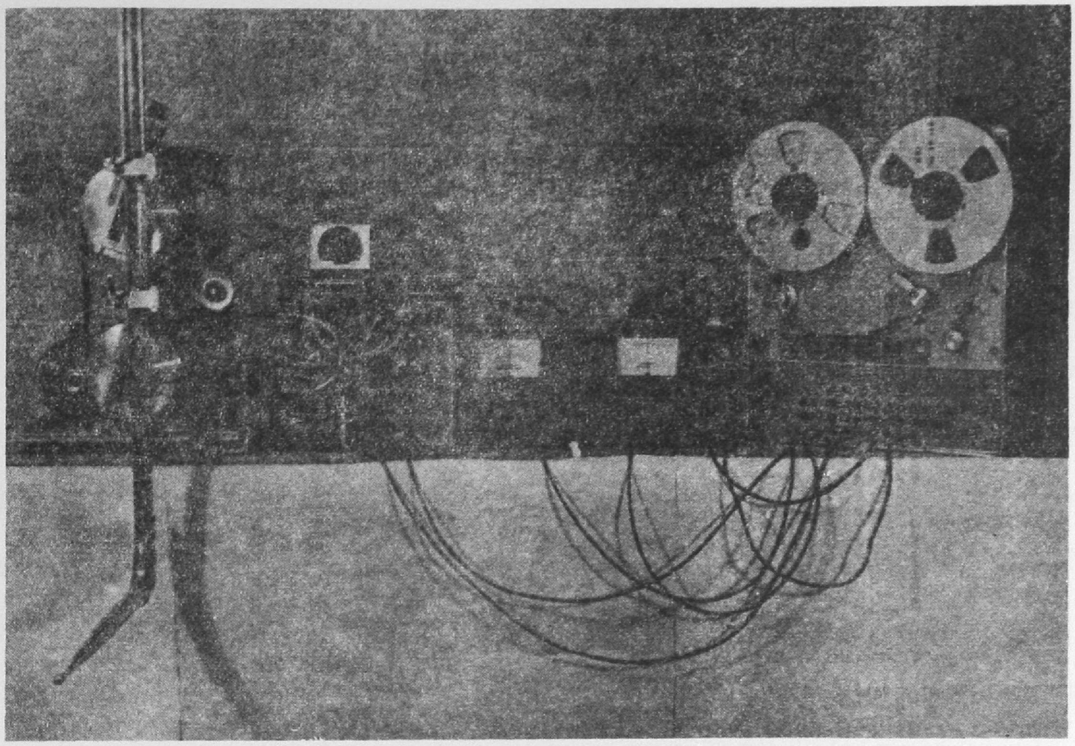


Fig. 19 Wake Survey Instrumentation.

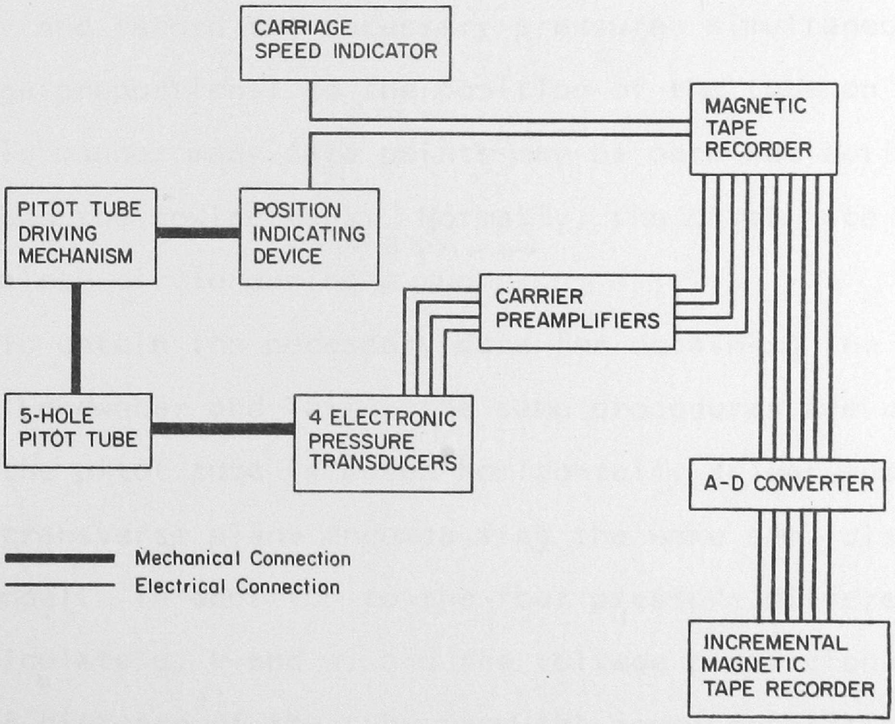


Fig. 20 Schematic of Wake Survey Instrumentation.

III. Data Acquisition and Handling

Before getting too far into the details of acquiring and handling wake survey data it should be made clear exactly what data must be recorded in order to make the subsequent analysis meaningful. To satisfy the basic requirements of the propeller designer, the three cartesian velocity components u, v and w must be known everywhere in the propeller disc area. Practically, this involves moving the pitot tube to a finite number of points in the disc area and recording four pressure differences at each point in the disc. Obviously, the points must be located such that the velocity components at any other point in the propeller plane may be found accurately by interpolation. The most convenient and fastest way to obtain this data is to move the pitot tube continuously in concentric circles with the propeller shaft as their center and record the necessary pressures simultaneously with a voltage proportional to the position of the tube on each circle. In this manner many data points may be obtained during a single run down the towing tank. Normally, the pitot tube can describe a complete circle during a run.

To obtain the necessary data for obtaining the viscous drag after Landweber and Tulin, the same procedures are used except that the pitot tube is moved horizontally at various fixed depths in a transverse plane encompassing the wake some distance aft of the model. In addition to the four pressure differences needed to calculate u, v and w, and the voltage proportional to the horizontal distance of the tube from the model centerline, the static pressure must also be recorded.

To facilitate the subsequent digitizing manipulation of the data recorded in the manner described above, calibration data is also recorded on tape at regular intervals during the testing program.

Upon completion of the tank test the magnetic tape containing all the necessary data in analog form is automatically converted to digital form. To perform the conversion, three pieces of equipment in addition to the SP-300 tape recorder are utilized: an AD-2-64PBC Analog Computer, a CD-8000 Analog to Digital Multiplexer, and a Control Data 160A computer. The data is played back into the analog computer, which multiplies each input voltage by ten. The A to D converter which samples each channel in sequence at a rate which the operator specifies and writes a digital number on a computer tape proportional to the voltage sampled, is directly linked to the analog computer.

The magnetic computer tape which is the end result of the analog to digital conversion contains all of the calibration data together with the actual data from the data runs but in a form that can only be read by the Control Data 160A Computer. Therefore, a short program is available entitled "Fortran Tape Rewrite" which reads the data from the original data tape, and merely rewrites it on another tape in a form compatible with the IBM 7090 tape handling equipment. This tape is then used as input to the MAD program "DATASORT" which averages the sampled calibration data, the zeros for both the calibration data and the data run which follows the calibration, and finally calculates X for each sampled data point according to Equation (8) below.

$$X = \frac{X\ \text{volts} - X_o\ \text{volts}}{CAL_{\text{volts}} - CALo_{\text{volts}}} \times \text{Scale Factor} \qquad (8)$$

where

X	is the value of a piece of data (in ft., psi, etc.)
X_{volts}	is the recorded voltage proportional to the piece of data
Xo_{volts}	is the recorded voltage proportional to the zero value of the piece of data
CAL_{volts}	is the recorded voltage proportional to the scale factor
$CALo_{\text{volts}}$	is the recorded voltage proportional to the zero during calibration
Scale Factor	is the fixed physical quantity to which the transducer is calibrated, (5 in. H_2O 4 ft., etc.) depending on the maximum value of the quantity during the data run.

This is done for all seven channels of the SP 300 recorder whether useful data is on all channels or not and the X's are either printed, punched, or written on a magnetic tape whichever is desired. The X's are in the correct physical units and arranged in consecutive data runs. The number of data points sampled during a given run is also punched, printed or written on tape before that data is punched, printed or written.

Listings of all the computer programs can be found in Appendix I. Step by step instructions for digitizing data and using "DATASORT" can be found in Reference 5. The output of the program "DATASORT" is then used as input to the program "PITSEAR", described previously, which calculates u, v and w for each sampled data point.

The output of "PITSEAR" is then in turn used as input to the program entitled "PWAKE". "PWAKE" is a plotting routine which plots u, v and w versus the distance in inches from the centerline of the model for various depths below the free surface. Some examples of the output of "PWAKE" are shown in Figures 26, 27 and 28. These figures are direct reproductions of the computer output except for some of the labeling.

A program has not as yet been written to plot propeller plane wake survey data but this is not difficult to do.

The program would plot in vector form the sum of the radial and tangental velocity at selected points and also contours of constant axial velocity.

IV. The Determination of the Viscous Drag by Means of the Wake Survey

Landwever and Wu(2) derived expressions for the viscous drag of surface ships based on the momentum theorem and continuity. In this report the expressions will be derived using a slightly different approach which illuminates the main assumption made by Landweber and Wu and hence shows that the viscous drag and the wavemaking drag are inseparable using only the momentum theorem.

Consider a ship model being towed down a relatively wide and deep towing tank at a constant speed U. We now enclose the model in a control volume moving with the model bounded by the tank walls on the sides and bottom, the free surface, a transverse plane far ahead of the model and a transverse plane fairly close behind the model. (see Figure 21).

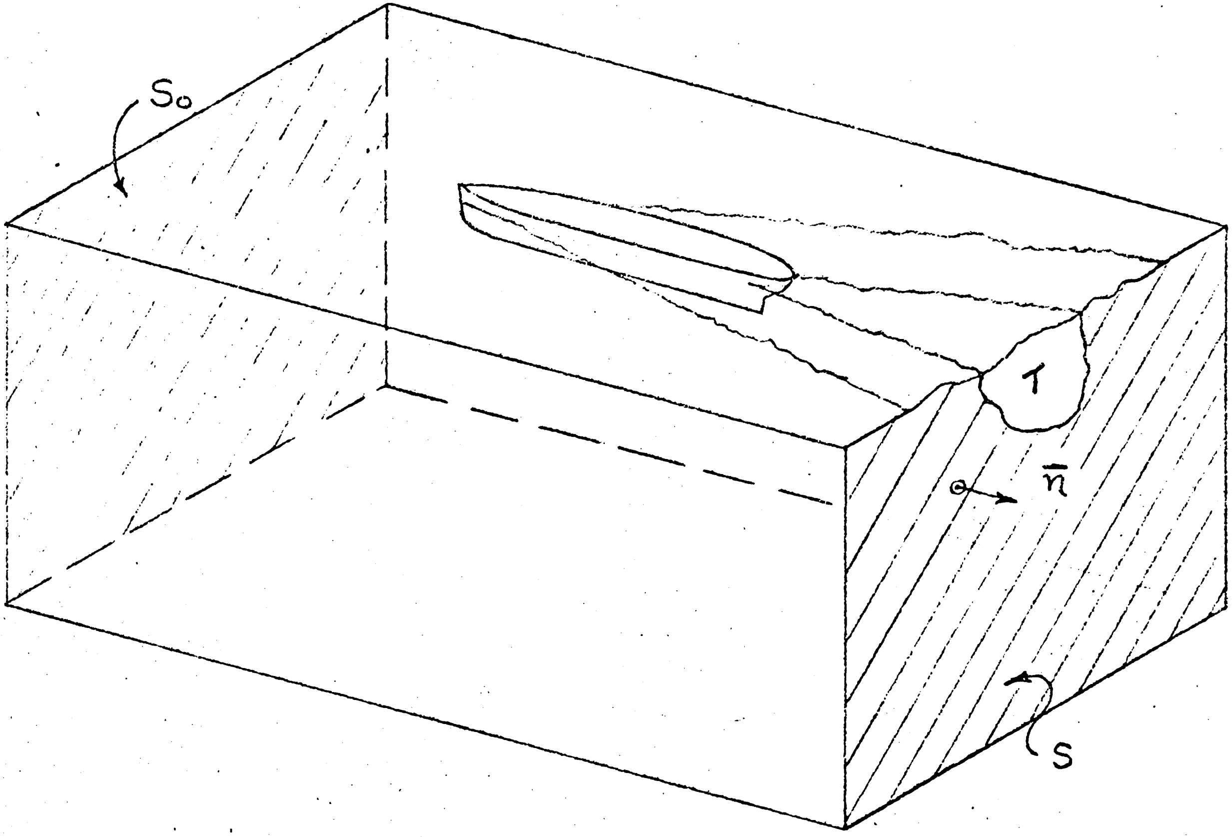


Figure 21. Momentum Control Volume

If we assume that the tank walls and bottom are far enough away from the model so that the water is not moving relative to the walls, the total drag force on the model can be written by means of the momentum equation:

$$D = \iint_{S_o} (p_o + \;U^2)\, dS_o - \iint_S (p+\rho u^2)\, dS \qquad (9)$$

If we now assume that the total drag is the sum of the wave-making drag and the viscous drag we can write

$$D = D_W + D_V \qquad (10)$$

In order to progress to an expression for the viscous drag alone we must, at this juncture, formulate a new problem. Consider now a potential flow in the finite region described above and shown in Figure (21). This flow is governed by the following conditions

$$\nabla^2\phi = 0 \quad \text{inside the region R (except at a finite number of points)} \qquad (11)$$

$$\frac{\partial \phi}{\partial n} = 0 \quad \text{on the free surface, sides and bottom} \qquad (12)$$

$$\frac{\partial \phi}{\partial n} = -U \quad \text{on the transverse plane in front of the model} \qquad (13)$$

$$\frac{\partial \phi}{\partial n} = u \quad \text{on the transverse plane behind the model everywhere } \underline{\text{outside}} \text{ the wake area T} \qquad (14)$$

$$R'_W = R_W \qquad (15)$$

The condition that the wave resistance of the potential flow be the same as the wave resistance of the real flow is necessary to insure that the potential flow problem has a unique solution since the boundary condition $\frac{\partial \phi}{\partial n} = u$ on the rear boundary can not be specified over the entire boundary due to the fact that the real flow is not potential inside the wake area.

Writing the momentum equation in the direction of flow for the potential flow problem we get

$$D' = \iint_{S_o} (p_o + \rho U^2)\, dS_o - \iint_S (p_1 + \rho u_1^2)\, dS \qquad (16)$$

D', however, has no viscous component since it results from the application of the momentum equation to a potential flow problem. It is made up of the wavemaking drag D'_W (which is the same as D_W) and a force called D_S which arises in order to satisfy boundary condition (14). That is, in order for the flow to be potential in the region R it cannot satisfy continuity. Hence, the total strength of the source sink distribution in R cannot add up to zero.

Since the real flow satisfies continuity the excess outflow can be written

$$\iint_S (u_1 - u)\, dS$$

and the force due to this excess outflow is then

$$D_S = -\rho U \iint_S (u_1 - u)\, dS \qquad (17)$$

If we now substract (16) from (9) and add (13) we get

$$D_V + D_W - D_W - D_S + D_S = D_V = \iint_S p_1 - p + \rho\left[(u_1^2 - u^2) - U(u_1 - u)\right] dS \qquad (18)$$

Now since p_1 is the pressure in an ideal flow we can apply Bernoulli's equation to get

$$p_1 = p_o + \tfrac{1}{2}\rho\left[(U^2 - u_1^2) - v_1^2 - w_1^2\right] \qquad (19)$$

Putting (19) into (18) we obtain

$$D_V = \iint_S p_o - p + \rho u(U - u) + \tfrac{1}{2}\rho\left[(U - u_1)^2 - v_1^2 - w_1^2\right] dS \qquad (20)$$

The integrand in (20) disappears outside the wake area T and therefore we need only integrate (20) inside the wake.

At this point it will be useful to derive the expression for the wavemaking drag.

$$D_W = D - D_V = \iint_{S_o} (p_O + \rho U^2) dS_O - \iint_S (p + \rho u^2) dS - \iint_S p_O - p + \rho u(U-u) + \tfrac{1}{2}\rho \left[(U-u_1)^2 - v_1^2 - w_1^2 \right] dS \qquad (21)$$

using the continuity equation in the form

$$\iint_{S_o} U \, dS_O = \iint_S u dS \qquad (22)$$

and the fact that the surface S differs from the surface S_O by the amount

$$S - S_O = \iint_{S-S_o} dzdy \qquad (23)$$

we get

$$D_W = \rho/_2 \iint_S \left[v_1^2 + w_1^2 - (U-u)^2 \right] dS + \frac{\gamma}{2} \int_{-b}^{b} \zeta^2 \, dy \quad (24)$$

where ζ is the free surface elevation in the plane S.

The expression for D_W in (24) depends on u, v and w just a does the expression for D_V. But as we have seen before u, v and w are obtained as solutions to a boundary value problem with the boundary conditions (11), (12), (13), (14) and (15). Condition (14) requires that we somehow specify a wave resistance before we get a solution. When we then find the singularity distribution satisfying the required conditions (which is no easy task) we can calculate u, v and w, substitute them into (24) and we will necessarily get the same D_W that we assumed at the start, since (24) is merely an application of the momentum equation. Using the same values of u, v and w in (20) the resulting viscous drag will merely be the total drag minus the assumed wavemaking drag.

As a practical matter the boundary value problem is much too difficult to solve. Actually it is unnecessary to solve it anyway since D_W must be assumed at the outset. It is much simpler to merely extrapolate the value of u, v and w into the wake area. In effect, this is the same as assuming a value of the wave resistance and depending on the method of extrapolation any value of the apparent viscous resistance may be calculated.

In conclusion, it can be stated that the wake survey method is not an independent method for determining the viscous drag of surface ships since it is based only on momentum considerations and relies on the assumption of the correct wavemaking drag for its success.

V. Experimental Wake Survey Results

Various wake surveys have been performed using the instrumentation described in this report. Surveys were performed on a 14 foot Series 60 model with a block coefficient of .75 at a distance of 2.25 ft. behind the model at a model speed of 4.2'/sec. The velocity components u, v and w and the static pressure versus the distance from the centerplane at various depths are plotted in Figures (22) - (25). Using this data the viscous drag was calculated from the following expressions:

$$D_{v_1} = \iint_{\text{wake}} \left[p_o - p + \rho u (U-u) \right] dS \qquad (25)$$

$$D_{v_2} = \iint_{\text{wake}} \left[p_o - p + \rho u (U-u) + \tfrac{1}{2} \rho \, (U-u_1)^2 \right] dS \qquad (26)$$

$$D_{v_3} = \iint_{\text{wake}} \left[p_o - p + \rho u (U-u) + \tfrac{1}{2} \rho \, (U-u_2)^2 \right] dS \qquad (27)$$

where

$$p + \tfrac{1}{2}\rho u_2^2 = p_o + \tfrac{1}{2}\rho U^2 \qquad (28)$$

$$D_{v4} = \iint \text{ wake } \left(\rho(u_1^2 - u^2) - \rho U(u_1 - u)\right) \; dS \qquad (29)$$

$$D_{v5} = \iint \text{ wake } \left(p_o - p + \rho u(U-u) + \tfrac{1}{2}\rho\{(U-u_1) - v_1^2 - w_1\}\right) \qquad (30)$$

Equations (25) and (26) were given by Landweber (2), equation (27) is the approximation given by Betz(2), equation (29) is nearly the same as the one used by Wu(3), and equation (30) is the "exact" solution providing the correct values of u_1, v_1 and w_1 are chosen. Notice that the other expressions are arrived at by essentially assuming values for u_1, v_1 and w_1 and thus D_w.

Unfortunately, the static pressure measurements were not accurate enough to permit the calculation of D_v. More surveys must be made.

As a last example, results of a wake survey taken behind a model of the Mohole Drilling Platform are shown in Figures (26), (27) and (28). This survey was taken in the propeller plane of the model but was carried out using horizontal cuts instead of moving the pitot tube in concentric circles about the center of the propeller shaft.

FIGURE 22

U/UO vs Y

FOR VARIOUS VALUES OF Z

WAKE SURVEY-14FT. SERIES 60..75BLOCK MODEL000
UO=4.20 FT/SEC
CUT TAKEN 2.25FT BEHIND MODEL

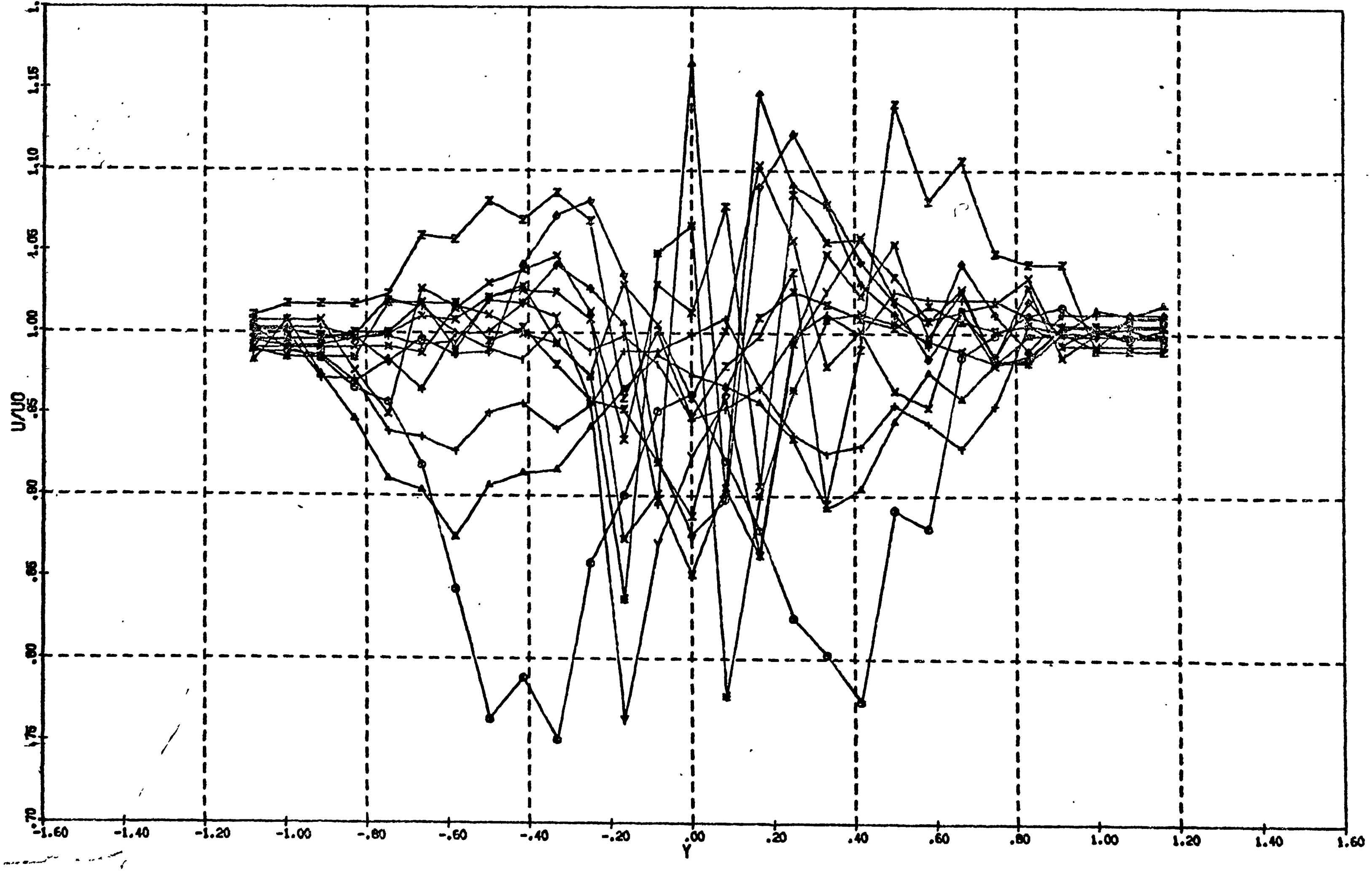


FIGURE 23

V/UO vs Y

FOR VARIOUS VALUES OF Z

WAKE SURVEY-14FT. SERIES 60..75BLOCK MODELOOO
UO=4.20 FT/SEC
CUT TAKEN 2.25FT BEHIND MODEL

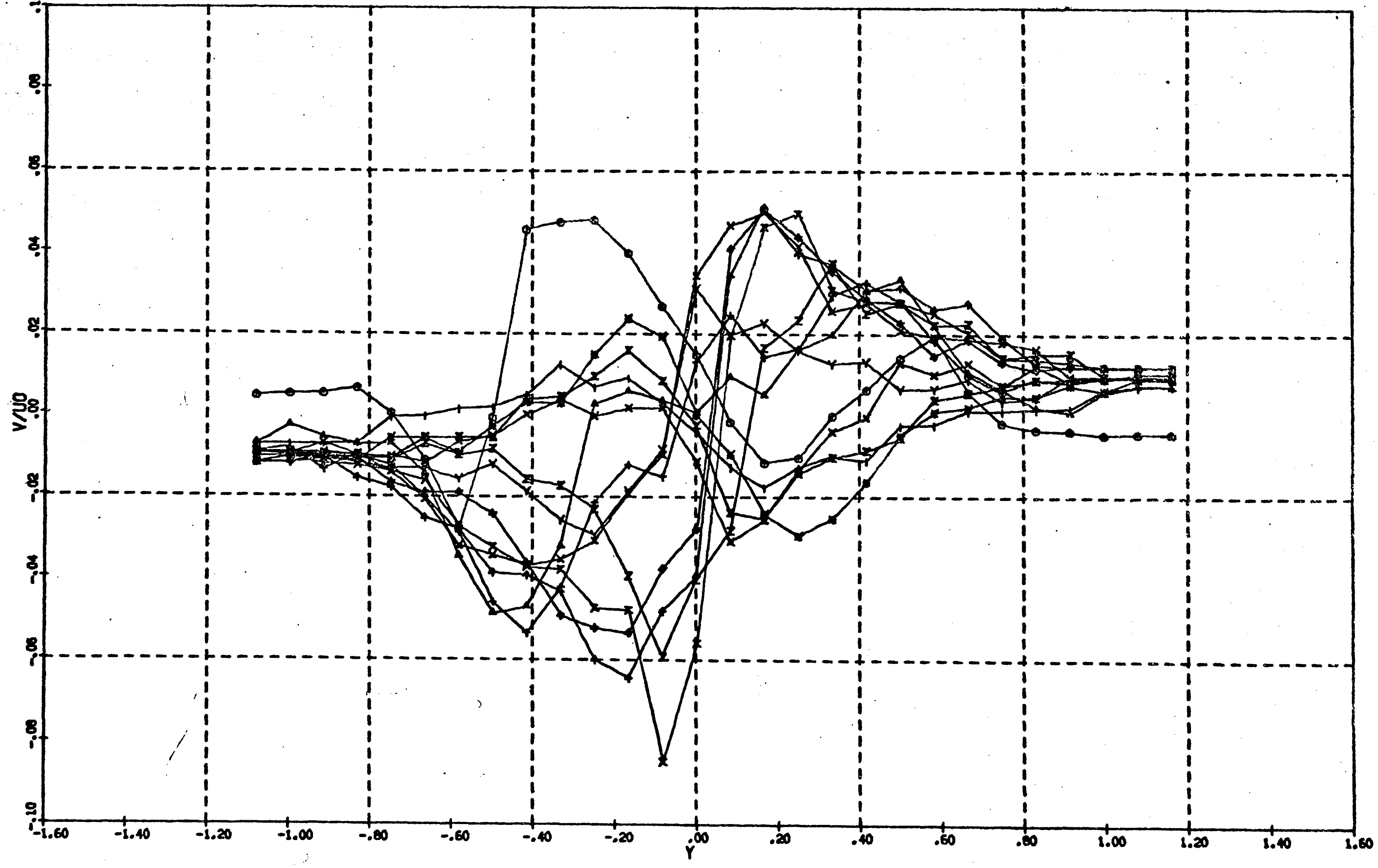


FIGURE 24

W/UO vs Y

FOR VARIOUS VALUES OF Z

WAKE SURVEY-14FT. SERIES 60..75BLOCK MODEL000
UO=4.20 FT/SEC
CUT TAKEN 2.25FT BEHIND MODEL

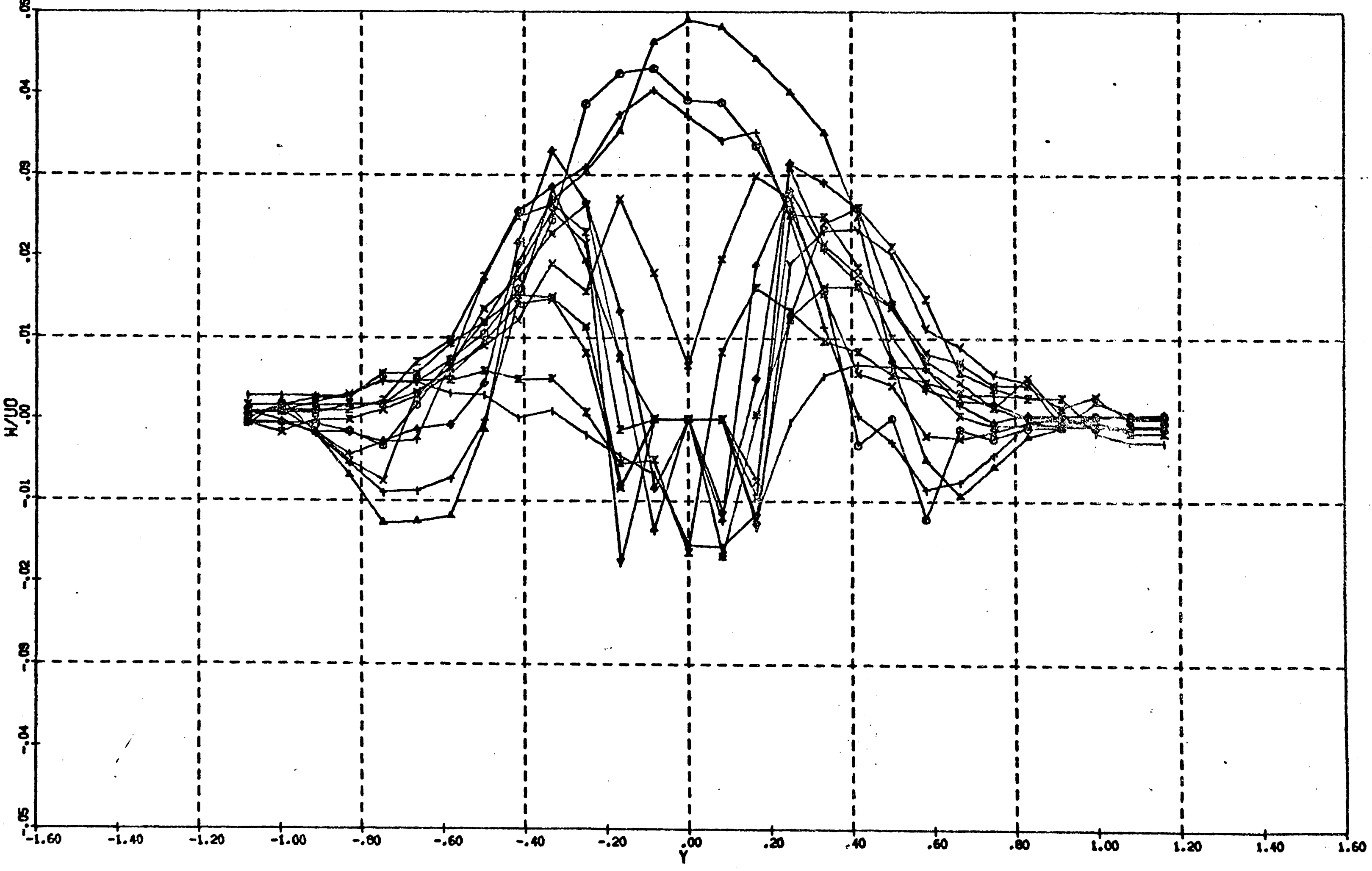
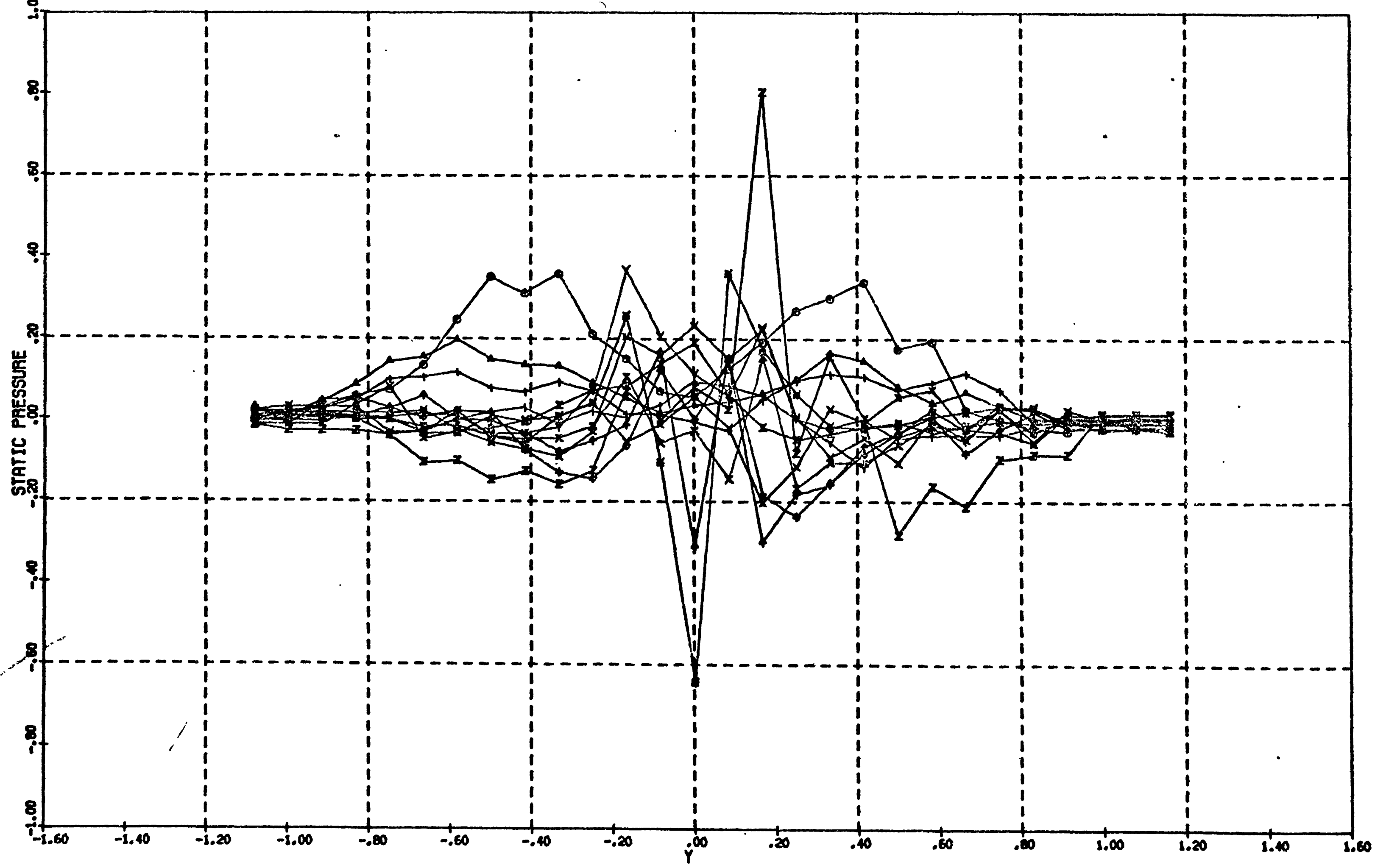


FIGURE 25

STATIC PRESSURE vs Y

FOR VARIOUS VALUES OF Z

WAKE SURVEY-14FT. SERIES 60..75BLOCK MODEL000
U0=4.20 FT/SEC
CUT TAKEN 2.25FT BEHIND MODEL



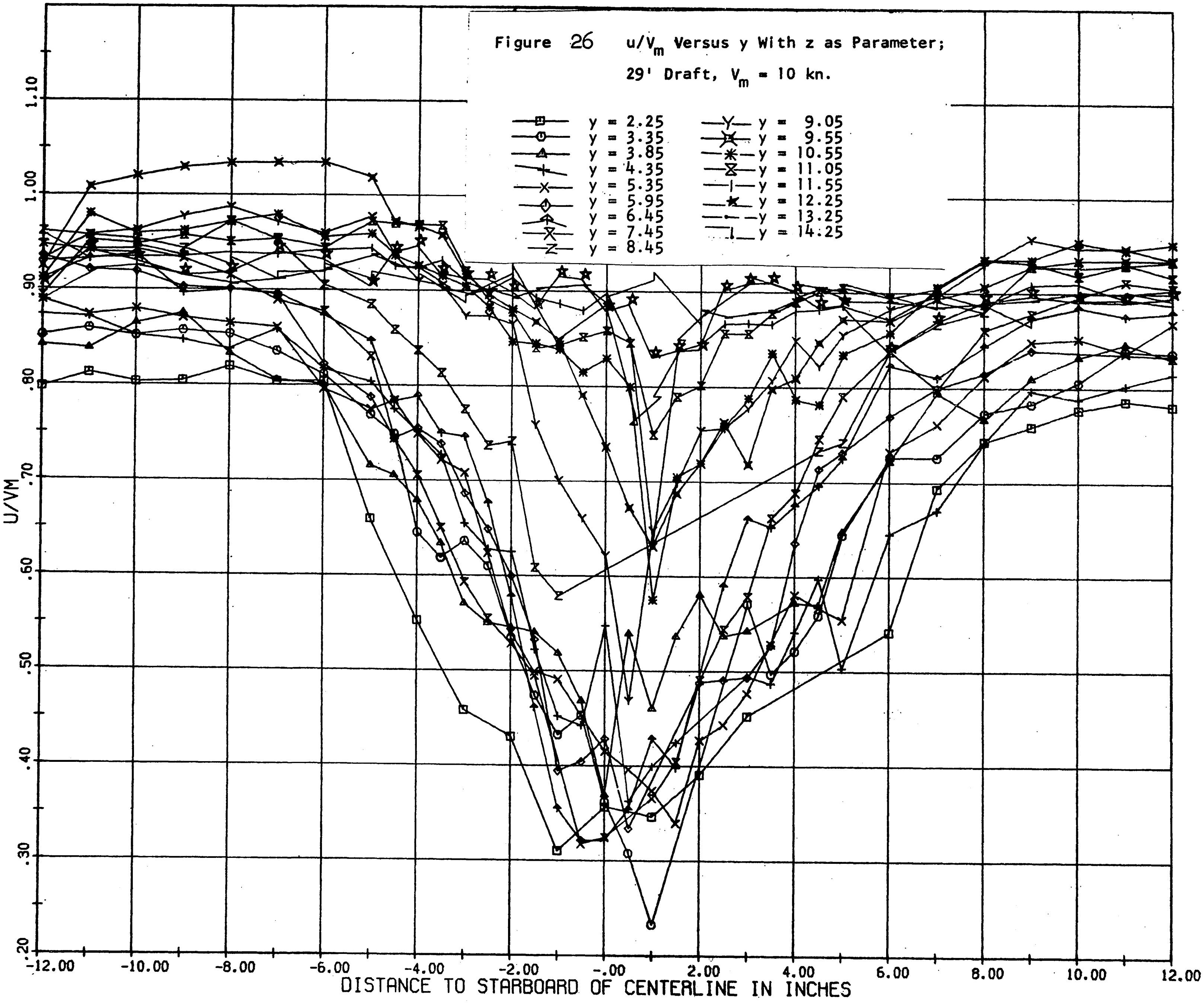


Figure 26 u/V_m Versus y With z as Parameter; 29' Draft, V_m = 10 kn.

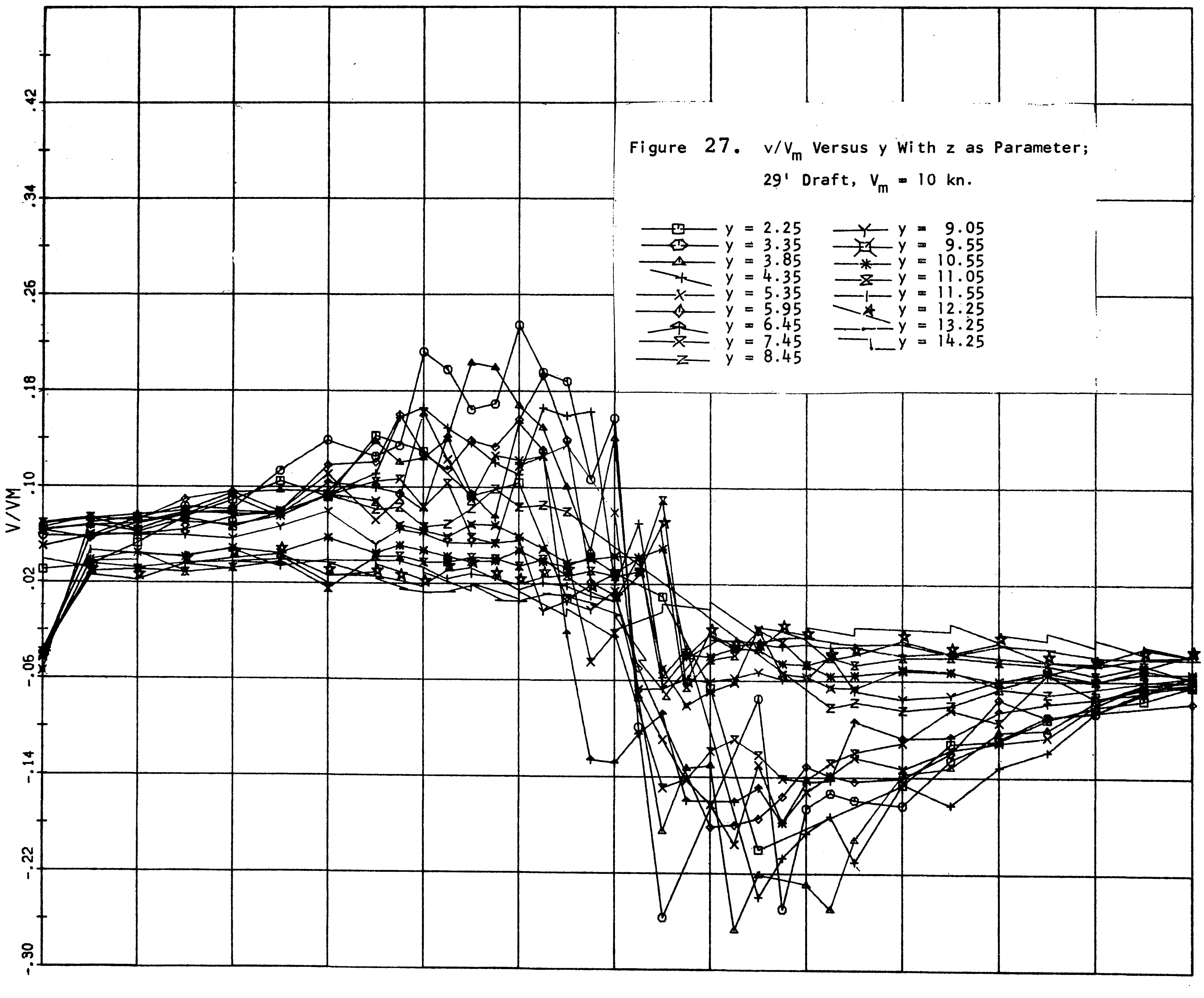


Figure 27. v/V_m Versus y With z as Parameter; 29' Draft, V_m = 10 kn.

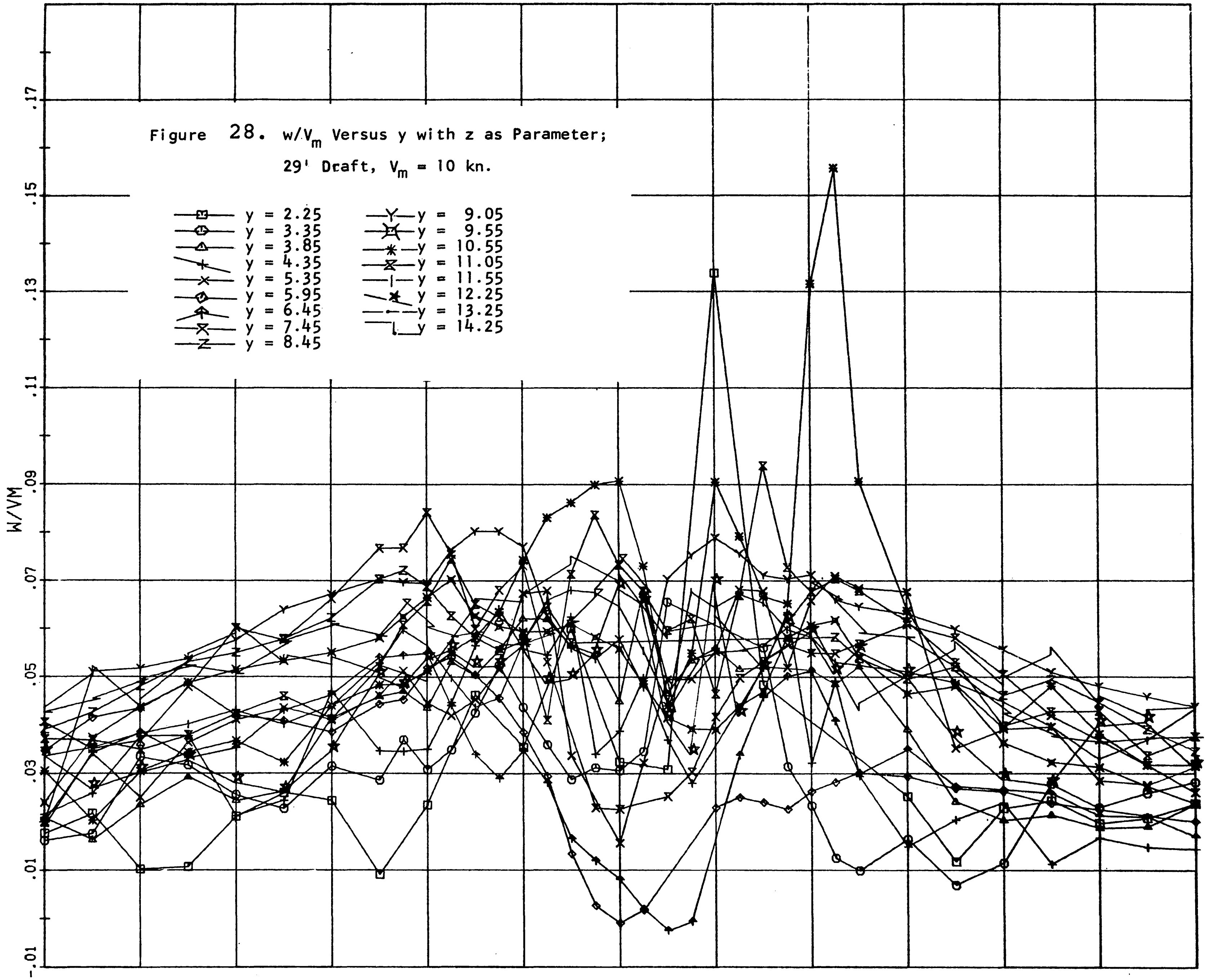


Figure 28. w/V_m Versus y with z as Parameter;
29' Draft, V_m = 10 kn.

References

1. Tulin, Marshal P., "The Separation of Viscous Drag and Wave Drag by Means of the Wake Survey", DTMB Report 772, July 1951.
2. Landweber, L. and Wu, J., "The Determination of the Viscous Drag of Submerged and Floating Bodies by Wake Surveys", Journal of Ship Research, Vol. 7, No. 1, June 1963.
3. Wu, J., "The Separation of Viscous Form Wave-Making Drag of Ship Forms", Journal of Ship Research, Vol. 6, No. 1, June 1962.
4. Pien, P. C.,"The Five-Hole Spherical Pitot Tube", DTMB Report No. 1229, May 1958.
5. Westervelt, Franklin H., "A Study of Automatic System Simulation Programming and the Analysis of Behavior of Physical Systems Using an Internally Stored Program Computer", Thesis, University of Michigan Industry Program, IP 470, October 1960.
6. Gebhardt, John C. and Michelsen, Finn C., "High Response Pitot Tube Techniques for Measurements of Time and Space Variations of the Velocity Distribution in the Wake of a Ship Model", University of Michigan Ship Hydrodynamics Laboratory Report presented at United States-Japanese Seminar on Instrumentation for Hydraulic Research, Tokyo, April 1965.
7. Gebhardt, John C., Michelsen, Finn C., and Moss, James L., "Experimental Techniques for Determination of Viscous Resistance Using the High Response Five-Hole Pitot Tube", University of Michigan Report presented at 14th A.T.T.C., Webb Institute of Naval Architecture, September 1965.
8. Gebhardt, John C., "The Direct Conversion of Analog Data to a Form Usable by the IBM 7090 Computer", University of Michigan Report, May 1966.
9. Gebhardt, John C. and Michelsen, Finn C., Project Mohole Performance Tests", University of Michigan Report 07833-I-F, July 1966.

Appendix I: Computer Programs

```
                    rem                analog to fortran binary
      0000          bnk0
      0000          con      0
0000  7101          jfi      1
0001  0173                   setup
0002  0000  recnr                      record counter
0003  0000  wctr                       words per record
0004  0000  sw                         bank switch
0005  0000  high                       high order word
0006  0000  low                        low order word
0007  0000  ctr                        channel counter
                    rem                interrupt 10 to terminate file
      0011          org      11
0011  2200  end     ldf      0
0012  7016          jpi      end3      set up exit
0013  4260          stf      clear
0014  7101          jfi      1
0015  0304                   wend
0016  0320  end3             wen3
0017  0000  rcnrl
                    rem                ignore interrupt 20
      0021          org      21
0021  0400          ldn
0022  4025          std      eofflg
0023  0120          cil
0024  6600          pjb      0
0025  0000  eofflg
0026  0000  joeoff
                    rem                interrupt 30 for read
      0031          org      31
0031  6202  int30   pjf      2
0032  0000          err                timing trap
0033  7500          exc      2515      clear 8000
0034  2515
0035  7600          ina
0036  7500          exc      2564
0037  2564
0040  0503  chan    lcn      3         ----preset by setup----
0041  4007          std      ctr
0042  7600  loop    ina                read analog
0043  0600          adn      0
0044  4106          sti      low
0045  6203          pjf      3
0046  0500          lcn      0
0047  6302          njf      2
0050  0400          ldn      0
0051  4105          sti      high      high order is 0 or 7777
0052  0403          ldn      3
0053  5006          rad      low       update addresses
0054  0403          ldn      3
0055  5005          rad      high
0056  5407          aod      ctr       check channel counter
0057  6515          nzb      loop
0060  5403          aod      wctr      check word counter
0061  6016          zjf      write
0062  2025          ldd      eofflg
0063  6010          zjf      clear
0064  7500  wteof   exc      1113
0065  1113
0066  2200          ldc      400
```

At halt at loc 205 enter into "A" reg in octal number of channels desired converted.

Do not use jump switches 1 or 2 during operation except to generate an interrupt 10 to end program.

interrupt 10 is generated by depressing any one jump switch and any one stop switch at the same time.

program will write an EOF and halt showing in "A" reg number of records written - restart program by cycling run switch

```
0067    0400
0070    4021            std     21
0071    0400            ldn
0072    4025            std     eofflg
0073    0120    clear   cil                         remove lockout
0074    0501            lcn     1                   set trap
0075    0401            ldn     1                   remove trap
0076    6600            pjb     0                   wait
0077    7710    write   slj1    ustu3               select tape write
0100    0157
0101    2200            ldc     2112
0102    2112
0103    4211            stf     tsfw    1
0104    2200            ldc     1113
0105    1113
0106    4321            stb     wteof   1
0107    2200            ldc     1112
0110    1112
0111    4100            stm     wen3    3
0112    0323
0113    7500    tsfw    exc     2112
0114    2112
0115    4404            srd     sw
0116    6204            pjf     4                   set bank controls
0117    0020            sic0
0120    0141            sbu1
0121    6303            njf     3
0122    0021            sic1
0123    0140            sbu0
0124    0514    word    lcn     12d                 ----preset by setup----
0125    4003            std     wctr
0126    2200    wra     ldc     record              set ber
0127    0333
0130    0105            ate     wra
0131    0126
0132    0601            adn     1
0133    4005            std     high                set high and low
0134    0601            adn     1
0135    4006            std     low
0136    7300    wrb     ibo     wrb                 start write
0137    0136
0140    7740            slj4    begeof              write end of file
0141    0150
0142    0400            ldn
0143    4026            std     joeoff
0144    5402    brc     aod     recnr
0145    6552            nzb     clear               go back to clear
0146    5417            aod     rcnr1
0147    6554            nzb     clear
0150    2026    begeof  ldd     joeoff
0151    6505            nzb     brc
0152    2200            ldc     401
0153    0401
0154    4021            std     21
0155    4026            std     joeoff
0156    6612            pjb     brc
0157    2200    ustu3   ldc     1112
0160    1112
0161    4065            std     wteof   1
0162    2200            ldc     1113
```

```
0163    1113
0164    4100              stm     wcr3    3
0165    0323
0166    2200              ldc     2113
0167    2113
0170    4354              stb     tsfw    1
0171    7101              jfi     1
0172    0113                      tsfw
                          rem                       initializing routine
0173    0060    setup     sid0
0174    2200              ldc     2525
0175    2525
0176    4004              std     sw
0177    2200              ldf     0
0200    0120              cil
0201    4073              std     clear
0202    0400              ldn     0
0203    4026              std     joeoff
0204    4025              std     eofflg
0205    7700              hlt                       enter number of channels
0206    6404              zjb     4                 0 thru 8
0207    0710              sbn     10                check if less than 9
0210    6002              zjf     2
0211    6607              pjb     7
0212    3200              adc     510               generate lcn command
0213    0510
0214    4100              stm     chan
0215    0040
0216    3600              sbc     500
0217    0500
0220    4002              std     recnr
0221    0450              ldn     50
0222    4017              std     rcnrl
0223    5426              aod     joeoff
0224    2017              ldd     rcnrl
0225    3402              sbd     recnr
0226    4017              std     rcnrl
0227    6604              pjb     4
0230    2026              ldd     joeoff
0231    0701              sbn     1                 number of passes per record
0232    7701              slsl
0233    3200              adc     500               the product of these two numbers
0234    0500
0235    4100              stm     word              must be less than 40d
0236    0124
0237    4201              stf     1
0240    0514              lcn     14
0241    4003              std     wctr
0242    0437              ldn     37                mandatory first word
0243    4100              stm     record
0244    0333
0245    0021              sicl
0246    4100              stm     record
0247    0333
0250    2200              ldc     record  123d
0251    0526
0252    0106              atx     setup
0253    0173
0254    2200              ldc     record  1
0255    0334
```

```
0256   4005            std    high
0257   0601            adn    1
0260   4006            std    low
0261   0400            ldn
0262   4002            std    recnr
0263   4017            std    rcnrl
0264   4026            std    joeoff
0265   7500            exc    1142
0266   1142
0267   7600            ina
0270   0240            lpn    40
0271   6003            zjf    3
0272   7500            exc    1112
0273   1112
0274   7500            exc    1171
0275   1171
0276   7500            exc    2515
0277   2515
0300   7600            ina
0301   0401            ldn    1
0302   0120            cil
0303   6600            pjb
0304   2005    wend    ldd    high              test if record empty
0305   3600            sbc    record  1
0306   0334
0307   6011            zjf    wen3
0310   0400    wenl    ldn    0                 no then fill with zeros
0311   4105            sti    high
0312   5405            aod    high
0313   3600            sbc    record  122d
0314   0525
0315   6505            nzb    wenl
0316   7101    wen2    jfi    1
0317   0077                   write
0320   0105    wen3    ate    wen3              wait till buffer finished
0321   0320
0322   7500            exc    1112              write file mark
0323   1112
0324   2017            ldd    rcnrl             if wrote more than 4095 records will halt
0325   6002            zjf    2                 and show multiples of 4095 record groups writ
0326   7700            hlt
0327   2002            ldd    recnr
0330   7700            hlt
0331   7101            jfi    1
0332   0173                   setup
0333   0000    record  blr    121d
       0000            end
```

```
c       program to rewrite fortran binary data on BCD tape
c
c
1       pause1
        dimension n(35)
300     read tape 2,(N(i),i=1,35)
        if(xeof(1))100,200,100
200     write output tape 1,400,n
        go to 300
100     endfile 1
        go to 300
400     format(21i5/14i5)
        stop
        end
```

Write Fortran test tape

1-5-65

```
1          pause 1
           endfile 2
           ntab=0
199        ncount=1
           ntab=ntab+1
           dimension m(40),N(40),l(40),mm(40),nn(40),ll(40)
           do 100,i=1,36
           m(i)=1500
100        n(i)=100
           do 101,i=1,6
           mm(i)=1500
101        nn(i)=100
           do 102 ,i=7,36
           mm(i)=0
102        nn(i)=0
           do 103,i=1,36
103        l(i)=1000
           do 104,i=1,12
104        ll(i)=1000
           do 105,i=13,36
105        ll(i)=0
200        go to (201,202,202,201,202,201,203,300)ncount
201        write tape 2,(n(i),i=1,36)
           write tape 2,(nn(i),i=1,36)
           end file 2
           ncount=ncount+1
           go to 200
202        write tape 2,(m(i),i=1,36)
           write tape 2,(mm(i),i=1,36)
           end file 2
           ncount=ncount+1
           go to 200
203        do 204,j=1,5
204        write tape 2,(l(i),i=1,36)
           write tape 2,(ll(i),i=1,36)
           end file 2
           end file 2
           ncount=ncount+1
           pause2
           go to 199
300        if(ntab-3)199,199,301
301        stop
           end
```

```
                 THIS PROGRAM PLOTS THE OUTPUT OF PITSEAR
                 ON THE CALCOMP PLOTTER.

              DIMENSION U(600),V(600),W(600),YDIS(600),CAT( 50),ZDIS(20),
          1 ),AXISY(5)                                                  AXISX(5
             INTEGER I,CAT,J ,II,CATT
             VM=3.22
             THROUGH S1, FOR I=1,1,I.G.573
             READ FORMAT DATA, U(I),V(I),W(I)
             U(I)=U(I)/VM
             V(I)=V(I)/VM
   S1        W(I)=W(I)/VM
              THROUGH S4, FOR I=  0,1,I.G.31
   S4         READ FORMAT DATA1,YDIS((18*I)+1)...YDIS((18*I)+18)
               PRINT RESULTS YDIS(1)...YDIS(37)
             J=1
             THROUGH S3, FOR I=1,1,I.G.573
              WHENEVER .ABS.(YDIS(I)+12.).L..01
             CAT(J)=I
             J=J+1
             OTHERWISE
             CONTINUE
             END OF CONDITIONAL
   S3        YDIS(I)=-YDIS(I)
             CAT(19)=574
             PRINT RESULTS CAT(1)...CAT(25)
             VECTOR VALUES VV=$V/VM$
             VECTOR VALUES WW=$W/VM$
             VECTOR VALUES Y=$DISTANCE TO STARBOARD OF CENTERLINE IN INCHES$
             VECTOR VALUES UU=$U/VM$
             VECTOR VALUES DATA=$3F10.4*$
             VECTOR VALUES DATA1= $18F4.1*$
             VECTOR VALUES ZDIS(1)=2.25,3.35,3.85,4.35,5.35,5.95,6.45,7.45,
          2 ,9.55,10.55,11.05,11.55,12.25,13.25,14.25               8.45,9.05
             VECTOR VALUES AXISX(1)=1.0,13.0
             PLTXMX.(20.0)
             PGRID.(1.0,1.0,12.0,16.0,1,1)
             PSCALE.(12.0,.5,YMIN,DY,YDIS(1),50,1)
             PSCALE.(4.0,.5,VMIN,DV,V(1),573,1)
             PAXIS.(1.0,1.0,Y,-45,12.0,0.0,YMIN,DY,.5)
             PSYMB.(.8,7.0,.15,$V/VM$,90.0,4)
             THROUGH R2,FOR I=1,1,I.G.17
             ZDIS(I)=17.0-ZDIS(I)
             AXIS Y(1)=ZDIS(I)
             AXIS Y(2)=ZDIS(I)
             PLINE.(AXISX(1),AXISY(1),2,1,0,1,0.)
             CAT T=CAT(I+1)-CAT(I)
             II=I-1
             PLTOFS.(YMIN,DY,0.   ,DV,1.0,ZDIS(I))
   R2        PLINE.(YDIS(CAT(I)),V(CAT(I)),CATT,1,1,II,1.)
             PLTEND.
             PLTXMX.(20.0)
```

```
                    PGRID.(1.0,1.0,12.0,16.0,1,1)
                    PSCALE.(12.0,.5,YMIN,DY,YDIS(1),50,1)
                    PSCALE.(4.0,.5,WMIN,DW,W(1),573,1)
                    PAXIS.(1.0,1.0,Y,-45,12.0,0.0,YMIN,DY,.5)
                    PSYMB.(.8,7.0,.15,$W/VM$,90.0,4)
                    THROUGH R3,FOR I=1,1,I.G.17
                    AXIS Y(1)=ZDIS(I)
                    AXIS Y(2)=ZDIS(I)
                    PLINE.(AXISX(1),AXISY(1),2,1,0,1,0.)
                    CAT T=CAT(I+1)-CAT(I)
                    II=I-1
                    PLTOFS.(YMIN,DY,0.  ,DW,1.0,ZDIS(I))
        R3          PLINE.(YDIS(CAT(I)),W(CAT(I)),CATT,1,1,II,1.)
                    PLTEND.
                     END OF PROGRAM

THE FOLLOWING NAMES HAVE OCCURRED ONLY ONCE IN THIS PROGRAM.
COMPILATION WILL CONTINUE.

        VMIN           *034
        WMIN           *050
```

```
 $COMPILE MAD, PRINT OBJECT, PUNCH OBJECT

MAD
(02 NOV 1966 VERSION) PROGRAM LISTING ... ... ...


                TFS=0
                TES=1
                EXECUTE FTRAP.
                WHENEVER KCOUNT .E.3,UNLOAD TAPE 3
                PAUSE NO. 1
                SET LOW DENSITY TAPE 2
                SET LOW DENSITY TAPE 3
                REWIND TAPE  2
                REWIND TAPE  3
                NUM=0
                INTEGER CAL,I,J,A,N,MCOUNT,N1, V,TES,ZCOUNT,L,LL,M,NUM,KCOUNT
              1 ,CH,ZOO,CHI1,CHI,CHECK,QCOUNT
              1 ,ZAT
                DIMENSION N(40),M1Z(99),M2Z(99),M3Z(99),M4Z(99),M5Z(99),M6Z(99),M7Z
              1 (99),M1X(99),M2X(99),M3X(99),M4X(99),M5X(99),M6X(99),M7X(99),
              1      M1C(99),M2C(99),M3C(99),M4C(99),M5C(99),M6C(99),M7C(99),
              1 M1(2000),M2(2000),M3(2000),M4(2000),M5(2000),M6(2000),M7(2000
              1 ),A((1...10)*(1...7)),V(50)
              1 ,VM(500)
              1 ,CHECK(60)
                SETDIM.(A,10,7)
                SETPRT.(1)
                SETERR.(T1,ERR)
                TRANSFER TO T9
 T1             PRINT FORMAT OUT,ERR
                VECTOR VALUESOUT=$85H ERROR ENCOUNTERED DURING READ. THE NEXT
              1 MAY BE IN ERROR. THE ERROR NUMBER =,I4*$                FEW DATA
                EXECUTE SKIP.(0,1,2)
                WHENEVER GOTO.E.1,TRANSFER TO S4
                WHENEVER GOTO.E.2,TRANSFER TO T2
                TRANSFER TO T3
 T9             READ FORMAT D,CAL,CAL1,CAL2,CAL3,CAL4,CAL5,CAL6,CAL7,KCOUNT,Q
              1 COUNT
                VECTOR VALUES D=$I2,7F6.3,S3,I1,S3,I1*$
                THROUGH R1,FORI=1,1,I.G.10
                THROUGH R1,FORJ=1,1,J.G.7
 R1             A(I,J)=0
                THROUGH S6,FOR I=1,1,I.G.7
 S6             READ FORMAT D1,A(I,1)...A(I,7)
                VECTOR VALUES D1=$7I1*$
                EXECUTE SKIP.(1,0,2)
                MCOUNT=1
                VECTOR VALUES D2=$21I5/14I5*$
                THROUGH S22,FOR M=1,1,M.G.50
                CHECK(M)=0
 S22            CONTINUE
                THROUGH S23, FOR M=1,1,A(M,1).E.0
 S23            CONTINUE
                CHECK(2*M)=1
                CHI=2*(M+CAL-1)
                THROUGH S24,FOR I=2*M,2,I.G.CHI
 S24            CHECK(I)=1
                CHI1=CHI-2
```

```
              ZOO=0
  START       I=1
              N1=0
              ZCOUNT=0
              WHENEVER MCOUNT.L.CAL,CHI=CHI1
  S4          EXECUTE SETEOF.(S1)
               GOTO=1
              READ BCD TAPE 2,D2,N(1)...N(35)
              TRANSFER TO S8
  S1          ZCOUNT=ZCOUNT+1
              CH=0
              WHENEVER ZOO.L.CHI
              ZOO=ZOO+1
              OTHERWISE
              ZOO=1
              END OF CONDITIONAL
              N1=0
 T2            EXECUTE SETEOF.(S15)
               GOTO=2
 S21          READ BCD TAPE 2,D2,N(1)...N(35)
              WHENEVER (ZCOUNT/2)*2      .E.ZCOUNT,I=I+1
  S8          WHENEVER (MCOUNT/CAL)*CAL.E.MCOUNT.OR.MCOUNT.E.1,TRANSFER TO SORT
 S5           THROUGH S2,FORM=1,1,M.G.5
              L=M+N1
              LL=7*(M-1)
              WHENEVER (ZCOUNT/2)*2      .NE.ZCOUNT,TRANSFER TO S3
              M1Z(L)=N(LL+1)
              M2Z(L)=N(LL+2)
              M3Z(L)=N(LL+3)
              M4Z(L)=N(LL+4)
              M5Z(L)=N(LL+5)
              M6Z(L)=N(LL+6)
              M7Z(L)=N(LL+7)
              TRANSFER TO S2
 S3           M1(L)=N(LL+1)
              M2(L)=N(LL+2)
              M3(L)=N(LL+3)
              M4(L)=N(LL+4)
              M5(L)=N(LL+5)
              M6(L)=N(LL+6)
              M7(L)=N(LL+7)
              NUM=L
 S2           CONTINUE
              N1=N1+5
              TRANSFER TO S4
 SORT         WHENEVER A(I,1).E.0,TRANSFER TO S5
              THROUGH S11,FOR M=1,1,M.G.5
              L=M+N1
              LL=7*(M-1)
              THROUGH S9,FOR J=1,1,J.G.7
              WHENEVER A(I,J).E.0,TRANSFER TO S11
              WHENEVER (ZCOUNT/2)*2.NE.ZCOUNT,TRANSFER TO S12
              TRANSFER TO SS(A(I,J))
 S12          TRANSFER TO SSS(A(I,J))
 SS(1)        M1X(L)=N(LL+1)
              TRANSFER TOS9
 SS(2)        M2X(L)=N(LL+2)
              TRANSFER TOS9
 SS(3)        M3X(L)=N(LL+3)
              TRANSFER TOS9
```

```
SS(4)           M4X(L)=N(LL+4)
                TRANSFER TOS9
SS(5)           M5X(L)=N(LL+5)
                TRANSFER TOS9
SS(6)           M6X(L)=N(LL+6)
                TRANSFER TOS9
SS(7)           M7X(L)=N(LL+7)
                TRANSFER TOS9
SSS(1)          M1C(L)=N(LL+1)
                TRANSFER TOS9
SSS(2)          M2C(L)=N(LL+2)
                TRANSFER TOS9
SSS(3)          M3C(L)=N(LL+3)
                TRANSFER TOS9
SSS(4)          M4C(L)=N(LL+4)
                TRANSFER TOS9
SSS(5)          M5C(L)=N(LL+5)
                TRANSFER TOS9
SSS(6)          M6C(L)=N(LL+6)
                TRANSFER TOS9
SSS(7)          M7C(L)=N(LL+7)
S9              CONTINUE
S11             CONTINUE
                N1=N1+5
                TRANSFER TO S4
S7              M1XA=0.
                M2XA=0.
                M3XA=0.
                M4XA=0.
                M5XA=0.
                M6XA=0.
                M7XA=0.
                M1CA=0.
                M2CA=0.
                M3CA=0.
                M4CA=0.
                M5CA=0.
                M6CA=0.
                M7CA=0.
                M1ZA=0.
                M2ZA=0.
                M3ZA=0.
                M4ZA=0.
                M5ZA=0.
                M6ZA=0.
                M7ZA=0.
                THROUGH S13,FOR I=1,1,I.G.10
                M1ZA=((I-1)*M1ZA+M1Z(I))/I
                M2ZA=((I-1)*M2ZA+M2Z(I))/I
                M3ZA=((I-1)*M3ZA+M3Z(I))/I
                M4ZA=((I-1)*M4ZA+M4Z(I))/I
                M5ZA=((I-1)*M5ZA+M5Z(I))/I
                M6ZA=((I-1)*M6ZA+M6Z(I))/I
                M7ZA=((I-1)*M7ZA+M7Z(I))/I
                M1XA=((I-1)*M1XA+M1X(I))/I
                M2XA=((I-1)*M2XA+M2X(I))/I
                M3XA=((I-1)*M3XA+M3X(I))/I
                M4XA=((I-1)*M4XA+M4X(I))/I
                M5XA=((I-1)*M5XA+M5X(I))/I
                M6XA=((I-1)*M6XA+M6X(I))/I
```

```
               M7XA=((I-1)*M7XA+M7X(I))/I
               M1CA=((I-1)*M1CA+M1C(I))/I
               M2CA=((I-1)*M2CA+M2C(I))/I
               M3CA=((I-1)*M3CA+M3C(I))/I
               M4CA=((I-1)*M4CA+M4C(I))/I
               M5CA=((I-1)*M5CA+M5C(I))/I
               M6CA=((I-1)*M6CA+M6C(I))/I
               M7CA=((I-1)*M7CA+M7C(I))/I
   S13         CONTINUE
               WHENEVER (KCOUNT.E.2.AND.QCOUNT.E.1).OR.(KCOUNT.E.3.AND.QCOUN
             1 T.E.2), M1CA=M1ZA+700.
               WHENEVER KCOUNT.E.1
               PRINT COMMENT $1$
               PRINT RESULTS NUM
               OR WHENEVER KCOUNT.E.2.AND.QCOUNT.E.1
               PUNCH FORMATKIM,NUM,M2CA-M2ZA
               VECTOR VALUES KIM=$I6,H*SCALE FACTOR=*,E12.4*$
               OR WHENEVER KCOUNT .E.2
               PUNCH FORMAT D3,NUM
               OR WHENEVER KCOUNT.E.3.AND.QCOUNT.E.2
               WRITE BCD TAPE 3, KIM, NUM, M2CA-M2ZA
               PRINT FORMAT BLUE,NUM,M2CA-M2ZA
               PRINT COMMENT $1$
               VECTOR VALUES BLUE=$H*NUM=*I8,S5,H*THE SCALE FACTOR FOR THE F
             1 OLLOWING RUN IS*E14.4*$
               OR WHENEVER KCOUNT .E.3
               WRITE BCD TAPE 3,D3,NUM
               PRINT FORMAT BLUE1,NUM
               VECTOR VALUES BLUE1=$H*NUM=*I8*$
               OTHERWISE
               CONTINUE
               END OF CONDITIONAL
               WHENEVER KCOUNT.E.1.OR.KCOUNT.E.3, PRINT COMMENT$0 CHANNEL 1
             1     CHANNEL 2     CHANNEL 3     CHANNEL 4     CHANNEL 5     C
             2 HANNEL 6     CHANNEL 7$
               THROUGH S14,FOR I=1,1,I.G.NUM
               M1(I)=((M1(I)-M1ZA)/(M1CA-M1XA))*CAL1
               M2(I)=((M2(I)-M2ZA)/(M2CA-M2XA))*CAL2
               M3(I)=((M3(I)-M3ZA)/(M3CA-M3XA))*CAL3
               M4(I)=((M4(I)-M4ZA)/(M4CA-M4XA))*CAL4
               M5(I)=((M5(I)-M5ZA)/(M5CA-M5XA))*CAL5
               M6(I)=((M6(I)-M6ZA)/(M6CA-M6XA))*CAL6
               M7(I)=((M7(I)-M7ZA)/(M7CA-M7XA))*CAL7
               WHENEVER KCOUNT.E.1
               PRINT FORMAT D5   ,         M1(I),M2(I),M3(I),M4(I),M5(I),M6(I),M7(
               OR WHENEVER KCOUNT.E.2
               PUNCH FORMAT D4,  M1(I),M2(I),M3(I),M4(I),M5(I),M6(I),M7(I)
               OR WHENEVER KCOUNT.E.3
               WRITEBCDTAPE3,D4, M1(I),M2(I),M3(I),M4(I),M5(I),M6(I),M7(I)
               PRINT FORMAT D5, M1(I),M2(I),M3(I),M4(I),M5(I),M6(I),M7(I)
               OTHERWISE
               CONTINUE
   S14         END OF CONDITIONAL
               WHENEVER KCOUNT .E.4
   CALC         CONTINUE
               EXECUTE ZERO.(CP,CS,CT,CB)
               THROUGH S18,FOR I=20,1,I.G.(NUM-20)
                J=I-19
                CP=((J-1)*CP+M1(I))/J
                CS=((J-1)*CS+M2(I))/J
```

```
           CT=((J-1)*CT+M3(I))/J
           CB=((J-1)*CB+M4(I))/J
 S18      CONTINUE
          WHENEVER COUN   .G..5,TRANSFER TO QUIT
          THROUGH S19, FOR I=20,1,I.G.(NUM-20)
           J=I-19
          DEVP=(.ABS.(M1(I)-CP)+(J-1)*DEVP)/J
          DEVS=(.ABS.(M2(I)-CS)+(J-1)*DEVS)/J
          DEVT=(.ABS.(M3(I)-CT)+(J-1)*DEVT)/J
          DEVB=(.ABS.(M4(I)-CB)+(J-1)*DEVB)/J
 S19      CONTINUE
          THROUGH S20,FORI=20,1,I.G.(NUM-20)
          WHENEVER .ABS.(M1(I)-CP).G.DEVP,M1(I)=CP
          WHENEVER .ABS.(M2(I)-CS).G.DEVS,M2(I)=CS
          WHENEVER .ABS.(M3(I)-CT).G.DEVT,M3(I)=CT
          WHENEVER .ABS.(M4(I)-CB).G.DEVB,M4(I)=CB
 S20      CONTINUE
          COUN=COUN+.2
          TRANSFER TO CALC
 QUIT     CONTINUE
          XARCAL=(CS-CP)/(CS+CP)
          YARCAL=(CT-CB)/(CT+CB)
          VMS=VM(MCOUNT)*VM(MCOUNT)
          VCALH1=CS/VMS
          VCALH2=CP/VMS
          VCALV1=CT/VMS
          VCALV2=CB/VMS
          WHENEVER MCOUNT .E.1,PRINT COMMENT $0CS-CP/CS+CP, CT-CB/CT+CB, CS/
        1 VSQUARED, CP/VSQUARED, CT/VSQUARED, CB/VSQUARED, RUN NO. $
           PRINT FORMAT D6, XARCAL,YARCAL,VCALH1,VCALH2,VCALV1,VCALV2,MCOUNT
          PUNCH FORMAT D7,XARCAL,YARCAL,VCALH1,VCALH2,VCALV1,VCALV2,
        1 MCOUNT
          VECTOR VALUES D6=$6E14.6,1I10*$
          VECTOR VALUES D7=$6E11.4,1I6*$
          OTHERWISE
           CONTINUE
           END OF CONDITIONAL
          VECTOR VALUES D3=$I4*$
          VECTOR VALUES D4=$7E11.4*$
          VECTOR VALUES D5=$1H ,7E14.6*$
          WHENEVER TES.E.0.OR.ZAT.E.1,TRANSFER TOS17
          SCALEF=((MCOUNT-1)*SCALEF+M2CA-M2ZA)/MCOUNT
          MCOUNT=MCOUNT+1
          TRANSFER TO START
 S15      CH=1
          WHENEVER CHECK(ZOO).NE.CH,TRANSFER TO S21
 T3       EXECUTE SETEOF.(S16)
           GOTO=3
          READ BCD TAPE 2,D2,V(1)...V(35)
          BACKSPACE RECORD OF TAPE 2
          BACKSPACE RECORD OF TAPE 2
          TRANSFER TO S7
 S16      ZAT=1
          TRANSFER TO S7
 S17      WHENEVER KCOUNT.E.3.AND.QCOUNT.E.2
          WRITE BCD TAPE3, SCA, SCALEF
          PRINT FORMAT SCA, SCALEF
          END OF FILE TAPE3
          REWIND TAPE3
          OR WHENEVER KCOUNT.E.3
```

```
END OF FILE TAPE 3
REWIND TAPE 3
OR WHENEVER KCOUNT.E.2.AND.QCOUNT.E.1
PUNCH FORMAT SCA,SCALEF
VECTOR VALUES SCA=$H*THE AVERAGE SCALE FACTOR=*,E12.4*$
 END OF CONDITIONAL
 REWIND TAPE 2
END OF PROGRAM
```

```
$ COMPILE MAD      ,PRINT OBJECT   ,PUNCH OBJECT

MAD (17 MAY 1967 VERSION) PROGRAM LISTING ... ... ...


           SUBROUTINE TO PLOT GRAPHS ON CALCOMP
            EXTERNAL FUNCTION (S,YHD,NY,XHD,NX,PHD1,N1,PHD2,P2,N2,
          1 PHD3,P3,N3,PHD4,P4,N4,X,Y,N,M,K,XM,XS,YM,YS,F2)
            ENTRY TO PLOT.
            DIMENSION GRX(2),GRY(2)
            INTEGER I,J,K,N,M,NX,NY,N1,N2,N3,N4
            R=.5*S
            PLTXMX.(21.)
            PLTSIZ.(.ABS.(R))
           DRAW GRID
            PENUP.(2.,2.)
            PENDN.(2.,12.)
            PENDN.(18.,12.)
            PENDN.(18., 2.)
            PENDN.( 2., 2.)
            THROUGH S1, FOR GRX(1)=4.,2.,GRX(1).G.17.
            GRX(2)=GRX(1)
            GRY(1)=2.
            GRY(2)=12.
S1          PDSHLN.(GRX(1),GRY(1),2,1,0.,0.)
            THROUGH S2, FOR GRY(1)=4.,2.,GRY(1).G.11.
            GRY(2)=GRY(1)
            GRX(1)=2.
            GRX(2)=18.
S2          PDSHLN.(GRX(1),GRY(1),2,1,0.,0.)
           SET SCALES, AXES, AND HEADINGS
            WHENEVER S.G.0.
            PSCALE.(16.,1.,XMIN,DX,X(1),N,1)
            PSCALE.(10.,1.,YMIN,DY,Y(1),N*M,1)
            WHENEVER XM.NE.0.,XMIN=XM
            WHENEVER YM.NE.0.,YMIN=YM
            WHENEVER XS.NE.0.,DX=XS
            WHENEVER YS.NE.0.,DY=YS
            PAXIS.(2.,2.,XHD,-NX,16.,0.,XMIN,DX,1.)
            PAXIS.(2.,2.,YHD,NY,10.,90.,YMIN,DY,1.)
            NUMS=12.8
            FIGS=13.25
            WHENEVER N1.E.0,FIGS=FIGS-.5
            WHENEVER N4.E.0,NUMS=NUMS-.3
            WHENEVER N3.E.0,NUMS=NUMS-.3
            PSYMB.(2.,FIGS,.25,FIG,0.,6)
            VECTOR VALUES FIG=$FIGURE$
            PSYMB.(2.,FIGS-.5,.23,YHD,0.,NY)
            PSYMB.(-0.,-0.,.15,VS,0.,6)
            VECTOR VALUES VS=$  VS   $
            PSYMB.(-0.,-0.,.23,XHD,0.,NX)
            WHENEVER N1.E.0,TRANSFER TOS4
             PSYMB.(2.,FIGS-1.,.23,FORV,0.,22)
            VECTOR VALUES FORV=$FOR VARIOUS VALUES OF $
            PSYMB.(-0.,-0.,.23,PHD1,0.,N1)
S4          WHENEVER N2.E.0, TRANSFER TO S5
            PSYMB.(10.,NUMS,.2,PHD2,0.,.ABS.(N2))
            WHENEVER N2.L.0,TRANSFER TO S5
```

```
          PNUMBR.(-0.,-0.,.2,P2,0.,F2)
S5        WHENEVER N3 .E. 0, TRANSFER TO S6
          NUMS=NUMS-.3
          PSYMB.(10.,NUMS,.2,PHD3,0.,.ABS.(N3))
          WHENEVER N3.L.0,TRANSFER TO S6
          PNUMBR.(-0.,-0.,.2,P3,0.,F2)
S6        WHENEVER N4.E.0,TRANSFER TO S7
          NUMS=NUMS-.3
          PSYMB.(10.,NUMS,.2,PHD4,0.,.ABS.(N4))
          WHENEVER N4.L.0,TRANSFER TO S7
          PNUMBR.(-0.,-0.,.2,P4,0.,F2)
S7        CONTINUE
          OTHERWISE
           PSCALE.(10.,1.,XMIN,DX,X(1),N,1)
           PSCALE.(16.,1.,YMIN,DY,Y(1),N*M,1)
          WHENEVER XM.NE.0.,XMIN=XM
          WHENEVER YM.NE.0.,YMIN=YM
          NORMAL MODE IS INTEGER XS.NE.0.,DX=XS
          WHENEVER YS.NE.0.,DY=YS
          PAXIS.(18.,2.,XHD,-NX,10.,90.,XMIN,DX,1.)
          PAXIS.(18.,2.,YHD,NY,16.,180.,YMIN,DY,1.)
           NUMS=1.2
          FIGS=.75
          WHENEVER N1.E.0, FIGS=FIGS+.5
          WHENEVER N4.E.0, NUMS=NUMS+.3
          WHENEVER N3.E.0, NUMS=NUMS+.3
          PSYMB.(FIGS,2.,.25,FIG,90.,6)
          PSYMB.(FIGS+.5,2.,.23,YHD,90.,NY)
          PSYMB.(-0.,-0.,.15,VS,90.,6)
          PSYMB.(-0.,-0.,.23,XHD,90.,NX)
          WHENEVER N1.E.0, TRANSFER TO S3
          PSYMB.(FIGS+1.,2.,.23,FORV,90.,22)
          PSYMB.(-0.,-0.,.23,PHD1,90.,N1)
S3        WHENEVER N2.E.0, TRANSFER TO S9
          PSYMB.(NUMS,8.2,.2,PHD2, 90.,.ABS.(N2))
          WHENEVER N2.L.0,TRANSFER TO S9
          PNUMBR.(-0.,-0.,.2,P2,90.,F2)
S9        WHENEVER N3.E.0, TRANSFER TO S10
          NUMS=NUMS+.3
          PSYMB.(NUMS,8.2,.2,PHD3, 90.,.ABS.(N3))
          WHENEVER N3.L.0,TRANSFER TO S10
          PNUMBR.(-0.,-0.,.2,P3,90.,F2)
S10       WHENEVER N4.E. 0, TRANSFER TO S11
          NUMS=NUMS+.3
          PSYMB.(NUMS,8.2,.2,PHD4, 90.,.ABS.(N4))
          WHENEVER N4.L.0,TRANSFER TO S11
          PNUMBR.(-0.,-0.,.2,P4,90.,F2)
S11       CONTINUE
          END OF CONDITIONAL
         PLOT GRAPHS
          WHENEVER S.G.0.
          PLTOFS.(XMIN,DX,YMIN,DY,2.,2.)
          THROUGH S12, FOR I=1,1,I.G.M
S12       PLINE.(X(1),Y((I-1)*N+1),N,1,K,I,1.)
           OTHERWISE
          PLTOFS.(YMIN,DY,XMIN,DX,18.,2.)
           THROUGH S13, FOR I=1,1,I.G.M*N
S13       Y(I)=2.*YMIN-Y(I)
          THROUGH S14, FOR I=1,1,I.G.M
S14       PLINE.(Y((I-1)*N+1), X(1),N,1,K,I,1.)
```

```
                 THROUGH S15,FOR I=1,1,I.G.M*N
       S15       Y(I)=2.*YMIN-Y(I)
                 END OF CONDITIONAL
                 PLTEND.
                 FUNCTION RETURN
                 END OF FUNCTION
```

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$COMPILE MAD, EXECUTE, PRINT OBJECT, DUMP, I/ODUMP


MAD (17 MAY 1967 VERSION) PROGRAM LISTING ... ... ...


                  PROGRAM TO DETERMINE THE VISCOUS DRAG FROM WAKE
                         SURVEY DATA
               READ FORMAT DATA,CODE,M,N,RHO,XDIS,YSTA,ZSTA,CHECK
               VECTOR VALUES DATA=$3I3,4F10.4,I4*$
               M1=M/18
               M2=M+1-M1*18
               MN=((M+1)*(N+1)-1)/14
               MN1=(M+1)*(N+1)-MN
               SETDIM.(U   ,0...M,0...N)
               SETDIM.(V   ,0...M,0...N)
               SETDIM.(W   ,0...M,0...N)
               SETDIM.(UONE,0...M,0...N)
               SETDIM.(VONE,0...M,0...N)
               SETDIM.(WONE,0...M,0...N)
               SETDIM.(PRESS,0...M,0...N)
               SETDIM.(UV,0...M,0...N)
               SETDIM.(UW,0...M,0...N)
               INTEGER I,J,CODE,M,N,MN,M1,MN1,M2,CHECK
               DIMENSION U((0...30)*(0...30)),V((0...30)*(0...30)),W((0...30
             1 )*(0...30)),PRESS((0...30)*(0...30)),UONE((0...30)*(0...30)),
             2 VONE((0...40)*(0...40)),WONE((0...40)*(0...40)),UCAP(20),
             3 UV((0...40)*(0...40)),UW((0...40)*(0...40))
             4 ,FF1(40),FF2(40),FF3(40),FF4(40),FF5(40),FF6(40),FF7(40)
             1   ,UP(13*28),Y(28)
    JUDY       READ FORMAT DATA1,UU,VV,WW,UVV,UWW,PSTAT,UCAPP,I,J,UONEE
               VECTOR VALUES DATA1=$7F8.4,2I3,F 8.3*$
               U(I,J)=UU
               V(I,J)=VV
               W(I,J)=WW
               UV(I,J)=UVV
               UW(I,J)=UWW
               UCAP(I)=UCAPP
               WHENEVER UONEE.L.1.0E-10
               UONE(I,J)=UU
               OTHERWISE
               UONE(I,J)=UONEE
               END OF CONDITIONAL
               WHENEVER UONE(I,J).L.1.
               UONE(I,J)=UONE(I,J)*10.
               OTHERWISE
               CONTINUE
               END OF CONDITIONAL
               PRESS(I,J)=PSTAT*14.7*12./34.
               WHENEVER I.E.M.AND.J.E.N
               TRANSFER TO START
               OTHERWISE
               TRANSFER TO JUDY
               END OF CONDITIONAL
    START      PRINT FORMAT HEAD
               WHENEVER CHECK .E.1,PRINT RESULTS U(0,0)...U(12,27),V(0,0)...V(12
             1 W(0,0)...W(12,27)
               THROUGH DALE,FOR I=0,1,I        .G.M
               FF1(I)=        (U(I,0)+U(I, N))/2.
```

```
               FF2(I)=(V(I,0)+V(I,N))/2.
               FF3(I)= (W(I,0)+W(I,N))/2.
               FF4(I)=   (UV(I,0)+UV(I,N))/2.
               FF5(I)=   (UW(I,0)+UW(I,N))/2.
               FF6(I)=        (PRESS(I,0)+PRESS(I,N))/2.
DALE           FF7(I)=        (UONE (I,0)+UONE (I,N))/2.
               THROUGH COTTON,FOR I=0,1,I.G.M
               THROUGH COTTON,FOR J=0,1,J.G.N
               UONE (I,J)=-FF7(I)                          +UONE(I,J)+UCAP(I)
               PRESS(I,J)=-FF6(I)                          +PRESS(I,J)
               UW(I,J)=-FF5(I)                    +UW(I,J)
               UV(I,J)=-FF4(I)                    +UV(I,J)
               W(I,J)=-FF3(I)              +W(I,J)
               V(I,J)=-FF2(I)+V(I,J)
               U(I,J)=UCAP(I)-FF1(I)                 +U(I,J)
               WONE(I,J)=UW(I,J)*UONE(I,J)
COTTON         VONE(I,J)=UV(I,J)*UONE(I,J)
               WHENEVER CHECK .E.1,PRINT RESULTS U(0,0)...U(12,27),V(0,0)...V(12,
             1 W(0,0)...W(12,27)
BEGIN          SUMI=0.
               VM=0.
               WHENEVER M/2*2.E.M
               M=M
               OTHERWISE
               M=M-1
               END OF CONDITIONAL
               WHENEVER N/2*2.E.N
               N=N
               OTHERWISE
               N=N-1
               END OF CONDITIONAL
               THROUGH ALPHA, FOR I=0,1,I.G.M
               VM=(UCAP(I)+I*VM)/(I+1)
               WHENEVER I.E.0.OR.I.E.M
               SMI=1.
               OR WHENEVER I/2*2.E.I
               SMI=2.
               OTHERWISE
               SMI=4.
               END OF CONDITIONAL
               SUMJ=0.
               THROUGH LOOP2, FOR J=0,1,J.G.N
               WHENEVER J.E.0.OR.J.E.N
               SMJ=1.
               OR WHENEVER J/2*2.E.J
               SMJ=2.
               OTHERWISE
               SMJ=4.
               END OF CONDITIONAL
               WHENEVER CODE .E.1, TRANSFER TO FWU
               WHENEVER CODE .E.2, TRANSFER TO FLAND1
               WHENEVER CODE .E.3, TRANSFER TO FBETZ
               WHENEVER CODE .E.4, TRANSFER TO FLAND2
               F=-PRESS(I,J)+.5*RHO*((UCAP(I)-UONE(I,J)).P.2.-VONE(I,J).P.2.
             1 -WONE(I,J).P.2.)+RHO*U(I,J)*(UCAP(I)-U(I,J))
               TRANSFER TO LOOP2
FWU            F=RHO*(UONE(I,J).P.2.-U(I,J).P.2.)-
             1 RHO*UCAP(I)*(UONE(I,J)-U(I,J))
               TRANSFER TO LOOP2
FLAND1         F=-PRESS(I,J)+RHO*U(I,J)*(UCAP(I)-U(I,J))
```

```
                   TRANSFER TO LOOP2
       FBETZ       UTWO=SQRT.(2.*(.5*RHO*UCAP(I).P.2.-PRESS(I,J))/RHO)
                   F=-PRESS(I,J)+RHO*U(I,J)*(UCAP(I)-U(I,J))
                 1 +.5*RHO*(UCAP(I)-UTWO    ).P.2.
                   TRANSFER TO LOOP2
       FLAND2      F=-PRESS(I,J)+RHO*U(I,J)*(UCAP(I)-U(I,J))
                 1 +.5*RHO*(UCAP(I)-UONE(I,J)).P.2.
       LOOP2       SUMJ=SUMJ+SMJ*F*YSTA/3.
       ALPHA       SUMI=SUMI+SMI*SUMJ*ZSTA/3.
                   DRAG=SUMI
                   WHENEVER CODE .E.1,TRANSFER TO PNT1
                   WHENEVER CODE .E.2,TRANSFER TO PNT2
                   WHENEVER CODE .E.3,TRANSFER TO PNT3
                   WHENEVER CODE .E.4,TRANSFER TO PNT4
                   PRINT FORMAT GEBHT
                   TRANSFER TO PNT
       PNT1        PRINT FORMAT WU
                   TRANSFER TO PNT
       PNT2        PRINT FORMAT LANDR
                   TRANSFER TO PNT
       PNT3        PRINT FORMAT BETZ
                   TRANSFER TO PNT
       PNT4        PRINT FORMAT LANDR1
       PNT         PRINT FORMAT RESULT,DRAG,XDIS,VM
                   VECTOR VALUES HEAD=$1H1,S10,13HAPPROXIMATIONS10,24HVISCOUS
                 1 DRAG IN POUNDSS10,26HDISTANCE BEHIND MODEL, FT.S10,13HSPEED,
                 2 FT/SEC///*$
                   VECTOR VALUES WU=$1H+S16,2HWU*$
                   VECTOR VALUES LANDR1=$1H+S12, 11H LANDWEBER2*$
                   VECTOR VALUES LANDR=$1H+S12,10HLANDWEBER1*$
                   VECTOR VALUES BETZ=$1H+S13,4HBETZ*$
                   VECTOR VALUES GEBHT=$1H+S12,8HGEBHARDT*$
                   VECTOR VALUES RESULT=$1H+S38,F8.3,S28,F4.2,S23,F4.2///*$
                   CODE=CODE+1
                   WHENEVER CODE.G.5,TRANSFER TO GO
                   TRANSFER TO BEGIN
       GO          Y=-1.162
                   M=M+1
                   N=N+1
                   THROUGH VAL, FOR I=0,1,I.G.M
                   THROUGH VAL ,FOR J=0,1,J.G.N
                   Y(J+1)=Y(J)+.083
                   UP(I+1,J+1)=U(I,J)/UCAP(I)
       VAL         WHENEVER UP(I+1,J+1).L..7.OR.UP(I+1,J+1).G.1.2,UP(I+1,J+1)=1.
                   WHENEVER CHECK.E.1, PRINT RESULTS UP(1,1)...UP(M+1,N+1)
                   PLOT.(1.,$U/UO$,4,L4,1 ,$Z$,1,L1,0., -45,L2,0.,-15,L3,0.,-29,
                 1 Y,UP,N,M,1,-1.6,0.,.7,.05,FMT)
                   THROUGH VAL1,FOR I=0,1,I.G.M
                   THROUGH VAL1,FORJ=0,1,J.G.N
       VAL1        UP(I+1,J+1)=V(I,J)/UCAP(I)
                   WHENEVER CHECK.E.1, PRINT RESULTS UP(1,1)...UP(M+1,N+1)
                   PLOT.(1.,$V/UO$,4,L4,1 ,$Z$,1,L1,0.,-45,L2,0.,-15,L3,0.,-29,
                 1 Y,UP,N,M,1,-1.6,0.,0.,0.,FMT)
                   THROUGH VAL2,FOR I=0,1,I.G.M
                   THROUGH VAL2,FOR J=0,1,J.G.N
                   UP(I+1,J+1)=W(I,J)/UCAP(I)
       VAL2        WHENEVER UP(I+1,J+1).L.-.038,UP(I+1,J+1)=0.
                   WHENEVER CHECK.E.1, PRINT RESULTS UP(1,1)...UP(M+1,N+1)
                   PLOT.(1.,$W/UO$,4,L4,1 ,$Z$,1,L1,0., -45,L2,0.,-15,L3,0.,-29,
                 1 Y,UP,N,M,1,-1.6,0.,-.05 ,.01,FMT)
```

```
                 THROUGH VAL3,FOR I=0,1,I.G.M
                 THROUGH VAL3,FOR J=0,1,J.G.N
                 UP(I+1,J+1)=PRESS(I,J)/(.5*RHO*UCAP(I)*UCAP(I))
     VAL3         WHENEVER UP(I+1,J+1).L.-1.20,UP(I+1,J+1)=0.
                 WHENEVER CHECK.E.1, PRINT RESULTS UP(1,1)...UP(M+1,N+1)
                 PLOT.(1.,L5,15    ,L4,1 ,$Z$,1,L1,0., -45,L2,0.,-15,L3,0.,-29,
               1 Y,UP,N,M,1,-1.6,0.,0.,0.,FMT)
                 VECTOR VALUES L1=$WAKE SURVEY-14FT, SERIES 60,.75BLOCK MODEL$
                 VECTOR VALUES L2=$U0=4.20 FT/SEC $
                 VECTOR VALUES L3=$CUT TAKEN 2.25FT BEHIND MODEL$
                  VECTOR VALUES L4=$Y$
                 VECTOR VALUES L5=$STATIC PRESSURE$
     VAL4        CONTINUE
                 END OF PROGRAM

THE FOLLOWING NAMES HAVE OCCURRED ONLY ONCE IN THIS PROGRAM.
COMPILATION WILL CONTINUE.

     VAL4          *167
     M2            *004
     MN1           *006
```

Appendix II

Theory of the Four Hole Spherical Pitot Tube

Consider a sphere in a uniform potential flow. It can be shown that

$$p - p_o = \frac{\rho}{2} V^2 (1-9/4 \sin^2 Z) \qquad (1a)$$

where

p = pressure at a given point on the sphere

p_o = pressure in the uniform stream

V = velocity of the undisturbed flow

Z = angle between the given point and stagnation point

ρ = mass density of the fluid

Applying equation (1a) to a four hole spherical head pitot tube with holes located at A, B, C and D and stagnation point at P (see Figure 1a), we can write

$$p_a - p_o = \frac{\rho}{2} V^2 (1-9/4 \sin^2 POA) \qquad (2a)$$

$$p_b - p_o = \frac{\rho}{2} V^2 (1-9/4 \sin^2 POB) \qquad (3a)$$

$$p_c - p_o = \frac{\rho}{2} V^2 (1-9/4 \sin^2 POC) \qquad (4a)$$

$$p_d - p_o = \frac{\rho}{2} V^2 (1-9/4 \sin^2 POD) \qquad (5a)$$

where p_o is the pressure in the uniform stream. Since the pressure p_o is unknown we will eliminate it by forming the difference between the pressure at point A and the pressures at points B, C and D. Doing this and letting

$$p_a - p_b = AB$$

$$p_a - p_c = AC$$

$$p_a - p_d = AD$$

Equations (2) - (5) become,

$$AB = \frac{9}{8} \rho V^2 (\sin^2 POB - \sin^2 POA) \qquad (6a)$$

$$AC = \frac{9}{8} \rho V^2 (\sin^2 POC - \sin^2 POA) \qquad (7a)$$

$$AD = \frac{9}{8} \rho V^2 (\sin^2 POD - \sin^2 POA) \qquad (8a)$$

We now desire to get expressions for the angles POA, POB, POC and POD in terms of X and Y, the unknown direction angles of the velocity vector V and the positions of points A, B, C and D. From spherical trigonometry we can write

$$\cos POA = \sin l_a \sin Y + \cos l_a \cos Y \cos (X - m_a) \qquad (9a)$$

$$\cos POB = \sin l_b \sin Y + \cos l_b \cos Y \cos (X - m_b) \qquad (10a)$$

$$\cos POC = \sin l_c \sin Y + \cos l_c \cos Y \cos (X - m_c) \qquad (11a)$$

$$\cos POD = \sin l_d \sin Y + \cos l_d \cos Y \cos (X - m_d) \qquad (12a)$$

where l_i and m_i are the latitude and longitude respectfully of the i^{th} hole. (See Figure 1a for sign conventions)

Also, since $1 - \sin^2 X = \cos^2 Z$, Equations (6a), (7a) and (8a) can be written

$$AB = U^2 (\cos^2 POA - \cos^2 POB) \qquad (13a)$$

$$AC = U^2 (\cos^2 POA - \cos^2 POC) \qquad (14a)$$

$$AD = U^2 (\cos^2 POA - \cos^2 POD) \qquad (15a)$$

where

$$U^2 = \frac{9}{8} \rho V^2$$

Now, squaring Equations (9a) - (12a) and substituting into Equations (13a) - (15a) we get the final equations describing the behavior of the four hole pitot tube.

$$\begin{aligned} AB = U^2 \Big[& \sin^2 Y \left[\sin^2 l_a - \sin^2 l_b\right] + \cos^2 Y \Big[\cos^2 l_a \\ & \cos^2(X-m_a) - \cos^2 l_b \cos^2(X-m_b)\Big] + 2 \cos Y \sin Y \\ & \Big[\cos l_a \sin l_a \cos (X-m_a) - \cos l_b \sin l_b \cos \\ & (X-m_b)\Big]\Big] \end{aligned} \tag{16a}$$

$$\begin{aligned} AC = U^2 \Big[& \sin^2 Y \left[\sin^2 l_a - \sin^2 l_c\right] + \cos^2 Y \Big[\cos^2 l_a \\ & \cos^2 (X-m_a) - \cos^2 l_c \cos^2 (X-m_c)\Big] + 2 \cos Y \sin \\ & Y \Big[\cos l_a \sin l_a \cos (X-m_a) - \cos l_c \cos (X-m_c)\Big]\Big] \end{aligned} \tag{17a}$$

$$\begin{aligned} AD = U^2 \Big[& \sin^2 Y \left[\sin^2 l_a - \sin^2 l_d\right] + \cos^2 Y \Big[\cos^2 l_a \cos^2 \\ & (X-m_a) - \cos^2 l_d \cos^2 (X-m_d)\Big] + 2 \cos Y \sin Y \Big[\cos \\ & l_a \sin l_a \cos (x-m_a) - \cos l_d \cos (x-m_d)\Big]\Big] \end{aligned} \tag{18a}$$

For water these equations take the form

$$AB = AB(X, Y, V^2) \tag{19a}$$

$$AC = AC(X, Y, V^2) \tag{20a}$$

$$AD = AD(X, Y, V^2) \tag{21a}$$

in principle we can solve (19a), (20a) and (21a) and get relations of the form

$$X = X(AB, AC, AD) \tag{22a}$$

$$Y = Y(AB, AC, AD) \tag{23a}$$

$$V^2 = V^2(AB, AC, AD) \tag{24a}$$

Therefore, by measuring AB, AC and AD we can determine from (22a), (23a) and (24a) the magnitude and direction of the velocity uniquely.

It is not practically possible to make the above transformation analytically. However, we can find approximate relations of the form (22a), (23a) and (24a) by evaluating Equations (16a), (17a) and (18a) for various values of X, Y and V^2 and then using these values in a stepwise regression scheme to generate a polynomial to approximate the functions (22a), (23a) and (24a).

Presently, work being done to generate these functions using Westervelt's "Stepwise Regression with Simple Learning" computer program. This can be done for various hole configurations to determine the best one before a four hole pitot tube is actually built. After the tube is manufactured it will then have to be completely calibrated by running it at various velocities at various angles to the flow. The resulting data will then be used to determine new functions of the form 22a, 23a and 24a.