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COLLEGE OF ENGINEERING

Department of Naval Architecture and Marine Engineering

BLOCKAGE EFFECT OR EFFECT OF TANK BOUNDARIES ON MODEL TEST RESULTS

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Under contract with:
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Introduction.

When testing a ship model in a conventional tank, and then scaling the results to the full scale ship it is generally assumed that there is no effect of the restricted depth, width and length of tank. In other words the model is assumed to run in open water, in the same way as the full scale ship. To be as near as possible to the open water condition there are certain emperical relations between the dimensions of the model and the width of the particular tank in which it is tested. There is also a certain relation between the maximum speed of the model and the depth of the tank in order to avoid what is called bottom effect. The speed of the model will then be such that $V_c^2/gh \geq 0.5$. To keep the size of the model within certain specified ratios to that of the tank, leads in many cases to models of small size which in turn are susceptible to laminar flow particularly at low speeds. In addition, in such cases, for self propulsion tests the size of the propeller will be small to the extent that scale effect will be inevitable. If the model size is increased to avoid the scale effect in self propulsion tests and/or to ensure turbulent flow at low speed, then in many cases the tank boundary effects show up and affect the measured results. Unless the measured results are corrected for tank boundaries, the ship results would be subject to errors which may be of appreciable magnitude.

Recently, theory has developed to the point where the wave making resistance may be estimated from analysing measured wave heights behind the model. The wave heights are affected by tank boundaries, which differs from tank to tank and even between different parts of the same tank.

The general blockage problem has been under consideration for many years. Corrections based on theoretical considerations have been derived such as that of Stretenski [1] [2] which involves calculation of the wave making resistance of a mathematical body in restricted water. Another theoretical approach is based on what is called the mean flow hypothesis based on calculating the mean increase of the relative speed between model and water and modifying the frictional and wave making resistance accordingly. The increase in relative velocity is calculated applying the continuity and Bernouilli equations. Comparison between results obtained for the same model from tanks of different size has been used to arrive at the appropriate increase in relative velocity round the model as compared with the mean increase obtained by the application of this method.

Among those who worked along these lines are Kreitner[3], Schuster[4], Van Lammeren [5], Conn and Lackenby [6], Emmerson [7] and Hughes [8]. Up to the present the probelm is far from being finally solved. The theoretical solutions based on the linear potential flow theory cannot be considered at the moment the happiest solution as the results of wave making resistance obtained from applying this theory in various ways, differ considerably from the measured results. In addition the linear theory neglects the height of waves which influence the local flow around the model. For a more detailed review of the theoretical approach to the blockage problem, see H. C. Kim [8a].

Still another method suffers many weaknesses. Essentially it is emperical in nature and is based on testing a model in different tanks, testing geosim models in the same tank, or testing the same model(s) in a tank where the cross section of the tank can be changed at will. None of these methods are immune to possible sources of errors. Testing a model in different tanks entails the differences in instrumentation and some other minor factors which may assume importance when we consider that the differences in results with which we are concerned are in themselves small. In addition it is known that the turbulent flow around a certain model is established at a lower speed in smaller tanks than in larger ones. In the case of geosim models tested in the same tank the sources of error are, model surface condition, small manufacturing differences and different degrees of turbulence as well as unknown scale effects in comparing results of models of different size. In altering a tank cross-section by the use of movable boundaries the possibility of deformation of the walls and bottom may affect the results, and again there may be a different degree of turbulence as the size of the tank is varied.

The equation representing the mean increase of relative speed, in any of the forms it was presented, suffers from the difficulty of a proper solution except a graphical one. This adds to the inaccuracy of the results obtained by this method. In addition testing the accuracy of this method is usually made by comparison with results taken from large tanks, yet there is still apt to be some influence on the results arising from the tank boundaries.

In the present paper the author discusses the problem of tank boundaries. The present methods for correction are examined, and criteria for tank dimensions relative to model size and speed in order to render boundary effects negligible, are introduced. A new method of approach is proposed based on the known shift of the humps and hollows of the resistance curve as compared with open water.

PART I: TANK BOUNDARY EFFECTS ON A MODEL:

A. The Mean Flow Equation:

Practially all model tanks in existence work on the principle of moving or running a model along the tank in otherwise stationary water.

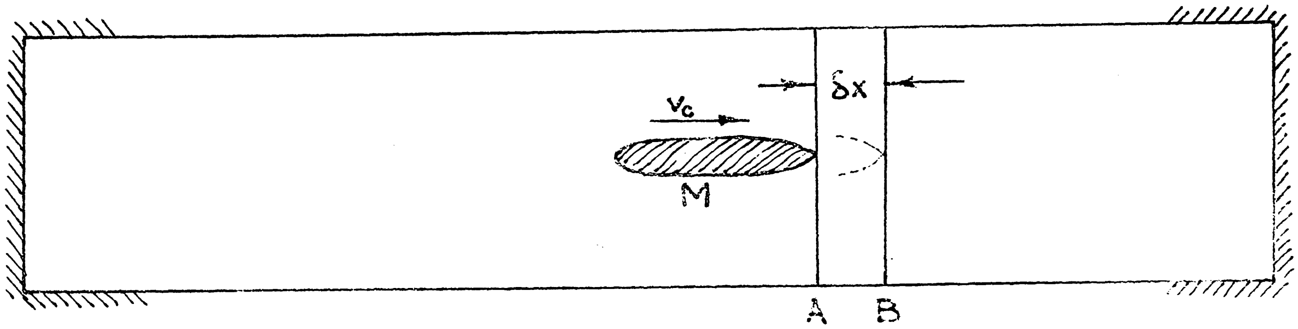


Figure 1

Referring to Figure 1, the model M runs along the center of the tank at a speed v_c relative to space which is considered uniform. When the model moves from position A to position B, i.e. for a distance δx, the model will push the water forward towards the right hand side by a certain amount and evacuates the same amount behind it, the so called piston effect. Had the water been unbounded in all directions then the displacement ahead of the model and the depression after it will cause no flow from ahead to astern as both the displacement and depression will be spread over an infinite surface, the back flow spreading over an infinite sectional area. But with the tank closed at both ends, the sides and the bottom , the level of water ahead is raised and that astern of the model is lowered. A back flow past the model takes place due to the difference of water level. In the steady state the velocity of the back flow will be constant but not necessarily uniformly distributed across the tank. If the distance the model moves in a certain time equals its length then the volume of the water displaced ahead of the position (t = o) will be the volume of displacement of the model which equals the volume of water evacuated astern. The time t for the model to move a distance equal to its own length is l/v_c, (v_c = the carriage speed). Hence the mean speed across the tank at a point where the model sectional area $= \frac{l \times B \times d \times C_b}{l}$ (= $B \times d \times C_b = a_m$) is given by $\left\{ \frac{\nabla_m}{t} \div [A_t - a_m - \delta A] \right\}$, where A_t equals the sectional area of the tank away from the model, ∇_m = volume of the model, δA = a small area caused by the depression of the water level due to the creation of this mean speed, and t = the time = $\frac{l}{v_c}$. The mean speed δ_v is then given by:

$$\delta v = \frac{l \times B \times d \times C_b}{l/v_c \, [A_t - a_m - \delta A]} = \frac{a_m \cdot v_c}{[A_t - a_m - \delta A]} \quad (1)$$

At any other point on the model where the sectional area is a_x, δv_x is given by.

$$\delta v_x = \frac{a_x \cdot v_c}{[A_t - a_x - \delta A_x]} \quad (2)$$

Equation (1) can be put in the following way

$$\frac{\delta v}{v_c} = \frac{m}{1 - m - \frac{\delta A}{A_t}}, \quad \text{where} \quad m = \frac{a_m}{A_t} \quad (3)$$

This is the same equation as that adopted by Hughes [8] except for the term representing the depression of the tank water level which will be considered at a later stage. Equations (1), (2) and (3) are all based on the mean back flow assumption.

B. The Perturbation Velocity in the Tank Due to the Movement of the Model.

1. To examine the question of tank wall effect, the starting point will suitably be the study of the change of velocity around the model. This question has been the subject of several investigations before. However, most work was done in connection with bodies of rotation, Rankine Ovoid, Spheroid, Ellipsoid, etc., moving in "closed tunnels". Although such investigations cannot be applied directly to ship models, the results can be useful in giving some insight into the question of the change of velocity caused by the tank walls in the vicinity of the model. This requires overlooking the free surface effect and the effect of the boundary layer on the model itself.

Taking the case of a Rankine ovoid we can represent this rotational body by a source of strength Q and a sink of the same strength, placed at a distance 2a apart, in a uniform flow of velocity-v_c, in unrestricted water.

The axial velocity component at a point whose cylindrical co-ordinates relative to the body are x, r is given by

$$-v_c + \delta v_r = -v_c + \frac{Q}{4\pi}\left\{\frac{x-a}{R_1^3} - \frac{x+a}{R_2^3}\right\} \quad (4)$$

where:

$$R_1^2 = (x-a)^2 + r^2 , \quad R_2^2 = (x+a)^2 + r^2$$

For x = o, i.e. midship, we have

$$\delta v = aQ \div 2\pi R^3, \text{ where } R^2 = a^2 + r^2$$

The stream function ψ will be given by

$$\psi = \frac{1}{2} v_c r^2 + \frac{Q}{4\pi}\left\{\frac{x-a}{R_1} - \frac{x+a}{R_2}\right\} \quad (5)$$

On the surface of the body $\psi = o$, the breadth (or diameter) of the body at $x = o$ is given by:

$$\frac{d^2}{4}\sqrt{\left(\frac{d^2}{4} + a^2\right)} = \frac{Qa}{\pi V_c} \qquad (6)$$

The relative axial velocity at $x = o$ is given by

$$V_c + \delta v_r = \frac{1}{r}\frac{d\psi}{dr} = -V_c + \frac{Q}{4\pi}\left\{\frac{x-a}{R_1^3} - \frac{x+a}{R_2^3}\right\}$$

$$\frac{V_c + \delta v_r}{V_c} = 1 + \frac{Qa}{2\pi V_c (r^2 + a^2)^{\frac{3}{2}}} \qquad (7)$$

On the surface of the body $r = d/_2$, and substituting from (6) we find

$$\frac{V_c + \delta v}{V_c} = 1 + \frac{d^2}{(d^2 + 4a^2)} \qquad (8)$$

When the body is considered moving in a closed circular tunnel of diameter D, then according to Lamb (9), the stream line equation of the source and sink at any point (x, r) is given by , for $a > x > -a$

$$\psi = \frac{1}{2}V_c r^2 + \frac{2Q}{\pi D^2}\left(r^2 - \frac{D^2}{4}\right) - \frac{4Q}{\pi D^2}\sum \frac{r J_0'(Kr)}{K J_0^2 K \cdot \frac{D}{2}} e^{-Kc} \cosh Kx \quad (9)$$

which vanishes for r = zero, i.e. along the axis. $J_0'(Kr)$, $J_0(K\frac{D}{2})$ are Bessel Functions. $(K\frac{D}{2})$ in the summation are the roots of the equation $J_1(K\frac{D}{2}) = o$. The corresponding axial velocity on the surface of the body at midship is then given by

$$V_c + \delta v = V_c + \frac{Q}{\frac{\pi D^2}{4}}\left\{1 + \sum \frac{J_0(K\frac{d}{2})}{J_0(K\frac{D}{2})} e^{-Ka}\right\} \quad (10)$$

The value of Q can be calculated from the following equation obtained by Lamb from equation (9) by substituting $d/_2$ for

r, at x = o and equating ψ = zero on the surface of the body:

$$\frac{\pi a^2 v_c}{Q} = \frac{4a^2}{d^2} - 1 + 2\sum \frac{J_0'\left(K\frac{d}{2}\right)}{\frac{Kd}{2}\cdot J_0^2\left(\frac{KD}{2}\right)} \qquad (11)$$

Applying the above equation for the case of ovoid with $\frac{2a}{d}$ = 5, Lamb obtained the following table.

Table 1

$\frac{D}{2a}$	$\frac{4Q}{\pi D^2 v_c}$	$\frac{v_c + \delta v}{v_c}$
½	.1906	1.1909
1	.04191	1.0469
2	.010237	1.0233
	.0000	1.0135

It is to be noticed that the change of velocity at midship is a function of the fineness of the body represented by the ratio $2l/d$, where l is half the length of the body, and of the degree of blockage represented by $\frac{a}{A_t}$ or $\left(\frac{d}{D}\right)^2$. Finally Lamb concludes that if the fineness and the blockage are at all considerable, the following equation holds

$$\frac{v_c + \delta v}{v_c} = \frac{D^2}{D^2 - d^2} \qquad (12)$$

as if the velocity at the middle section were uniform in the space between the model and the walls of the tunnel or tank.

2. The work of Lamb (9), Lock (10) and Lock and Johansen (11) examined the change of speed of the surface of a body of

rotation in a uniform stream in a closed tunnel of arbitrary cross-section. As a final result Lock and Johansen consider that a stream line body of revolution in a tunnel will be equivalent to the same body in unbounded stream of a velocity $v_c + \delta v_1$.

where δv_1, is given by the equation:

$$\frac{\delta v_1}{v_c} = \tau \lambda_1 (2d^2 l \div D^3) Q_1 \qquad (13)$$

where Q_1 is a factor = unity for $\frac{2l}{D} \leq 1$ and equals $\left[1 - q_c \left(\frac{2l}{D}\right)^2\right]$ for $\frac{2l}{D}$ > unity; τ is a coefficient depending on the shape of the cross-section of the tunnel or tank and λ_1 is a coefficient depending on the proportions of the body. λ_1 is given by Lock and Johansen, for the Rankine Ovoid, as

$$\lambda_1 = \frac{d}{2l}\left(1 + \frac{4a^2}{d^2}\right)^{\frac{1}{2}} \qquad (14)$$

Although τ depends on the shape of the cross-section of the tunnel, it varies within narrow limits with the variation of the cross-section as can be seen from the following table given by Lock and Johansen in Reference (11).

Table 2
Values of τ

		τ Closed Jet
Two Dimensions		.82
Three Dimensions	Circular	.797
	Square	.809
	Duplex (width = 2 x height)	1.030

In summary, Lock and Johansen put their results for the Rankine Ovoid as follows:

$$\frac{\delta v_\infty}{v_c} = \frac{d}{2(d^2 + 4a^2)} \qquad \text{(unbounded water)}$$

$$\frac{\delta v_1}{v_c} = \frac{\delta v - \delta v_\infty}{v_c + \delta v_\infty}$$

$$\frac{\delta v_1}{v_c} = \tau \lambda_1 \left(\frac{d}{D}\right)^2 \frac{2l}{D} \left\{ 1 - q_c \left(\frac{2l}{D}\right)^{-2} \right\} \qquad (15)$$

In the above equations δv_∞ equals the increase of velocity on the surface of the body amidship in unbounded water above the uniform speed of the stream, δv_1 is the additional speed which when added to the uniform flow speed will bring about conditions on the surface of the body similar to those caused by tunnel or tank interference, and δv is the increase of speed on the surface of the body amidship when in a closed tunnel, above the uniform speed of the stream. The influence of the tunnel wall interference on such matters as frictional resistance, and pressure distribution on the surface of the body will be that corresponding to an increase of velocity δv_1, over that of the uniform stream v_c. It can be seen from Lock's results that δv_1 depends on the fineness of the body, i.e. on its prismatic coefficient. Again the speed on the surface of the body amidship, in general, differs from the axial speed at other points on the surface in accordance with the equation of continuity, however for a body with practically uniform section this change is very small, such that the mean speed increase contributing to an increase in friction resistance of the body in a closed tunnel over that in unrestricted water may be taken as that amidship unless the body is much tapered towards the ends.

3. If we neglect the free surface, a model in a tank will be exactly equivalent to a double model in a closed tunnel of the same width of the tank and whose height is double the water depth. Subject to these limitations the formulae of Lock and Johansen may be applied for the case of towing tanks at very low Froude number. Also, the velocity increase contributing to the increase in frictional resistance may be taken the increase of speed amidship or more accurately that obtained by assuming the body to have a uniform section area $= a_m = \frac{\nabla}{2l}$, where ∇ is the volume and $2l$ is the length of the body.

4. Recently, Dr. Maria Kirsch [12], investigated the increase of speed on the surface of a Rankine Ovoid, a rotational ellipsoid and two other bodies of rotation representing these bodies by sources and sinks or dipoles of appropriate strength and distribution. The main results of this work may be summarized as follows:

a. The increase in speed amidship above the uniform flow speed is not the maximum, the maximum increase of speed lies at a point away from the midship.

b. The mean increase in speed above the uniform speed of the stream is a function of the fineness of the body and the ratio of the water depth to the diameter of the body, i.e. $\frac{h}{d}$.

c. The presence of the bottom causes an increase of velocity on the surface of the body, over the corresponding speed in water of unrestricted depth. However, this increase of speed can hardly be noticed for $h/d > 3$ and is definitely nil at $h/d = 5$, being less than 0.1% of the speed of the stream, a result which agrees with D.W. Taylor [13]. This additional increase of speed decreases slowly with the increase of $\frac{a}{d}$ or $\frac{l}{d}$, i.e. as the body becomes finer and finer, the change with variation of fineness ratio is more noticable for lower values of $\frac{h}{d}$. For example, for $h/d = 1$ the mean percentage increase of velocity due to shallow water over the corresponding speed in deep water is given by Dr. Kirsch as 2.04% for $a/d = 3$ for the Rankine Ovoid.

5. Examining the results which we obtain by applying Lamb's work [9], to the case of an ovoid in a circular tunnel, which corresponds to the same ovoid in a semi-circular towing tank neglecting the free surface and the viscosity of the water, we get the following table for an ovoid whose $\frac{a}{d} = 2.5$. Table 3 shows that the speed obtained by assuming the flow to be uniformly distributed across the space between the body (i.e. mean flow) and the tunnel or tank walls (as suggested by Lamb) is very near the speed on the surface of the body at low values of D/d or high blockages only, as appears from columns (2) and (3). At lower blockage ratios the difference is large reaching 100% in unrestricted water. If the increase of speed obtained by the previous assumption is taken

Table 3

(1) D/d	(2) $\frac{\delta v}{v_c}$ From Eq. (10) According to Lamb	(3) $\delta v/v_c = \frac{D^2}{D^2-d^2} - 1$ Eq. of Mean Vel.	(4) Difference $\frac{(2)-(3)}{(2)}$ %	(5) $\frac{\delta v - \delta v_\infty}{v_c}$ v from Col. (2)	(6) $\frac{\delta v + \delta v_\infty}{v_c}$ v from Col. (3)	(7) $\frac{\delta v - \delta v_\infty}{v_c}$ v from Col. (3)	(8) $\frac{\text{Col. (6)}}{\text{Col. (2)}}$	(9) $\frac{\text{Col. (7)}}{\text{Col. (5)}}$
2.5	.1909	.1905	.2	.1717	.2097	.1713	109.5	99.8
5.0	.0469	.0417	10.5	.0277	.0609	.0228	130.0	82.3
10.0	.0233	.010	57.3	.0041	.0292	-.0092	125.5	-225
	.0192 = v	.000	100.0	.000	.0192	-.0192	100.0	-

as the speed increase on the surface of the body in a tunnel, over the corresponding speed in unrestricted water, the increase of speed on the body over the speed of the uniform flow will be the sum of the values in column (3) plus the value in column (2) at $\frac{D}{d} = \infty$. We find that the application of the mean increase of speed assumption overestimates the speed on the surface of the body and consequently overestimates increases in frictional resistance and other things which depend on the speed on the surface of the body. Considering the speed increase in column (3) as the total perturbation speed on the surface of the body we find that the increase on the surface of the body obtained by this assumption becomes negative at low values of blockage as appears from columns (7) and (9).

6. Table 4 gives the results of the application of Lamb's work [9] and Lock's work [10], [11] to the case of a Rankine Ovoid ($2\ a/d = 6$, $\frac{2f}{d} = 6.503$) in a circular tunnel and in a square tunnel of equal cross-sectional area.

By definition Lock considers the increase of speed δv_1 obtained from equation (15) as an additional value which, when added to the uniform stream, will bring conditions on the surface of the body in unrestricted water similar to the conditions on the surface of the body in closed tunnel. The values obtained by the application of Lock's results should then compare with the results in column (5) in Table 4 for a circular tunnel and column (3) for a square tunnel.

Table 4

(1) $\frac{D}{d}$	(2) $\frac{\delta v}{v_c}$ C.T.	(3) $\frac{\delta v_\infty}{v_c}$ C.T.&Sq.T mean speed	(4) $\delta v_1 = \frac{(\delta v - \delta v_\infty)}{v_c}$ C.T.	(5) $\frac{(\delta v - \delta v_\infty)}{v_c}$ from Col. (2) C. T.	(6) $\frac{(\delta v - \delta v_\infty)}{v_c}$ Sq.T.	(7) $\frac{\text{Equiv. Width}}{d}$ Sq.T. $\frac{W}{d}$	(8) Col. 5 × $\frac{\tau_1}{\tau_2}$ +.0135	(9) $\frac{\delta v_1 - \delta v}{v_c}$ Col. 5 $\frac{\tau_1}{\tau_2}$
2	.333	.333	-.703	.3198	-.712	1.775	.3375	.324
3	.125	.125	.00628	.1115	.00638	2.625	.1265	.113
4	.0675	.0666	-.0345	.0540	.0351	3.550	.0682	.0547
6	.032	.0286	.0169	.0185	.0172	5.250	.03225	.01875
8	.0192	.01588	.00946	.0057	.0096	7.10	.01928	.00578
10	.0161	.010	.00485	.0026	.0049	8.875	.01614	.00264
∞	.0135	000	000	000	000	∞	.0135	000
	Fr. Eq. (10) Lamb		Eq. (15) Lock	Eq. (10) Lamb	Eq. (15) Lock		τ_1 for Sq.T τ_2 for C. T.	

From Table 4 it appears that equation (15) cannot be used for low values of D/d i.e. for high blockage. For low blockage (D/d = 8→10), equation (15) gives values which practically equal about double the values obtained by application of equation (10) for the case of Rankine Ovoid. Using Lock's results then may lead to speed increments higher than those obtained by more rigorous methods; when we neglect the viscosity and free surface effect.

Table 5 shows a comparison between the results of unbounded shallow water as obtained from Kirsch's report and those of a square tunnel obtained in the following way. It is stated by Lock that the results of a circular tunnel can be approximately applied to other shapes of tunnels by substituting a suitable value of τ. Hence the results from a square tunnel equal the results from a circular tunnel times $\frac{\tau_1}{\tau_2}$ where τ_1 corresponds to a square tunnel and τ_2 corresponds to a circular tunnel. Columns (8) and (9) of Table 4 were obtained from column (2) on this basis. Table 5 shows that for very high blockage ratios the effect of tank walls (defined as total effect of walls minus the effect in shallow water) has a major contribution while for low blockage ratios the contribution of the tank walls is comparatively less important.

It appears from the above considerations that the mean flow assumption has its limitations as it does not represent the speed on the surface of the body, except when the blockage is large. However, while the mean flow assumption gives, for

Table 5

(1) W/d 2h/d	(2) $\frac{\delta v_\infty}{v_c}$ Sq. T Col. (8) Table 4	(3) $\frac{(\delta v - \delta v_\infty)_\infty}{v_c}$ Sq. T Col. (9) Table 4	(4) $\frac{\delta v_\infty}{v_c}$ Shallow Water	(5) $\frac{(\delta v - \delta v_\infty)_\infty}{v_c}$ Shallow Water	(6) Effect of Walls Col. (2)- Col.(4)
1.775	.3375	.324	.048	.0345	.29
2.625	.1265	.113	.0325	.0190	.094
3.550	.0682	.0547	.0243	.0108	.044
7.10	.01928	.00578	.0160	.0025	.0033
9.875	.01614	.00264	.0147	.0012	.0014
∞	.0135	000	.0135	000	000

large blockage, values of speed increase on the surface of the body comparable with those obtained with equation (10), it remains unknown whether the speed increase obtained by the equation of mean increase of flow speed represents the effect of tunnel walls on speed perturbation, i.e. the difference between the speed in the field (including the body surface) in a closed tunnel and in unrestricted water, or whether it represents the total increase of speed over the speed of the uniform flow. In the first case the speed increase obtained by the mean flow equation should be added to the speed of uniform flow to represent the case of unrestricted water. It is clear that by so doing the speed on the surface of the body will be higher than that obtained by equation (10). In the second case the speed on the surface of the body will be underestimated. The error in either case depends on the blockage ratio, or $\frac{a_m}{A_t}$ (area of model ÷ area of tank). Secondly, if the speed increase obtained by equation (12) is taken to represent the increase on the surface of the body over the corresponding speed in unrestricted water, and then if the frictional resistance is calculated according to a formulation which includes a form factor, it would be found that the mean speed assumption would give higher frictional resistance than actually occurs on the body surface. If it is considered as the total increase of speed over the uniform flow then it will underestimate the frictional resistance if the latter is taken as that of the equivalent plate.

7. Another way of looking at the distribution of the increase in speed across the tunnel or tank is to assume a hypothetical surface in place of the walls around the body in unrestricted water, and to assume that the volume of fluid flowing outside this hypothetical surface to be uniformly distributed over the cross-section inside this surface. This is shown in Appendix I. Comparing the speed increase over the surface of the ovoid obtained in this way with the corresponding increase obtained by applying equation (10) (column 5, Table 4) we find that, still, the uniform distribution or mean increase of flow gives values higher than those obtained by equation (10), particularly at low blockage ratios.

A further point to consider is to calculate the increase at the tunnel wall over the corresponding speed at the same place in unrestricted water. For the case of ovoid a/d = 3 we obtain Table 6. Table 6 shows that the contribution of the walls to the increase in speed is greater at the walls than at the body itself, particularly at low blockage. It also shows that even at high values of D/d, or blockage of about one percent, the speed ratio at the tank walls is of the same order as the blockage ratio, and that the increase of speed due to the tank walls is practically the same as that obtained from the mean flow assumption. The corresponding speed increase on the surface of the body is relatively smaller particularly for very small blockage when the increase of speed is small.

Table 6

D/d	(1) $\frac{\delta v_\infty}{v_c}$ unrest. at pl. of T. walls	(2) $\frac{\delta v_\infty}{v_c}$ at tunnel walls	(3) Difference Col. (2) - Col. (1)	(4) $\frac{\delta v_\infty}{v_c}$ on Surface of Ovoid, unrest.	(5) $\frac{\delta v_\infty}{v_c}$ on Surface of Ovoid in tunnel	(6) Difference Col. (5) - Col. (4)	$\frac{\delta v_\infty}{v_c}$ mean flow Eq. (12)
2	.0115	.3333	.3218	↑	.333	.3198	.333
3	.0101	.125	.1149		.125	.1115	.125
4	.00815	.0672	.059		.0675	.054	.0666
6	.00498	.0291	.0241	.0135	.0320	.0185	.0286
8	.003	.0181	.0151		.0192	.0057	.01588
10	.00192	.01255	.01063		.0161	.0026	.0101
∞	000	000	000	↓	.0135	000	000

8. The Effect of Wake: All studies discussed before are made on the assumption that the water is an incompressible inviscid fluid. However the drag of the body including frictional drag causes a wake behind the body. This means that a certain portion of the water moves behind the body relative to the ocean. As seen from Appendix II, the effect of the wake is equivalent to an additional quantity of fluid which is displaced ahead and which causes lowering of level behind the body. In other words if ∇_m is the volume which has to move between the body and tank walls in the nonviscous condition, then the actual volume will be $\nabla_m + \delta\nabla$, where $\delta\nabla$ is an additional volume caused to move by the wake. In effect the body may be looked upon as if its volume has been increased by $\delta\nabla$. To the first approximation all values given in Tables 3 through 6 can be augmented by the ratio $\frac{\nabla_m + \delta\nabla}{\nabla_m}$.

However, if we consider the effect of the friction wake on the distribution of the increase of velocity at different sections along the body we find that near the bow the effect of drag is negligible, while near the stern the effect is appreciable. That is, the effect of friction drag on the increase in flow velocity is not the same at different sections along the body. As shown in Appendix II, the various speed increases given in the above tables for a ship model (neglecting free surface effect) should be increased by the effect of friction drag at midship i.e. by about 15 percent. At the stern of the model the increase should be about 30 percent. (This is based on an average wake factor $\equiv .3$).

9. Effect of Free Surface: The effect of free surface is two-fold:

a. Due to the presence of free surface and the induced velocity between the model and tank there will be a drop of pressure and consequently a lowering of the water level in the tank in accordance with Bernouilli's theorem.

b. Due to the free surface, waves are generated and there is water level elevation at the bow and stern which are not of equal magnitude. Such a difference in elevation will result in static pressure differences between the bow and stern, which in turn will alter the flow around the model.

The first part has been dealt with by different writers in the following way.

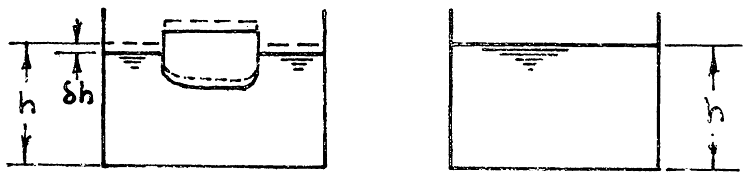


Figure 2

If v_c is the speed of the model and δv is the average induced speed between the model and tank walls then, according to various authors

$$\delta h = \frac{(V_c + \delta v)^2 - V_c^2}{2g}$$

$$= \frac{2\,\delta v \cdot V_c + \delta v^2}{2g} \qquad (16)$$

Equation (16) is obtained by applying Bernouilli's equation relative to the model. It is correct only in the case of a stationary model in flowing water, as in the case of a circulating channel. This equation is also applied to determine the additional sinkage, δh, of the model.

With the conventional tank, the water is stationary and the model is moving. δh shall then be obtained by applying Bernouilli's equation relative to a fixed reference datum in space say the tank bottom, and not relative to the model. Far ahead of the model the water velocity is zero and the depth is h. At the model the water depth is $(h-\delta h)$ and the velocity is δv. By applying Bernouilli's equation it follows that

$$(h - \delta h) + \frac{\delta v^2}{2g} = h$$

or $$\delta h = \frac{\delta v^2}{2g}$$

It follows that δh as given by equation (16) is overestimated. It also follows that unless δv is large, as may result from very large blockage or high velocity, or both, the water depression near the model can safely be neglected. Equation (1) can then be written as

$$\delta v = \frac{a_m \cdot v_c}{[A_t - a_m]}$$

$$\frac{\delta v}{v_c} = \frac{m}{1-m} \qquad (17)$$

The second part of the free surface effect which causes the differential head between bow and stern, thus modifying the flow around the model has not been investigated before to the author's knowledge. When considering the flow very near or on the model surface it is necessary to take into consideration the effect of the presence of a wave elevation, i.e. the non-linear effects which can assume relative importance. From observations such as seen in Reference (14) it is clear that the elevation of the water at the bow is higher than at the stern. See Figures (3a, b, c) which are taken from this reference. The difference in elevation, i.e. in pressure head, will create an additional flow aftwards, which in turn augments the increase in speed over the uniform flow as obtained before. This additional speed will increase the viscous resistance of the model. It will also reduce the time taken by the bow wave to reach the stern in the vicinity of the model. Away from the model the difference in water level becomes smaller and the effect of this difference on the variation in speed becomes less as we go from the model toward the tank walls.

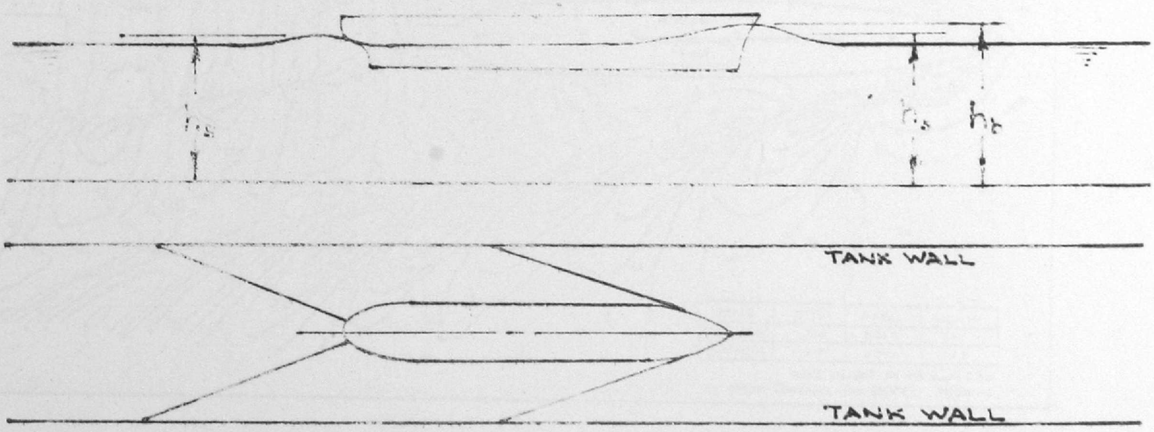
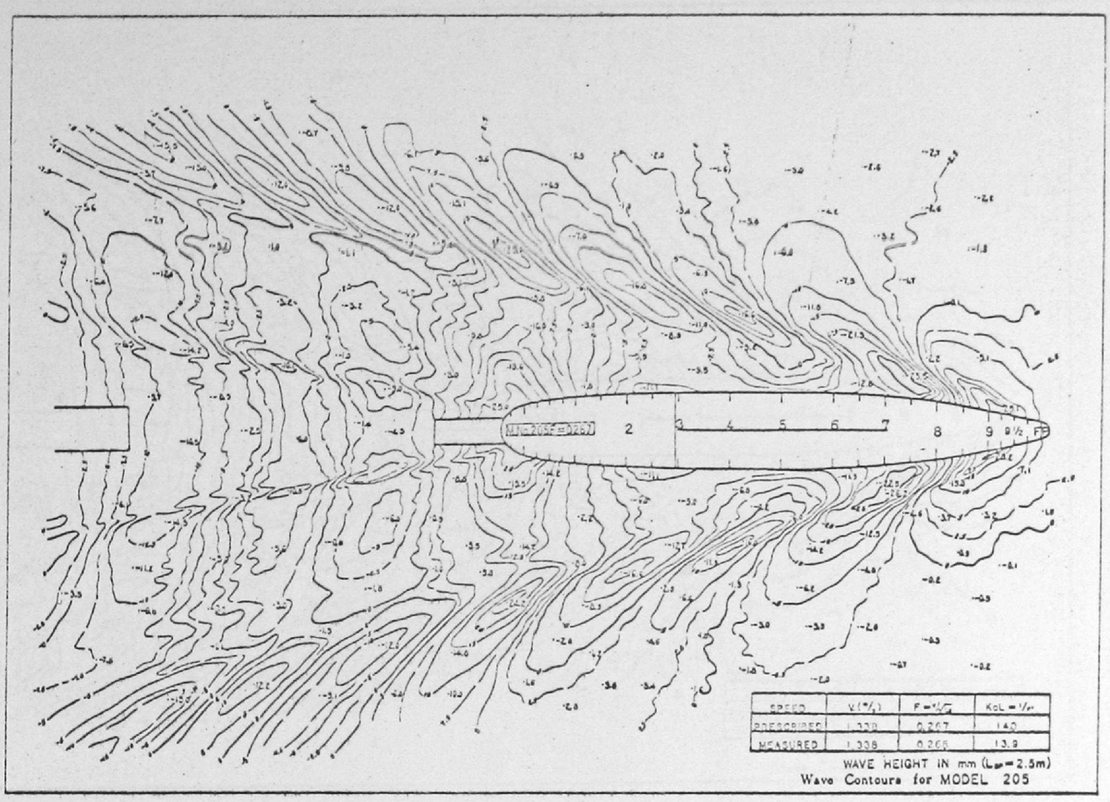
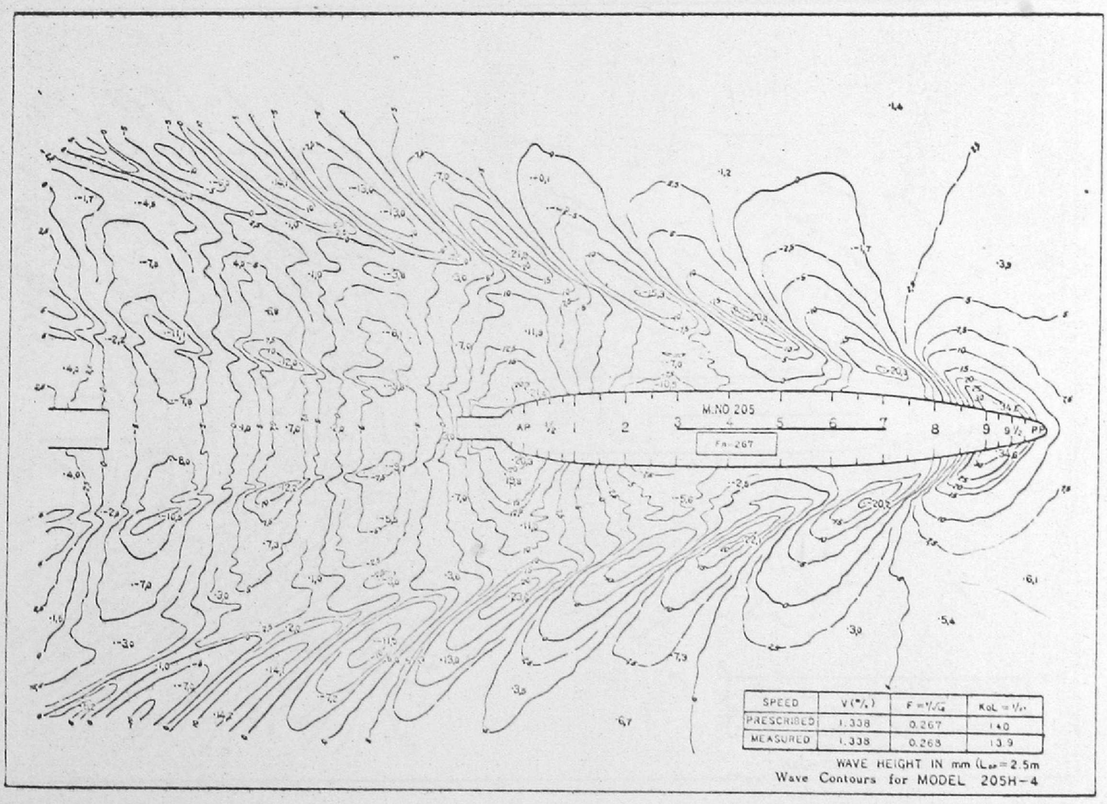


Figure 4



SPEED	V (m/s)	$F = V/\sqrt{gL}$	KoL = 1/F²
PRESCRIBED	1.338	0.267	14.0
MEASURED	1.338	0.266	13.9

WAVE HEIGHT IN mm (L_{pp} = 2.5m)
Wave Contours for MODEL 205

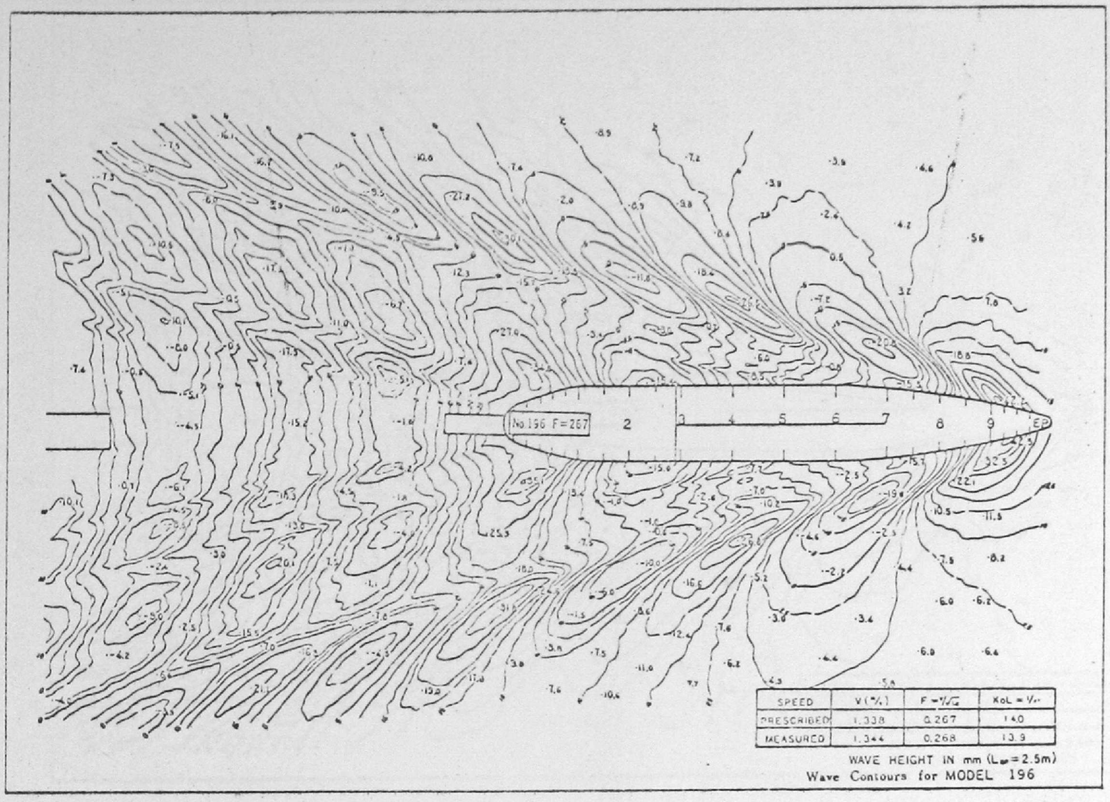


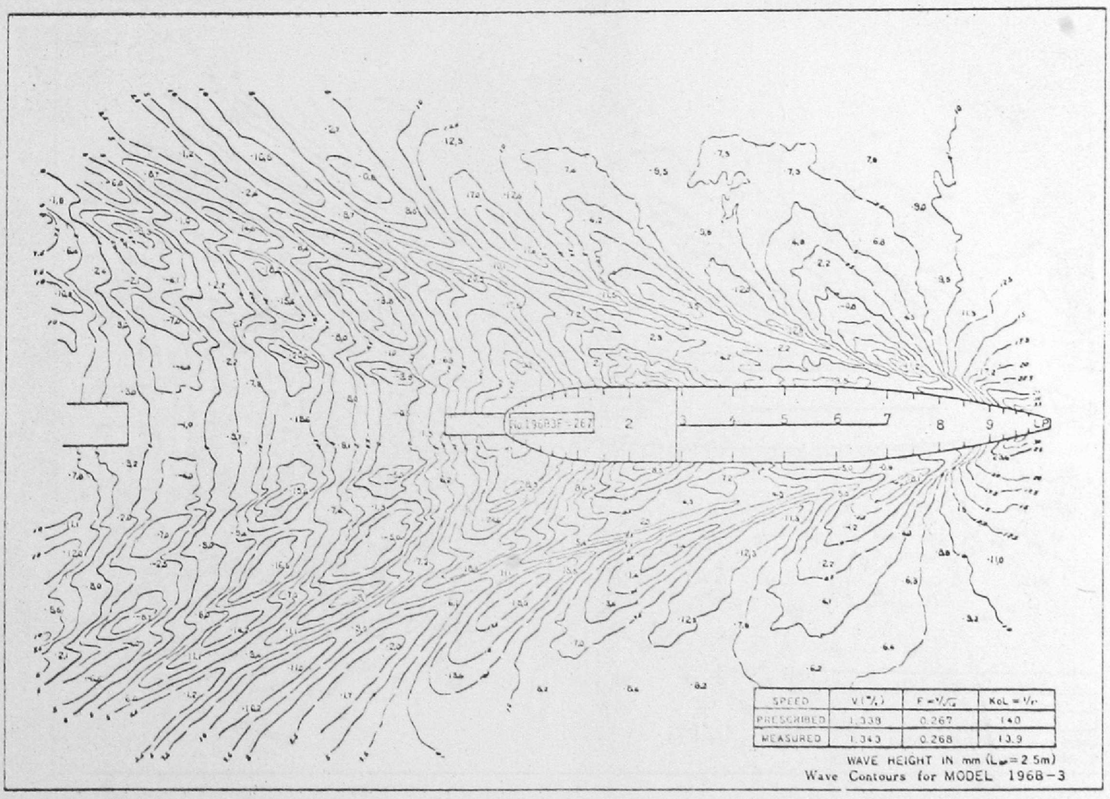
SPEED	V (m/s)	$F = V/\sqrt{gL}$	KoL = 1/F²
PRESCRIBED	1.338	0.267	14.0
MEASURED	1.338	0.268	13.9

WAVE HEIGHT IN mm (L_{pp} = 2.5m
Wave Contours for MODEL 205H-4

Wave-Contours of M. No. 205 (without bulb), upper, and M.No. 205H-4 (with bulb), lower.

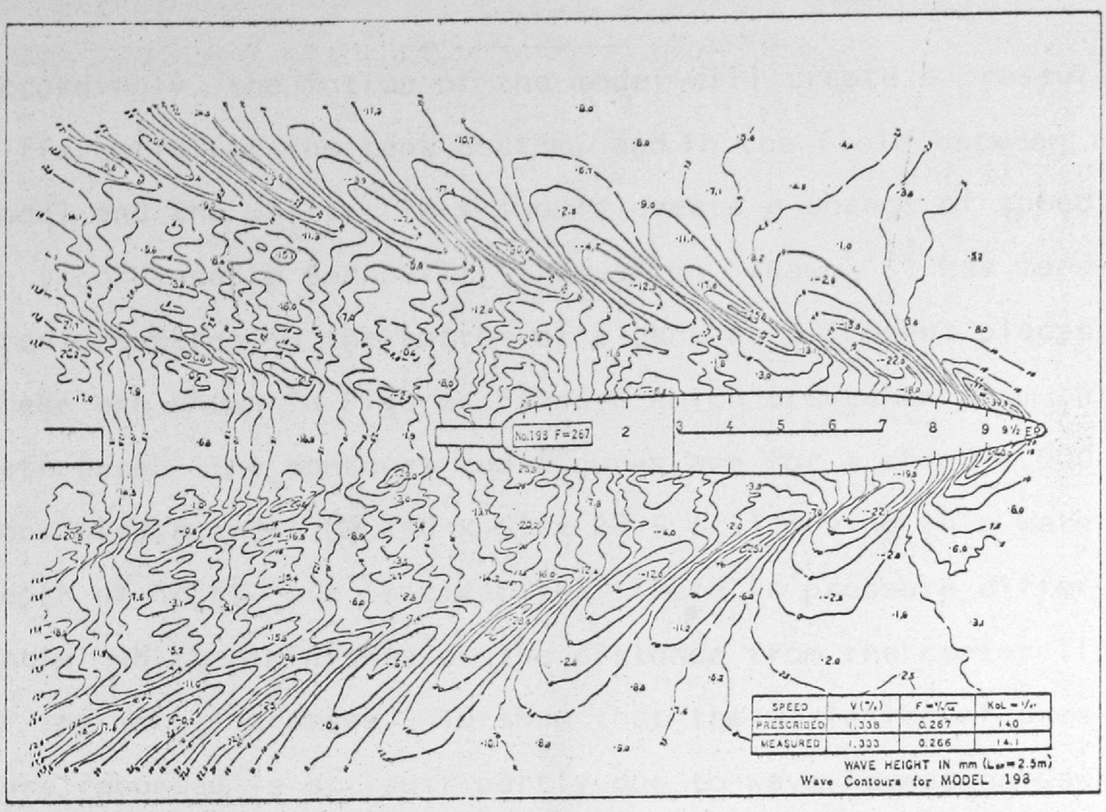
Taken From Ref. 14

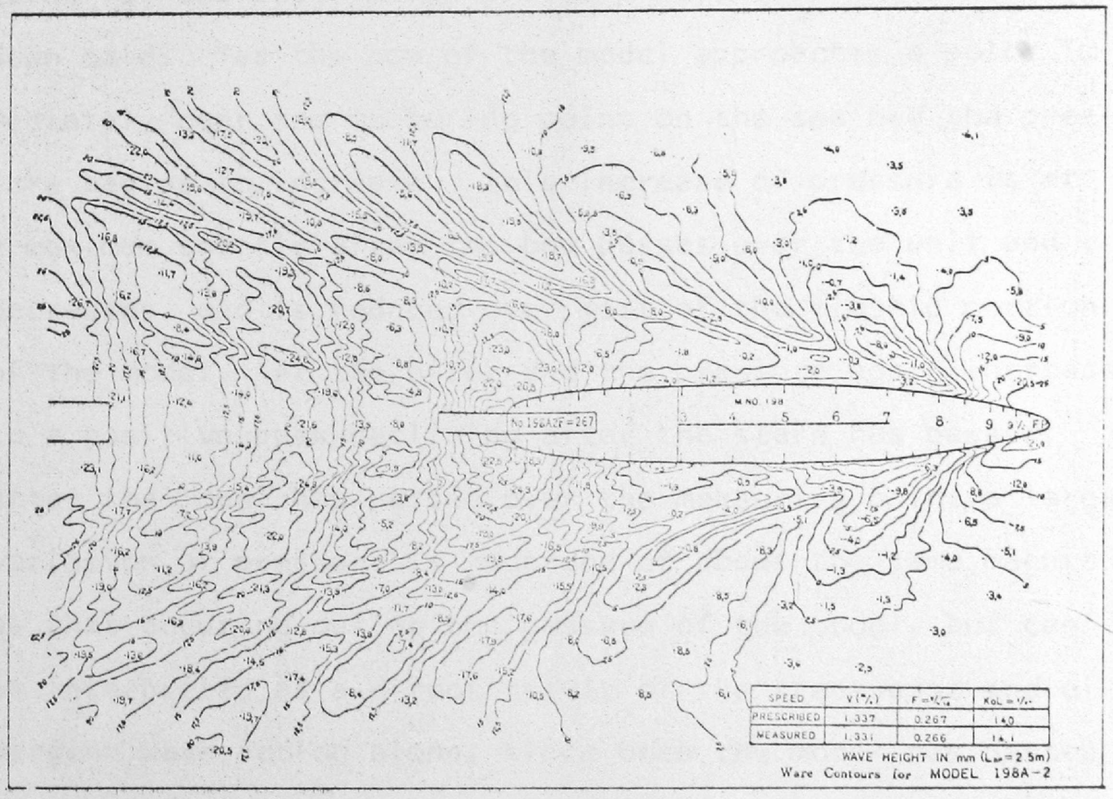




Wave-Contours of M.No. 196 (without bulb), upper, and M. No. 196B-3 (with bulb), lower.

Taken From Ref. 14

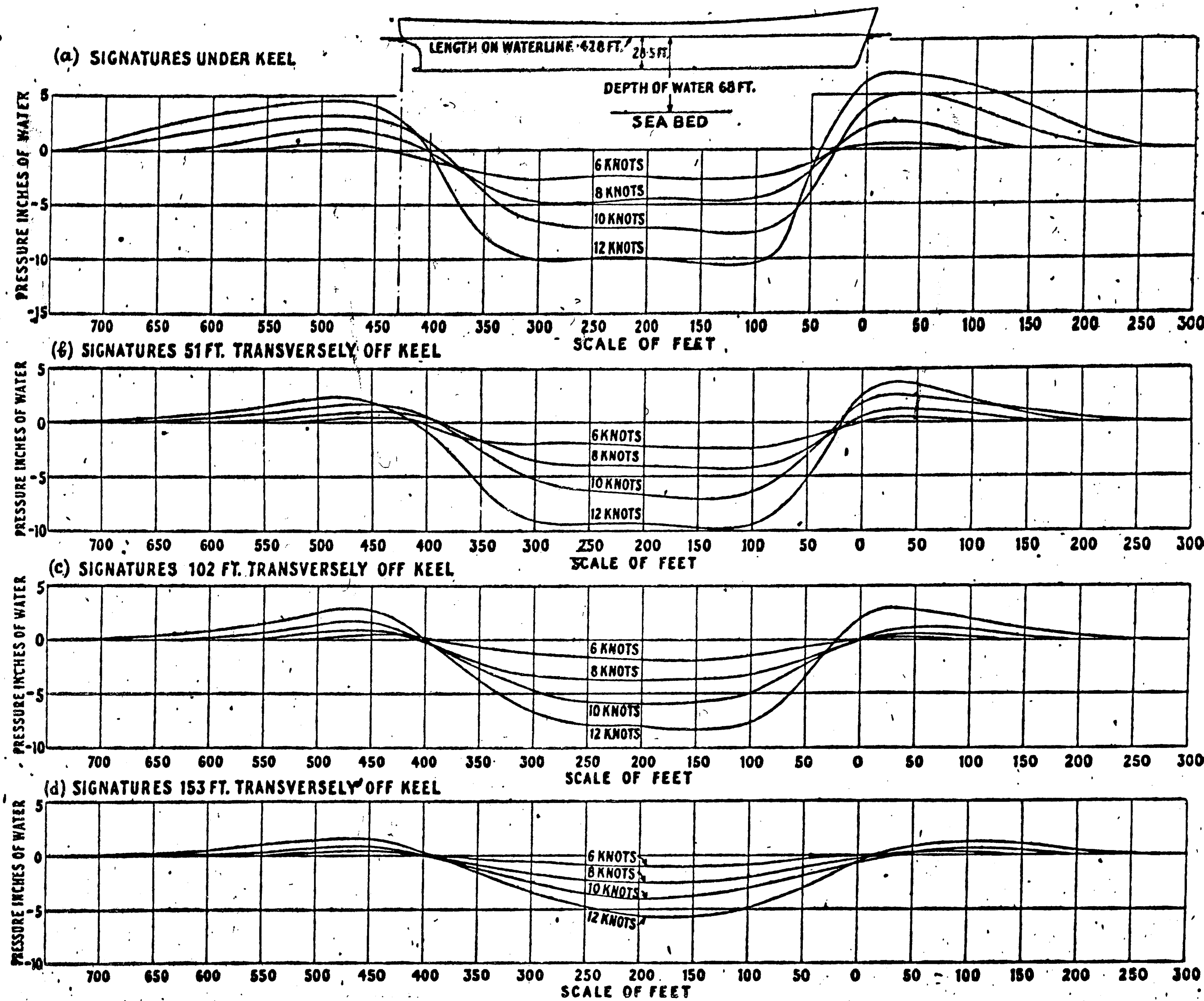




Wave-Contours of M.No. 198 (without bulb), upper, and M.No. 198A-2 (with bulb), lower.

Taken From Ref. 14

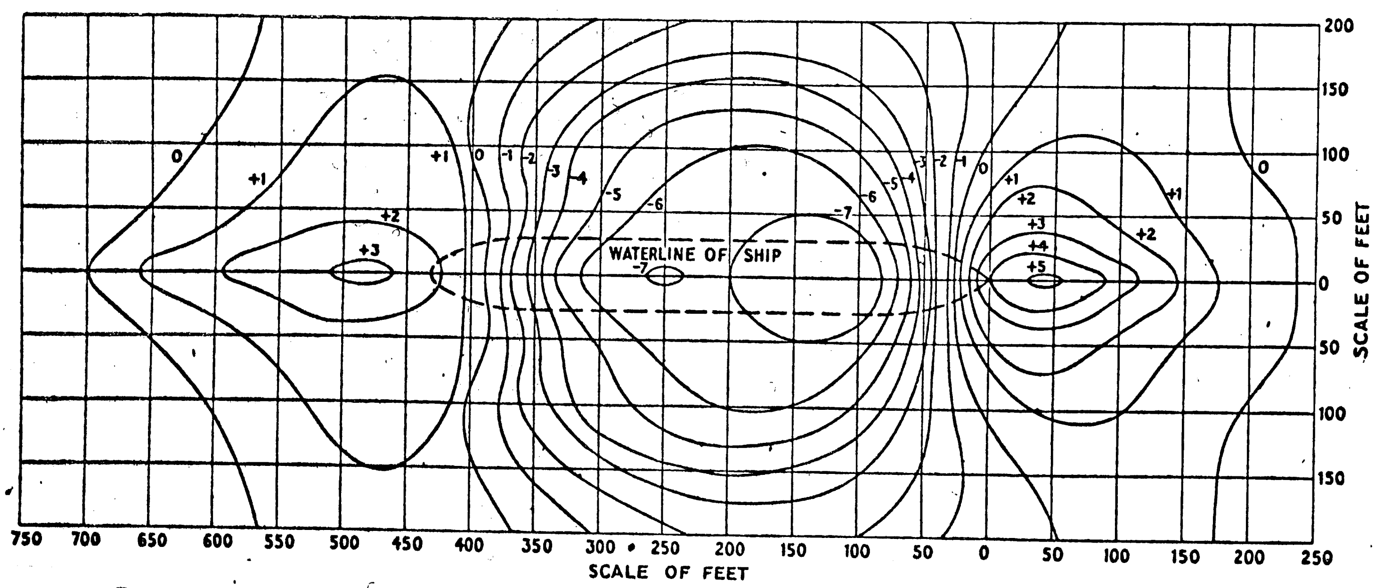
Accordingly, the motion of the model will create a pressure differential at the tank bottom, and in the field between the model and the walls. This should create a change of speed in accordance to Bernouilli's equation. Gawn[(15)] has measured pressures on the bottom of a tank at different places. These are shown in Figures 5 and 6 which are taken from this reference. The contours and figures are for a ship 14,000 tons displacement ($\cong$ 430 x 51 x 28.5 ft.) moving in a water depth of 68 ft. It is clear that there is pressure differential which diminishes as the distance from the center line of the ship increases. To show that the variation of pressure recorded is at least partly due to wave formation Gawn's words for the description of the pressure variation are given. Gawn said: "as the bow of the model approaches a point immediately over the measuring point on the sea-bed the pressure begins to increase. This increase of pressure is at a maximum shortly after the bow passes over the unit and then decreases, and is reduced over most of the midship portion of the model. At the after end the pressure again increases to a positive peak declining after the stern has passed. After the model has passed over the measuring point a large variation of pressure is recorded of about the same magnitude as that occuring during the passage of the model, but cannot be interpreted as a direct result of the transverse and divergent wave action alone, since once the model has passed the measuring point the resultant wave formation is confused by the reflection of divergent waves from the walls of the



-PRESSURE SIGNATURES IN PROFILE FORM

NOTE: The curves are deduced from experiments with ship model scale: 1/?? full size.

Figure 5



-PRESSURE SIGNATURES IN CONTOUR FORM. PRESSURE CONTOURS ON SEA BED PRODUCED BY A SHIP OF 14,000 TONS DISPLACEMENT PROCEEDING AT 10 KNOTS IN 68 FEET DEPTH OF WATER

Figures on contour curves relate to inches of water head.

Figure 6

Taken From Ref. 15

tank water way. Nevertheless, the records subsequent to the passage of the model confirmed theoretical considerations that the pressure changes are consistent with surface wave formation and are in fact simulated in a swell".

The presence of this additional flow caused by the wave pattern can also be verified indirectly by comparing the wake factors of two similar ships, one with a conventional bow and the other with bulbous bow. The presence of a bulbous bow causes cancelation of the bow wave, accordingly the elevation of water at the bow is much reduced while the effect on the elevation of the stern wave is nil. This means that the differential pressure between bow and stern is reduced. The flow around the model should be less, then, in the case of bulbous bow ships than with conventional ships. Accordingly, the wake factor of a bulbous bow ship should be higher for a ship with bulb than for the comparable conventional ship. Wake factors as derived from propeller diagrams for self propulsion tests show that this is the case, as shown in Figure 32 of Reference 21.

As the differential pressure is at its maximum around the model the excess flow caused by it is expected to be concentrated in this area. Therefore, the majority of the volume of water which has to pass aftwards between the model and tank boundaries, which is fixed (= volume of model + volume due to the foreward flow of wake), is concentrated near the model with little left to flow through the rest of tank area. Pressure and wave profile measurements by Hogben (24) indicate that the pressure differential between bow and stern is greatest near the load water line. Further the pressure difference due to wave profile has an oscillatory character dependent on Froude number.

The additional flow caused by this pressure differential is considerable. This can be seen from the fact that it requires only about 3/16" difference in head to create a flow of one foot per second. It is also clear that this additional flow is very small at low Froude numbers when the wave pattern has not developed to an extent sufficient enough to create a large head differential.

It is realized that the additional flow due to the difference in wave height occurs in the open sea and as well as in the bounded tank. However, in a tank which is closed at both ends the effect is much greater than in unrestricted water, since in the latter case the flow goes in all directions while in the tank any flow in any direction other than the aftward direction will ultimately and continuously flow backwards to fill the cavity created by the model.

It is difficult to evaluate the flow due to the difference in the height of the bow and stern waves. It is more difficult to evaluate the difference between the condition in a tank and the condition in open sea. However, we shall come to this point once again.

The additional flow due to the difference in wave heights makes the resultant induced velocity concentrated around the model, i.e. near the tank axis, while at points between the model and tank walls this induced velocity will be much less than near the model. Consequently the induced velocity at the model will be higher than the mean induced velocity in the tank. This point has been verified experimentally by Hughes (8), and Emmerson (7). Hughes obtained the actual C_T of the same model in a small tank and in a large tank at very low Froude numbers, while Emmerson verified this point by direct measurement. Assuming the large tank as a practically unbounded ocean, Hughes found that the induced velocity around a model is double the mean induced velocity and according to Emmerson the actual δv equalled 1.65 times the mean value. These values of Hughes and Emmerson are based on analysis of model results at very low Froude numbers in different tanks. However at large Froude numbers the wave amplitudes increase and the concentration of velocity around the model becomes relatively greater and greater with possible oscillation at humps and hollows. In addition, none of Hughes's or Emmerson's models were tested in unrestricted water. Whatever width the tank has, still it is not an open sea. Accordingly it is probable that the induced speed around the model in a tank will be higher than twice or 1.65 the mean induced speed if the comparison were made between results in a tank and in open sea.

10. As can be seen from the previous analyses, the condition in the tank is such that the major back flow takes place around the model. As the volume of water flowing back per unit time in the steady condition is fixed, the induced velocity at points far from the tank axis will be relatively small.

C. Wave Pattern of a Model in a Tank.

The wave pattern created by a model moving at a uniform speed can be looked upon to be composed of two parts; as was suggested by Eggers (16).

i. Local disturbance which accompanies the model in its motion and

ii. Free waves which trail behind the model.

For a pressure point the free waves are dominant within the Kelvin angle but outside the Kelvin angle the elevation due to the free waves decays exponentially with radial distance times $\frac{g}{v_c^2}$, where g is gravitational acceleration and v_c is the advance speed. The elevation due to the local disturbance dies out only as the inverse third power of the radial distance r (16).

1. Looking on the model as composed of multiple pressure points we may conclude that outside the Kelvin angle of the bow we have only the local disturbance while within this angle we have both the local disturbance and free waves. For a

symmetrical ship or model advancing in nonviscous fluid the local disturbance in the steady condition is symmetrical with respect to midship and contributes nothing to the resistance of the ship. The local disturbance or local rise of water level at the ship's bow and stern and the local sinking of the same amidship may be analogized from the pressure distribution around the hull in a boundless fluid without the free surface. The local disturbance has no contribution to wave making resistance because it does not contribute to the transport of wave energy, and the wave making resistance as measured is that due to the free waves left behind the ship.

The condition in a tank bounded by two walls can be represented by the image method as shown in Figure (7). The number of images such as A, B--should be infinite.

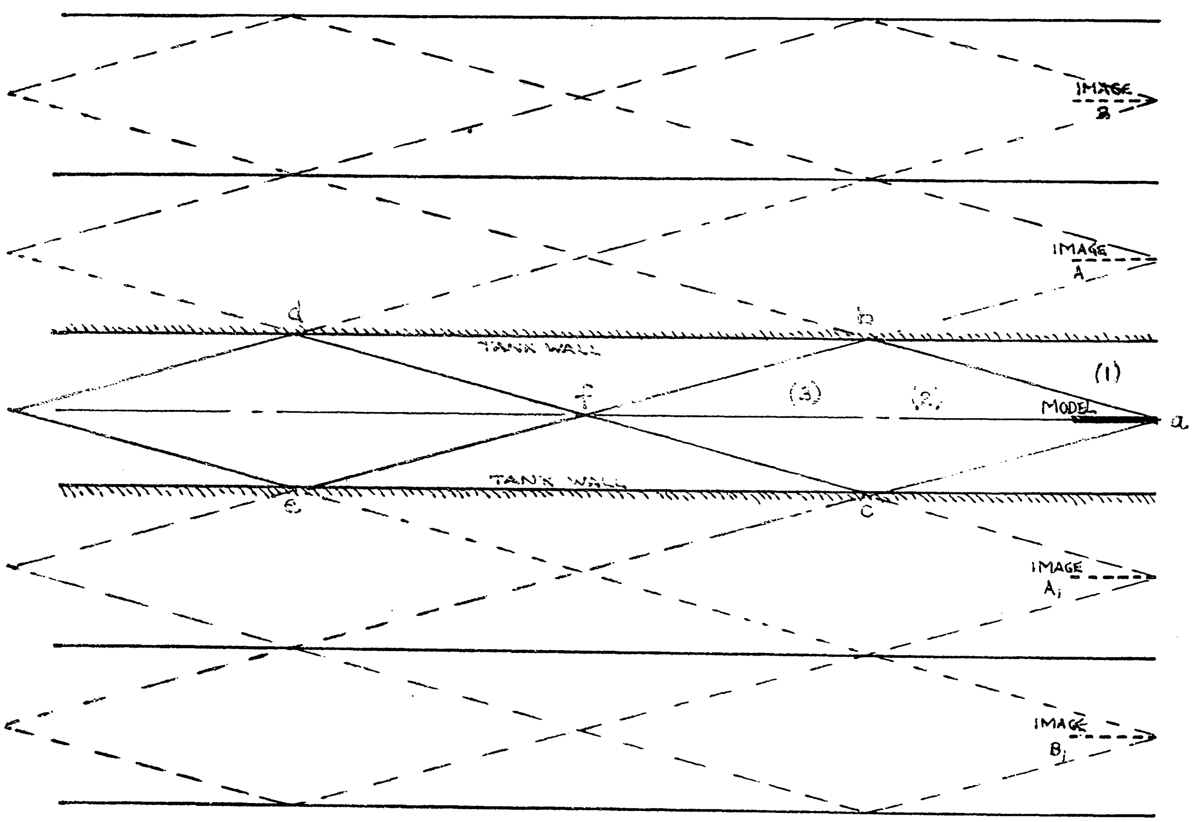


Figure 7

If the elevation of the wave is designated by $\mathcal{L}$, and putting

$\mathcal{L}_{l,M}$ = Local elevation due to the model in open sea

$\mathcal{L}_{l,A,B,C...}$ = Local elevation due to images A, B, C... in open sea

$\mathcal{L}_{F,M}$ = Free wave elevation due to the model in open sea

$\mathcal{L}_{F,A,B,C...}$ = Free wave elevation due to images A, B, C etc. in open sea,

then:

In region (1), i.e. outside the Kelvin angle of the bow wave system, the elevation of the wave will be

$$\mathcal{L}_{l,M} + \left[\mathcal{L}_{l,A} + \mathcal{L}_{l,A_1} + \mathcal{L}_{l,B} + \mathcal{L}_{l,B_1} + \cdots\right]$$

In region (2), i.e. in the triangle abc, the wave elevation will be

$$\mathcal{L}_{F,M} + \mathcal{L}_{l,M} + \left[\mathcal{L}_{l,A} + \mathcal{L}_{l,A_1} + \mathcal{L}_{l,B} + \mathcal{L}_{l,B_1} + \cdots\right]$$

The terms between brackets form the contribution of the images i.e. the wall effect. It is clear that the contribution of the images ahead of the line bc, where the reflection of the divergent waves takes place is all caused by the local disturbances of the images, and their free waves do not come into the picture. In the steady condition the local disturbance of the model and its images is constant and advances with the model, and thus does not contribute to the wave making resistance of the model. In effect the model will be as if it were moving in a statically elevated water level which elevation is the local disturbance due to the images. In arriving at this result we do not take into account the effect of the local disturbances caused by the images on the modification of flow around the model which, in turn, may have an effect on the wave making resistance of certain aft parts such as the shoulders, etc.

It is relevant that the contribution of an image to the wave elevation diminishes as its distance from the point of reference increases, the most important contribution comes from the closer images A and A_1. The effect of the images then increases as the width of the tank decreases, i.e. the change of water level around the model increases as the tank width decreases.

Now we consider the effect of the walls or images on the wave elevation along the model. Taking the bow wave of the

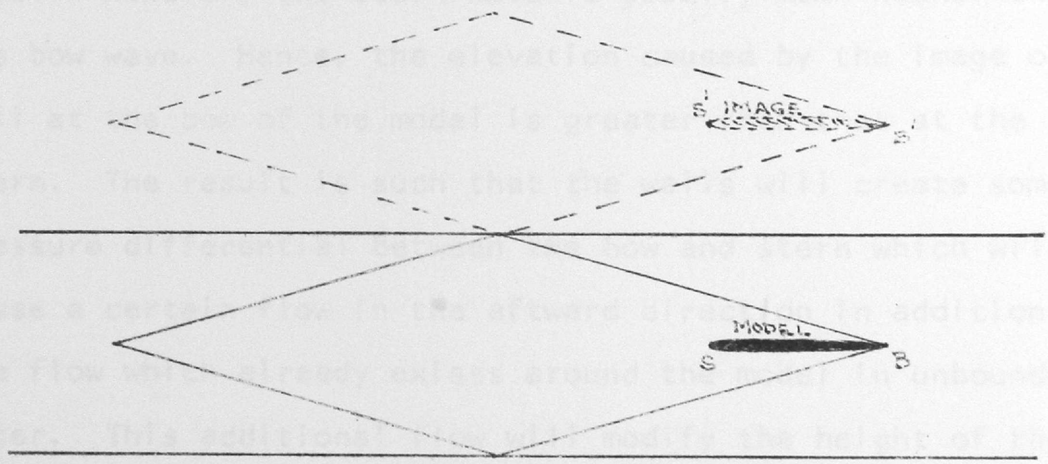


Figure 8

image, we find that its influence on the local wave elevation at point B, Figure (8), is a function of the inverse third power of the distance B'B, where as its influence at the stern is a function of the inverse third power of the distance B'S which is larger than the distance B'B. It follows that the influence of the image on the bow is greater than on the stern. The difference depends on the ratio of B'B to B'S, which in turn is a function of the length of the model and width of tank. If the model length is increased while the tank width is kept constant, then the difference in the elevation of the water between the bow and the stern, caused by the image created bow wave, increases, and vice versa. The same thing may be said for the effect of the stern wave image. If the bow wave and stern wave are of equal strength then the total effect of the model images (wall effect) will be the same at the bow and stern of the model. However, the stern wave is usually much weaker than the bow wave. Hence, the elevation caused by the image or wall at the bow of the model is greater than that at the stern. The result is such that the walls will create some pressure differential between the bow and stern which will cause a certain flow in the aftward direction in addition to the flow which already exists around the model in unbounded water. This additional flow will modify the height of the waves created by different parts behind the bow such as foreward and aft shoulders and the stern.

From the above analyses it appears that the wall effect on the flow around the model is to create an additional flow caused by the local disturbance of the images and the strength of this additional flow for a given speed increases as the ratio of model length to tank width increases.

2. Effect of Walls on Free Waves.

The elevation of water surface caused by an advancing pressure point or disturbance is a function of the square power of the uniform speed of advance of the disturbance relative to the fluid.

The energy of an advancing wave is half potential and half kinetic. The energy per wave length is given by [17].

$$
\begin{aligned}
\text{Pot. Energy } E_P &= \tfrac{1}{4}\varrho g a^2 \lambda \\
\text{Kinetic Energy } E_K &= \tfrac{1}{4}\varrho g a^2 \lambda \qquad (18) \\
\text{Total Energy } E_T &= \tfrac{1}{2}\varrho g a^2 \lambda
\end{aligned}
$$

where a and λ represent the amplitude and wave length respectively.

The total energy as given above is constant for a given wave, i.e. the energy included in a wave length of a wave group is the same irrespective of the position of the particular wave in the group. Hence, the flow of energy per unit time coming from the right of the particular wave is equal to the flow of energy going out of the same wave to the left. If the wave is created by a disturbance, then the work done per unit time equals the energy flowing out the wave, i.e. the energy which is left behind.

The total energy per wave length of an advancing wave is $E_T = 1/2\, g \rho a^2 \lambda b$. As shown by Lamb [17], the Kinetic energy of an element δx of the wave length may be given as

$$\delta E_K = 1/2\, \rho \phi \left(\frac{d\phi}{dy}\right)_{(at\ y = 0)} dx \cdot b$$

where ϕ is the potential function given by

$$\phi = -\frac{ga}{\sigma} \frac{\cosh K(y+h)}{\cosh Kh} \cos Kx \cos(\sigma t + \epsilon) \quad (19)$$

where $\sigma^2 = gK \tanh Kh$, h = water depth, and $K = \frac{2\pi}{\lambda}$

Then $\delta E_K = \frac{1}{2} \rho g a^2 \cos^2(Kx) \cos^2(\sigma t + \epsilon)\, dx$

The potential energy = $dE_p = \frac{1}{2} \rho g \xi^2\, dx \cdot b$

$$= \frac{1}{2} \rho g a^2 \cos^2(Kx) \cos^2(\sigma t + \epsilon)\, dx \cdot b$$

The total energy of an element dx

$$= \frac{1}{2} \rho g a^2 \cos^2(Kx)\, dx \cdot b$$

$$= \frac{1}{2} \rho g a^2 \cos^2\left(\frac{2\pi x}{\lambda}\right) dx \cdot b \quad (20)$$

At the crest or hollow the energy of a wave element dx

$$= \frac{1}{2} \rho g a^2 b\, dx$$

A wave created by an advancing disturbance usually starts by a crest or by a hollow. In the steady condition the work done by the disturbing point per unit time $= \frac{1}{2} \rho g a^2 b \frac{dx}{dt}$. For a time $t = \frac{\lambda}{V_c}$, V_c = speed of advance the work done by the disturbing point $= \frac{1}{2} \rho g a^2 b \frac{dx}{dt} \times \frac{\lambda}{V_c} = \frac{1}{2} \rho g a^2 b \lambda$. During this time t a single wave is created having a total energy $= \frac{1}{2} \rho g a^2 \lambda b$ which equals the work done by the disturbance.

As the energy per wave length is constant $= \frac{1}{2} \rho g a^2 \lambda b$ then it follows that for the same λ, $a^2 b$ = constant. If the

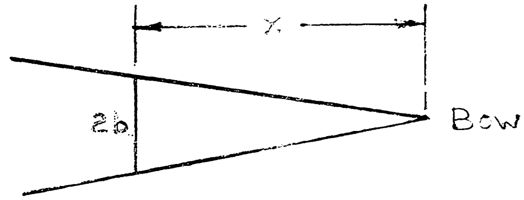


Figure 9

wave is created by a disturbance then $a_x^2 b_x$ = constant or $a_x^2 x$ = const. i.e. the amplitude of the transverse wave is continuously reduced such that $a_x^2 x$ is constant. The same thing may be said about the divergent wave system.

In the steady condition, the energy of an element δx of both the divergent wave and the transverse wave is constant in the x direction, and is equal to the energy supplied by the disturbing point in a corresponding time element δt where $\delta x = v\delta t$. The energy supplied by the disturbing point depends on the conditions at this point, i.e. at the birth of the wave, viz. the relative speed, the depth of the disturbance below the free surface and, in an actual fluid, on the viscosity. It does not depend on what happens to the wave after its birth. Thus if the wave is damped, as all waves are, at some distance after its birth then this will not influence the rate of energy supplied by the body per unit time and consequently the wave making resistance is not influenced by such damping. Consequently the wave making resistance is independent of how the energy of the wave is dissipated, whether by viscosity if the wave is left to spread over the surface of the ocean as in the case of a ship in open sea, or by the use of mechanical means, beaches, walls or otherwise to dampen the waves. It follows also that the wave making resistance is independent from the reflection of the divergent waves provided the reflection does not take place at a point where the wave is being formed, or where any other wave contributing to the wave making resistance is being formed.

3. Coming now to the effect of the gradual increase of speed of water relative to disturbing point after the birth of the wave. In this case the wave is stretched in the X direction relative to the disturbing point.

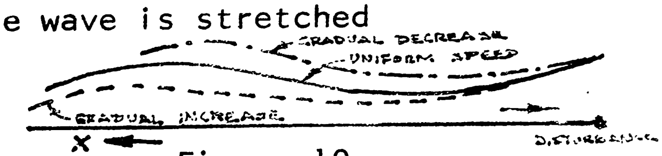


Figure 10

Still the rate of energy supply in the x direction is constant and is dependent only on the conditions at the disturbing point. But due to the stretching of the wave, its amplitude is reduced, and the wave making resistance of the disturbing point is not affected. If the disturbing body is the bow of a ship or model the wave making resistance of the various points depends on the speed at each point. In a tank, when blockage is considered we may conclude that the wave making resistance caused by the points at the stem is not influenced by blockage as the speed at these points is not influenced by the blockage. As we go aft, the speed of the water relative to the various points increases. Consequently the contribution of these points to wave making resistance is affected by the change of speed i.e. by blockage. With small blockage ratios (about 0.01→0.02) the change of speed around the model in the bow area is very small, ranging from zero at the stem to a small value at the foreward shoulder. For practical applications it can be said that for small blockages the wave making resistance of the bow is unaffected.

4. After the point where the divergent waves are reflected from the tank walls, the breadth b of the transverse waves remains constant. As the energy per wave length is constant = $\frac{1}{2}\varrho g a_x^2 b_x \lambda$, then the wave amplitude should remain constant and equal to the amplitude of the transverse wave at the point where the reflection takes place, provided that the effect of viscosity is neglected.

The divergent waves are reflected and directed towards the tank center. Their amplitude depends on the degree by which the walls reflect the waves. The reflection of the divergent waves usually destroys part of the energy of these waves particularly if the walls are provided by beaches or other damping devices.

5. The stern wave is formed under the following conditions:

   a. The speed of the stern relative to the water is reduced due to the frictional wake, hence the amplitude is reduced as the square of the speed.

   b. The speed of the water relative to the disturbing stern is reduced by the speed of the wake, hence the wave length of the stern wave is less than in a perfect fluid.

   c. Due to the pressure differential between the bow and stern there will be an additional flow around the model which will cancel part of the speed reduction caused by friction and form drag. The measured wake is the resultant of the reduction in speed due to frictional and form drag and the increase of flow due to the pressure differential caused by the unequal wave elevation at the bow and stern.

These conditions happen for a ship or model in an infinitely wide ocean. The energy supplied per unit time by the stern to the fluid is dependent on the conditions at the disturbing stern, and equals $\frac{1}{2}\rho g a_s^2 b_s \lambda_s \frac{dx}{dt} = \frac{1}{2}\rho g a_s^2 b_s v_c \lambda_s$ where a, λ_s and b_s are the amplitude, length and breadth of the stern wave system. If v_s is the speed of the stern relative to the water then $a_s^2 \propto v_s^4$ and $\lambda_s \propto v_s^2$. Then the energy per unit time is $\propto v_s^6$. Putting $v_s = (1-w)v_C$ where w is the wake factor, then for a symmetrical ship the energy supplied by the stern = $(1-w)^6$ x the energy supplied by the bow. Putting $w \cong 0.3$ as a mean representative value, the energy supplied by the stern, or the wave making resistance of the stern will be about 10% of the total wave making resistance.

d. When the model is running in a tank bounded with two walls there will be an additional flow due to the presence of the walls as explained before, caused by the local disturbance of the images, and by the constriction or blockage due to the presence of the model. This will increase the speed of the water relative to the stern. In consequence the wave making resistance caused by the stern increases. To form an idea about the increase in stern wave resistance due to blockage Table 7 is prepared.

Table 7

Assumed Inc. of flow due to blockage $\varepsilon = \delta v/v_c$	Resultant wake factor $w - \varepsilon$	$1-w+\varepsilon = \frac{v_s}{v_c}$	$(1-w+\varepsilon)^6 = \frac{R_{ws}}{R_{wb}}$	$\frac{\text{Inc. in } R_{ws}}{R_{wb}}$ (blockage effect)	$\frac{\text{Inc. in } R_{ws}}{R_w \text{ open water}}$ (interference term ignored)	$\frac{\text{Inc. in } R_{ws}}{R_w \text{ in tank}}$ (interference term ignored)
0	.3	.7	w = .3 .1175	0	0	0
.01	.29	.71	.1275	.01	.00895	.00888
.02	.28	.72	.1395	.022	.0197	.0193
.04	.26	.74	.1646	.047	.042	.0403
.06	.24	.76	.194	.077	.0656	.0645
.08	.22	.78	.225	.1075	.0915	.0877
.1	.2	.80	.2625	.145	.1235	.1147
0	.25	.75	w = .25 .1785	0	0	0
.01	.24	.76	.194	.0155	.01315	.013
.02	.23	.77	.2085	.03	.0255	.0249
.04	.21	.79	.243	.0645	.0548	.0518
.06	.19	.81	.282	.1035	.088	.0807
.08	.17	.83	.327	.1485	.127	.112
.1	.15	.85	.443	.2645	.2245	.183

Table 7 shows that the proportional increase of wave making resistance of the stern wave system is of the order of the additional flow caused by blockage except when this additional flow is appreciable. It also shows that the proportional increase of wave making resistance of the stern wave system increases as the wake factor decreases. The Figures in Table 7 indicate the influence of blockage on the wave making resistance of the stern. For an actual ship or model the wave making resistance of the stern may be different from that of the bow even in an ideal fluid. The picture as indicated by the numerical values in the table may then be different, however it is believed that the difference is not appreciable.

6. Some authors claim that the energy of the bow wave system and consequently the wave resistance caused by it, is influenced by the fact that part of the transverse waves of the system trail behind the model in the wake belt which has a certain speed relative to space in the direction of motion of the model. As shown before this has no effect on the energy supplied by the bow as a disturbing body. What happens in such a case is that the wave length is contracted, however the energy per wave length will remain the same, the amplitude of the wave in the region of the wake belt increases such that the total energy per wave length will remain constant.

7. The Interference Between Bow and Stern Systems.

The stern wave is formed in a water whose surface is being disturbed by the bow transverse waves. Considering two waves of equal length, of amplitudes a_1 & a_2, and assume that the distance between the crest of one wave and the crest of the other next to it = Z, then the amplitude a of the resulting wave is given by

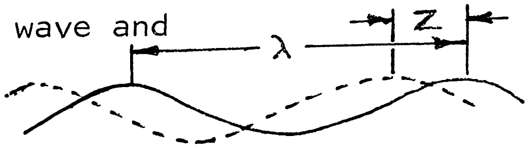


Figure 11

$$a^2 = a_1^2 + a_2^2 + 2a_1a_2 \cos \frac{2\pi Z}{\lambda} \qquad (21)$$

The resultant wave will have the same length λ. At the stern the wave length is $\lambda_s = \frac{2\pi V_s^2}{g}$. The bow wave length in the vicinity of the stern will be also λ_s, for the wave, being stationary relative to the model, will have a reduced speed relative to the water and hence its length is reduced, but the energy included remains unchanged. The wave amplitude of the bow wave near the stern will be reduced such that $a_{bs}^2 \times_s \lambda_s$ = constant as explained before, or the amplitude of the bow wave near the stern in an actual fluid will be equal to $\frac{\lambda}{\lambda_s}$ of its amplitude near the stern in an ideal fluid.

If the amplitude of the bow wave at the point of birth of the stern wave is a_{bl} and the amplitude of the stern wave at its birth is a_s then the resultant amplitude will be given by

$$a^2 = a_{bl}^2 + a_s^2 + 2a_{bl}a_s \cos \frac{2\pi Z}{\lambda_s} \qquad (22)$$

where λ_s= length of the stern wave at its birth = length of the resultant wave = length of the bow wave at the stern. The third term in equation (22) causes the humps and hollows in the resistance curve. The position of humps and hollows depends on the phase difference between the two waves. For a given model in an ideal fluid it depends on $\frac{Z}{\lambda_s}$, or $\frac{Z}{\lambda}$ as $\lambda = \lambda_s$ in this case, i.e. on the speed and length of model. For an actual fluid the length of the bow wave system is reduced by the wake which means that Z is greater for a model in an actual fluid than in an ideal fluid for a given speed of advance.

If the speed around the model is increased by blockage then Z will be shorter than when the model is tested in unbounded water. This means that the humps and hollows are shifted towards the lower speed as the additional flow caused by the blockage increases, i.e. as the blockage increases. This result is exhibited experimentally. The precise location of the humps and hollows is a very important consideration in hull design.

If the position of humps and hollows is known for a model in unbounded water, then the shift of the humps and hollows can be taken as a measure of the additional flow caused by blockage. A method using this shift of humps and hollows for the determination of the additional flow will be given later.

8. The Reflection of the Wave by the Walls:

As shown before, the reflection of a wave from the walls will not affect the flow of energy of that wave because such reflection does not influence the conditions under which the wave is being formed. However, after the reflection the transverse wave is prevented from spreading in a lateral direction, hence the energy included in the wave will be distributed over a certain fixed breadth. Therefore the amplitude of the wave will cease to diminish as we go aft of the point of reflection. If the reflection of the bow wave occurs ahead of the place where the stern wave is formed then such reflection will influence the conditions under which the stern wave is born. Consequently, the stern wave will be formed under different condtions than the case where there has been no reflection. If the reflection of the bow wave takes place some distance further aft of the stern, then the stern wave will be formed under conditions similar to the case where no reflection takes place. To avoid the influence of reflections on the formation of the stern wave, and on the energy necessary to form it, then it is important to make the reflection of the bow wave take place such that the formation of the stern wave is not affected.

To ensure this we may refer to the diagram, Figure 12, which represents one of the Kelvin waves created by a moving pressure point. Assuming the bow wave to be represented by Kelvin waves, we find that the transverse waves are composed of curved waves such as CAC. Assuming an origin traveling with the pressure point of o, the line CAC may be represented in polar coordinates by

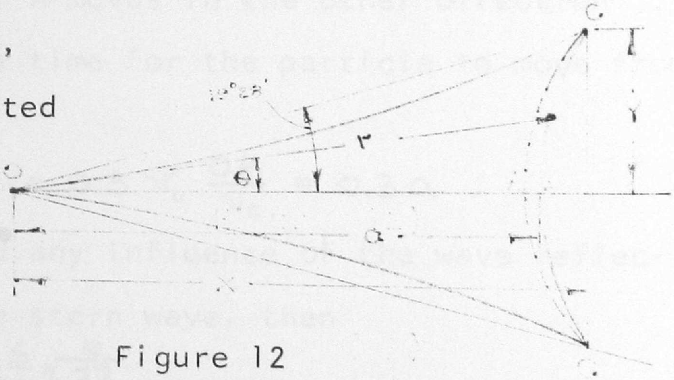


Figure 12

$$r^4 - a^2 r^2 (1 + 18 \sin^2\theta - 27 \sin^4\theta) + 16 a^4 \sin^2\theta = 0 \qquad (23)$$

At points C where the transverse waves meet the divergent waves, $r = OC = \sqrt{\frac{4}{3}}\, a$, hence $Y = r \sin = \sqrt{\frac{4}{3}}\, a \times \frac{1}{3} = \sqrt{\frac{4}{27}}\, a$. Considering the point at which the bow divergent wave is reflected from the wall we find that $Y = 1/2\, b = \sqrt{\frac{4}{27}}\, a$ or $b = (\frac{4}{3}\sqrt{\frac{1}{3}} \times a)$ = breadth of the tank. It has been shown that the transverse wave amplitude will remain unchanged after the divergent waves hit the walls, i.e. after a distance a from the traveling point given by $b = \frac{4}{3}\sqrt{\frac{1}{3}} \cdot a$. If the traveling point is the bow of the ship and a is the length of its water line, then, in order that the first line of the bow transverse wave affected by wave reflection will not be at the stern where the stern wave is formed, the wideth b must be equal to or more than $\frac{4}{3} \times \sqrt{1/3}\, a$, i.e. $b \geq 0.77\, a$. This is true for a perfect inviscid fluid. For an actual fluid, point A in Figure 12, is situated somewhat to the left. To estimate the distance AA', and assuming the average wake factor = 0.3, we can assume that the point A of the transverse wave has moved foreward with the pressure point at a speed of $v_W \equiv .3\, v_C$, i.e. during the time a particle moves from o to A at a relative speed v_C in a perfect fluid, A moves in the other direction a distance AA'. If t is the time for the particle to move from point o to A then

$$AA' = 0.3\, v_c \times t = 0.3\, v_c \frac{OA}{v_c} = 0.3\, a .$$

Therefore, in order to avoid any influence of the wave reflection on the formation of the stern wave, then

$$l + 0.3a \leqq a \leqq \frac{b}{1.77}$$

where l = distance between the farthest foreward point of the immersed bow to the farthest aft point of the immersed stern, both points participating in the wave formation.

Hence $$l \leqq 0.7 \times \frac{b}{1.77} \leq .9\,b \qquad (24)$$

9. An attempt has been made to approximate the influence of wave reflection from the walls on wave making resistance, if the reflection takes place ahead of the stern. Certain approximate assumptions are made, viz,

a. In an ideal fluid the bow and the stern waves are of the same amplitude, and the flow is assumed potential.

b. Due to the wake effect, the velocity relative to the stern is less than the velocity relative to the bow, hence the amplitude of the stern wave = $(1-w)^4$ x amplitude of the bow wave and the stern wave length = λ_s = $(1-w)^2$ x bow wave length .

c. The width of the bow wave and stern wave at the birth = 1/12 x beam of ship or model. (See Appendix III).

d. The ship or model is assumed to advance in deep water and only the reflection from the walls is considered, blockage being ignored.

e. The bow transverse wave in the vicinity of the stern will have the same wave length λ_s as the stern wave. By these assumptions and as shown in Appendix III, the effect of the wave reflection, when it takes place at an assumed distance = 0.75 x length of model from the bow, is only +2.7% of the total wave making resistance at a hump in the resistance curve and -2.8 at a hollow.

These figures are by necessity approximate, however they indicate the order of magnitude of the correction to the wave making resistance needed. They are for a ship whose $l/B = 7.5$. Obviously the correction increases as l/B decreases. The correction is apparently small and decreases as the point of reflection moves aft towards the stern until it becomes zero when the width of the tank = 1.1 length of model, as shown before. If the oscillating term is referred to the monotonic wave making resistance of the bow and stern, its percentage becomes ± 10.9% when there is no wall reflection and ± 14.2% when reflection takes place as assumed above, a difference of about ± 3% only.

10. Effect of Water Depth:

The effect of water depth on wave making resistance has been dealt with in detail by various authors. It is generally accepted that such effect is nil if the speed of the moving body is related to the water depth by the following equation

$$\frac{v_c^2}{gh} < 0.5 \quad \text{OR} \quad \frac{v_c^2}{h} < 16. \qquad (25)$$

For a tank 10 feet deep, $v_c < 12.5$ ft./sec. If the speed has to be greater than that given by eq. (25), a correction of the wave making resistance has to be made following one or the other methods found in the literature, for example the method given by Schlichting[18] which leads in many cases to a reasonably good approximation in the velocity range below the critical speed[19].

For the present work it is assumed that the speed of the model will be within the limits given by eq. (25). If it is greater, a separate correction will be made as by Schlichting or any other proven method.

PART II:

A. DETERMINIATION OF THE SPEED AROUND A MODEL FROM THE SHIFT OF HUMPS AND HOLLOWS OF THE RESISTANCE CURVES, AND CORRECTIONS TO RESISTANCE COMPONENTS:

1. Proposed Method.

As has been mentioned before, the resistance curves for restricted waters are similar to those in deep unbounded water. Humps and hollows of restricted water curves are shifted toward the lower speeds as the channel becomes more restricted.

The reason for the shift of humps and hollows can be seen from the following figure. The distance between points B and S which represent the nearest bow and stern wave crests, respectively, is a fixed portion of the model length. The

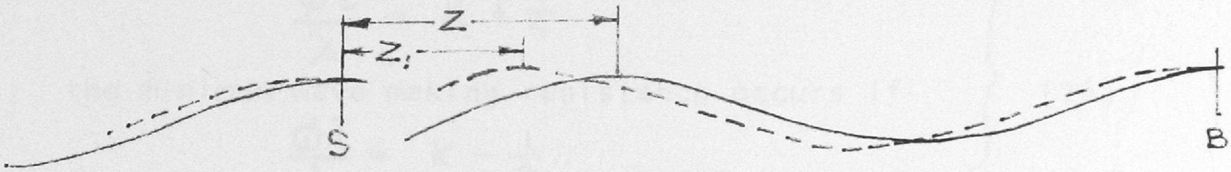


Figure 13

continuous lines represent the bow and stern waves in deep unrestricted water while the dotted lines represent the same in restricted water. The bow wave length λ in unrestricted water is proportional to v_c^2 where v_c is the advance speed of the model. In restricted water the wave length is proportional to $(v_c + \delta_v)^2$ where δv is the additional speed due to blockage. In other words the bow wave length in restricted water is greater than the bow wave length in unrestricted water. Hence the phase difference $Z_1 < Z$. As the hump occurs when the phase difference vanishes, and as B and S are considered fixed, then the hump occurs in both cases at the same bow wave length λ, i.e. the same wave speed. But the wave speed in unrestricted water is assumed to be equal to the speed of the carriage v_c (or a function of it) while in restricted water it equals $(V_c + \delta v)$. Hence, in restricted water the speed of the carriage at which the hump occurs must be less than the speed of the carriage at which the hump occurs in unrestricted water. If both speeds are known with a sufficient degree of accuracy then the difference may be taken as the average additional speed due to blockage.

According to Lap[19], after Baker and Kent [20], the maximum wave making resistance occurs if

$$\frac{\phi l}{\lambda} = k + \frac{1}{4}$$

the minimum wave making resistance occurs if

$$\frac{\phi l}{\lambda} = k - \frac{1}{4} \qquad (26)$$

where k is an integer, 0, 1, 2, 3,....

ϕl is a function of the model length, ϕ being a certain coefficient of length, and λ is the wave length. ϕ is given as the prismatic coefficient.

The above result can be put as follows

$$\frac{\phi l}{k+\frac{1}{4}} = \lambda \propto v_c^2 \quad \text{for maximum wave resistance or a hump,}$$

$$\frac{\phi l}{k-\frac{1}{4}} = \lambda \propto v_c^2 \quad \text{for minimum wave resistance or a hollow.} \qquad (27)$$

If we know k for the successive humps and hollows of the resistance curve, we can determine the values of v_c at which the humps and hollows occur in unrestricted water. Lap[19] gives a diagram from which (k + 1/4) and (k - 1/4) can be obtained once we know ϕ and $\frac{v_c}{v}$. Knowing the speed of the carriage at which the humps and hollows of the resistance curve in restricted water occur we can determine the increase of speed due to blockage, being the difference between the speed obtained from eq. (27) and the speed obtained from the resistance curve. To obtain the speed from eq. (27), it is necessary to know the exact value of the length term in the equation. However, in actual application this is not necessary as is clear from the following:

Suppose we obtain the values of k_1 and k_2 for the first and second humps, respectively. Hence, assuming the length term to be independent of the speed of the model, we have

$$\frac{\lambda_1}{\lambda_2} = \frac{k_2 + \frac{1}{4}}{K_1 + \frac{1}{4}} = \frac{v_{1c}^{\,2}}{v_{2c}^{\,2}}$$

$$\text{OR} \quad \frac{v_{1c}}{v_{2c}} = \left(\frac{k_2 + \frac{1}{4}}{K_1 + \frac{1}{4}}\right)^{\frac{1}{2}} = \left(\frac{4K_2 + 1}{4K_1 + 1}\right)^{\frac{1}{2}} \qquad (28)$$

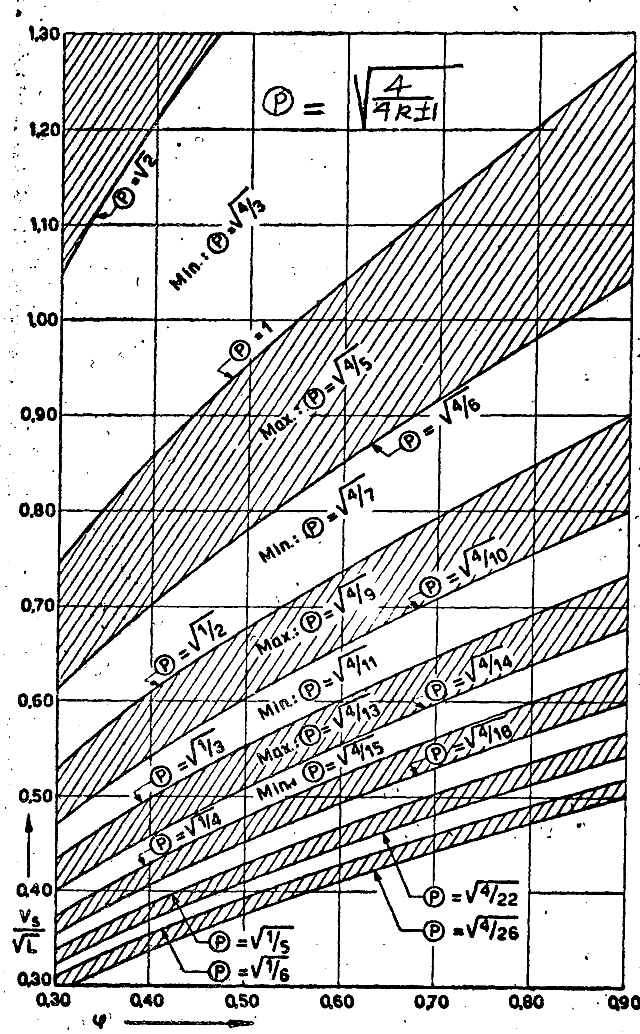
Similarly if k_3 is the value of k at the first hollow then

$$\frac{v_{1c}}{v_{3c}} = \left(\frac{4K_3 - 1}{4K_1 + 1}\right)^{\frac{1}{2}} \qquad (28a)$$

$(k_1 + 1/4)$, $(k_2 + 1/4)$, $(k_3 - 1/4)$ ... etc. may be obtained from the diagram given by Lap(19) which is reproduced in Figure (14). For restricted water, equations (28) and (28a) become

$$\frac{v_{1c} + \delta_1 v}{v_{2c} + \delta_2 v} = \left(\frac{4K_2 + 1}{4K_1 + 1}\right)^{\frac{1}{2}}$$

$$\frac{v_{1c} + \delta_1 v}{v_{3c} + \delta_3 v} = \left(\frac{4K_3 - 1}{4K_1 + 1}\right)^{\frac{1}{2}} \qquad (29)$$



Humps and hollows according to the (P)-theory
(V_s in kn, *L* in ft)

Figure 14

Taken From Ref. 19

v_{1c}, v_{2c}, v_{3c} ... are the carriage velocities at which the humps and hollows occur in restricted water.

The solution of eq. (29) is not possible, however, experimental tests by Hughes at very low Froude numbers show that $\delta v \cong$ 2 x mean speed obtained by the mean speed equation. While Hughes used the factor 2 for the whole speed range, we shall assume it applicable up to the first clear hump or hollow only. Hence $\delta_1 v$ in eq. (29), can be approximated from the mean flow equation as obtained before, i.e. $\delta_1 v = \frac{2 a_m}{A_t - a_m} \cdot v$ It may be argued that the use of $\lambda_1, \lambda_2, \lambda_3$ ... etc. in the above analyses, where λ_1, λ_2 ... are based on the speed of advance of the model is incorrect, since at the ship side the wave length is influenced by the friction wake. However, as we obtain the equation for the relative speed on the surface of the body we use the ratios $\frac{\lambda_1}{\lambda_2}$, $\frac{\lambda_1}{\lambda_3}$, etc. i.e. the influence of the wake on the wave length is cancelled, being practically a fixed ratio from the wave length in an ideal fluid.

2. Advantage and Difficulties of the Proposed Method: Obtaining the additional speed around a model caused by blockage by the proposed method has many merits. On the otherhand there are some difficulties in applying it.

The proposed method allows the determination of the blockage effect from the test results of the same model in the same tank. Therefore it is not necessary to test geosim models in the same tank, or the same model in different tanks, both cases involve experimental and manufacturing difference. As for testing the same model in a tank whose size can be changed, the expenses encountered and the uncertainty of the behavior of the temporary walls and bottom makes such a procedure rather expensive and with uncertain results.

It is clear that the proposed method is far cheaper than any of the other methods which have been used. In fact, it does not require more than careful determination of the humps and hollows of the resistance curves, which, with some understanding by those running the tests, can easily be achieved.

It is realized that the resistance curves of some models do not exhibit the humps and hollows very clearly. However, an average value of additional speeds obtained from other models of the same proportions and dimensions may be used for those whose humps and hollows are not clear. Data accumulated from model tests in a certain tank may be analyzed to arrive at standard corrections to be used for routine work in such a tank.

The method has been applied to the models reported by Hughes(8) in his 1961 INA paper. Froude numbers at which the humps and hollows occur were taken from those given by Hughes. When the Froude number at the lowest hump is not given the mean value of other models was used. No mention was given in Hughes' paper regarding the particulars of the tanks in which the models were tested. The results of the application are given in Table 8. The values of $\varepsilon = \frac{\delta v}{v_c}$ given in Table 8 are erratic and look very high compared with those

TABLE 8

Values of $\varepsilon = \frac{\delta v}{v_c}$ for Hughes Models

Model No. Hughes	l/C_b	Block Ratio Hughes	Hump (1)	Hollow (2)	Hump (3)	Hollow (4)	Hump (5)	Hollow (6)	Hump (7)	Hollow (8)
18	$\frac{19.38'}{.799}$	0.61	.0122	.0348	.046	.06				
29	$\frac{18.11'}{.7}$	.49	.0098*		.0412	.0476	.0673	.1065		
30	$\frac{20.83'}{.671}$	.67	.0134*			.003	.048	.127		
31	$\frac{16.09'}{.496}$	.5	.01*			.0853	.0394	.07	.105	
32	$\frac{15.25'}{.532}$	.57	.0114*		.045	-.0045	.0412	.0466	.209	
33	$\frac{15.02'}{.485}$	.44	.0088*		.075	.043	.0575	.13	.28	
40	$\frac{15.18'}{0.65}$	.34	.0068*	.028	.0055	.012	.04	.0622		
49	$\frac{20'}{.444}$	.30	.006	-.0185	00	.0135	.0093	.057	.084	.217
73	$\frac{15.84'}{.65}$	.4	.008	-.0075	.0178	.03	.087	.1455		

*Froude number was taken as the mean value given by Hughes.

obtained by applying Hughes' or Emmerson's methods. They are particularly large at high Froude numbers. The large variations of $\delta v / v_c$ as given in the table may be due to inaccurate determination of the humps and hollows which have effect on the results. However, the possibility of determining the positions of humps and hollows by the use of the wave making length concept, within such narrow differences (if for the sake of argument the values in the table are considered as errors and not due to blockage), indicates that the proposed method may lead to encouraging and sufficiently accurate results. At high Froude numbers $\delta v / v_c$ appears to be large. Whether this is due to a strong additional flow due to strong wave formation or due to a shortening of the length factor in eq. (27), or due to both, remains to be seen and needs further investigation.

The proposed method was also applied to the results of the Lucy Ashton tests given by Conn and Lackenby(6), in INA 1953. The positions of humps and hollows were determined by drawing the curves of C_R versus $v/\sqrt{L}$. C_R was obtained by applying Schoenherr friction line. The results are shown in Table 9. The results in this table look much better. However, it is to be noticed that $\varepsilon = \frac{\delta v}{v_c}$ caused by blockage is larger at high Froude numbers. The results of a single model are comparable with each other, whereas the results of models of different lengths do not compare in all cases. In this connection we find that the 24 ft. and 20 ft. model results generally compare with each other but neither compare with

TABLE 9

$\frac{\delta v}{v_c}$ of Lucy Ashton Tests

Model Length	Hump (1)	Hollow (2)	Hump (3)	Hollow (4)	Hump (5)	Hollow (6)	Hump (7)	Hollow (8)	Hump (9)	$\frac{a_\phi}{A_{T/2} - a_\phi}$
24'	.006	.014	.075	.042	.05	.03	.035	.0395	.075	.3
20'	--	--	.05	.025	.014	.028	.035	.037	.081	.208
16'	--	--	--	.021	.0085	.028	.0441	.0523	.1025	.134
12'	--	--	.0080	.0052	.0005	.010	.0463	.054	.10	.075
9'		.0084	.040	--	--	.04	.025	.035	.084	.042

the other models. All models were tested in a 30 ft. width tank. Hence it is expected that the wave reflection by the walls will influence the results for the 24 ft. and possibly the 20 ft. and 16 ft. models. Whether the reflection from the walls has any effect on the additional flow due to blockage cannot be answered at this stage. The values shown in Table 9 indicate that the size of the model relative to the tank seems not to be the only major factor in creating the additional flow around the model. As all models were tested in the same tank and as the values obtained do not follow exactly the general trend dictated by the change of model size, then it may be that another factor depending on the width of the tank and the speed of the model alone plays an important role. This point has to be investigated. It may be found that only the width of the tank is more important and not the constriction of the tank section provided that the speed of the model is less than that at which the bottom effect shows up.

B. EFFECT OF BLOCKAGE ON DIFFERENT COMPONENTS OF RESISTANCE AND PROPULSIVE COEFFICIENT--CORRECTIONS NEEDED:

1. Frictional Resistance:

The frictional resistance approximately varies as the square of the average relative speed, i.e. as $(v_c + \delta v)^2$. More accurately, we may obtain the frictional resistance which occurs at a model speed v_c by using $(v_c + \delta v)$ to calculate Reynolds number and C_f. The correction to R_f is straight forward.

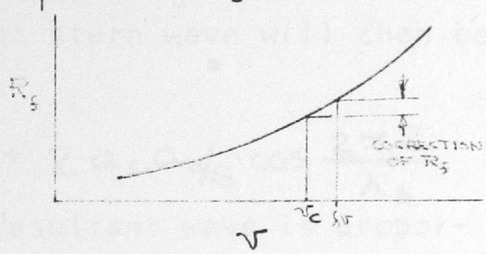


Figure 15

2. Wave Making Resistance:

As explained before, the energy given to the bow free wave is not affected by blockage. Hence the wave resistance of the bow, R_{wb}, will not be influenced.

The stern wave, as explained in Appendix III will have an amplitude given by $a_s = (1 - \omega + \varepsilon)^2 a_b$, where a_b = amplitude of the bow wave, w is the average wake factor at the stern taken over the whole projectional area of the ship in an axial direction, and $\varepsilon = \frac{\delta v}{v_c}$. The length of the free wave of the stern is $\lambda_s = (1 - \omega + \varepsilon)^2 \lambda_b$ where λ_b = the wave length of the bow wave.

The amplitude of the resultant stern wave is given by

$$a^2 = a_s^2 + a_{bl}^2 \pm 2 a_s a_{bl} \cos \frac{2\pi Z}{\lambda_s}$$

a_{bl} is the amplitude of the bow wave in the vicinity of the stern. It is related to the amplitude of the bow wave by the following equation.

$$a_{b2}^2 \times 2 \tan 19°28' \cdot \lambda_s = a_b^2 \lambda_b \times \text{(breadth of wave)}.$$

Assuming that the breadth of the bow wave at its birth = B/12 where B = ship or model beam, then,

$$a_{b2} \times \left\{ \frac{12 2}{B} \tan 19°28' \frac{\lambda_s}{\lambda_b} \right\} = a_b^2$$

Denoting the term inside the brackets by S^2, then

$$a_{b2}^2 \cdot S^2 = a_b^2$$

The amplitude of the resultant stern wave will then be given by:

$$a^2 = a_s^2 + a_b^2/S^2 + 2\, a_s . a_b/S \cos \frac{2\pi Z}{\lambda_s}$$

The energy included in this resultant wave is proportional to $a^2 \lambda_s$ or energy in the resultant stern wave is proportional to

$$\lambda_s a_s^2 + \lambda_s a_b^2/S^2 + 2 \lambda_s a_s . a_b/S . \cos \frac{2\pi Z}{\lambda_s}$$

i.e. proportional to

$$(1-\omega+\varepsilon)^2 \lambda_b (1-\omega+\varepsilon)^4 a_b^2 + (1-\omega+\varepsilon)^2 \lambda_b a_b^2/S$$
$$+ 2(1-\omega+\varepsilon)^2 \lambda_b (1-\omega+\varepsilon)^2 \times \frac{1}{S} a_b^2 \cos \frac{2\pi Z}{\lambda_s}$$

i.e. proportional to

$$(1-\omega+\varepsilon)^6 \lambda_b a_b^2 + (1-\omega+\varepsilon)^2 \lambda_b a_b^2/S$$
$$+ \frac{2}{S} (1-\omega+\varepsilon)^4 a_b^2 \lambda_b \cos \frac{2\pi Z}{\lambda_s}$$

i.e. equal to

$$(1-\omega+\varepsilon)^6 E_{wb} + \frac{(1-\omega+\varepsilon)^2}{S} E_{wb} + \frac{2}{S}(1-\omega+\varepsilon)^4 E_{wb} \cos \frac{2\pi Z}{\lambda_s}$$

The energy supplied by the stern is that represented by the first and third terms only, as the second term represents part of the energy already supplied by the bow. Hence the total energy of the model wave pattern E_w is given by

$$E_w = E_{wb} + (1-\omega+\varepsilon)^6 E_{wb} + \frac{2}{S}(1-\omega+\varepsilon)^4 E_{wb} \cos \frac{2\pi Z}{\lambda_s}$$

S^2 by assumption $= \frac{12l}{B} \tan 19°28' \frac{\lambda_s}{\lambda_b} =$

$$\frac{12l}{B} \tan 19°28' (1-\omega+\varepsilon)^2$$

Hence $S = \sqrt{\frac{12l \tan 19°28'}{B}} \cdot (1-w+\varepsilon)$. Then,

$$E_w = E_{wb} + (1-\omega+\varepsilon)^6 E_{wb} + 2\sqrt{\frac{B}{12l \tan 19°28'}}(1-\omega+\varepsilon)^3 \cos \frac{2\pi Z}{\lambda_s} \cdot E_{wb} \quad (30)$$

or

$$R_w = R_{wb} + (1-\omega+\varepsilon)^6 R_{wb} + 2\sqrt{\frac{B}{12l \tan 19°28'}}(1-\omega+\varepsilon)^3 \cos \frac{2\pi Z}{\lambda_s} \cdot R_{wb} \quad (31)$$

Knowing the ratio $\frac{l}{B}$, w and ε, eq. (31) can be solved at the humps, hollows and at the point when the oscillating term equals zero. It can, therefore, be used for determining the increase in wave making resistance of a given model by assuming ε = zero, then $\varepsilon = \frac{\delta v}{v_c}$ obtained separately. This was done for three values of $\frac{l}{B}$ and varying values of ε. Tables 10 and 11 give the ratio of the variation of wave resistance due to assumed values of ε. When the oscillating term of the wave resistance R'_w is zero, the variation of total resistance based on the previous assumptions will be as in Table 11. These values do not depend on the ratio $\frac{l}{B}$ except in so far as it may influence the value of ω. Table 10 shows that the change in wave making resistance considered as a ratio of the wave resistance in open or in restricted water is much higher at a hump than at a hollow. At the hump the effect of blockage on the stern wave and on the oscillating term add together, while at a hollow they oppose each other. Based on the assumptions detailed in different sections of

Table 10

Assumed Value of ε	Result wake factor w - ε	Rws / Rwb	R'w / Rwb	Inc. in Rws / Rwb	Inc. in R'w / Rwb	Inc. in Rws / Rw open water	Inc. in R'w / Rw open water	Inc. in Rw total / Rw open water	Inc. in Rw total / Rw measured
		Hump, w = .3		L/B = 7.5					
0	.3	.1175	.1215	0	0	0	0	0	0
.01	.29	.1275	.1265	.01	.005	.00807	.00403	.0121	0.01195
.02	.28	.1395	.1325	.022	.011	.01775	.00887	.0266	0.0259
.04	.26	.1646	.1436	.047	.0221	.0379	.0178	.0557	0.0529
.06	.24	.194	.1565	.077	.035	.0621	.0282	.0903	0.0830
.08	.22	.275	.1685	.1075	.047	.0867	.0379	.1246	0.110
.1	.2	.2625	.182	.145	.0605	.1170	.0488	.1658	0.142
		Hollow, w =	.3	L/B =	7.5				
0	.3	.1175	-.1215	0	0	0	0	0	0
.01	.29	.1275	-.1265	.01	-.005	.01	-.005	.005	.005
.02	.28	.1395	-.1325	.022	-.011	.0221	-.01105	.01105	.0109
.04	.26	.1646	-.1436	.047	-.0221	.0472	-.0222	.0250	.0245
.06	.24	.194	-.1565	.077	-.035	.0774	-.0352	.0422	.0405
.08	.22	.225	-.1685	.1075	-.047	.108	-.0472	.0608	.0573
.1	.2	.2625	-.182	.145	-.0605	.1455	-.0608	.085	.0782

Table 11

ε	Inc. in Rws / Rw (open water) w = 0.3	Inc. in Rws / Rw (Measured) in restricted water
0	0	0
.01	.00895	.00888
.02	.0197	.0193
.04	.042	.0403
.06	.0656	.0645
.08	.0915	.0877
.10	.1235	.1147

this report, the figures in Tables 10 and 11 can be taken as representative of the change of wave making resistance for a model whose $\frac{l}{B}$ = 7.5 and has an average wake factor of 0.3. At points between humps, hollows and the points of zero oscillating term, interpolation may be used to arrive at the required correction of the wave making resistance.

The numerical values in Tables 10 and 11 were calculated on the assumption that the walls will not influence the stern wave, i.e., the length of the model will be less than 0.9 × the tank width. If this is not the case, a further correction shall be made as given in Appendix III.

3. Correction of the Resistance Curve:

Knowing the values of $\frac{\delta v}{v_c}$ at several points corresponding to the humps and hollows, a curve may then be drawn, giving the values of $\frac{\delta v}{v_c}$ at different values of the model speed. The correction for R_f is straight forward. However, the correction for Rw needs some consideration. Figure (16) shows how the correction for frictional resistance, δR_f, can be made at a particular carriage speed. δR_w is the

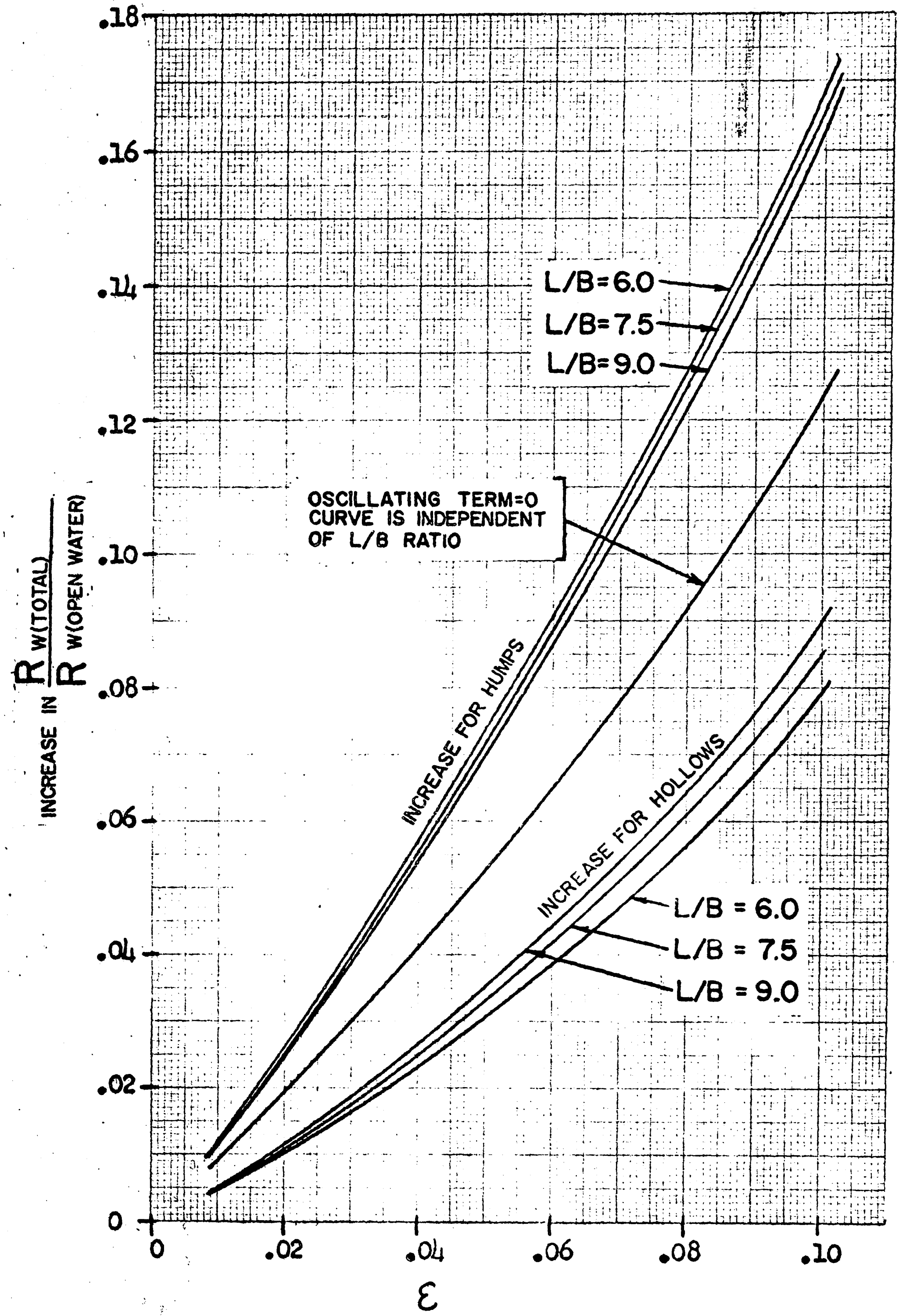


Figure 17 a.

Curves are for:

Average Wake Factor = .3

Effective Wave Width $b = \frac{B}{12}$.

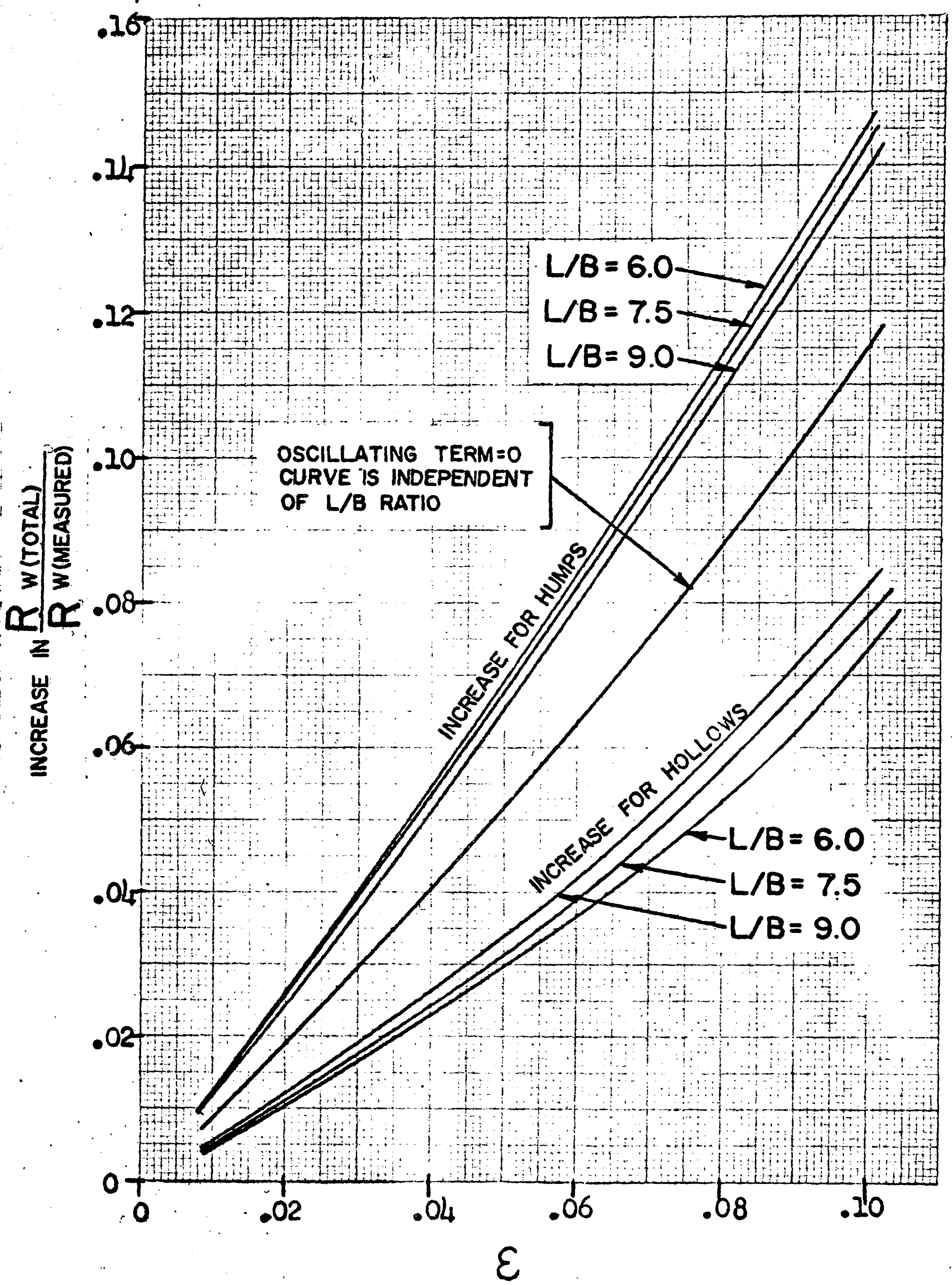


Figure 17 b.

Curves are for:
Average Wake Factor = .3

Effective Wave Width $b = \frac{B}{12}$.

correction to the wave making resistance due to blockage which may be obtained from Tables such as (10) and (11) or from the graphs in Figure (17a,b). Then R_w will be the net open water wave making resistance at a speed v_c. However, since the humps and hollows have to change position to eliminate their shift due to blockage, then this net open water resistance must move to a position $(v_c + \delta v)$. Its magnitude R_{1w} will be given by $R_{1w} \cong R_w \left(1 + \frac{\delta v}{v_c}\right)^5$. This is because the energy included in the wave is proportional to v_c^6 and hence the resistance will be proportional to v_c^5.

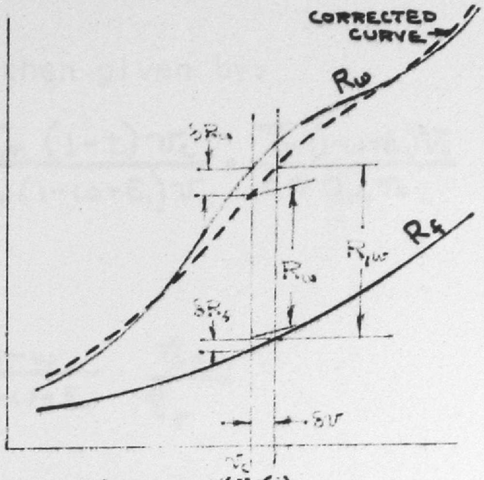


Figure (16)

4. Effect of Blockage on Propulsive Coefficient:

Due to the increase of flow in the vicinity of the propeller caused by blockage the RPM of the screw will increase to make up for the reduced hydrodynamic pitch angle at the different propeller sections. Assume that

R_m = Total measured resistance

T_m = Measured thrust

Q_m = Measured torque

v_c = Speed of model = Speed of carriage

n = Measured RPS

ω = Wake factor, no blockage

$\varepsilon_1 = \frac{\delta v_w}{v_c}$ caused by blockage $= \omega \times \varepsilon$

t = Thrust deduction factor assumed constant

η_p = Propeller efficiency, (no blockage).

The propulsive coefficient $(P.C.)_m$ is then given by:

$$(P.C.)_m = \frac{R_m \times V_c}{2\pi n Q_m} = \frac{T_m(1-t)V_c}{2\pi n Q_m} = \frac{T_m(1-t)V_c}{T_m(1-\omega+\varepsilon_1)V_c} \cdot \frac{T_m(1-\omega+\varepsilon_1)V_c}{2\pi Q_m n}$$

$$= \frac{1-t}{1-\omega+\varepsilon_1} \cdot \eta_{p_m}$$

$$= \frac{1-t}{1-\omega} \cdot \frac{1-\omega}{1-\omega+\varepsilon_1} \eta_{p_m} = \frac{1-t}{1-\omega} \cdot \eta_p \cdot \frac{1-\omega}{1-\omega+\varepsilon_1} \cdot \frac{\eta_{pm}}{\eta_p}$$

where η_{p_m} = propeller efficiency at a torque, Q_m and RPS, n, which may be obtained from the usual K_T, K_Q, and η versus J, diagrams. From this diagram we can also obtain $(\omega - \varepsilon_1)$ which is the net wake factor in way of propeller including blockage effect, Knowing ε we obtain ω. As the thrust deduction factor t can be assumed to be unaltered with blockage, then the measured thrust will be corrected in the same ratio as the resistance, or we can obtain the thrust T in the no-blockage condition. With the already known values of $T, V_c, (1-\omega)$ and propeller diameter we can assume three values of n and obtain J and K_T, for each value of n. The point of intersection drawn to the base of J with the propeller K_T - J curve will give the required values of J, K_T, n, K_Q and η_p in the no blockage condition. Finally the propulsive coefficient in the no blockage condition may be calculated.

It is expected that η_{pm} will not differ considerably from η_p, unless the blockage effect is considerably high. If $\eta_p \cong \eta_{pm}$ the P.C. in the no blockage condition will be given by

$$P.C. = \frac{1-\omega+\varepsilon_1}{1-\omega}(P.C.)_m$$

For w = 0.3 and ε_1 = 0.03, P.C. = 1.042 $(P.C.)_m$, i.e., about four percent higher than the measured propulsive coefficient.

5. Summary and Conclusions:

a. The presence of tank walls and bottom influences the flow around the model, and thus affects both the measured frictional and wave making resistance of the model.

b. The increase of velocity around rotational bodies due to constriction in a closed tunnel as compared with open water is very small for small blockage ratios. This is also the case for a rotational body moving in a tank if we neglect the free surface effect.

c. The effect of the shape of the tank cross section on the increase in flow around a rotational body, with the effect of the free surface neglected, is very small and can be corrected for by a method given by Lock(11).

d. Due to frictional drag, the body tends to pull the water with it, hence the displaced volume which has to flow back between the model and tank boundaries will increase. Accordingly, the friction wake increases the flow around the model in a tank as compared to unbounded water.

e. The effect of free surface is two-fold:

1) An additional flow due to the unequal wave elevation at the bow and stern. This occurs in restricted and unrestricted water.

2) The effect of the walls on the local disturbance around the model. This is equivalent to the effect of infinite images on both sides of the model and is greater at the bow than at the stern due to the weakness of the stern wave. Hence an additional flow due to this effect takes place.

f. Measurements show that the additional flow around the model is higher than the mean flow obtained by the mean flow equation. In other words the increase in velocity is concentrated around the model, and as the volume of water to flow back is a fixed quantity, we conclude that the increase in flow near the walls is comparatively small.

g. The change in flow around the model influences the positions of humps and hollows of the resistance curve. The humps and hollows appear at smaller carriage speeds as the blockage effect increases.

h. The reflection of waves from walls has no effect on the wave making resistance provided that it does not influence the stern wave at its birth. The reflection of waves will influence the stern wave at its birth if the length of model $l > 0.9b$ where b is the tank width. In this case a correction to the wave making resistance can be made as explained in Appendix III. This correction amounted to + 2.7% of the wave making resistance at a hump and -2.8% approximately at a hollow when the reflection was assumed to influence the model at three quarters of its length from the bow. This correction decreases as the point at which the wave reflection influences the model moves aftwards and increases as it moves forward, by increasing or decreasing the tank width for the same model. The correction is zero when the oscillating term is zero.

i. In addition to the influence of the wave reflection on the wave making resistance mentioned in (h), it seems that there is some influence on the additional flow around the model. However, this point needs further investigation.

j. The resistance due to the bow wave is not affected by the tank boundary as the conditions of flow in this area are not changed by blockage to any degree to influence the bow wave at its birth.

k. On the assumptions given in Appendix III and the text, the influence of the increase in flow speed around a model on wave making resistance is calculated. This is shown in Tables 10 and 11 and Figure (17). The influence on the wave making resistance at a hump is much higher than at a hollow. On the assumptions given, such a correction increases at the hump as the ratio $\frac{l}{B}$ decreases where l = length of model and B = its beam. At the hollow the influence decreases as $\frac{l}{B}$ increases. If $\frac{l}{B}$ of a given model is different from those in Figure 17, the correction may be made by interpolation or separately estimated. At the points where the oscillating term is zero, the correction may be taken from Table 11. These corrections do not include the correction due to wave reflection or due to water depth.

l. The shift of humps and hollows can be used as a measure for determining the additional flow around a model. A method is given in the text by which this can be done. This method was applied to the available data with encouraging results. However, the numerical values obtained are much higher than those obtained by the use of the mean flow speed. The numerical values tend to increase considerably at high Froude numbers. Whether this considerable increase at high Froude numbers is due to a shortening of the effective length between the bow and stern waves or due to actual increase of flow cannot be answered at this stage. The shortening of the effective length between the bow and stern waves happens because the waves created aft of the stem by the bow, and by the stern ahead of the after end of the water plane becomes stronger as the speed increases. Thus the effective center of the bow wave moves aft and the effective center of the stern wave moves forward, and the distance between the resultant bow and stern waves becomes shorter. A further investigation of this point is necessary.

m. It seems that the walls have more effect on the change in flow than the bottom provided $\frac{v}{\sqrt{gh}} \leq 0.7$, or, the shallow water effect is absent. This leads to the conclusion that increasing the depth beyond that limit barely alters the change in flow around the model...hence the degree of constriction may not be the main factor in changing the flow around a model. Consequently, the width of the tank may be the most important factor. This point also needs further

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MAIN SYMBOLS

A_t	=	Cross sectional area of tank
a	=	1/2 distance between source and sink representing a Rankine Ovoid
a_b, a_s	=	Amplitude of transverse bow and stern waves at birth
$a_{b\ell}$	=	Amplitude of transverse bow wave at the place where the stern wave starts
a_m	=	Mean area of body or model
a_m	=	$C_b \times B \times d$
a_x	=	Section area at any point along the model axis
a_x	=	Transverse wave amplitude at a distance x from disturbance
B	=	Model beam
b, bx, ...	=	Wave width
b	=	Tank width
C_b	=	Block coefficient
D	=	Diameter of a circular tank
d	=	Draft of a model
d	=	Diameter of a rotational body
F	=	Froude number
g	=	Acceleration due to gravity
h, h_b, h_s	=	Water depth
ℓ	=	Length of model
ℓ	=	1/2 length of rotational body

MAIN SYMBOLS Continued

m	$= \frac{a_m}{A_t}$
n	= r.p.s. of propeller
Q	= Torque
R_f	= Frictional resistance
R_w	= Total wave making resistance
R_{wb}	= Wave making resistance of bow
R_{ws}	= Wave making resistance of stern
$R'w$	= Oscillating term of the wave making resistance
T	= Propeller thrust
t	= Time
v_c	= Speed of carriage
	= Speed of model relative to water in open sea
v_b, v_s	= Speed of water relative to model at bow and stern respectively
$\delta v, \delta v_m, \delta v \ldots$	= Change of water speed relative to model
$w, w_1, \ldots$	= wake factors
$\lambda, \lambda_b, \lambda_s \ldots$	= Wave length
ρ	= Mass density of water
∇_m	= Model volume
ϕ	= Presmatic coefficient
ε	$= \frac{\delta v}{v_c}$

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APPENDIX I

For a source and sink of strength Q in a uniform flow of a general velocity $-v_c$.

$$\psi = \frac{1}{2} v_c^2 r^2 + \frac{Q}{4\pi}\left(\frac{x-a}{R_1} - \frac{x+a}{R_2}\right) \qquad (1)$$

for $\psi = 0$

$$\frac{d^2}{4}\sqrt{\frac{d^2}{4} + a^2} = \frac{Qa}{\pi v_c} \qquad (2)$$

$$Q = \pi v_c \frac{d^2}{4a}\sqrt{\frac{d^2}{4} + a^2} \qquad (3)$$

$$-v_c + \delta v = -v_c + \frac{Q}{4\pi}\left(\frac{x-a}{R_1^3} - \frac{x+a}{R_2^3}\right) \qquad (4)$$

$$-\frac{\delta v}{v_c} = -\frac{d^2}{16a}\sqrt{\frac{d^2}{4} + a^2} \cdot \frac{2a}{(a^2+r^2)^{3/2}}$$

$$= -\frac{d^2}{16\frac{a}{d}}\sqrt{\frac{1}{4} + \frac{a^2}{d^2}} \cdot \frac{(2a/d)\cdot d}{(a^2+r^2)^{3/2}}$$

$$= -\frac{1}{16\frac{a}{d}}\sqrt{\frac{1}{4} + \frac{a^2}{d^2}} \cdot \frac{(2a/d)\cdot d^3}{\left(\frac{a^2}{d^2} + \frac{r^2}{d^2}\right)^{3/2} d^3}$$

$$= -\frac{1}{16}\sqrt{1 + \frac{4a^2}{d^2}} \cdot \frac{1}{\left(\frac{a^2}{d^2} + \frac{r^2}{d^2}\right)^{3/2}}$$

$$= -\frac{1}{16}\sqrt{1 + \frac{4a^2}{d^2}} \cdot \frac{8}{\left(\frac{4a^2}{d^2} + \frac{4r^2}{d^2}\right)^{3/2}}$$

$$= -\frac{1}{16}\sqrt{1 + \frac{4a^2}{d^2}} \cdot \frac{8}{\left(\frac{4a^2}{d^2} + \frac{D^2}{d^2}\right)^{3/2}}$$

$$= -\frac{1}{2}\sqrt{1 + \frac{4a^2}{d^2}} \cdot \frac{1}{\left(\frac{4a^2}{d^2} + \frac{D^2}{d^2}\right)^{3/2}}$$

For $\frac{2a}{d} = 6.0 \qquad \frac{4a^2}{d^2} = 36.0$

$\frac{D}{d}$	$\frac{D^2}{d^2}$	$(36 + \frac{D^2}{d^2})$	$(36 + \frac{D^2}{d^2})^{\frac{3}{2}}$	* $\frac{\delta v_{\infty}}{v_c}$	$\frac{\delta v_c}{v_c}$ (at walls)
2	4	40	263	.0115	.333
3	9	45	302	.0101	.125
4	16	52	374	.00815	.0672
6	36	72	611	.00498	.0291
8	64	100	1000	.003	.0181
10	100	136	1585	.00192	.01255
∞	∞	∞	∞	000	.000

* = increase of speed at place of tunnel walls.

Volume of water passing between body and hypothetical tank walls due to the body.

$$v_c + \delta v = v_c + \frac{Q}{4\pi}\left[\frac{x-a}{R_1^3} - \frac{x+a}{R_2^3}\right]$$

at midship $x = 0$, $R_1^2 = R_2^2 = r^2 + a^2$

$$\therefore \delta v = \left(\frac{Q}{4\pi}\right) \times \frac{-2a}{(a^2+r^2)^{\frac{3}{2}}}$$

volume of water flowing between body and place of walls per second.

$$= \int_{d/2}^{D/2} 2\pi r\, dr\, \frac{Q}{4\pi} \cdot \frac{-2a}{(a^2+r^2)^{3/2}}$$

$$= Qa \int_{d/2}^{D/2} \frac{-r\, dr}{(a^2+r^2)^{3/2}}$$

$$= Qa \left\{\frac{1}{(a^2+r^2)^{1/2}}\right\}_{d/2}^{D/2}$$

$$= Q\frac{a}{d}\left[\frac{1}{\left(\frac{a^2}{a^2} + \frac{r^2}{a^2}\right)^{1/2}}\right]_{d/2}^{D/2}$$

If D is made = nd.

$$\text{volume} = \frac{Qa}{d}\left[\frac{1}{\left(\frac{a^2}{d^2}+\frac{n^2}{4}\right)^{1/2}}\right]_{n=1}^{n=\frac{D}{d}}$$

Then we have

$$\frac{V}{V_b} = \frac{a}{d}\left[\frac{1}{\left(\frac{a^2}{d^2}+\frac{n^2}{4}\right)^{1/2}}\right]_{n=1}^{n=\frac{D}{d}}$$

where V = volume passing between the body and the hypothetical tank walls due to the source and sink and V_b = Volume flowing per sec. = Q.

For an Ovoid $\frac{a}{d}$ = 3, then we have

$\frac{D}{d} = n$	$\frac{\delta v}{v_b}$	Volume passing within the body ÷ V_b	Total volume within walls ÷ V_b
1	0.00	.012	.012
2	.039	.012	.051
3	..093	.012	.105
4	.153	.012	.165
6	.279	.012	.291
8	.387	.012	.399
10	.471	.012	.483
∞	.988	.012	1.000

$\frac{D}{d} = n$	$\frac{\text{Volume Outside Tunnel Walls}}{V_b}$	$\frac{\delta v_{(1)}}{v_c}$
1	.988	∞
2	.949	.3163
3	.895	.112
4	.835	.0556
6	.709	.0202
8	.601	.00955
10	.517	.00523
∞	00	000

APPENDIX II

As stated by Prandtl[23], the body and its wake pushes the fluid away to the side so that it causes in the fluid outside the wake a flow apparently caused by a source. The strength of the source Q_1 is given by Prandtl as

$$Q_1 = \frac{D}{\rho v_c}$$

where D is the drag force, v_c is the speed of the uniform flow.

In case of a ship model D is the frictional resistance + form resistance.

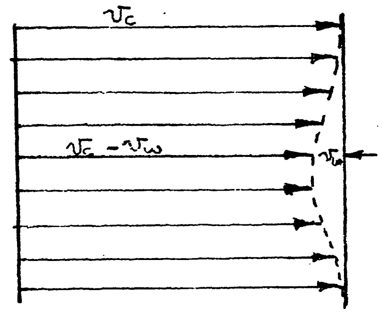

Figure 1

Hence $Q_1 = \frac{R_f + R_{Form}}{\rho v_c}$

Based on the discussion by Prandtl[23] pages 126 and 127.

$$Q_1 = \iint^{T} v_w \, da$$

The integral is taken over the trough in the velocity distribution behind the body or model.

For a ship model the average value of $\frac{v_w}{v_c}$ across the flow behind the ship model $\cong 0.3$

Hence

$$Q_1 = 0 \cdot 3 \; v_c \; a_m$$

where a_m is the model cross sectional area,

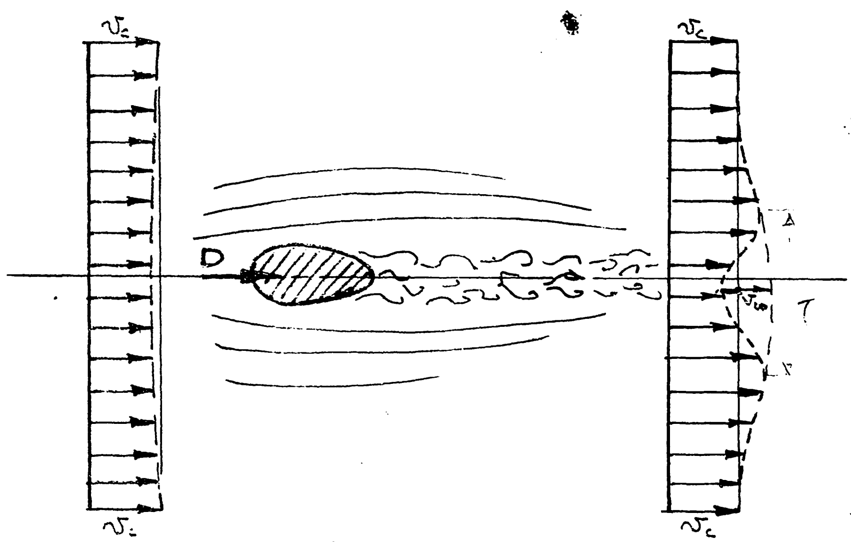

Figure 2

or the additional volume of water due to form drag of the body is about 30% of the flow per second due to the body in nonviscous flow.

Considering the longitudinal distribution of the effect of the drag we can assume that Q_1 is uniformly distributed along the length of the model. Hence at any section we have

$$\int_0^{r_1} 2\pi r(v_c + \delta v)\,dr = Q + Q_{11}$$

where Q_{11} is the total algebraic sum of the volume of liquid issuing per second due to source distribution ahead of the section, r_1 lies on some stream line or (surface). Q = Source representing the body.

Accordingly the effect of drag at midship will be approximately 1/2 the same effect at the tail.

APPENDIX III

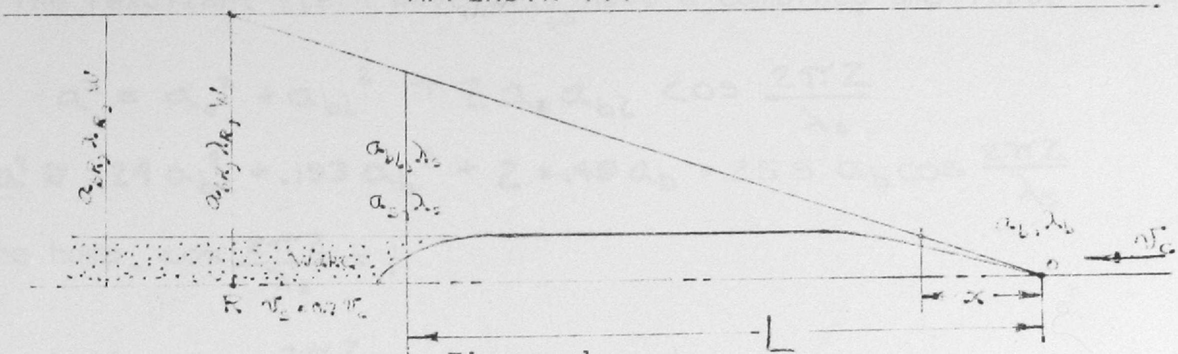

Figure 1

Assuming the origin at the F.P., 0, we have from consideration of the energy of the transverse waves, $a_{bx}^2 b_x \lambda_b$ = constant. Where a_{bx} is the amplitude of the wave, b_x is the width of wave at x, and λ_{bx} is the corresponding wave length. At the stern, the bow transverse wave will have, an amplitude a_{bl}, a width b_l and a length λ_s, such that $a_{bl}^2 b_l \lambda_s$ = constant. Assuming that the bow wave is represented by the conditions at a point of the bow such that b = 1/12 x beam of the ship or model from the center line, then

$$a_{bl}^2 \times b_l \times \lambda_s = a_b^2 \times \frac{B}{12} \times \lambda_b, \text{ where } B = \text{beam}$$

$$b_l \cong l \tan 19^\circ 28'$$

$$\lambda_s = (1-w)^2 \lambda_b. \quad w = \text{average wake factor} = .3$$

Taking $l/B = 7.5$ (an AVERAGE VALUE)

$$a_{bl}^2 \times l \tan 19^\circ 28' \times (1-w)^2 \lambda_b = a_b^2 \times \frac{B}{12} \lambda_b.$$

$$\text{or } a_{bl}^2 \times 90 \tan 19^\circ 28' \times (1-w)^2 = a_b^2$$

PUTTING $(1-w) \cong .7$ we get,

$$a_{bl}^2 \cong 0.0641\ a_b^2$$

$$a_s^2 \cong (.7)^4\ a_b^2$$ where a_s = stern wave amplitude.

The resultant stern wave will have a combined amplitude given by

$$a^2 = a_s^2 + a_{b2}^2 + 2 a_s a_{b2} \cos \frac{2\pi Z}{\lambda_s}$$

or $$a^2 \cong .24 a_b^2 + .193 a_b^2 + 2 \times .49 a_b \times .253 a_b \cos \frac{2\pi Z}{\lambda_s}$$

At the hump $\cos \frac{2\pi Z}{\lambda_s} = 1$

At the hollow $\cos \frac{2\pi Z}{\lambda_s} = -1$

Hence at the hump the oscillating term becomes .248 a_b^2 and at the hollow it becomes $-0.248\ a_b^2$. The energy due to wave resistance of the model (bow and stern) is given by

$$E_w = E_b + E_s + E' \quad \text{where } E' \text{ is the oscillating term}$$

or

$$R_w = R_{wb} + R_{ws} \pm R'_w$$

or $$R_w = R_{wb} + 0.24 \frac{\lambda_s}{\lambda_b} R_{wb} \pm .248 \frac{\lambda_s}{\lambda_b} R_{wb}$$

$$= R_{wb} + (0.24) .49 R_{wb} \pm .248 \frac{\lambda_s}{\lambda_b} R_{wb}$$

$$= 1.1175 R_{wb} \pm 0.1215 R_{wb}$$

Hence the ratio of the oscillating term to the total wave making resistance $\cong$ 9.8% at a hump and $\cong$ −12.2% at a hollow.

Assume the reflection from the wall to take place for the sake of argument at a distance = .75ℓ from the bow. After the reflection, the bow wave is prevented from increasing its width, hence the amplitude of the bow transverse wave will not change materially after the point of reflection. It can be shown that in this case,

$$a^2 = 0.24\, a_b^2 + 0.26\, a_b^2 \pm 2(0.49\, a_b)(0.331\, a_b) \cos \frac{2\pi Z}{\lambda_s}$$

The oscillating factor = $\pm$ 0.324 a^2_b

$$R_w = R_{wb} + R_{ws} \pm R_w'$$

$$= R_{wb} + .24 \frac{\lambda_s}{\lambda_b} R_{wb} \pm .324 \frac{\lambda_s}{\lambda_b} R_{wb}$$

$$= R_{wb} + .1175 R_{wB} \pm .1585 R_{wb}$$

Hence the ratio of the oscillating term to total wave resistance will be 12.5% at a hump and -15% at a hollow, or the reflection at a point 1/4 l ahead of the stern will cause an increase of 2.7% only of the wave making resistance at a hump and a decrease of about -2.8% at a hollow. These figures are by no means accurate, however they indicate the order of magnitude of the effect of wave reflection on the measured wave making resistance.

If we neglect the viscosity we get

$$(a_{bl}^2)\, l\, (\tan 19°28')\lambda_b = a_b^2 \cdot l \cdot \frac{B}{12} \lambda_b$$

$$a_{bl}^2\; 90 \tan 19°28' = a_b^2$$

$$a_{bl}^2 = .0315\, a_b^2$$

$$a_s^2 = a_b^2$$

The combined amplitude of the resultant stern wave will be given by

$$a^2 = a_s^2 + a_{bl}^2 + 2 a_s a_{bl} \cos \frac{2\pi Z}{\lambda}$$

$$= a_b^2 + 0.0315\, a_b^2 + 2(.178\, a_b^2) \cos \frac{2\pi Z}{\lambda}$$

$$R_w = R_{wb} + R_{ws} \pm R_w'$$

$$= 2R_{wb} + 2(.178 R_{wb}) \cos \frac{2\pi Z}{\lambda}$$

$$= 2R_{wb} + 0.356 R_{wb} \cos \frac{2\pi Z}{\lambda}$$

At the hump the oscillating term is + 0.356 R_{wb} or 15.2% of the total wave making resistance and at the hollow it becomes -0.356 R_{wb} or 21.6%. Now assume the reflection to take place at .75 l from the bow as before.

$$a_{b2}^2 = 0.042\, a_b^2$$

$$a^2 = a_s^2 + a_{b2}^2 \pm 2 a_s a_{b2} \cos \frac{2\pi Z}{\lambda}$$

$$= a_b^2 + .042 a_b^2 \pm 2\left(.205\, a_b^2\right) \cos \frac{2\pi Z}{\lambda}$$

$$R_w = 2 R_{wb} \pm 2(.205 R_{wb}) \cos \frac{2\pi Z}{\lambda}$$

At the hump the oscillating term becomes + .41 R_{wb} or 17.0% of the total wave making resistance i.e. about 2% higher than when no wave reflection takes place. At the hollow the oscillating term becomes -0.41 R_{wb} or -25.8% of the total wave resistance, or -4% less than when there is no wave reflection.

If the oscillating term is taken as a percentage of the monotonic wave resistance curve, then in the ideal fluid it becomes

$\cong \pm$ 17.8% no reflection

$\cong \pm$ 22.5% with reflection taking place at quarter length from the stern.

For actual fluid the oscillating term becomes

$\cong$ 10.9% no reflection

$\cong$ 14.2% with reflection taking place at quarter length from the stern.

The effective width of the bow wave at its birth may be defined as the width of a two dimensional wave whose elevation is that of the wave at its birth and which contains the same energy as the transverse wave generated by the bow. Theoretically the elevation of the wave at the disturbance is infinite, thus the effective width tends to be zero. In actual cases the elevation of the wave at its birth is large but not infinite, hence the effective width will have a positive value, which will be small. In this work the effective width of the wave is taken as 1/12 x Beam of ship, which gives ratios of the oscillating term of the wave resistance to the total wave resistance, of the same order as those obtained by experiment.

In the present consideration the energy in the divergent waves is neglected, as the divergent waves have little contribution to wave making resistance.