SCHOENHERR FRICTION COEFFICIENTS
[Smooth Surface Turbulent Flow.]

by

FEB. 82

The University of Michigan Naval Tank
(inc. "Modern Methods for Computing the Surface Friction of Ships," by K. C. Barnaby.

SCHOENHERR FRICTION COEFFICIENTS

[SMOOTH SURFACE TURBULENT FLOW]

UNIVERSITY OF MICHIGAN NAVAL TANK

SCHOENHERR FRICTION COEFFICIENTS
SMOOTH SURFACE – TURBULENT FLOW
UNIVERSITY OF MICHIGAN NAVAL TANK

$$\frac{0.242}{\sqrt{C_f}} = \log_{10}(Re \times C_f)$$

Re 1-5.5
× 10^5

Re 10^5 ×	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
1.0	7.179	7.163	7.146	7.130	7.114	7.098	7.083	7.067	7.052	7.037	7.022
1.1	7.022	7.008	6.993	6.979	6.965	6.951	6.937	6.923	6.910	6.896	6.883
1.2	6.883	6.870	6.857	6.844	6.831	6.819	6.806	6.794	6.782	6.770	6.758
1.3	6.758	6.716	6.735	6.723	6.712	6.700	6.689	6.678	6.667	6.656	6.645
1.4	6.645	6.635	6.624	6.613	6.603	6.593	6.583	6.572	6.562	6.552	6.543
1.5	6.543	6.533	6.523	6.514	6.504	6.493	6.485	6.476	6.467	6.458	6.449
1.6	6.449	6.440	6.431	6.422	6.413	6.404	6.396	6.387	6.379	6.370	6.362
1.7	6.362	6.354	6.345	6.337	6.329	6.321	6.313	6.305	6.297	6.289	6.282
1.8	6.282	6.274	6.266	6.259	6.251	6.244	6.236	6.229	6.222	6.214	6.207
1.9	6.207	6.200	6.193	6.186	6.179	6.172	6.165	6.158	6.151	6.144	6.137
2.0	6.137	6.131	6.124	6.117	6.111	6.104	6.098	6.091	6.085	6.078	6.072
2.1	6.072	6.066	6.060	6.053	6.047	6.041	6.035	6.029	6.023	6.017	6.011
2.2	6.011	6.005	5.999	5.993	5.987	5.982	5.976	5.970	5.964	5.959	5.953
2.3	5.953	5.948	5.942	5.936	5.931	5.926	5.920	5.915	5.909	5.904	5.899
2.4	5.899	5.893	5.888	5.883	5.878	5.872	5.837	5.862	5.857	5.852	5.847
2.5	5.847	5.842	5.837	5.832	5.827	5.822	5.817	5.812	5.807	5.802	5.798
2.6	5.798	5.793	5.788	5.783	5.779	5.774	5.769	5.765	5.760	5.756	5.751
2.7	5.751	5.746	5.742	5.737	5.733	5.728	5.724	5.720	5.715	5.711	5.706
2.8	5.706	5.702	5.698	5.694	5.689	5.685	5.681	5.676	5.672	5.668	5.664
2.9	5.664	5.660	5.656	5.652	5.648	5.643	5.639	5.635	5.631	5.627	5.623
3.0	5.623	5.619	5.615	5.612	5.608	5.604	5.600	5.596	5.592	5.588	5.584
3.1	5.584	5.581	5.577	5.573	5.569	5.566	5.562	5.558	5.554	5.551	5.547
3.2	5.547	5.544	5.540	5.536	5.533	5.529	5.525	5.522	5.518	5.515	5.511
3.3	5.511	5.508	5.504	5.501	5.497	5.494	5.490	5.487	5.484	5.480	5.477
3.4	5.477	5.474	5.470	5.467	5.464	5.460	5.457	5.454	5.450	5.447	5.444
3.5	5.444	5.440	5.437	5.434	5.431	5.428	5.424	5.421	5.418	5.415	5.412
3.6	5.412	5.409	5.405	5.402	5.399	5.396	5.393	5.390	5.387	5.384	5.381
3.7	5.381	5.378	5.375	5.372	5.369	5.366	5.363	5.360	5.357	5.354	5.351
3.8	5.351	5.348	5.345	5.342	5.340	5.337	5.334	5.331	5.328	5.325	5.322
3.9	5.322	5.319	5.317	5.314	5.311	5.308	5.305	5.303	5.300	5.297	5.294
4.0	5.294	5.292	5.289	5.286	5.284	5.281	5.278	5.275	5.273	5.270	5.267
4.1	5.267	5.265	5.262	5.259	5.257	5.254	5.252	5.249	5.246	5.244	5.241
4.2	5.241	5.239	5.236	5.234	5.231	5.228	5.226	5.223	5.221	5.218	5.216
4.3	5.216	5.213	5.211	5.208	5.206	5.204	5.201	5.199	5.196	5.194	5.191
4.4	5.191	5.189	5.186	5.184	5.182	5.179	5.177	5.174	5.172	5.170	5.167
4.5	5.167	5.165	5.163	5.160	5.158	5.156	5.153	5.151	5.149	5.146	5.144
4.6	5.144	5.142	5.140	5.137	5.135	5.133	5.131	5.128	5.126	5.124	5.122
4.7	5.122	5.119	5.117	5.115	5.113	5.111	5.108	5.106	5.104	5.102	5.100
4.8	5.100	5.098	5.095	5.093	5.091	5.089	5.087	5.085	5.082	5.080	5.078
4.9	5.078	5.076	5.074	5.072	5.070	5.068	5.066	5.064	5.062	5.059	5.057
5.0	5.057	5.055	5.053	5.051	5.049	5.047	5.045	5.043	5.041	5.039	5.037
5.1	5.037	5.035	5.033	5.031	5.029	5.027	5.025	5.023	5.021	5.019	5.017
5.2	5.017	5.015	5.013	5.011	5.010	5.008	5.006	5.004	5.002	5.000	4.998
5.3	4.998	4.996	4.994	4.992	4.990	4.988	4.987	4.985	4.983	4.981	4.979
5.4	4.979	4.977	4.975	4.974	4.972	4.970	4.968	4.966	4.964	4.962	4.961

MULTIPLY TABULATED VALUES BY 10^{-3}

SCHOENHERR FRICTION COEFFICIENTS
SMOOTH SURFACE - TURBULENT FLOW
UNIVERSITY OF MICHIGAN NAVAL TANK

Re 5.5 - 10
× 10^5

Re 10^5 ×	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
5.5	4.961	4.959	4.957	4.955	4.953	4.952	4.950	4.948	4.946	4.944	4.943
5.6	4.943	4.941	4.939	4.937	4.936	4.934	4.932	4.930	4.929	4.927	4.925
5.7	4.925	4.923	4.922	4.920	4.918	4.916	4.915	4.913	4.911	4.910	4.908
5.8	4.908	4.906	4.904	4.903	4.901	4.900	4.898	4.896	4.894	4.893	4.891
5.9	4.891	4.889	4.888	4.886	4.884	4.883	4.881	4.880	4.878	4.876	4.875
6.0	4.875	4.873	4.871	4.870	4.868	4.867	4.865	4.863	4.862	4.860	4.859
6.1	4.859	4.857	4.855	4.854	4.852	4.851	4.849	4.848	4.846	4.844	4.843
6.2	4.843	4.841	4.840	4.838	4.836	4.835	4.833	4.832	4.830	4.829	4.827
6.3	4.827	4.826	4.824	4.823	4.821	4.820	4.818	4.817	4.815	4.814	4.812
6.4	4.812	4.811	4.809	4.808	4.806	4.805	4.803	4.802	4.800	4.799	4.797
6.5	4.797	4.796	4.794	4.793	4.792	4.790	4.789	4.787	4.786	4.784	4.783
6.6	4.783	4.781	4.780	4.778	4.777	4.776	4.774	4.773	4.771	4.770	4.768
6.7	4.768	4.767	4.766	4.764	4.763	4.762	4.760	4.759	4.757	4.756	4.754
6.8	4.754	4.753	4.752	4.750	4.749	4.748	4.746	4.745	4.744	4.742	4.741
6.9	4.741	4.739	4.738	4.737	4.735	4.734	4.733	4.731	4.730	4.729	4.727
7.0	4.727	4.726	4.725	4.723	4.722	4.721	4.719	4.718	4.717	4.715	4.714
7.1	4.714	4.713	4.711	4.710	4.709	4.707	4.706	4.705	4.704	4.702	4.701
7.2	4.701	4.700	4.698	4.697	4.696	4.694	4.693	4.692	4.691	4.690	4.688
7.3	4.688	4.687	4.686	4.684	4.683	4.682	4.681	4.679	4.678	4.677	4.676
7.4	4.676	4.674	4.673	4.672	4.671	4.670	4.668	4.667	4.666	4.665	4.663
7.5	4.663	4.662	4.661	4.660	4.658	4.657	4.656	4.655	4.654	4.652	4.651
7.6	4.651	4.650	4.649	4.648	4.646	4.645	4.644	4.643	4.642	4.640	4.639
7.7	4.639	4.638	4.637	4.636	4.634	4.633	4.632	4.631	4.630	4.629	4.628
7.8	4.628	4.626	4.625	4.624	4.623	4.622	4.621	4.620	4.618	4.617	4.616
7.9	4.616	4.615	4.614	4.613	4.612	4.610	4.609	4.608	4.607	4.606	4.605
8.0	4.605	4.604	4.602	4.601	4.600	4.599	4.598	4.597	4.596	4.595	4.594
8.1	4.594	4.592	4.591	4.590	4.589	4.588	4.587	4.586	4.585	4.584	4.582
8.2	4.582	4.581	4.580	4.579	4.578	4.577	4.576	4.575	4.574	4.573	4.572
8.3	4.572	4.570	4.569	4.568	4.567	4.566	4.565	4.564	4.563	4.562	4.561
8.4	4.561	4.560	4.559	4.558	4.557	4.556	4.554	4.553	4.552	4.551	4.550
8.5	4.550	4.549	4.548	4.547	4.546	4.545	4.544	4.543	4.542	4.541	4.540
8.6	4.540	4.539	4.538	4.537	4.536	4.535	4.535	4.533	4.532	4.531	4.530
8.7	4.530	4.529	4.528	4.527	4.526	4.525	4.524	4.523	4.522	4.521	4.520
8.8	4.520	4.519	4.518	4.517	4.516	4.515	4.514	4.513	4.512	4.511	4.510
8.9	4.510	4.509	4.508	4.507	4.506	4.505	4.504	4.503	4.502	4.501	4.500
9.0	4.500	4.499	4.498	4.497	4.496	4.495	4.494	4.493	4.492	4.491	4.490
9.1	4.490	4.489	4.488	4.488	4.487	4.486	4.485	4.484	4.483	4.482	4.481
9.2	4.481	4.480	4.479	4.478	4.477	4.476	4.475	4.474	4.473	4.472	4.472
9.3	4.472	4.471	4.470	4.469	4.468	4.467	4.466	4.465	4.464	4.463	4.462
9.4	4.462	4.461	4.460	4.460	4.459	4.458	4.457	4.456	4.455	4.454	4.453
9.5	4.453	4.452	4.452	4.451	4.450	4.449	4.448	4.447	4.446	4.445	4.444
9.6	4.444	4.443	4.442	4.442	4.441	4.440	4.439	4.438	4.437	4.436	4.435
9.7	4.435	4.434	4.434	4.433	4.432	4.431	4.430	4.429	4.428	4.427	4.427
9.8	4.427	4.426	4.425	4.424	4.423	4.422	4.421	4.420	4.420	4.419	4.418
9.9	4.418	4.417	4.416	4.415	4.414	4.414	4.413	4.412	4.411	4.410	4.409

MULTIPLY TABULATED VALUES BY 10^{-3}

SCHOENHERR FRICTION COEFFICIENTS

SMOOTH SURFACE - TURBULENT FLOW

UNIVERSITY OF MICHIGAN NAVAL TANK

Re 1-5.5 x 10^6

Re 10^6x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
1.0	4.409	4.401	4.393	4.384	4.376	4.368	4.360	4.352	4.345	4.337	4.330
1.1	4.330	4.322	4.315	4.307	4.300	4.293	4.286	4.279	4.272	4.265	4.258
1.2	4.258	4.252	4.245	4.238	4.232	4.226	4.219	4.213	4.207	4.200	4.194
1.3	4.194	4.188	4.182	4.176	4.170	4.165	4.159	4.153	4.148	4.142	4.136
1.4	4.136	4.131	4.125	4.120	4.115	4.109	4.104	4.099	4.094	4.088	4.083
1.5	4.083	4.078	4.073	4.068	4.063	4.058	4.054	4.049	4.044	4.039	4.035
1.6	4.035	4.030	4.025	4.021	4.016	4.012	4.007	4.003	3.998	3.994	3.990
1.7	3.990	3.985	3.981	3.977	3.972	3.968	3.964	3.960	3.956	3.952	3.948
1.8	3.948	3.944	3.940	3.936	3.932	3.928	3.924	3.920	3.916	3.912	3.909
1.9	3.909	3.905	3.901	3.898	3.894	3.890	3.886	3.883	3.879	3.876	3.872
2.0	3.872	3.869	3.865	3.862	3.858	3.855	3.851	3.848	3.844	3.841	3.838
2.1	3.838	3.834	3.831	3.828	3.825	3.822	3.818	3.815	3.812	3.809	3.806
2.2	3.806	3.802	3.799	3.796	3.793	3.790	3.787	3.784	3.781	3.778	3.775
2.3	3.775	3.772	3.769	3.766	3.763	3.760	3.758	3.755	3.752	3.749	3.746
2.4	3.746	3.743	3.741	3.738	3.735	3.732	3.730	3.727	3.724	3.722	3.719
2.5	3.719	3.716	3.714	3.711	3.708	3.706	3.703	3.700	3.698	3.695	3.693
2.6	3.693	3.690	3.688	3.685	3.683	3.680	3.678	3.675	3.673	3.670	3.668
2.7	3.668	3.665	3.663	3.661	3.658	3.656	3.654	3.651	3.649	3.646	3.644
2.8	3.644	3.642	3.640	3.637	3.635	3.633	3.630	3.628	3.626	3.624	3.622
2.9	3.622	3.619	3.617	3.615	3.613	3.611	3.608	3.606	3.604	3.602	3.600
3.0	3.600	3.598	3.596	3.594	3.592	3.589	3.587	3.585	3.583	3.581	3.579
3.1	3.579	3.577	3.575	3.573	3.571	3.569	3.567	3.565	3.563	3.561	3.559
3.2	3.559	3.557	3.555	3.553	3.551	3.549	3.547	3.546	3.544	3.542	3.540
3.3	3.540	3.538	3.536	3.534	3.532	3.530	3.529	3.527	3.525	3.523	3.521
3.4	3.521	3.520	3.518	3.516	3.514	3.512	3.510	3.509	3.507	3.505	3.503
3.5	3.503	3.502	3.500	3.498	3.496	3.495	3.493	3.491	3.490	3.488	3.486
3.6	3.486	3.484	3.483	3.481	3.479	3.478	3.476	3.474	3.473	3.471	3.470
3.7	3.470	3.468	3.466	3.465	3.463	3.461	3.460	3.458	3.457	3.455	3.453
3.8	3.453	3.452	3.450	3.449	3.447	3.446	3.444	3.442	3.441	3.439	3.438
3.9	3.438	3.436	3.435	3.433	3.432	3.430	3.429	3.427	3.426	3.424	3.423
4.0	3.423	3.421	3.420	3.418	3.417	3.415	3.414	3.412	3.411	3.410	3.408
4.1	3.408	3.407	3.405	3.404	3.402	3.401	3.400	3.398	3.397	3.395	3.394
4.2	3.394	3.393	3.391	3.390	3.388	3.387	3.386	3.384	3.383	3.382	3.380
4.3	3.380	3.379	3.377	3.376	3.375	3.373	3.372	3.371	3.369	3.368	3.367
4.4	3.367	3.365	3.364	3.363	3.362	3.360	3.359	3.358	3.356	3.355	3.354
4.5	3.354	3.352	3.351	3.350	3.349	3.347	3.346	3.345	3.344	3.342	3.341
4.6	3.341	3.340	3.339	3.337	3.336	3.335	3.334	3.333	3.331	3.330	3.329
4.7	3.329	3.328	3.326	3.325	3.324	3.323	3.322	3.320	3.319	3.318	3.317
4.8	3.317	3.316	3.314	3.313	3.312	3.311	3.310	3.309	3.308	3.306	3.305
4.9	3.305	3.304	3.303	3.302	3.301	3.299	3.298	3.297	3.296	3.295	3.294
5.0	3.294	3.293	3.292	3.290	3.289	3.288	3.287	3.286	3.285	3.284	3.283
5.1	3.283	3.282	3.280	3.279	3.278	3.277	3.276	3.275	3.274	3.273	3.272
5.2	3.272	3.271	3.270	3.269	3.268	3.266	3.265	3.264	3.263	3.262	3.261
5.3	3.261	3.260	3.259	3.258	3.257	3.256	3.255	3.254	3.253	3.252	3.251
5.4	3.251	3.250	3.249	3.248	3.247	3.246	3.245	3.244	3.243	3.242	3.241

MULTIPLY TABULATED VALUES BY 10^{-3}

SCHOENHERR FRICTION COEFFICIENTS
SMOOTH SURFACE - TURBULENT FLOW
UNIVERSITY OF MICHIGAN NAVAL TANK

Re 5.5 - 10
× 10⁶

Re 10⁶×	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
5.5	3.241	3.240	3.239	3.238	3.237	3.236	3.235	3.234	3.233	3.232	3.231
5.6	3.231	3.230	3.229	3.228	3.227	3.226	3.225	3.224	3.223	3.222	3.221
5.7	3.221	3.220	3.219	3.218	3.217	3.216	3.215	3.214	3.214	3.213	3.212
5.8	3.212	3.211	3.210	3.209	3.208	3.207	3.206	3.205	3.204	3.203	3.202
5.9	3.202	3.202	3.201	3.200	3.199	3.198	3.197	3.196	3.195	3.194	3.193
6.0	3.193	3.192	3.192	3.191	3.190	3.189	3.188	3.187	3.186	3.185	3.184
6.1	3.184	3.184	3.183	3.182	3.181	3.180	3.179	3.178	3.178	3.177	3.176
6.2	3.176	3.175	3.174	3.173	3.172	3.172	3.171	3.170	3.169	3.168	3.167
6.3	3.167	3.166	3.166	3.165	3.164	3.163	3.162	3.161	3.161	3.160	3.159
6.4	3.159	3.158	3.157	3.156	3.156	3.155	3.154	3.153	3.152	3.152	3.151
6.5	3.151	3.150	3.149	3.148	3.147	3.147	3.146	3.145	3.144	3.143	3.143
6.6	3.143	3.142	3.141	3.140	3.139	3.139	3.138	3.137	3.136	3.135	3.135
6.7	3.135	3.134	3.133	3.132	3.132	3.131	3.130	3.129	3.128	3.128	3.127
6.8	3.127	3.126	3.125	3.125	3.124	3.123	3.122	3.122	3.121	3.120	3.119
6.9	3.119	3.119	3.118	3.117	3.116	3.116	3.115	3.114	3.113	3.113	3.112
7.0	3.112	3.111	3.110	3.110	3.109	3.108	3.107	3.107	3.106	3.105	3.104
7.1	3.104	3.104	3.103	3.102	3.102	3.101	3.100	3.099	3.099	3.098	3.097
7.2	3.097	3.096	3.096	3.095	3.094	3.094	3.093	3.092	3.092	3.091	3.090
7.3	3.090	3.090	3.089	3.088	3.087	3.087	3.086	3.085	3.085	3.084	3.083
7.4	3.083	3.083	3.082	3.081	3.080	3.080	3.079	3.078	3.078	3.077	3.076
7.5	3.076	3.076	3.075	3.074	3.074	3.073	3.072	3.072	3.071	3.070	3.070
7.6	3.070	3.069	3.068	3.068	3.067	3.066	3.066	3.065	3.064	3.064	3.063
7.7	3.063	3.062	3.062	3.061	3.060	3.060	3.059	3.058	3.058	3.057	3.056
7.8	3.056	3.056	3.055	3.054	3.054	3.053	3.053	3.052	3.051	3.051	3.051
7.9	3.050	3.049	3.049	3.048	3.047	3.047	3.046	3.046	3.045	3.044	3.044
8.0	3.044	3.043	3.042	3.042	3.041	3.041	3.040	3.039	3.039	3.038	3.037
8.1	3.037	3.037	3.036	3.036	3.035	3.034	3.034	3.033	3.032	3.032	3.031
8.2	3.031	3.031	3.030	3.029	3.029	3.028	3.028	3.027	3.026	3.026	3.025
8.3	3.025	3.025	3.024	3.023	3.023	3.022	3.022	3.021	3.020	3.020	3.019
8.4	3.019	3.019	3.018	3.018	3.017	3.016	3.016	3.015	3.014	3.014	3.013
8.5	3.013	3.013	3.012	3.012	3.011	3.010	3.010	3.009	3.009	3.008	3.008
8.6	3.008	3.007	3.006	3.006	3.005	3.005	3.004	3.004	3.003	3.002	3.002
8.7	3.002	3.001	3.001	3.000	3.000	2.999	2.998	2.998	2.997	2.997	2.996
8.8	2.996	2.996	2.995	2.994	2.994	2.993	2.993	2.992	2.992	2.991	2.991
8.9	2.991	2.990	2.990	2.989	2.988	2.988	2.987	2.987	2.986	2.986	2.985
9.0	2.985	2.985	2.984	2.984	2.983	2.982	2.982	2.981	2.981	2.980	2.980
9.1	2.980	2.979	2.979	2.978	2.978	2.977	2.977	2.976	2.976	2.975	2.974
9.2	2.974	2.974	2.973	2.973	2.972	2.972	2.971	2.971	2.970	2.970	2.969
9.3	2.969	2.969	2.968	2.968	2.967	2.967	2.966	2.966	2.965	2.964	2.964
9.4	2.964	2.963	2.963	2.962	2.962	2.961	2.961	2.960	2.960	2.959	2.959
9.5	2.959	2.958	2.958	2.957	2.957	2.956	2.956	2.955	2.955	2.954	2.954
9.6	2.954	2.953	2.953	2.952	2.952	2.951	2.951	2.950	2.950	2.949	2.949
9.7	2.949	2.948	2.948	2.947	2.947	2.946	2.946	2.945	2.945	2.944	2.944
9.8	2.944	2.943	2.943	2.942	2.942	2.941	2.941	2.940	2.940	2.940	2.939
9.9	2.939	2.939	2.938	2.938	2.937	2.937	2.936	2.936	2.935	2.935	2.934

MULTIPLY TABULATED VALUES BY 10^{-3}

SCHOENHERR FRICTION COEFFICIENTS
SMOOTH SURFACE - TURBULENT FLOW
UNIVERSITY OF MICHIGAN NAVAL TANK

Re 1-5.5 $\times 10^7$

Re $10^7 \times$	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
1.0	2.934	2.930	2.925	2.920	2.916	2.911	2.907	2.902	2.898	2.893	2.889
1.1	2.889	2.885	2.881	2.877	2.873	2.868	2.864	2.861	2.857	2.853	2.849
1.2	2.849	2.845	2.841	2.838	2.834	2.830	2.827	2.823	2.820	2.816	2.813
1.3	2.813	2.809	2.806	2.802	2.799	2.796	2.792	2.789	2.786	2.783	2.780
1.4	2.780	2.776	2.773	2.770	2.767	2.764	2.761	2.758	2.756	2.752	2.749
1.5	2.749	2.746	2.744	2.741	2.738	2.735	2.732	2.730	2.727	2.724	2.721
1.6	2.721	2.719	2.716	2.713	2.711	2.708	2.706	2.703	2.701	2.698	2.696
1.7	2.696	2.693	2.691	2.688	2.686	2.683	2.681	2.678	2.676	2.674	2.672
1.8	2.672	2.669	2.667	2.665	2.662	2.660	2.658	2.656	2.653	2.651	2.649
1.9	2.649	2.647	2.645	2.642	2.640	2.638	2.636	2.634	2.632	2.630	2.628
2.0	2.628	2.626	2.624	2.622	2.620	2.618	2.616	2.614	2.612	2.610	2.608
2.1	2.608	2.606	2.604	2.602	2.600	2.599	2.597	2.595	2.593	2.591	2.589
2.2	2.589	2.588	2.586	2.584	2.582	2.580	2.579	2.577	2.575	2.573	2.572
2.3	2.572	2.570	2.568	2.567	2.565	2.563	2.562	2.560	2.558	2.557	2.555
2.4	2.555	2.553	2.552	2.550	2.549	2.547	2.545	2.544	2.542	2.541	2.539
2.5	2.539	2.538	2.536	2.534	2.533	2.531	2.530	2.528	2.527	2.525	2.524
2.6	2.524	2.522	2.521	2.519	2.518	2.516	2.515	2.514	2.512	2.511	2.509
2.7	2.509	2.508	2.507	2.505	2.504	2.502	2.501	2.500	2.498	2.497	2.496
2.8	2.496	2.494	2.493	2.491	2.490	2.489	2.488	2.486	2.485	2.484	2.482
2.9	2.482	2.481	2.480	2.478	2.477	2.476	2.475	2.473	2.472	2.471	2.470
3.0	2.470	2.468	2.467	2.466	2.465	2.463	2.462	2.461	2.460	2.459	2.457
3.1	2.457	2.456	2.455	2.454	2.453	2.451	2.450	2.449	2.448	2.447	2.446
3.2	2.446	2.444	2.443	2.442	2.441	2.440	2.439	2.438	2.436	2.435	2.434
3.3	2.434	2.433	2.432	2.431	2.430	2.429	2.428	2.427	2.426	2.424	2.423
3.4	2.423	2.422	2.421	2.420	2.419	2.418	2.417	2.416	2.415	2.414	2.413
3.5	2.413	2.412	2.411	2.410	2.409	2.408	2.407	2.406	2.405	2.404	2.403
3.6	2.403	2.402	2.401	2.400	2.399	2.398	2.397	2.396	2.395	2.394	2.393
3.7	2.393	2.392	2.391	2.390	2.389	2.388	2.387	2.386	2.385	2.384	2.383
3.8	2.383	2.382	2.382	2.381	2.380	2.379	2.378	2.377	2.376	2.375	2.374
3.9	2.374	2.373	2.372	2.372	2.371	2.370	2.369	2.368	2.367	2.366	2.365
4.0	2.365	2.364	2.364	2.363	2.362	2.361	2.360	2.359	2.358	2.358	2.357
4.1	2.357	2.356	2.355	2.354	2.353	2.352	2.352	2.351	2.350	2.349	2.348
4.2	2.348	2.347	2.347	2.346	2.345	2.344	2.343	2.342	2.342	2.341	2.340
4.3	2.340	2.339	2.338	2.338	2.337	2.336	2.335	2.334	2.334	2.333	2.332
4.4	2.332	2.331	2.330	2.330	2.329	2.328	2.327	2.327	2.326	2.325	2.324
4.5	2.324	2.324	2.323	2.322	2.321	2.321	2.320	2.319	2.318	2.318	2.317
4.6	2.317	2.316	2.315	2.315	2.314	2.313	2.312	2.312	2.311	2.310	2.310
4.7	2.310	2.309	2.308	2.307	2.307	2.306	2.305	2.304	2.304	2.303	2.302
4.8	2.302	2.302	2.301	2.300	2.300	2.299	2.298	2.298	2.297	2.296	2.295
4.9	2.295	2.295	2.294	2.293	2.293	2.292	2.291	2.291	2.290	2.289	2.289
5.0	2.289	2.288	2.287	2.287	2.286	2.285	2.285	2.284	2.283	2.283	2.282
5.1	2.282	2.281	2.281	2.280	2.279	2.279	2.278	2.277	2.277	2.276	2.276
5.2	2.276	2.275	2.274	2.274	2.273	2.272	2.272	2.271	2.270	2.270	2.269
5.3	2.269	2.268	2.268	2.267	2.267	2.266	2.265	2.265	2.264	2.264	2.263
5.4	2.263	2.262	2.262	2.261	2.261	2.260	2.259	2.259	2.258	2.253	2.257

MULTIPLY TABULATED VALUES BY 10^{-3}

SCHOENHERR FRICTION COEFFICIENTS

SMOOTH SURFACE - TURBULENT FLOW

UNIVERSITY OF MICHIGAN NAVAL TANK

Re 5.5 - 10
× 10^7

Re 10^7 ×	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
5.5	2.257	2.256	2.256	2.255	2.255	2.254	2.253	2.253	2.252	2.252	2.251
5.6	2.251	2.250	2.250	2.249	2.249	2.248	2.248	2.247	2.246	2.246	2.245
5.7	2.245	2.245	2.244	2.244	2.243	2.242	2.242	2.241	2.241	2.240	2.240
5.8	2.240	2.239	2.238	2.238	2.237	2.237	2.236	2.236	2.235	2.234	2.234
5.9	2.234	2.233	2.233	2.232	2.232	2.231	2.231	2.230	2.230	2.229	2.229
6.0	2.229	2.228	2.228	2.227	2.226	2.226	2.225	2.225	2.224	2.224	2.223
6.1	2.223	2.223	2.222	2.222	2.221	2.221	2.220	2.220	2.219	2.219	2.218
6.2	2.218	2.218	2.217	2.216	2.216	2.216	2.215	2.214	2.214	2.213	2.213
6.3	2.213	2.212	2.212	2.211	2.211	2.210	2.210	2.209	2.209	2.208	2.208
6.4	2.208	2.207	2.207	2.206	2.206	2.205	2.205	2.204	2.204	2.203	2.203
6.5	2.203	2.202	2.202	2.202	2.201	2.201	2.200	2.200	2.199	2.199	2.198
6.6	2.198	2.198	2.197	2.197	2.196	2.196	2.195	2.195	2.194	2.194	2.193
6.7	2.193	2.193	2.192	2.192	2.192	2.191	2.190	2.190	2.190	2.189	2.189
6.8	2.189	2.188	2.188	2.187	2.187	2.186	2.186	2.185	2.185	2.184	2.184
6.9	2.184	2.184	2.183	2.183	2.182	2.182	2.181	2.181	2.180	2.180	2.180
7.0	2.180	2.179	2.179	2.178	2.178	2.177	2.177	2.176	2.176	2.176	2.175
7.1	2.175	2.175	2.174	2.174	2.173	2.173	2.172	2.172	2.172	2.171	2.171
7.2	2.171	2.170	2.170	2.170	2.169	2.169	2.168	2.168	2.167	2.167	2.166
7.3	2.166	2.166	2.166	2.165	2.165	2.164	2.164	2.164	2.163	2.163	2.162
7.4	2.162	2.162	2.161	2.161	2.161	2.160	2.160	2.159	2.159	2.159	2.158
7.5	2.158	2.158	2.157	2.157	2.156	2.156	2.156	2.155	2.155	2.154	2.154
7.6	2.154	2.154	2.153	2.153	2.152	2.152	2.152	2.151	2.151	2.150	2.150
7.7	2.150	2.150	2.149	2.149	2.148	2.148	2.148	2.147	2.147	2.146	2.146
7.8	2.146	2.146	2.145	2.145	2.144	2.144	2.144	2.143	2.143	2.142	2.142
7.9	2.142	2.142	2.141	2.141	2.141	2.140	2.140	2.140	2.139	2.139	2.138
8.0	2.138	2.138	2.138	2.137	2.137	2.136	2.136	2.136	2.135	2.135	2.135
8.1	2.135	2.134	2.134	2.133	2.133	2.133	2.132	2.132	2.132	2.131	2.131
8.2	2.131	2.130	2.130	2.130	2.129	2.129	2.129	2.128	2.128	2.128	2.127
8.3	2.127	2.127	2.126	2.126	2.126	2.125	2.125	2.125	2.124	2.124	2.124
8.4	2.124	2.123	2.123	2.122	2.122	2.122	2.122	2.121	2.121	2.120	2.120
8.5	2.120	2.120	2.119	2.119	2.119	2.118	2.118	2.118	2.117	2.117	2.116
8.6	2.116	2.116	2.116	2.115	2.115	2.115	2.114	2.114	2.114	2.113	2.113
8.7	2.113	2.113	2.112	2.112	2.112	2.111	2.111	2.111	2.110	2.110	2.110
8.8	2.110	2.109	2.109	2.109	2.108	2.108	2.108	2.107	2.107	2.107	2.106
8.9	2.106	2.106	2.106	2.105	2.105	2.105	2.104	2.104	2.104	2.103	2.103
9.0	2.103	2.103	2.102	2.102	2.102	2.101	2.101	2.101	2.100	2.100	2.100
9.1	2.100	2.099	2.099	2.099	2.098	2.098	2.098	2.097	2.097	2.097	2.096
9.2	2.096	2.096	2.096	2.096	2.095	2.095	2.094	2.094	2.094	2.094	2.093
9.3	2.093	2.093	2.093	2.092	2.092	2.092	2.091	2.091	2.091	2.090	2.090
9.4	2.090	2.090	2.089	2.089	2.089	2.088	2.088	2.088	2.088	2.087	2.087
9.5	2.087	2.087	2.086	2.086	2.086	2.085	2.085	2.085	2.084	2.084	2.084
9.6	2.084	2.084	2.083	2.083	2.083	2.082	2.082	2.082	2.082	2.081	2.081
9.7	2.081	2.081	2.080	2.080	2.080	2.079	2.079	2.079	2.078	2.078	2.078
9.8	2.078	2.078	2.077	2.077	2.077	2.076	2.076	2.076	2.076	2.075	2.075
9.9	2.075	2.075	2.074	2.074	2.074	2.073	2.073	2.073	2.072	2.072	2.072

MULTIPLY TABULATED VALUES BY 10^{-3}

SCHOENHERR FRICTION COEFFICIENTS
SMOOTH SURFACE – TURBULENT FLOW
UNIVERSITY OF MICHIGAN NAVAL TANK

Re 1-5.5
× 10^8

Re 10^8 ×	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
1.0	2.072	2.069	2.066	2.063	2.061	2.058	2.055	2.052	2.050	2.047	2.045
1.1	2.045	2.042	2.039	2.037	2.034	2.032	2.029	2.027	2.025	2.022	2.020
1.2	2.020	2.018	2.015	2.013	2.011	2.008	2.006	2.004	2.002	2.000	1.998
1.3	1.998	1.995	1.993	1.991	1.989	1.987	1.985	1.983	1.981	1.979	1.977
1.4	1.977	1.975	1.973	1.972	1.970	1.968	1.966	1.964	1.962	1.960	1.959
1.5	1.959	1.957	1.955	1.953	1.952	1.950	1.948	1.946	1.945	1.943	1.941
1.6	1.941	1.940	1.938	1.936	1.935	1.933	1.932	1.930	1.928	1.927	1.925
1.7	1.925	1.924	1.922	1.921	1.919	1.918	1.916	1.915	1.913	1.912	1.911
1.8	1.911	1.909	1.908	1.906	1.905	1.904	1.902	1.901	1.899	1.898	1.897
1.9	1.897	1.895	1.894	1.893	1.891	1.890	1.889	1.887	1.886	1.885	1.884
2.0	1.884	1.882	1.881	1.880	1.878	1.877	1.876	1.875	1.874	1.872	1.871
2.1	1.871	1.870	1.869	1.868	1.866	1.865	1.864	1.863	1.862	1.861	1.860
2.2	1.860	1.858	1.857	1.856	1.855	1.854	1.853	1.852	1.851	1.850	1.848
2.3	1.848	1.847	1.846	1.845	1.844	1.843	1.842	1.841	1.840	1.839	1.838
2.4	1.838	1.837	1.836	1.835	1.834	1.833	1.832	1.831	1.830	1.829	1.828
2.5	1.828	1.827	1.826	1.825	1.824	1.823	1.822	1.821	1.820	1.820	1.819
2.6	1.819	1.818	1.817	1.816	1.815	1.814	1.813	1.812	1.811	1.810	1.810
2.7	1.810	1.809	1.808	1.807	1.806	1.805	1.804	1.803	1.803	1.802	1.801
2.8	1.801	1.800	1.799	1.798	1.797	1.797	1.796	1.795	1.794	1.793	1.792
2.9	1.792	1.792	1.791	1.790	1.789	1.788	1.788	1.787	1.786	1.785	1.784
3.0	1.784	1.784	1.783	1.782	1.781	1.781	1.780	1.779	1.778	1.778	1.777
3.1	1.777	1.776	1.775	1.774	1.774	1.773	1.772	1.772	1.771	1.770	1.769
3.2	1.769	1.769	1.768	1.767	1.766	1.766	1.765	1.764	1.764	1.763	1.762
3.3	1.762	1.762	1.761	1.760	1.760	1.759	1.758	1.757	1.757	1.756	1.755
3.4	1.755	1.755	1.754	1.753	1.753	1.752	1.751	1.751	1.750	1.749	1.749
3.5	1.749	1.748	1.748	1.747	1.746	1.746	1.745	1.744	1.744	1.743	1.742
3.6	1.742	1.742	1.741	1.740	1.740	1.739	1.739	1.738	1.737	1.737	1.736
3.7	1.736	1.736	1.735	1.734	1.734	1.733	1.733	1.732	1.731	1.731	1.730
3.8	1.730	1.730	1.729	1.728	1.728	1.727	1.727	1.726	1.725	1.725	1.724
3.9	1.724	1.724	1.723	1.722	1.722	1.721	1.721	1.720	1.720	1.719	1.719
4.0	1.719	1.718	1.718	1.717	1.716	1.716	1.715	1.715	1.714	1.714	1.713
4.1	1.713	1.713	1.712	1.712	1.711	1.710	1.710	1.709	1.709	1.708	1.708
4.2	1.708	1.707	1.707	1.706	1.706	1.705	1.705	1.704	1.704	1.703	1.703
4.3	1.703	1.702	1.702	1.701	1.701	1.700	1.700	1.699	1.699	1.698	1.698
4.4	1.698	1.697	1.697	1.696	1.696	1.695	1.695	1.694	1.694	1.693	1.693
4.5	1.693	1.692	1.692	1.691	1.691	1.690	1.690	1.689	1.689	1.688	1.688
4.6	1.688	1.688	1.687	1.687	1.686	1.686	1.685	1.685	1.684	1.684	1.683
4.7	1.683	1.683	1.682	1.682	1.682	1.681	1.681	1.680	1.680	1.679	1.679
4.8	1.679	1.678	1.678	1.677	1.677	1.677	1.676	1.676	1.675	1.675	1.674
4.9	1.674	1.674	1.674	1.673	1.673	1.672	1.672	1.671	1.671	1.670	1.670
5.0	1.670	1.670	1.669	1.669	1.668	1.668	1.668	1.667	1.667	1.666	1.666
5.1	1.666	1.665	1.665	1.665	1.664	1.664	1.663	1.663	1.662	1.662	1.662
5.2	1.662	1.661	1.661	1.660	1.660	1.660	1.659	1.659	1.658	1.658	1.658
5.3	1.658	1.657	1.657	1.656	1.656	1.656	1.655	1.655	1.654	1.654	1.654
5.4	1.654	1.653	1.653	1.653	1.652	1.652	1.651	1.651	1.651	1.650	1.650

MULTIPLY TABULATED VALUES BY 10^{-3}

SCHOENHERR FRICTION COEFFICIENTS

SMOOTH SURFACE — TURBULENT FLOW

UNIVERSITY OF MICHIGAN NAVAL TANK

Re 5.5 - 10
× 10^8

Re 10^8 ×	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
5.5	1.650	1.650	1.649	1.649	1.648	1.648	1.648	1.647	1.647	1.646	1.646
5.6	1.646	1.646	1.645	1.645	1.645	1.644	1.644	1.644	1.643	1.643	1.642
5.7	1.642	1.642	1.642	1.641	1.641	1.641	1.640	1.640	1.640	1.639	1.639
5.8	1.639	1.638	1.638	1.638	1.637	1.637	1.637	1.636	1.636	1.636	1.635
5.9	1.635	1.635	1.635	1.634	1.634	1.634	1.633	1.633	1.632	1.632	1.632
6.0	1.632	1.631	1.631	1.631	1.630	1.630	1.630	1.629	1.629	1.629	1.628
6.1	1.628	1.628	1.628	1.627	1.627	1.627	1.626	1.626	1.626	1.625	1.625
6.2	1.625	1.625	1.624	1.624	1.624	1.623	1.623	1.623	1.622	1.622	1.622
6.3	1.622	1.621	1.621	1.621	1.620	1.620	1.620	1.619	1.619	1.619	1.618
6.4	1.618	1.618	1.618	1.618	1.617	1.617	1.617	1.616	1.616	1.616	1.615
6.5	1.615	1.615	1.615	1.614	1.614	1.614	1.613	1.613	1.613	1.613	1.612
6.6	1.612	1.612	1.612	1.611	1.611	1.611	1.610	1.610	1.610	1.610	1.609
6.7	1.609	1.609	1.609	1.608	1.608	1.608	1.607	1.607	1.607	1.606	1.606
6.8	1.606	1.606	1.606	1.605	1.605	1.605	1.604	1.604	1.604	1.604	1.603
6.9	1.603	1.603	1.603	1.602	1.602	1.602	1.602	1.601	1.601	1.601	1.600
7.0	1.600	1.600	1.600	1.600	1.599	1.599	1.599	1.598	1.598	1.598	1.598
7.1	1.598	1.597	1.597	1.597	1.596	1.596	1.596	1.596	1.595	1.595	1.595
7.2	1.595	1.594	1.594	1.594	1.594	1.593	1.593	1.593	1.592	1.592	1.592
7.3	1.592	1.592	1.591	1.591	1.591	1.590	1.590	1.590	1.590	1.589	1.589
7.4	1.589	1.589	1.589	1.588	1.588	1.588	1.588	1.587	1.587	1.587	1.586
7.5	1.586	1.586	1.586	1.586	1.585	1.585	1.585	1.585	1.584	1.584	1.584
7.6	1.584	1.584	1.583	1.583	1.583	1.583	1.582	1.582	1.582	1.582	1.581
7.7	1.581	1.581	1.581	1.580	1.580	1.580	1.580	1.579	1.579	1.579	1.579
7.8	1.579	1.578	1.578	1.578	1.578	1.577	1.577	1.577	1.577	1.576	1.576
7.9	1.576	1.576	1.576	1.575	1.575	1.575	1.575	1.574	1.574	1.574	1.574
8.0	1.574	1.574	1.573	1.573	1.573	1.573	1.572	1.572	1.572	1.572	1.571
8.1	1.571	1.571	1.571	1.571	1.570	1.570	1.570	1.570	1.569	1.569	1.569
8.2	1.569	1.569	1.568	1.568	1.568	1.568	1.568	1.567	1.567	1.567	1.567
8.3	1.567	1.566	1.566	1.566	1.566	1.565	1.565	1.565	1.565	1.564	1.564
8.4	1.564	1.564	1.564	1.564	1.563	1.563	1.563	1.563	1.562	1.562	1.562
8.5	1.562	1.562	1.562	1.561	1.561	1.561	1.561	1.560	1.560	1.560	1.560
8.6	1.560	1.559	1.559	1.559	1.559	1.559	1.558	1.558	1.558	1.558	1.558
8.7	1.558	1.557	1.557	1.557	1.557	1.556	1.556	1.556	1.556	1.556	1.555
8.8	1.555	1.555	1.555	1.555	1.554	1.554	1.554	1.554	1.554	1.553	1.553
8.9	1.553	1.553	1.553	1.552	1.552	1.552	1.552	1.552	1.551	1.551	1.551
9.0	1.551	1.551	1.551	1.550	1.550	1.550	1.550	1.550	1.549	1.549	1.549
9.1	1.549	1.549	1.548	1.548	1.548	1.548	1.548	1.547	1.547	1.547	1.547
9.2	1.547	1.547	1.546	1.546	1.546	1.546	1.546	1.545	1.545	1.545	1.545
9.3	1.545	1.544	1.544	1.544	1.544	1.544	1.543	1.543	1.543	1.543	1.543
9.4	1.543	1.542	1.542	1.542	1.542	1.542	1.541	1.541	1.541	1.541	1.541
9.5	1.541	1.540	1.540	1.540	1.540	1.540	1.539	1.539	1.539	1.539	1.539
9.6	1.539	1.538	1.538	1.538	1.538	1.538	1.537	1.537	1.537	1.537	1.537
9.7	1.537	1.536	1.536	1.536	1.536	1.536	1.535	1.535	1.535	1.535	1.535
9.8	1.535	1.534	1.534	1.534	1.534	1.534	1.534	1.533	1.533	1.533	1.533
9.9	1.533	1.533	1.532	1.532	1.532	1.532	1.532	1.531	1.531	1.531	1.531

MULTIPLY TABULATED VALUES BY 10^{-3}

SCHOENHERR FRICTION COEFFICIENTS

SMOOTH SURFACE TURBULENT FLOW

UNIVERSITY OF MICHIGAN NAVAL TANK

Re 1–5.5
$\times 10^9$

Re $10^9 \times$	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
1.0	1.531	1.529	1.527	1.526	1.524	1.522	1.520	1.518	1.517	1.515	1.513
1.1	1.513	1.512	1.510	1.508	1.507	1.505	1.503	1.502	1.500	1.499	1.497
1.2	1.497	1.496	1.494	1.493	1.491	1.490	1.488	1.487	1.485	1.484	1.482
1.3	1.482	1.481	1.480	1.478	1.477	1.476	1.474	1.473	1.472	1.470	1.469
1.4	1.469	1.468	1.467	1.466	1.464	1.463	1.462	1.461	1.459	1.458	1.457
1.5	1.457	1.456	1.455	1.454	1.452	1.451	1.450	1.449	1.448	1.447	1.446
1.6	1.446	1.445	1.444	1.443	1.442	1.441	1.440	1.438	1.437	1.436	1.435
1.7	1.435	1.434	1.433	1.432	1.431	1.430	1.429	1.428	1.428	1.427	1.426
1.8	1.426	1.425	1.424	1.423	1.422	1.421	1.420	1.419	1.418	1.417	1.416
1.9	1.416	1.416	1.415	1.414	1.413	1.412	1.411	1.410	1.410	1.409	1.408
2.0	1.408	1.407	1.406	1.405	1.405	1.404	1.403	1.402	1.401	1.401	1.400
2.1	1.400	1.399	1.398	1.397	1.397	1.396	1.395	1.394	1.394	1.393	1.392
2.2	1.392	1.391	1.391	1.390	1.389	1.388	1.388	1.387	1.386	1.386	1.385
2.3	1.385	1.384	1.383	1.383	1.382	1.381	1.381	1.380	1.379	1.379	1.378
2.4	1.378	1.377	1.376	1.376	1.375	1.374	1.374	1.373	1.373	1.372	1.371
2.5	1.371	1.371	1.370	1.369	1.369	1.368	1.367	1.367	1.366	1.366	1.365
2.6	1.365	1.364	1.364	1.363	1.363	1.362	1.361	1.361	1.360	1.360	1.359
2.7	1.359	1.358	1.358	1.357	1.357	1.356	1.356	1.365	1.354	1.354	1.353
2.8	1.353	1.353	1.352	1.352	1.351	1.350	1.350	1.349	1.349	1.348	1.348
2.9	1.348	1.347	1.347	1.346	1.346	1.345	1.345	1.344	1.343	1.343	1.342
3.0	1.342	1.342	1.341	1.341	1.340	1.340	1.339	1.339	1.388	1.338	1.337
3.1	1.337	1.337	1.336	1.336	1.336	1.335	1.334	1.334	1.333	1.333	1.332
3.2	1.332	1.332	1.331	1.331	1.330	1.330	1.330	1.329	1.329	1.328	1.328
3.3	1.328	1.327	1.327	1.326	1.326	1.325	1.325	1.324	1.324	1.324	1.323
3.4	1.323	1.323	1.322	1.322	1.321	1.321	1.320	1.320	1.320	1.319	1.319
3.5	1.319	1.318	1.318	1.317	1.317	1.316	1.316	1.316	1.315	1.315	1.314
3.6	1.314	1.314	1.314	1.313	1.313	1.312	1.312	1.312	1.311	1.311	1.310
3.7	1.310	1.310	1.310	1.309	1.309	1.308	1.308	1.308	1.307	1.307	1.306
3.8	1.306	1.306	1.306	1.305	1.305	1.304	1.304	1.304	1.303	1.303	1.302
3.9	1.302	1.302	1.302	1.301	1.301	1.300	1.300	1.300	1.299	1.299	1.299
4.0	1.299	1.298	1.298	1.297	1.297	1.297	1.296	1.296	1.296	1.295	1.295
4.1	1.295	1.294	1.294	1.294	1.293	1.293	1.293	1.292	1.292	1.292	1.291
4.2	1.291	1.291	1.291	1.290	1.290	1.290	1.289	1.289	1.289	1.288	1.288
4.3	1.288	1.288	1.287	1.287	1.287	1.286	1.286	1.286	1.285	1.285	1.285
4.4	1.285	1.284	1.284	1.284	1.283	1.283	1.283	1.282	1.282	1.282	1.281
4.5	1.281	1.281	1.281	1.280	1.280	1.280	1.279	1.279	1.279	1.278	1.278
4.6	1.278	1.278	1.277	1.277	1.277	1.276	1.276	1.276	1.276	1.275	1.275
4.7	1.275	1.275	1.274	1.274	1.274	1.273	1.273	1.273	1.272	1.272	1.272
4.8	1.272	1.272	1.271	1.271	1.271	1.270	1.270	1.270	1.269	1.269	1.269
4.9	1.269	1.269	1.268	1.268	1.268	1.267	1.267	1.267	1.267	1.266	1.266
5.0	1.266	1.266	1.265	1.265	1.265	1.265	1.264	1.264	1.264	1.263	1.263
5.1	1.263	1.263	1.263	1.262	1.262	1.262	1.262	1.261	1.261	1.261	1.260
5.2	1.260	1.260	1.260	1.260	1.259	1.259	1.259	1.258	1.258	1.258	1.258
5.3	1.258	1.25	1.257	1.257	1.257	1.256	1.256	1.256	1.256	1.255	1.255
5.4	1.255	1.255	1.254	1.254	1.254	1.254	1.253	1.253	1.253	1.253	1.252

MULTIPLY TABULATED VALUES BY 10^{-3}

SCHOENHERR FRICTION COEFFICIENTS
SMOOTH SURFACE TURBULENT FLOW
UNIVERSITY OF MICHIGAN NAVAL TANK

Re 5.5-10
× 10^9

Re 10^9×	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
5.5	1.252	1.252	1.252	1.252	1.251	1.251	1.251	1.251	1.250	1.250	1.250
5.6	1.250	1.250	1.249	1.249	1.249	1.249	1.248	1.248	1.248	1.248	1.247
5.7	1.247	1.247	1.247	1.247	1.246	1.246	1.246	1.246	1.245	1.245	1.245
5.8	1.245	1.245	1.244	1.244	1.244	1.244	1.243	1.243	1.243	1.243	1.242
5.9	1.242	1.242	1.242	1.242	1.242	1.241	1.241	1.241	1.241	1.240	1.240
6.0	1.240	1.240	1.240	1.240	1.239	1.239	1.239	1.239	1.238	1.238	1.238
6.1	1.238	1.238	1.237	1.237	1.237	1.237	1.237	1.236	1.236	1.236	1.236
6.2	1.236	1.235	1.235	1.235	1.235	1.235	1.234	1.234	1.234	1.234	1.233
6.3	1.233	1.233	1.233	1.233	1.233	1.232	1.232	1.232	1.232	1.232	1.231
6.4	1.231	1.231	1.231	1.231	1.230	1.230	1.230	1.230	1.230	1.229	1.229
6.5	1.229	1.229	1.229	1.229	1.228	1.228	1.228	1.228	1.228	1.227	1.227
6.6	1.227	1.227	1.227	1.227	1.226	1.226	1.226	1.226	1.226	1.225	1.225
6.7	1.225	1.225	1.225	1.224	1.224	1.224	1.224	1.224	1.223	1.223	1.223
6.8	1.223	1.223	1.223	1.222	1.222	1.222	1.222	1.222	1.222	1.221	1.221
6.9	1.221	1.221	1.221	1.220	1.220	1.220	1.220	1.220	1.220	1.219	1.219
7.0	1.219	1.219	1.219	1.218	1.218	1.218	1.218	1.218	1.218	1.217	1.217
7.1	1.217	1.217	1.217	1.217	1.216	1.216	1.216	1.216	1.216	1.215	1.215
7.2	1.215	1.215	1.215	1.215	1.214	1.214	1.214	1.214	1.214	1.214	1.213
7.3	1.213	1.213	1.213	1.213	1.213	1.212	1.212	1.212	1.212	1.212	1.212
7.4	1.212	1.211	1.211	1.211	1.211	1.211	1.210	1.210	1.210	1.210	1.210
7.5	1.210	1.210	1.209	1.209	1.209	1.209	1.209	1.208	1.208	1.208	1.208
7.6	1.208	1.208	1.208	1.207	1.207	1.207	1.207	1.207	1.207	1.206	1.206
7.7	1.206	1.206	1.206	1.206	1.206	1.205	1.205	1.205	1.205	1.205	1.204
7.8	1.204	1.204	1.204	1.204	1.204	1.204	1.203	1.203	1.203	1.203	1.203
7.9	1.203	1.203	1.202	1.202	1.202	1.202	1.202	1.202	1.201	1.201	1.201
8.0	1.201	1.201	1.201	1.201	1.200	1.200	1.200	1.200	1.200	1.200	1.199
8.1	1.199	1.199	1.199	1.199	1.199	1.199	1.198	1.198	1.198	1.198	1.198
8.2	1.198	1.198	1.197	1.197	1.197	1.197	1.197	1.197	1.196	1.196	1.196
8.3	1.196	1.196	1.196	1.196	1.196	1.195	1.195	1.195	1.195	1.195	1.195
8.4	1.195	1.194	1.194	1.194	1.194	1.194	1.194	1.194	1.193	1.193	1.193
8.5	1.193	1.193	1.193	1.193	1.192	1.192	1.192	1.192	1.192	1.192	1.192
8.6	1.192	1.191	1.191	1.191	1.191	1.191	1.191	1.190	1.190	1.190	1.190
8.7	1.190	1.190	1.190	1.190	1.189	1.189	1.189	1.189	1.189	1.189	1.189
8.8	1.189	1.188	1.188	1.188	1.188	1.188	1.188	1.188	1.187	1.187	1.187
8.9	1.187	1.187	1.187	1.187	1.186	1.186	1.186	1.186	1.186	1.186	1.186
9.0	1.186	1.185	1.185	1.185	1.185	1.185	1.185	1.185	1.184	1.184	1.184
9.1	1.184	1.184	1.184	1.184	1.184	1.183	1.183	1.183	1.183	1.183	1.183
9.2	1.183	1.183	1.182	1.182	1.182	1.182	1.182	1.182	1.182	1.181	1.181
9.3	1.181	1.181	1.181	1.181	1.181	1.181	1.180	1.180	1.180	1.180	1.180
9.4	1.180	1.180	1.180	1.180	1.179	1.179	1.179	1.179	1.179	1.179	1.179
9.5	1.179	1.178	1.178	1.178	1.178	1.178	1.178	1.178	1.177	1.177	1.177
9.6	1.177	1.177	1.177	1.177	1.177	1.177	1.176	1.176	1.176	1.176	1.176
9.7	1.176	1.176	1.176	1.176	1.175	1.175	1.175	1.175	1.175	1.175	1.175
9.8	1.175	1.174	1.174	1.174	1.174	1.174	1.174	1.174	1.174	1.173	1.173
9.9	1.173	1.173	1.173	1.173	1.173	1.173	1.172	1.172	1.172	1.172	1.172

MULTIPLY TABULATED VALUES BY 10^{-3}

DENSITY OF WATER

IN ENGLISH ENGINEERING UNITS

Fresh Water	Temp. DEG. F.	Sea Water 3.5% Salinity	Fresh Water	Temp. DEG. F.	Sea Water 3.5% Salinity
1.9399	32	1.9947	1.9381	61	1.9901
1.9399	33	1.9946	1.9379	62	1.9898
1.9400	34	1.9946	1.9377	63	1.9895
1.9400	35	1.9945	1.9375	64	1.9893
1.9401	36	1.9944	1.9373	65	1.9890
1.9401	37	1.9943	1.9371	66	1.9888
1.9401	38	1.9942	1.9369	67	1.9885
1.9401	39	1.9941	1.9367	68	1.9882
1.9401	40	1.9940	1.9365	69	1.9879
1.9401	41	1.9939	1.9362	70	1,9876
1.9401	42	1.9937	1.9360	71	1.9873
1.9401	43	1.9936	1.9358	72	1.9870
1.9400	44	1.9934	1.9355	73	1.9867
1.9400	45	1.9933	1.9352	74	1.9864
1.9399	46	1.9931	1.9350	75	1.9861
1.9398	47	1.9930	1.9347	76	1.9858
1.9398	48	1.9928	1.9344	77	1.9854
1.9397	49	1.9926	1.9342	78	1.9851
1.9396	50	1.9924	1.9339	79	1.9848
1.9395	51	1.9923	1.9336	80	1.9844
1.9394	52	1.9921	1.9333	81	1.9841
1.9393	53	1.9919	1.9330	82	1.9837
1.9392	54	1.9917	1.9327	83	1.9834
1.9390	55	1.9914	1.9324	84	1.9830
1.9389	56	1.9912	1.9321	85	1.9827
1.9387	57	1.9910	1.9317	86	1.9823
1.9386	58	1.9908			
1.9384	59	1.9905			
1.9383	60	1.9903			

1 FOOT = 0.30479449 METER

1 POUND = 453.59243 GRAMS

$g = 9.80665$ M./SEC.^{2}

DENSITY = $\frac{\text{LB. X SEC.}}{\text{FT.}^4}$ = 1.94018 x $\frac{\text{GMS.}}{\text{CM.}^3}$

KINEMATIC VISCOSITY OF WATER

In English Engineering Units

Fresh Water	Temp.	Sea Water 3.5% Salinity	Fresh Water	Temp.	3.5% Salinity Sea Water
$\nu \times 10^5$(FT.^{2}/SEC.)	DEG.F.	$\nu_s \times 10^5$(FT^2/SEC.)	$\nu \times 10^5$(FT.^{2}/SEC.)	DEG.F.	$\nu_s \times 10^5$(FT./SEC.)
1.9291	32		1.1937	61	1.2470
1.8922	33		1.1769	62	1.2303
1.8565	34		1.1605	63	1.2139
1.8219	35		1.1444	64	1.1979
1.7883	36		1.1287	65	1.1822
1.7558	37		1.1133	66	1.1669
1.7242	38		1.0983	67	1.1519
1.6935	39		1.0836	68	1.1372
1.6638	40		1.0692	69	1.1229
1.6349	41	1.6846	1.0552	70	1.1088
1.6068	42	1.6568	1.0414	71	1.0951
1.5795	43	1.6298	1.0279	72	1.0816
1.5530	44	1.6035	1.0147	73	1.0684
1.5272	45	1.5780	1.0018	74	1.0554
1.5021	46	1.5531	0.98918	75	1.0427
1.4476	47	1.5289	0.97680	76	1.0303
1.4538	48	1.5053	0.96466	77	1.0181
1.4306	49	1.4823	0.95276	78	1.0062
1.4080	50	1.4599	0.94111	79	0.99447
1.3860	51	1.4381	0.92969	80	0.98299
1.3646	52	1.4168	0.91850	81	0.97172
1.3437	53	1.3961	0.90752	82	0.96067
1.3233	54	1.3758	0.89676	83	0.94982
1.3034	55	1.3561	0.88621	84	0.93917
1.2840	56	1.3368	0.87586	85	0.92873
1.2651	57	1.3180	0.86570	86	0.91847
1.2466	58	1.2996			
1.2285	59	1.2817			
1.2109	60	1.2641			

POISE = DYNE-SEC. PER CM.^{2}

KINEMATIC VISCOSITY = $\frac{\text{VISCOSITY}}{\text{DENSITY}} = \frac{\mu}{\rho}$

1 FOOT = 0.30479449 METER

1 POUNDS = 453.59243 GRAMS

= 9.80665 M./SEC.^{2}

KIN. VIS.: $\frac{\text{FT.}^2}{\text{SEC.}}$ = 10.764230 x $\frac{\text{M.}^2}{\text{SEC.}}$

DENSITY: $\frac{\text{LB. X SEC.}^2}{\text{FT.}^4}$ = 1.94018 x $\frac{\text{gms.}}{\text{CM.}^3}$

WEIGHT OF WATER

Pounds per Cubic Foot
$g = 32.174$ ft. /sec.^{2} at sea level
$g = 32.1616$ ft. /sec.^{2} at Ann Arbor

Salinity of sea water is 3.5%

TEMP °F	F.W. SEALEVEL	F.W. A.A.	S.W.
32	62.415	62.391	64.177
33	62.416	62.392	64.176
34	62.417	62.393	64.174
35	62.418	62.394	64.171
36	62.419	62.395	64.168
37	62.420	62.396	64.165
38	62.421	62.397	64.161
39	62.421	62.397	64.157
40	62.421	62.397	64.153
41	62.421	62.397	64.150
42	62.420	62.396	64.146
43	62.419	62.395	64.142
44	62.418	62.394	64.137
45	62.417	62.393	64.132
46	62.415	62.392	64.127
47	62.413	62.389	64.122
48	62.411	62.387	64.116
49	62.409	62.385	64.110
50	62.406	62.381	64.103
51	62.403	62.377	64.100
52	62.399	62.374	64.094
53	62.395	62.370	64.087
54	62.391	62.366	64.080
55	62.386	62.362	64.072
56	62.381	62.358	64.065
57	62.376	62.353	64.058
58	62.371	62.348	64.051
59	62.366	62.343	64.043
60	62.362	62.338	64.036
61	62.356	62.332	64.929
62	62.350	62.326	64.021
63	62.343	62.320	64.012
64	62.337	62.313	64.004
65	62.331	62.307	63.996
66	62.324	62.300	63.989
67	62.318	62.294	63.978
68	62.311	62.287	63.968
69	62.305	62.280	63.959
70	62.297	62.272	63.949
71	62.289	62.265	63.940
72	62.282	62.258	63.930
73	62.273	62.249	63.920
74	62.263	62.240	63.910
75	62.526	62.232	63.901
76	62.247	62.223	63.891
77	62.238	62.214	63.880
78	62.230	62.205	63.869
79	62.221	62.197	63.858
80	62.212	62.188	63.847
81	62.202	62.178	63.836
82	62.192	62.168	63.824
83	62,183	62.159	63.813
84	62.173	62.149	63.801
85	62.163	62.139	63.791
86	62.152	62.128	63.779

F.W. - Sea Level - 50^{O}F.; 35.894 cu. ft. = 1 long ton
S.W. - Sea Level - 50^{O}F.; 34.944 cu. ft. = 1 long ton
F.W. - Ann Arbor - 50^{O}F.; 35.908 cu. ft. = 1 long ton
F.W. - Ann Arbor - 68^{O}F.; 35.963 cu. ft. = 1 long ton

ROUGHNESS FACTORS OF SHIPS' SURFACES

VESSEL OR EXPERIMENTER AND REFERENCE	CONDITION OF SURFACE	LENGTH M.	LOG R	C_{fk} $\times 10^3$	C_f $\times 10^3$	C_k $\times 10^3$	C_k/C_f	L/k_s	K_s	
									MM.	FT×10³
KEMPF (1)	CLEAN NORMAL STEEL PLATING, TWO COATS SHIP PAINT	67.	8.5	1.90	1.82	.08	.045	2×10^6	.03	.10
" "	SAME WITH BUTTS 20 MM. HIGH SPACED 5 M.	67.	8.5	2.03	1.82	.21	.115	1.1×10^6	.06	.20
" "	CLEAN, PAINTED, WITH RIVETS AND TWO SEAM LAPS	21.	8.0	2.50	2.13	.37	.17	2.6×10^5	.08	.26
" "	SAME WITH BUTTS 20 MM. HIGH SPACED 5 M.	21.	8.0	2.50	2.13	.37	.17	2.6×10^5	.08	.26
" "	CLEAN, PAINT ROUGHENED WITH SAND IMPRESSIONS	21.	8.0	3.10	2.13	.97	.45	9.5×10^4	.22	.72
SCHULTZ-GRUNOW (3)	NORMAL RUSTED; BLACK OR GALV.; ORIG. PAINT 2 COATS ANTI-CORR., 1 COAT ANTI-FOUL.; PAINT AFTER EXPOSURE 2 COATS ANTI-FOUL., EXPOSURE:									
	14 DAYS, SALT WATER AND SALT AIR								.318	1.04
	42 DAYS, CLEANED WITH WIRE BRUSH OR CHIPPING								.195	.64
"YUDACHI"-HIRAGA (1)(2)	OLD HULL RUSTED AND NEWLY PAINTED	71.	8.75	2.45	1.68	.77	.45	4.5×10^5	.16	.52
"CLAIRTON" (1)(5)	" " " " " "	120.	8.75	2.18	1.68	.50	.30	7.5×10^5	.16	.52
"HAMBURG" (1)	NEW HULL NORMAL RUSTED	201.	9.11	2.30	1.52	.78	.51	7.2×10^5	.28	.92
"GREYHOUND"-FROUDE (1)(2)	DETERIORATED COPPER SHEATHING	52.5	8.45	2.83	1.85	.98	.53	1.75×10^5	.30	1.00
"BREMEN" (1)(2)	NEW HULL NORMAL RUSTED	275.	9.41	2.23	1.39	.84	.60	7.8×10^5	.35	1.15
TUGBOAT-HIRAGA (1)	NORMAL RUSTED	35.	8.04	3.10	2.10	1.00	.48	9.5×10^4	.37	1.21
KEMPF (1)(2)	CLEAN, PAINTED, BARNACLES 1.2 MM. OVER 75% AREA	21.	8.00	4.35	2.13	2.22	1.04	1.85×10^4	1.14	3.74
" "	" " " 4 " " " "	21.	8.00	5.47	2.13	3.34	1.57	6.25×10^3	3.36	11.0

KEMPF { AV. NEW STEEL CLEAN SHIP - 0.50
" OLD " " " - 1.00 } $\times 10^{3}$
BUTTS FLUSH WELDED —— 0.30

DEFINITIONS

R REYNOLD'S NUMBER OF TEST
C_{fk} COEFFICIENT OF TOTAL FRICTIONAL RESISTANCE, INCLUDING ROUGHNESS, OBTAINED FROM TEST
C_f " " SMOOTH " " CORRESPONDING TO R (SCHLICHTING FORMULA)
C_k " " ADDED " " DUE TO ROUGHNESS, $C_k = C_{fk} - C_f$
C_k/C_f RELATIVE INCREASE OF " " " " "
L/k_s EQUIVALENT RELATIVE SAND ROUGHNESS CORRESPONDING TO C_{fk} AND R
k_s " SAND ROUGHNESS

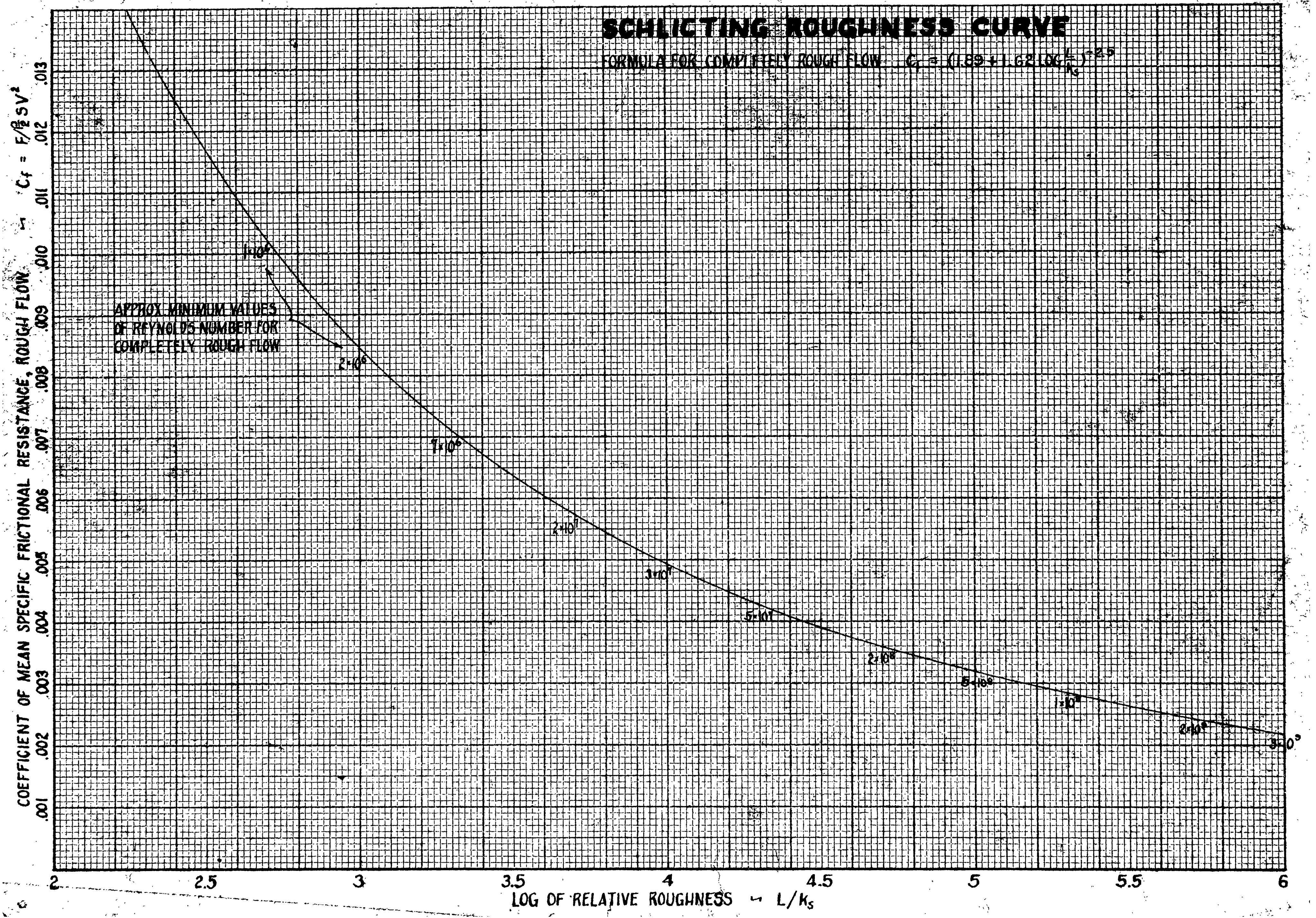
AV. $C_K = 0.55$ TO 0.65×10^{-3}

COMMENT

THE CURRENT THEORY OF SURFACE ROUGHNESS CONSIDERS THAT ANY GIVEN ROUGHNESS MAY BE EXPRESSED IN TERMS OF THE SAND ROUGHNESS WHICH GIVES THE SAME FRICTIONAL RESISTANCE AT THE SAME LENGTH AND REYNOLD'S NUMBER. THE GIVEN ROUGHNESS IS THEN CONSIDERED TO FOLLOW THE SAME LAWS AS THE EQUIVALENT SAND ROUGHNESS; I.E., THE RESISTANCE OF THE GIVEN ROUGHNESS AT ANY OTHER LENGTH AND REYNOLD'S NUMBER IS CONSIDERED TO BE THE SAME AS THAT OF THE EQUIVALENT SAND ROUGHNESS UNDER THOSE CONDITIONS. THE LATTER MAY BE DETERMINED FROM SCHLICTING'S SAND ROUGHNESS CURVES AND FORMULAE (4).

REFERENCES

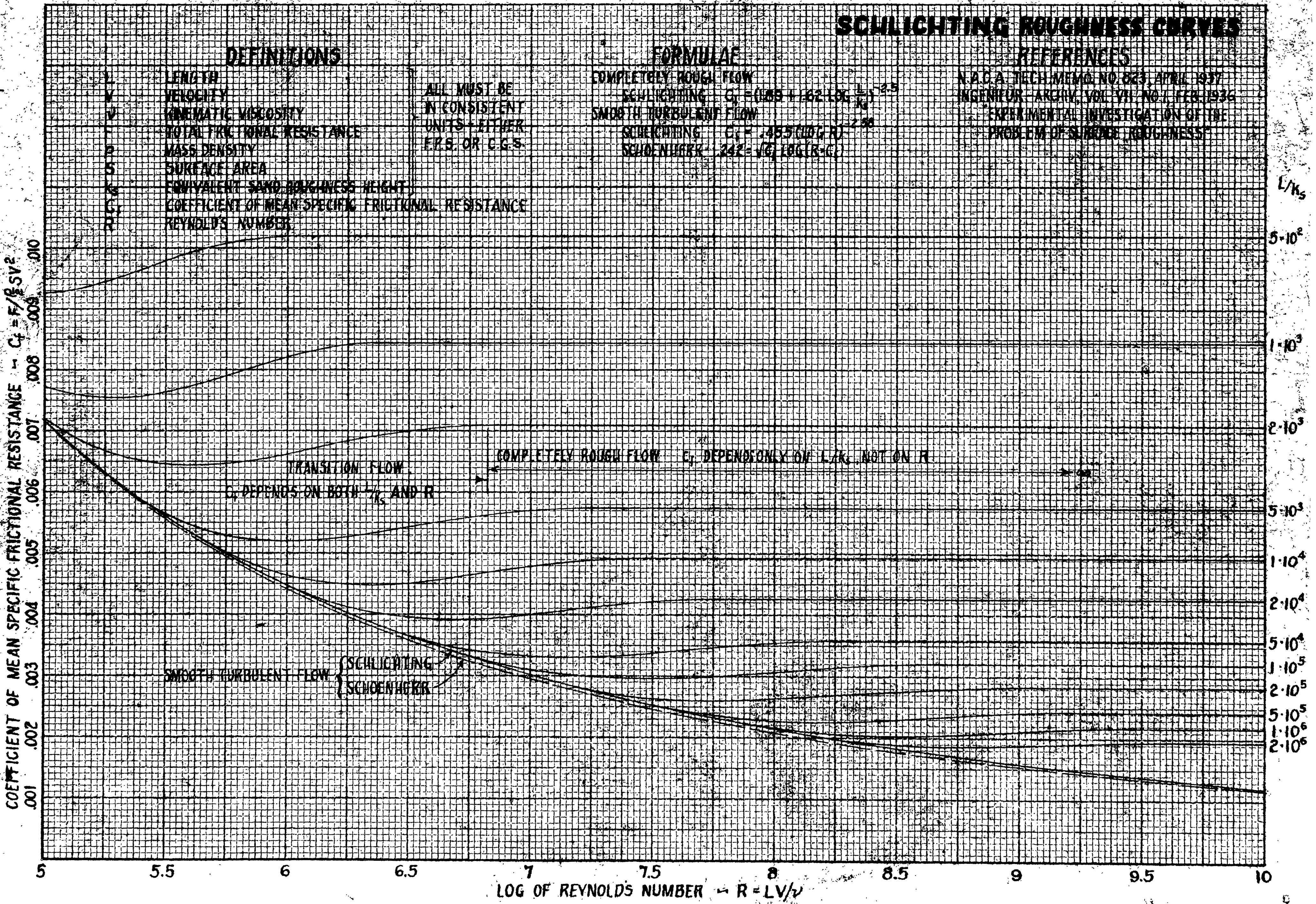
(1) KEMPF - "ON THE EFFECT OF ROUGHNESS ON THE RESISTANCE OF SHIPS," I.N.A. 1937
(2) KEMPF - "SHIP PERFORMANCE IN SMOOTH AND ROUGH WATER", S.N.A. & M.E. 1936
(3) SCHULTZ-GRUNOW - "FRICTIONAL RESISTANCE OF MODERATELY ROUGH SURFACES", SHIPBUILDER, APRIL 1938
(4) SCHLICHTING - "EXPERIMENTAL INVESTIGATION OF THE PROBLEM OF SURFACE ROUGHNESS," N.A.C.A. TECH. MEMO. NO. 823, APRIL 1937
(5) SCHOENHERR - "RESISTANCE OF FLAT SURFACES", S.N.A. & M.E. 1932



SCHLICTING ROUGHNESS CURVE

FORMULA FOR COMPLETELY ROUGH FLOW $C_f = (1.89 + 1.62 \log \frac{L}{k_s})^{-2.5}$

SCHLICHTING ROUGHNESS CURVES



DEFINITIONS

L LENGTH
V VELOCITY
ν KINEMATIC VISCOSITY
F TOTAL FRICTIONAL RESISTANCE
ρ MASS DENSITY
S SURFACE AREA
k_s EQUIVALENT SAND ROUGHNESS HEIGHT
C_f COEFFICIENT OF MEAN SPECIFIC FRICTIONAL RESISTANCE
R REYNOLDS NUMBER

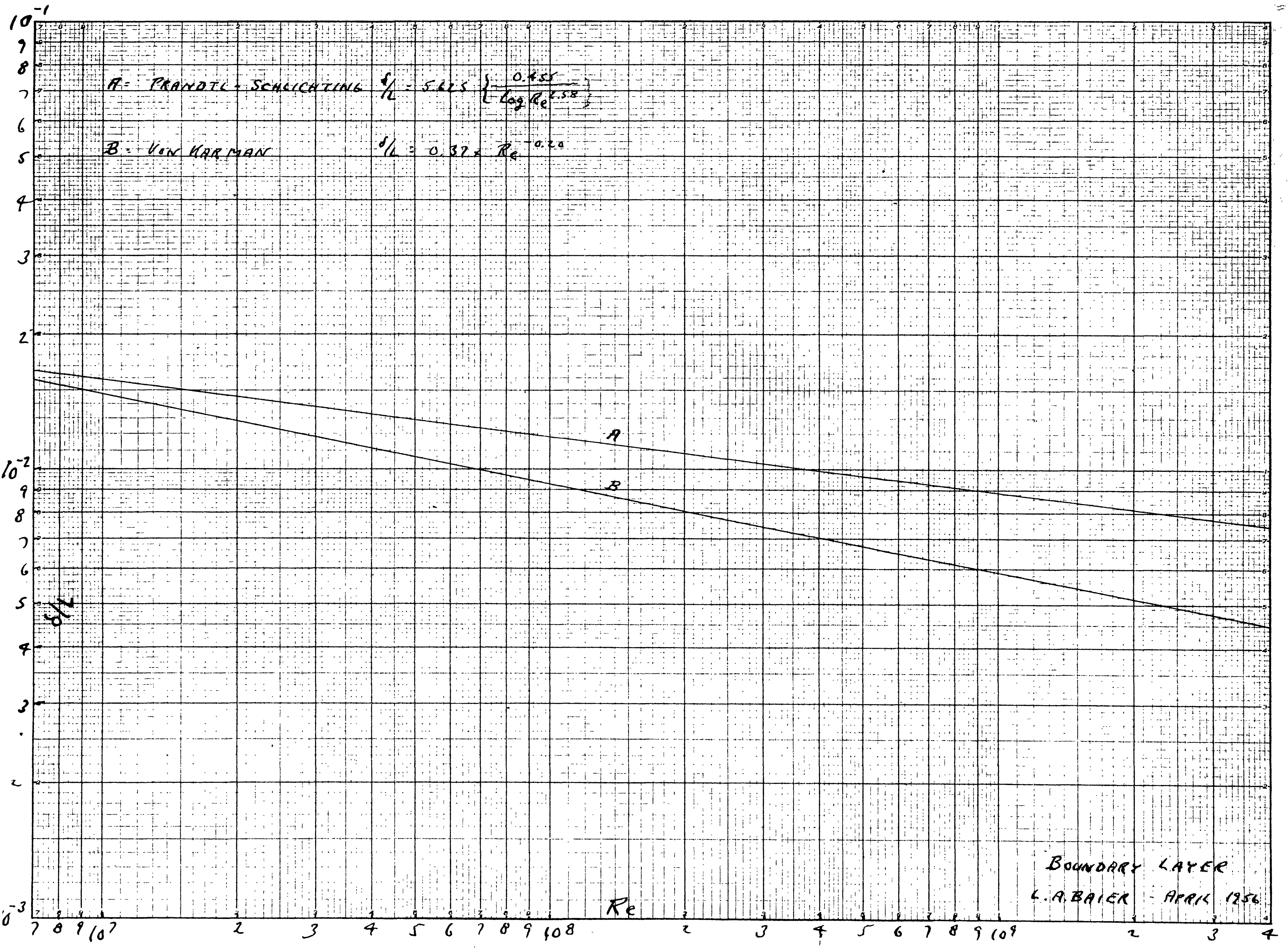
ALL MUST BE IN CONSISTENT UNITS - EITHER F.P.S. OR C.G.S.

FORMULAE

COMPLETELY ROUGH FLOW
SCHLICHTING $C_f = (1.89 + 1.62 \log \frac{L}{k_s})^{-2.5}$
SMOOTH TURBULENT FLOW
SCHLICHTING $C_f = .455 (\log R)^{-2.58}$
SCHOENHERR $.242 = \sqrt{C_f} \log (R \cdot C_f)$

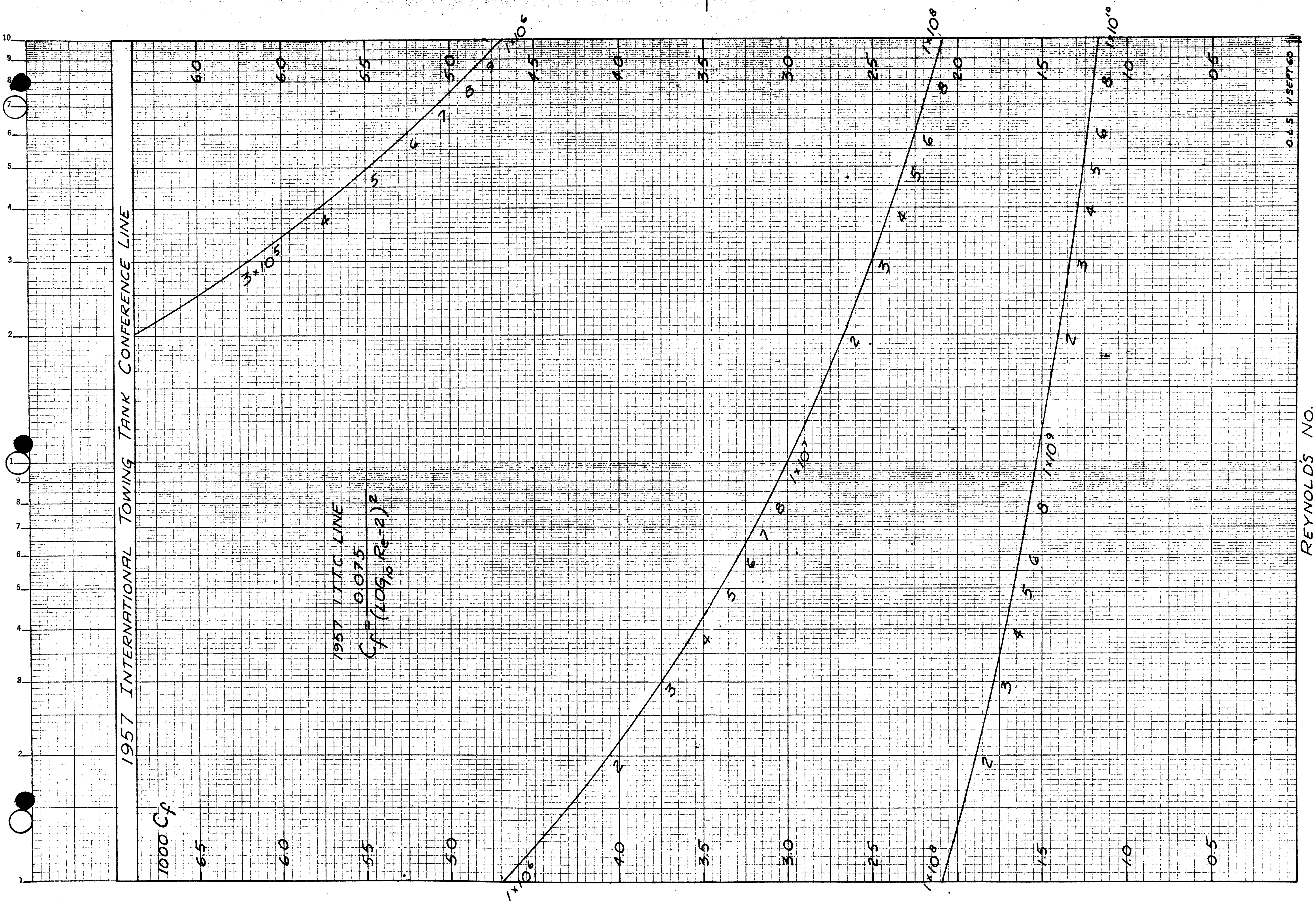
REFERENCES

N.A.C.A. TECH. MEMO. NO. 823, APRIL 1937
INGENIEUR - ARCHIV, VOL. VII, NO. 1, FEB. 1936
"EXPERIMENTAL INVESTIGATION OF THE PROBLEM OF SURFACE ROUGHNESS"



BOUNDARY LAYER

L.A. BAIER - APRIL 1956



1957 INTERNATIONAL TOWING TANK CONFERENCE LINE

1957 I.T.T.C LINE

$$C_f = \frac{0.075}{(\log_{10} R_e - 2)^2}$$

EHP CALCULATION FORM

	LENGTH (L) FEET	DRAFT (H) FEET	DISPLACEMENT (Δ)	WETTED SURF(S) FT.^{2}	TEMP °F	DENSITY(ρ) LB.SEC^2/FT^4	KINEMATIC VISCOSITY FT^2/SEC.
MODEL		FWD AFT					
SHIP		FWD AFT					

APPENDAGES:

$\lambda =$

$C_B =$

$C_X =$

$C_P =$

$L/B_X =$

$B_X/H =$

$\Delta/(0.01L)^3 =$

	MODEL	SHIP
L_{WL}		
L_{BP}		
B_X		

MODEL Nº

TEST DATE

TEST BY

STIMULATION

PROJ. Nº

SHIP

DATE

CALC. BY

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
v	v^2	R_t/v^2	R_t	Re_{MODEL}	av^2	C_t	C_f	C_r	$V_K/\sqrt{L_M}$	$V_K/\sqrt{L_S}$	V_K	v	v^2	Re_{SHIP}	C_f	$C_f+\Delta C_f$	C_r	C_t	Av^2	R_t	EHP					

1 - MODEL TEST VALUES OF SPEED IN FEET/SEC.

2 - SQUARE COL.1

3 - MODEL TEST VALUES OF RESISTANCE/SPEED^2

4 - MODEL RESISTANCE IN POUNDS

5 - $Re_{MODEL} = \frac{vL_M}{\nu_m} = \text{COL. 1} \times \frac{L_M}{\nu_M} = \text{COL. 1} \times \frac{(\quad)}{(\quad)} = \text{COL. 1} \times (\qquad)$

6 - $a = \frac{\rho}{2} S = \frac{(\quad)}{2}(\qquad) = (\qquad)$

7 - $C_t = \frac{R_t}{\frac{\rho}{2} S v^2} = \frac{\text{COL. 4}}{\text{COL. 6}}$

8 - FROM TABLE OF FRICTIONAL RESISTANCE COEFF. USING R_e FROM COL.5

9 - $C_r = C_t - C_f$ = COL.7 - COL.8

10 - $\frac{V_K}{\sqrt{L_M}} = \frac{v}{1.6889\sqrt{L_M}} = \frac{\text{COL. 1}}{1.6889\sqrt{L_M}} = \frac{\text{COL. 1}}{(\qquad)}$

11 - SAME AS COL.10

12 - $V_K = \frac{V_K}{\sqrt{L_S}} \times \sqrt{L_S}$ = COL.11 × $\sqrt{L_S}$ = COL.11 × ( )

13 - $v = 1.6889 V_K$ = 1.6889 × COL.12

14 - SQUARE COL.13

15 - $Re_{SHIP} = \frac{vL_S}{\nu_S} = \text{COL. 13} \times \frac{L_S}{\nu_S} = \text{COL. 13} \times \frac{(\quad)}{(\quad)} = \text{COL. 13} \times (\qquad)$

16 - FROM TABLE OF FRICTIONAL RESISTANCE COEFF. USING R_e FROM COL. 15

17 - COL.16 + ΔC_f

18 - SAME AS COL 9

19 - $C_t = C_f + \Delta C_f + C_r$ = COL.17 + COL.18

20 - $A = \frac{\rho}{2} S = \frac{(\quad)}{2}(\qquad) = (\qquad)$

21 - $R_t = C_t \frac{\rho}{2} S v^2$ = COL.19 × COL.20

22 - EHP = $0.00307\, V_K R_t$ = 0.00307 × COL.12 × COL.21

REMARKS:

The INSTITUTE of MARINE ENGINEERS
Transactions

1950, Vol. LXII, No. 8.

Modern Methods for Computing the Surface Friction of Ships*

K. C. BARNABY, O.B.E., B.Sc. (Member)

SURFACE FRICTION

Water is very far from being the perfect fluid that one is able to assume in hydrostatics. It may seem rather anomalous that in this branch of Naval Architecture it can be assumed that water is incompressible, incapable of resisting deformation and entirely frictionless, when in fluid dynamics surface friction is the major element in the resistance of all large vessels. There may even be a feeling that nature was a trifle careless with her specifications and that she might have made water rather more of a perfect fluid with a corresponding benefit to all forms of propulsion including the fish.

Table 1 examines the first point. It will be seen that the frictional drag for commercial vessels may be less than 0·1 per cent of the displacement and that even on express Atlantic liners it may not exceed 0·4 per cent. These low ratios justify the neglect of viscosity in static calculations and suggest that such computations will remain accurate until very high speeds are attained. It will also be noted from the table that for commercial types surface friction accounts for from 60 per cent to just over 70 per cent of the total resistance. These results are, of course, for normal types. The most extreme cases within the author's experience were a floating aerodrome with the net

TABLE 1.—FRICTIONAL DRAG AND TOTAL RESISTANCE FOR VARIOUS TYPES

Type of ship	Length, feet (l)	Speed, knots (v)	Displacement, tons, (Δ)	Frictional drag, tons	Frictional drag, per cent of displacement	Total resistance to propulsion, tons	Friction, per cent of total resistance	Speed length ratio $v/\sqrt{l}$
Cargo vessel	300	9·5	5,965	5·4	0·091	8·2	66	0·55
Cargo vessel	450	13·0	16,000	17·9	0·112	24·7	72	0·614
Coaster	195	10·5	1,640	2·72	0·166	4·85	56	0·752
Intermediate liner	465	17·0	15,820	30·4	0·192	45·4	67	0·788
Express liner	762	25·5	37,080	124·0	0·335	203·0	61	0·923
Express liner	962	31·0	79,000	288·0	0·364	458·0	62·8	1·0
Sailing yacht	50	7·25	32	0·140	0·438	0·26	54	1·027
Steam yacht	178	13·8	768	3·12	0·406	6·64	47	1·034
Motor yacht	115	13·0	212	1·19	0·562	3·29	36·2	1·210
Motor launch	49	11·0	15	0·167	1·11	0·88	19	1·57
Destroyer (deep)	350	32·5	2,455	34·7	1·41	109·0	31·8	1·70
Destroyer	364	36·57	1,962	39·2	1·99	96·5	40·5	1·92
Destroyer	295	36·045	1,140	26·2	2·30	64·6	40·6	2·098
Destroyer	271	37·52	884	23·9	2·70	55·8	42·8	2·28

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* Read before the Southern Junior Branch I.N.A. and I.Mar.E. at Portsmouth Municipal College on 24th November 1948.

inconsiderable displacement of two million tons and a hydroplane with a speed of over 100 knots. The corresponding figures are given in Table 2. It will be clear that on account of the high drag/displacement ratio, static methods would be completely out of place for the fast skimmer. Owing largely to the increasing ranges of modern aeroplanes the floating aerodrome never got beyond model and drawing board stages. This was a considerable relief to the author as he viewed the launching calculations—among other matters—with considerable trepidation.

TABLE 2

Item	Type	
	Floating aerodrome	Skimmer
Length, ft. (L)	2,000	35
Displacement, tons (Δ)	2,000,000	6·2
Speed, knots (V)	7·0	103·7
$\frac{V}{\sqrt{L}}$	0·157	17·55
Total drag, tons	300	3·4
Drag, per cent of Δ	0·015	55

Turning to the suggestion that nature might have made water rather less viscous, reflection shows that this would have had very unpleasant consequences. Storm waves would take longer to die out than the normal interval between gales so that the seas would be perpetually rough. Wave energy would, in fact, have to be largely dissipated on erosion of our foreshores and coast line. Ships would also roll more violently and capsize more readily so that one could be sure that not only would coastal areas be unpleasant and residential districts wasting assets, but shipbuilding itself would be a very depressing and discredited industry.

TABLE 3.—TYPICAL VALUES OF ρ, μ AND ν

Temperature, deg.		Density, ρ ML^{-3} = lb. × sec.^{2}/ft.^{4}		Viscosity $\mu \times 10^5$ $MT^{-1}L^{-1}$ = lb. × sec./ft.^{2}		Kinematic viscosity $\nu \times 10^5$. Units = L^2T^{-1} = ft.^{2}/sec.			
		American towing tank data				Lyle and Hosking values*		1942 values†	
1	2	3	4	5	6	7	8	9	10
deg. C.	deg. F.	F.W.	S.W.	F.W.	S.W.	F.W.	S.W.	F.W.	S.W.
0	32	1·9399	1·9947	3·74	—	1·931	1·916	1·9291	—
10	50	1·9396	1·9924	2·73	2·91	1·409	1·419	1·4080	1·4599
15	59	1·9384	1·9905	2·38	2·55	1·232	1·247	1·2285	1·2817
20	68	1·9367	1·9882	2·10	2·26	1·088	1·103	1·0836	1·1372
30	86	1·9317	1·9823	1·67	1·82	0·868	0·888	0·8657	0·91847

* As adopted at the 1935 Paris conference of tank superintendents

† As adopted at the 1942 American towing tank conference

It must, therefore, be recognized that nature knew what she was about in making water an imperfect fluid and that she went as far as was desirable in giving it only about one-thirteenth of the kinematic viscosity of air.

Before dealing with specific problems concerning the flow of liquids and the computation of surface friction, it would seem desirable to examine the actual qualities of water and the terms and units that are to be employed. This may seem elementary but, on the other hand, it is as well to be quite definite in the groundwork. There are two major differences as well as a number of minor differences between a real fluid such as water and an ideal fluid. The two main points are (1) the absence of slip at all rigid boundaries and (2) the presence of viscosity or shear resistance. It may be possible to insert small highly polished plates into a real fluid and obtain incomplete wetting. This is impossible with ordinary model or ship surfaces. The immersed portions are completely wetted and a film of water is attached to the surface, presumably by molecular attraction and interlocking. As a corollary, it is impossible to insert a plate, however thin, into a real fluid without changing the flow conditions in the vicinity of the plate. These changes will be discussed under the heading of "boundary layer".

THE DENSITY OF SEA WATER

This depends on the salinity and the temperature and varies appreciably from place to place and at various seasons of the year. The variation of surface salinity in the oceans is primarily dependent on the difference between the average evaporation and the average rainfall. Minimum values occur in high latitudes towards the poles and maximum values at about the two tropics with a slight reduction at the equator. Owing to the effect of temperature, the less saline water near the poles is often denser than at the equator. The mean value normally assumed for ship calculations is 1·026 which is equivalent to 64lb. per cu. ft. In this paper the author uses gravitational and not absolute units so that (ρ) for salt water = 64 ÷ 32·2 = 1·99 and for fresh water = 62·2 ÷ 32·2 = 1·94.

THE VISCOSITY AND KINEMATIC VISCOSITY OF FRESH AND SALT WATER

Real fluids can withstand a slight amount of tension owing to molecular attraction between the particles. This sets up an apparent resistance to shear between any two adjacent streamlines or layers and is termed the viscosity of the fluid. The coefficient of viscosity μ is defined as the ratio between the viscous stress and the angle of distortion. Typical values for μ are given in Table 3.

KINEMATIC VISCOSITY

The viscous stresses in a fluid are dependent on μ and on the velocity gradients but the effects on the motion are determined by the ratio of μ to the mass density of the fluid. This ratio $\frac{\mu}{\rho}$ is known as the kinematic viscosity and is denoted by the symbol ν. Typical values of ν are given in Table 3. It will be noted that two differing sets of values are given for both fresh and salt water. The first pair (7 and 8) are the Lyle and Hosking figures as adopted at the 1935 Paris Conference of Towing Tank Superintendents. The second pair are the figures adopted by the 1942 American Towing Tank Conference. There is little difference in the fresh water values but the salt water figures are about 3 per cent higher than the British values. The Americans state that the "fifth significant figures are doubtful". This implies an accuracy of the order of 0·05 per cent.

If this high standard of accuracy has really been attained, the British figures for the kinematic viscosity of salt water are obviously much too small. It seems that this large difference ought to be examined.

LAWS OF DIMENSIONAL SIMILARITY AND COMPARISON

The comparison of small inclined plates with large ones, or of models with full sized vessels, can only be undertaken with confidence providing the connecting equation is known for each source of resistance. If a plate is inclined at a small angle to the stream flow, the tangential component is more important than the normal component and is mainly a viscosity effect. As the angle of inclination is increased, the reverse applies and the normal component becomes much more important than the tangential component and is mainly a pressure effect. Before dealing with the extent to which these results can be applied to a ship's hull, it is desirable to examine the form that the connecting equation must take (1) when both viscosity and gravity are included (2) when gravity is omitted and (3) when viscosity is omitted. For this purpose the law of dimensional similarity is an invaluable tool.

This law elaborates the rather obvious point that the dimensions on both sides of an equation must be identical. If a resistance is to be evaluated, the various factors must be so assembled as to give an answer with the dimensions of a force, that is in terms of MLT^{-2}. For flat plates, aerofoils or ship-shaped forms moving in a real fluid, the various influencing factors can be set out as below.

	Factor	*Dimensions*
(1)	Length L as determining sizes	L
(2)	Velocity V as determining speed ...	LT^{-1}
(3)	Mass density ρ as determining inertia ...	ML^{-3}
(4)	Viscosity μ as influencing frictional drag	$ML^{-1}T^{-1}$
(5)	Gravity g as affecting weight and energy	LT^{-2}
(6)	Shape factors r_1, r_2, etc., as affecting the character and type of the stream flow pattern	(non-dimensional)
(7)	Roughness of surface r_0 as affecting frictional drag and expressed as the ratio of the size of grain to the length ...	(non-dimensional)

The total resistance can then be written $R = \phi[LV\rho\mu g r_1 r_2 r_0]$ where ϕ is an unknown function. This expression can be put in the following equations:—

(1) Including both viscositv and gravity

$$R = L^2V^2\rho\phi\left[\left(\frac{\nu}{LV}\right)^d\left(\frac{Lg}{V^2}\right)^e\right]$$

(2) Including viscosity only

$$R = L^2V^2\rho\phi\left[\frac{\nu}{LV}\right]$$

(3) Including gravity only

$$R = L^2V^2\rho\phi\left[\frac{Lg}{V^2}\right]$$

If a resistance is entirely viscous, equation (2) applies and shows that the essential requirement for dynamical similarity is the constancy of the unknown function $\phi\left[\frac{\nu}{LV}\right]$ Hence comparisons at different scales are valid only if made at constant $\left[\frac{\nu}{LV}\right]$ values.

If a resistance is entirely due to gravity (i.e., to wave-making) and is unaffected by viscosity, equation (3) is applicable and shows that dynamical similarity is obtainable only when the function $\phi\left[\frac{Lg}{V^2}\right]$ is constant. The more convenient reciprocal form used in practice is $\frac{LV}{\nu}$ which is termed the "Reynolds number" after Professor Osborne Reynolds who discovered its significance in connexion with laminar and turbulent flow and $\frac{V}{\sqrt{Lg}}$ termed the Froude number. As g is sensibly constant the more usual form for the latter is, of course, the familiar speed length ratio $\frac{V}{\sqrt{L}}$.

It should be specially noted that the law of dimensional similarity affords no justification whatever for the usual practice of treating viscous and gravity resistances as independent and additive, unless these sources of resistance are confined to different bodies of water. With smooth models, this is certainly the case. Viscous resistance is confined to a relatively narrow boundary layer, whereas gravity resistance or wave-making affects a very large body of water that is mainly outside the boundary layer. Under these circumstances it was undoubtedly correct to treat viscous and gravity resistances on models as additive and independent and this is, of course, the basis of all tank experiments.

FROUDE'S EXPERIMENTS

It is just eighty years since William Froude submitted to the Admiralty his proposals for setting-up an experiment tank where the laws of ship resistance could be investigated. There was a certain amount of opposition as at that date model tests were apparently in considerable disrepute owing to their seeming inconsistency. This defect was attributed by Froude to the results not being compared at the corresponding speeds given by his law of comparison and to the general crudity of the towing and recording apparatus. After some delay the proposal was accepted and the pioneer tank built on land adjoining Froude's house at Torquay. Extremely valuable results soon flowed from the new tank and amply vindicated the innovation.

Only one month after the completion of his Torquay tank, Froude started his classical experiments on the surface friction of planks. It was necessary to find the variation of resistance with speed and with the type and length of surface, since at that time (1872) these laws were completely unknown and it was assumed that the resistance varied with the square of the speed. Ships were in fact powered on the basis of an "Admiralty formula" which was $\text{speed}^3 \times \frac{\text{(immersed midship section area)}}{\text{i.h.p.}} = \text{constant}$. The appropriate values for this constant were supposed to vary from about 500 for a "short vessel" to about 600 for a long vessel. Both the significance of the Reynolds number and the dimensional argument of Lord Rayleigh had still to be demonstrated.

Froude took thin planks coated in various ways and varying from two to fifty feet in length. These were towed at increasing speeds and the corresponding resistances carefully measured. It was then found that the results could be expressed in an empirical formula $R = fSV^n$. These results are given in Table 4 and can be summarized as follows:—

(1) The coefficient f decreases with length, except at very short lengths.

(2) The index n is appreciably less than 2 except for rough surfaces when it is constant at 2·0. It is greater than 2·0 in the case of a very short and very smooth surface.

(3) The degree of roughness has a great influence on the magnitude of f.

Despite some "perplexing anomalies" as Froude termed them, the general pattern was clear. The new formula $R = fSV^n$ was not only established with suitable coefficients but was supposed to have solved the problem of surface friction. Certainly the formula has been a very satisfactory guide and as late as 1935 the Paris Conference of Tank Superintendents

TABLE 4.—FROUDE'S ORIGINAL FRICTION CONSTANTS CONVERTED TO KNOTS IN FORMULA $R = fSV^n$

Nature of surface	Length of surface							
	2 feet		8 feet		20 feet		50 feet	
	n	*f*	*n*	*f*	*n*	*f*	*n*	*f*
Varnish	2·0	0·0117	1·85	0·0121	1·85	0·0104	1·83	0·0097
Paraffin wax	1·95	0·0118	1·94	0·0100	1·93	0·0088	—	—
Tinfoil	2·16	0·0064	1·99	0·0081	1·90	0·0089	1·83	0·0095
Calico	1·93	0·0281	1·92	0·0206	1·89	0·0184	1·87	0·0170
Fine sand	2·0	0·0231	2·0	0·0166	2·0	0·0137	2·06	0·0104
Medium sand	2·0	0·0257	2·0	0·0178	2·0	0·0152	2·0	0·0139
Coarse sand	2·0	0·0314	2·0	0·0204	2·0	0·0168	—	—

R = Frictional resistance in lb. S = Wetted surface in sq. ft.

agreed to the use of the Froude standard constants as given in Table 4(a) (abridged).

These constants were based on the Froude values as modified and extended by Froude's son and successor, R. E. Froude, who had also introduced his "O" values. With the latter f is defined as being equal to $\frac{\text{"O"} \times L^{0.0875}}{14{\cdot}165}$. "O" values are also given in Table 4(a).

Froude had not explained the extremely complicated mechanism of surface friction. He had produced a thoroughly practical and, on the whole, a very successful method for dealing with the problem. The full explanation started with Osborne Reynolds and is not complete even now particularly as regards the effect of roughness.

TABLE 4(A).—FROUDE'S FRICTION CONSTANTS IN THE FORMULA $R = fSV^{1\cdot825}$ AND FROUDE'S "O" VALUES, BOTH AS ADOPTED AT THE 1935 PARIS CONFERENCE OF TANK SUPERINTENDENTS

Length, feet	*f*	"O"
10	0·011579	0·13409
20	0·010524	0·1147
30	0·010068	0·1059
40	0·009791	0·10043
50	0·009607	0·09664
60	0·009475	0·0938
80	0·009309	0·08987
100	0·009207	0·08716
120	0·009135	0·08511
160	0·009046	0·08219
200	0·008992	0·08102
300	0·008902	0·07655
400	0·008832	0·07406
500	0·008776	0·07217
600	0·008726	0·07062
700	0·008680	0·06931
800	0·008639	0·06818
900	0·008608	0·06724
1,000	0·008574	0·06636

R is in lb., S in sq. ft. and V in knots.
f is for sea water; for fresh water multiply by 0·975.

REYNOLDS NUMBER

Reynolds discovered the different laws governing laminar or streamline flow and turbulent or non-streamline flow and established the critical velocities at which these changes occurred. His formula for the lower critical velocity in straight parallel pipes and which is applicable to all fluids is $V_c = \frac{2000\mu}{d\rho}$ where d is the diameter. The above formula can be written as $V_c = \frac{2000\nu}{d}$ and differs from that already discussed (Equation 2) only by the substitution of d for L. With a parallel pipe, the mean velocity must remain constant. The actual velocity will vary from zero at the walls to a maximum at the centre or towards the centre in the case of large diameter pipes of short length. Both the mean velocity and the perimeter of the pipe are controlled by the diameter, hence the primary importance of d. In the case of long planks or of ship forms there is no such restriction and the boundary layer (to be described later) can be regarded as a diverging pipe causing reduced velocity gradients with increase of length. The governing linear dimension is therefore length and not girth or draught. If a turbulent fluid is introduced into a pipe line at such a rate of flow that the Reynolds number is below the lower critical velocity the eddies will die out and the flow become streamline. A more important point for naval architects is the higher velocity at which the flow is definitely turbulent. For full models the Reynolds number for this is probably about 4×10^6 and for fine models about 6×10^6*. The somewhat astronomical size of Reynolds number has militated against its general use. A convenient way of quickly calculating the R_n value is to base it on that of a ship of 100 feet and 7·384 knots. The R_n value for such a ship in salt water at 15 deg. C. (59 deg. F.) is $1{\cdot}0 \times 10^8$. For a ship of different speed and length, it is then only necessary to multiply by the new speed and new length and to divide by the old figures. Thus the R_n for a ship of 400ft. and 18 knots in 15 deg. C. salt water would be $9{\cdot}75 \times 10^8$.

The above relation will be found to simplify the use of Reynolds number in the normal case of standard temperature conditions. If R_n is to be calculated *ab initio*, speeds must, of course, be in ft. per sec.

GEBERS' EXPERIMENTS

After Lord Rayleigh's demonstration of the connexion between the Froude number and the Reynolds number, there was an uneasy feeling that the latter ought to be used as a basis for the estimation of surface friction, despite the apparent success of the empiric formula $R = fSV^n$. Dr. G. S. Baker† in 1915 was one of the first to plot friction on a basis of R_n. The matter has more recently assumed great importance for two reasons. There have been the publication of numerous friction "lines" and formulæ by Gebers, Prandtl, Schlichting, Von Karman, Hiraga, Schoenherr and others. In addition, there is an increasing body of evidence that under certain

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* See postscript, p. 289.

† Baker, G. S. 1915. Trans.N.E.C.I.E.S., Vol. 32, p. 41, "Notes on Model Experiments".

special conditions of length, speed and surface condition, the conventional methods give results that are not in harmony with the recorded thrusts and powers.

Dr. Friedrich Gebers carried out an extensive series of friction tests in the Vienna tank. Starting with the idea that the Froude results contained a certain amount of residual resistance, Gebers took very special precautions to ensure smooth and straight surfaces with well faired endings. The results obtained did show an appreciable reduction from Froude's coefficients but this was due primarily to the greater smoothness of Gebers' models.

Gebers found that if he ignored some specially low readings at low R_n values the remaining results could be expressed by the formula (put into English units):—

$$C_f = \frac{R_f}{\frac{\rho}{2}SV^2} = 0{\cdot}02058(R_n)^{-\frac{1}{8}}$$

If this expression is plotted on log-log scale, it gives a straight line as in Fig. 1.

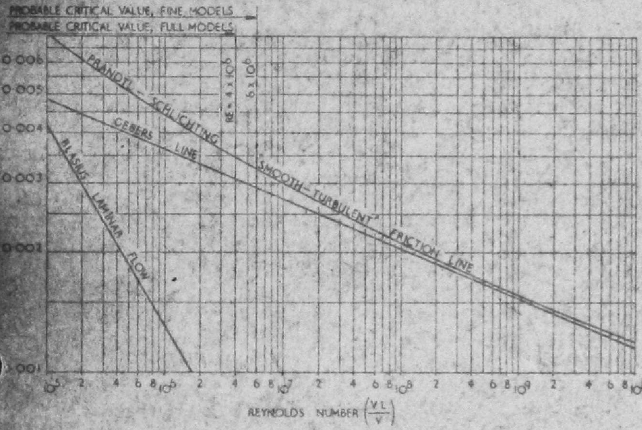

FIG. 1—*Curves of skin friction resistance*

The abscissæ are Reynolds numbers and the ordinates C_f. In the above formula R_f is in lb., S in sq. ft., and V in ft. per sec. Suppose one examines the deviation from the Froude values for a typical case. Let R_n be $4{\cdot}6 \times 10^8$ and the length of vessel 200 feet. Then by re-arranging the relation already given, $V = \frac{7{\cdot}384 \times 4{\cdot}6 \times 100}{200} = 16{\cdot}98$ knots. If the vessel is 500 feet long, the speed would be $7{\cdot}384 \times 4{\cdot}6 \div 5 = 6{\cdot}795$ knots. The calculated resistances per sq. ft. are then as follows:—

(1) By Gebers formula $R_f = 1{\cdot}39$lb. for 200 feet and 16·98 knots.
(2) By Froudes formula $R_f = 1{\cdot}58$lb. for the above conditions.
(3) By Gebers formula $R_f = 0{\cdot}22$lb. for 500 feet and 6·795 knots.
(4) By Froudes formula $R_f = 0{\cdot}29$lb. for the above conditions.

The very considerable difference from the Froude values will be noted.

LAMINAR AND TRANSITIONAL FLOW

The probable critical value for Reynolds number for fine models has been given as 6×10^6. Plane surfaces for friction tests are nearest to the finest possible form so that the shorter surfaces used by Froude and Gebers were well within the danger zone for transitional flow. The equation for laminar flow has been established by Blasius as $C_f = 1{\cdot}327\,R_n^{-\frac{1}{2}}$ and is shown in Fig. 1. It follows that a high index n or a low coefficient in the Froude formula is *prima facie* evidence of transitional flow, since the coefficient C_f has to rise from the Blasius curve during the transitional régime.

If the Froude results given in Table 4 are examined, it will be seen that the tin foil surface shows these indications of transitional flow for the shorter lengths. The tin foil was noted as being appreciably smoother than the other surfaces, so the reason for some of Froude's "perplexing anomalies" seems clear. Lackenby in 1937 gave convincing evidence of the existence of transitional flow in both Froude's and Gebers' experiments. By plotting the actual trial spots in log-log form Lackenby showed that at low R_n values the curves were definitely drooping below straight lines drawn through the high R_n spots.

"SMOOTH-TURBULENT" FRICTION LINES

This somewhat contradictory description refers to fully turbulent flow over ideally smooth surfaces. Von Kármán realized the need for completely turbulent flow at low R_n numbers if a satisfactory basis for extrapolation is to be obtained. His curve is higher than that of Gebers at low R_n values but falls under it at high R_n values. This would give a still lower estimate of skin friction, whereas trial reports seem to show that this would be inadmissible. Schoenherr from an analysis of existing data and certain trials of his own produced a formula $\frac{0{\cdot}242}{\sqrt{C_f}} = \log_{10}(R_n C_f)$. The Prandtl-Schlichting formula is $C_f = 0{\cdot}455\,\log_{10}(R_n)^{-2{\cdot}58}$. This gives almost identi-

TABLE 5.—MODERN FRICTION "LINES"

Reynolds' number R_n	Log R_n	$C_f = \frac{\text{Resistance in lb.}}{\rho/2\,SV^2}$			
		Gebers	Schoenherr	Prandtl-Schlichting	Lackenby*
1×10^6	6·0	0·00366	0·00441	0·004470	0·00495
$3{\cdot}162 \times 10^6$	6·5	0·003169	0·003567	0·003637	0·00401
1×10^7	7·0	0·002744	0·002937	0·003004	0·00328
$3{\cdot}162 \times 10^7$	7·5	0·002377	0·002452	0·002514	0·00271
1×10^8	8·0	0·002058	0·002073	0·002128	0·00225
$3{\cdot}162 \times 10^8$	8·5	0·001782	0·001772	0·001820	0·00190
1×10^9	9·0	0·001544	0·001532	0·001571	0·00162
$3{\cdot}162 \times 10^9$	9·5	0·001336	0·001333	0·001366	0·00140
1×10^{10}	10·0	0·001157	0·001172	0·001197	0·00123

* From formula $C_f = 0{\cdot}0006 + 0{\cdot}0791 R_n^{-{\cdot}21}$ for Froude-Kempf data

cal results to the Schoenherr formula and is shown in Fig. 1. The values for four friction lines are given in Table 5 and it will be noted that the differences are very small for the normal working range, despite the apparent large variation in the formulæ.

BOUNDARY LAYER FLOW CONDITIONS

It is assumed that all measurable frictional resistance takes place within a relatively narrow belt of water termed the "boundary layer" since the velocity of flow at the outer fringe is approximately that of the fluid. The boundary layer can be imagined as consisting of a number of thin stream bands each with gradually decreasing velocity from the surface to the outer fringe. Normal pressures are transmitted without loss across the stream bands, but no tangential stress is assumed beyond the outer boundary.

Flow within the boundary layer can be entirely laminar, mainly turbulent, or entirely turbulent. The various changes are shown in diagrammatic form in Fig. 2 which exaggerates the thickness of the laminar film for the sake of clearness.

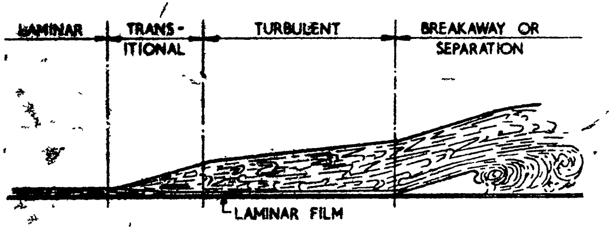

FIG. 2—*Flow within the boundary layer*

If the model is short there is a serious risk of laminar or at all events of transitional flow near the bow. When laminar flow breaks down there is a thickening of the boundary layer and a portion of the original laminar boundary layer will continue as a thin laminar film which may be less than 1/1,000 inch in thickness. The layer then continues in what is called the "smooth turbulent" condition. If there are roughnesses exceeding the laminar film, the latter will break down in wake of the roughnesses. The film will be re-formed providing there is a sufficient length of smooth portion.

Owing to its extreme thinness, the laminar film cannot be detected by Pitot-tube measurements. Any such measurements are taken in the turbulent area outside the film and when taken as close as possible to the surface, they indicate a velocity of about 50 per cent of that of the hull or surface. The inferences to be drawn are that (1) about 50 per cent of the total variation in velocity at any section of the boundary layer must occur within the thin laminar film and (2) that all momentum changes can be considered as taking place within the turbulent region outside the film and affecting the remaining 50 per cent of the velocity.

Important experiments on the flow through artificially roughened pipes were carried out by Nikuradse at Gottingen. Pipes of various diameters were coated with sand of different grades of coarseness. The scale of roughness was taken as being represented by the factor $\frac{r}{k}$ where r is the radius of the pipe and k is the average height of the sand grains. The results of the tests may be summarized as follows:—

(1) At sufficiently low Reynolds numbers, the resistance curve followed the ordinary curve for laminar flow over smooth surfaces.
(2) Laminar flow broke down at the same R_n values for all grades of roughness.
(3) The resistance coefficient followed down the smooth-turbulent line for a considerable range of R_n and then became constant.
(4) The coarsest grade of coatings gave coefficients that passed directly from the transitional flow period region to a constant value and did not follow the smooth-turbulent line at all.

The inferences to be drawn are as follows:—

(a) Roughness has no effect on resistance provided the flow remained laminar.
(b) So long as the roughness does not protrude through the thin laminar layer film at high R_n values or through laminar boundary layer at low R_n values, the surface remains "hydraulically smooth" and there is no increase in resistance.
(c) Outside the limits indicated in (a) and a certain transitional period the resistance coefficient for any grade of roughness becomes constant at all values of R_n. This implies that at sufficiently high speeds, the effect of viscosity becomes negligible in comparison with that of roughness.

COMPARISON OF FROUDE'S CONSTANTS WITH MODERN FORMULATIONS

If the Froude formula $R_f = fSV^{1\cdot825}$ is inserted in the equation for the resistance constant, the latter can be written as

$$C_f = \frac{fSV^{1.825}}{\frac{\rho}{2}Sv^2} = \frac{f}{2{\cdot}824V^{0.175}}.$$

In the above equations V = speed in knots and v = speed in ft. per sec.. With the Froude constants, f was seen to vary with length. By calculating C_f from the above equation and taking a fixed length and varying speeds, a series of values are obtained which correspond with definite Reynolds numbers. Plotted in log-log form these values lie on a straight line as in Fig. 3. Thus at 400 feet length $f = 0{\cdot}008832$ and at 20 knots R_n by the relation method already stated $= 1{\cdot}0 \times 10^8 \times \frac{400}{100} \times \frac{20}{7{\cdot}384} = 1{\cdot}085 \times 10^9$. Hence $C_f = 0{\cdot}00185$.

In this way a series of parallel straight lines will be obtained each corresponding to some fixed length. If the Froude lines are accepted as being reasonably accurate for normal ship surfaces and the smooth-turbulent line for ideal surfaces, any differences must be due to roughness. Also, no fixed constant will bring the two methods into agreement, except at one length. It will either be too great at short lengths or too small at long lengths.

Fig. 3 can be used for the calculation of surface friction by either the smooth-turbulent line and roughness corrections or by the Froude constants. Each intersection of a Froude line and an R_n ordinate corresponds with a definite speed. Fixed speed lines can thus be drawn and they enable the diagram to be used without having to first work out the R_n value.

Suppose one requires the frictional horse-power on a Froude constant basis for 15 knots and a 400-feet ship of 9,500 tons. The C_f value from Fig. 3 is 0·00195 and the wetted surface is 30,000 sq. ft. Conversion factors will be needed as the C_f values are for ft. per sec. These factors are as follows:—

Resistance, R in lb. $= C_f \times \frac{\rho}{2} SV^2 \times \left(\frac{6080}{3600}\right)^2 = 2{\cdot}824\, C_f SV^2$

Power, f.h.p. $= C_f \times \frac{\rho}{2} SV^3 \times \left(\frac{6080}{3600}\right)^3 \div 550 = 0{\cdot}00867\, C_f SV^3$

The above equations give for 15 knots $R = 37{,}200$ and frictional horsepower $= 1{,}715$.

It is often convenient to express the frictional horse-power in terms of circular C and obtain this from C_f by means of a relation factor. Using the Taylor expression for wetted surface $Ⓒ_F = 3{\cdot}7033\, C_f c\sqrt{L \div \Delta}$. Putting c at 15·39 this becomes $Ⓒ_F = 57 C_f \sqrt{L \div \Delta}$.

Summing up:—

(1) Surface friction is not affected by the kind of surface such as wax, steel, tin foil or wood but only by the smoothness of that surface.
(2) There is no slip at the surface itself and all measureable frictional resistance takes place within the boundary layer.
(3) The frictional resistance of a hydraulically smooth surface can be expressed, with considerable accuracy by either the Schoenherr or the Prandtl-Schlichting

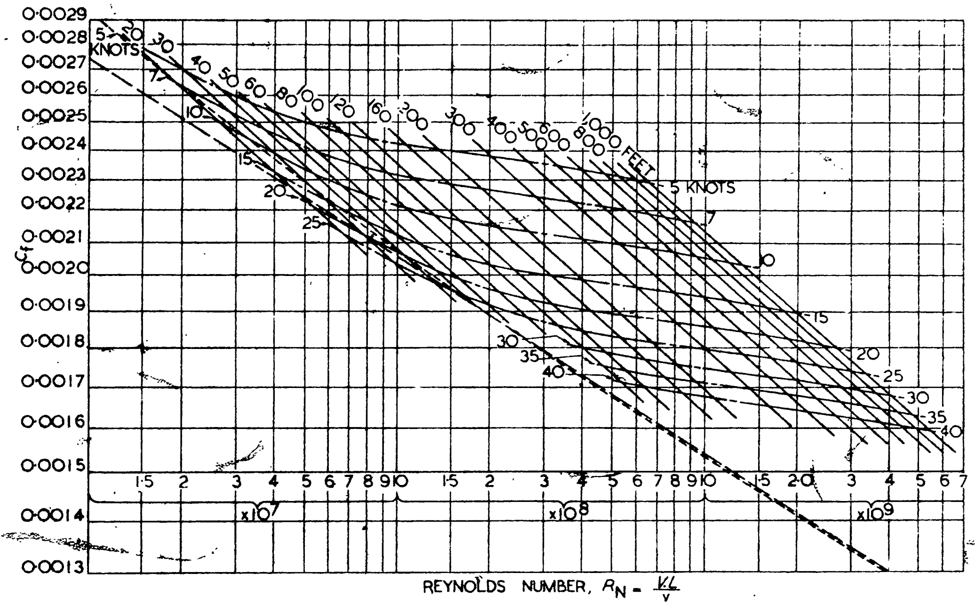

FIG. 3—*Based on a kinematic viscosity* ν *for salt water at 15 deg. C. or 59 deg. F.*

Frictional resistance, $R = C_f \frac{\rho}{2} S V^2$ lb.

$R = 0{\cdot}99\, C_f S V^2$ for salt water
S = area of wetted surface, sq. ft.
V = speed in ft. per sec.

Froude's coefficients ————
Geber's line — — — — —
Shoenherr's smooth-turbulent line - - - - - - - -

formula. Additional roughness coefficients have then to be added to C_f to represent actual ship surfaces. Precise values for these are a matter of great difficulty.

(4) The frictional resistance of a smooth freshly painted ship surface can be expressed with reasonable accuracy by the Froude formula $R = fSV^{1.825}$ where f has the values of Table 4(a). At least 6 per cent must, however, be added to represent the resistance of rivets and plate edges, etc., on normal commercial vessels.

At the recent conference of towing tank superintendents, the American delegation made a strong plea for the adoption of the Schoenherr method with the fixed roughness addition of 0·0004. This addition was admitted to be only tentative. The British speakers took the view that while one might have to come to a smooth-turbulent method, the time was hardly ripe as it was not yet certain what was the precise formula to be used. It was therefore agreed that for the present both the Schoenherr and the Froude methods could be used. The whole question is to be brought up at the next conference when further evidence will be available.

These two methods will be used for the next few years. Both cannot be equally correct, so it seemed desirable to try and devise a method of plotting surface friction which will indicate quite clearly the differences between Schoenherr and Froude and, if possible, enable judgment to be made as to which is the more correct method. The author suggests that one can accept the following: (1) That the law of dimensional similarity must be followed; (2) That the standard Froude coefficients do give with very considerable accuracy the resistance of the smooth freshly painted surface of a 400 feet vessel provided a suitable allowance is added for rivet projection, plate edges, etc.; (3) That no ship surface can be "hydraulically smooth".

The reason for postulating a 400 feet ship is that so many vessels of about this size have been tank tested, built and have given a satisfactory agreement between tank tests and trial results. Proceeding on these assumptions and from first principles start by trying to comply with dimensional similarity. Both μ and g must be retained since a resistance due to both viscosity and to gravity has to be evaluated. In air, or when very deeply submerged in water, one would not have to worry about wave-making resistance as there would be no free surface to be affected by the pressure changes consequent on diverting the flow lines over or around an obstacle or inequality. This would have enormously simplified the problem. Having to retain both μ and g, the resistance equation becomes

$$R_T = L^2V^2\rho\phi\left[\left(\frac{VL}{\nu}\right)^{-d}\left(\frac{V^2}{Lg}\right)^{-e}\right]$$

The indices d and e are not known and neither is the relationship between the two expressions based respectively on the Reynolds number and on the Froude number. If the relationship is assumed to be additive as is usual and also that x units are based on R_n and y units are based on F_n

$$R_T = L^2V^2\rho\phi\left[x(R_n)^{-d} + y\left(\frac{V^3}{\nu g}\times\frac{1}{R_n}\right)^{-e}\right]$$

This is based on the arithmetical relation $F_n = \frac{V^3}{\nu g} \times \frac{1}{R_n}$

This conversion may not help, but it suggests trying a plot on $\frac{V^3}{\nu g}$ as this is the relation factor by which one can convert F_n into R_n or *vice versa*. This has been done in Fig. 4. Resistance coefficients have been plotted in log-log form precisely as for a smooth-turbulent line and for both the Schoenherr and Froude methods.

The diagram shows at a glance at any point the precise difference between Schoenherr and Froude. One can read off at any speed the C_f values to be used by either method for a ship of any given length. The information is the same as in Fig. 3 but it is presented in a clearer fashion.

On examination of the diagram it will be noted that for short lengths, both methods show very similar coefficients which

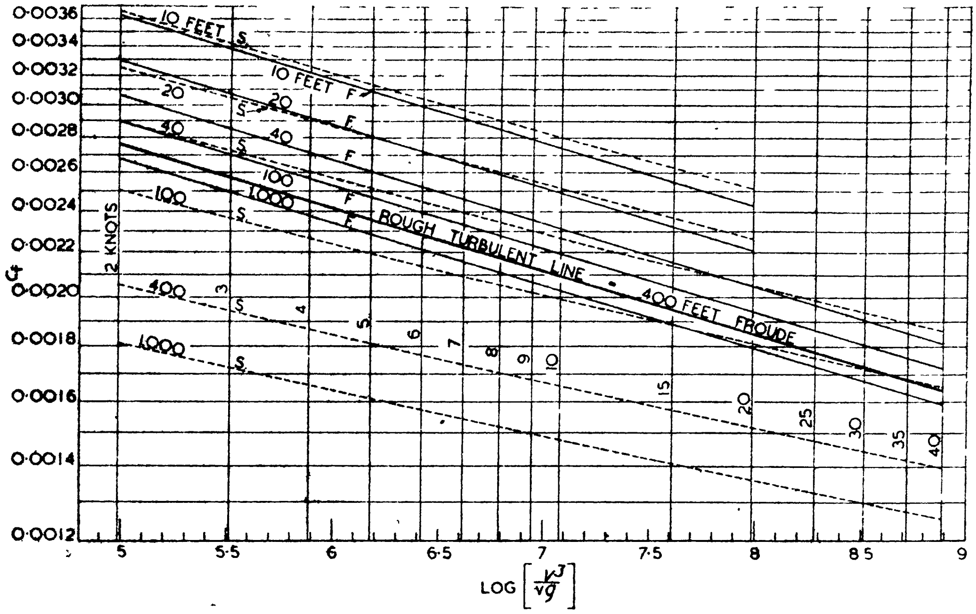

FIG. 4—*Based on a kinematic viscosity ν for salt water at 61 deg. F.*

Frictional resistance, $R = C_f \frac{\rho}{2} SV^2$ lb.

$R = 0{\cdot}99\ C_f SV^2$ for salt water
S = area of wetted surface, sq. ft.
V = speed in ft. per sec.
R = frictional resistance, lb.

Schoenherr's coefficients - - - S - - -
Froude's coefficients ——— F ———

vary very rapidly with length. At long lengths there are very large differences. The Froude values are varying very slowly with length, whereas the Schoenherr values are still varying more rapidly. Thus in going from 20 feet to 400 feet, the Schoenherr decrease is about 34 per cent compared with about 16 per cent for Froude. In going from 400 to 1,000 feet the Schoenherr decrease is about 10 per cent against only 3 per cent for Froude. These differences are obviously very serious.

The effect of roughness has been represented in the dimensional formula by a roughness ratio in the form of height of inequality divided by length. It is then non-dimensional and should only affect C_f by a numerical addition. Now paint is applied in the same grades, by the same painters and by the same brushes for both short and long vessels. Paint texture ratio th[illegible] decreases by about 60 per cent in going from a 400-feet ship to a 1,000-feet vessel.

Roughness due to rivet projection and plate edges will be approximately proportional to the thickness of the immersed shell plating. If the latter is assumed to be about 0·25 inch at 100 feet, 0·52 inch at 400 feet and 0·71 inch at 600 feet, this would be in line with normal commercial practice. This would give relative roughness ratios still decreasing with length but at about half the rate of the paint ratio.

Whatever proportion of the total C_f is assumed to be affected by these roughness changes it still follows that for a *fixed* and constant roughness ratio, the 1,000 feet line must be lifted by a greater amount than for the 400 feet line. This would decrease the already small gap between the two lines and render f in the Froude formula almost entirely independent of length and varying mainly with roughness ratio. This relative independence is not a theoretical result, it can be deduced by mere inspection of the Froude standard constants in Table 4. This will show the variation in f in going from 400 feet to 1,000 feet to be just 3 per cent or, as shown by Fig. 4. If, on the other hand, one is using Schoenherr for say 20 knots in both cases, there is first a deduction of about 10 per cent for length in place of 3 per cent and then a very large addition for roughness. For a fixed addition of 0·0004 for the case in point about 30 per cent is being added.

Summing up the position, it can be said that the Froude lines for long vessels are empiric rough-turbulent lines which are not very far from the truth but which involve the assumption that the roughness addition nearly equals the length deduction and so nearly cancels it. The Schoenherr line is correct dimensionally for the smooth surfaces of models, but it involves the use of very large corrections. If these are fixed amounts, the combined formula can no longer be dimensionally correct for two reasons: (1) The grade of roughness should, as shown, decrease with length, and (2) some account should be taken of the parasitic wave-making effect of roughness at high speed length ratios.

Doubt may be entertained about the necessity of the second correction, but this point could be easily cleared up by tank experiment. The late Admiral Dyson produced curves showing the enormously increased appendage resistance at certain speed length ratios. There seems no reason to suppose that surface roughness does not produce a similar effect though this has been disregarded by both Schoenherr and Froude. If one accepts, as the author would propose, the necessity for such a correction then it must be based on the Froude number not on the Reynolds number.

What seems required is a basic rough-turbulent line to include some standard grade of roughness expressed as a ratio of the length. The quite small corrections needed for difference in roughness ratio could be made appropriate to the length and character of the surface. This is a job for research workers and the author can only make suggestions which may or may not be helpful. It would seem, however, that one ought to abandon trying to use a unique line connecting both a model and a ship and equally applicable at all Reynolds num-

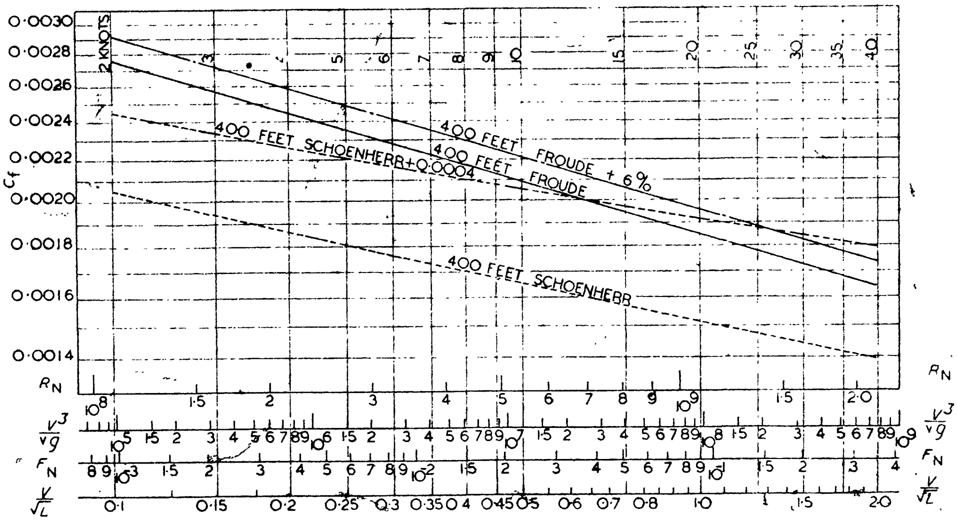

FIG. 5—*Based on a kinematic viscosity ν for salt water at 59 deg. F. (Lyle and Hosking values)*

bers. Coming to the ship, V and L in the Reynolds number can be divorced by plotting logarithmically on $\frac{V^3}{\nu g}$. It is then possible to consider one length only at a time as with Froude and the base line can be marked off in terms of both R_n and F_n or, if preferred, as R_n and $\frac{V}{\sqrt{L}}$. It is known that at low $\frac{V}{\sqrt{L}}$ values the effect of wave-making is very small so the gap at low values should be merely a numerical addition dependent on grade of roughness. At very high $\frac{V}{\sqrt{L}}$ values, the gap should also be a numerical addition since wave-making effect should again be nearly constant.

The mean line should in some way diminish with speed. This is indicated by the general trend of Figs. 4 and 5 and by the following consideration. Any resistance varying with the Froude number can be expressed arithmetically in terms of the Reynolds number by means of the relation factor $V^3 \div \nu g$ as in the equation on p. 287. It should vary, therefore, with $\frac{1}{R_n}$ that is a decrease with speed.

In Fig. 5, the Schoenherr and Froude lines for 400 feet have been lifted from Fig. 4 and provided with additional scales for both R_n and F_n. This combination can only be shown on a fixed length basis and it serves to remind one that if a resistance becomes independent of the Reynolds number it must also be independent of the Froude number. The practical naval architect is mainly interested in the fixed length of some ship or design and for this reason it is hoped that Figs. 3, 4 and 5 will be found useful.

POSTSCRIPT

Since this paper was written, there have been some very important developments in connexion with surface friction. Walker, Allan and Conn (see references) have shown quite clearly that laminar flow may persist to much higher Reynolds numbers than those formerly thought to be limiting values. It has also been shown that the liability to laminar flow is specially marked with rather full models running at deep draughts. This has been proved, not merely by the abnormal increase of resistance when trip-wires or other turbulence stimulators are used, but also visually by means of chemical detection methods. These latter are described by Walker.

The general condition favouring the persistence of laminar flow is a negative pressure gradient, that is one in which the pressure decreases in the direction of motion. The potential flow is then accelerating and the boundary layer tends to become thinner. This delays the transition point because, as already seen (Fig. 2) a boundary layer becomes thicker and not thinner at the transition point. A sharply raking bow also seems to delay transition.

It is probable that all tank models will in future be fitted with trip wires, or at least check tested to ensure that they are not needed. These precautions are no doubt more necessary with commercial models than with naval models which are usually of finer form and higher speeds. Fears have been expressed that, owing to the absence of turbulence stimulation in the past, many of the earlier tank experiments and the derived data are inaccurate. This is probably an over-statement but the early experiments on bow form and angle of entrance, etc., must be regarded with some suspicion until repeated with turbulence stimulation. Mr. Emerson of the N.P.L. is now doing work of this character. It is also possible that some of the residuary resistance values in Taylor's standard series have been under-estimated owing to laminar flow. It is known that Taylor had a masterful way in fairing out "rogue spots" and it is quite possible that these were sometimes connected with a delayed transition.

The other development that should be chronicled is due to Telfer. His "Extrapolator" method did not meet with much approval when it was first introduced, but in 1949 Dr. Telfer put forward a greatly improved version. A fundamental assumption is the very reasonable suggestion that one should divide turbulent friction into two portions, one affected by viscosity (but not, of course, by gravity) and the other by neither viscosity nor gravity. Putting this idea through the dimensional mill, one can add a fourth equation to the three given on p. 284. This is:—

$$R = L^2V^2\rho k_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Adding this to the viscosity effect equation (2), one gets for the combined equation:—

$$R = L^2V^2\rho\left(k_1 + k_2\left[\frac{\nu}{LV}\right]^{d}\right) \quad \ldots\ldots\ldots\ldots (5)$$

From Gebers and other results Telfer deduced the following values, $k_1 = 0{\cdot}0012$, $k_2 = 0{\cdot}34$ and the index $d = \frac{1}{3}$. The specific resistance coefficient according to Telfer should therefore be:—

$$C_f = 0{\cdot}0012 + 0{\cdot}34(R_n)^{-\frac{1}{3}}.$$

This formula will be found to give almost identical values to those of Schoenherr at about $R_n = 4 \times 10^6$ and at about $R_n = 10^9$, that is at about the normal small model range and also at about the normal ship range. At intermediate points, the Telfer values lie rather below either the Schoenherr or Prandtl-Schlichting values.

Dr. Telfer's treatment of roughness is on somewhat similar lines. As with his "Extrapolator" values, he plots the additional roughness C_{fr} on a basis of the inverse cube root of the Reynolds number. He then finds that for most actual ships the values lie between a "unity roughness ray" having the equation $C_{fr} = 0{\cdot}34(R_n)^{-\frac{1}{3}}$ and a "double roughness ray" of $C_{fr} = 0{\cdot}68(R_n)^{-\frac{1}{3}}$.

The author has pleaded for a basic "rough-turbulent" line which would avoid the very large increases that are necessary in going from a smooth model surface to an actual plated ship hull. If one was to accept the Telfer "unity roughness ray" for this purpose one would have an expression:—

$$C_f = 0{\cdot}00112 + 0{\cdot}68(R_n)^{-\frac{1}{3}}.$$

Such a line would give values remarkably near the Froude values and would have the advantages of dimensional accuracy and great simplicity. It would also avoid a fundamental defect of all one term lines which reduce C_f to zero value at an infinite R_n—an obvious absurdity.

Space precludes examination of Telfer's further and very ingenious suggestions for predicting ship C_r direct from model C_r, that is without first having to separate out the frictional component. Time alone will show if this is destined to supersede the conventional Froude method.

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