Online appendix to accompany:

Accounting for Price Dependencies in Simultaneous Sealed-Bid Auctions

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A. ONLINE APPENDIX

A.1. Proofs regarding bounds

LEMMA A.1. *Given joint price prediction $f_{\boldsymbol{Q}}$ and bid vector $\boldsymbol{b}$, $\mathbb{E}[c_j(\boldsymbol{b},\boldsymbol{Q})] = \mathbb{E}[c_j(\boldsymbol{b},Q_j)]$.*

PROOF.

$$\begin{aligned}
\mathbb{E}[c_j(\boldsymbol{b},\boldsymbol{Q})] &= \int_{\boldsymbol{q}} f_{\boldsymbol{Q}}(\boldsymbol{q}) c_j(\boldsymbol{b},\boldsymbol{q}) d\boldsymbol{q} && (4)\\
&= \int_{q_j}\int_{\boldsymbol{q}_{-j}} f_{Q_j}(q_j) f_{\boldsymbol{Q}_{-j}|Q_j}(q_{-j}|q_j) c_j(\boldsymbol{b},\boldsymbol{q}) dq_j d\boldsymbol{q}_{-j} && (5)\\
&= \int_{q_j}\int_{\boldsymbol{q}_{-j}} f_{Q_j}(q_j) f_{\boldsymbol{Q}_{-j}|Q_j}(q_{-j}|q_j) c_j(\boldsymbol{b},q_j) dq_j d\boldsymbol{q}_{-j} && (6)\\
&= \int_{q_j} f_{Q_j}(q_j) c_j(\boldsymbol{b},q_j) dq_j \int_{\boldsymbol{q}_{-j}} f_{\boldsymbol{Q}_{-j}|Q_j}(q_{-j}|q_j) d\boldsymbol{q}_{-j} && (7)\\
&= \int_{q_j} f_{Q_j}(q_j) c_j(\boldsymbol{b},q_j) dq_j && (8)\\
&= \mathbb{E}[c_j(\boldsymbol{b},Q_j)] && (9)
\end{aligned}$$

Eq. 4 holds by the definition of expectation. Eq. 5 holds by the chain rule. Eq. 6 holds by the definition of c_j. Eq. 7 follows from algebra. Eq. 8 holds because the integral of a pdf is 1. Eq. 9 holds by the definition of expectation. □

PROOF OF THEOREM 3.1.

$$\begin{aligned}
\mathbb{E}[c(\boldsymbol{b},\boldsymbol{Q})] &= \mathbb{E}\left[\sum_{j=1}^{m} c_j(\boldsymbol{b},\boldsymbol{Q})\right] && (10)\\
&= \sum_{j=1}^{m} \mathbb{E}[c_j(\boldsymbol{b},\boldsymbol{Q})] && (11)\\
&= \sum_{j=1}^{m} \mathbb{E}[c_j(\boldsymbol{b},Q_j)] && (12)\\
&= \sum_{j=1}^{m} \mathbb{E}[c_j(\boldsymbol{b},\boldsymbol{Q}')] && (13)\\
&= \mathbb{E}\left[\sum_{j=1}^{m} c_j(\boldsymbol{b},\boldsymbol{Q}')\right] && (14)\\
&= \mathbb{E}[c(\boldsymbol{b},\boldsymbol{Q}')] && (15)
\end{aligned}$$

Eqs. 10 and 15 follow from the definition of c. Eqs. 11 and 14 follow from linearity of expectations. Eq. 12 follows from Lemma A.1. Eq. 13 follows from the fact that $\boldsymbol{Q}'$ is the product of marginals: i.e., the prices across goods are independent.

PROOF OF THEOREM 3.2. The proof follows exactly the same reasoning as the proof of Thm. 3.1, which shows that $f_{\boldsymbol{Q}'}$ is sufficient to compute expected costs (which are additive across goods, like valuations are in this case).

PROOF OF THEOREM 3.3. To maximize $\mathbb{E}[v(\boldsymbol{b},\boldsymbol{Q}')]-\mathbb{E}[v(\boldsymbol{b},\boldsymbol{Q})]$, it can be shown that the adversary's optimal play is to spread the joint probability evenly amongst bundles that are missing a single item. That is, each w_X whose X is missing only a single item gets probability $1/m$. In such a case, the marginal probability of winning each good is $(m-1)/m$. Perceived expected value is $\overline{v}\prod_{j=1}^{m}\frac{m-1}{m} = \overline{v}(\frac{m-1}{m})^m$. Actual expected value is 0, since the agent will never actually receive all m goods, and that is the only time it gets any value.

To minimize $\mathbb{E}[v(\boldsymbol{b},\boldsymbol{Q'})] - \mathbb{E}[v(\boldsymbol{b},\boldsymbol{Q})]$, the adversary's optimal play is to put weight $m^{1/(1-m)}$ onto the bundle in which all items are won, and puts the remaining weight onto the bundle in which no items are won. The marginal probability of winning each good is $m^{1/(1-m)}$. Perceived expected value is $\overline{v}\prod_{j=1}^{m} m^{1/(1-m)} = \overline{v}\left(m^{m/(1-m)}\right)$. Actual expected value is $\overline{v}\left(m^{1/(1-m)}\right)$.

Proof of Theorem 3.4. To maximize $\mathbb{E}[v(\boldsymbol{b},\boldsymbol{Q'})] - \mathbb{E}[v(\boldsymbol{b},\boldsymbol{Q})]$, it can be shown that adversary's optimal play is to put weight $m^{1/(1-m)}$ onto the bundle in which no items are won, and to put the remaining weight on the bundle in which all items are won. In such a case, the marginal probability of winning each good is $1 - m^{1/(1-m)}$.

To minimize $\mathbb{E}[v(\boldsymbol{b},\boldsymbol{Q'})] - \mathbb{E}[v(\boldsymbol{b},\boldsymbol{Q})]$, the adversary's optimal play is to spread the joint probability evenly amongst all bundles that only win a single good, so that each has weight $1/m$. In such a case, the marginal probability of winning each good is $1/m$. Perceived expected value is $\overline{v}\left[1 - \prod_j (1-1/m)\right] = \overline{v}[1-(1-1/m)^m]$. Actual expected value is $\overline{v}$. The difference between these is $\overline{v}(1-1/m)^m$.

Proof of Theorem 3.5. Let $\boldsymbol{b}^* \in \operatorname{argmax}_{\boldsymbol{b}} \mathbb{E}[u(\boldsymbol{b},\boldsymbol{Q})]$; let $\boldsymbol{b}' \in \operatorname{argmax}_{\boldsymbol{b}} \mathbb{E}[u(\boldsymbol{b},\boldsymbol{Q'})]$; and let $\boldsymbol{b}$ be the bid vector that ensues by following strategy $s(f_{\boldsymbol{Q'}}, v)$.

$$\begin{aligned}
\mathbb{E}[u(\boldsymbol{b}^*,\boldsymbol{Q})] - \mathbb{E}[u(\boldsymbol{b},\boldsymbol{Q})] &\leq [\mathbb{E}[u(\boldsymbol{b}^*,\boldsymbol{Q'})] + \underline{\alpha}(u)] - [\mathbb{E}[u(\boldsymbol{b},\boldsymbol{Q'})] - \overline{\alpha}(u)] && (16)\\
&\leq [\mathbb{E}[u(\boldsymbol{b}',\boldsymbol{Q'})] + \underline{\alpha}(u)] - [\mathbb{E}[u(\boldsymbol{b},\boldsymbol{Q'})] - \overline{\alpha}(u)] && (17)\\
&\leq [(\mathbb{E}[u(\boldsymbol{b},\boldsymbol{Q'})] + \beta) + \underline{\alpha}(u)] - [\mathbb{E}[u(\boldsymbol{b},\boldsymbol{Q'})] - \overline{\alpha}(u)] && (18)\\
&= \beta + \underline{\alpha}(u) + \overline{\alpha}(u)
\end{aligned}$$

Eq. 16 holds by the definitions of $\overline{\alpha}(u)$ and $\underline{\alpha}(u)$. Eq. 17 holds because $\boldsymbol{b}'$ is optimal under $f_{\boldsymbol{Q'}}$, so it must have expected utility at least that of $\boldsymbol{b}^*$ under $f_{\boldsymbol{Q'}}$. Eq. 18 holds because strategy s places bids within β of optimal under $f_{\boldsymbol{Q'}}$.

A.2. Details of bounds on $\overline{\alpha}$ and $\underline{\alpha}$

Combining the error bounds for expected value and expected costs, we achieve bounds on the maximum error (in terms of both over and underestimation) of using $f_{\boldsymbol{Q'}}$ to compute expected utility for both perfect complements and perfect substitutes.

$$\overline{\alpha}(u) = \max_{f_{\boldsymbol{Q}},\boldsymbol{b}} \{\mathbb{E}[v(\boldsymbol{b},\boldsymbol{Q'})] - \mathbb{E}[v(\boldsymbol{b},\boldsymbol{Q})]\} \qquad \underline{\alpha}(u) = \min_{f_{\boldsymbol{Q}},\boldsymbol{b}} \{\mathbb{E}[v(\boldsymbol{b},\boldsymbol{Q'})] - \mathbb{E}[v(\boldsymbol{b},\boldsymbol{Q})]\}$$

Let u_{comp} represent a utility function that has perfect complements, and u_{sub} a utility function that has perfect substitutes. Plugging in the (tight) bounds from Thms. 3.3 and 3.4, we get

$$\overline{\alpha}(u_{comp}) = \overline{v}\left(\frac{m-1}{m}\right)^m \qquad \underline{\alpha}(u_{comp}) = -\overline{v}\left(\frac{m-1}{m(m^{1/(m-1)})}\right)$$

$$\overline{\alpha}(u_{sub}) = \overline{v}\left(\frac{m-1}{m(m^{1/(m-1)})}\right) \qquad \underline{\alpha}(u_{sub}) = -\overline{v}\left(\frac{m-1}{m}\right)^m$$

A.3. Proofs regarding local searches

Proof. The proof of Proposition 5.3 relies on the following observations:

(1) $w(\boldsymbol{b},\boldsymbol{q}) = \cup_k w(b_k, q_k) = w(\boldsymbol{b}_{-j}, \boldsymbol{q}_{-j}) \cup w(b_j, q_j)$
(2) $w(\boldsymbol{b},\boldsymbol{q}) \cup \{j\} = w(\boldsymbol{b}_{-j}, \boldsymbol{q}_{-j}) \cup w(b_j, q_j) \cup \{j\} = w(\boldsymbol{b}_{-j}, \boldsymbol{q}_{-j}) \cup \{j\}$
(3) $w(\boldsymbol{b},\boldsymbol{q}) \setminus \{j\} = (w(\boldsymbol{b}_{-j}, \boldsymbol{q}_{-j}) \cup w(b_j, q_j)) \setminus \{j\} = w(\boldsymbol{b}_{-j}, \boldsymbol{q}_{-j})$

JointLocal.

$$\begin{aligned}
b_j &\leftarrow \mathbb{E}[v(w(\boldsymbol{b},\boldsymbol{Q}) \cup \{j\})] - \mathbb{E}[v(w(\boldsymbol{b},\boldsymbol{Q}) \setminus \{j\})] \\
&= \int_{q_1=0}^{\infty} \int_{q_2=0}^{\infty} \cdots \int_{q_m=0}^{\infty} [v(w(\boldsymbol{b},\boldsymbol{q}) \cup \{j\}) - v(w(\boldsymbol{b},\boldsymbol{q}) \setminus \{j\})]\, f_{\boldsymbol{Q}}(q_1, q_2, \ldots, q_m) dq_1 dq_2 \cdots dq_m \\
&= \int_{q_j} \int_{\boldsymbol{q}_{-j}} [v(w(\boldsymbol{b}_{-j}, \boldsymbol{q}_{-j}) \cup \{j\}) - v(w(\boldsymbol{b}_{-j}, \boldsymbol{q}_{-j}))]\, f_{\boldsymbol{Q}_{-j}|Q_j}(\boldsymbol{q}_{-j} \mid q_j) f_{Q_j}(q_j) d\boldsymbol{q}_{-j} dq_j \\
&= \int_{\boldsymbol{q}_{-j}} [v(w(\boldsymbol{b}_{-j}, \boldsymbol{q}_{-j}) \cup \{j\}) - v(w(\boldsymbol{b}_{-j}, \boldsymbol{q}_{-j}))]\, f_{\boldsymbol{Q}_{-j}}(\boldsymbol{q}_{-j}) d\boldsymbol{q}_{-j} \int_{q_j} f_{Q_j}(q_j) dq_j && (19) \\
&= \int_{\boldsymbol{q}_{-j}} [v(w(\boldsymbol{b}_{-j}, \boldsymbol{q}_{-j}) \cup \{j\}) - v(w(\boldsymbol{b}_{-j}, \boldsymbol{q}_{-j}))]\, f_{\boldsymbol{Q}_{-j}}(\boldsymbol{q}_{-j}) d\boldsymbol{q}_{-j} \\
&= \int_{\boldsymbol{q}_{-j}} [v(w(\boldsymbol{b}_{-j}, \boldsymbol{q}_{-j}) \cup \{j\}) - v(w(\boldsymbol{b}_{-j}, \boldsymbol{q}_{-j}))] \prod_{k \neq j} f_{Q_k}(q_k) d\boldsymbol{q}_{-j} && (20)
\end{aligned}$$

We apply the independence assumption first in deriving line 19, and then again in deriving line 20.

CondMVLocal.

$$\begin{aligned}
b_j &\leftarrow \mathbb{E}\left[v(w(\boldsymbol{b},\boldsymbol{Q}) \cup \{j\}) \mid Q_j \leq b_j - v(w(\boldsymbol{b},\boldsymbol{Q}) \setminus \{j\}) \mid Q_j \leq b_j\right] \\
&= \mathbb{E}\left[v(w(\boldsymbol{b}_{-j}, \boldsymbol{Q}_{-j}) \cup \{j\}) - v(w(\boldsymbol{b}_{-j}, \boldsymbol{Q}_{-j})) \mid Q_j \leq b_j\right] \\
&= \int_{\boldsymbol{q}_{-j}} [v(w(\boldsymbol{b}_{-j}, \boldsymbol{q}_{-j}) \cup \{j\}) - v(w(\boldsymbol{b}_{-j}, \boldsymbol{q}_{-j}))] \frac{\int_{q_j=0}^{b_j} f_{\boldsymbol{Q}}(\boldsymbol{q}_{-j}, q_j) dq_j}{\int_{\boldsymbol{q}_{-j}} \int_{q_j=0}^{b_j} f_{\boldsymbol{Q}}(\boldsymbol{q}_{-j}, q_j) dq_j \boldsymbol{q}_{-j}} d\boldsymbol{q}_{-j} \\
&= \int_{\boldsymbol{q}_{-j}} [v(w(\boldsymbol{b}_{-j}, \boldsymbol{q}_{-j}) \cup \{j\}) - v(w(\boldsymbol{b}_{-j}, \boldsymbol{q}_{-j}))] \frac{f_{\boldsymbol{Q}}(\boldsymbol{q}_{-j}, q_j \leq b_j)}{f_{Q_j}(q_j \leq b_j)} d\boldsymbol{q}_{-j} \\
&= \int_{\boldsymbol{q}_{-j}} [v(w(\boldsymbol{b}_{-j}, \boldsymbol{q}_{-j}) \cup \{j\}) - v(w(\boldsymbol{b}_{-j}, \boldsymbol{q}_{-j}))] \frac{f_{\boldsymbol{Q}_{-j}|Q_j}(\boldsymbol{q}_{-j} \mid q_j \leq b_j) f_{Q_j}(q_j \leq b_j)}{f_{Q_j}(q_j \leq b_j)} d\boldsymbol{q}_{-j} \\
&= \int_{\boldsymbol{q}_{-j}} [v(w(\boldsymbol{b}_{-j}, \boldsymbol{q}_{-j}) \cup \{j\}) - v(w(\boldsymbol{b}_{-j}, \boldsymbol{q}_{-j}))]\, f_{\boldsymbol{Q}_{-j}|Q_j}(\boldsymbol{q}_{-j} \mid q_j \leq b_j) d\boldsymbol{q}_{-j} \\
&= \int_{\boldsymbol{q}_{-j}} [v(w(\boldsymbol{b}_{-j}, \boldsymbol{q}_{-j}) \cup \{j\}) - v(w(\boldsymbol{b}_{-j}, \boldsymbol{q}_{-j}))]\, f_{\boldsymbol{Q}_{-j}}(\boldsymbol{q}_{-j}) d\boldsymbol{q}_{-j} && (21) \\
&= \int_{\boldsymbol{q}_{-j}} [v(w(\boldsymbol{b}_{-j}, \boldsymbol{q}_{-j}) \cup \{j\}) - v(w(\boldsymbol{b}_{-j}, \boldsymbol{q}_{-j}))] \prod_{k \neq j} f_{Q_k}(q_k) d\boldsymbol{q}_{-j} && (22)
\end{aligned}$$

We apply the independence assumption first in deriving line (21), and then again in deriving line (22). □

A.4. Counterexamples for local search heuristics

The local search heuristics introduced in this paper are not optimal. In this appendix, we present counterexamples that demonstrate their suboptimality.

Example 1. Suppose $m = 2$; valuations are perfect complements with value 50 for winning both goods and value 0 otherwise; price predictions are such that two price vectors are equally likely: $(30,0)$ and $(0,30)$; and $\boldsymbol{b} = (25,25)$. Further, suppose JointLocal is currently updating its bid for good $j = 1$. In such a case, JointLocal's bid for good j is:

$$\begin{aligned}
b_1 &\leftarrow \mathbb{E}\left[v(w(\boldsymbol{b},\boldsymbol{Q}) \cup \{1\})\right] - \mathbb{E}[v(w(\boldsymbol{b},\boldsymbol{Q}) \setminus \{1\})] \\
&= [.5v(w((25,25),(30,0)) \cup \{1\}) + .5v(w((25,25),(0,30)) \cup \{1\})] - \\
&\quad\ [.5v(w((25,25),(30,0)) \setminus \{1\}) + .5v(w((25,25),(0,30)) \setminus \{1\})] \\
&= [.5v(\{2\} \cup \{1\}) + .5v(\{1\} \cup \{1\})] - [.5v(\{2\} \setminus \{1\}) + .5v(\{1\} \setminus \{1\})] \\
&= [.5(50) + .5(0)] - [.5(0)] \\
&= 25
\end{aligned}$$

First, observe that this setup is symmetric, so that updating its bid on good 2 would also yield 25, meaning JointLocal converges to $\boldsymbol{b} = (25,25)$. Second, the bid vector $\boldsymbol{b} = (25,25)$ yields an expected utility of 0: the agent will never win both goods, so it will never receive any value. A better bid on good 1 would have been 31, because the bid vector $\boldsymbol{b} = (31,25)$ yields an expected utility of 10.

Example 2. As above, suppose $m = 2$; valuations are perfect complements with value 50 for winning both goods and value 0 otherwise; and $\boldsymbol{b} = (25,25)$. But this time assume that three price vectors are equally likely: $(30,10)$, $(10,30)$, and $(10,10)$. Finally, as above, suppose CondMVLocal is currently updating its bid for good $j = 1$. In such a case, CondMVLocal's bid for good $j = 1$ (and $j = 2$, by symmetry) is:

$$\begin{aligned}
b_1 &\leftarrow \mathbb{E}\left[v(w(\boldsymbol{b},\boldsymbol{Q}) \cup \{1\}) - v(w(\boldsymbol{b},\boldsymbol{Q}) \setminus \{1\}) \mid Q_1 \le b_1\right] \\
&= .5[v(w((25,25),(10,30)) \cup \{1\}) - v(w((25,25),(10,30)) \setminus \{1\})] + \\
&\quad\ .5[v(w((25,25),(10,10)) \cup \{1\}) - v(w((25,25),(10,10)) \setminus \{1\})] \\
&= .5[v(\{1\} \cup \{1\}) - v(\{1\} \setminus \{1\})] + .5[v(\{1,2\} \cup \{1\}) - v(\{1,2\} \setminus \{1\})] \\
&= .5[0 - 0] + .5[50 - -0] \\
&= 25
\end{aligned}$$

First, observe that this setup is symmetric, so that updating its bid on good 2 would also yield 25, meaning CondLocal converges to $\boldsymbol{b} = (25,25)$. Second, the bid vector $\boldsymbol{b} = (25,25)$ yields an expected utility of $\frac{10}{3}$: the agent wins both goods with probability $\frac{1}{3}$, in which case it earns a utility of 30 (50-20), and it wins one good with probability $\frac{2}{3}$, in which cases it incurs costs of 10. A better bid on good 1 would have been 31, which yields an expected utility of 10: $\frac{1}{3}(30) + \frac{1}{3}(10) + \frac{1}{3}(-10)$.

A.5. Optimization Experiments: Raw Data

Table VI presents the mean and standard deviations of expected utility for the optimization experiments described in Sec. 7.

Table VI. Local Search Optimization Results (mean,std)

Env	TC	MargLocal	MargDownHill	JointLocal	JointDownHill
L_1[2,5]	0.07447	(1.481, 3.048)	(1.479, 3.047)	(1.481, 3.048)	(1.483, 3.049)
S[2,5]	0.3135	(4.147, 6.295)	(4.136, 6.3)	(4.148, 6.295)	(4.173, 6.284)
L_1[3,5]	0.161	(1.276, 3.239)	(1.274, 3.239)	(1.276, 3.239)	(1.284, 3.245)
S[3,5]	0.8411	(4.901, 7.603)	(4.832, 7.636)	(4.875, 7.617)	(4.973, 7.578)
L_1[4,5]	0.2663	(1.231, 3.266)	(1.227, 3.264)	(1.231, 3.267)	(1.231, 3.266)
S[4,5]	1.283	(5.149, 8.242)	(5.071, 8.264)	(5.138, 8.254)	(5.134, 8.271)
S[5,2]	4.477	(8.983, 11.04)	(9.006, 11.3)	(9.686, 11.74)	(10.08, 11.56)
S[5,4]	2.136	(6.645, 9.785)	(6.593, 9.838)	(6.673, 9.794)	(6.67, 9.84)
L_1[5,5]	0.3989	(1.892, 4.585)	(1.89, 4.588)	(1.892, 4.585)	(1.886, 4.583)
S[5,5]	1.835	(5.341, 8.687)	(5.22, 8.648)	(5.319, 8.687)	(5.29, 8.694)
S[5,6]	1.627	(4.903, 8.231)	(4.828, 8.236)	(4.909, 8.247)	(4.918, 8.235)
S[5,8]	1.381	(4.022, 7.567)	(4.0, 7.606)	(4.014, 7.58)	(4.006, 7.617)
L_1[6,5]	0.5486	(2.492, 5.523)	(2.489, 5.524)	(2.496, 5.521)	(2.492, 5.526)
S[6,5]	2.583	(6.206, 9.583)	(6.153, 9.601)	(6.235, 9.583)	(6.205, 9.687)
L_1[7,5]	0.5605	(3.852, 6.958)	(3.838, 6.962)	(3.854, 6.958)	(3.835, 6.954)
S[7,5]	3.302	(6.482, 10.17)	(6.449, 10.26)	(6.62, 10.28)	(6.587, 10.33)
S[8,5]	4.126	(6.584, 10.84)	(6.56, 10.89)	(6.664, 10.91)	(6.636, 10.94)
L_1[8,5]	0.6069	(4.598, 7.693)	(4.571, 7.692)	(4.586, 7.688)	(4.571, 7.706)
L_1[9,5]	0.6522	(5.818, 8.501)	(5.786, 8.497)	(5.811, 8.495)	(5.782, 8.494)
S[9,5]	5.058	(6.948, 11.17)	(6.853, 11.32)	(7.035, 11.24)	(6.949, 11.36)
L_1[10,5]	0.6593	(6.023, 9.03)	(5.975, 9.028)	(6.01, 9.019)	(5.975, 9.013)
S[10,5]	5.969	(6.557, 11.12)	(6.477, 11.2)	(6.767, 11.23)	(6.59, 11.37)