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Spin-chain-based full quantum computation by accessing only two spins

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Abstract. We apply quantum control techniques to a long spin chain by acting only on two qubits at one of its ends, thereby implementing universal quantum computation by a combination of quantum gates on these qubits and indirect SWAP operations across the chain. It is shown that the control sequences can be computed and implemented efficiently.We discuss the application of these ideas to physical systems such as superconducting qubits in which full control of long chains is challenging.

Keywords: Quantum computing, spin chains, limited access
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INTRODUCTION

Controlling quantum systems at will has been a dream for physicists for a long time. In particular, the future success of quantum information processing depends largely on our ability of taming many body quantum systems that are highly fragile. Although the progress of technology allows us to manipulate a small number of quanta quite well, controlling larger systems seems still to be an enormous challenge. Unless we overcome difficulties towards the control over many body systems, the benefits we can enjoy with the quantumness will be severely limited.

Problems we need to contemplate before attempting to build a quantum computer using quantum control are as follows. Firstly, the control criterion is generally not computable for large systems. Secondly, even if the question of controlability can be answered positively for *specific* systems, the precise sequence of actual controls (or 'control pulses') are realistically not computable. And thirdly, even if they can be computed, the theory of control tells us nothing about the overall *duration* of the control pulses to achieve a given task, and it might take far too long to be practically meaningful.

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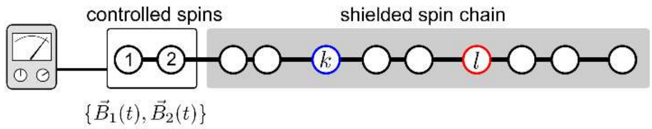

FIGURE 1. Our scheme for universal computation works on a chain of N spins. By modulating the magnetic fields in the "accessible area" (spins 1 and 2) we induce effective swap gates on the chain, e.g., between 1 and k, or 2 and l. The gates from the quantum algorithm are then performed on the control qubits only by operations U_{12}.

QUANTUM CONTROLLABILITY AND CLASSICAL COMPUTABILITY

We consider the problem of controlling a 1-dimensional chain of N spin-1/2 particles (Fig. 1). If we take the most generic Heisenberg interaction with a time-dependent magnetic field at spin 1,

$$\begin{aligned} H &= H_0 + H_1(t), \\ \text{where } H_0 &= \sum_{n=1}^{N-1} c_n (X_n X_{n+1} + Y_n Y_{n+1} + Z_n Z_{n+1}) \text{ and } H_1(t) = b_1(t) Z_1, \end{aligned}$$

then any unitary operation $U \in \mathrm{SU}(2^N)$ can be realisable by modulating the field intensity $b_1(t)$ only. Here, X, Y, and Z are the Pauli operators and we take the standard basis so that $Z|0\rangle = |0\rangle$ and $Z|1\rangle = -|1\rangle$. The full controllability can be shown by utilising a theorem in the theory of quantum coherent control in terms of Lie algebra, which states as follows. The set of realisable unitary operations are those generated by the dynamical Lie algebra $\mathscr{L}$ that can be constructed with H_m (in this case $m = 0, 1$). All we need to do to obtain the dynamical Lie algebgra $\mathscr{L}$ is simply to repeat taking commutators of generators, such as $[H_m, H_{m'}]$, $[H_m, [H_{m'}, H_{m''}]]$, $[[H_m, H_{m'}], [H_{m''}, H_{m'''}]]$, etc. Because we are considering Hilbert spaces of finite dimension, the number of independent generators we can obtain this way is finite and thus the process terminates at certain point. The theorem says that if $\mathscr{L}$ has the same dimensionality as $\mathrm{su}(2^N)$ the entire system is fully controllable, which is the case with H_0 and H_1 above.

However, the optimisation of the control pulses $b_1(t)$ would be inefficient. That is, the (classical) computing time will be exponential with respect to the system size N, since the Hamiltonian cannot be diagonalised straightforwardly. In such a case, we would need a quantum computer to design the pulses to run a quantum computer! This difficulty can be circumvented by employing the (time-independent) nearest-neighbour XX Hamiltonian,

$$H_0^{XX} = \frac{1}{2} \sum_{n=1}^{N-1} c_n [(1+\gamma) X_n X_{n+1} + (1-\gamma) Y_n Y_{n+1}] \qquad (1)$$

which, through Jordan-Wigner transformation, can be efficiently diagonalised for large systems, and it is still a Hamiltonian with practical relevance in realistic systems. Hence, the control pulses $b_1(t)$ with this Hamiltonian can be computed efficiently by using standard numerical algorithms developed in the field of optimal quantum controls, such as Krotov's.

THE MAIN RESULTS

Yet, the dynamical Lie algebra obtained from H_0^{XX} in Eq. (1) and $H_1(t) = b_1(t)Z_1$ does not span the whole $\mathrm{su}(2^N)$, thus the spin chain is not fully controllable by modulating the field $b_1(t)$ only. Nevertheless, a closer look into the algebra brings us the following generators, $ih_{kl} = a_k^\dagger a_l - a_l^\dagger a_k$ $(k-l$ even$), ih_{kl} = i(a_k^\dagger a_l + a_l^\dagger a_k)$ $(k-l$ odd$), Z_k = 1 - 2a_k^\dagger a_k$, where a_n and $a_n^\dagger$ are the annihilation and creation operators of Jordan-Wigner fermions. The operators h_{kl} work essentially as SWAP operations for the k-th and the l-th spins (precisely, JW fermions). Therefore, if we can access the two spins at the chain end (spins 1 and 2 in Fig. 1), any operations of $\mathrm{SU}(2^N)$ can be possible. That is, if we want to apply a two-spin operation between the k-th and the l-th spins, no matter where they reside in the chain, we can swap spins 1 and k, and 2 and l to bring both to the controllable area. Then, the two-spin operation could be applied to the spins 1 and 2 fast enough (compared with the spin-chain dynamics), and by swapping two pairs again we will have completed the desired two-spin operation effectively. Single-spin operations can be performed in a similar manner, hence any $\mathrm{SU}(2^N)$ operations can be implemented.

The above mentioned scheme would work, however, if the operators ih_{kl} generate a genuine SWAP operation for spin states. But this is not the case. For example, what can be realised as a time evolution of ih_{kl} (for $k-l$ even) is a unitary operator

$$e^{-\pi i h_{kl}/2} = (|00\rangle\langle 00| + |11\rangle\langle 11|)_{kl} \otimes I + (|01\rangle\langle 10| - |10\rangle\langle 01|)_{kl} \otimes \prod_{k<j<l} Z_j, \tag{2}$$

where $(...)_{kl}$ is an operation only on the k-th and the l-th spins. The operator $\prod_{k<j<l} Z_j$, which arises from the nonlocal tail of the Jordan-Wigner transformation, acts on the spins between k and l, being controlled by the parity of the spins k and l. Such 'controlled' operations together with one- and two-spin operations at sites 1 and 2 will produce undesired complicated entanglement over the chain and mess the whole state up.

In order to fix this problem, we encode a logical qubit with two physical spins as $|0\rangle_L := |01\rangle$ and $|1\rangle_L := |10\rangle$. The rough idea behind this encoding is that by restricting ourselves in the subspace spanned by $\{|01\rangle, |10\rangle\}$ we can always be sure about the value the operator $\prod_{k<j<l} Z_j$ acting between two logical qubits will induce. Single qubit operations then can be performed on any qubit without transferring the state to the accessible area. This is because the dynamical Lie algebra contains Z_k and the SWAP between the neighbouring spins, which corresponds to the X_k operation on the k-th qubit. Note also that the cumbersome operator $\prod_{k<j<l} Z_j$ does not appear in the SWAP for neighbouring k and j. Therefore, the spin chain is fully controllable for quantum computation by controlling only two qubits (1and 2) [1].

The remaining problem is how long/short the overall control pulses should be. To show this, we used one of the numerical techniques, called Krotov's method, for optimising the control pulses. We found out that the overall timescale of the operations is only proportional to N^2, which is polynomial with respect to the system size N. In order to estimate the necessary accuracy and the required speed of modulation, we have carried out a Fourier analysis of the optimised $b_1(t)$ for the SWAP operation between the spins 1 and 29 in the chain of 30 spins. The dominant frequency components are in the slower region compared with the intrinsic dynamics of the chain, and neglecting (cutting off) the components whose frequencies are higher than $\sim J$ does not impair the resulting operation fidelity substantially (cutoff frequencies higher than $2J$ still gives the fidelity higher than $1-10^{-4}$). We refer interested readers to ref.[1] for more details.

Meanwhile, obtaining the precise knowledge on the coupling strengths J_k is also a nontrivial problem under the condition of restricted access. Without accurate information on J_k, full quantum control can by no means be possible. We have also analysed this problem and found a positive answer; not only J_k, but also the strengths of local magnetic fields B_k can be determined by accessing only the end spin of the chain [2]. Therefore the whole scenario here is applicable to physical situations, where we can have only a limited access to a large system for some technical/physical reasons.

CONCLUSIONS

We have shown how to efficiently compute control pulses for large spin chains described by a vast class of Hamiltonians. The pulses are computed for a 2N-dimensional system but can be applied to the full 2N-dimensional system. Full quantum computation is possible by controlling only two spins at one end of the chain. The only price for this indirect control is that the quantum computation takes quadratically longer than for direct control. Given the large benefit of requiring so little control for a quantum computer, we believe that this scheme would be very useful for future implementations.

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REFERENCES

1. D. Burgarth, K. Maruyama, M. Murphy, S. Montangero, T. Calarco, F. Nori, and M. B. Plenio, Phys. Rev. A 81, 040303(R) (2010)
2. D. Burgarth, K. Maruyama, and F. Nori, Phys. Rev. A79, 020305(R) (2009); D. Burgarth and K. Maruyama, New J. Phys. 11, 103019 (2009).