

A numerical investigation of surface crevasse propagation in glaciers using nonlocal continuum damage mechanics

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[1] We use a nonlocal viscoelastic damage model to investigate the conditions that enable water-free surface crevasse propagation in grounded marine-terminating glaciers. Our simulations, on idealized rectangular ice slabs in contact with the ocean, show that crevasses propagate faster in thicker ice slabs. We find that: (1) the fraction of ice slab thickness penetrated by surface crevasses decreases with increasing seawater depth near the terminus; (2) a no-slip (fixed) basal boundary condition retards crevasse growth; and (3) crevasses form closer to the terminus when the seawater depth is larger or when the glacier base is fixed to the bedrock, which could lead to calving of smaller icebergs. However, water-free surface crevasses can penetrate (nearly) the entire ice thickness only in thicker ice slabs terminating in shallow seawater depths. This leads us to the conclusion that surface crevasses alone are not responsible for calving events in marine-terminating and thin glaciers. **Citation:** Duddu, R., J. N. Bassis, and H. Waisman (2013), A numerical investigation of surface crevasse propagation in glaciers using nonlocal continuum damage mechanics, *Geophys. Res. Lett.*, 40, 3064–3068, doi:10.1002/grl.50602.

1. Introduction

[2] Calving is an important process in marine-terminating glaciers, including Antarctic ice shelves and Greenland outlet glaciers, accounting for as much as half of the mass lost from the Antarctic and Greenland ice sheets [e.g., *Jacobs et al.*, 1992; *Biggs*, 1999; *Rignot et al.*, 2008]. The processes that cause iceberg detachment, however, are complex, involving the initiation and propagation of fractures over spatial scales ranging from crystal-sized defects to rifts systems in ice shelves that span hundreds of kilometers. Although the initiation and propagation of crevasses and rifts within glaciers and ice shelves are clearly linked to flow dynamics over a wide range of environmental conditions and flow regimes, we do not yet have a well-formulated mathematical model of iceberg calving [*Van der Veen*, 2002; *Bassis et al.*, 2005; *Benn et al.*, 2007; *Bassis*, 2011; *Bassis*

and *Walker*, 2012]. This suggests that success in incorporating iceberg calving into the next-generation ice sheet models hinges on formulating (and testing) a physical model of the fracture process that enables us to simulate crevasse propagation.

[3] The depth to which surface crevasses penetrate was first estimated using the Nye zero-stress model in which surface and bottom crevasses penetrate to the depth where the net horizontal stress vanishes [*Nye*, 1957; *Jezeq*, 1984; *Nick et al.*, 2010]. The Nye zero-stress model typically assumes that glacier ice has no tensile strength, a hypothesis at odds with both field- and laboratory-derived studies of the strength of ice [*Schulson and Duval*, 2009]. An alternative approach uses linear elastic fracture mechanics (LEFM) which assumes that ice behaves like a brittle elastic solid and fractures initiate from small, sharp “starter cracks” assumed to be always present within the ice [*Lawn*, 1993]. However, LEFM is only appropriate for modeling instantaneous crevasse propagation over a shorter time scale associated with brittle elastic behavior but not for modeling creep-induced crevasse propagation over a longer time scale associated with significant viscous deformation. Moreover, the discontinuous nature of (linear elastic) fracture makes it difficult to self-consistently integrate crevasse initiation and propagation into viscous ice dynamics models.

[4] An alternative approach involves the use of continuum damage mechanics. Unlike other methods that focus on initiation and propagation of individual fractures in ice, this technique models the evolution of many distributed “flaws” within the ice and their effect on its bulk rheology. Previously, *Pralong et al.* [2003, 2005] employed the notion of continuum damage and assumed that ice behaves as an incompressible viscous fluid to study crevasse propagation. However, this approach neglects the elastic stress effects and uses a local creep damage formulation that yields mesh-dependent and inconsistent results [*Murakami and Liu*, 1995]. A nonlocal damage formulation eliminates the unrealistic singular localization of damage and alleviates the pathological mesh dependence of finite element calculations by introducing a length scale for damage [*Bazant and Pijaudier-Cabot*, 1988].

[5] In this paper, our aim is to investigate the conditions that enable surface crevasse propagation using a nonlocal viscoelastic damage model. We consider the nonlinear viscoelastic material behavior of ice [*Glen*, 1955] and creep damage evolution due to the progressive accumulation of microcracks in tension [*Murakami*, 1983], under sustained gravity loading. Eventually, the microcracks coalesce to form one macrocrack, leading to tensile creep fracture. The main advantage of this computational model is that it is only necessary to specify glacier geometry and boundary conditions (at the glacier base and on the ice-ocean boundary); the

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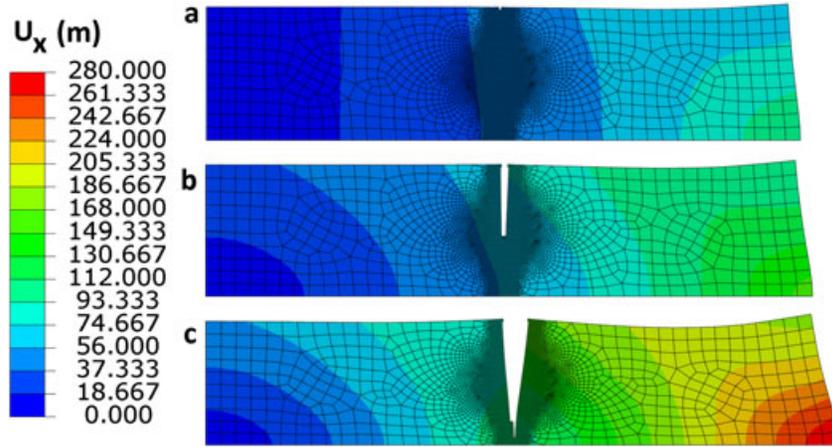


Figure 1. Contour plots of the displacement component in the x direction (in meters) showing a crevasse opening at times (a) $t = 16$ hours, (b) $t = 22$ hours, and (c) $t = 33$ hours. The ice slab has the dimensions $2000 \text{ m} \times 500 \text{ m}$. The depth-varying flow velocity field is evident from these plots particularly near the terminus. The deformations and crack opening displacements are not drawn to scale.

stress field in the glacier and its damage state, due to gravity and hydrostatic seawater pressure, are computed using the finite element method. Additionally, using the displacement field results from the simulations, we can estimate the strain rate and flow velocity of ice.

2. Model Geometry and Formulation

[6] We approximate an idealized glacier as a rectangular ice slab (see Figure II in the supporting information for a schematic diagram). The ice slab of dimensions $l \times h$ is assumed to be under plane strain; hence, the model is most appropriate for glaciers in wide fjords. For example, most outlet glaciers in Greenland exist in fjords that are 1–3 km wide with ice thickness (depth) less than 1 km. Since the width is larger compared to the thickness, a plane strain assumption is appropriate. Seawater depth near the terminus is denoted by h_w , and crevasse depth is denoted by d . In all the simulations, the density of ice, $\rho = 910 \text{ kg/m}^3$, and the ice temperature, $T = -10^\circ\text{C}$, are assumed to be constant within the slab and the density of seawater $\rho_w = 1020 \text{ kg/m}^3$. The polycrystalline ice that constitutes glaciers is modeled using a viscoelastic constitutive damage model with damage accumulating only under tension. The damage rate under tension is given by a power law dependence on the Cauchy stress and current state of damage. In this study we do not consider any material deterioration (damage) under compression, both because our focus is on understanding surface crevasse propagation driven by tensile fracture mechanisms and because of our limited knowledge on the actual mechanisms of compression failure in glaciers. Therefore, we only considered viscoelastic behavior of ice under compression. All material parameters of the model under tension at $T = -10^\circ\text{C}$ are calibrated in *Duddu and Waisman*, [2012] using the experimental data of *Mahrenholtz and Wu* [1992]. The constitutive model equations and parameter values are summarized in the supporting information. Although the model is calibrated using existing laboratory experimental data, it can easily be recalibrated once better laboratory or field data become available; however, we emphasize that the conclusions of this study would still be valid as the final crevasse propagation depth is not sensitive to model param-

eters (viscosity or damage parameters only affect the rate of propagation). We employ a Lagrangian finite element formulation and a nonlocal continuum damage representation to simulate crevasse propagation in creeping slabs of ice [*Duddu and Waisman*, 2013]. The nonlocal formulation introduces a damage length scale of 10 m for thermodynamic (numerical) consistency, which is assumed to be equal to the fracture process zone size estimated by *Pralong et al.* [2005] for a temperate alpine glacier. In all the simulation results shown in this paper, the crack path is given by fully damaged zones that can be identified from the contour plots of the damage variable.

3. Results

3.1. Crevasse Propagation in a Dry Environment

[7] We first investigated the influence of ice thickness on calving in a dry environment ($h_w = 0$) by performing simulations with ice slab thicknesses of $h = 125, 250,$ and 500 m . We take a slab length of $l = 4h$ (in all the simulations) so that it is sufficiently long to fully develop maximum tensile stresses at midlength, due to differential creep flow of the slab across the thickness (or depth). We prescribe a notch of depth $d = 10 \text{ m}$ at midlength near the glacier's top surface to initialize a single isolated crevasse so as to consider the worst-case scenario when there is a preexisting defect at the location of maximum tensile stress. Alternatively, a statistical distribution of preexisting damage or defects can be prescribed; however, initiating damage using a stress concentration feature (e.g., a notch or a hole) is a common practice in damage mechanics [*Desmorat et al.*, 2007; *Jirasek and Grassl*, 2008]. Our simulations indicate that crevasse depth is determined by the depth where tensile stresses vanish and so the conclusions of our study are not affected by the consideration of a small initial notch. Our future work will focus on studying the propagation of multiple crevasses assuming an initial distribution of defects based on field observations. A more detailed discussion on simulating multiple crevasses is presented in the supporting information.

[8] Figure 1 shows a crevasse opening within the contour plots of the horizontal displacement for the thickest glacier

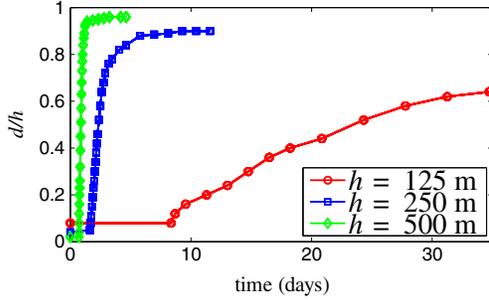


Figure 2. Curves of crevasse depth (d) normalized with slab thickness (h) versus time, for ice slabs of thicknesses $h = 125, 250,$ and 500 m when $h_w = 0$. The results indicate that crevasses only penetrate (nearly) the entire thickness in thicker glaciers.

($h = 500$ m) computed using a free slip basal boundary condition. The surface crevasse penetrates (nearly) the entire ice thickness over 33 hours (≈ 1.5 days). Figure 2 shows normalized crack length to slab thickness ratio versus time curves. Simulations suggest that crevasses propagate much faster and penetrate to greater depths in thicker ice slabs, which is in agreement with the theoretical findings of *Bassis and Walker* [2012]. However, we are faced with the conundrum that crevasses only penetrate (nearly) the entire ice thickness for the thickest slabs ($h = 500$ m) while observations indicate that calving occurs for much smaller ice thicknesses [*Benn et al.*, 2007; *Bassis and Walker*, 2012]. This suggests that additional mechanisms, such as meltwater-enhanced hydrofracture, basal crevassing, or shear failure, must be considered to explain full thickness failure.

3.2. Influence of Seawater Depth and Basal Boundary Condition

[9] We estimated the depth to which surface crevasses penetrate in a marine-terminating glacier as a function of water depth and ice thickness. The problem setup is similar to that described in the previous section. The water depth to slab thickness ratio (h_w/h) is varied from 0 to 0.8 (corresponding to a range that varies from fully grounded to near floatation), and the corresponding surface crevasse penetration ratios (d/h) obtained are shown in Figure 3. The results indicate that surface crevasses propagate deeper in thick grounded glaciers and that the (compressive) seawater pressure acts as a stabilizing force retarding crack growth

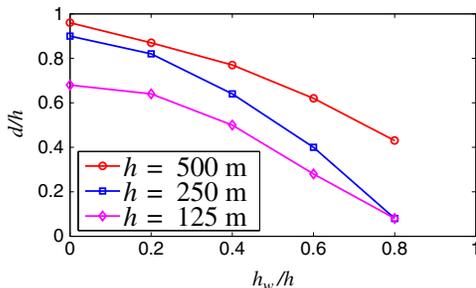


Figure 3. Fraction of the ice slab penetrated by surface crevasses (d/h) as a function of the ratio of seawater depth (h_w) to ice slab thickness (h).

and preventing surface crevasses from penetrating as deeply into the ice as in the land-terminated case. Again, this study leads us to the hypothesis that surface crevasse penetration alone is not sufficient to trigger iceberg calving, but it can lead to faster flow of glaciers downstream due to the opening of crevasses upstream in addition to viscous creep flow (see the flow velocity contour plots in Figure 1).

[10] We next performed a sequence of experiments with marine-terminating glaciers of thickness $h = 500$ m. We considered the extreme cases of the basal boundary condition: free slip (no basal friction) and no slip (frozen or fixed to the bed), where the former condition gives the upper bound on the depth to which crevasses penetrate while the latter boundary condition gives the lower bound. Additionally, the seawater depth is changed from $h_w = 0$ to $h_w = 0.5h$ to investigate the combined effect of water pressure and basal boundary condition. The curves of ice thickness normalized crack length versus time are given in Figure 4. The predicted equilibrium crevasse depths are $d = 0.6h$ when $h_w = 0.5h$ and $d = 0.94h$ when $h_w = 0$ with free slip at the base, whereas the equilibrium crevasse penetration depths are $d = 0.28h$ when $h_w = 0.5h$ and $d = 0.35h$ when $h_w = 0$ with no slip at the base. Thus, the basal boundary condition had a more prominent effect on the equilibrium crevasse depth than the water depth for $h_w = 0.5h$, illustrating the importance of the sliding law in determining crevasse penetration depths. In Figure 4, we also compare the equilibrium crevasse depth estimated directly from crack growth simulations with that from the Nye zero-stress model [*Nye*, 1957]. To determine the Nye crevasse depths, we simulated the stress field in notched rectangular ice slabs due to viscoelastic deformation without any damage evolution and found the depths at which the longitudinal stress vanishes. We found that the Nye zero-stress model predicted shallower crevasses compared to the direct crack growth simulations, illustrating the important role played by creep damage evolution and stress concentration effects in the fracture of glaciers.

3.3. Size of Icebergs

[11] In the previous set of experiments, we initialized the simulations using a notch at the midlength at the top surface. We also investigated the most favorable location for crevasse initiation and subsequent propagation in glaciers that is closest to the terminus as a function of the applied/assumed boundary conditions. Since crevasse propagation is driven

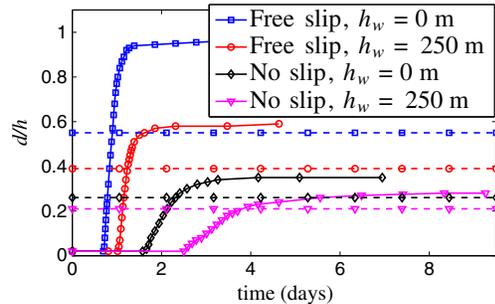


Figure 4. Curves of normalized crack length (d/h) versus time showing the effect of hydrostatic pressure and basal boundary condition on crevasse propagation. The corresponding Nye depths are marked by dashed horizontal lines.

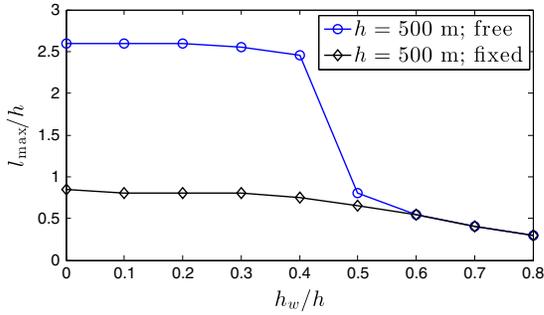


Figure 5. The normalized distance of maximum tensile stress from the terminus (l_{\max}/h) varies as a function of the seawater depth to slab thickness ratio (h_w/h).

by tensile stresses, we assume that the location of maximum tensile stress l_{\max} that is closest to the terminus is the most favorable location for crevasse initiation. The distance l_{\max} gives us an estimate of the size of the calving icebergs as a function of the boundary conditions. In the numerical experiments, we considered an ice slab of thickness $h = 500$ m and varied the water depth from $h_w = 0$ to $h_w = 0.8h$. From the simulation result shown in Figure 5, we can see that as the water depth h_w increases, the distance l_{\max} decreases. The results suggest that for a dry glacier ($h_w = 0$), the iceberg size is $l_{\max} > 2h$, whereas for a marine-terminating glacier with $h_w = 0.5h$, the iceberg size $l_{\max} < h$. This suggests that glaciers terminating in deep water would generate smaller icebergs than those grounded in shallow water. We also investigated the effect of the basal boundary condition. We performed full creep fracture simulations assuming free slip and no-slip conditions on two identical ice slabs with two initial notches placed at distances less than and greater than the thickness of the slab from the terminus (right edge). The results shown in Figure 6 indicate that the crack farther away from the terminus grows under free slip basal condition, whereas the nearer crack grows under fixed basal condition. This study suggests that basal boundary condition and seawater depth both play a prominent role in determining the size of the calving icebergs by establishing favorable locations for crevasse propagation.

4. Conclusion

[12] By employing a viscoelastic constitutive damage model, we studied surface crack propagation in grounded

slabs of ice. Our results suggest that surface crevasses propagate deeper in thicker grounded glaciers and seawater pressure retards surface crevasse propagation. The basal boundary condition and seawater depth not only affect the crevasse propagation rate but also affect the favorable location for surface crevasse formation. Our study suggests that crevasses form closer to the terminus when the seawater depth is larger or when the glacier base is fixed to the bedrock, highlighting the important role that basal sliding plays in iceberg calving. Our computations suggest that crevasses can propagate deeper than those predicted by the Nye zero-stress model, illustrating the dominant role of creep damage (fracture) evolution. However, even in the case of dry thick glaciers, water-free surface crevasses do not propagate the full glacier thickness. This suggests that meltwater pressure driven surface crevasse propagation or seawater pressure driven basal crevasse propagation may be plausible mechanisms that lead to the breakup of glaciers. The main advantage of our computational model is that it is not constrained by any simplistic assumptions and it can help consider realistic scenarios so as to gain new insights into the processes leading to ice fracture and glacial calving.

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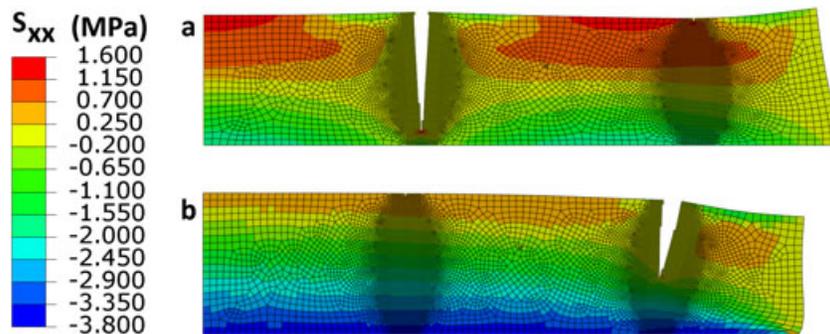


Figure 6. Contour plots of the longitudinal stress in the x direction (in MPa) showing different preferred locations for a crevasse opening with (a) free slip at the base and (b) no slip (fixed) at the base. The ice slab has the dimensions 2000 m \times 500 m. The deformations and crack opening displacements are not drawn to scale.

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