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We are interested in whether preventing resale of tickets benefits the capacity providers for sporting and entertainment events. Common wisdom suggests that ticket resale is harmful to event organizers' revenues and event organizers have tried to prevent resale of tickets. For instance, Ticketmaster has recently proposed paperless (non-transferrable) ticketing which would severely limit the opportunity to resell tickets. We consider a model that allows resale from both consumers and speculators with different transaction costs for each party. Surprisingly, we find that this wisdom is incorrect when event organizers use fixed pricing policies, in fact event organizers benefit from reductions in consumers' (and speculators') transaction costs of resale. Even when multiperiod pricing policies are used, we find that an event organizer may still benefit from ticket resale if his capacity is small. While paperless ticketing is suggested as a way to reduce ticket resale and prevent speculators from buying tickets, our results suggest that it may reduce the capacity providers' revenues in many situations. Instead, we propose ticket options as a novel ticket pricing mechanism. We show that ticket options (where consumers would initially buy an option to buy a ticket and then exercise at a later date) naturally reduce ticket resale significantly and result in significant increases in event organizers' revenues. Furthermore, since a consumer only risks the option price (and not the whole ticket price) if she cannot attend the event, options may face less consumer resistance than paperless tickets.

Key words: events; ticket resale; fixed pricing; multiperiod pricing; options

1. Introduction

Consumer resale behavior plays an important role in ticket sales of concerts and sporting events. For live music and sporting events, ticket sales in the primary markets generate $20 billion per year in the US. On the other hand, resale markets generate $3 billion each year in the US, and this number is expected to grow over the next several years (Mulpuru et al. 2008). For popular concerts, the resale market revenue can be as much as 37% of the primary market revenue, and 46% of the resale activity is generated by consumers (Leslie and Sorensen 2011). Consumer resale is prevalent in event ticket sales for the following reasons. First, event capacity providers make tickets available early in advance to satisfy the needs of those highly dedicated fans who want to secure the rights to attend the events they are interested in (Courty 2003a and Moe et al. 2011). Second, event tickets are usually transferrable. Third, most tickets are non-refundable and consumers purchasing event tickets usually have high valuation uncertainties. A sports fan may not know whether her favorite team will get into the final game or not when she buys the ticket for it. A consumer may also find the event conflicting with some other appointment of higher priority after she buys the ticket. In addition to consumer resale, there may be speculators who purchase tickets solely for the purpose of reselling later hopefully at a higher price[1].

A consumer who cannot attend the event can resell the ticket directly to another consumer or through a broker, among which StubHub, eBay, RazorGator are major players. Brokers obtain profits by charging transaction fees that can be as high as 25% of the ticket resale value to the seller and the buyer. The development of online transactions on the Internet has provided more opportunities for such brokers to thrive. No matter how consumers resell their tickets, traditionally the perception is that resale (secondary) markets are bad for the event organizers and ticket distributors and need to be prevented. As the largest ticket sales and distribution company in the US, Ticketmaster attempted to prevent resale of tickets by influencing ticketing legislation. The battle between firsthand ticket sellers and brokers has produced two nonprofit groups (Sisario 2011). The Fans First Coalition, financed by Live Nation Entertainment which is the parent company of Ticketmaster, supports paperless ticketing. On the other hand, The Fan Freedom Project, financed by StubHub, supports the use of paper tickets. Paperless ticketing works like an airline e-ticket, with no traditional ticket printed when a customer places an order. Instead, a fan shows his credit card at the box office to enter the event, guaranteeing that the person who originally placed the order is the same one attending the event. Paperless ticketing is an instrument to make the tickets non-transferrable while paper tickets are transferrable. However, in 2010, Ticketmaster failed to prevent a change to New York's scalping law which required that consumers have the option for transferrable tickets. So far, there is no federal regulation regarding event ticket resale in the US. Some states restrict resale, but anti-scalping laws are rarely enforced. In 2010, non-transferrable tickets made up only 0.01 percent of all the tickets Ticketmaster processed (Rovell 2011). Moreover, it is not clear when and under what conditions resale markets are harmful to event capacity providers, as many college athletics departments have recently partnered with brokers to create fan-to-fan ticket exchange marketplaces and encouraged their fans to use these platforms to resell their tickets.

There are two major goals of event capacity providers in order to maximize revenue in this challenging environment: first, tie prices to demand; second, capture the revenues from the resale markets. Indeed, Nathan Hubbard, the CEO of Ticketmaster, said that 2010 taught them they have real challenges as an industry and one of them is pricing (Smith 2011). While the level of analytics and technology in event revenue management is far behind travel and retail revenue management, in recent years, event capacity providers started to use multiperiod pricing (i.e., changing the

[1] Speculators can be thought as consumers with zero valuations for attending the event.

ticket price over time) which has been used by airlines for 30 years. For example, Ticketmaster has partnered with MarketShare to bring multiperiod pricing to events[2]. The event capacity providers are hoping that rather than fixed pricing (i.e., keeping the same ticket price over time) which was used as the major pricing strategy, a more flexible pricing strategy can help them capture more of the revenue potential, especially the revenue generated by the resale markets. Recent dynamics of the event ticketing industry and the resale markets motivate our research questions: (i) How does ticket resale affect the event capacity providers' prices and revenues (i.e., is resale harmful to event capacity providers?), and ii) Which pricing strategy is more effective in capturing the resale market revenues?

Table 1 Pricing strategies studied in this paper

	Period 1: tickets	Period 1: ticket options
Price fixed over time	Fixed pricing (Section 4)	N/A
Price changes over time	Multiperiod pricing (Section 5)	Ticket options (Section 6)

To answer these questions, we study whether an event capacity provider is indeed harmed by or in fact can benefit from resale of tickets from consumers as well as speculators under different pricing strategies. As described by Table 1, we consider whether the capacity provider keeps the price fixed over the selling period (fixed pricing) or can change the price (multiperiod pricing) and whether the capacity provider actually sells tickets or ticket options in period 1 (clearly, if the capacity provider sells ticket options initially but tickets later, the prices over time cannot be fixed). We note that fixed pricing is the pricing mechanism used by most college athletics departments and concert organizers, and multiperiod pricing has started to be used by professional sports teams. We find that the capacity provider's optimal revenue from fixed pricing increases when ticket resale is easier for either consumers or speculators, and paperless (non-transferrable) ticketing actually hurts the capacity provider's revenue. Under multiperiod pricing, when the provider's capacity is small, similar to fixed pricing, he benefits from consumer resale. On the other hand, if an event capacity provider uses multiperiod pricing and his capacity is large, then he indeed may benefit from making tickets non-transferrable. Finally, motivated by recent industry practice, we study ticket options that are offered by OptionIt. When an event capacity provider sells options, the consumer pays a fee to get an option to buy a ticket later, and she pays an execution fee when she finally buys the ticket. An advantage to the consumer is that if the consumer cannot attend the event, she only loses the option fee instead of the whole ticket price. We show that options generate higher revenues for event capacity providers by significantly reducing ticket resale and capturing

[2] According to LiveAnalytics (March 3, 2012 MIT Sports Analytics Conference Presentation), 57% of NBA, 50% of MLB, 37% of NHL teams use multiperiod pricing.

the resale market revenues. However, the capacity provider improves revenues further if tickets are non-transferrable under option pricing. Our numerical results indicate that while switching to selling options from multiperiod pricing results in a large revenue increase, making tickets non-transferrable in addition does not result in a revenue increase that is as significant. Therefore, the revenue gains from switching to selling options can be very significant for event capacity providers. Thus, our paper offers a different route to increasing revenues and shrinking the resale market (than non-transferrable tickets) that is likely to generate less adverse consumer reaction.

2. Literature Review

This paper is related to the general revenue management literature (see Talluri and van Ryzin 2005 for a review). In particular, the advance pricing literature, Gale and Holmes (1993), DeGraba (1995), Dana (1998), Shugan and Xie (2000), is relevant to event ticket sales. However, these papers assume tickets are non-transferrable and there is no secondary market. There is not much literature in operations management that deals with issues regarding event ticket pricing in particular. To our knowledge, this paper is one of the few that study event ticket pricing (Su 2010, Balseiro et al. 2011, Tereyagoglu et al. 2012) and the first one that studies the consumer resale behavior in the context of event ticket pricing (perishable product pricing).

Streams of economics and marketing literature investigate several aspects of the ticket industry. Table 2 summarizes the papers, including our paper, that study ticket resale and are closely related to event revenue management. Courty (2003b) studies monopolistic ticket selling to consumers who learn new information about their demands over time. He assumes no capacity constraint and shows that rationing and inter-temporal sales are never optimal. He also shows that the monopolist cannot do strictly better by allowing resale. We assume the provider has limited capacity and the resellers incur resale transaction costs, and study how the capacity level and the resale transaction cost influence the provider's optimal pricing decisions. Moreover, we study a general ticket options model and show that options help event capacity providers capture more resale market revenues. Leslie and Sorensen (2011) study a similar problem empirically and find that while consumer resale improves allocative efficiency, some of the welfare gain from reallocation is offset by increases in efforts and transaction costs in the resale market. Moller and Watanabe (2010) briefly study consumer resale with price commitment and with period 1 arrivals only. They show that the relative profitability of clearance sales with respect to advance purchase discounts increases with resale.

Table 2 Comparison of papers studying ticket resale

Paper	Source of resale	Resale transaction cost	Capacity constraint	Pricing strategies	Findings
Courty (2003b)	- Consumers	No	No	- Multiperiod pricing	- The provider cannot do strictly better by allowing resale.
Moller and Watanabe (2010)	- Consumers	No	Yes	- Multiperiod pricing	- Resale makes clearance sales (i.e., high-to-low pricing) more profitable. - Resale increases the relative profitability of clearance sales with respect to advance purchase discounts (i.e., low-to-high pricing).
Geng et al. (2007)	- Consumers	No	Yes	- Multiperiod pricing	- Partial resale (i.e., consumers can resell tickets till the capacity provider announces the period 2 price) is beneficial to the capacity provider only if the seller sells advance tickets at a premium.
Karp and Perloff (2005)	- Speculators	Yes	Yes	- Multiperiod pricing	- If the speculators are able to perfectly price discriminate and extract all consumer surplus, then speculators may increase capacity provider profits when their transaction costs are low.
Su (2010)	- Speculators	No	Yes	- Fixed pricing - Dynamic pricing	- Speculators increase capacity provider profits under fixed pricing. - Speculators do not increase capacity provider profits under dynamic pricing.
This paper	- Consumers - Speculators	Yes (consumers and speculators can incur different resale transaction costs)	Yes	- Fixed pricing - Multiperiod pricing - Option pricing	- Option pricing is the most effective pricing strategy to reduce consumer and speculator resale and to capture resale market revenues. - Consumer and speculator resale always increases provider profits under fixed pricing and always decreases under option pricing. - Speculator resale decreases provider profits under multiperiod pricing; consumer resale increases provider profits under multiperiod pricing unless the capacity is large.

Geng et al. (2007) study a two-period model where the capacity provider changes the price in period 2 (multiperiod pricing) and assume consumers are only allowed to resell before the capacity provider's price change (they call this pricing scheme "partial resale"). In contrast, our paper assumes that initially tickets can only be sold by the capacity provider, but after a later date, tickets are also available from the secondary market till the event takes place (currently it is possible to buy a ticket from StubHub only a few hours before the start of an event). Furthermore, Geng et al. (2007) assume no resale transaction cost. In contrast, we are interested in whether increases in the resale transaction cost benefit or hurt the capacity provider. These differences in modeling lead to different conclusions. For example, Geng et al. (2007) predict that resale before the price change is beneficial to the capacity provider only if he sells advance tickets at a premium. If advance tickets are discounted, they find that resale should not be allowed. We find that premium advance selling is not an equilibrium if resale occurs till event takes place, and the only equilibrium is discounted advance selling. Finally, we also study ticket resale in the context of fixed pricing and option pricing, in addition to multiperiod pricing. Therefore, the focus and the insights of our paper are different.

There is also a stream of literature on ticket scalping and speculative behavior. Different from consumer resale, speculators purchase tickets solely for the purpose of reselling later hopefully at a higher price. Courty (2003a) provides a survey of this literature. Karp and Perloff (2005) assume scalpers are able to perfectly price discriminate and extract maximal consumer surplus. Therefore, they find that speculators do not reduce and may increase monopoly profits when their transaction costs are low under multiperiod pricing. We find that if the speculators cannot perfectly price discriminate and consumer resale is possible, speculator resale is never beneficial to the capacity provider under multiperiod pricing. Different from Karp and Perloff (2005), Su (2010) captures the possibility that scalpers may incur a loss (e.g., if demand turns out to be weak). He finds that the presence of speculators increases the firm's expected profits from fixed pricing but does not change the profits if dynamic pricing is used. Our paper is complementary to Su (2010), as we study resale from both consumers and speculators. We show that while his finding regarding speculator resale remains true for consumer resale as well if fixed pricing is used, consumer resale can sometimes be a benefit to the capacity provider when multiperiod pricing is used. Under multiperiod pricing, consumer resale can create competition in the secondary market and drive down the capacity provider's price, but it also increases consumers' willingness to pay in the advance selling period. Thus, consumer resale can sometimes be beneficial to the capacity provider. On the other hand, speculator resale is never beneficial and may even decrease the revenues of the provider under multiperiod pricing. Therefore, interestingly, we find that effects of consumer resale and speculator resale on provider revenues are not identical. Moreover, unlike previous papers, we allow consumers to have inter-temporal valuation uncertainties, and allow both consumers and speculators to incur transaction costs for ticket resale. Finally, for the first time in the literature, we show that ticket options result in higher revenues for event capacity providers than fixed and multiperiod pricing due to significant reduction of the resale markets.

Finally, there are a few papers that study options for services. Xie and Gerstner (2007) show that a capacity-constrained service provider can profit from offering partial refunds for service cancellations. Selling ticket options is similar to allowing service cancellations, with the advance price equal to the sum of the option price and the execution fee, and the refund equal to the execution fee. However, in Xie and Gerstner (2007), the refund is set upon receiving a cancellation notification. With ticket options, the service provider commits to the refund upfront as he pre-announces both the option price and the execution fee. It is easy to show that commitment results in higher profits. More importantly, our focus is the benefit of tickets options in capturing the resale market revenues. Xie and Shugan (2001) show that with infinite capacity, advance selling with refund is more profitable than both advance selling without refund and spot selling. Gallego and Sahin (2010) study real options with limited capacity. They show that the capacity provider earns significantly higher revenues by selling real options on capacity than low-to-high pricing. Similarly, Balseiro et al. (2011) show that offering team-based options for sporting events benefits the provider and the consumers. Sainam et al. (2010) find that consumer options can protect consumers from the downside risk related to uncertain outcomes and enhance seller profits by enabling superior market segmentation and increasing consumer willingness to pay. They empirically demonstrate that consumer willingness to pay increases and profits from option pricing can exceed those from advance selling and spot selling. However, none of these papers considers consumer or speculator resale in secondary markets. Our main focus is how resale markets and transaction costs affect the capacity providers' revenues and optimal pricing strategies. We are interested in whether the capacity provider has an incentive to prevent ticket resale under different pricing strategies where pricing with ticket options is one of these strategies. We show that the capacity provider can significantly reduce resale hence capture more resale market revenues with options, while under fixed and multiperiod pricing, he has only limited control over resale markets.

3. Model

We consider an event capacity provider that sells his capacity C over two periods. As in Courty (2003a), we "assume that the audience is composed only of two types of consumers: 'diehard fans,' who plan their social calendars well in advance, and 'busy professionals,' who make decisions at the last minute. This consumer characterization does not suggest that busy professionals enjoy the event less than diehard fans, only that these two market segments plan their social calendars differently. Indeed, a consumer could qualify as a diehard fan for one event and as a busy professional for another." λ_1 consumers arrive in period 1 to purchase advance tickets. λ_2 consumers arrive in period 2, who make their purchasing decisions at the last minute because they may want to wait until some uncertainties in their schedules or regarding the event are settled (e.g., a diehard soccer fan can buy a ticket for the World Cup final without knowing who will be in the final; others will only buy if their country is in the final). $\lambda_1 + \lambda_2$ measures the magnitude of the consumer base for the event. In our analysis, we focus on the case of $C < \lambda_1 + \lambda_2$ which is the more realistic and interesting scenario. When $C \geq \lambda_1 + \lambda_2$, the prices decrease to the lowest possible level $v_{\min}$ (i.e., consumer valuation lower bound) no matter which strategy is used because the market is over-supplied. In that case, every pricing strategy results in the same outcome.

Consumers have an ex ante i.i.d. valuation V which has a continuous support and is bounded below by $v_{\min} > 0$. Let $F(\cdot)$ and $f(\cdot)$ denote the cumulative distribution function and probability density function of V, respectively. Without loss of generality, we assume that consumers arriving in different periods have the same valuation distribution, our analysis can be easily generalized to the case where period 2 consumers' valuations follow a different distribution[3]. Consumers learn their valuations at the beginning of period 2. If a period 1 consumer purchases an advance ticket, she can either use the ticket to attend the event or resell it in period 2, depending on her realized valuation. A period 1 consumer may also decide to postpone her purchasing decision to period 2 when she gains more information about her valuation. In this case, she can buy from either the capacity provider or the resale market. We assume efficient rationing, i.e., given the same price, consumers who value the ticket the most are served first and resellers who value the ticket the least make sales first. This assumption is common in economics literature and is also made in papers studying event ticketing such as Su (2010).

Consumers incur a transaction cost when they resell tickets. This transaction cost can represent the commission paid to the broker and can also represent the search or inconvenience cost when looking for the buyer. In reality, brokers charge commissions which are typically percentages of the ticket resale prices. For example, StubHub charges a 15% commission to the seller and a 10% commission to the buyer. To make sales, the resellers have to reduce the resale price so that buyers find the price competitive to the capacity provider's price after paying the buyers' commission. Without loss of generality, we use a single transaction cost $\tau > 0$ which is a percentage of the resale price and define the resale price as the one in the case where only the resellers pay the commission[4].

[3] All our results in Sections 4 - 6 hold and all our managerial insights remain valid if the valuations of the two classes of consumers are different but both follow uniform or shifted exponential distributions.

[4] To see the equivalence of using a single transaction cost and using separate transaction costs, let r denote the resale price, and let τ_s and τ_b denote the transaction costs (as percentages) that the broker charges to the seller and the buyer, respectively. With separate transaction costs, the actual price resellers can charge is $r/(1+\tau_b)$, hence the net gain from resale is $r(1-\tau_s)/(1+\tau_b)$. Thus, using separate transaction costs is equivalent to using a single transaction cost of $\tau = 1-(1-\tau_s)/(1+\tau_b)$. For StubHub, this single transaction cost is equal to $\tau = 1-(1-15\%)/(1+10\%) = 22.73\%$.

Besides regular consumers who have a genuine interest in potentially attending the event, we also allow an infinite pool of speculators who do not value attending the event but may purchase tickets in period 1 and resell tickets in period 2. Since speculators only enter the market if the net payoff from resale is greater than the capacity provider's period 1 price, the number of speculators entering the market in equilibrium is endogenously determined[5]. We use τ' to denote the speculators' resale transaction cost and assume $\tau' \leq \tau$ to capture the fact that speculators usually have less costly channels to resell tickets (e.g., speculators may not have to sell their tickets through well-established brokers such as StubHub but create their own cheaper channels to sell tickets directly to consumers).

The capacity provider's goal is to maximize his revenue from selling his capacity over two periods[6]. To reflect the event ticketing industry practice, we assume that the capacity provider makes tickets available in advance to satisfy the needs of those highly dedicated fans who want to secure the rights to attend events they are interested in (Courty 2003a, Moe et al. 2011). (Under fixed pricing, in our model, the capacity provider may increase his revenues even further by not allowing advance sales, whereas under multiperiod or option pricing, advance sales can be endogenously optimal. We note that it may not be realistic to sell event tickets only on the spot before the event. For example, many college football fans travel from out of state to see their team play. Last minute airfares and hotel prices are a lot more expensive typically. Thus, if the capacity provider does not make tickets available in advance, these fans may not attend the event.) We also assume the provider does not strategically hold back capacity in either period. This is consistent with the practice of most college sports teams, professional sports teams and artists as they intentionally offer all seats available to maximize the entertainment value of the event which is highly correlated with the size of the audience: the bigger the audience, the more enjoyable the experience (Becker 1991). In section 7.1, we study the case where the provider may hold back part of his capacity in period 1 to sell in period 2 as a model extension.

We first study the pricing strategies that have been commonly used in practice by event organizers. In Section 4, we study fixed pricing where the capacity provider sells tickets at price p_f throughout two periods. In Section 5, we study multiperiod pricing where the capacity provider sells tickets at price p_1 in period 1 and at price p_2 in period 2. The sequence of events is as follows. First, at the beginning of period 1, the capacity provider announces his advance ticket price. After that, period 1 consumers decide whether to purchase tickets immediately or wait, and speculators

[5] Without loss of generality, in the model we do not include arbitrageurs who buy tickets in period 2 and resell tickets immediately. Similarly, we do not allow period 2 consumers to buy and resell tickets in period 2. It is easy to show that such behavior cannot occur in equilibrium.

[6] We ignore the variable cost because from a production standpoint, events have high fixed costs and low variable costs (Connolly and Krueger 2006).

decide whether to enter the market or not. Then, in period 2, after consumers realize their valuations, the period 1 consumers who have purchased tickets decide whether to resell or use them, and those choosing to resell the tickets as well as speculators determine the resale price. If the capacity provider uses multiperiod pricing, he determines his period 2 price at the same time. Figure 1 describes the period 1 consumers' inter-temporal decision process and the payoff from each decision under fixed and multiperiod pricing. A speculator's decision process is a special case of Figure 1 where $V = 0$ with probability one and the resale transaction cost is τ' instead of τ. Throughout the paper, we add subscripts to the notations to specify which pricing strategy we are considering: "f" for fixed pricing, "m" for multiperiod pricing, "o" for ticket options.

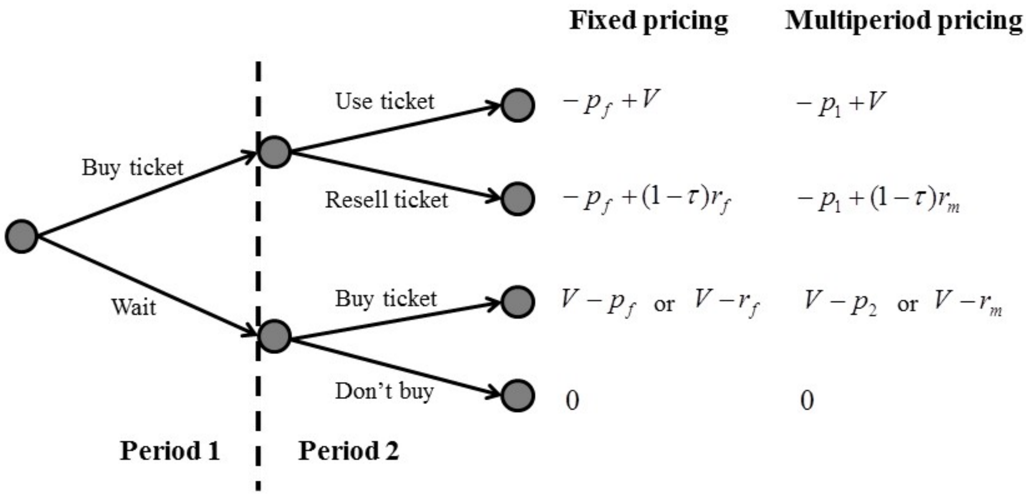

Figure 1 **Consumer choice model under fixed and multiperiod pricing**

As described above, our main interest is in the effect of ticket resale on the capacity provider's revenues where the capacity provider's goal is to extract as much revenue as possible while selling out the tickets to maximize the entertainment value of the event. On the one hand, allowing resale (or a decrease in resale transaction costs) can increase the value of tickets for consumers since consumers know that they have an option to resell tickets if for some reason they cannot attend the event. On the other hand, resale markets (as well as speculators buying tickets when resale is allowed) may increase competition with the capacity provider and may result in a decrease of ticket revenues. This is the fundamental high-level tradeoff that we are interested in and that we are going to analyze under fixed, multiperiod and option pricing in the following sections.

4. Fixed Pricing

In this section, we study the fixed pricing strategy that has been commonly used by event capacity providers such as college sports teams and concert organizers in practice. Our result here is that event capacity providers are always hurt by an increase in the transaction costs that either consumers or speculators incur in reselling the tickets, that is, an event capacity provider using fixed pricing prefers consumers and speculators to be able to use resale markets with no transaction cost at all. To analyze this case, we use backward induction to find the subgame perfect equilibrium of the game between the capacity provider, consumers and speculators. More specifically, we first characterize the equilibrium resale price in period 2, then characterize the purchasing decisions of consumers and speculators in period 1, and finally determine the capacity provider's optimal fixed price.

THEOREM 1. *(i) The equilibrium resale price* r_f^* *is given by* $[(\lambda_1-C)^+ + \lambda_2]\bar{F}(r_f^*) = (C-\lambda_1)^+ + \min(\lambda_1, C)F((1-\tau)r_f^*)$.

(ii) Define p_s *as the solution to* $(\lambda_1+\lambda_2)\bar{F}(p_s) = C$ *and define* p_f^n *as the solution to* $p_f^n + \int_{p_s}^{\infty}(v - p_f^n)\,\mathrm{d}F(v) = E[\max(V,(1-\tau)r_f^*)]$. *The capacity provider's optimal fixed price is* $p_f^* = p_f^n$ *if* $p_f^n \geq (1-\tau')p_s$ *and* $p_f^* = \min\big(E[\max(V,(1-\tau)r_f^*)] - E(V-p_s)^+, (1-\tau')p_s\big)$ *otherwise. Moreover,* $p_f^* < r_f^*$.

(iii) For a given τ*, speculators enter the market in equilibrium if and only if* $\tau' < \bar{\tau}_f'(\tau) = 1 - p_f^n/r_f^*$.

Theorem 1 characterizes the equilibrium outcome under fixed pricing. Given that a period 1 consumer obtains a ticket from the capacity provider, in period 2, if her valuation is smaller than the net payoff from resale, $(1-\tau)r_f^*$, she resells the ticket; otherwise, she uses the ticket herself. Note that the equilibrium resale price is higher than the capacity provider's optimal fixed price. This price inflation in resale markets close to event dates is often observed in reality[7] and is one of the reasons event capacity providers are sometimes interested in eliminating resale markets. However, as we will show in Theorem 2 below, this would actually hurt event capacity providers.

Theorem 1(iii) states that speculators enter the market in equilibrium if their resale transaction cost is small enough. In this case, speculators keep entering the market in period 1 until the provider's capacity is depleted, and they resell the tickets in period 2 instead of the capacity provider at the resale price which is higher than the capacity provider's fixed price. This result provides one explanation for why we see speculators in reality – their transaction cost to resell tickets is smaller than the transaction cost incurred by regular consumers. A reduced transaction cost to resell tickets gives speculators an advantage and makes speculators more likely to enter the market.

[7] In 2012, the average ticket resale price is $139.71 for the Southeastern Conference and $132.65 for the Big Ten Conference (Rovell 2012), which is almost double the original ticket price.

On the other hand, if speculators' resale transaction cost is large enough so that their willingness to pay for advance tickets is lower than regular consumers, by charging a price higher than speculators' willingness to pay, the capacity provider may shut speculators out of the market. Of course, in many events where the capacity provider uses fixed pricing, we see speculators and they are not shut out of the market. A second reason for speculators' existence may be underpricing by event capacity providers. Note that in our model, if $\tau' \geq \bar{\tau}'_f(\tau)$, the capacity provider can shut speculators out of the market if he uses "optimal" pricing. However, it is not clear that event capacity providers always set prices optimally in reality. For example, in the 2012 college football season, the Ohio State University charged $75 or $85 per seat for every game ($85 was charged for better seats) even though some games are known to be much more popular than others, such as the game against the University of Michigan. Even though the Michigan – Ohio State game is one of the most popular games in college football, Ohio State did not charge more for this game. Consequently, ticket prices on the resale markets were at a minimum double the original ticket price, which would offer a great opportunity for speculators to make profits. Thus, underpricing may be another reason for speculators' existence in the market. There is some evidence in the literature that until recently, teams were afraid of offending loyal fans by changing prices according to demand. For example, as Courty (2003a) pointed out, "a constant price (same price for all events in a season) may be necessary to attract loyal team fans". Similarly, Krueger (2001) cited the NFL vice president for public relations who stated that the league tries to set "a fair, reasonable price" because it wants to maintain an "ongoing relationship with fans and business associates". The NFL vice president for public relations stated that although the NFL could increase its "present-day profit" by raising ticket prices, it prefers to take "a long-term strategic view"[8]. The underpricing potentially motivated by these considerations, however, can lead to speculators buying tickets under fixed pricing as we showed above. Interestingly, under fixed pricing at least, speculator and consumer resale do not hurt the capacity provider's optimal revenues, as we show below.

THEOREM 2. *Under fixed pricing, the capacity provider's optimal price and optimal revenue are decreasing in τ and τ'. Thus, the capacity provider achieves the highest revenue when $\tau = \tau' = 0$, and selling non-transferrable tickets harms the capacity provider.*

Our primary interest is in whether the capacity provider benefits from a larger or smaller resale transaction cost and whether the capacity provider should prevent resale of tickets. We answer this question by analyzing the most favorable resale transaction costs incurred by consumers and speculators from the capacity provider's point of view. Theorem 2 states that the capacity provider's

[8] Modeling how long-term demand may change because loyal fans may be offended by more demand-driven pricing is beyond the scope of this paper. It is an interesting future research direction.

optimal fixed price and optimal revenue from fixed pricing are decreasing[9] in both τ and τ'. The decreasing result regarding τ holds for any τ' and is independent of the existence of speculators in the market, and vice versa. Thus, the existence of speculators never hurts the capacity provider under fixed pricing. If τ' is small enough (i.e., $\tau' < 1 - p_f^n/p_s$), the capacity provider's optimal fixed price and revenue are higher with the existence of speculators. This is because when speculators' transaction cost is small enough, they will enter the market even when period 1 consumers do not buy tickets immediately. In this case, if period 1 consumers wait, then in period 2, they will have to buy tickets from speculators at a higher price than the capacity provider. Seeing this threat, period 1 consumers will accept a higher price for advance tickets from the capacity provider, hence the capacity provider can earn more revenue.

Moreover, Theorem 2 implies that the capacity provider actually loses money when it is more costly for consumers to resell tickets. This is exactly the opposite of the belief of many event capacity providers in practice. As τ becomes larger, period 1 consumers value advance tickets less because their payoff in period 2 if purchasing advance tickets, $E[\max(V, (1-\tau)r_f^*)]$, decreases. On the other hand, their payoff from waiting and purchasing in period 2 may increase. To induce them to buy tickets, the capacity provider has to decrease his price[10]. The capacity provider can charge a higher price and earn more revenue when resale is less costly[11]. When $\tau = \tau' = 1$, the net payoff from resale is zero for both consumers and speculators, so this corresponds to the case of selling non-transferrable tickets[12]. We have clearly shown that an event capacity provider using fixed pricing would be hurt by non-transferrable tickets and always benefits from ticket resale, even if some of the tickets will be bought by speculators. Thus, the increase in how much consumers value tickets (and thus the capacity provider being able to charge consumers more because of this increased valuation) dominates the effect of increased competition with the capacity provider from the resale market. We now analyze how these results are affected if the capacity provider charges different prices over time.

[9] In this paper, we use increasing/decreasing in the weak sense.

[10] Note that the amount that the capacity provider decreases his price is not always equal to the amount that period 1 consumers' payoff from buying tickets decreases. Period 1 consumers' payoff from waiting is $\int_{p_s}^{\infty}(v - p_f^n)\,\mathrm{d}F(v)$ if $p_f^n \geq (1-\tau')p_s$ and $E(V-p_s)^+$ otherwise. $E(V-p_s)^+$ is independent of τ while $\int_{p_s}^{\infty}(v-p_f^n)\,\mathrm{d}F(v)$ is increasing in τ, hence as the capacity provider decreases his price, period 1 consumers' payoff from waiting may increase. Thus, as τ becomes larger, if period 1 consumers' payoff from waiting increases, the capacity provider may have to decrease his price more than the amount that period 1 consumers' payoff from buying tickets decreases.

[11] The University of Michigan signed an agreement with StubHub in July 2011 that makes the company the official fan-to-fan ticket exchange marketplace for Wolverine Athletics. In the following season, Michigan raised ticket prices for the first time in seven seasons (Shea 2012). In fact, StubHub is now the secondary ticketing partner of 20 colleges. In addition to Michigan, StubHub has partnered with the Big Ten Conference, North Carolina, Florida State and Virginia Tech.

[12] Strictly speaking, for every C, selling non-transferrable tickets is equivalent to $\tau \geq \hat{\tau}(C) = \inf\{0 \leq \tau \leq 1 : [(\lambda_1 - C)^+ + \lambda_2]\bar{F}(v_{\min}/(1-\tau)) \leq (C-\lambda_1)^+\}$ and $\tau' \geq 1 - p_f^n/r_f^*$. For the sake of readability, we refer to $\tau = \tau' = 1$ as selling non-transferrable tickets in the main text.

5. Multiperiod Pricing

In this section, we study the multiperiod pricing strategies where capacity providers change their ticket prices over time. Multiperiod pricing has started to become the dominant strategy used by capacity providers such as professional sports teams. To study the effects of price changes and demonstrate whether the capacity provider should try to prevent resale or not, we analyze a two-period model. We assume the capacity provider announces his advance ticket price p_1 at the beginning of period 1 and can adjust his price to p_2 in period 2, after consumers learn their valuations, to sell the remaining capacity. In this section, we assume the capacity provider cannot commit to the period 2 price upfront. (We have also analyzed the case where the capacity provider can commit to the period 2 price, and omit this case for space considerations. The insights regarding whether the capacity provider should prevent resale or not do not change if he can commit to the period 2 price under the multiperiod pricing setting.) Clearly, being able to charge different prices over time gives the capacity provider more flexibility, so the fact that multiperiod pricing results in higher revenues than fixed pricing is not too surprising. However, we are more interested in whether the capacity provider benefits from a larger or smaller resale transaction cost for consumers and speculators under multiperiod pricing. Recall that under fixed pricing, we showed that the capacity provider always benefits from a smaller transaction cost. As we will show in this section, this is no longer true under multiperiod pricing.

THEOREM 3. *(i) The capacity provider's optimal period 2 price p_2^* and the equilibrium resale price r_m^* are $p_2^* = r_m^* = r_f^*$.*

(ii) The capacity provider's optimal period 1 price is $p_1^* = E[\max(V, (1-\tau)p_2^*)] - E(V - p_s)^+$. Moreover, $p_1^* < p_2^*$.

(iii) For a given τ, speculators enter the market in equilibrium if and only if $\tau' < \bar{\tau}'_m(\tau) = 1 - p_1^*/p_2^*$.

Similar to fixed pricing, we use backward induction to find the subgame perfect equilibrium of the game between the capacity provider, consumers and speculators. The difference is that in period 2, to determine p_2^*, the capacity provider plays a simultaneous game with the consumers and speculators who have purchased tickets in period 1. Theorem 3 characterizes the equilibrium outcome under multiperiod pricing. In period 2, the capacity provider's price is equal to the resale price, so the equilibrium outcome in period 2 is equivalent to the situation where the capacity provider participates in the resale market and determines its market clearing price together with the resellers. This is because the capacity provider and the resale market are competitors in period 2. When their prices are different, the party with the lower price will raise the price to gain more margin, and if the party with the higher price is not making sales, it will decrease the price to gain market share. As $p_1^* < p_2^*$, the capacity provider implements a "low-to-high" pricing. He offers a discount for advance tickets but captures a higher margin close to the event date.

Note that Theorem 3(iii) characterizes the condition on τ' for a given τ such that speculators enter the market in equilibrium. For a given τ', we can also characterize the condition on τ such that speculators enter the market in equilibrium. Define $\bar{\tau}_m(\tau')$ as the τ solving $\tau' = \bar{\tau}'_m(\tau)$, i.e., $\bar{\tau}_m(\tau')$ is the inverse function of $\bar{\tau}'_m(\tau)$. Speculators enter the market in equilibrium if $\tau > \bar{\tau}_m(\tau')$ and do not enter the market in equilibrium if $\tau \leq \bar{\tau}_m(\tau')$. This is because when consumers' transaction cost becomes larger, fewer consumers would like to resell tickets, hence speculators have less competition in the resale market and can make more profits. Recall that under fixed pricing, we showed that the existence of speculators does not hurt the capacity provider and may in fact benefit the capacity provider. Under multiperiod pricing, the result is exactly the opposite – the capacity provider's revenue decreases when speculators enter the market in equilibrium. Speculators hurt the capacity provider's revenue under multiperiod pricing because they force the capacity provider to sell more tickets in period 1 at a lower price than period 2. Without speculators, if the provider has sufficient capacity to satisfy the period 1 consumers and has leftovers, he will then sell the remaining tickets in period 2 at a higher price and earn more revenue. Under fixed pricing, however, the capacity provider does not have the flexibility to change the price and capture a higher margin close to the event date in the first place. Therefore, with the additional price flexibility under multiperiod pricing, an event capacity provider no longer needs speculators as an instrument to boost revenue, he is better off in the absence of speculators. Given these interesting dynamics under multiperiod pricing, the result on whether the capacity provider would like consumer resale to be less or more costly is more complex than under fixed pricing and we characterize it below.

THEOREM 4. *(i) Under multiperiod pricing, the capacity provider's optimal period 1 price is decreasing in τ, while the optimal period 2 price is increasing in τ.*

(ii) For $\tau > \bar{\tau}_m(\tau')$, the optimal revenue from multiperiod pricing is decreasing in τ.

(iii) Assume $f(\cdot)$ is decreasing. For $\tau \leq \bar{\tau}_m(\tau')$, there exists a threshold $\bar{C} > \lambda_1$ such that if $C \leq \lambda_1$, the optimal revenue from multiperiod pricing is decreasing in τ; if $C \geq \bar{C}$, it is increasing in τ; otherwise, it may be decreasing or first decreasing then increasing in τ. The capacity provider achieves the highest revenue either when $\tau = \tau' = 0$ or $\tau = \tau' = 1$. If $C \leq \lambda_1$, $\tau = \tau' = 0$ results in the highest revenue; if $C \geq \bar{C}$, $\tau = \tau' = 1$ results in the highest revenue (i.e., the capacity provider benefits from selling non-transferrable tickets).

We have shown that under multiperiod pricing, the capacity provider prefers speculators' resale transaction cost to be large enough to prevent them from entering the market. Now we analyze what resale transaction cost incurred by consumers is most favorable to the capacity provider.

For a given τ', Parts (ii) and (iii) of Theorem 4 characterize how the capacity provider's optimal revenue from multiperiod pricing changes with respect to τ when speculators exist and do not exist in equilibrium, respectively. When speculators exist in equilibrium (i.e., $\tau > \bar{\tau}_m(\tau')$), decreases in the consumer resale transaction cost increase the capacity provider's revenue. This is because in this case, the provider will sell out his capacity in period 1 (with speculators' help) and he can increase his period 1 price and earn more revenue if consumers incur a smaller resale transaction cost. On the other hand, when speculators do not enter the market in equilibrium (i.e., $\tau \leq \bar{\tau}_m(\tau')$), how the capacity provider's optimal revenue from multiperiod pricing changes with respect to τ depends on the capacity C. If the provider's capacity is small, he sells out his capacity early and most sales occur in period 1. Since a smaller τ results in a higher period 1 price, the capacity provider achieves a higher revenue when the consumers' resale transaction cost is smaller. Thus, if an event capacity provider has a small capacity or the event is popular (a sufficient condition is $C \leq \lambda_1$), we have the same result from fixed pricing that the capacity provider will be better off when consumer resale is less costly. On the contrary, if the provider's capacity is large enough so that the majority of his revenue comes from ticket sales in period 2 (a sufficient condition is when $C \geq \bar{C}$), the effect of a larger τ on the period 2 price will dominate. As we show in Part (i) of Theorem 4, a larger τ results in a higher period 2 price. Thus, the capacity provider prefers larger resale transaction costs in this case, as he has sufficient remaining tickets to sell in period 2 at a higher margin and the competition from consumers that resell tickets can harm his revenue. This is different from what we found under fixed pricing.

To summarize, our result indicates that the capacity provider may sometimes benefit from non-transferrable tickets when using multiperiod pricing, unlike the fixed pricing case when he will always be hurt by non-transferrable tickets. Whether the capacity provider benefits or not depends on the actual values of demand and capacity. For example, if demand significantly exceeds capacity, then non-transferrable tickets are again a bad idea for capacity providers. However, the problem is that most capacity providers have more than one event in the same venue during a season with each event having a different demand level. For example, an NBA team (where multiperiod pricing is commonly used) typically plays 82 games in a regular season. The Detroit Pistons (who have been performing pretty badly in the last few years), for example, cannot sell out capacity for most games except the games where they play against very popular teams such as Miami Heat. It would be very difficult for a team like Pistons to allow ticket resale for the Miami Heat game but sell non-transferrable tickets for another game.

Interestingly, the primary reason that a team would want to make resale more difficult (or sell non-transferrable tickets) is to increase revenues. In fact, in the next section, we show that for that purpose, there is a much better pricing mechanism than multiperiod pricing. We will show that ticket options always dominate multiperiod pricing in revenue generation for event capacity providers. Furthermore, ticket options naturally reduce ticket resale. Thus, there is in fact a way for capacity providers to reduce the resale market and capture its revenue without resorting to paperless ticketing.

6. Ticket Options

So far, we have analyzed fixed and multiperiod pricing which are the pricing strategies that have been commonly used by event capacity providers in practice. We have found that consumer resale is actually beneficial to an event capacity provider in most cases unless he has a large capacity to sell and is using multiperiod pricing. Speculators may benefit the capacity provider under fixed pricing, but they may hurt the capacity provider under multiperiod pricing. Thus, under multiperiod pricing, if the provider has a large capacity (or the event is not popular), he achieves the highest revenue without any ticket resale, where paperless ticketing proposed by Ticketmaster is one way to make tickets non-transferrable and eliminate the resale markets. However, to achieve this benefit in practice, an event capacity provider would have to enforce paperless ticketing for only unpopular events and allow ticket resale for other events in the same season. In this section, we study a novel pricing strategy with ticket options that has emerged recently in practice (e.g., OptionIt sells online ticket options for events). As we will show, this novel pricing strategy is generally more profitable than the current strategies used in practice. It also has the benefit of giving consumers more flexibility, that is, consumers initially only buy an option to attend the event at a much lower price than the regular ticket price and can exercise the option when they know their valuations for the event.

Consumers expose themselves to low valuation risks by purchasing advance tickets as the event may conflict with their schedules that are not known in advance. Also, many sports employ elimination type tournaments, and an advance ticket may become worthless to a consumer if the athlete/team she supports does not qualify for the event (e.g., US Open men's final). On the other hand, if consumers do not purchase tickets in advance, they risk paying high prices in the resale markets or seats being sold out. Options can be very attractive to consumers because options can help them hedge against the valuation uncertainties. For example, a search for tickets for the ice hockey game of Florida Panthers vs. Montreal Canadiens on March 10, 2013 resulted in tickets at \$76.75 – \$87 on Ticketmaster for seats on the lower level of the stadium. On the other hand, OptionIt allows consumers to buy an option (i.e., to reserve a seat) for the seats in the same region for \$8 and pay an additional \$100 if later deciding to actually buy the ticket. By purchasing an option, if a consumer later finds herself unable to attend the event, she loses at most \$8 (she may even be able to resell the ticket and incur a smaller loss if the resale price is high enough), while she may lose up to \$87 if purchasing a regular ticket. With options, consumers can purchase the right but not the obligation to buy tickets closer to the event date. A consumer can pay a relatively small amount (option price) to secure the right of purchase and make her final purchasing decision after the uncertainties are resolved. She needs to pay an additional amount (strike price) if she exercises the option to obtain a real ticket later. A consumer may exercise the option because her valuation is high enough (e.g., her favorite tennis player qualifies for the final) so she will use the ticket to attend the event, or the resale price is high enough so she will resell the ticket. Otherwise, the consumer will find an event ticket unattractive and let the option expire.

We study a pricing scheme where the capacity provider sells (x, p) options in period 1 and regular tickets at price p_o in period 2. x is the option price. i.e., the price to purchase a ticket option; p is the strike price, i.e., the extra amount to pay if one decides to exercise the option to obtain a real ticket. Both x and p are announced at the beginning of period 1. To reflect the fact that consumers would want to decide whether to exercise the options or not as their uncertainties are resolved, we assume options can be exercised in period 2 after consumers learn their valuations. The capacity provider can sell the expired options again as tickets in period 2. We assume the capacity provider announces his period 2 ticket price p_o after consumers learn their valuations, that is, our ticket options model also has the multiperiod pricing feature. At the same time, the consumers and speculators who choose to resell tickets after exercising the options determine the resale price r_o. The capacity provider's goal is to optimally set the option price, the strike price (both are announced in period 1) and the period 2 price (announced in period 2) so that his revenue is maximized. We do not allow the capacity provider to sell more options than his capacity although one might increase revenues by doing so in the short term. The reason is that there have been consumer backlashes to firms (e.g., Yoonew and FirstDibz) that have sold more options than their available capacities and had to deny consumers' requests to exercise the options. Compared to airline tickets where overselling is standard, event tickets are much less substitutable because an event usually occurs only once.

6.1. Consumer Choice Model

Consumers make their purchasing decisions in period 1 based on their expectations on the realizations of valuations and the prices in period 2. Period 1 consumers' inter-temporal decision process and the corresponding payoffs are illustrated in Figure 2. A speculator's decision process is a special case of Figure 2 where $V = 0$ with probability one and the resale transaction cost is τ' instead of τ. In period 2, the option price x becomes sunk cost; the period 1 consumers who have purchased options decide whether to exercise the options or not and whether to resell or use the tickets. A consumer exercises the option if her valuation is greater than the strike price or the payoff from reselling the ticket is greater than the strike price, i.e., $\max(V,(1-\tau)r_o) > p$; she lets the option expire otherwise. On the other hand, as speculators never use the tickets to attend the event, they exercise the options and resell the tickets if $(1-\tau')r_o > p$ and let the options expire otherwise.

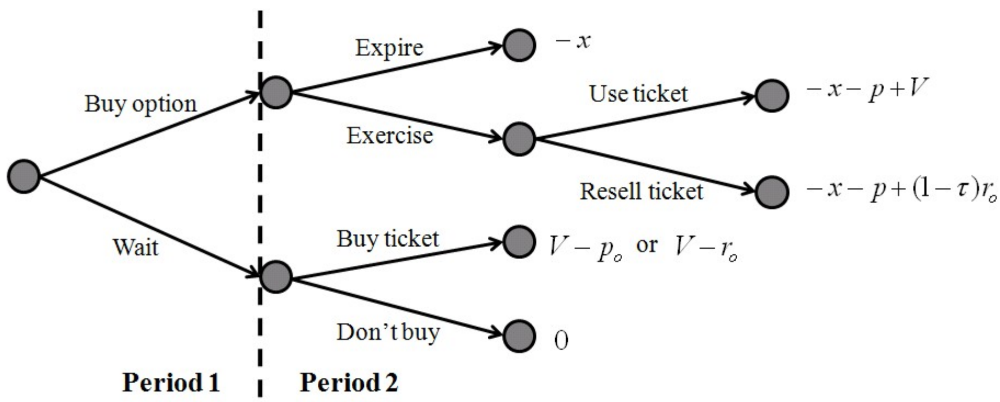

Figure 2 **Consumer choice model under option pricing**

6.2. Optimal Option Pricing

We again use backward induction to solve the game between the capacity provider, consumers and speculators. In this section, we assume $F(\cdot)$ has an increasing failure rate. We will show that selling ticket options in period 1 instead of regular tickets can indeed improve the capacity provider's revenue in the multiperiod pricing framework, and we discuss where the benefit of ticket options comes from. Theorem 5 characterizes the optimal pricing strategy with options as well as how the capacity provider's optimal prices and revenue change as consumers' and speculators' transaction costs are changed. Similar to multiperiod pricing, the capacity provider's period 2 price is equal to the resale price in equilibrium due to competition. Speculators enter the market in equilibrium if their resale transaction cost is small enough. If speculators buy options in period 1, then in period 2, they exercise the options and resell the tickets because they would not enter the market in the first place if they later let the option expire and incur a net loss. In Section 5, we showed that with the flexibility to change the price in period 2, the capacity provider prefers the absence of speculators. This is still true if the capacity provider sells ticket options, as without speculators, the capacity provider can sell more tickets in period 2 at a higher margin (i.e., $x^* + p^* < p_o^*$) and increase the revenue.

THEOREM 5. *(i) The capacity provider's optimal strike price* p^* *is decreasing in* τ[13]. *The optimal options price is* $x^* = E(V-p^*)^+ - E(V-p_s)^+$ *which is increasing in* τ. *In equilibrium, period 1 consumers do not choose to resell tickets in period 2.*

[13] The characterization of p^* is complicated, therefore we omit it in the theorem statement. It can be found in the proof of Theorem 5 in Appendix.

(ii) The capacity provider's optimal period 2 price p_o^* *and the equilibrium resale price* r_o^* *are* $p_o^* = r_o^* = \inf\{r \geq v_{\min} : [(\lambda_1 - C)^+ + \lambda_2]\bar{F}(r) \leq (C - \lambda_1)^+ + \min(\lambda_1, C)F(p^*)\}$*.* p_o^* *is increasing in* τ*. Moreover,* $x^* + p^* < p_o^*$*.*

(iii) For a given τ*, there exists a threshold* $\bar{\tau}_o'(\tau)$ *such that speculators enter the market in equilibrium if and only if* $\tau' < \bar{\tau}_o'(\tau)$*.*

(iv) The capacity provider's optimal revenue from option pricing is increasing in τ*. The capacity provider achieves the highest revenue when* $\tau = \tau' = 1$ *(i.e., the capacity provider benefits from selling non-transferrable tickets).*

Different from speculators that exercise the options in equilibrium, the equilibrium number of period 1 consumers that choose to exercise the options after learning their valuations is influenced by the strike price p. If $p < (1-\tau)r_o$, since the payoff from reselling a ticket exceeds the strike price, all consumers exercise the options. In this case, the capacity provider's optimal period 2 price is equal to the optimal period 2 price under multiperiod pricing and selling (x, p) options is equivalent to multiperiod pricing with the period 1 price equal to $x + p$. Thus, the pricing strategy with options we are analyzing cannot result in a lower optimal revenue than multiperiod pricing. On the other hand, if $p \geq (1-\tau)r_o$, since the payoff from reselling a ticket does not exceed the strike price, a consumer will exercise an option only because her valuation is higher than the strike price so that she will use the ticket herself. In this case, consumer resale is eliminated. We find that it is indeed optimal for the capacity provider to set the strike price high enough to eliminate consumer resale (i.e., the optimal strike satisfies $p^* \geq (1-\tau)r_o^*$), as he can achieve a higher revenue without the resale competition from consumers. Moreover, the capacity provider should set the optimal option price x^* at the minimum possible level that induces period 1 consumers to purchase options. Therefore, by appropriately choosing the prices, the capacity provider can prevent resale from consumers with the use of ticket options.

Given the fact that the capacity provider's optimal strike price is high enough to dominate the payoff from resale so that consumers are prevented from reselling tickets in equilibrium, we can explain why the optimal prices and revenue change with respect to the consumers' resale transaction cost τ in the way stated in Theorem 5. Observing the high strike price, when a period 1 consumer purchases the option, she knows that her chance of reselling the ticket after exercising the option in period 2 is very low. Thus, her payoff in period 2 is merely her realized valuation. As τ becomes larger, the payoff from ticket resale becomes lower, so the capacity provider is able to prevent consumer resale with a lower strike price (i.e., p^* is decreasing in τ). Having a lower strike price to pay later, a period 1 consumer is willing to pay more to purchase an option. Thus, when τ is larger, the capacity provider can charge a higher option price (i.e., x^* is increasing in τ). Since consumers do not resell tickets in equilibrium, a period 1 consumer exercises the option in period 2 if her realized valuation is greater than the strike price. When τ is larger, the capacity provider's optimal strike price is lower, hence more consumers will exercise the options and fewer consumers will let the options expire. As the capacity provider can sell the expired options again as regular tickets in period 2, when τ is larger, he has fewer tickets left to sell and the total supply in period 2 becomes smaller. As a result, the equilibrium resale price as well as the capacity provider's period 2 price is driven up (i.e., p_o^* is increasing in τ).

Finally, Theorem 5(iv) states that unlike fixed or multiperiod pricing, the capacity provider always benefits when consumers have larger resale transaction costs if he sells ticket options. Recall that under multiperiod pricing, if the provider's capacity is small enough, his revenue increases when consumer resale becomes less costly because he can charge a higher period 1 price. This is not true with ticket options because for all levels of capacity, while the provider's optimal strike price is high enough to eliminate consumer resale, it also guarantees that there are enough consumers letting the options expire in period 2 so that the capacity provider can sell a significant amount of tickets in period 2 at a higher price. Thus, the effect that the optimal period 2 price is increasing in τ dominates and the optimal revenue from option pricing is increasing in τ. Therefore, we have shown that with ticket options, the capacity provider loses revenue when resale is less costly for either consumers or speculators. The capacity provider achieves the highest revenue when $\tau = \tau' = 1$ in which case ticket resale from both speculators and consumers are precluded, that is, if an event capacity provider sells ticket options, he always benefits from making tickets non-transferrable.

Theorem 6. $\bar{\tau}'_o(\tau) \leq \bar{\tau}'_m(\tau) < \bar{\tau}'_f(\tau)$.

Theorem 6 points out another interesting feature of option pricing. We have stated before that whereas speculators can benefit the capacity provider under fixed pricing, they can hurt the capacity provider's revenues under multiperiod pricing and option pricing. For speculators to profitably buy and resell tickets, their transaction cost has to be lower than a certain threshold $\bar{\tau}'_i(\tau),\ i = f, m, o$. Theorem 6 shows that this threshold is lowest under option pricing. Thus, an event capacity provider is most likely to be able to shut speculators out of the market under option pricing.

Finally, we discuss why option pricing is beneficial to event capacity providers. First, option pricing is more effective in reducing resale of tickets, hence the capacity provider can capture more revenue from the resale markets. We have shown that with ticket options, the capacity provider can eliminate consumer resale regardless of the consumers' resale transaction cost. Moreover, Theorem 6 indicates that speculators are less likely to exist under option pricing, as speculators enter the market in equilibrium for a smaller range of τ' compared to other pricing strategies. Second, as the capacity provider can sell the expired options as tickets in period 2 and can use the strike price to control the number of expired options, this additional price decision gives the capacity provider more flexibility that he can "virtually" allocate capacity to the two periods and earn more revenue from selling more tickets in period 2 at a higher price.

Note that our comparison between multiperiod pricing and option pricing has been for the same τ and τ', that is, if an event capacity provider is currently using multiperiod pricing, he can increase revenues by switching to selling ticket options while consumers and speculators incur the same resale transaction costs. Theorem 5(iv) indicates that the capacity provider could increase his revenues even more by switching to ticket options and making tickets non-transferrable. Note that under fixed and multiperiod pricing, consumers lose a lot if they buy non-transferrable tickets and then cannot attend the event, as they lose the whole value of the ticket in this case. Thus, generally, even discussions to initiate non-transferrable tickets have led to significant consumer backlashes (e.g., in a June 18, 2012 op-ed, the Consumer League of New Jersey President Bob Russo stated that "Ticketmaster paperless tickets are anti-consumer and is new ploy by company to take more of fans' hard-earned money"). Negative consumer reaction usually focuses on the fact that consumers would lose the whole value of the ticket if they could not attend the event for some reason. However, with ticket options, a consumer will only lose the option price (which is much less than the regular ticket price) if she buys an option and then decides she does not want the ticket. Even more interestingly, by only switching to ticket options from multiperiod pricing while still allowing resale of tickets, the capacity provider may capture most of the total benefit that he could obtain from option pricing with non-transferrable tickets. For example, if $\lambda_1 = 150$, $\lambda_2 = 100$, $C = 120$, $V \sim U[10, 100]$, $\tau = 0.25$, $\tau' = 0.1$, by switching from multiperiod pricing to option pricing, the capacity provider improves his revenue from 6339 to 7166 (increased by 13%); by further making tickets non-transferrable, the capacity provider's revenue is increased to 7253 (increased by only 1.2% additionally). Thus, compared to making tickets non-transferrable which may result in significant consumer backlashes, the novel pricing strategy of option pricing may be a good choice for event capacity providers to consider.

7. Extensions

7.1. Strategic Capacity Rationing

In this section, we consider the case where the capacity provider can strategically hold back some of his capacity in period 1 to sell later in period 2. This, however, isn't common for events in practice. As we have stated before, consumers can get very upset if the capacity provider sells tickets later when he claimed tickets were sold out earlier. Although Ticketmaster explicitly claims on its website that it does not divert inventory designated by clients for primary sales into the resale market, the possibility that Ticketmaster does this still has worried consumers and there have been consumer complaints[14]. Nevertheless, it is of interest to understand if any of our main findings

[14] See http://www.consumeraffairs.com/entertainment/ticketmaster.htm.

regarding whether the capacity provider should prevent resale or not in the previous sections would change in this case.

We define the decision variable $0 < b \leq C$ as the provider's designated capacity to be sold in period 1. For any b, the previous equilibrium analysis for each pricing strategy still holds. Thus, to analyze the optimal pricing problem with strategic capacity rationing, we can write all optimal prices as functions of b and optimize on the b-dimension. For fixed pricing, we can easily show that the optimal revenue is increasing in b. As Theorem 1 indicates, the equilibrium resale price $r_f^*(b)$ is given by $[(\lambda_1 - b)^+ + \lambda_2]\bar{F}(r_f^*) = C - \min(\lambda_1, b) + \min(\lambda_1, b)F((1-\tau)r_f^*)$. Since $r_f^*(b)$ is increasing in b, p_f^* is also increasing in b. Thus, under fixed pricing, even if the capacity provider can hold back some capacity, it is optimal to sell as many tickets in period 1 as possible (i.e., not to ration any capacity). Thus, strategic capacity rationing does not improve the capacity provider's revenue and we have the same results in Section 4. The capacity provider still benefits when resale of tickets are easier for consumers as well as speculators, and selling non-transferrable tickets hurts his revenue.

Theorem 7. *If the consumer valuations are uniformly distributed over* $[v_{\min}, v_{\max}]$*, the optimal revenue from multiperiod pricing with strategic capacity rationing is increasing in* τ*.*

In Section 5, we showed that under multiperiod pricing, the capacity provider may still prefer consumers to have a zero resale transaction cost if his capacity is small. Interestingly, Theorem 7 states that this is no longer true when the provider can strategically ration capacity in period 1. With the additional flexibility from capacity rationing, we find that the optimal revenue from multiperiod pricing is always increasing in τ. Therefore, if an event capacity provider can ration capacity in period 1, he will never benefit from a resale market in period 2. In this case, the capacity provider achieves a higher revenue if the resale market is precluded (e.g., by the enforcement of non-transferrable tickets).

Finally, if the capacity provider sells ticket options, all our numerical results indicate that the optimal revenue is still increasing in τ with strategic capacity rationing. So the capacity provider still benefits when consumers have larger resale transaction costs, and he achieves the highest revenue by making tickets non-transferrable. Moreover, for any b, option pricing reduces to multiperiod pricing if the strike price is low enough (i.e., $p < (1-\tau)r_o(b)$), and the capacity provider can improve his revenue by choosing a high enough strike price that dominates the payoff from ticket resale so that consumer resale is prevented. Therefore, our previous insight that ticket options can help event capacity providers prevent consumers resale of tickets and increase revenues carries through to a capacity rationing provider. As we noted at the beginning, holding back capacity to sell later may cause significant consumer dissatisfaction and may be very hard to implement in practice. Thus, it is interesting to compare its benefit to other strategies (such as ticket options) that we have discussed. Consider the example given at the end of Section 6 ($\lambda_1 = 150$, $\lambda_2 = 100$, $C = 120$, $V \sim U[10, 100]$, $\tau = 0.25$, $\tau' = 0.1$). By strategically rationing capacity, the provider can improve his multiperiod pricing revenue from 6339 to 6762, whereas his revenue is increased to 7166 by switching to option pricing. Moreover, after switching to ticket options, the capacity provider does not further increase the revenue by rationing capacity, because it is indeed optimal for the capacity provider to sell as many options as possible in period 1 in this example. Therefore, compared to increasing revenues through rationing capacity, switching to option pricing may be a better way to increase revenues and avoid risking upsetting the fan base.

7.2. Heterogeneous Period 1 Consumers

Similar to other papers in the literature (e.g., Geng et al. 2007, Courty 2003b), our model considered a situation where all period 1 customers have ex ante symmetric valuations. In this section, we consider the case of two types of consumers in period 1 to explore whether the insights from our model are affected. In this case, we assume that among the λ_1 consumers who arrive in period 1, λ_{1H} consumers (the super fans) have higher ex ante valuations (V_H) than the rest λ_{1L} consumers (V_L), where V_H is stochastically larger than V_L. The λ_2 consumers who arrive in period 2 have ex ante valuations V_L. For each consumer type, all the equilibrium analysis in our model still holds. However, characterizing the optimal pricing policy becomes much more complicated, because in period 1 the capacity provider may want only one type or both types of consumers to buy tickets, resulting in a much more complex revenue function. Nevertheless, our numerical results indicate that the main insights regarding when resale markets are beneficial or harmful to the capacity provider do not seem to be affected. For example, suppose the capacity provider is using multiperiod pricing and the problem parameters are as follows: $\lambda_{1H} = 90$, $\lambda_{1L} = 60$, $\lambda_2 = 100$, $V_H \sim U[50, 100]$, $V_L \sim U[10, 80]$. If $C \leq 104$, the capacity provider achieves the highest revenue when $\tau = \tau' = 0$, that is, if the capacity is small enough, the capacity provider's most favorable scenario is when tickets can be resold with zero transaction cost. On the other hand, if $C > 104$, the capacity provider achieves the highest revenue when $\tau = \tau' = 1$, that is, if the capacity is large enough, the capacity provider benefits from making tickets non-transferrable. These observations are consistent with our results given by Theorem 4. Moreover, intuitively, the capacity provider would like to induce more consumer types to purchase tickets in period 1 when he has a larger capacity. In the above example, when $\tau = \tau' = 1$ which is the best scenario for the capacity provider for $C > 104$, the optimal multiperiod pricing policy induces only the high-valuation consumers to purchase tickets in period 1 if $C \leq 193$; if $C > 193$, the optimal multiperiod pricing policy induces both types of consumers to purchase tickets in period 1. Thus, as the numerical results clearly indicate, our main insights with respect to whether event capacity providers should prevent resale of tickets or not do not change significantly with more complex assumptions about the number of period 1 consumer types.

8. Conclusion

In this paper, we studied three pricing strategies, fixed pricing, multiperiod pricing, and option pricing, for an event capacity provider that faces resale of tickets. One major contribution of this paper is that we find how the behavior of optimal prices and revenues depend on the resale transaction costs incurred by the consumers and speculators, which indicates whether the capacity provider should prevent resale of tickets or not. We have found that contrary to what common wisdom suggests, event capacity providers do not always benefit from restricting resale.

By appropriately choosing the prices associated with ticket options (i.e., option price and strike price), an event capacity provider can eliminate consumer resale of tickets and significantly reduce the magnitude of the resale market. We conjecture that compared to enforcing paperless ticketing under multiperiod pricing, event capacity providers would have a much easier time convincing consumers to switch to buying options. Ticket options also benefit consumers, because if a consumer buys an option and cannot attend the event, she is risking only the option price instead of the whole ticket price. Furthermore, under multiperiod pricing, whether paperless ticketing is beneficial or not depends on the event's demand, which would imply that to obtain the highest benefit, the capacity provider would have to make some events' tickets paperless and allow ticket resale for other events. This is clearly impractical. While going to paperless ticketing with options would increase the capacity provider's revenues even more, our numerical results indicate that this additional revenue gain is small compared to switching to option pricing from multiperiod or fixed pricing.

Thus, our paper suggests that efforts to move to paperless ticketing are likely to hurt not only consumers but also event capacity providers in many cases. A reason given by Ticketmaster to introduce paperless ticketing is to prevent speculators from entering the market. However, our paper argues that speculators may actually be beneficial to event providers when they use fixed pricing. While speculators are indeed never beneficial to capacity providers under multiperiod pricing, the capacity provider may still lose revenues overall by introducing paperless ticketing. Moreover, we provide the insight that option pricing not only results in the highest revenues for event capacity providers but also has the highest likelihood of shutting down speculators, while giving consumers much greater choice than paperless ticketing. Thus, our research indicates that event organizers should not support paperless ticketing but instead consider novel pricing strategies such as ticket options.

Appendix. Proofs of Theorems and Lemmas

LEMMA A1. *Under fixed pricing, given that the capacity provider's price is p_f and that z consumers and y speculators have purchased tickets in period 1, the equilibrium resale price in period 2 is*

$$r_f(z,y)=\begin{cases}\bar{r}(z,y) & \text{if } p_f>\bar{r}(z,y),\\ p_f & \text{if } \underline{r}(z,y)<p_f\le\bar{r}(z,y),\\ \underline{r}(z,y) & \text{if } p_f\le\underline{r}(z,y),\end{cases}$$

where $\bar{r}(z,y)$ is the solution to $(\lambda_1-z+\lambda_2)\bar{F}(\bar{r})=zF((1-\tau)\bar{r})+y$ and $\underline{r}(z,y)$ is the solution to $(\lambda_1-z+\lambda_2)\bar{F}(\underline{r})=(C-z-y)+zF((1-\tau)\underline{r})+y$.

Proof of Lemma A1 In period 2, speculators resell tickets at the same price with consumers, because otherwise, the party with the lower price will raise it to gain more margin and if the party with the higher price cannot make sales, it will reduce the price to make sales. The provider has $C-z-y$ remaining capacity, and $\lambda_1-z+\lambda_2$ consumers arrive, including the period 1 consumers that were not satisfied or decided to wait. $(\lambda_1-z+\lambda_2)\bar{F}(p_f)$ is the number of consumers that are willing to buy the ticket if the ticket price is p_f. Also, $zF((1-\tau)r_f)$ is the number of period 1 consumers that would like to resell their tickets if the resale market price is r_f, because a period 1 consumer will want to resell her ticket if her valuation is smaller than the payoff from resale, $(1-\tau)r_f$.

If $p_f>\bar{r}(z,y)$, we have $(\lambda_1-z+\lambda_2)\bar{F}(p_f)<zF((1-\tau)p_f)+y$. In this case, the equilibrium resale price is $r_f(z,y)=\bar{r}(z,y)$ which is lower than the capacity provider's price p_f. All demand is satisfied by the resale market. If $\underline{r}(z,y)<p_f\le\bar{r}(z,y)$, we have $zF((1-\tau)p_f)+y\le(\lambda_1-z+\lambda_2)\bar{F}(p_f)<C-z+zF((1-\tau)p_f)$. In this case, resellers enter the resale market in the order of increasing valuations up to the one with valuation $(1-\tau)p_f$, because otherwise the resale price will be higher than p_f, hence the capacity provider will make sales first and the resellers with high valuations will not be able to make sales. Thus, the equilibrium resale price is $r_f(z,y)=p_f$ in this case. If $p_f\le\underline{r}(z,y)$, we have $(\lambda_1-z+\lambda_2)\bar{F}(p_f)\ge C-z+zF((1-\tau)p_f)$. In this case, the equilibrium resale price is $r_f(z,y)=\underline{r}(z,y)$ which is higher than or equal to the capacity provider's price p_f. The capacity provider sells tickets first and he sells out his capacity, the resale market captures the residual demand. □

Proof of Theorem 1 In the first part of the proof, we derive the period 1 consumers' purchasing decisions in equilibrium based on the equilibrium resale price given by Lemma A1. Then, in the second part of the proof, we derive the capacity provider's optimal fixed price p_f^*. If a period 1 consumer buys a ticket, her payoff in period 2 is the maximum of her payoff from using the ticket, V, and her payoff from reselling the ticket, $(1-\tau)r_f(z,y)$. Thus, her payoff from buying a ticket in period 1 is $S_f^1(z,y)=-p_f+E[\max(V,(1-\tau)r_f(z,y))]$. If a period 1 consumer waits, then she can obtain a ticket in period 2 only if her valuation is high enough. As Lemma A1 indicates, if $p_f>\bar{r}(z,y)$, she can buy a ticket from the resale market at price $\bar{r}(z,y)$ if $V>\bar{r}(z,y)$. If $\underline{r}(z,y)<p_f\le\bar{r}(z,y)$, she can buy a ticket from either the resale market or the capacity provider at price p_f if $V>p_f$. If $p_f\le\underline{r}(z,y)$, she can buy a ticket from the capacity provider at price p_f if $V>\tilde{r}(z,y)$ where $\tilde{r}(z,y)$ is the solution to $(\lambda_1-z+\lambda_2)\bar{F}(r)=C-z-y$, she can buy a ticket from the resale market at a higher price $\underline{r}(z,y)$ if $\underline{r}(z,y)<V\le\tilde{r}(z,y)$, and she may not obtain a ticket otherwise. Thus, a period 1 consumer's payoff from waiting is

$$S_f^2(z,y)=\begin{cases}E[V-\bar{r}(z,y)]^+ & \text{if } p_f>\bar{r}(z,y),\\ E(V-p_f)^+ & \text{if } \underline{r}(z,y)<p_f\le\bar{r}(z,y),\\ \int_{\tilde{r}(z,y)}^{\infty}(v-p_f)\,\mathrm{d}F(v)+\int_{\underline{r}(z,y)}^{\tilde{r}(z,y)}[v-\underline{r}(z,y)]\,\mathrm{d}F(v) & \text{if } p_f\le\underline{r}(z,y).\end{cases}$$

Note that the upper bound of z is $\bar{z} = \min(\lambda_1, C)$, as z cannot exceed the total number of period 1 consumers or the provider's capacity.

Next, we will show that any $z \in (0, \bar{z})$ cannot be an equilibrium. The Implicit Function Theorem gives that $\bar{r}(z,y)$ is decreasing in z, $\underline{r}(z,y)$ and $\tilde{r}(z,y)$ are increasing in z:

$$\begin{aligned}
\frac{\partial \bar{r}}{\partial z} &= -\frac{\bar{F}(\bar{r}) + F((1-\tau)\bar{r})}{(\lambda_1 - z + \lambda_2) f(\bar{r}) + (1-\tau) z f((1-\tau)\bar{r})} \leq 0, \\
\frac{\partial \underline{r}}{\partial z} &= \frac{\bar{F}((1-\tau)\underline{r}) - \bar{F}(\underline{r})}{(\lambda_1 - z + \lambda_2) f(\underline{r}) + (1-\tau) z f((1-\tau)\underline{r})} \geq 0, \\
\frac{\partial \tilde{r}}{\partial z} &= \frac{F(\tilde{r})}{(\lambda_1 - z + \lambda_2) f(\tilde{r})} > 0.
\end{aligned}$$

First, consider the case of $p_f > \underline{r}(z,y)$. $y^* = 0$ in this case because speculators will incur a loss if entering the market. If $p_f > \bar{r}(z,0)$, $S_f^1(z,0) - S_f^2(z,0) = -p_f + E[\max(V,(1-\tau)\bar{r}(z,0))] - E[V - \bar{r}(z,0)]^+ < -\bar{r}(z,0) + E[\max(V,(1-\tau)\bar{r}(z,0))] - E[V - \bar{r}(z,0)]^+ = E[\max(V,(1-\tau)\bar{r}(z,0))] - E[\max(V,\bar{r}(z,0))] < 0$. Also, $S_f^1(z,0)$ is decreasing in z and $S_f^2(z,0)$ is increasing in z. Thus, we have $\sup_{0\leq z\leq \bar{z}}(S_f^1(z,0)) < \inf_{0\leq z\leq \bar{z}}(S_f^2(z,0))$, hence $z^* = 0$. Similarly, if $\underline{r}(z,y) < p_f \leq \bar{r}(z,0)$, $S_f^1(z,0)$ and $S_f^2(z,0)$ stay constant with respect to z and $S_f^1(z,0) - S_f^2(z,0) = -p_f + E[\max(V,(1-\tau)p_f)] - E(V - p_f)^+ < 0$, hence we also have $z^* = 0$. Second, consider the case of $p_f \leq \underline{r}(z,y)$. Note that in this case, the equilibrium resale price is $\underline{r}(z,y)$ which is independent of y, hence $S_f^1(z,y)$ and $S_f^2(z,y)$ are also independent of y. We have $y^*(z) = C - z$ if $p_f < (1-\tau')\underline{r}(z,y)$ and $y^*(z) = 0$ otherwise. $S_f^1(z,y)$ is increasing in z and since

$$\frac{\partial S_f^2}{\partial z} = \frac{\partial \tilde{r}}{\partial z}[p_f - \underline{r}(z,y)] f(\tilde{r}(z,y)) - \frac{\partial \underline{r}}{\partial z}[F(\tilde{r}(z,y)) - F(\underline{r}(z,y))] \leq 0,$$

$S_f^2(z,y)$ is decreasing in z. Thus, if a small portion of period 1 consumers who are currently waiting deviate to buying tickets, more such deviations will occur; and vice versa. Therefore, $z^* = \bar{z}$ and $z^* = 0$ are the only possible equilibria. To induce $z^* = \bar{z}$, p_f needs to satisfy[15] $p_f \leq \underline{r}(\bar{z}, y^*(\bar{z}))$ and $S_f^1(\bar{z}, y^*(\bar{z})) \geq S_f^2(0, y^*(0))$. The equilibrium resale price is $\underline{r}(\bar{z}, y^*(\bar{z})) = r_f^*$, hence Part (i) of the theorem is proved. Additionally, since the equilibrium resides in the case of $p_f \leq \underline{r}(z,y)$, the proof of Lemma A1 indicates that the provider sells out his capacity, hence his revenue is $p_f C$.

Now we derive p_f^*. Note that $\underline{r}(0, y^*(0)) = p_s$ and $\underline{r}(\bar{z}, y^*(\bar{z})) = r_f^*$. For $p_s < p_f \leq r_f^*$, $S_f^2(0, y^*(0)) = E(V - p_f)^+$. In this case, $S_f^1(\bar{z}, y^*(\bar{z})) \geq S_f^2(0, y^*(0))$ becomes $-p_f + E[\max(V,(1-\tau)r_f^*)] \geq E(V - p_f)^+$, or equivalently, $E[\max(V,(1-\tau)r_f^*)] \geq E[\max(V,p_f)]$, which can be simplified to $p_f \leq \max((1-\tau)r_f^*, v_{\min})$. Consider $[(\lambda_1 - C)^+ + \lambda_2]\bar{F}(r_f^*) = (C - \lambda_1)^+ + \min(\lambda_1, C) F((1-\tau)r_f^*)$ which defines r_f^*. As τ increases, the rhs decreases, so we need to increase r_f^* to maintain equality. Both the lhs and the rhs become smaller when the equality is reached again. Thus, as τ increases, r_f^* increases and $(1-\tau)r_f^*$ decreases. When $\tau = 0$, $(1-\tau)r_f^* = p_s$, hence $p_s > (1-\tau)r_f^*$ for any $\tau > 0$. Since $p_s > v_{\min}$, we have $p_f > p_s > \max((1-\tau)r_f^*, v_{\min})$ which contradicts $S_f^1(\bar{z}, y^*(\bar{z})) > S_f^2(0, y^*(0))$. Therefore, $p_s < p_f \leq r_f^*$ is not feasible.

For $p_f \leq p_s$, $y^*(0) = C$ if $p_f < (1-\tau')p_s$ and $y^*(0) = 0$ otherwise; $y^*(\bar{z}) = (C - \lambda_1)^+$ if $p_f < (1-\tau')r_f^*$ and $y^*(\bar{z}) = 0$ otherwise. Since $p_s < r_f^*$, when $y^*(\bar{z}) = 0$, we must also have $y^*(0) = 0$. In this case, $S_f^2(0, y^*(0)) =$

[15] We assume when period 1 consumers are indifferent between buying tickets and waiting, they buy immediately. The capacity provider can resolve the consumer indifference by reducing p_f by an infinitesimally small amount.

$\int_{p_s}^{\infty}(v-p_f)\,\mathrm{d}F(v)$, and we can rewrite $S_f^1(\bar{z},y^*(\bar{z}))\geq S_f^2(0,y^*(0))$ as $p_f+\int_{p_s}^{\infty}(v-p_f)\,\mathrm{d}F(v)\leq E[\max(V,(1-\tau)r_f^*)]$, the binding solution of which is p_f^n (the superscript of "n" refers to no speculators in equilibrium). Since the lhs of the above inequality is increasing in p_f, $p_f^*=p_f^n$ in this case. $y^*(\bar{z})=0$ indeed occurs if $p_f^n\geq(1-\tau')r_f^*$ or $\tau'\geq\bar{\tau}_f'(\tau)$. Thus, $y^*(\bar{z})=0$ if $\tau'\geq\bar{\tau}_f'(\tau)$ and $y^*(\bar{z})=(C-\lambda_1)^+$ otherwise, Part (iii) of the theorem is proved. Finally, we consider the case of $\tau'<\bar{\tau}_f'(\tau)$ where $y^*(\bar{z})=(C-\lambda_1)^+$. If $p_f^n\geq(1-\tau')p_s$, $y^*(0)=0$ hence we still have $p_f^*=p_f^n$ in this case. If $p_f^n<(1-\tau')p_s$, $y^*(0)=C$ hence we have $S_f^2(0,y^*(0))=E(V-p_s)^+$ and $S_f^1(\bar{z},y^*(\bar{z}))\geq S_f^2(0,y^*(0))$ becomes $p_f\leq E[\max(V,(1-\tau)r_f^*)]-E(V-p_s)^+$. Thus, we have $p_f^*=\min\big(E[\max(V,(1-\tau)r_f^*)]-E(V-p_s)^+,(1-\tau')p_s\big)$ when $\tau'<\bar{\tau}_f'(\tau)$. Combining all the cases above, we obtain the optimal fixed price given in Part (ii) of the theorem. Finally, we have shown that r_f^* is increasing in τ and we prove in Theorem 2 that p_f^* is decreasing in τ. Then, since $p_f^*=r_f^*=p_s$ when $\tau=0$, it follows that $p_f^*<r_f^*$. The proof is completed. □

Proof of Theorem 2 We showed in the proof of Theorem 1 that $(1-\tau)r_f^*$ is decreasing in τ. Recall that p_f^n is the solution to $p_f^n+\int_{p_s}^{\infty}(v-p_f^n)\,\mathrm{d}F(v)=E[\max(V,(1-\tau)r_f^*)]$. Since the lhs is increasing in p_f^n, p_f^n is decreasing in τ. Thus, p_f^* is decreasing in τ. p_f^* is decreasing in τ' because p_f^* is decreasing in τ' when $p_f^*=(1-\tau')p_s$ and stays constant in τ' otherwise. The remaining results follow directly. □

Lemma A2. *Under multiperiod pricing, given that z consumers and y speculators have purchased tickets in period 1, in equilibrium, the capacity provider's period 2 price $p_2(z,y)$ and the resale price $r_m(z,y)$ are given by $p_2(z,y)=r_m(z,y)=\underline{r}(z,y)$.*

Proof of Lemma A2 $p_2(z,y)=r_m(z,y)$ in equilibrium because otherwise, the party with the lower price will raise it to gain more margin, and if the party with the higher price cannot make sales, it will reduce the price to make sales. Moreover, the equilibrium prices are equal to $\underline{r}(z,y)$ which is the marketing clearing price in period 2 where the total supply comes from both the capacity provider and the resale market. If the prices are lower than $\underline{r}(z,y)$, both parties have the incentive to increase the price and earn more revenue. If the prices are higher than $\underline{r}(z,y)$ so that the provider has un-sold capacity, since he does not ration capacity in period 2, he will decrease $p_2(z,y)$ to sell more tickets. In this case, more resellers will enter the market and $r_m(z,y)$ is decreased to $p_2(z,y)$ until the market is cleared. □

Proof of Theorem 3 We follow the same approach of deriving the optimal pricing policy under fixed pricing. Period 1 consumers' payoffs from purchasing tickets in period 1 and waiting under multiperiod pricing are $S_m^1(z,y)=-p_1+E[\max(V,(1-\tau)\underline{r}(z,y))]$ and $S_m^2(z,y)=E[V-\underline{r}(z,y)]^+$, respectively. $\underline{r}(z,y)$ is increasing in z as derived in the proof of Theorem 1. Thus, $S_m^1(z,y)$ is increasing in z while $S_m^2(z,y)$ is decreasing in z, hence the only possible equilibria are $z^*=\bar{z}$ and $z^*=0$. To induce $z^*=\bar{z}$, p_1 needs to satisfy $S_m^1(\bar{z},y^*(\bar{z}))\geq S_m^2(0,y^*(0))$ or $p_1\leq E[\max(V,(1-\tau)\underline{r}(\bar{z},y^*(\bar{z})))]-E[V-\underline{r}(0,y^*(0))]^+=E[\max(V,(1-\tau)p_2^*)]-E(V-p_s)^+$ (note that $\underline{r}(z,y)$ is independent of y). Part (i) of the theorem follows from Lemma A2. Moreover, $y^*(\bar{z})=(C-\lambda_1)^+$ if $p_1<(1-\tau')p_2^*$ and $y^*(\bar{z})=0$ otherwise. As the provider sells out the capacity with $z^*=\bar{z}$, his revenue is $\Pi_m(p_1)=p_1C$ if $p_1<(1-\tau')p_2^*$ and $\Pi_m(p_1)=p_1\min(\lambda_1,C)+p_2^*(C-\lambda_1)^+$ otherwise. Since $\Pi_m(p_1)$ is increasing in p_1, $p_1^*=E[\max(V,(1-\tau)p_2^*]-E(V-p_s)^+$. In the proof of Theorem 1, we showed that p_2^* is increasing in τ and $(1-\tau)p_2^*$ is decreasing in τ, hence p_1^* is decreasing in τ. Then, since $p_1^*=p_2^*=p_s$ when $\tau=0$, we have $p_1^*<p_2^*$. Part (ii) of the theorem is proved. Finally, Part (iii) holds because $p_1^*<(1-\tau')p_2^*$ is equivalent to $\tau'<\bar{\tau}_m'(\tau)$. □

Proof of Theorem 4 Part (i) is proved in the proof of Theorem 3. When $\tau > \bar{\tau}_m(\tau')$, the optimal revenue from multiperiod pricing is $\Pi_m^* = p_1^* C$, hence it is decreasing in τ. Part (ii) is proved.

Now we prove Part (iii). As $\tau \le \bar{\tau}_m(\tau')$, we have $y^*(\bar{z}) = 0$. If $C \le \lambda_1$, $\Pi_m^* = p_1^* C$ which is decreasing in τ. Next, consider $\lambda_1 < C < \lambda_1 + \lambda_2$. For $\tau \ge \hat{\tau}(C)$, $(1-\tau)p_2^* \le v_{\min}$ and p_2^* is the solution to $\lambda_2 \bar{F}(p_2^*) = C - \lambda_1$. In this case, p_2^* is independent of τ and so is $\Pi_m^* = E[\min(V, p_s)]\lambda_1 + p_2^*(C - \lambda_1)$. For $\tau < \hat{\tau}(C)$, $(1-\tau)p_2^* > v_{\min}$ and $\Pi_m^* = \{E[\max(V, (1-\tau)p_2^*)] - E(V - p_s)^+\}\lambda_1 + p_2^*(C - \lambda_1)$, where p_2^* is the solution to $\lambda_2 \bar{F}(p_2^*) = C - \lambda_1 \bar{F}((1-\tau)p_2^*)$. Taking derivative with respect to τ on both sides of this equation yields

$$\frac{\mathrm{d}[(1-\tau)p_2^*]}{\mathrm{d}\tau} = -\frac{\lambda_2 f(p_2^*)}{\lambda_1 f((1-\tau)p_2^*)} \frac{\mathrm{d}p_2^*}{\mathrm{d}\tau}.$$

Thus

$$\frac{\mathrm{d}\Pi_m^*}{\mathrm{d}\tau} = \lambda_1 F((1-\tau)p_2^*) \frac{\mathrm{d}[(1-\tau)p_2^*]}{\mathrm{d}\tau} + (C - \lambda_1)\frac{\mathrm{d}p_2^*}{\mathrm{d}\tau} = \frac{\mathrm{d}p_2^*}{\mathrm{d}\tau}\left[C - \lambda_1 - \frac{\lambda_2 F((1-\tau)p_2^*) f(p_2^*)}{f((1-\tau)p_2^*)}\right]. \tag{A1}$$

Since $f(\cdot)$ is decreasing, as τ increases, $f(p_2^*)$ decreases, $F((1-\tau)p_2^*)$ decreases and $f((1-\tau)p_2^*)$ increases, hence the terms within the bracket in (A1) are increasing in τ. Then, since $\mathrm{d}p_2^*/\mathrm{d}\tau > 0$, Π_m^* is quasi-convex in τ for $\tau \le \bar{\tau}_m(\tau')$. When $\tau = 0$, the terms in the bracket become $C - \lambda_1 - \lambda_2 F(p_s)$, as $p_2^* = p_s$ when $\tau = 0$. We consider $C - \lambda_1 - \lambda_2 F(p_s)$ as a function of C. Since p_s is decreasing in C, $C - \lambda_1 - \lambda_2 F(p_s)$ is increasing in C. If $C = \lambda_1$, $C - \lambda_1 - \lambda_2 F(p_s) = -\lambda_2 F(p_s) < 0$; if $C = \lambda_1 + \lambda_2$, $p_s = v_{\min}$ and $C - \lambda_1 - \lambda_2 F(p_s) = \lambda_2 > 0$. Thus, there exists a threshold $\bar{C}$ ($\lambda_1 < \bar{C} < \lambda_1 + \lambda_2$) such that, Π_m^* is decreasing in τ at $\tau = 0$ if $\lambda_1 < C < \bar{C}$ and Π_m^* is increasing in τ at $\tau = 0$ if $\bar{C} \le C < \lambda_1 + \lambda_2$. Thus, due to quasi-convexity, we conclude that for $\tau \le \bar{\tau}_m(\tau')$, Π_m^* is decreasing in τ if $C \le \lambda_1$ and increasing in τ if $C \ge \bar{C}$; otherwise, Π_m^* may be decreasing or first decreasing then increasing in τ. The result in Part (iii) of the theorem regarding which τ gives the highest revenue follows from the monotonicity results we obtained above as well as Part (ii). The τ' that maximizes the revenue is $\tau' = \tau$ because we already know that the revenue decreases when speculators enter the market. □

LEMMA A3. *Under option pricing, given that the strike price is p and that z consumers and y speculators have purchased options in period 1, in equilibrium, the capacity provider's period 2 price $p_o(z,y)$ and the resale price $r_o(z,y)$ are given by $p_o(z,y) = r_o(z,y) = \inf\{r \ge v_{\min} : (\lambda_1 - z + \lambda_2)\bar{F}(r) \le C - z\bar{F}(\max((1-\tau)r, p))\}$.*

Proof of Lemma A3 Define $\bar{V} = \max(V, (1-\tau)r_o)$ and $\bar{F}_{\bar{V}}(t) = P(\bar{V} > t)$. In period 2, $z\bar{F}_{\bar{V}}(p)$ consumers exercise the options, $z\bar{F}_{\bar{V}}(p)P(V \le (1-\tau)r_o | \bar{V} > p)$ consumers resell the tickets, $y\mathbf{1}_{p<(1-\tau)r_o}$ speculators exercise the options then resell the tickets. Thus, the provider's remaining capacity to sell in period 2 is $C - z\bar{F}_{\bar{V}}(p) - y\mathbf{1}_{p<(1-\tau)r_o}$. Following the proof of Lemma A2, we have $p_o(z,y) = r_o(z,y) = \inf\{r \ge v_{\min} : (\lambda_1 - z + \lambda_2)\bar{F}(r) \le [C - z\bar{F}_{\bar{V}}(p) - y\mathbf{1}_{p<(1-\tau)r_o}] + z\bar{F}_{\bar{V}}(p)P(V \le (1-\tau)r | \bar{V} > p)\} + y\mathbf{1}_{p<(1-\tau)r_o}$. For any r, $[C - z\bar{F}_{\bar{V}}(p) - y\mathbf{1}_{p<(1-\tau)r_o}] + z\bar{F}_{\bar{V}}(p)P(V \le (1-\tau)r | \bar{V} > p) + y\mathbf{1}_{p<(1-\tau)r_o} = C - z\bar{F}_{\bar{V}}(p)P(V > (1-\tau)r | \bar{V} > p) = C - zP(V > (1-\tau)r, \bar{V} > p) = C - z\bar{F}(\max((1-\tau)r, p))$. □

Proof of Theorem 5 To improve readability, we divide this long proof into four steps. In Step 1, we derive period 1 consumers' purchasing decisions in equilibrium. In Step 2, we show that consumers do not resell tickets in equilibrium. In Step 3, we derive the optimal strike price p^*. In Step 4, we show how the optimal prices and revenue change with respect to τ.

Step 1: Period 1 consumers' payoffs from buying options in period 1 and waiting are $S_o^1(z,y) = -x + E[\max(V,(1-\tau)p_o(z,y)) - p]^+$ and $S_o^2(z,y) = E[V - p_o(y,z)]^+$, respectively, where $p_o(z,y)$ is independent of y. Note that $p < (1-\tau)p_o(z,y)$ is equivalent to $p < (1-\tau)\underline{r}(z,y)$. If $p < (1-\tau)\underline{r}(z,y)$, as shown in the main text, option pricing is equivalent to multiperiod pricing, so the proof of Theorem 3 implies that the only possible equilibria are $z^* = \bar{z}$ or $z^* = 0$. If $p \geq (1-\tau)\underline{r}(z,y)$, $S_o^1(z,y) = -x + E(V-p)^+$ and $p_o(z,y) = \inf\{r \geq v_{\min} : (\lambda_1 - z + \lambda_2)\bar{F}(r) \leq C - z\bar{F}(p)\}$. We have several subcases to discuss for $p \geq (1-\tau)\underline{r}(z,y)$. Define $\hat{p}(z) = \inf\{p \geq v_{\min} : \lambda_1 + \lambda_2 \leq C + zF(p)\}$. If $p > \hat{p}(z)$, $p_o(z,y) = v_{\min}$ and $S_o^1(z,y) < S_o^2(z,y)$, hence $z^* = 0$. If $p \leq \hat{p}(z)$, the provider sells out his remaining capacity in period 2. In this case, $p > p_o(z,y)$ if and only if $p > p_s$. If $p > p_s$, we still have $S_o^1(z,y) < S_o^2(z,y)$, hence $z^* = 0$. Otherwise, $p_o(z,y)$ is increasing in z, so $S_o^2(z,y)$ is decreasing in z. Thus, if a small portion of period 1 consumers who are currently buying options deviate to waiting, more such deviations will occur, then we know $z^* = \bar{z}$ and $z^* = 0$ are the only possible equilibria. Combining all the cases discussed above, we conclude that for any p, the only possible equilibria are $z^* = \bar{z}$ and $z^* = 0$. $z^* = 0$ is always a possible equilibrium. $z^* = \bar{z}$ is a possible equilibrium only if $p \leq \min(\hat{p}(\bar{z}), p_s) = p_s$ as we can easily prove $\hat{p}(\bar{z}) > p_s$; in this case, the provider's capacity is sold out.

To induce $z^* = \bar{z}$, x and p need to satisfy $p \leq p_s$ as well as $S_o^1(\bar{z}, y^*(\bar{z})) \geq S_o^2(0, y^*(0))$ or $-x + E[\max(V,(1-\tau)p_o(\bar{z}, y^*(\bar{z}))) - p]^+ \geq E(V-p_s)^+$. The capacity provider's revenue is $\Pi_o(x,p) = x[\bar{z} + y^*(\bar{z})] + p[\bar{z}\bar{F}_V(p) + y^*(\bar{z})] + p_o(\bar{z}, y^*(\bar{z}))[C - \bar{z}\bar{F}_V(p) - y^*(\bar{z})]$, where $y^*(\bar{z}) = (C-\lambda_1)^+$ if $x + p < (1-\tau')p_o(\bar{z}, y^*(\bar{z}))$ and $y^*(\bar{z}) = 0$ otherwise. Since $\Pi_o(x,p)$ is increasing in x, $x^*(p) = E[\max(V,(1-\tau)p_o(\bar{z}, y^*(\bar{z}))) - p]^+ - E(V-p_s)^+$. Note that $p \leq p_s$ ensures $x \geq 0$. We focus on $\Pi_o(p) = \Pi_o(x^*(p), p)$ from now on.

Step 2: Next, we show that in equilibrium, we must have $p^* \geq (1-\tau)p_o(\bar{z}, y^*(\bar{z}))$, or equivalently, $p^* \geq (1-\tau)p_2^*$, so that consumers do not resell tickets. When $p < (1-\tau)p_2^*$, $\Pi_o(p) = \{E[\max(V,(1-\tau)p_2^*)] - E(V-p_s)^+\}[\bar{z} + y^*(\bar{z})] + p_2^*[C - \bar{z} - y^*(\bar{z})]$. Since $x^*(p) + p$ is increasing in p and $p_o(\bar{z}, y^*(\bar{z})) = p_2^*$ is independent of p in this case, as we increase p, $y^*(\bar{z})$ may shift from $(C-\lambda_1)^+$ to 0. When this occurs, $\Pi_o(p)$ becomes larger because $x^*(p) + p = E[\max(V,(1-\tau)p_2^*)] - E(V-p_s)^+ < E[\max(V, p_2^*)] - E(V-p_2^*)^+ = p_2^*$. Other than when $y^*(\bar{z})$ shifts from $(C-\lambda_1)^+$ to 0, $\Pi_o(p)$ is constant in p. Thus, $\Pi_o(p)$ is increasing in p for $p < (1-\tau)p_2^*$. On the other hand, when $p \geq (1-\tau)p_2^*$, $\Pi_o(p) = [E(V-p)^+ - E(V-p_s)^+][\bar{z} + y^*(\bar{z})] + p[\bar{z}\bar{F}(p) + y^*(\bar{z})] + p_o(\bar{z}, y^*(\bar{z}))[C - \bar{z}\bar{F}(p) - y^*(\bar{z})]$. Then, we have

$$\lim_{p \downarrow (1-\tau)p_2^*} \Pi_o(p) - \lim_{p \uparrow (1-\tau)p_2^*} \Pi_o(p) = \tau p_2^* \bar{z} F((1-\tau)p_2^*) \geq 0.$$

Thus, $p \geq (1-\tau)p_2^*$ results in a higher revenue than $p < (1-\tau)p_2^*$, hence the optimal strike price satisfies $p^* \geq (1-\tau)p_o(\bar{z}, y^*(\bar{z}))$, in which case $x^* = E(V-p^*)^+ - E(V-p_s)^+$ and $p_o(\bar{z}, y^*(\bar{z}))$ is indeed given by Part (ii) of the theorem. The characterization of p_o^* and r_o^* then follows from Lemma A3.

Step 3: Now we derive p^*. We first derive p^n which is the optimal strike price in the absence of speculators. If $p \leq v_{\min}$, p_o is independent of p, and so is $\Pi_o(p) = E[\min(V, p_s)]\min(\lambda_1, C) + p_o(C-\lambda_1)^+$. We then restrict p^n to be no less than $v_{\min}$, hence the feasible region of p becomes $\max((1-\tau)p_2^*, v_{\min}) \leq p \leq p_s$ and we have $\Pi_o(p) = [E(V-p)^+ - E(V-p_s)^+ + p\bar{F}(p)]\min(\lambda_1, C) + p_o(p)[C - \min(\lambda_1, C)\bar{F}(p)]$. When $p_o(p) = v_{\min}$ which occurs for larger enough p, it is easy to see that $\Pi_o(p)$ is decreasing in p, hence this case does not result in the optimal solution. We then know that at optimality, p_o is the solution to $[(\lambda_1-C)^+ + \lambda_2]\bar{F}(p_o) = C - \min(\lambda_1, C)\bar{F}(p)$. The Implicit Function Theorem gives

$$\frac{\mathrm{d}p_o}{\mathrm{d}p} = -\frac{\min(\lambda_1, C)f(p)}{[(\lambda_1-C)^+ + \lambda_2]f(p_o)}.$$

Taking derivative of $\Pi_o(p)$ with respect to p gives

$$\frac{\mathrm{d}\Pi_o}{\mathrm{d}p} = \min(\lambda_1, C)f(p)(p_o - p) - \frac{\min(\lambda_1, C)f(p)[C - \min(\lambda_1, C)\bar{F}(p)]}{[(\lambda_1-C)^+ + \lambda_2]f(p_o)} = \min(\lambda_1, C)f(p)\left[p_o - p - \frac{\bar{F}(p_o)}{f(p_o)}\right].$$

Note that the second equality follows from $[(\lambda_1-C)^+ + \lambda_2]\bar{F}(p_o) = C - \min(\lambda_1, C)\bar{F}(p)$. Since p_o is decreasing in p and $F(\cdot)$ has an increasing failure rate, $p_o - p - \bar{F}(p_o)/f(p_o)$ is decreasing in p, hence $\Pi_o(p)$ is quasi-concave. Then, if $\Pi_o(p)$ is decreasing at $p = \max((1-\tau)p_2^*, v_{\min})$, we have $p^n = \max((1-\tau)p_2^*, v_{\min})$; otherwise, we have $p^n > \max((1-\tau)p_2^*, v_{\min})$.

We need to determine the sign of $p_o - p - \bar{F}(p_o)/f(p_o)$ at $p = (1-\tau)p_2^*$. At $p = (1-\tau)p_2^*$, $p_o = p_2^*$ and $p_o - p - \bar{F}(p_o)/f(p_o) = \tau p_2^* - \bar{F}(p_2^*)/f(p_2^*)$. Consider $\tau p_2^* - \bar{F}(p_2^*)/f(p_2^*)$ as a function of τ. Since p_2^* is increasing in τ, $\tau p_2^* - \bar{F}(p_2^*)/f(p_2^*)$ is increasing in τ. When $\tau = 0$, $\tau p_2^* - \bar{F}(p_2^*)/f(p_2^*) = -\bar{F}(p_2^*)/f(p_2^*) < 0$. When $\tau = 1$, $\tau p_2^* - \bar{F}(p_2^*)/f(p_2^*) = p_2^* - \bar{F}(p_2^*)/f(p_2^*) > 0$ if $C \le \lambda_1$ because $p_2^* = \infty$ if $C \le \lambda_1$. However, if $\lambda_1 < C < \lambda_1 + \lambda_2$, p_2^* is given by $\bar{F}(p_2^*) = (C - \lambda_1)/\lambda_2$, hence p_2^* is finite. Then it may occur that $p_2^* - \bar{F}(p_2^*)/f(p_2^*) \le 0$. Define $\hat{r}$ as the solution to $\hat{r}f(\hat{r}) = \bar{F}(\hat{r})$ and define $\hat{C} \in (\lambda_1, \lambda_1 + \lambda_2)$ as the solution to $\bar{F}(\hat{r}) = (C - \lambda_1)/\lambda_2$. Then, when $\tau = 1$, if $\lambda_1 < C < \hat{C}$, we have $p_2^* > \hat{r}$ and $p_2^* - \bar{F}(p_2^*)/f(p_2^*) > 0$ which also occurs if $C \le \lambda_1$; if $\hat{C} \le C < \lambda_1 + \lambda_2$, we have $p_2^* \le \hat{r}$ and $p_2^* - \bar{F}(p_2^*)/f(p_2^*) \le 0$. Thus, if $\hat{C} \le C < \lambda_1 + \lambda_2$, $p_o - p - \bar{F}(p_o)/f(p_o) \le 0$ for any τ. If $C < \hat{C}$, define $\tilde{\tau}(C) \in (0,1)$ as the solution to $\tau p_2^* = \bar{F}(p_2^*)/f(p_2^*)$. Then $p_o - p - \bar{F}(p_o)/f(p_o) \le 0$ for $\tau \le \tilde{\tau}(C)$ and $p_o - p - \bar{F}(p_o)/f(p_o) > 0$ for $\tau > \tilde{\tau}(C)$.

The above analysis implies that if $\hat{C} \le C < \lambda_1 + \lambda_2$ or if $C < \hat{C}$ and $\tau < \tilde{\tau}(C)$, $\mathrm{d}\Pi_o/\mathrm{d}p < 0$ for all $p > (1-\tau)p_2^*$. However, whether $p^n = (1-\tau)p_2^*$ or not depends on whether $(1-\tau)p_2^* \ge v_{\min}$ or not, so we need to determine whether $\tilde{\tau}(C)$ or $\hat{\tau}(C)$ is larger. We evaluate the sign of $\hat{\tau}(C)p_2^* - \bar{F}(p_2^*)/f(p_2^*)$ by varying C. If $C \ge \hat{C}$, $p_2^* \le \bar{F}(p_2^*)/f(p_2^*)$ hence $\hat{\tau}(C)p_2^* < \bar{F}(p_2^*)/f(p_2^*)$, so $\hat{\tau}(C) < \tilde{\tau}(C)$. If $C \le \lambda_1$, $p_2^* = \infty$ hence $\hat{\tau}(C)p_2^* > \bar{F}(p_2^*)/f(p_2^*)$, so $\hat{\tau}(C) > \tilde{\tau}(C)$. Moreover, as C increases, p_2^* decreases and $\hat{\tau}(C)$ decreases, hence $\hat{\tau}(C)p_2^* - \bar{F}(p_2^*)/f(p_2^*)$ decreases. Thus, there exists a threshold $\tilde{C} \in (\lambda_1, \hat{C})$ such that $\hat{\tau}(C) > \tilde{\tau}(C)$ if $C < \tilde{C}$ and $\hat{\tau}(C) \le \tilde{\tau}(C)$ if $\tilde{C} \le C < \lambda_1 + \lambda_2$.

Based on the above results, we can characterize p^n for different levels of C and τ. If $C < \tilde{C}$, $p^n = (1-\tau)p_2^*$ when $\tau \le \tilde{\tau}(C)$. When $\tau > \tilde{\tau}(C)$, $\Pi_o(p)$ first increases then decreases in p for $p > (1-\tau)p_2^*$. Thus, $p^n > (1-\tau)p_2^*$ and p^n is the solution to the first-order condition, $p_o - p^n - \bar{F}(p_o)/f(p_o) = 0$. Note that in this case, p^n is independent of τ; also, p^n is indeed feasible (i.e., $p^n \le p_s$), because when $\tau = \tilde{\tau}(C)$, $p^n = [1 - \tilde{\tau}(C)]p_2^* < p_s$. On the other hand, if $\tilde{C} \le C < \lambda_1 + \lambda_2$, since $\hat{\tau}(C) \le \tilde{\tau}(C)$, when $\tau \le \hat{\tau}(C)$, we have $p^n = (1-\tau)p_2^*$. When $\hat{\tau}(C) < \tau \le \tilde{\tau}(C)$, since $(1-\tau)p_2^* < v_{\min}$, we have $p^n = v_{\min}$. When $\tau > \tilde{\tau}(C)$, p^n remains constant at $v_{\min}$. Therefore, if $\tilde{C} \le C < \lambda_1 + \lambda_2$, $p^n = \max((1-\tau)p_2^*, v_{\min})$.

Now that we have found p^n, we proceed to characterize p^* by incorporating the case where speculators enter the market in equilibrium. Consider p_o as a function of p. If $x^*(p^n) + p^n = E[\max(V, p^n)] - E(V - p_s)^+ \ge (1-\tau')p_o(p^n)$ or $\tau' \ge 1 - \{E[\max(V, p^n)] - E(V - p_s)^+\}/p_o(p^n)$, we have $y^*(\bar{z}) = 0$ and $p^* = p^n$. If

$\tau' < 1 - \{E[\max(V, p^n)] - E(V - p_s)^+\}/p_o(p^n)$, define $\bar{p}$ as the solution to $E[\max(V, p)] - E(V - p_s)^+ = (1 - \tau')p_o(p)$. Since $E[\max(V, p)] - E(V - p_s)^+$ is increasing in p while $(1-\tau')p_o(p)$ is decreasing in p, $\bar{p} > p^n$. For $(1-\tau)p_2^* \le p < \bar{p}$, we have $y^*(\bar{z}) = (C - \lambda_1)^+$ and $\Pi_o(p) = [E(V-p)^+ - E(V-p_s)^+]C + p[\min(\lambda_1, C)\bar{F}(p) + (C-\lambda_1)^+] + p_o(p)\min(\lambda_1, C)F(p)$. Denote $p^s = \arg\max_{(1-\tau)p_2^* \le p < \bar{p}}(\Pi_o(p))$ as the optimal strike price when speculators exist in equilibrium. For $p \ge \bar{p}$, we have $y^*(\bar{z}) = 0$. Since $\Pi_o(p)$ is decreasing in p for $p \ge \bar{p}$, $\arg\max_{p \ge \bar{p}}(\Pi_o(p)) = \bar{p}$. Now we need to compare $\Pi_o(p^s)$ and $\Pi_o(\bar{p})$ to determine p^*. Since $\bar{p}$ is decreasing in τ', $\Pi_o(\bar{p})$ is increasing in τ' while $\Pi_o(p^s)$ is decreasing in τ'. Thus, there exists a threshold $\bar{\tau}_o'(\tau) \le 1 - \{E[\max(V, p^n)] - E(V-p_s)^+\}/p_o(p^n)$ such that $p^* = \bar{p}$ and $y^*(\bar{z}) = 0$ if $\bar{\tau}_o'(\tau) \le \tau' < 1 - \{E[\max(V, p^n)] - E(V-p_s)^+\}/p_o(p^n)$, $p^* = p^s$ and $y^*(\bar{z}) = (C-\lambda_1)^+$ if $\tau' < \bar{\tau}_o'(\tau)$. Part (iii) of the theorem is proved.

By now, we have fully characterized the optimal prices: 1) $p^* = p^n$ if $\tau' \ge 1 - \{E[\max(V, p^n)] - E(V - p_s)^+\}/p_o(p^n)$, $p^* = \bar{p}$ if $\bar{\tau}_o'(\tau) \le \tau' < 1 - \{E[\max(V, p^n)] - E(V-p_s)^+\}/p_o(p^n)$, $p^* = p^s$ if $\tau' < \bar{\tau}_o'(\tau)$; 2) $x^* = E(V-p^*)^+ - E(V-p_s)^+$; 3) $p_o^* = r_o^* = \inf\{r \ge v_{\min} : [(\lambda_1 - C)^+ + \lambda_2]\bar{F}(r) \le (C-\lambda_1)^+ + \min(\lambda_1, C)F(p^*)\}$. Moreover, $y^*(\bar{z}) = (C-\lambda_1)^+$ if $\tau' < \bar{\tau}_o'(\tau)$ and $y^*(\bar{z}) = 0$ otherwise.

Step 4: To complete the proof, we derive how the optimal prices and revenue change in τ. p^* is decreasing in τ because either $p^* = (1-\tau)p_2^*$ which is decreasing in τ or p^* stays constant in τ. Thus, x^* and p_o^* are increasing in τ. Part (i) of the theorem is proved. Furthermore, $x^* + p^*$ is decreasing in τ. Since $x^* + p^* = p_o^*$ when $\tau = 0$, we have $x^* + p^* < p_o^*$, hence Part (ii) of the theorem is proved.

Finally, we derive how the optimal revenue Π_o^* changes in τ. First, consider the case of $p^* = p^n$. When $\tau > \tilde{\tau}(C)$ or $\tau > \hat{\tau}(C)$, p^n is independent of τ, hence so is Π_o^*. On the other hand, when $\tau \le \min(\tilde{\tau}(C), \hat{\tau}(C))$, $p^n = (1-\tau)p_2^*$ and $p_o = p_2^*$, hence we have $\Pi_o^* = \{E[V - (1-\tau)p_2^*]^+ - E(V-p_s)^+ + (1-\tau)p_2^*\bar{F}((1-\tau)p_2^*)\}\min(\lambda_1, C) + p_2^*[C - \min(\lambda_1, C)\bar{F}((1-\tau)p_2^*)]$. Using a similar approach to derive $\mathrm{d}\Pi_m^*/\mathrm{d}\tau$ in the proof of Theorem 4, we obtain

$$\frac{\mathrm{d}\Pi_o^*}{\mathrm{d}\tau} = \frac{\mathrm{d}p_2^*}{\mathrm{d}\tau} \cdot [(\lambda_1 - C)^+ + \lambda_2] \cdot \left[\bar{F}(p_2^*) - \tau p_2^* f(p_2^*)\right].$$

For $\tau \le \tilde{\tau}(C)$, we have $\tau p_2^* \le \bar{F}(p_2^*)/f(p_2^*)$, then $\mathrm{d}p_2^*/\mathrm{d}\tau > 0$ implies $\mathrm{d}\Pi_o^*/\mathrm{d}\tau \ge 0$. Second, if $p^* = \bar{p}$, p^* is independent of τ, hence so is Π_o^*. Third, if $p^* = p^s$, p^* and Π_o^* are independent of τ if $p^s > (1-\tau)p_2^*$; if $p^s = (1-\tau)p_2^*$, we have

$$\frac{\mathrm{d}\Pi_o^*}{\mathrm{d}\tau} = \left.\frac{\mathrm{d}\Pi_o}{\mathrm{d}p}\right|_{p=(1-\tau)p_2^*} \cdot \frac{\mathrm{d}[(1-\tau)p_2^*]}{\mathrm{d}\tau} \ge 0,$$

because $p^s = (1-\tau)p_2^*$ implies $(\mathrm{d}\Pi_o/\mathrm{d}p)|_{p=(1-\tau)p_2^*} \le 0$ and we already know $\mathrm{d}[(1-\tau)p_2^*]/\mathrm{d}\tau < 0$. Therefore, Π_o^* is increasing in τ overall. Furthermore, because the existence of speculators decreases Π_o^*, we conclude that Π_o^* is maximized when $\tau = \tau' = 1$. Part (iv) of the theorem is proved. □

Proof of Theorem 6 Since $p^n \ge (1-\tau)p_2^*$ and $p_o(p^n) \le p_2^*$, we have $\bar{\tau}_o'(\tau) \le 1 - \{E[\max(V, p^n)] - E(V - p_s)^+\}/p_o(p^n) \le 1 - p_1^*/p_2^* = \bar{\tau}_m'(\tau)$. $\bar{\tau}_m'(\tau) < \bar{\tau}_f'(\tau)$ because $p_2^* = r_f^*$ and $p_1^* = E[\max(V, (1-\tau)p_2^*)] - E(V - p_s)^+ = p_f^n + \int_{p_s}^{\infty}(v - p_f^n)\,\mathrm{d}F(v) - E(V-p_s)^+ = p_f^n + \int_{p_s}^{\infty}(p_s - p_f^n)\,\mathrm{d}F(v) > p_f^n$. □

Proof of Theorem 7 First of all, we must have $b^* \le \min(\lambda_1, C)$. If $C > \lambda_1$, the revenue does not change if speculators do not enter the market in equilibrium; if speculators enter the market in equilibrium, as we decrease b from C to λ_1, the capacity provider shifts sales from period 1 to period 2 where the price is higher, hence his revenue increases. The analysis in Section 5 implies that the revenue is $\Pi_m(b) = \{E[\max(V,(1-\tau)p_2^*(b))] - E(V-p_s)^+\}b + p_2^*(b)(C-b)$, where $p_2^*(b)$ is given by $(\lambda_1 - b + \lambda_2)\bar{F}(p_2^*) = C - b\bar{F}((1-\tau)p_2^*)$.

Next, we show that $\Pi_m(b)$ is concave in b. Taking derivative gives

$$\frac{\mathrm{d}\Pi_m}{\mathrm{d}b} = [(1-\tau)F((1-\tau)p_2^*)b + C - b]\cdot\frac{\mathrm{d}p_2^*}{\mathrm{d}b} + E[\max(V,(1-\tau)p_2^*)] - E(V-p_s)^+ - p_2^*,$$

where $\mathrm{d}p_2^*/\mathrm{d}b = \partial\underline{r}/\partial z$ is derived in the proof of Theorem 1. Define $\Delta v = v_{\max} - v_{\min}$. With uniform distribution, we have $p_s = v_{\max} - C\Delta v/(\lambda_1+\lambda_2)$ and

$$\begin{aligned}\frac{\mathrm{d}\Pi_m}{\mathrm{d}b} &= \frac{(1-\tau)^2(\lambda_1+\lambda_2)}{[(\lambda_1+\lambda_2)v_{\max} - C\Delta v]\Delta v}\cdot(p_2^*)^3 \\ &\quad + \left\{\frac{\tau C\Delta v - (\lambda_1+\lambda_2)(v_{\max}-\tau v_{\min})}{[(\lambda_1+\lambda_2)v_{\max} - C\Delta v]\Delta v} - \frac{(1-\tau)^2}{2\Delta v}\right\}\cdot(p_2^*)^2 + \frac{1}{2\Delta v}\left[v_{\max}^2 - \frac{C^2\Delta v^2}{(\lambda_1+\lambda_2)^2}\right].\end{aligned}$$

Then we have

$$\frac{\mathrm{d}}{\mathrm{d}p_2^*}\left(\frac{\mathrm{d}\Pi_m}{\mathrm{d}b}\right) = \frac{p_2^*}{[(\lambda_1+\lambda_2)v_{\max} - C\Delta v]\Delta v}\cdot B_0,$$

where $B_0 = 3(1-\tau)^2(\lambda_1+\lambda_2)p_2^* - (1-\tau)^2[(\lambda_1+\lambda_2)v_{\max} - C\Delta v] - 2(\lambda_1+\lambda_2)(v_{\max}-\tau v_{\min}) + 2\tau C\Delta v$. When $\tau = 0$, $B_0 = -2C\Delta v < 0$; when $\tau = 1$, $B_0 = 2(C-\lambda_1-\lambda_2)\Delta v < 0$. Thus, if we can show B_0 is convex in τ, we know $B_0 < 0$ for all τ. Taking derivative of B_0 with respect to τ gives

$$\frac{\partial B_0}{\partial\tau} = \frac{(\lambda_1+\lambda_2)v_{\max} - C\Delta v}{(\lambda_1+\lambda_2-\tau b)^2}\cdot B_1,$$

where $B_1 = (1-\tau)[-6(\lambda_1+\lambda_2)(\lambda_1+\lambda_2-\tau b) + 3(1-\tau)(\lambda_1+\lambda_2)b + 2(\lambda_1+\lambda_2-\tau b)^2]$. Since $(\lambda_1+\lambda_2-\tau b)^2$ is decreasing in τ, it remains to show B_1 is increasing in τ, or equivalently,

$$\frac{\partial B_1}{\partial\tau} = -6b^2\tau^2 + 2[(\lambda_1+\lambda_2)b + 2b^2]\tau + 4(\lambda_1+\lambda_2)(\lambda_1+\lambda_2-b) \geq 0.$$

This is true because $\partial B_1/\partial\tau = 4(\lambda_1+\lambda_2)(\lambda_1+\lambda_2-b) > 0$ when $\tau = 0$, $\partial B_1/\partial\tau = 0$ when $\tau = 1$, and $\partial B_1/\partial\tau$ is concave in τ. By now we have shown that $\mathrm{d}\Pi_m/\mathrm{d}b$ is decreasing in p_2^*. Since $\mathrm{d}p_2^*/\mathrm{d}b \geq 0$, the chain rule then gives $\mathrm{d}^2\Pi_m/\mathrm{d}b^2 \leq 0$, hence we conclude that $\Pi_m(b)$ is concave in b.

Now that we have proved concavity, we derive the monotonicity of Π_m^* with respect to τ. First, if b^* is attained at an interior point, the Envelope Theorem gives

$$\frac{\mathrm{d}\Pi_m^*}{\mathrm{d}\tau} = \left.\frac{\partial\Pi_m}{\partial\tau}\right|_{b=b^*} = -\frac{\mathrm{d}p_2^*}{\mathrm{d}\tau}\cdot\frac{(\lambda_1+\lambda_2)v_{\max} - C\Delta v}{\tau(p_2^*)^2}\cdot[\alpha(p_2^*)^2 + \beta p_2^* + \gamma],$$

where

$$\alpha = \frac{1-\tau^2}{2\Delta v} \geq 0, \qquad \beta = -\frac{v_{\max}}{\Delta v} < 0, \qquad \gamma = \frac{1}{2\Delta v}\left[v_{\max}^2 - \frac{C^2\Delta v^2}{(\lambda_1+\lambda_2)^2}\right] > 0.$$

Note that in the above derivation, the first-order condition is used to simplify the algebra. We already know $\mathrm{d}p_2^*/\mathrm{d}\tau \geq 0$ and it is easy to see that $(\lambda_1+\lambda_2)v_{\max} > C\Delta v$, hence we only need to show $\alpha(p_2^*)^2 + \beta p_2^* + \gamma \leq 0$ to conclude that Π_m^* is increasing in τ. Recall that $p_2^* > p_s$. As the problem degenerates for $p_2^* > v_{\max}$, we restrict p_2^* to $p_2^* \leq v_{\max}$. Moreover, $-\beta/2\alpha = v_{\max}/(1-\tau^2) > v_{\max}$. Thus, it suffices to show $\alpha(p_2^*)^2 + \beta p_2^* + \gamma \leq 0$ at $p_2^* = p_s = v_{\max} - C\Delta v/(\lambda_1+\lambda_2)$. This is true as $\alpha p_s^2 + \beta p_s + \gamma = -\tau^2 p_s^2/(2\Delta v) < 0$. Second, if $b^* = \min(\lambda_1, C)$, we have the same optimal revenue function as in the basic model. For $\tau \geq \hat{\tau}(C)$, Π_m^* stays constant in τ. For $\tau < \hat{\tau}(C)$, to show Π_m^* is increasing in τ, (A1) implies that with uniform distribution, we need to show $(C-\lambda_1)^+ - [(\lambda_1 - C)^+ + \lambda_2]F((1-\tau)p_2^*) \geq 0$. Note that $b^* = \min(\lambda_1, C)$ implies $\mathrm{d}\Pi_m/\mathrm{d}b \geq 0$ at $b = \min(\lambda_1, C)$. With uniform distribution, this results in

$$\begin{aligned}
&[(1-\tau)F((1-\tau)p_2^*)\min(\lambda_1,C) + (C-\lambda_1)^+]\cdot \frac{\tau p_2^*}{[(\lambda_1-C)^+ + \lambda_2] + (1-\tau)\min(\lambda_1,C)} \\
&\geq -E[\max(V,(1-\tau)p_2^*)] + E(V-p_s)^+ + p_2^* \\
&> -E[\max(V,(1-\tau)p_2^*)] + E[\max(V,p_2^*)] \\
&\geq \tau p_2^* F((1-\tau)p_2^*),
\end{aligned}$$

where the second inequality follows from $p_s < p_2^*$ and the third inequality follows from the fact that the derivative of $E[\max(V,t)]$ is $F(t)$. Thus, $(C-\lambda_1)^+ - [(\lambda_1-C)^+ + \lambda_2]F((1-\tau)p_2^*) \geq 0$. Therefore, overall Π_m^* is increasing in τ[16]. $\square$

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[16] Note that the feasible region for b is $b > 0$. For any $\epsilon > 0$ where ϵ can be arbitrarily small, when b^* is attained at $b^* = \epsilon$ for the optimization problem over $\epsilon \leq b \leq \min(\lambda_1, C)$, Π_m^* is constant in τ, so overall Π_m^* is still increasing in τ.

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