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Reply to "Comment on: 'Flexural Strength by Fractography in Modern Brittle Materials'"

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We thank Mecholsky *et al.*[1] for their intense interest in our paper and their appreciation of the great importance and novelty of our results. We are also grateful for this opportunity to place our findings in a more comprehensive context to better discuss their comments.

Orr[2] is normally credited with developing the now commonly used equation relating the strength of brittle materials to the mirror radius at which 'mist' forms on fracture surfaces, $\sigma\sqrt{R_m} = A$, where R_m is the mirror radius, σ the strength of the sample, and A the 'mirror constant'. The idea of investigating the limitations of Orr's equation came to us during the course of a decade of fractographic analyses performed on a variety of brittle materials. It became apparent to us that, for fracture surfaces of submillimeter-thick glass plates, the mirror radius does not accurately predict the experimentally measured stress at failure when using published bulk values for 'A'. In addition, flexural tests by four-point bending (4PBT) and ring-on-ring (RoR) consistently showed a relatively large, nonzero "residual stress": the thinner the sample, the larger the apparent residual stress. We were surprised by this unexpected result until reading Quinn's paper[3] on fracture of glass plates subject to bi-axial stresses. Quinn independently confirmed that annealed glass plates fractured by RoR displayed a clear, positive y-intercept (i.e., apparent residual stress) when plotted against $1/\sqrt{R_m}$. We recognized that the tangent to the 'σ vs. $\sqrt{R_m}$ curve' would intersect the y-axis to produce a false apparent "residual stress" if the mirror constant were not truly a constant.

It is well-known that mist forms when a critical stress intensity factor (SIF) is reached at the crack front. This SIF depends on the local stress, the length of the crack (R_m), and a shape factor, Y, related to the geometry of the crack front. At the limit, a crack very shallow compared to the sample's thickness, H, in a sample loaded in bending ($R_m/H \ll 1$) will have a shape factor that is relatively uniform along the nearly semicircular crack front. Importantly, a nearly identical shape factor applies to samples tested in uniaxial tension, as in these cases the crack front is also nearly semicircular. Conversely, a very long through-the-thickness crack in bending ($R_m/H \gg 1$) will have a significantly different shape factor due to the elongated geometry of the crack front. For instance, Sherman *et al.*[4] observed a crack in bending with an aspect ratio of $c/a \approx 3.125$. It is therefore clear that the mirror constant of a given material, $A \cong K_{Im}/Y$, must be nearly equal for shallow cracks fractured in bending ($R_m/H < 1$) and for samples fractured in uniaxial tension. On the other hand, 'A' is expected to be larger for long through-the-thickness cracks fractured in bending.

Until recently, the majority of glass applications utilized relatively thick geometries (> 1 mm). The strength of glasses with untreated edges is normally on the order of 100 MPa (i.e., $R_m \approx$ 0.3–0.4 mm) and hence $R_m/H \ll 1$. In these cases, no significant difference between the values of 'A' in tension and 'A' in bending is expected, explaining the apparent thickness independence of 'A' historically observed. Interestingly, the 1966 work of Kerper and Scuderi[5] explored the possibility that the sample's thickness might have an effect on 'A', but regretfully the hypothesis was tested on glass rods. This choice of sample geometry was unfortunate, because the condition $R_m/H < 1$ necessarily always applies.

Having placed this discussion in a greater context, we can directly address the comments by Mecholsky *et al.*[1] Their main concern appears to be the validity of the data reported in Fig. 2. The astute reader will note that our findings are not in any way based on these data. As these data merely provide a historical frame of reference, we did not emphasize the minutiae of how the values were extracted from the literature. Nevertheless, we are pleased to now be given the opportunity to provide the details needed for addressing this apparent discrepancy.

Since the literature spans various authors and many decades of evolving knowledge, the mirror constants were originally extracted with a range of fitting equations presented in an incoherent fashion. In order to provide the consistency needed to compare across works, we therefore regressed the raw data for each reference by using a single fitting equation. In particular, these standardized values of 'A' were obtained as recommended by both ASTM C-158[6] and the NIST Recommended Practice Guide (NIST-RPG), Appendix D[7] using the equation

$$\sigma = \frac{A}{\sqrt{R_m}} + C \tag{1}$$

For glass samples, both references require that the constant C should not be forced to zero when $C > 10$ MPa. In the case of Mecholsky *et al.*,[8] $A = 1.9$ MPa·$\sqrt{m}$ is obtained by forcing $C = 0$, but $A = 1.31$ MPa·$\sqrt{m}$, $C = 18$ MPa are computed when complying with ASTM C-158 and the NIST-RPG. Similarly, both Choi *et al.*[9] and Ruggero *et al.*[10] incorrectly forced $C = 0$, although in both cases $C > 10$ MPa (i.e., 13.4 and 15.0 MPa respectively when using Equation 1). Table I in our paper[11] clearly reports all values of 'A' and the intercept extracted based on this fitting equation, consistent with accepted standards.

Mecholsky *et al.*[1] also assert that Zaccaria and Overend[12] had not corrected for large deflections and propose $A = 1.8$ MPa·$\sqrt{m}$, $C = 0$ as the appropriate regressing constants. However, based on a recent personal communication with Zaccaria,[13] we learned that test dimensions of 20 and 40 mm were used for the loading and the reaction spans respectively. Zaccaria[13] indicated that the observed deflection was in all cases less than 1 mm. Based on this span/deflection

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Table 1. Summary of Fitting Constants and Test conditions for Flexural Strength Data on Borosilicate (BSG), Sodalime (SLG), Aluminosilicate (ASG), and Flint Glass.

		H	Orr eq.		Proposed fit			
Author	Glass type	(mm)	A_m (MPa√m)	$\Delta\sigma_o$ (MPa)	K_{Im} (MPa√m)	$\Delta\sigma_d$ (MPa)	Samples tested	Test conditions
Dugnani	BSG	0.3	1.19	88.0	2.18 ± 0.16	-0.7	35	4PBT
Dugnani	BSG	0.7	1.37	31.6	2.26 ± 0.12	-21.4	12	4PBT
Dugnani	ASG	0.7	1.14	35.0	1.88 ± 0.05	-12.1	28	4PTB
Gulati[19]	BSG	0.9	1.94	11.9	2.63 ± 0.08	-25.5	32	RoR
Dugnani	ASG	1.0	1.48	34.1	2.37 ± 0.15	-15.6	140	4PTB
Ruggero[20]	SLG	1.0	1.41	15.0	2.06 ± 0.12	-22.7	41	4PTB, annealed
Choi[21]	SLG	1.5	1.52	13.4	1.77 ± 0.27	0.0	12	RoR, annealed
Mecholsky[22]	SLG	2.0	1.31	18.4	2.34 ± 0.16	-19.5	22	4PTB
Gaume[23]	SLG	2.2	1.47	19.5	1.90 ± 0.17	0.0	24	4PTB
Mecholsky[24]	BSG /silicate	2.5	2.13	8.7	2.62 ± 0.43	-6.9	21	RoR
Zacaria[30]	Flint glass	3.0	1.37	11.0	n/a	n/a	33	4PTB
Kerper[25]	BSG	4.1	1.85	16.6	2.48 ± 0.16	-2.4	22	Flexure, rods
Schwartz[13]	SLG	4.8	1.84	10.3	2.53 ± 0.19	-9.3	25	4PTB, annealed
Kirchner[26]	SLG	4.8	1.88	10.1	1.88	2.7	2	Flexure, rods
Kirchner[9]	Flint	5.0	1.88	25.0	2.09 ± 0.16	26.3	25	Flexure, rods
Quinn[27]	BSG	5.3	1.98	9.6	2.60 ± 0.12	-5.0	45	RoR, annealed
Kirchner[9]	Flint	6.0	1.68	24.7	2.16 ± 0.10	11.2	13	Flexure, rods
Kerper[25]	BSG	6.1	2.00	7.0	2.51 ± 0.16	0.3	20	Flexure, rods
Orr[3]	SLG	6.4	2.01	4.1	2.81 ± 0.05	-12.8	46	RoR
Gaume[23]	SLG	7.9	1.92	6.9	2.22 ± 0.29	0.0	25	4PTB
Kerper[25]	BSG	9.9	1.78	8.1	2.17 ± 0.14	4.9	25	Flexure, rods
Shand[28]	BSG	11.6	2.23	35.1	2.59 ± 0.19	0.0	19	4PTB, annealed rods
Ball[29]	SLG	12.5	1.80	1.4	2.11 ± 0.11	0.3	16	3PTB, annealed
Kerper[25]	BSG	19.1	1.87	7.1	2.13 ± 0.27	8.4	63	Flexure, rods
Kerper[25]	BSG	25.4	1.98	6.9	2.53 ± 0.17	2.2	39	Flexure, rods
Kerper[25]	BSG	38.1	1.77	9.1	2.28 ± 0.25	4.8	39	Flexure, rods

ratio, a correction for large displacement is inappropriate, and the original results valid, to the best of our knowledge.

Mecholsky *et al.*[1] state that we did not provide enough details of our experimental measurements. However, Section V of our manuscript indicates that all tests were done by 4PTB using articulated fixtures and loading rates complying with ASTM C158-02. The sample dimensions are reported in Table I. As explained in the paper,[11] standard corrections for large deflections were in some cases necessary using common, commercially available Finite Element Analysis (FEA) software. Neither raw data nor details on the FEA were included in our original manuscript in light of the large number of samples tested. We regret the opinion of Mecholsky *et al.*,[1] because we believe the reference to ASTM C158-02 is sufficient to allow anyone skilled in the field to easily replicate the results obtained in our work.

Mecholsky *et al.*[1] allege that "the paper omits reference data that contradict their results", and selfreference works describing two metal-clad glass-fiber composite samples fractured by wrapping around a mandrel. We must respectfully disagree with this statement. As explained in our original manuscript,[11] and further discussed above, no differences in '*A*' for flexural and tensile tests are expected when $R_m/H < 1$. Although our manuscript did not treat composite materials, we believe their allegation might refer to the fact that these two anisotropic samples may fall far from other points on Fig. 2. Yet Fig. 2 is merely a survey of historical data from the literature and was not used in deriving our model. Figure 2 loosely follows a trend with the thickness H, since often the values of R_m fall in the same range for most data sets. Nevertheless, we invite Mecholsky *et al.* to include

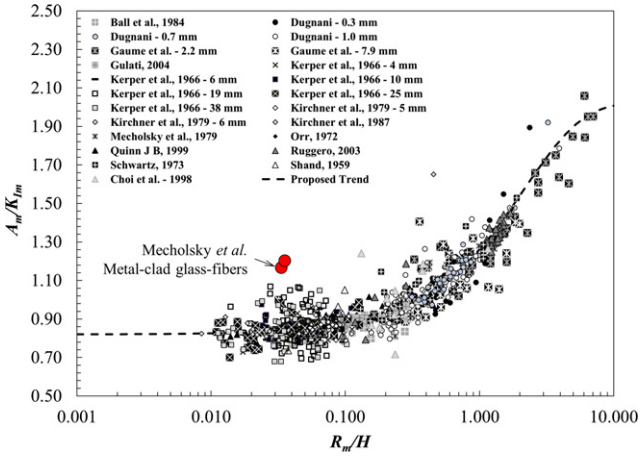

Fig. 1. A_m/K_{Im} vs. R_m/H adapted from Figure 4 of the original manuscript to include glass-fiber data from Mecholsky *et al.*[15]

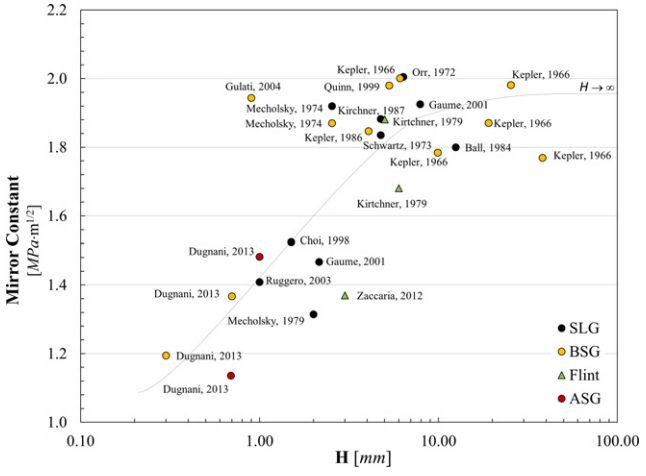

Fig. 2. Average mirror constant vs. the thickness of the glass plates, H, as reported by various authors.

these two additional results, along with the data from other authors, in Fig. 4 to confirm that they follow the trend predicted by our crack evolution model rather nicely (see Fig. 1 in this response). They will find the assertion that our data "disagrees with virtually all investigators" to therefore, be entirely incorrect.

We appreciate the added comments by Mecholsky *et al.*[1] that help define some subtleties, although we believe they are largely editorial in nature and likely provide little additional insight for the attentive reader. For example, we describe the length '*a*' as the 'crack depth' rather than the more restrictive term 'semi-minor axis'; we contend that our nomenclature is not incorrect, but merely more general. In our original manuscript, we referenced Mecholsky and Freiman[14] (Ref. [7] in our original manuscript) to support our claim that the SIF is not constant along the crack front ("Note that although the mist appears at R_m, no mist is necessarily present at a_m, as the stress intensity is not constant along the crack front"). The length, a_m, was defined in Section I as the depth of the crack when the half-width of the crack $c = R_m$. Clearly, mist appearing at c does not necessarily require mist to appear at a_m, as the SIF is often different at the two locations. The reference concerns the equation for the SIF at branching (Equation 3 in Mecholsky and Freiman,[14] $K_{Bj} = \sigma_j \cdot Y(\theta)\sqrt{r_j}$). These authors further discuss that the border correction factor, $Y(\theta)$, in the equation is necessary to account for the fact that the stress intensity is not constant along the crack front. We interpret these statements as indicating that the SIF is not constant along the crack front and therefore, we consider the citation valuable.

We also agree with Mecholsky *et al.*[1] that the SIF is constant along the mirror-mist boundary. It is, therefore, not clear to us where in our manuscript we may have stated otherwise, hence we are unable to respond to this comment. Finally, Mecholsky *et al.*[1] claim that the SIF at branching, instead of at the mirror boundary, may also be used to predict the strength of a sample, but our paper never investigated the relative merits of one method over another.

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