RESEARCH BULLETIN

AN UNFOLDING ANALYSIS OF COGNITIVE PREFERENCES

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Abstract

This study represents a reanalysis of Heath's (1963) data using Coombs' unfolding technique. Some additional objectives of this study are to find the dimensions underlying the cognitive preferences, to test Heath's first two hypotheses in terms of individuals as well as clusters, and to compare cluster and individual analyses.

The first dimension, which goes from Applications to Memory to Principles to Questioning, accounts for over half of the individuals and over two-thirds of the clusters. A two-dimensional configuration accounts for 90% of the individuals and 99% of the clusters. The ordering along the second dimension goes from Memory to Application to Questioning to Principles. The analyses of clusters and individuals were shown to give identical solutions.

With respect to the first dimension alone as well as with consideration of both dimensions, the following hypotheses were supported:

(a) that the Physical Science Study Curriculum group and the control group would be generally located in different regions of the same joint space, the PSSC group being more densely distributed than controls in regions near Principles and Questioning, the controls being more densely distributed than the PSSC group in regions near Memory and Applications;

(b) that achievement scores for PSSC clusters would be higher in regions near Principles and Questioning than in regions near Applications and Memory; and

(c) that the region-achievement relationship hypothesized in "b" would be stronger in the PSSC group than in the control group.

Implications following from the analysis were discussed.

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INTRODUCTION

Heath (1963) conducted a curriculum studies experiment in which physics students of thirty-one physics teachers (thirty-one clusters) learning by the Physical Science Study Committee (PSSC) method, a modern approach stressing concepts and fundamental principles, comprised the experimental group and fifty physics clusters, parallel to the PSSC group on certain demographic characteristics, learning by the conventional method served as controls. All students in both groups took:

1) The School and College Ability Test (SCAT)

2) The Cooperative Physics Test - a traditionally oriented comprehensive final examination

3) The PSSC Comprehensive Final Examination - a test oriented to the PSSC goals

4) A Cognitive Preference Test - Four correct options were given to each of twenty items. "Each of the four options was designed to demonstrate a different form of cognitive preference in physics. One option shows preferences for memory of specific facts or terms. Another provides a practical application of the information given in the item stem. A third choice reflects some challenging or critical questioning of the information given. The fourth is a statement of a fundamental principle of physics underlying the data." (Heath, 1963) The subjects were told that all answers were correct, but to pick the one which they preferred. Each subject had a sum of twenty points which were distributed over the four categories as desired. The four scores were thereby interdependent, or ipsàtive.

Eliminating one experimental and one control cluster from the analysis, Heath found:

"a) that PSSC students demonstrate a stronger preference for fundamental principles and questioning than non-PSSC students,

b) that non-PSSC students prefer memory for facts and terms and for practical application to a greater degree than PSSC students,

c) that preference for fundamental principles and questioning is more positively related to achievement test scores for PSSC students than for the control group students, and

d) that preference for facts and terms and for practical application is more negatively related to achievement test scores for PSSC students than for control group students." (Heath, 1963)

This report basically represents a reanalysis of Heath's data using Coombs' unfolding technique. Some additional objectives of this study are to find the dimensions underlying the cognitive preferences, to test Heath's first two hypotheses in terms of individuals as well as clusters, and to compare analyses of clusters with those of individuals. After reformulating Heath's hypotheses in unfolding theory terms, I will test them from this perspective. Such an analysis should clarify what the Cognitive Preference Test is measuring and explicate the pattern of Heath's results.

METHOD

The relative preference for Applications and Memory items for the control group in contrast to the desirability of Principles and Questioning items for the experimentals, combined with the pattern of correlations between cognitive preference and achievement,suggested that the four stimuli could be ordered on a continuum with Memory and Applications on one side, Principles and Questioning on the other. However, the ipsative scoring procedure complicates a factor analytic solution.

Coombs' (1964) unfolding technique which uses only rank orders of preferences seemed to be an appropriate analytic tool for recovering the underlying dimensions. The experimenter assumed, to obtain cognitive preference orderings, that the rank order of endorsement frequencies for the four styles reflected the individual's or cluster's preference ordering; i.e., an individual who endorsed Memory items eight times, Principles six times, Questioning four times, and Practical Applications twice would have a preference ordering: Memory, one; Principles, two; Questioning, three; and Practical Applications, four.

By means of the unfolding technique I sought a dominant dimension on which individuals and stimuli could be located. To determine the dimensionality of the space and to find additional dimensions, Bennett and Hays' (in Coombs, 1964) multidimensional extension of the unfolding technique was utilized, whereas the set of geometrical solutions provided by McElwain and Keats (1961) was used to determine the stimulus configuration.

The basic idea underlying the unfolding technique is that "stimuli and individuals can be represented by points in a common space called a Joint Space and that each person's preference ordering of the stimuli from most to least preferred corresponds to the rank order of the absolute distance of the stimulus points from the individual point, the nearest being most preferred." (Coombs, 1964) In one dimension, preference orderings of individuals, which are called I-scales or Individual scales, can be numbered left to right from 1 to $\binom{n}{2}+1$, where n indicates the number of stimuli.

This idea is illustrated in Figure 1. The letters "A", "B", and "C" represent three stimuli. With three stimuli there are three possible pairs -- AB, AC, and BC. For each of these pairs there exists a midpoint such that those individuals to the right of the midpoint prefer the member of the stimulus pair to the right; those to the left prefer the stimulus located left of the midpoint. Location on a midpoint corresponds to indifference, or tied ordering with respect to the stimulus pair. Given the side of each midpoint an individual is on, we can determine his rank ordering of the stimuli and vice versa. For example, the individuals in the region labelled I_1 in Figure 1 are to the left of the AB, AC, and BC midpoints. Being to the left of the AB and AC midpoints signifies that A is preferred to B and C and being to the left of BC points out that B is preferred to C. The rank ordering of the three stimuli is thereby ABC. All subjects who, as I_1 subjects, are to the left of AC and BC, but to the right of AB, reverse the ordering and thereby the preferences for A and B, but otherwise have the same ordering - BAC. This preference ordering corresponds to I_2. As each midpoint is passed the preferences for the relevant pair reverses since, having passed that midpoint, the subject is closer to the member of the stimulus pair on the right. The I-scales can, in the case of three stimuli, be numbered from I_1 to I_4. Unfolding theory actually works backwards from the preference orderings of individuals to the order of midpoints, which in turn gives the stimulus ordering on the latent dimension and, possibly, some metric information on interpoint distances.

In the unidimensional case a midpoint between stimuli suffices to partition preference orderings into two sets, those on one side of the midpoint preferring one of the stimuli, those on the other side preferring the other. In the two-dimensional case, perpendicular bisectors of the lines joining the stimulus pairs serve as boundaries, separating individuals with opposite preference orderings for the relevant pairs. Figure 2 illustrates three points in two dimensions. Here "A", "B", and "C" are stimuli and "x" is a subject. Since x is on B's side of the BC and BA perpendicular bisectors and on C's side of the AC perpendicular bisector, x's ordering is BCA.

Returning to the data, in addition to analyzing the cluster preference orderings, I analyzed preferences of individuals in both experimental and control groups. Initially, I selected eighteen of the experimental clusters and eighteen of the control clusters randomly, and from each of these thirty-six clusters randomly selected eighteen individuals. The preferences of the 324 experimental subjects and the 324 control subjects were then analyzed in two separate analyses. It was hypothesized that the analyses of control and experimental clusters and control and experimental individuals would all yield the same solution. If the configuration recovered from individuals did not accommodate clusters, generalization of the results from clusters to individuals would be inadvisable. If different solutions arose from the analysis of control and experimental groups, it could be concluded that the PSSC treatment had altered the relations between the four stimuli for the PSSC students. Such a difference would suggest that the stimuli had, in the course of the school year, acquired different meaning for the PSSC students.

Although similar cognitive spaces were expected for the two groups, it was hypothesized that the PSSC and control groups would be generally located in different regions of the Joint Space, the experimentals being more densely distributed than controls in regions near Principles and Questioning, the controls being more densely distributed than experimentals in regions near Memory and Application. It was also hypothesized that achievement test scores for PSSC clusters would be higher in regions near Principles and Questioning than in regions near Applications and Memory, and that this region-achievement relationship would, in fact, be stronger in the PSSC group than in the control group.

RESULTS

Table 4 shows that the first dimension is a strong one, accounting for over half of the individuals' and over two-thirds of the clusters' preference orderings. The ordering on the first dimension,which is the same for all four analyses, is Applications to Memory to Principles to Questioning. This dimension can be interpreted as going from an applied to a theoretical, or an engineering to a scientific orientation. An interesting bit of metric information follows from the preference orderings: the distance between Memory and Applications is greater than the distance between Principles and Questioning.

With respect to the first dimension the hypotheses were tested. Given the ordering along the first dimension the first hypothesis becomes that the average I-scale number of experimentals will be higher than the average I-scale number of controls. Table 2 gives strong support for this prediction for both clusters and individuals. For clusters the mean I-scale number of 3.73 for experimentals is significantly greater than the value of 2.97 for controls. Even more significant is the difference between the mean I-scale number of PSSC individuals, 4.10, and of control individuals, 3.24.

In terms of unidimensional unfolding the second hypothesis becomes that I-scale number will be positively correlated with achievement for PSSC clusters and that the correlation between I-scale number and achievement will be more positive in the PSSC group than in the control group.

For PSSC clusters Table 3 shows that the correlation between I-scale number and achievement on the PSSC Final of .37 and of .49 between I-scale number and the Cooperative Physics Test are both significant. Such a relationship is absent in the control group clusters. In fact these two correlations are slightly negative in the control group. Although the correlations, for PSSC clusters, between I-scale number and the PSSC Final is not significantly greater than the same correlation in the control group, the correlation between I-scale number and the Cooperative Physics Test is significantly greater in the PSSC group than in the control group. After partialling out SCAT, the difference in correlations between the control and PSSC group is significant on both tests.

Table 4 points out the finding that the same two-dimensional cognitive space accommodates 89% of the individuals in the PSSC group, 90% of the control students, 98% of the control clusters and 100% of the PSSC clusters. On the second dimension the most satisfactory ordering appears to Memory to Applications to Questioning to Principles. This dimension seems to go from what are regarded as lower mental processes --rote learning memory tasks-- to higher mental processes required in critical examination and understanding of materials. With respect to the location of individuals in the Joint Space, about half of the subjects are either in the region occupied by their respective clusters or are in regions immediately adjacent to their clusters region. This indicates that the preference ordering of a given cluster represents well the preference ordering of the students within that cluster.

To test the hypothesis concerning the distribution of PSSC and control clusters and individuals in the two-dimensional space, I partitioned the space into three regions. Region I was so constructed that all clusters or individuals whose preferences were mapped into it would be nearer Memory and Applications than Principles and Questioning on both dimensions. Operationally, this region is on the Applications side of the Applications-Questioning perpendicular bisector, or boundary, and on the Memory side of the Memory-Principles boundary. The opposite region of the space was designated Region III. The subjects and clusters in this region, being on the Principles side of the Memory-Principles boundary and the Questioning side of the Applications-Questioning boundary, are nearer Principles and Questioning than Memory and Applications on both dimensions. The remaining area of the space, the intermediate region, was labelled Region II. Given this partitioning of the space, the hypothesis concerning predicted preferences for the control and PSSC groups becomes that the PSSC and control groups will be distributed differently in the space, with PSSC's denser than controls near Region III and the controls denser than PSSC's near Region I.

Figure 3 illustrates the distribution of clusters in the Joint Space and Table 5 indicates that the distribution difference for clusters as well as individuals is highly significant. Clearly PSSC's prefer Principles and Questioning more than controls, whereas the Controls prefer Memory and Applications more than the PSSC's. Table 6 shows that this effect cannot be attributed to aptitude differences -- aptitude being measured by SCAT -- for SCAT scores are not significantly different in different regions of the space.

In light of the division of the space into three regions, the hypotheses relating preferences and achievement become that Region number will be positively correlated with achievement for PSSC clusters and that the correlation between Region number and Achievement will be higher in the PSSC group than in the control group. Table 7 shows that the data support the hypotheses. The correlation, for PSSC clusters, between Region number and achievement as measured by both the PSSC Final and the Cooperative Physics Test is significant and it remains significant after SCAT is partialled out. Moreover, these correlations in the PSSC group are significantly greater than in the control group, both before and after SCAT is partialled out.

DISCUSSION

The difference in spatial location for controls and experimentals leads to a challenging of one of Heath's interpretations. He states (Heath, 1963, p. 17) that "These differences (in correlations) may signify that practical application has a rather different meaning to students in the two courses." In view of the results from the unfolding analysis, the stimuli seem to bear the same relationship to each other and also have the same meaning for the two groups. The difference in correlations between applications and other stimuli can be explained in terms of three effects - laterality, remoteness, and distance on the I-scales.

Laterality refers to the location of stimuli in relation to a given individual. If, in the space, both stimuli are located on one side of an individual, the stimuli are said to be unilateral for that individual. For example, in Figure 1, B and C are unilateral to individuals in I_1. If, however, a pair of stimuli are on opposite sides of the individual, they are said to be bilateral to the individual. In Figure 1, A and C are bilateral to individuals in I_2 and I_3. It can be easily shown that inconsistency is greater for bilateral than for unilateral pairs (Coombs, 1963). With respect to correlations, bilaterality imposes negative correlations on the pairs of stimuli, for, as individuals move closer to one member of the pair, they depart from the other stimulus. So individuals bilateral to A and B will have low scores on A if they have high scores on B and vice versa. Over the AB interval the stimuli will therefore be quite highly negatively correlated. This does not occur when the stimuli are unilateral, for moving close to one of the stimuli implies moving closer to the other stimulus. Therefore high endorsement for one of the pair tends to accompany high endorsement of the other.

Remoteness refers to the average absolute distance of the pair of stimuli from an individual. Numerous studies have shown that as stimuli become more distant from individuals they are perceived to be closer together. In terms of a monotonic relation between correlations and distance (the smaller the distance, the higher positively the correlation), this would indicate that the stimuli would be more highly correlated for individuals further away from the stimuli.

Distance on the I-scale refers to the difference in absolute distance of the stimuli from the individual. If the stimuli are unilateral, distance on the I-scale corresponds to the distance between the stimuli. If they are bilateral, however, distance between stimuli becomes a function of the point at which the scale (or space) is folded. This rather than just the distance between the stimuli functions when the data is preferential choice data.

Since the two groups are distributed differently in the space, these three variables have differential effects on the two groups. An interpretation of the differences in interstimulus correlations for controls and experimentals should therefore consider laterality, remoteness, and I-scale distance.

In terms of the unfolded dimensions, the pattern of results becomes quite clear. Heath's hypotheses "a" and "b" can actually be considered one hypothesis, as can "c" and "d." Surely the ipsative nature of the scales would make each pair dependent in any case. Considering that both dimensions place Applications and Memory on one side, Principles and Questioning on the other, it follows that, if the first group prefers one side of the continuum more than the second, the second will prefer the other side more than the first group does. Furthermore, if there is a higher positive correlation between achievement and one side of the continuum for the experimentals, there must be a higher negative correlation between achievement and the other side of the continuum for the experimentals.

That the dimensions are related to achievement and curriculum in the expected direction simultaneously supports the PSSC curriculum claims, demonstrates the construct validity of the Cognitive Preference Test, and gives psychological meaning to the dimensions. Further exploration utilizing unfolding analyses may suggest how future Cognitive Preference Tests might be constructed so as to increase discrimination, eliminate redundant score categories, and measure along other dimensions.

References

Coombs, C. H., Theory of Data, John Wiley and Sons, Inc., New York, 1964.

Heath, R. W., "Curriculum, Cognition, and Educational Measurement," Educational Testing Service Research Bulletin 63-6, March, 1963.

McElwain, D. W. and Keats, J. A., "Multidimensional Unfolding: Some Geometrical Solutions," Psychometrika, Vol. 26, No. 3, September, 1961.

Figure 1

Hypothetical Example: Three Stimuli in One Dimension

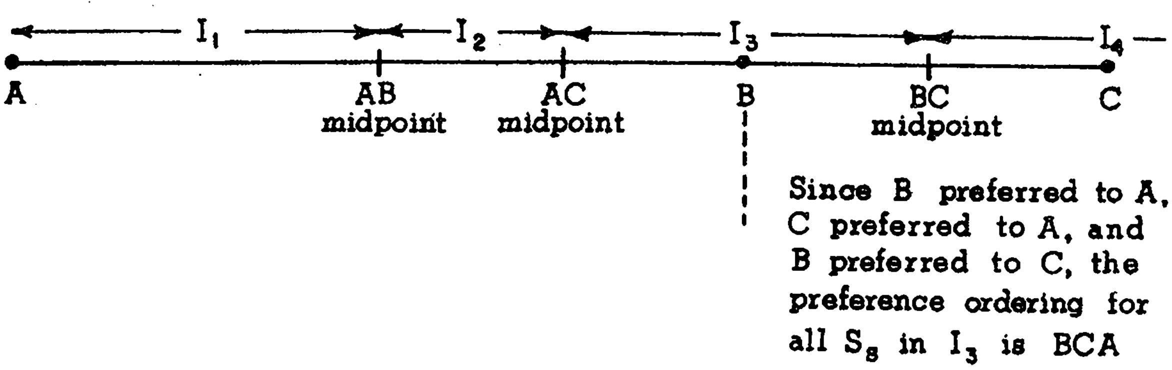


Figure 2

Hypothetical Example: Three Stimuli in Two Dimensions

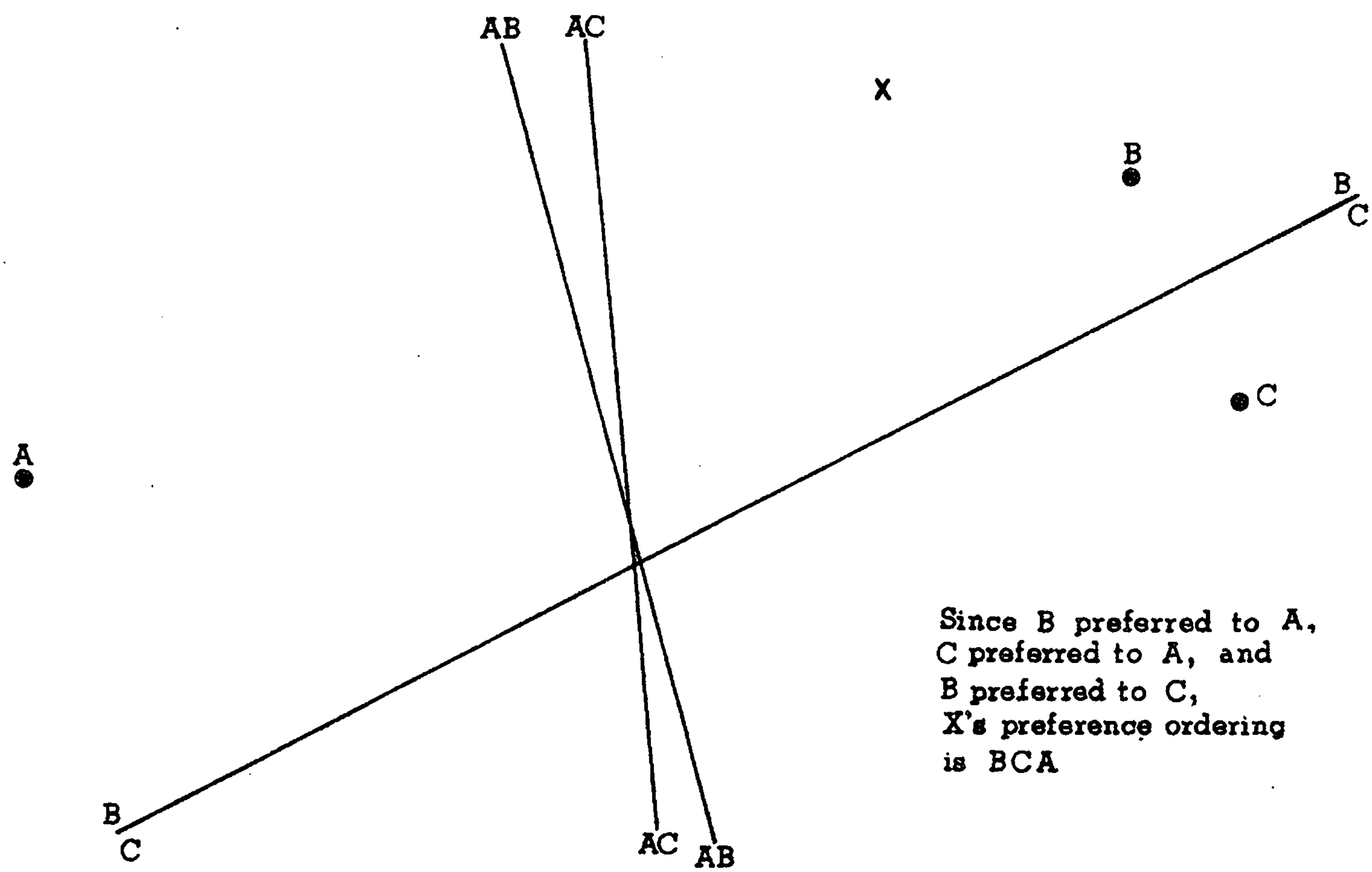


TABLE 1a

PREFERENCE ORDERINGS OF CLUSTERS

	PSSC	CONTROLS
Region III		
PMQA-I_5	2	0
PQMA-I_6	3	0
QPMA-I_7	1	0
PQAM-X	0	0
Region II		
PMAQ-I_4	3	0
MPQA-I_4	5	12
MQAP-X	1	2
MQPA-X	3	4
QMPA-X	0	0
PAQM-X	1	0
APQM-X	0	1
APMQ-X	0	1
PAMQ-X	1	0
P(MA)Q-I_4	0	1
Region I		
AMPQ-I_1	2	4
MAPQ-I_2	4	7
MPAQ-I_3	3	7
MAQP-X	1	5
AMQP-X	0	4
Violations of 2-dimensional space		
AQMP	0	1
QAMP	0	0
QMAP	0	0
AQPM	0	0
QAMP	0	0
QPAM	0	0

Code

A - Applications

M - Memory

P - Principles

Q - Questioning

X - Violation of first dimension

( ) - tied ordering

TABLE 1b

PREFERENCE ORDERING OF INDIVIDUALS

	PSSC	CONTROLS
Region III		
PMQA	13	3
PQMA	9	2
QPMA	11	2
PQAM-X	7	3
P(QM)A	6	5
(QP)(MA)	2	1
QP(MA)	7	1
PQ(MA)	4	3
(PQ)AM-X	3	2
(PQ)MA	3	0
Region II		
MQAP-X	5	8
MQPA-X	7	16
QMPA-X	11	4
PAQM-X	4	1
APQM-X	5	11
APMQ-X	8	3
MPQA	17	25
PMAQ	5	3
PAMQ-X	3	1
(MQ)(AP)-X	2	1
(MQ)AP-X	1	5
MQ(AP)-X	4	3
QM(PA)-X	1	3
P(MA)Q	1	4
(AP)(MQ)-X	3	2
PA(MQ)-X	5	2
(MQ)PA-X	2	5
M(PQ)A	16	16
A(PQ)M-X	7	9
AP(MQ)-X	4	0
(AP)MQ-X	2	0

TABLE 1b (continued)

	PSSC	CONTROLS
Region I		
MAQP-X	2	10
AMPQ	8	6
AMQP-X	4	6
MAPQ	8	19
MPAQ	11	18
(MA)(QP)	2	1
MA(PQ)	3	11
(MQ)QP-X	1	5
AM(PQ)	2	8
A(MQ)P-X	4	4
M(AP)Q	9	8
(MA)PQ	3	3
Indeterminate Region (Bounding 2 or 3 regions)		
(MAPQ)	1	2
M(AQ)P-X	0	6
M(APQ)	2	7
A(MPQ)	4	2
A(MP)Q	4	8
(MP)(AQ)	5	3
MP(AQ)	13	9
(MP)AQ	1	2
P(MAQ)	0	1
PM(AQ)	1	2
(MPQ)A	1	1
Q(PM)A	8	2
(MP)QA	4	1
(AQP)M-X	1	1
P(AQ)M-X	5	1
Q(PMA)	2	0
Violations of 2-dimensional space		
AQMP	5	6
QAMP	2	2
QMAP	2	4
AQPM	5	3

TABLE 1b (continued)

	PSSC	CONTROLS
QAPM	2	1
QPAM	6	5
(AQ)(MP)	0	1
AQ(MP)	4	6
(AQ)MP	1	1
QA(MP)	3	1
(AQ)PM	1	2
Q(AP)M	1	1
Q(MA)P	5	0

Code

A - Applications

M - Memory

P - Principles

Q - Questioning

X - Violations of first dimension

( ) - tied ordering

TABLE 2

Between Groups Comparison of I-Scale Numbers

Clusters:	Mean I-Scale Number		Standard Deviation
PSSC	3.73		1.73
Control	2.97		1.42
		$t=2.00$ $P<.05$	
Individuals:			
PSSC	4.10		1.67
Control	3.24		1.09
		$t=3.38$ $P<.001$	

TABLE 3

Correlation Between I-Scale Number and Aptitude and Between I-Scale Number and Achievement

Correlations	PSSC Clusters (N=23)	Control Clusters (N=31)	Significance of Correlation difference between groups
I-scale, PSSC Final	$r=.37$ $P<.05$ *$r=.63$ $P<.001$	$r=-.02$ *$r=.00$	$t=1.41$ $t=2.55$ $P<.01$
I-scale, Coop Test	$r=.49$ $P<.01$ *$r=.61$ $P<.01$	$r=-.11$ *$r=-.08$	$t=2.24$ $P<.05$ $t=2.73$ $P<.01$
I-scale, SCAT	$r=-.08$	$r=-.08$	$t=0$

* Correlations with SCAT partialled out

TABLE 4

PROPORTION OF PREFERENCE ORDERINGS SATISFYING DIMENSIONS

	PSSC Clusters	Control Clusters	PSSC Individuals	Control Individuals
Dimension I	$\frac{23}{30}$	$\frac{31}{49}$	$\frac{186}{324}$	$\frac{179}{324}$
Dimensions I and II	$\frac{30}{30}$	$\frac{48}{49}$	$\frac{287}{324}$	$\frac{291}{324}$

Dimension I: Applications — Memory — Principles — Questioning
Dimension II: Memory — Applications — Questioning — Principles
Metric Implications: Distance from Applications to Memory greater than Distance from Principles to Questioning

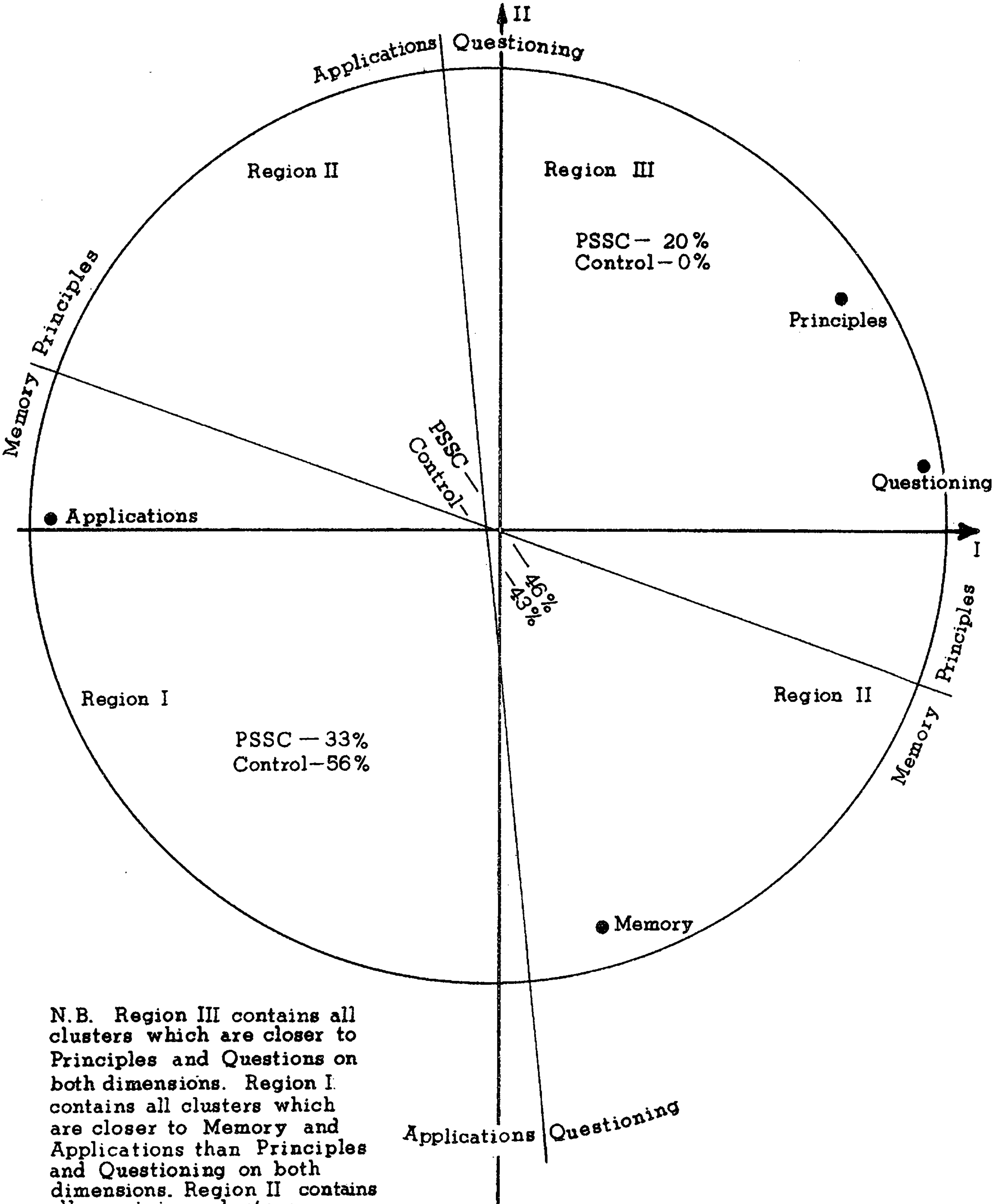
TABLE 5

LOCATIONS OF CLUSTERS AND INDIVIDUALS IN COGNITIVE SPACE

		Region I	Region II	Region III	
Clusters	PSSC	10	14	6	$X^2 = 10.95$
	Control	27	21	0	$p < .01$
Individuals	PSSC	57	113	65	$X^2 = 31.63$
	Control	99	122	22	$p < .0000001$

Figure 3

Distribution of Clusters in Space



N.B. Region III contains all clusters which are closer to Principles and Questions on both dimensions. Region I contains all clusters which are closer to Memory and Applications than Principles and Questioning on both dimensions. Region II contains all remaining clusters.

TABLE 6

SCAT SCORES IN REGIONS OF SPACE

	Region I	Region II	Region III	Regions I and II
PSSC Clusters	42.6	42.2 $t_{I,II} = -.29$	44.4	42.4 $t_{I \text{ and } II, III} = 1.28$
Control Clusters	40.2	38.6 $t_{I,II} = -1.28$		39.3

TABLE 7

CORRELATIONS BETWEEN REGION NUMBER AND APTITUDE AND BETWEEN REGION NUMBER AND ACHIEVEMENT

Correlations:	PSSC Clusters (N = 30)	Control Clusters (N = 48)	Significance of Differences Between Groups After r to z Transformation
Region Number, PSSC Final	r = .44 p < .01 *r = .54 *p < .001	r = .06 *r = .11	t = 1.69 p < .05 *t = 2.00 *p < .05
Region Number, Coop Test	r = .48 p < .01 *r = .50 *p < .01	r = −.08 *r = .10	t = 2.48 p < .01 *t = 1.87 *p < .05
Region Number, SCAT	r = .09	r = −.19	t = 1.17

*Correlations with SCAT partialled out