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A Model of Aggregate Reporting Quality

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Abstract

We characterize firms' aggregate reporting quality in an economy where a rational capital market as well as a regulator discipline firms' reporting choices. When the regulator is resource constrained, multiple aggregate reporting choices are possible in equilibrium. This multiplicity is driven not just by the regulatory constraint, but also by how this constraint interacts with managerial incentives and the level of reporting discretion available to firms. These results obtain despite firms trying to signal their type through private voluntary activities such as dividends. This link between aggregate reporting and aggregate signaling activities is under-emphasized in the literature. The model's results thus offer a way to jointly interpret empirical results on aggregate reporting, aggregate signaling, and underlying institutions.

A Model of Aggregate Reporting Quality

1   Introduction

The role of underlying legal and regulatory institutions in determining aggregate properties of accounting reports is a vital subject of accounting research. This research has documented variation in aggregate accounting patterns that still elude explanations. For example, Ball et al. (2000, Figure 1) documents that countries with similar legal origin have different accounting properties. Likewise, Dechow et al. 2010 (Sections 4 and 5) review the empirical earnings literature, and note that the links between reporting quality and the underlying institutions are only partly understood. Theoretical accounting literature has developed models of aggregate accounting properties, but the main economic force tying all firms together in these models are production factors such as exogenous CAPM-like correlations among the firms' cash flows (e.g., Leuz et al. 2007; Strobl 2013). This study develops a theoretical model of aggregate reporting choices where the main economic force tying all the firms together is regulation. As a result, the model speaks directly to the role of legal institutions in driving reporting choices.

As a first step, we introduce a capital market economy with both "good" and "bad" firms whose reporting choices are overseen by a central regulator. Firms in this economy are run by managers who have an interest in the stock price, and have reporting discretion over the assessed value of their firms' assets. The regulator investigates firms and forces any misreporting manager to publicly correct his disclosures and pay a penalty. However, the regulator has limited resources and cannot investigate all firms' reporting choices (Becker 1968).[1] Investors then price all the firms rationally, taking into account the possibility that the regulator may have missed some misreporting firms.

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[1]Becker's model appears to be especially relevant in the recent deregulatory era where regulators had been curtailed substantially (e.g., our Exhibit 1; Rajan 2005; Akerlof and Shiller (2009, Ch. 3)). Labaton (2008b) documents that the S.E.C.'s office set up to oversee the use of complex securities in the $4 trillion banking industry had not had a director and, as of September 2008, had not completed a single inspection in the last year and a half.

Managers in our model maximize share price less the expected penalty for misreporting. Reporting choices affect both the share price and the expected penalties. As more firms choose to misreport, the less likely it is that a given firm will be caught by the regulator. On the other hand, the stock price benefit to over-reporting also changes with the number of misreporting firms in the economy, because investors are rational. Thus, for individual managers, both the share price benefit and the regulatory penalty costs of misreporting vary with misreporting prevalence in the economy. This feature leads to multiple reporting equilibria in our model.

To the extent the reporting equilibrium allows "bad" firms to mimic the "good" firms in our economy, the good firms would like to separate from the bad firms through signaling activities such as dividend payments. As good firms choose to differentiate in this manner and opt out of the reporting game, the combination of the regulatory oversight effects and the investor valuation effects on the remaining firms causes their values to change in a non-monotonic manner. Multiplicity in the subsequent reporting choices can thus still obtain.[2]

Investors in our model are rational; therefore prices are always correct on average, regardless of the equilibrium level of aggregate misreporting. The true real effect manifests itself in the sensitivity of prices to new information: when aggregate misreporting levels are high, new information does not get into prices efficiently. This phenomenon has important welfare implications because prices' ability to aggregate dispersed private information efficiently is what makes them key drivers of real resource allocation in a free market economy (e.g., Hayek 1945). This crucial allocation mechanism of capital markets is impaired in the "bad" equilibrium where many firms misreport.

The above welfare result makes it important to understand the conditions under

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[2]Our model differs from studies such as Povel et al. (2007) and Shleifer and Wolfenzon (2002), where the cost of detecting misbehavior in a given firm is an exogenously specified parameter. In our model, the strength of the regulatory force on any firm is endogenous, and depends on the reporting behavior of other firms. Consequently, the aggregate level of misreporting in Povel et al. (2007) depends on the ex ante investor bullishness about the economy, whereas the same initial conditions in our model can generate different aggregate misreporting levels.

which such multiplicity obtains. The conditions in our model are: constrained regula-
tors, the presence of reporting discretion, and strong stock-based incentives for managers.
These factors have been widely implicated in the systematic shift from disciplined bal-
ance sheets to overvalued balance sheets in the U.S. banking industry in the period
leading up to the mortgage crisis (e.g., Blinder 2013, Ch. 3; Donaldson et al. 2008;
Kouwe 2009; Labaton 2008a; Mollenkamp et al. 2008). In such situations, our model
therefore suggests that empirical studies may not be able to uniquely map reporting
institutions to reporting quality, no matter how carefully researchers construct their
empirical measures (e.g., Ball et al. 2000, Figure 1).

Our reporting results hold even when "good" firms credibly signal their way out of
the reporting game. To the extent one can think of dividends as such a signal, our model
speaks to aggregate dividend patterns, which are a well-analyzed but partly-understood
phenomenon (Fama and French 2001). Recent empirical research suggests earnings are
a important factor (Skinner 2008). Our model suggests another important factor over
and above earnings, namely the nature of the entire financial reporting regime itself.
This link, to the best of our knowledge, has not been demonstrated before, despite the
fact that signals and financial reporting serve the same aggregate purpose: to separate
good firms from bad.

The externality in our model arises from constrained regulatory resources. How-
ever, it is well understood in economics that congestion effects in deterrence can by
themselves drive multiplicity in a traditional crime setting: i.e., a low crime level is a
deterrence in itself, because the lone criminal knows that the police have ample spare
resources to catch him. However, this line of reasoning has not been extensively used to
understand aggregate financial reporting patterns in the empirical accounting literature,
in part because, in addition to regulation, one has to account for additional disciplin-
ing mechanisms such as rational pricing in capital markets and managerial incentives.
The value of our model is that it shows how these additional disciplining mechanisms
change the nature of deterrence congestion. As a result, our model is better suited to

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explain aggregate empirical patterns of financial reporting choices than a pure crime and deterrence model.

Section 2 reviews the literature. Section 3 develops the model. Section 4 computes the equilibrium. Section 5 concludes.

2 Literature Review

The traditional focus of misreporting models has been on single-firm misreporting (Strobl 2013, p. 452). An important step towards multi-firm reporting was taken by Leuz et al. (2007), who "unpacked" the CAPM by rewriting returns in terms of future cash flows, and introduced accounting as signals of these future cash flows. Subsequent studies such as Strobl (2013) built on Leuz et al. (2007) by allowing managers to manipulate these accounting signals. The key feature tying all the firms together is CAPM's exogenous correlation among the firms' cash flows, which drives both misreporting and investor pricing. Likewise, in Povel et al. (2007), investors' ex ante level of bullishness about the economy drives the aggregate level of misreporting.

A common feature of all the above models is that the cost of detecting misreporting is exogenously held constant. For example, the cost of detecting reporting fraud in Povel et al. (2007, p. 1229) is a constant m per firm. We relax this assumption, and, following Becker (1968), assume that public enforcers have finite fixed resources and cannot evaluate every potential violation.[3] Consequently, as more actors engage in violations, enforcers will run out of resources to detect, convict, and punish these malefactors. This enforcement thinning feature of Becker has not received sufficient attention, even in studies that apply Becker's idea to financial markets (e.g., Shleifer and Wolfenzon (2002)).

Our model generates multiplicities in equilibrium, so we offer a brief review of the literature. Interest in multiplicities arose in financial economics research from a desire

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[3]For theories of the existence and importance of a public regulator, see Rajan and Zingales (2004, Ch. 7) and Shleifer (2005).

to model excess volatility relative to fundamentals. Multiplicities imply that the same fundamentals can yield different outcomes, and this extra variation in outcomes is representative of excess volatility relative to fundamentals (e.g., Angeletos and Werning 2006). We first start a mathematical discussion of multiplicities and then show the various institutional settings to which prior economic models have applied this mathematics.

In their economics text, Mas-Colell et al. (2005, Section 17.D) note that, *"For a theorist, the best possible worlds is one in which the social situation being analyzed ... is parsimonious ... and manages to predict a unique outcome."* They then go on to explain the mathematical theory of intersection of manifolds and show that unless one is willing to impose very strong geometric restrictions on the problem, multiplicity of fixed points is likely.[4] Angeletos and Werning (2006) demonstrate this point by showing how geometrically relaxing the global games formulation removes the unique robust Nash equilibrium and introduces multiplicities: in fact, echoing Mas-Colell et al., Angeletos and Werning (2006) show that previous authors who had obtained uniqueness in global games settings had been considering only a limiting case of a much more general problem.

To date, papers exploring multiplicity in financial markets have largely focused on non-convexities in trading activities. Angeletos and Werning (2006) show that price signals improve information coordination among financially constrained traders, and that this information externality can lead to multiple equilibria. Barlevy and Veronesi (2003) show that a market with informed and uninformed traders can have price multiplicities due to the uninformed traders' backward bending demand curve: higher prices lead uninformed traders to infer higher fundamentals and demand more of the stock. [5]

A central feature of all the above papers is that they impose various exogenous re-

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[4]Uniqueness of fixed points is primarily a consequence of geometric assumptions and refinements such as convexity, which models of multiplicity relax (Cooper 1998). Even the simplest competitive 2-consumer 2-good competitive exchange equilibrium in an Edgeworth box admits multiplicity unless one makes specific geometric refinements such as "gross substitutes" utility functions (Mas-Colell et al. 1995, Section 17.D).

[5]The traders in these models are rational. Separately, note that behavioral trading patterns such as positional heuristics or herding, if present in a large measure, can also cause multiplicity and excess volatility in prices.

strictions on individual trading behavior: individual traders in Angeletos and Werning (2006) do not have enough resources to attack the asset, while individual arbitrageurs in Barlevy and Veronesi (2003) face financial constraints that limit their stock holding and maximum short positions. Traders' limited resources force them to coordinate, and this collective action externality, in conjunction with information asymmetry, leads to multiplicities and excess volatility in prices. From our perspective, however, the key institutional feature of these papers is that the asset characteristics themselves are taken as given. As a result, regulatory institutions have little role to play. The externality in our model occurs in a different part of the economy, namely the legal institutions supporting management disclosures. In fact, traders in our model are financially unconstrained and rational, which eliminates both trading externalities and the ensuing multiplicities in price dynamics.[6]

3 Model

We lay out the model and then motivate the various assumptions. We first model an economy of "good" and "bad" firms, where the bad firms can mimic the good firms' financial reports. We then solve this reporting game. We then backwards induct to a prior-period signaling game where the good firms can credibly signal their type and thus separate or opt out of the reporting game. We will show that not all good firms always will always want to opt out, and the bad firms will sometimes mimic the reports of the remaining good firms. Multiplicity of outcomes is possible in both games.

3.1 The layout of the reporting game model

We assume an economy of $N > 0$ independent firms run by independent managers. These firms operate in a capital market with rational risk-neutral price-taking investors

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[6]Rajan (2005) discusses both information and regulation. Our incremental contribution is to show with an explicit analytical model how rational investors and regulation interact.
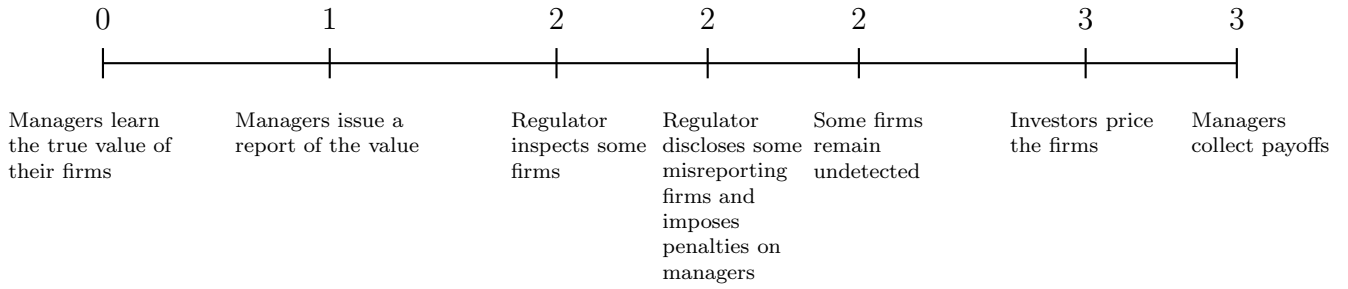
0	1	2	2	2	3	3
Managers learn the true value of their firms	Managers issue a report of the value	Regulator inspects some firms	Regulator discloses some misreporting firms and imposes penalties on managers	Some firms remain undetected	Investors price the firms	Managers collect payoffs

Figure 1: Timeline of Events in the Reporting Game

and a regulator. N_1 firms have a true valuation of 1, and $N_0 = N - N_1$ firms have a true valuation of 0.[7] This information is common knowledge. At Time 0 each manager learns the true value of his firm. Investors and the regulator are still uninformed. At time 1 all managers report the value of their firms. They have discretion to report either 1 or 0. At Time 2, the regulator investigates the firms for misreporting. It penalizes any manager who is caught misreporting, and forces him to publicly issue a restatement, which investors learn about.[8] At Time 3, the investors price all the disclosures, and the managers are paid off. Specifically, they get compensated on the stock price less any levied penalties for misreporting. Figure 1 provides a time-line.

Each manager's stock price interest in the firm is $\alpha > 0$.[9] A manager of a 0 value firm who reports 1 without being caught makes α times the price of the firm in Time 3. A manager of a 0 value firm who reports 1 and subsequently is caught makes $-c$ in Time 3 because the regulator levies the penalty, and investors drive the stock price of that firm to zero. A manager who truthfully reports 0 makes 0 in Time 3.

The 0 manager who reports 1 at Time 1 therefore computes his expected payoff at Time 3 as follows: α times expected Time 3 stock price less the expected penalty c multiplied by the probability of being caught for misreporting in Time 2.

Note that $\alpha > 0$ guarantees that no manager will misreport "down". The only possibility is the true value of 0 being misreported as 1. All parties understand this, and

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[7]See Aghion and Bolton (1992) for a model of the firm whose value has binary support.

[8]The regulator does not disclose the identity of the firms that were inspected but did not misreport.

[9]One possible interpretation of α is the manager's ownership share.

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the regulator will focus its attention exclusively on firms that report 1.

We next discuss the regulatory action at Time 2. The regulator has $K > 0$ dollars in resources, and does not know in advance which of the firms reporting 1 are misreporting. Investigating each firm is assumed to cost the regulator one dollar; so the regulator can investigate at most K firms.[10] Since all firms reporting 1 are indistinguishable from each other, the regulator chooses randomly among firms that report 1. We assume that the regulator makes no errors in the investigation process (Schwartz 1997). Each investigation will therefore reveal the true value of the firm. Therefore, the only way for a misreporting firm to avoid getting caught is if the regulator never got around to investigating it in the first place.[11] The manager of the caught firm is personally fined $c > 0$. Investors learn about it in Time 3 and drive that firm's stock price to its true value of zero.

The extensive form of the game is portrayed in Figure 2.

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[10]Section 3.2 motivates this assumption.

[11]We assume that $K < N$; otherwise we have uninteresting case that the regulator checks all firms and ensures truthful reporting throughout the economy.
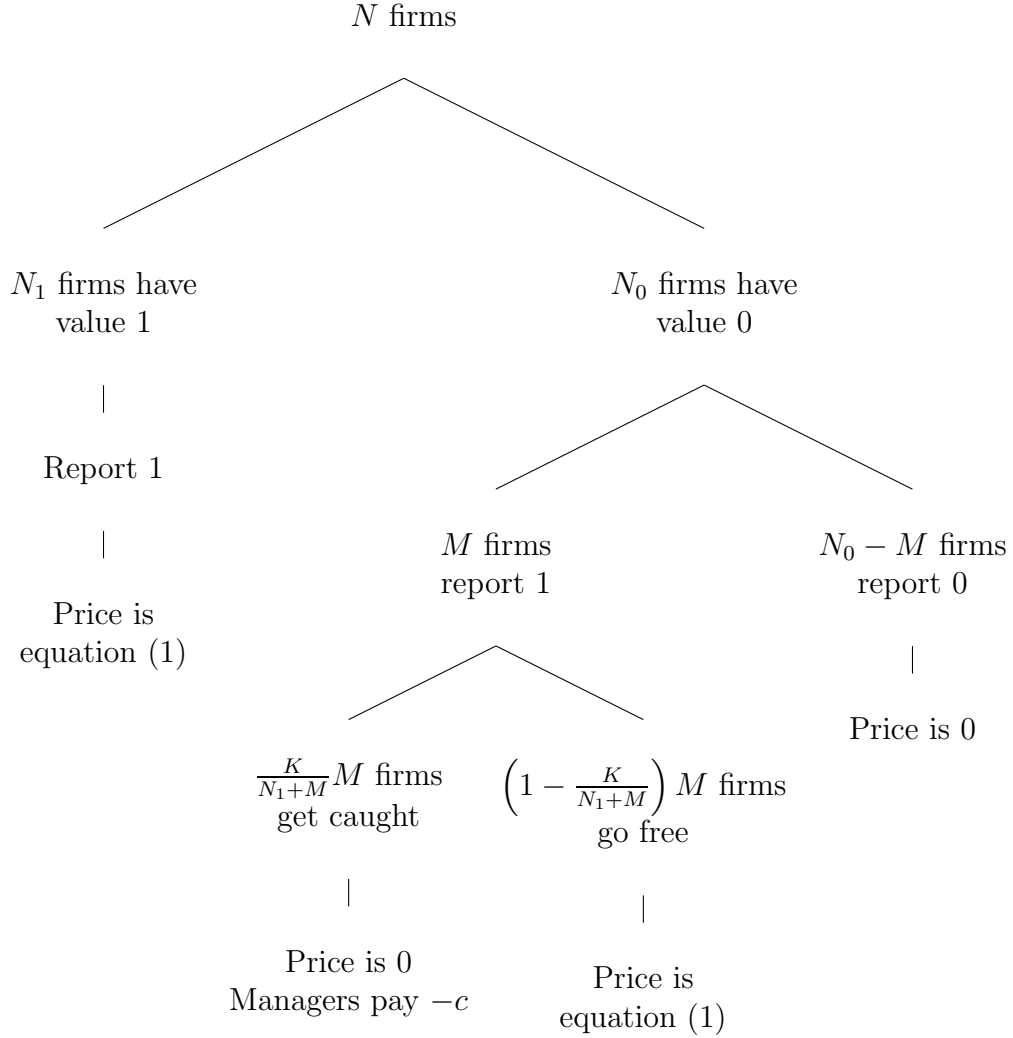
Figure 2: Payoff Tree in the Reporting Game. M, the number of 0 value firms reporting 1, is endogenous, All other variables are exogenous. N is the total number of firms. N_1 is the number of firms with value 1, and N_0 the number of firms with value 0. K is the number of firms examined by the regulator. c is the misreporting penalty, if assessed. The above tree is valid for equilibria where $M + N_1 > K$. The only feasible equilibrium value for M not satisfying this inequality is 0. In that case, no 0 value firm lies, and investors price all reports at face value.

Suppose M out of N_0 of the 0 value firms misreport 1. Our goal is to characterize the properties of M. First, note that the total number of firms reporting 1 is $N_1 + M$. The regulator can investigate K of these firms. If $K > N_1 + M$ in equilibrium, all M firms will be caught for sure. So the only possibility is $M = 0$. Therefore, if $M > 0$ in

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equilibrium, it has to be also true that the endogenous M satisfies $K < N_1 + M$. In that case, $M\frac{K}{N_1+M}$ misreporting firms will get caught, and the rest will go undetected.

The prices thus depend on whether the equilibrium $M = 0$ or the equilibrium M satisfies $K < N_1 + M$. We describe this process next. Investors know N_1, so they can check the number of firms reporting 1 to assess if equilibrium $M > 0$ or not. If the equilibrium $M = 0$, investors price a report of 0 as 0 and a report of 1 as 1. If the equilibrium $M > 0$, then $K < N_1 + M$, and investors know that $M(1 - \frac{K}{N_1+M})$ of the firms reporting 1 are worth zero. So they will price a report of 1 as:

$$\frac{N_1}{N_1 + M - \frac{KM}{N_1+M}} \tag{1}$$

Note that an equilibrium $M > 0$ must also satisfy $K < N_1 + M$, causing the price in equation (1) to be the range $(0, 1)$. In sum, in the equilibrium case $M > 0$, the manager of a 0 value firm who reports 1 at Time 1 computes the following as his expected payoff in Time 3:

$$\left(1 - \frac{K}{N_1 + M}\right) \alpha \frac{N_1}{N_1 + M - \frac{KM}{N_1+M}} + \left(\frac{K}{N_1 + M}\right)(-c) \tag{2}$$

We can divide the manager's net payoff by α. We denote the scaled penalty as $C \equiv \frac{c}{\alpha}$. So the expected scaled payoff for a manager of a 0 value firm to reporting 1 is:

$$\left(1 - \frac{K}{N_1 + M}\right) \frac{N_1}{N_1 + M - \frac{KM}{N_1+M}} + \left(\frac{K}{N_1 + M}\right)(-C) \tag{3}$$

On the other hand, a manager who reports 0 is always valued as 0 because managers have no incentives to report down (we will discuss this point later in Section 4). The scaling therefore applies to this manager as well.

3.2 Motivating the reporting model's assumptions

We first discuss the valuation process. The binary valuation model we use has a rich precedence in the finance literature (see Hart (1995, Ch.6) and Aghion and Bolton (1992)). This model is flexible enough to accommodate the needed reporting and regulatory actions in the front end while retaining a tractable model of investor valuation at the back end. In addition, the results generated by binary models have clear intuition that likely extends beyond these models.

In valuing a firm, investors pay attention to the outcome of the regulator's activities. The binary valuation model considerably simplifies our modeling of the regulator. Because the only misreporting possible is 0 being misreported as 1, the rational regulator need only scan the 1 firms. In addition, all 1 firms are otherwise indistinguishable. So the rational regulator can randomly pick K of them (at most). Additionally, since the firms are i.i.d, the regulator learns nothing about an unexamined firm from the examined ones. So we can assume that each firm costs the same to scan. This simple model of regulation is tractable, while reflecting Becker's (1968) idea of constrained regulation (also see Exhibit 1 of this study).

Rajan and Zingales (2004, Ch. 7) and Shleifer (2005) offer theories on the existence and importance of a public regulator. We therefore exogenously assume the existence of a public regulator. Most economies also supplement the public regulator with private auditors. The force of these auditors is zero in our model, but one could imagine adding a private cost of misreporting (Fischer and Verrecchia (2000)), which in our case would add an additional negative constant to equation (2). However, as we will show in the next section, a constant shift in the payoff structure does not in any way disturb the geometric structure of the model: we currently intersect a curve with the x-axis, and we just have to shift the x-axis up to accommodate the negative constant. This changes the computation of the exact intersection points but does not change the intuition. We therefore assume the private cost of misreporting to be zero.

Finally, our modeling of the manager incentives is standard in the accounting liter-

ature.

4   Equilibrium in the reporting game

Figure 2 shows that both the costs and benefits of misreporting vary with the number of misreporters. This can lead to multiple fixed points or equilibrium number of misreporters, as the following proposition makes explicit (the proof is in the Appendix):

Proposition 1   *1. If $N_1 \leq KC$ then there is a unique equilibrium where no 0 value firm reports its value as 1. All firms thus report truthfully.*

2. If $N_1 > KC$ then there are three cases to consider. Define:

$$\mathfrak{M} \equiv \frac{K}{2} + \frac{K}{2}\sqrt{1 + \frac{4N_1C}{N_1 - KC}} - N_1 \qquad (4)$$

(a) If $\mathfrak{M} \geq N_0$ then no 0 value firm reports its value as 1. All firms thus report truthfully.

(b) If $0 \leq \mathfrak{M} < N_0$ there are three equilibria:

  i. No 0 value firm reports its value as 1. That is, all firms report truthfully.

  ii. Exactly $\mathfrak{M}$ 0 value firms report their values as 1 (and the others report 0).

  iii. All 0 value firms report their values as 1.

(c) If $\mathfrak{M} < 0$ then there is a unique equilibrium where all 0 value firms report their value as 1.

First, note that N_1 is common knowledge. Therefore, by examining the number of firms reporting 1, everyone can ascertain the realized equilibrium in the multiple equilibrium case. Second, recall that K is the number of firms that the regulator can scan, and C is the penalty that the regulator imposes on a manager. KC can thus be viewed as a measure of regulatory strength. Intuitively, one would expect that all

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0 value firms will misreport when regulatory costs are low, no firms will misreport when regulatory costs are high, and multiplicity may arise when regulatory forces are moderate. What Proposition 1 does is to quantify these thresholds precisely.[12]

The first criterion that prevents misreporting is if $N_1 < KC$, where N_1 is the number of firms truthfully reporting 1 (see Figure 2). However, as Proposition 1 shows, this is not necessarily the tightest threshold. That is, if $KC = (N_1 - \epsilon)$ where ϵ is a very small positive number, $\mathfrak{M}$ in equation (4) can be very large and uniqueness can still obtain. Only when $\mathfrak{M} \in (0, N_0)$ does multiplicity obtain.

On the opposite side, when C becomes very small, $\mathfrak{M} \approx K - N_1$ in equation (4). If $K > N_1$, then $\mathfrak{M} > 0$ and multiplicity is still possible. To see this, consider the case where no 0 value firm misreports. If one 0 value firm misreports, the regulator will catch that firm $(K > N_1)$ and assess the penalty C. This is sufficient to make no misreporting an equilibrium. The multiplicity thus depends on $\mathfrak{M}$ in equation (4) in the manner that Proposition 1 makes precise.

An important assumption in the proof is that when all firms report 1, the investors' off-equilibrium path belief is that a report of 0 means it is a 0 firm. We show that this belief satisfies the intuitive criterion, a commonly-used criterion to assess the reasonableness of off-equilibrium beliefs (Kreps 1990, Section 12.7.4). The intuitive criterion requires that if for all off-equilibrium beliefs of investors upon receiving the report 0, a 1 firm (or a 0 firm) does strictly worse than its current reporting of 1, then the only allowable off-equilibrium investor belief upon receiving 0 is that the firm is a 0 firm (or a 1 firm). However, a 0 firm cannot do strictly worse: consider the case when investors value a report of 0 at 1; then a 0 firm pays no penalty and earns 1 for telling the truth, which is more than it earns now. If a similar dominance criterion fails to apply to the 1 firm as well (e.g., when investors believe a report of 0 means 1, and a 1 firm makes more by reporting 0 (net of regulatory penalties) than it is making now), the intuitive criterion is then trivially satisfied. Otherwise, the only admissible possibility is that

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[12]We separate the $N_1 \leq KC$ case explicitly from $\mathfrak{M} \geq N_0$ case because $\mathfrak{M}$ could be complex or undefined when $N_1 \leq KC$.

investors value a report of 0 at 0. Our investors' off-equilibrium beliefs thus satisfy the intuitive criterion.

Figure 3 provides an illustration of all the three possibilities. The multiplicity happens when C is at a moderate level ($C = 1$ in the example). This three-solution case has two corner solutions – no 0 value firm misreports and all 0 value firms misreport. The results follow the well-known properties of multiple equilibrium models (see Section 2.12 and 6.14 of Romer (1996)). In particular, the two corner or end solutions are stable. When every 0 value firm misreports, misreporting pays more than zero, ensuring stability. When no 0 value firm misreports, misreporting pays less than zero, ensuring stability. The interior or middle solution by, contrast, is not stable. If a few 0 value firms reporting 0 were to deviate to misreporting 1, the payoffs from misreporting rise for all 0 value firms, providing an incentive for every 0 value firm to misreport. One could therefore conjecture that when C is moderate, the most likely equilibria are all 0 value firms reporting 0, or all 0 value firms reporting 1. However, the model of multiplicity itself cannot specify which equilibrium the economy really picks (Romer (1996, Sections 2.12 and 6.14)). However, we can still compare the two equilibria on certain attributes and investigate how the parameters of the model need to change to make the preferred equilibrium more likely. This point is explored further in Section 4.2.

Figure 3 also makes clear why an additional constant private cost of misreporting would not affect the intuition of the results. As of now, the payoff from misreporting is compared to the payoff from reporting 0, which is 0, i.e., the x-axis. If misreporting carries an additional constant penalty F, the comparison of the curves will not be with the x-axis in Figure 3, but with the x-axis raised by F. This complicates the computation of the $\mathfrak{M}$ but does not change the geometric intuition of multiple intersections.

To see this more clearly, consider the corner equilibrium point where all 0 value firms misreport. It is easy to visualize in Figure 3 a raised horizontal line that is below the curve $C = 0.02$ for some range of M, but ends up above the curve when M reaches its maximum value. In this case, all 0 value firms reporting 1 is not an equilibrium:
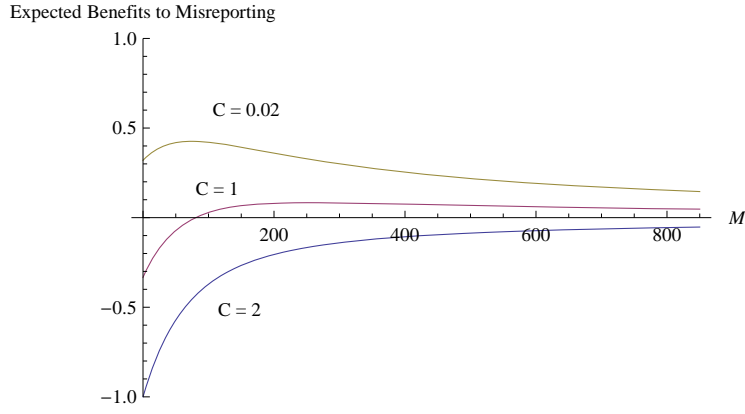
Figure 3: Expected payoffs of equation (3) to misreporting 0 as 1 as a function of M, the number of misreporting firms. $N_1 = 150, N_0 = 850, K = 100$. There are thus 150 firms with value 1 and 850 firms with value 0. The equilibrium number of misreporting firms $M \in [0, 850]$. If $C = 0.02$, all 0 value firms misreport. If $C = 2$, no 0 value firms misreport. If $C = 1$, there are three equilibria: all 0 value firms misreport, or no 0 value firms misreport, or 80.27 of the 850 firms misreport.

what is happening is that the capital market and regulatory net benefits when all 0 value firms misreport are too low for each misreporting firm relative to its private cost F to misreporting. The precise locations of the equilibrium points thus change with the introduction of private misreporting costs, but multiple intersections are still very much possible.

Finally, we demonstrate the presence of multiplicity in a pure deterrence congestion effect. In a pure crime and punishment model, the gross benefits of crime do not directly depend on the congestion effect, but the probability of getting caught does. To mimic that situation, suppose the benefits of misreporting are constant, i.e., investors (irrationally) value a 1 reports at some constant I and a 0 report at 0, and do not change their valuations subsequently. That is, the gross benefits of crime and honesty to a 0 firm are I and 0 respectively. Equation (3) would then become:

$$I - \frac{KC}{N_1 + M} \tag{5}$$

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This equation is monotonically increasing in M and will therefore yield at most one intersection with the x-axis. If such an intersection obtains, both $M = 0$ and $M = N_0$ are stable equilibria in a manner similar to Figure 3. However, the factors driving this multiplicity and the resulting comparative statics will be very different in that case, compared to the setting where there is an additional capital market force. We illustrate this fact next.

4.1   Signaling by the "good" firms prior to the reporting game

The model thus far allows for strategic action only by the 0 firms (and the regulator); the 1 firms are just bystanders. However, the equilibrium in Proposition 1 suggests that 1 firms would want to separate from the 0 firms, if possible, and receive higher valuations. We now extend the model where 1 firms can credibly signal their type. In particular, we insert a signaling game *prior* to the reporting game to allow 1 firms to separate out.

The separation of 1 firms can be effected in several ways: 1) an investor could collect information about the firm and trade on it (e.g., Grossman and Stiglitz 1980), or 2) stock price incentives for management of "good" firms drive these firms to engage in costly signaling activities such as dividend payments or building excess capacity. Investors are risk-neutral and are price-takers in our setting; in case 1) such an investor will trade infinite amounts and drive the Walrasian price to his private information estimate, thus losing his information advantage. To avoid this outcome, models with a positive cost of information acquisition typically assume non-linearity either in form of investor risk-aversion or investor pricing power. For simplicity and tractability, but without compromising on the intuition, we model case 2), assuming an exogenous signaling technology that costs $0 < s < 1$ and credibly reveals firm value. Only 1 firms will choose to employ this signaling technology (0 firms have no resources to pay s). In addition, we allow the 1 firms to signal *before* the reporting game. As a result, the regulator also observes these signals and, being rational, does not investigate the signaling firms. In other words, by paying s, any 1 firm can opt out of the reporting game and receive a sure

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valuation of $1 - s$. We investigate how this opt-out possibility changes the equilibrium.

The signaling game thus works as follows: in the first stage, all 1 firms simultaneously decide whether to signal or not (0 firms have no resources to pay the s signaling fee). Then, all firms who do not signal enter the reporting game. We now compute the Nash equilibrium for the signaling game, taking into account the equilibrium we have already derived for the reporting game.

Recall that N_0 is the intial number of 0 firms, and N_1 is the initial number of 1 firms. We wish to compute how many of the N_1 firms will opt out, and how many will remain in the reporting game. As a first backwards induction step, we compute the ex ante expected payoff to a 1 firm for choosing to stay in the reporting game. The only difficulty arises in Case 2(b) of Proposition 1, where there are three possible equilibria. The middle equilibrium, as discussed in the previous section, is unstable. So, we assume that the ex ante belief is that the two corner equilibria can occur with equal probability $\frac{1}{2}$.[13] We can then use Proposition 1 to write the ex ante value to a 1 firm for choosing to stay in the reporting game as ($\mathbb{1}$ is the indicator function that is one in the region denoted by its subscript and zero otherwise):

$$\mathbb{1}_{N_1 \leq KC} 1 + \mathbb{1}_{N_1 > KC} \left[\mathbb{1}_{\mathfrak{M} \geq N_0} 1 + \mathbb{1}_{0 \leq \mathfrak{M} < N_0} \left(\frac{1}{2} + \frac{1}{2} \frac{N_1}{N_1 + N_0 - \frac{KN_0}{N_1 + N_0}} \right) + \mathbb{1}_{\mathfrak{M} < 0} \frac{N_1}{N_1 + N_0 - \frac{KN_0}{N_1 + N_0}} \right] \tag{6}$$

For example, the first term $\mathbb{1}_{N_1 \leq KC} 1$ indicates that when $N_1 \leq KC$, the reporting payoff to a 1 firm is 1. This is Part 1 of Proposition 1. The other terms of equation (6) follow the subsequent parts of Proposition 1.

The payoff in equation (6) is plotted in Figure 4. Also plotted is the payoff $1 - s$ that a 1 firm receives for opting out by signaling. This amount is independent of the number of firms, so it is a constant. The optimal number of 1 firms choosing to opt out depends on nature of the intersections of the two payoff functions.

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[13]Our reasoning goes through with other strictly positive probability distributions as well.
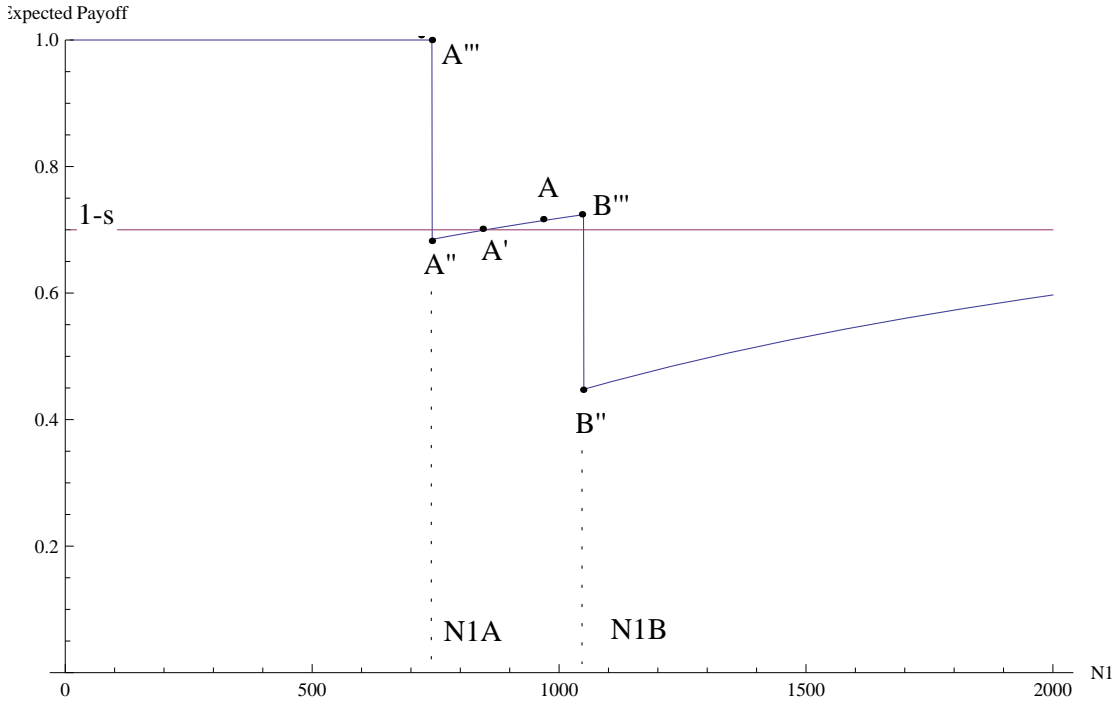
Figure 4: Equilibrium ex ante expected payoff to a 1 firm in the reporting game as in equation (6) as a function of N_1, the number of 1 firms. $N_0 = 1500, K = 350, C = 2$.

The key strategic players in the reporting game are the 0 firms, who switch their strategies at precise thresholds. This naturally leads to discontinuities in the ex ante expected payoffs to the 1 firms in equation (6) and Figure 4. Given these discontinuities, a geometric analysis is far more intuitive and approachable than an analytical one. We use Figure 4 as a guide, but we first illustrate several observations from the figure that are universal and not dependent on the specific parameters chosen in the figure. Once these observations are made, we can work with the figure in manner that is both intuitive and rigorous.

First, we note that here are two discontinuities separating the three regions delineated in Proposition 1. To identify these discontinuities, note that Equation (4) shows that $\mathfrak{M} = +\infty$ when $N_1 = KC$, and $\mathfrak{M} = -\infty$ when $N_1 = \infty$, and that $\mathfrak{M}$ is continuously decreasing in N_1. So we can think of $\mathfrak{M}$ as a function of N_1, and represent N_1 by the inverse function $\mathfrak{M}^{-1}$. The N_1 where the first discontinuity happens is therefore:

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$$N_{1A} = \mathfrak{M}^{-1}(N_0) > KC \tag{7}$$

$N_{1A} > KC$ by the intermediate value theorem. The value of N_1 where the second discontinuity happens is:

$$N_{1B} = \mathfrak{M}^{-1}(0) > \mathfrak{M}^{-1}(N_0) = N_{1A} \tag{8}$$

The discontinuities in the ex ante expected payoffs to the 1 firms for choosing to stay in the reporting game are also evident from equation (6) and Figure 4. As we cross N_{1A} from left to right in Figure 4, the payoff drops from 1 to a smaller value (we know from the proof of Proposition 1 that $K < N_1 + N_0$ in this region):

$$1 > \frac{1}{2} + \frac{1}{2} \frac{N_{1A}}{N_{1A} + N_0 - \frac{KN_0}{N_{1A}+N_0}} \tag{9}$$

Likewise, there is a discontinuous drop in payoffs as well when we cross N_{1B} from left to right in Figure 4 (we know from the proof of Proposition 1 that $K < N_1 + N_0$ in this region):

$$\frac{1}{2} + \frac{1}{2} \frac{N_{1B}}{N_{1B} + N_0 - \frac{KN_0}{N_{1B}+N_0}} > \left(\frac{1}{2} + \frac{1}{2}\right) \left(\frac{N_{1B}}{N_{1B} + N_0 - \frac{KN_0}{N_{1B}+N_0}}\right) \tag{10}$$

We can now use Figure 4 to compute the Nash equilibrium in the signaling game. In particular, for any initial N_1, we compute the equilibrium number of 1 firms who signal and opt out of the reporting game, and the number of 1 firms who remain in the subsequent reporting game, with all choices being made simultaneously. Note that the signaling firms are all 1 firms, so we just refer to them as "firms".

Consider Figure 4. If N_1, the initial number of 1 firms pre opt-outs, is such that the original equilibrium is at point A, then point A is itself a stable equilibrium: no 1 firm will want to individually ex-ante opt out of the reporting game and earn the strictly less amount $1 - s$. Subsequently, in this region, the reporting game exhibits multiplicities.

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In this situation, therefore, the ability to opt out does not remove multiplicities in the reporting game.[14]

On the other hand, if the initial N_1 is such that the payoff is slightly below A', firms will begin opting out: the ex ante payoff to staying in the reporting game is less than the opt-out payoff of $1 - s$. The above opt-out cascade ends at A''. We next show how the cascade ends in an opt out equilibrium.

By its definition in equation (7), N_{1A}, the x-coordinate of A'', will not be an integer with probability one. Let $\lfloor N_{1A} \rfloor$ be the largest integer smaller than N_{1A}.[15] One possible Nash equilibrium is as follows: let all firms be rank ordered by their publicly known CUSIP (Committee on Uniform Security Identification Procedures).[16] Firms with rank order greater than $\lfloor N_{1A} \rfloor$ opt out and earn $1 - s$. The remaining $\lfloor N_{1A} \rfloor$ firms stay in and earn 1. Given everyone else's choices, none of these latter firms will opt out, for then the opting out firm will earn $1 - s < 1$. Likewise, none of the former set of firms opting out will choose to stay in, for if one such firm does so, the number of firms in the reporting game will be $\lfloor N_{1A} \rfloor + 1 > N_{1A}$. The entry of one firm therefore will move the payoff to the reporting game to the point right of A'' (and to the left of A') and thus cause that firm to earn less than $1 - s$, the initial payoff to opting out. Thus no firm will individually deviate, and the equilibrium is Nash.[17]

The geometric way to think about the opting out behavior is to examine the payoff curve with the $1 - s$ line. If the payoff curve is initially above the $1 - s$ line, then no

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[14]We will show the existence of reporting multiplicities even when the reporting payoff at the initial N_1 is below $1 - s$.

[15]We assume that the exogenous parameters are such that $\lfloor N_{1A} \rfloor > 0$.

[16]There are $N_1!$ such possible rank orderings of the firms. The equilibrium we compute works for any such ordering.

[17]We assume that exogenous values of the parameters are such that payoff of equation (6) at the integer value $\lfloor N_{1A} \rfloor$ is greater than $1 - s$. That is, integers exist in the support of the region between 0 and A'' and the region between A'' and A'. Also note that the payoff term $\frac{N_1}{N_1 + N_0 - \frac{KN_0}{N_1 + N_0}}$ in equation (6) is not monotonic in N_1, the number of 1 firms: increasing N_1 increases the base line probability that any firm is a 1-firm, but it also decreases the probability that a misreporting 0-firm is detected. Investor valuation of a report of 1 balances both these effects. This non-monotonocity of the payoff curve can yield additional intersections with the $1 - s$ line, and assuming existence of integers in the support, similar Nash equilibria can be computed for all intersections. Finally, we recognize that we have not shown the uniqueness of our pure Nash equilibrium (modulo the ordering sequence).

firm will opt out. If the payoff curve is initially below the $1 - s$ line, firms will start opting out, till the payoff curve "intersects" the $1 - s$ line. The Nash equilibria we formulated above make the notion of intersection precise in terms of integer supports. More important, the ending point can either be in the unique reporting regime or the multiple reporting regime. The ability to opt out decreases, but does not eliminate, the possibility of multiplicity.

The reporting equilbirium with $\lfloor N_{1A} \rfloor$ 1 firms is unique. This may suggest that reporting multiplicity holds only when signaling is costly, i.e., $1 - s$ is too small. But this is not the case. If the initial number of 1 firms in Figure 4 is larger than N_{1B}, and thus each firm expects a payoff less than $1 - s$, firms will begin to opt out till the number of firms in the reporting game is $\lfloor N_{1B} \rfloor$ (assuming integer support exists in the region from A' to B'''). This is a Nash equilbrium, because following the same argument as above, no individual firm will want to deviate. Also note that this equilibrium leads to multiplicities in the reporting game.[18] The ability to opt out thus reduces, but does not eliminate, the possibility of reporting multiplicity.

In our model, therefore, prices both reflect and guide costly real decisions such as costly signaling, an idea that has had a rich history in the finance literature (see Bond et al. (2012)). To see the impact of the pricing process, note that the opt-out behavior can be very different in a pure congestion game where only the penalties, not the gross benefits of crime are typically subject to the congestion effect. For example, in equation (5), the payoff to reporting 1 is a constant I. If $I > 1 - s$, no 1 firm will opt out. If $I < 1 - s$, all 1 firms will opt out. This is an outcome substantially different from our results, suggesting that the equilibrium opt-out behavior and the resulting reporting game in our model is not a matter of pure congestion effect; it also matters how the capital market payoffs interact with the regulatory congestion. These two forces together determine the

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[18]$\lfloor N_{1B} \rfloor$ is not the unique Nash equilibrium in Figure 4, because the $1 - s$ line intersects the reporting payoff curve thrice. Again following the same argument as above, $\lfloor N_{1A} \rfloor$ is also a Nash equilibrium in that no single firm will individually deviate. The latter Nash equilibrium at $\lfloor N_{1A} \rfloor$ would not obtain if the $1 - s$ line in Figure 4 was slightly below A'', because, in that case, the $1 - s$ line would intersect the reporting payoff curve only once, not thrice.

configuration of the payoff function in Figure 4 and the resulting intersection points with $1 - s$. We next discuss the implications of our model, where the advantages of explicitly modeling capital market benefits along with congestion costs become even clearer.

4.2   Comparative statics

We first discuss the implications for the reporting game. We know that multiplicities can occur in the reporting regime. A model of multiplicity cannot predict which of the equilibria the economy will attain. However, comparative statics are still useful if we can compare the equilibrium points on certain attributes and establish what criteria make those equilibria more likely. In our setting, different equilibrium points have different welfare properties, even though investors price rationally at every equilibrium point. To see this, first note from equation (3) that investors price 0 as 0, but 1 as $\frac{N_1}{N_1 + M - \frac{KM}{N_1 + M}}$, where M is the number of misreporters. We can show that (proof in Appendix):

Lemma 1 *The valuation $\frac{N_1}{N_1 + M - \frac{KM}{N_1 + M}}$ is lower when the equilibrium level of M is higher.*

Investors thus do not react as much to a report of 1 when the equilibrium number of misreporters is high. In fact, the sensitivity of the price to earnings is maximum when $M = 0$, i.e., no firm misreports. Prices then take the reports at face value, and thus fully reflect the information available to managers. In contrast, when M is bigger, the price system does not respond as much to the earnings reports and thus does a poorer job of aggregating information in the economy.[19] It has been long known that such a price system cannot effectively direct the allocation of resources in a capitalistic economy (Hayek 1945). So even though the model cannot determine which equilibrium the economy will take, it is important to understand the conditions under which a "good" equilibrium is more likely.[20]

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[19]Dechow et al. (2010, Section 4.1) review the empirical research on cross-country variations in price response to earnings.

[20]We do not explicitly model, other than the costly opt-out signaling choice, resource allocation distortions due to uninformative prices. The extensive finance literature on how prices both reflect and guide real investment is surveyed in Bond et al. (2012). Also see Gao and Liang (2012). One pricing feature we cannot speak to is cost of capital, because our investors are risk-neutral.

Figure 4 shows that no 0 firm will misreport if $N_1 < N_{1A}$. One way to achieve this outcome is to make the signaling opt-out option more viable. For example, it has been shown that changing dividend taxes changes the propensity of firms to pay dividends (Chetty and Saez 2005; Hanlon and Hoopes 2013).[21] Of course, no improvement comes for free, and from a welfare perspective, the resource allocation benefits of informative prices have to be compared with the welfare losses arising from the extensive use of costly signaling techniques in the economy. Our model does not conduct this exercise; what we view as our model's contribution instead is that it shows how costs of ex ante signaling activities affect the equilibrium level of reporting quality and the resulting information asymmetry in the subsequent reporting game.

Another way to change the reporting game outcome is to increase regulatory resources. For example, note from equation (4) that $\mathfrak{M}$ is increasing in K and C. Consequently, a further observation is that as the misreporting penalty C increases, the points of discontinuity move to the right in Figure 4. A similar movement happens when the regulatory budget K increases; in addition, $-K$ is in the denominator of the payoff amount in equation (6), so the expected payoff to the 1 firms for choosing to stay in the reporting game also increases. It is in this *global* sense that increasing regulatory strength improves the valuation of the good firms, thereby increasing price sensitivity to earnings reports.

Increasing K has the standard interpretation of increasing regulatory resources. But increasing C has more interpretations than just increasing the penalties if caught. Recall from equations (2) and (3) that $C \equiv \frac{c}{\alpha}$. Decreasing α can therefore increase C as well. One interpretation of α is the sensitivity of the manager's total compensation to the stock price. Another possible interpretation is reporting discretion. That is, one can think of equation (2) as representing an economy where the binary payoffs are $\{\alpha, 0\}$. An economy with smaller α can thus also be viewed as an economy where managers have less reporting discretion.

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[21] One can think of the payment of the dividend tax to the government as a cost that the firm's owners incur for choosing to employ this signal.

However, note from Proposition 1 that the reporting equilibrium always has no 0 firms misreporting or all 0 firms misreporting. Furthermore, the switches in the number of equilibrium fixed points happens only when $\mathfrak{M}$ crosses any of its two discrete thresholds. However, for any given random values of the exogenous parameters, the value of $\mathfrak{M}$ is almost always strictly away from these two thresholds, implying:

Observation 1 *Because either all 0 firms or no 0 firms misreport in our equilibrium, small changes in regulatory resources or incentive regimes will almost always cause no change in the number of misreporting firms. However, when changes do happen, they will be typically be large changes in the number of misreporting firms.[22] Similarly, a small change in signaling costs either causes no change, or causes a large change in both the number of signaling and misreporting firms (e.g., when a small change in $1-s$ causes the initial point to move from above the $1-s$ line to below the $1-s$ line in Figure 4).*

More subtly, equation (6) indicates that increasing K raises the ex ante expected payoffs to the 1 firms in Figure 4, and increasing K or C shifts the location of discontinuities to the right, which also (weakly) raises the ex ante payoffs. This increase in the ex ante expected payoffs to 1 firms choosing to enter the reporting game renders the $1-s$ exit option (weakly) less profitable, implying that, in this context, one can consider the regulatory resources and signaling option as substitutes.

Some of the observations above do not explicitly rely on the multiplicity of fixed points. A significant comparative statics difference between unique equilibrium models and multiplicity models is that temporary shocks to exogenous variables in a multiplicity model can stably shift the economy from one equilibrium to another (Matsuyama 2005). This can be seen in our model as well. In Figure 4, if the economy is initially at A, a temporary change in $1-s$ can potentially be used to drive the economy to the point where only N_{1A} "good" firms remain in the reporting game. However, this new equilibrium

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[22]This observation will not always be true when there is a private cost of misreporting, because one can then obtain an interior equilibrium in the number of 0 firms misreporting. See the discussion following Figure 3.

continues to remain viable when $1 - s$ is returned back to its original value. Of course, a rigorous analysis of this process would require a full-fledged dynamic model, which is beyond the scope of this paper. Instead, we just state as a conjecture:

Conjecture 1 *A temporary change in signal benefits $1 - s$ or regulatory resources K and C can lead to permanent changes in the initial signaling and the resulting reporting equilibrium.*

4.3   Empirical implications

The above results speak to findings on cross-country empirical work on accounting quality. Initial work such as Ball et al. (2000) examined the impact of broader institutional factors such as legal origins on a country's accounting quality and found that similar institutions support different levels of accounting quality (see their Figure 1). Such results are consistent with reporting multiplicities predicted by our model. Subsequent empirical studies expanded the set of institutions considerably (e.g., Leuz et al. 2003). Our model suggests that interactions among various institutions factors is a natural area for further empirical work. For example, Burghstahler et al. (2006) examine the interactive effects of institutions and public capital markets on earnings management. Our model provides specific theoretical guidance for such empirical work on interactive effects, by showing how credible signaling activities, regulatory resources, and managerial compensation schemes together impact reporting quality. In particular, Observation 1 suggests that empirical research on institutional drivers of reporting quality may not discover any effects unless there is large variation in these drivers. There might thus be more empirical power in smaller settings, e.g., where a given country faced a large change in its institutions. Our model thus provides several empirical paths forward, despite the presence of multiplicities.

One potential signaling technique that is viewed as costly and credible, and thus relevant to our model, is dividend payments. Consistent with Observation 1, the U.S. has seen substantial shifts in aggregate dividend patterns over time, both in the number

of dividend payers and the aggregate dividends paid (Fama and French 2001). Several explanations have been advanced for this findings, a particular salient one being the emergence of repurchases as a factor that changes the costs and benefits of dividend signaling. This explanation could be viewed as changes in $1 - s$ changing the number of signalers in our model. In addition, Skinner (2008) provides descriptive evidence that the level of reported earnings is associated propensity to pay dividends. What our model suggests is that the entire reporting regime plays a significant role in aggregate signaling activities. This linkage could be worthy of further empirical investigation.

The findings of our model apply to empirical studies of individual firms as well. Specifically, our model highlights the importance of accounting for the contemporaneous activities of other firms. For example, Karpoff et al. (2008) empirically document the penalties to managers caught for misreporting. Our model suggests that the Karpoff et al. (2008) study could be extended to incorporate economy-wide factors such as regulatory resources, discretion in reporting rules, and estimates of the ex ante probability of being caught. Such empirical investigations have been undertaken by studies such as Huizinga and Laeven (2012), who recognize that the reporting choices of individual banks during the U.S. financial crisis took into account the reporting choices of other banks as well as the constraints faced by the banking regulators.

Finally, Conjecture 1 provides a policy-related suggestion as well. Much of the current policy debate has centered around the idea of strengthening regulation and curbing managerial incentives (e.g., Kashyap and Stein 2008; Blinder 2009; Department of Treasury 2009). However, the political support for such permanent regulatory expansion plans is typically far from unified (e.g., Wyatt 2010). A temporary expansion of regulatory oversight is more feasible politically, and according to our model, likely to be effective in shifting the reporting equilibrium. Likewise, Figure 4 shows that a temporary change in cost-benefits of dividend signaling (perhaps via a temporary dividend tax change) can have a permanent effect on reporting quality. Chetty and Saez (2005) and Hanlon and Hoopes (2013) show how the initiation and expiry of divided tax regime

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changed dividend payouts. What our model suggests (and this remains to be empirically tested) is that such temporary dividend tax changes could permanently change financial reporting quality.

5   Conclusion

This study builds a model where management incentives, rational traders, and regulatory constraints contribute to create an economy capable of multiplicities in aggregate reporting behavior. From an empirical perspective, this is an important theoretical result for two reasons. First, even after holding key institutional features constant, the reporting quality across similar countries shows high variation (e.g., Ball et al. 2000). A model of reporting multiplicities is a useful way of thinking about these cross-sectional phenomena. Second, the model also speaks to the time-series variation in reporting in the same country. For example, the widespread belief in the pre-crisis period in the US was that aggregate misreporting of balance sheets could not occur in rational capital markets, and any departure from this axiomatic belief (e.g., Rajan 2005) was met with considerable skepticism by mainstream economists (e.g., Knight 2005). Our model shows how these beliefs might be proven wrong. More important, our model shows that some of these outcomes have welfare implications in that new information fails to reach prices, substantially impairing the primary mechanism through which a free market system directs the allocation of its real resources.

A interesting feature of our model is that "good" firms can credibly signal their type with a costly signal and opt out of the reporting game. We show how this possibility changes, but does not eliminate, multiplicities. More interestingly, if one thinks of dividends as a credible signal, our model shows how the regulatory reporting regime links both aggregate dividend policies and reporting choices. This linkage, which demonstrates another way in which prices guide real costly decisions, could be of considerable empirical interest.

This study's approach to modeling reporting phenomena at the economy-wide level is a considerable departure from the traditional accounting analytical models that explore reporting choices in a single firm. The key tradeoff we make is to simplify the analytical treatment of each individual firm in order to aggregate the model at the economy-wide level. Such tradeoffs are necessary because single-firm models have limited relevance to the exploding empirical literature on reporting phenomena at the country-level (see Section 4 in Dechow et al.'s (2010) review). Simplifying individual-firm phenomena to speak to the aggregate is thus an important and increasingly necessary analytical endeavor (e.g., Nagar and Yu 2014).

Appendix

Proof of Proposition 1. The fundamental intuition behind the proof is present in Figure 2 which shows that both the benefits and the costs of misreporting varies with the number of misreporters. This can lead to multiple fixed points in the number of misreporters. However, as we will see, in order to identify these fixed points, one has to ennumerate a fair number of cases.

The net benefits to misreporters is a quadratic function, which we compare to 0 because our maintained assumption throughout the paper is that investors will always value a report of 0 at 0 (even when no firm reports 0 in equilibrium). In comparing a quadratic equation to zero, several cases arise because a quadratic equation can be convex or concave, and may not have real roots, and we have to check for all possibilities.

Recall that all exogenous constants $N_0, N_1, N \equiv N_0 + N_1, K, C$ are strictly positive and $K < N \equiv N_0 + N_1$.

The endogenous variable is M, where M is the number of the N_0 0 value firms that report 1 in equilibrium. That is, a total of $M + N_1$ firms report 1. Our goal is to characterize the equilibrium value of M. Specifically, we want to fully specify when M takes a corner solution and when M takes an interior solution.

Note that we also have to make sure that any solution $M > 0$ satisfies $K < N_1 + M$. This condition ensures that all probabilities and prices are positive in equation (11) below. In that case, M 0 value firms report 1, of which $\frac{KM}{N_1+M}$ get caught, and investors lump the remaining $M - \frac{KM}{N_1+M}$ with the N_1 firms that are truly worth 1, resulting in a price of $\frac{N_1}{N_1+M-\frac{KM}{N_1+M}}$ for firms that report 1. The ex ante payoff to the manager of a 0 value firm to reporting 1 is:

$$\left(1 - \frac{K}{N_1 + M}\right)\left(\frac{N_1}{N_1 + M - \frac{KM}{N_1+M}}\right) + \frac{K}{N_1 + M}(0 - C) \tag{11}$$

Note on Notation: It is easier to couch the equation (11) in terms of x where $x = N_1 + M$ is the total number of firms reporting 1. Our proof will largely use x, but occasionally for ease of exposition, we will use M. Note, however, that there is only one endogenous variable, and x and M are its two representations.

The payoff to the manager of a 0 firm from reporting 1 is:

$$\left(1 - \frac{K}{x}\right)\left(\frac{N_1}{x - \frac{K(x-N_1)}{x}}\right) + \frac{K}{x}(-C) \tag{12}$$

$$\frac{(x - K)N_1}{x^2 - Kx + KN_1} - \frac{KC}{x} \tag{13}$$

$$\frac{x^2(N_1 - KC) - xK(N_1 - KC) - KN_1KC}{(x^2 - Kx + KN_1)x} \tag{14}$$

In equilibrium, this payoff has to be ≥ 0. We therefore have to examine both the numerator and the denominator of equation (14).

We first note the following observations:

Observation 2 *The denominator of the fraction in equation (14) is positive when $x > K > 0$ because $x^2 - Kx + KN_1$ is positive when $x > K > 0$.*

We next consider the case when $x < K$.

Observation 3 *If $x < K$ in equilibrium, the regulator can inspect all firms reporting 1. No 0 firm will report 1 in that case and the equilibrium $M = 0$. Also note that equation (11) is no longer valid.*

We first attempt to ascertain the conditions under which an interior equilibrium solution M obtains. Such an equilibrium should satisfy:

1. $0 < M < N_0$

2. $K < N_1 + M$

3. $\left(1 - \frac{K}{N_1+M}\right)\left(\frac{N_1}{N_1+M-\frac{KM}{N_1+M}}\right) + \frac{K}{N_1+M}(-C) = 0$

The first condition is interiority. The second condition is that the regulator does not have enough resources to catch all misreporting firms (otherwise the equilibrium is $M = 0$). The third condition is that misreporting 1 should have the same ex ante payoff for a 0 firm as truthfully reporting 0.

Rewriting the equilibrium in terms of the endogenous variable x (the total number of firms reporting 1) the feasibility conditions become $N_1 < x < N \equiv N_0 + N_1$ and $K < x$. The equilibrium condition becomes:

$$\frac{x^2(N_1 - KC) - xK(N_1 - KC) - KN_1KC}{(x^2 - Kx + KN_1)x} = 0 \tag{15}$$

We have not yet proved the existence of an interior equilibrium point, but should such an equilibrium exist, Observation 3 applies, and we can only consider solutions for $x > K$. This implies that Observation 2 applies and the denominator in equation (15) is strictly greater than zero in an interior equilibrium (if any); we can thus clear the denominator.

We can therefore simply solve the numerator quadratic equation below to get the interior solution (if any):

$$x^2(N_1 - KC) - xK(N_1 - KC) - KN_1KC = 0 \tag{16}$$

We next investigate whether the numerator quadratic equation (16) has a solution $x > K$. The roots are:

$$\frac{K}{2} \pm \frac{K}{2}\sqrt{1 + \frac{4N_1C}{N_1 - KC}} \tag{17}$$

The roots of the above equation determine where there is an interior solution. However, note that interior solution is not the only possible solution. There are two corner solutions as well: all 0 value firms misreport ($x = N$) or no 0 value firms misreport ($x = N_1$). We have to check the feasibility of all these possibilities. We first begin with the following observation:

Observation 4 *When $N_1 - KC > 0$ equation (16) is a convex upward facing parabola in x (the second derivative with respect to x is positive). It also has two real roots because it is negative at $x = 0$. The parabola is negative between the these roots, and positive outside. By the same token, when $N_1 - KC < 0$, the parabola is downward facing and concave. The parabola has either two real roots or no real roots. If it has two real roots, it is positive between these two roots and negative outside. If it has no real roots, it is negative always (checked by evaluating the quadratic at zero). Finally, if $N_1 - KC = 0$, the quadratic is always negative with value $-KN_1KC$.*

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We now analyze all the possible solutions (interior and corner) for all the cases.

1. Case when $N_1 \leq KC$:

If $N_1 \leq KC$ then one can see from equation (17) that an interior solution greater than K is not possible: either the real roots are less than K or there are no roots. So we have excluded the interior solution. We now check for possible corner solutions. For this we need to examine the following cases:

1a. Case $N_1 \leq KC$ with real roots:

We show that the corner equilibrium $x = N > K$ cannot happen. Such an x is greater than both roots. Observation 4 tells us that the numerator quadratic of equation (15) is negative. Observation 2 tells us that the denominator of equation (15) is positive. It does not pay for any 0 firm to report 1; it is better to report 0 and gain 0. The optimal is therefore the corner solution $M = 0$; all firms report truthfully.

1b. Case $N_1 \leq KC$ with no real roots:

If there are no real roots, Observation 4 tells us the numerator quadratic in equation (15) is always less than zero for all x, and thus for all $x > K$, including the corner value $x = N > K$. This negativity condition also holds if $N_1 = KC$. Furthermore, Observation 2 yields a positive denominator. A 0 value firm is therefore always better off reporting 0 and $M = 0$.

The only equilibrium again is all firms reporting truthfully. This proves the first part of Proposition 1.

2. Case when $N_1 > KC$:

If $N_1 > KC$, we have two real roots:

$$x^* \equiv \frac{K}{2} + \frac{K}{2}\sqrt{1 + \frac{4N_1C}{N_1 - KC}} > \frac{K}{2} + \frac{K}{2} = K \tag{18}$$

$$x_* \equiv \frac{K}{2} - \frac{K}{2}\sqrt{1 + \frac{4N_1C}{N_1 - KC}} < \frac{K}{2} - \frac{K}{2} = 0 \tag{19}$$

The first root thus satisfies $x^* > K$, and by extension Observation 2. However, we cannot state that the interior equilibrium $M = x^* - N_1$ because we still do not know if $0 < x^* - N_1 < N_0$. We must check this by hand for the given values of the exogenous variables. But we first establish a simple lemma on the numerator quadratic (equation 16) based on Observation 4:

Lemma 2 *If $N_1 > KC$, the quadratic equation (16), $x^2(N_1 - KC) - xK(N_1 - KC) - KN_1KC$, in x crosses zero with a positive slope at x^*, and remains above zero for all $x > x^*$.*

Proof: Differentiating $x^2(N_1 - KC) - xK(N_1 - KC) - KN_1KC$ with respect to x and evaluated at x^* gives us:

$$2x^*(N_1 - KC) - K(N_1 - KC) = (2x^* - K)(N_1 - KC) > 0$$

The positive sign obtains because $x^* > K$ (this, as proved above, is a consequence of $N_1 > KC$). Since x^* is the larger root, and the parabola is convex (the coefficient $(N_1 - KC)$ on x^2 is greater than zero), it remains above zero for $x > x^*$ (Observation 4). ∎

Effectively, Lemma 2 tells us the behavior of a convex quadratic at its "right" or larger root x^*.

We now examine all possibilities for $x^* - N_1$:

2a. Case when $N_1 > KC$ and $0 \leq x^* - N_1 < N_0$:

There are three equilibria.

The first equilibrium is all firms telling the truth. If no 0 value firm is reporting 1, the equilibrium $x = N_1$, i.e., $M = 0$. Note that $x_* < 0 < x = N_1 \leq x^*$. That is, $x = N_1$ is between the two real roots. Therefore, when evaluated at $x = N_1$, the left hand side of equation (15) is negative because the numerator is $\leq$ zero (Observation 4) and the denominator is positive at $x = N_1$ (i.e., $(N_1)^2 - KN_1 + KN_1 > 0$). There is thus no benefit for a 0 value firm to reporting 1 when no other 0 value firm is. In the corner case when $x^* = N_1$, the 0 firms are indifferent, and we assume they tell the truth. Therefore all firms tell the truth.

The second equilibrium is all 0 value firms reporting 1. Now $x = N > x^*$, and the left hand side of equation (15) is positive (or zero in the corner case) because the numerator is above zero (or zero in the corner case) due to Lemma 2 and the denominator is positive (Observation 2 applies because $x = N \geq x^* > K$). There is a positive benefit for a 0 value firm to reporting 1 when all other 0 value firms are. Therefore, all firms in the economy report 1.

The third equilibrium is the knife-edge where the equilibrium $M = x^* - N_1$. That is, $M = x^* - N_1$ of the 0 value firms report 1 and the remaining 0 value firms report 0. All managers of 0 value firms earn 0 in expectation. Equation (15) as well as all the feasibility and interiority conditions are thus satisfied.

Finally, note that the number of firms reporting 1 is different in each possible equilibrium; therefore, everyone can ascertain which equilibrium the economy is in.

2b. Case when $N_1 > KC$ and $x^* - N_1 < 0$:

In this case, the minimum feasible value of x is greater than x^*, i.e., $x > x^* > K$. Therefore the left hand side of equation (15) is positive at the feasible values of x because the numerator is positive (Lemma 2) and the denominator is positive (Observation 2 applies because $x > K$). The numerator will never come back to zero as x increases all the way to N because the quadratic equation (16) has no zeroes larger than x^* (the only other zero x_* is smaller than x^*). Thus, reporting 1 is always better than reporting 0. The only equilibrium is therefore all

firms reporting 1.

2c. Case when $N_1 > KC$ and $x^* - N_1 \geq N_0$:

In this case, because $x \leq N$, the feasible range of x is $\leq x^*$. If x is in the range $x \leq K$, the regulator can inspect all firms and therefore $M = 0$. All firms report the truth.

Next consider the feasible range $N \geq x > K$. Because $x \leq x^*$ in this range, the left hand side of equation (15) is negative because Lemma 2 indicates that the numerator is below zero (or zero in the corner case). The numerator will stay below zero until x goes all the way down to the other root x_*. That root is smaller than zero and is thus below the positive feasible range of x. In addition, the denominator of equation (15) is positive because $x > K$ and Observation 2 applies. Therefore, the payoff for a 0 value firm for reporting 1 is strictly negative. Therefore no 0 value firm will report 1 and all firms in the economy report the truth. In the corner case when $x^* = N$, the 0 firms are indifferent; we assume that they report the truth. ■

Proof of Lemma 1. The key fact is that in equilibrium $K < N_1 + M$. Therefore $\frac{K}{N_1 + M} < 1$. Differentiating the denominator of the price function $\frac{N_1}{N_1 + M - \frac{KM}{N_1 + M}}$ with respect to M gives:

$$1 - \frac{KN_1}{(N_1 + M)^2} = 1 - \left(\frac{K}{N_1 + M}\right)\left(\frac{N_1}{N_1 + M}\right) > 0$$

The denominator of the price function is thus increasing in M, and the overall fraction is decreasing in M. Also note that the price function reaches its maximum value 1 when the number of misreporting firms M reaches its lowest value 0.

■

Exhibit 1

Excerpts from the S.E.C's Testimony Before The Subcommittee On Financial Services And General Government

Chairman Mary Schapiro, March 11, 2009

`http://www.sec.gov/news/testimony/2009/ts031109mls.htm`

The last year has been a wrenching time for the investors whom the SEC is charged with protecting ...

One of my very first actions as Chairman was to end the two-year "penalty pilot" program, which had required the Enforcement staff to obtain a special set of approvals from the Commission in cases involving civil monetary penalties against public companies as punishment for

securities fraud ... Another change I implemented to bolster the SEC's Enforcement program was to provide for more rapid approval of formal orders of investigation.

It is clear that, regardless of the ultimate findings of the Inspector General, the agency needs to improve its ability to process and pursue appropriately the more than 700,000 tips and referrals it receives annually ... In addition, the examination staffs of the SEC and FINRA are working together to identify better ways that incipient frauds might be detected at an early stage ...

SEC Resources

Few of the initiatives I have identified can be implemented with the SEC's existing resources. Most of this agenda will require additional funding, particularly to rebuild the agency's work-force and invest in new technologies ...

The agency has suffered a significant decline in staffing levels, due to several years of flat or declining budgets. Between 2005 and 2007, the agency lost 10 percent of its employees, a decline that inevitably affected all of the SEC's major programs ...

Yet as the SEC staff has declined, the securities markets grew dramatically. For example, since 2005 the number of investment advisers registered with the Commission has increased by 32 percent and their assets under management have jumped by over 70 percent (to now more than $40 trillion) ... The SEC oversees more than 30,000 registrants including 12,000 public companies, 4,600 mutual funds, 11,300 investment advisers, 600 transfer agencies, and 5,500 broker dealers. We do this with a total staff of 3,600 people.

In the context of such rapidly expanding markets, I believe the recent reductions in the SEC's staff seriously undermined the agency's ability to effectively oversee the markets and effectively pursue violations of the securities laws ...

Then, if we hope to restore the SEC as a vigorous and effective regulator, I believe we must go even further. The President is requesting a total of $1.026 billion for the agency in FY 2010, a 9 percent increase over the FY 2009 appropriation ... It will fund an additional 50 staff for the SEC, enhance our ability to uncover and prosecute fraud, and begin to build desperately needed technology.

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