

**Downriver Tunnel Surge Modeling**

**By**

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## Summary

SWMM modeling of flow inputs into the Downriver Tunnel system indicated the potential for hydraulic grade lines rising above ground level during conditions of high inflow. This prediction was generated in the model simulation during rapid filling conditions as the tunnel underwent a transition from an open channel to pressurized flow state. It was suspected that this prediction may be an artifact of the model's inability to correctly describe the dynamics of the transition associated with rapid filling as the EXTRAN model is not formulated to model these rapidly varied flow conditions. An investigation was performed to determine whether or not the model predictions were realistic.

A decision was made to estimate an upper bound on the surge in the upstream shaft by performing a simplified analysis that treated the filling of the tunnel as an upstream propagating surge due to filling towards the downstream end of the tunnel segment in question. This analysis indicates that the tunnel hydraulic grade line will not go to grade under the prescribed inflow condition. An analysis of the maximum air expulsion rate was also estimated from the model predictions.

In order to verify that the model was, in fact, conservative, a laboratory model was constructed that contained most of the essential features of the prototype configuration. Experiments were run at various filling rates at the downstream end and both the maximum surge and the time to reach this state were recorded. The numerical model was used to simulate these flow conditions and proved to slightly over-estimate the observed surge at the highest filling rates. At the lowest filling rate, the filling of the line was distinctly different than the conditions assumed in the numerical model and the maximum surge was significantly over-predicted for this situation. Even this lowest filling rate is greater when dynamically scaled to the prototype condition than that used in the original SWMM analysis.

It is therefore concluded that with the conservative assumptions employed in formulating the surge model and the conservative fashion in which the surge model represents the actual tunnel filling, there will be no problem with surges in the upstream shaft reaching ground elevation. In the Allen Park spur segment of the tunnel, connections to the sanitary sewer system occur at an elevation of 575 ft which is considerably below ground elevation and the numerical model does predict a surge in excess of that height. However, by extrapolating the differences between the physical and numerical models, it is not expected that this elevation will be reached even under the extreme inflow scenario assumed for the EXTRAN simulation.

## **Introduction**

The Downriver Tunnel system consists of approximately 54,000 feet of 6.5, 7.0 or 7.5 ft diameter tunnel, with an intended function to store and transport excess sanitary sewer wet weather flow to limit surcharging in the existing interceptor system. The storage tunnel consists of a lower tunnel segment approximately 26,000 feet long and an upper tunnel segment. The two are separated by an overflow weir which serves the purpose of backing water up in the upper tunnel segment before it overflows to the lower segment. The upper segment consists of approximately 14,500 feet of seven ft diameter tunnel upstream of the weir to a junction chamber in which two 6.5 ft diameter tunnel segments enter. One of these tunnel segments is 5850 ft long and ends in an eight foot diameter drop shaft while the other segment (Allen Park spur) is 7875 ft long.

Numerical modeling of the filling of the tunnel system was performed using the EXTRAN block of the USEPA Stormwater Management Model (SWMM). An extreme filling scenario was generated by assuming a uniform precipitation over the tributary area and routing the wet weather flow to various tunnel overflow points. Flow hydrographs were generated at each of these points and used with EXTRAN to model the dynamics of the filling of the tunnel. In this simulation, the hydraulic grade line in the upper tunnel segment was predicted to rise to the ground surface elevation at various locations. This occurrence was associated with the transition between open channel and pressurized flow in the tunnel system during rapid filling conditions. It is suspected that these predictions may not be physically realistic, and simply an artifact of the inability of SWMM to correctly model the dynamics of the rapidly varied flow conditions. This investigation was undertaken to determine whether or not there would be any potential for hydraulic grade lines to rise above the overflow levels during this design storm event.

## **Numerical Modeling Approach**

After an evaluation of the flow hydrographs, a decision was made to provide an upper bound on the potential for surge in the eight foot drop shaft at the upstream end of the system, a location where the maximum surge would occur. In order to accomplish this, a simplified model of the system was developed utilizing assumptions that would always make the surge more severe than in the actual system. By this approach, a prediction that the hydraulic grade line always remains below the overflow level could be used to infer that there will be no problem in the actual system for the storm event assumed in the original SWMM modeling.

The flow hydrographs at the upstream ends of both this segment and the Allen Park spur lag behind the hydrographs at intermediate and downstream locations and there is no inflow into these upper segments until after the other flows into the upper tunnel segment reach a maximum and have declined again. Therefore, the system

could be described as one filling from downstream and intermediate locations and the flow moving upstream through the tunnel to fill the upstream ends. In order to be conservative in modeling the system inflow, it was decided to model the tunnel filling at the maximum rate determined by summing the flows at all locations along the upper tunnel segment; this maximum rate is predicted to be approximately 200 cfs for a 3-inch, 2 hour rain storm over the entire Downriver collection system tributary area. Furthermore, this flow was concentrated at a single point, the furthest upstream inflow point which contributed to this peak discharge. By concentrating all flow at this point, the effect of system friction is minimized and the resulting surge would be over-estimated.

The particular overflow point in which the flow was concentrated was the 25 ft diameter Champaign and Pelham shaft into which the two 6.5 ft diameter tunnel segments connected. Along with concentrating the inflow at this point, all storage available in further downstream sections of the tunnel was ignored. Thus, the entire inflow was taken up by storage in the junction chamber and by filling the upstream tunnel segments. By ignoring the storage in the downstream sections, the predicted surge would over-estimate what would actually occur since the actual downstream storage would reduce the flow into the upstream tunnel segments.

Finally, the Allen Park spur is longer (7875 ft) than the tunnel segment of interest (5850 ft), and these tunnel segments otherwise have similar physical characteristics. The filling process would therefore be similar in both segments and the inflow would be split approximately equally between the two tunnel segments. Again, this neglects the greater storage in the Allen Park spur and therefore is conservative as the to surge in the eight ft diameter drop shaft at the upstream end of the main tunnel segment.

Under these limiting assumptions, the one upstream tunnel segment can be modeled by assuming filling from the downstream end of a 5850 ft long tunnel. The inflow rate into this tunnel is effectively half the maximum in the actual system and therefore a sustained inflow of effectively 100 cfs into the tunnel segment is prescribed in the model. Actually, the model is formulated by considering two parallel tunnel segments with exactly the same characteristics as the 5850 ft segment and an inflow rate of 200 cfs is specified. The inflow is used to both fill the 25 ft diameter shaft and the upstream tunnel segment. Once the junction chamber has filled, however, to the crest elevation of the weir between the upstream and downstream tunnel segments, the head in the shaft is assumed to be fixed at that elevation and no further storage in the shaft is considered.

The final aspect of the model formulation is controlled by specifying the nature of the tunnel filling process. At very low filling rates, the water surface elevation in the tunnel will be nearly horizontal and the tunnel will fill more or less uniformly along its length (except for the variation due to the tunnel slope). At higher filling rates, the fluid flowing upstream will move as a bore or moving hydraulic jump with a fairly

steep face. This condition will define the upper bound for surge in the upstream drop shaft once the bore reaches the upstream end of the system. Using this description of the flow, the tunnel has a sharp transition between the empty condition and the pressurized flow state. A rigid column or surge model can therefore compute the upstream propagation of the bore until it reaches the drop shaft. The inertia in the flow at this point then determines the rise of the water column in the upstream drop shaft.

### **Numerical Model Formulation**

The governing equations are developed in Appendix A. Basically, there are two phases in the flow development; 1.) the filling of the nearly horizontal tunnel, and 2.) the rise of the fluid in the upstream drop shaft. The first portion of the filling sequence is modeled by basically simultaneously solving a continuity equation for the shaft at the downstream end of the system and a momentum equation written over the length of the upstream propagating fluid column. This momentum equation includes the unsteady acceleration/deceleration of the fluid, the pressure forces driving the fluid, the pipe slope, and the resistance due to wall shear and an entrance loss into the tunnel from the junction chamber. The Darcy-Weisbach equation was used to prescribe the wall shear and an entrance loss coefficient of 0.5 was assumed. Simulation results in terms of the maximum predicted surge were fairly insensitive to the values of the friction factor and the loss coefficient. As mentioned above, the simulation was performed under the assumption of two identical parallel segments to include the effect of the Allen Park spur in the analysis in a conservative fashion. The governing equations were solved by a modified Euler integration until the surge reached the upstream end of the 5850 ft segment.

At this point, the velocity in the tunnel as well as the head in the junction chamber were extracted for input into the second phase of the simulation. The formulation for this second phase is somewhat similar except that a continuity and momentum equation is developed for the 8 ft diameter drop shaft. The momentum equation considers, however, the driving force through the tunnel segment including the pressure head differences and the head losses including a second entrance loss at the drop shaft. These two equations are also integrated by a modified Euler method until the rise velocity in the drop shaft goes to zero, at which point the maximum surge level is defined. The time to achieve this is also determined in the solution.

### **Numerical Model Results**

For a sustained flow of 200 cfs, the maximum predicted hydraulic grade line elevation in the model described above was 588.9 ft. This result was insensitive to the values of the friction factor and the entrance loss coefficients. Since the ground elevation at this drop shaft is 604.8 ft, it is concluded that the surge in the shaft should never go to grade and therefore that the SWMM predictions were invalid. However, the overflow weir at the upstream end of the Allen Park spur is at an elevation of 575 ft and

the predicted hydraulic grade line is higher than the weir elevation. Since the numerical model was formulated with a set of assumptions that conservatively predicted the surge, it is not possible to conclude whether or not the predicted surge would rise to the level of the overflow weir. Therefore, a physical model was constructed to investigate differences between the numerical model predictions and observations in the lab model.

### **Physical Model**

An additional effort was undertaken to verify the nature of the model predictions including the conservative nature of some of the model assumptions. A physical model of a system similar to that described by the numerical model was constructed and tested in the hydraulic laboratory of the University of Michigan. The model was basically a drop shaft at the upstream end, a straight section of pipe, and a storage box at the downstream end to mimic the effect of the shaft. Once the box was filled, water overflowed to a drain, maintaining a constant downstream head in a manner similar to the actual system. All system components were constructed of Plexiglas so that the nature of the filling process could be clearly visualized.

Although the model system was not an exact scaled replica of the prototype situation, relative sizes of the storage elements were roughly in the same proportion as in the prototype. One important departure is that the length of the tunnel segment could not be reproduced at an appropriate scale. Considering the tunnel diameter (3.75 inches in the model versus 6.5 ft in the prototype) as the scaling parameter, a model scale ratio of 20.8 is determined. The length of the model pipe was 30.38 feet, thereby corresponding only to a length of 632 ft at the prototype scale. A major difference between the physical model and the prototype system is therefore that the pipe crown elevation at the upstream end of the model pipe is lower than the overflow elevation in the downstream storage box.

The upstream shaft had an internal diameter of 3.5 inches and was tall enough to confine all surges. The downstream storage box had a cross-sectional area of 0.39 square feet and the top of the box was 0.927 feet above the entering pipe invert. This cross-sectional area would correspond to a prototype shaft diameter of 16 ft which is less than the actual 25 ft diameter.

Requirements for dynamic similarity between model and prototype will be controlled by Froude number scaling since the front propagation in the tunnel and the drop shaft is controlled by the displacement of a free surface. Under Froude scaling, the discharges in the model and prototype are different by a factor of  $1/20.8^{2.5}$  or 0.000507. In this fashion, it is possible to compare the model discharges investigated to the prototype discharge (effectively 100 cfs in one tunnel segment). The model discharge comparing directly to this flow rate is 0.0507 cfs.

## Physical Model Procedure

The model was constructed with a slight slope from upstream to downstream end to correspond to the prototype slope specified for the upstream tunnel segment. A steady flow was established in a 2 inch diameter flexible hose and metered by a venturi meter installed in the constant head delivery system. An experiment was initiated by inserting the flexible hose into the storage box. Water began filling the system at the specified inflow rate until this box began to overflow. The system then filled until water began to rise in the upstream drop shaft. The maximum water level attained in the drop shaft and the time since the commencement of the experiment were noted. Experiments at each flow rate studied were repeated several times to ensure repeatability of the measurements and the results of these replicates were average to give a model surge and time of rise.

Because of the nature of the process for initiating the flow, some amount of air was entrained into the tunnel. Although there is also very likely air entrainment in the prototype system, this process is not included in the numerical model nor is there necessarily any connection between the model and prototype in this regard. It was observed that the specific procedure in directing the inflow into the storage box influenced the amount of air entrained and ultimately the maximum surge. After some initial experimentation, a process to minimize the air entrainment was developed and the results of repetitions of the same experiment were quite consistent. It was observed that the more air entrained, the less the maximum surge will be, an observation which is intuitive as well. Therefore, it is likely that the laboratory experiment will result in a lower surge than predicted by the numerical model which does not consider this process. This was also observed as discussed below.

Initial experiments were performed at the scaled discharge of 0.0507 cfs which corresponds to the maximum filling rate expected in the prototype. However, the magnitudes of the surge in the upstream shaft were less than an inch above the downstream overflow elevation and subject to fluctuations so that the magnitude of this incremental rise would be difficult to estimate accurately. Consequently, flows in excess of the scaled discharge were established in order to measure sufficiently large surges so that a realistic test of the numerical model predictions could be performed.

Experiments were performed at three different model discharges, 0.147, 0.226, and 0.275 cfs. All three of these flow rates are much greater than the scaled prototype discharge. At the lowest flow rate of 0.147 cfs, the pipe filled more uniformly over the length without a clearly defined front while with the two higher flow rates, a more clearly defined front was observed. A summary of the experimental results and a comparison with the computations is presented in the following table. The surges are reported relative to the invert elevation at the downstream end of the pipe.

Table 1. Comparison between observed and predicted surges in the physical model.

Q cfs	H <sub>max</sub> observed ft	H <sub>max</sub> computed ft	H <sub>mobs</sub> /H <sub>mcom</sub>	t <sub>observed</sub> seconds	t <sub>computed</sub> seconds
0.147	1.40	1.94	0.72	16.0	16.8
0.226	2.75	3.05	0.90	11.3	12.25
0.275	3.32	3.68	0.90	11.4	10.6

It is observed that at the two higher flow rates, the observed surge heights are 90 percent of the computed values. This approximately four inch difference is within the range of variation in surge heights created by different methods of injecting the discharge so it is tempting to conclude that this discrepancy may be due to the entrainment of air in the model. On the other hand, the discrepancy at the lower flow rate is much greater than the variation observed in the model associated with different methods of injecting the flow (only about an inch and a half variation) and is therefore concluded to be a real effect associated with the fact that the pipe fills more slowly and in a way inconsistent with the model description. Filling times are fairly consistently computed in all cases which numerically confirms continuity as the storage box would overflow only in the final stages of the surge process.

The comparison between the predicted and observed surges in the model indicates that the numerical model over-predicts the maximum surge at lower filling rates and it is expected that this discrepancy would be even greater at smaller discharges than those tested in the physical model. Since the numerical model predicted a surge to a higher elevation than the overflow weir at the upstream end of the Allen Park spur, the deviations between the model results are used to extrapolate a maximum surge at that location. The model results for the lowest model discharge of 0.147 cfs are used for this purpose with the expectation that this will provide an overly conservative estimate of the differences that would have been observed at the even lower discharge of 0.507 cfs. The rise of the physical model surge above the overflow weir elevation in the storage box was 0.43 ft while the equivalent surge predicted in the numerical model is 0.97 ft or the observed surge was only 44 percent of the predicted. At the prototype flow rate of 200 cfs and geometry, the surge predicted by the numerical model was 28.9 feet above the overflow elevation. Scaling this surge height down by the 44 percent would result in a maximum surge of 572.7 ft at the upstream end of the system or 12.7 ft above the overflow weir elevation of 560 ft. Although this is only a little less than the overflow weir elevation of 575 ft, the cumulative nature of the conservative assumptions made in the analysis indicates that even this surge level is not likely to be



attained. Therefore it is concluded that the surge will not reach the elevation of the overflow weir at the upstream end of the Allen Park spur.

### **Air Venting Rates**

With the numerical model substantially verified as being conservative, it is also possible to use the model predictions to estimate upper bounds on air venting rates. This rate is developed on the basis of an assumption that the next manhole in the system is not more than 600 feet from the upstream end of the system. Therefore, one only needs to look at the maximum velocity predicted in the flow as it travels the last 600 feet of the tunnel. A check of the numerical model indicates that the predicted velocities are on the order of 3 ft/s which is basically consistent with a discharge rate through the tunnel of about 100 cfs. That is, most of the inflow into the shaft is used to fill the tunnel and indeed, the model does not predict the shaft to fill completely over the time of the surge.

In a worst case scenario, the Allen Park spur will still be filling as the shorter main tunnel segment reaches its maximum surge height and stops filling. The model was used to simulate this scenario by transferring all the flow to the Allen Park spur once the main tunnel segment reached its maximum surge level. However, in this situation, the hydraulic grade line at the downstream end of the system will quickly become regulated by the overflow weir at the downstream end. Using the procedure above to estimate the air venting rate, the maximum water velocity is predicted to be 4.3 ft/s and an air venting rate of 140 cfs is predicted. Because of the conservative approach taken in the formulation of the analysis, this is likely to be substantially larger than actual.

### **Conclusions**

The laboratory model produces observations that are consistent with the numerical model in the cases where the filling rate is sufficiently rapid. However, at lower filling rates, the model significantly over-predicts the surge. Since the prototype condition is for a scaled filling rate that is even less than the lowest tested in the laboratory investigation, it is concluded that the numerical model over-predicts the surge even for the idealized conditions simulated. Coupled with the fact that the model formulation itself is developed utilizing conservative assumptions at every juncture, the predicted surge up to a level of 588.9 is likely to be substantially too high. Since this is about fifteen feet below the ground surface at this location, it is concluded that the hydraulic grade line should never go above ground level. In addition, use of the physical model results to estimate a deviation between predictions and expected surges indicates that the surge will not reach to the elevation of the overflow at the upstream end of the Allen Park spur (elevation 575 ft).

## APPENDIX - MODEL EQUATIONS

Figure 1 is a schematic of the system described in the numerical model. For purposes of visualization, the system can be thought of as two parallel tunnel segments, but only one is indicated in the figure. The coordinate system is oriented along the pipe length from the downstream end. Specific model assumptions include:

- The upstream end of the flow propagates as a surge (essentially vertical) front completely filling the entire pipe.
- There is sufficient venting that the air pressure is atmospheric everywhere in the system.
- Hydraulic losses are confined to pipe friction, an entrance loss at the downstream end of the system and a bend loss at the upstream shaft (losses coefficients of 0.5 assumed for both local losses).

Under these assumptions, the general surge equation is developed by initially ignoring the local losses and later including them in the model equations if they are appropriate for the location of the upstream moving front. An unsteady momentum equation can be written as

$$\rho A x \frac{dV}{dt} = PA - \rho g Ax \sin \theta - \tau \pi D x$$

in which the variables are defined as follows:

$\rho$  = fluid density

$A$  = cross-sectional area of pipe

$D$  = pipe diameter

$x$  = length of surge from downstream end

$V$  = fluid velocity in pipe

$P$  = pressure head on downstream end of pipe controlled by water level in the downstream shaft (determined by water surface elevation)

$g$  = gravitational acceleration

$\sin \theta$  = pipe slope

$\tau$  = wall shear stress, assumed given by Darcy-Weisbach equation as  $\frac{f}{8} \rho V^2$

Dividing the equation through by  $\rho Ax$  yields an equation for the rate of change of velocity with time:

$$\frac{dV}{dt} = \frac{gH}{x} - g \sin \theta - \frac{f V^2}{2D}$$

in which  $H$  is the head in the downstream shaft. This equation is then modified to include appropriate local losses by replacing the friction factor  $f$  by

$$f + \frac{k D}{x}$$

in which  $k$  is the appropriate local loss coefficient(s).

This equation is solved by approximating the velocity derivative in finite difference form and using a modified Euler's method to perform the integration. However, the head in the downstream shaft must also be simultaneously solved for by using a continuity equation on the shaft:

$$A_c \frac{dH}{dt} = Q_{in} - 2VA$$

in which  $A_c$  is the cross sectional area of the shaft and  $Q_{in}$  is the specified flow rate into the chamber. The factor of 2 in the last term is inserted to accommodate the flow in the two upstream tunnel segments. This above continuity equation is used to compute the filling rate in the shaft until such time as it is filled to an overflow level after which the head  $H$  is held at a constant elevation.

The two equations are solved starting from an empty tunnel configuration and integrating until the surge occupied the entire length of the entire tunnel segment. At this point, the upstream traveling surge begins to rise in the shaft at the upstream end of the system. The original momentum equation must be modified to describe this condition and a continuity equation is written on the upstream shaft to compute its rate of rise. The continuity equation is:

$$A_s \frac{dH_s}{dt} = VA$$

where  $A_s$  is the cross sectional area of the upstream shaft and  $H_s$  is the water surface elevation. The momentum equation is modified as:

$$\frac{dV}{dt} = \frac{g(H - H_m)}{L} - g \sin \theta - \frac{f V^2}{2D}$$

in which  $L$  is the total length of the tunnel. Again, the friction factor  $f$  is modified to include appropriate local losses as

$$f + \frac{k D}{L}$$

and the resulting equations are numerically integrated until the velocity goes to zero, from which the maximum surge elevation is determined.

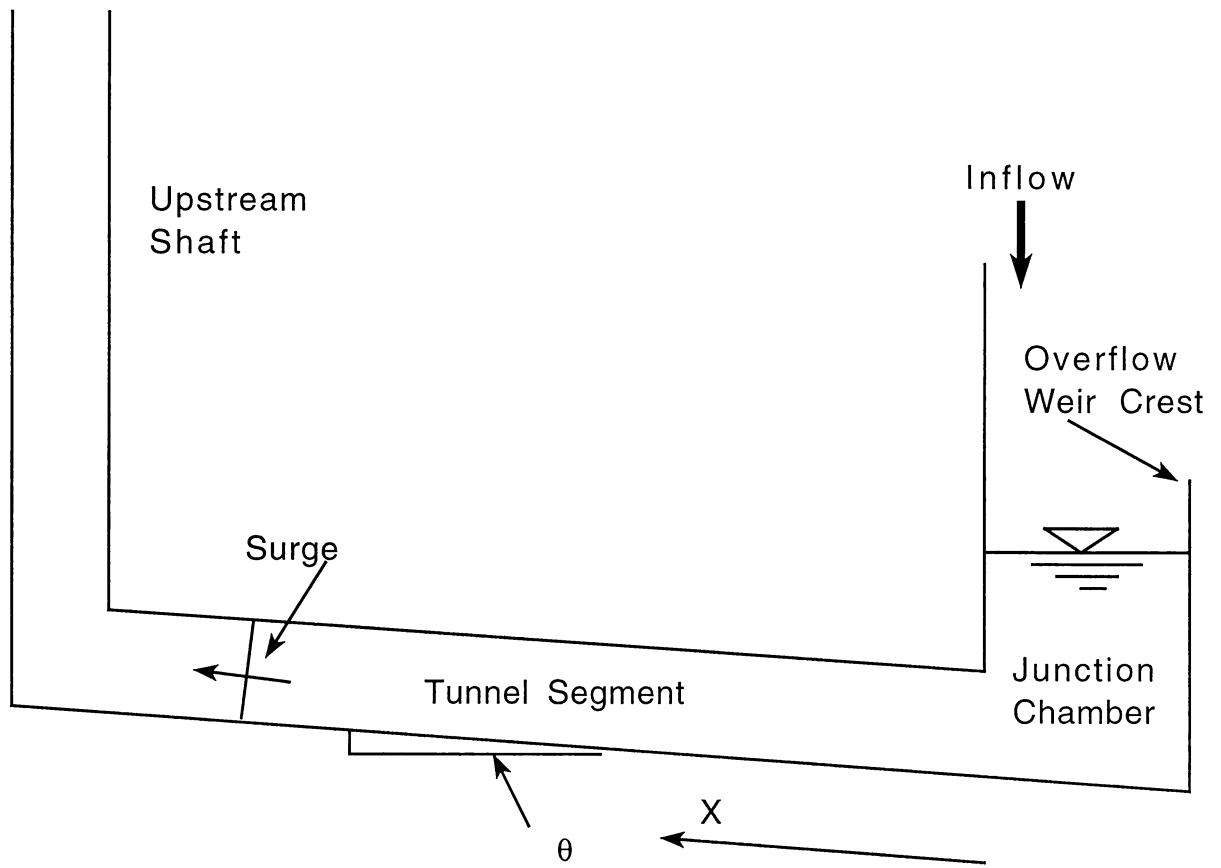


Figure 1. Schematic of system with surge filling tunnel segment.

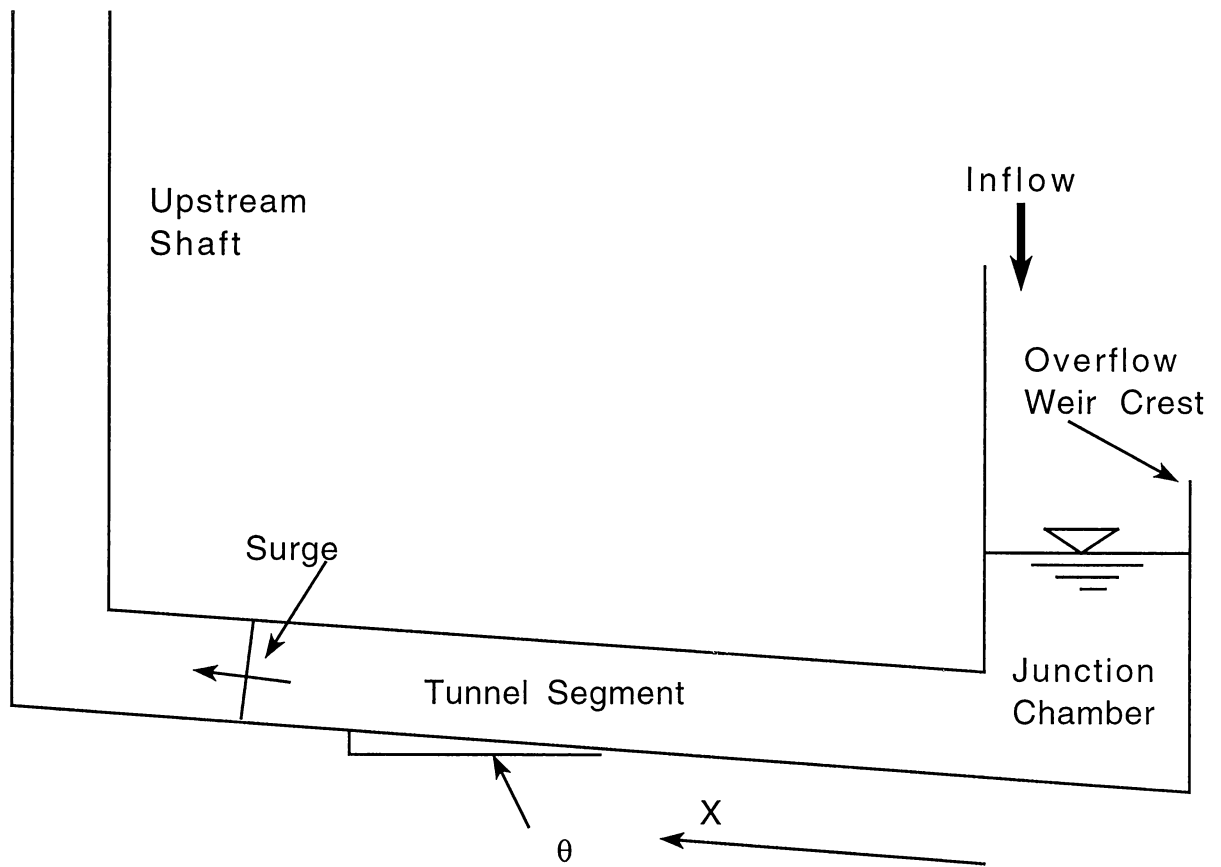


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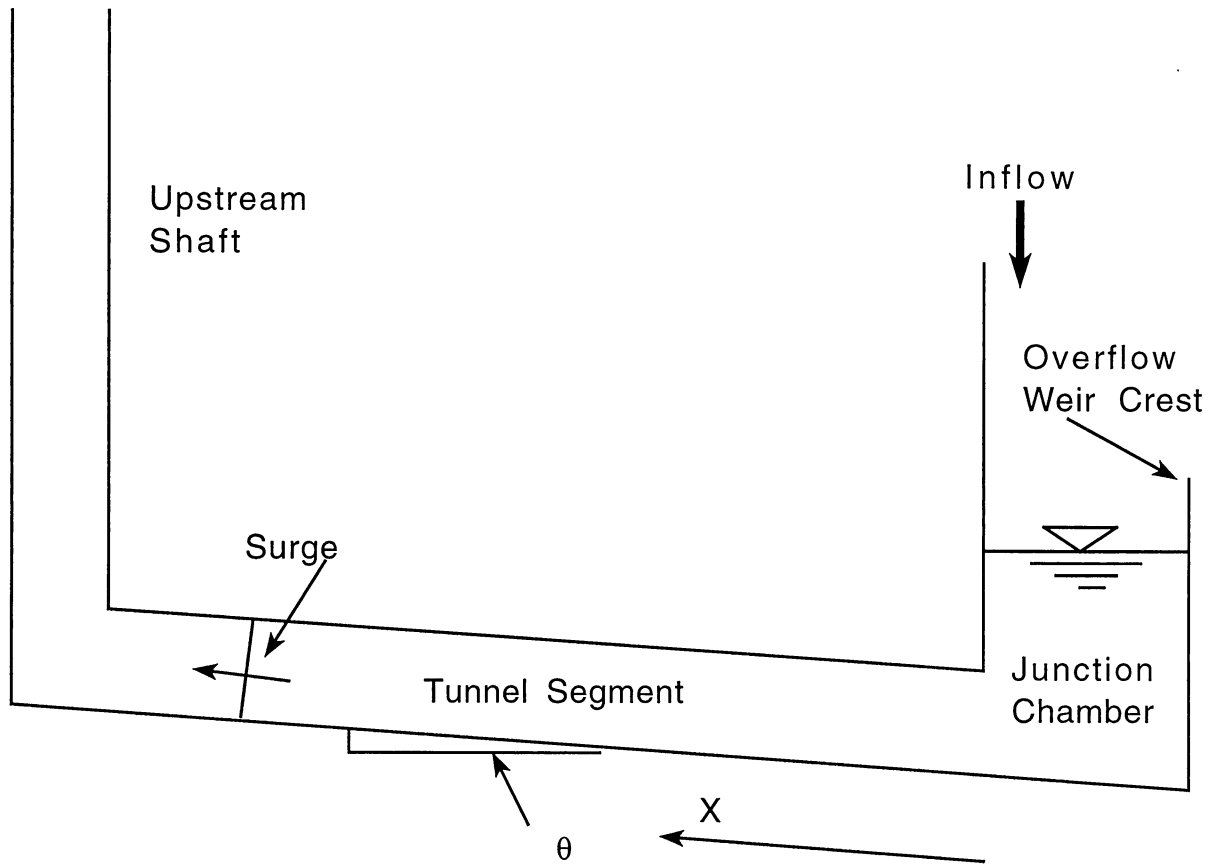


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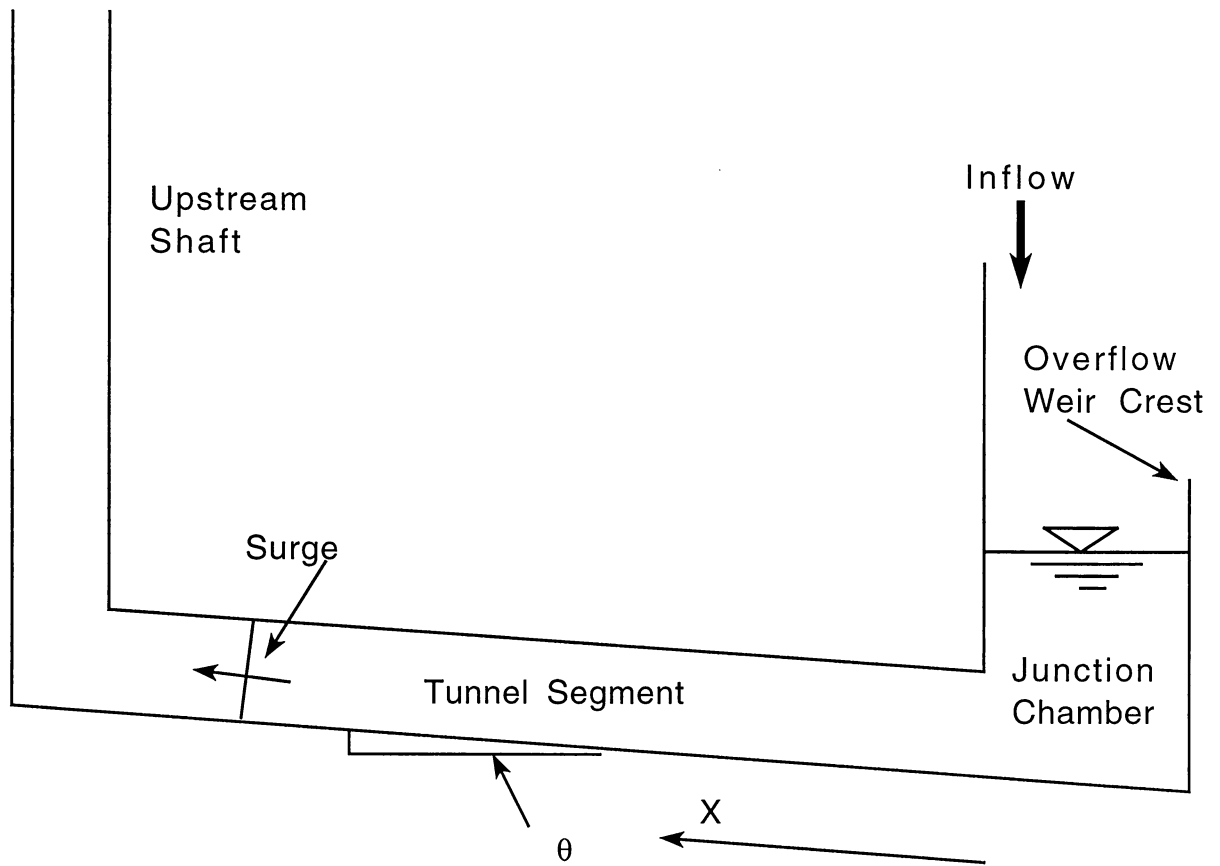


Figure 1. Schematic of system with surge filling tunnel segment.





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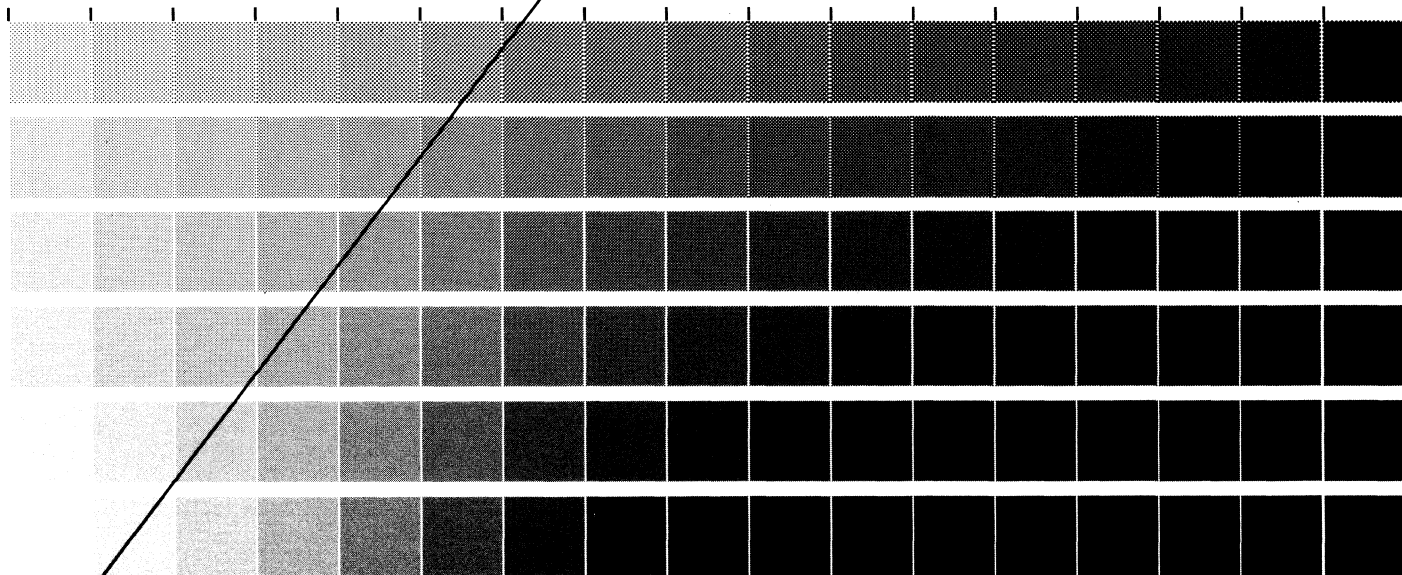
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RIT ALPHANUMERIC RESOLUTION TEST OBJECT, RT-171

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