



Market Segment Derivation and Profiling Via a Finite Mixture Model Framework

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Abstract

The Marketing literature has shown how difficult it is to profile market segments derived with finite mixture models, especially using traditional descriptor variables (e.g., demographics). Such profiling is critical for the proper implementation of segmentation strategy. We propose a new finite mixture modelling approach that provides a variety of model specifications to address this segmentation dilemma. Our proposed approach allows for a large number of nested models (special cases) and associated tests of (local) independence to distinguish amongst them. A commercial application to customer satisfaction is provided where a variety of different model specifications are tested and compared.

Key words: finite mixture models, market segmentation, concomitant variables, customer satisfaction

1. Introduction

Finite mixture models have been popular for deriving market segments (cf. Wedel and Kamakura 2000). In typical Marketing applications, one often investigates the relations of these derived market segments with a set of specified concomitant variables. This is particularly useful if segments are identified on the basis of core or basis variables that are costly to obtain such as needs, purchase, or life-style data. If the derived market segments are profiled with concomitant variables that are cheaper to collect or widely available (such as demographic data), once the segments are identified, new consumers can be classified using the demographic data only. However, profiling segments derived with finite mixture models using these concomitant variables has proven to be difficult.

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The contribution of this research is intended to be primarily methodological. We identify limitations in the assumptions on which existing finite mixture or latent class techniques for simultaneous segmentation and description are based, and propose a general approach. We first review the finite mixture models that deal with the simultaneous identification and description of market segments. Then, we provide a more general way to formulate and test finite mixture models with concomitant variables, and provide a commercial application involving an analysis of customer satisfaction data. Our approach is intended to contribute to resolving some of the difficulties of simultaneously deriving and profiling market segments using finite mixture models with concomitant variables.

2. Finite Mixture Models for Segmentation

Table 1 provides an overview of two models, the standard mixture model and the concomitant variable mixture model that have been used for simultaneous identification and description of market segments. To establish notation, we use y for the core or basis variables, x_1 for the predictor variables, and x_2 for concomitant or profiling variables. We denote the latent variable as z , with z a discrete variable with $s = 1, \dots, S$ classes/market segments. $f(\cdot)$ denotes a density function, the distribution of z is assumed to be multinomial with probabilities π_s , and $f_s(\cdot)$ is a conditional distribution given segment s , denoted as: $f(\cdot | z = s)$. In attempting to profile the segments identified with a mixture model with concomitant variables, several authors have included the y - and x_2 - variables simultaneously within the mixture framework. This approach is fairly common in latent class modeling (i.e. mixture models for discrete core variables) and is usually taken if there are no theoretical reasons to separate the variables into two types that are used to identify or discriminate segments. An application of this approach is LADI by Dillon and Mulani (1989). Other authors have proposed models that directly incorporate the concomitant variables into the specification of the mixing probabilities, the first being Dayton and McReady (1988). Similar models include those by Dillon, Kumar, and Smith de Borero (1993), Kamakura, Wedel, and Agrawal (1994), and Gupta and Chintagunta (1994). These restricted or reparameterized latent class models have been called concomitant variable mixture models.

Table 1. Comparison of Mixture Models with Concomitant Variables

	Standard Mixture ^a	Concomitant Variable
Model equation	$\sum \pi_s f_s(y) f_s(x_2)$	$\sum \pi_{s x_2} f_s(y)$
Parameterisation of class sizes	None	$\pi_{s x} = \exp(\gamma_{0s} + x_2' \gamma_s) / \sum_s \exp(\gamma_{0s} + x_2' \gamma_s)$
Independence relations	$y \perp x_2 z$	$y \perp x_2 z$
Factorisation of the likelihood	$f(y z) f(x_2 z) f(z)$	$f(y z) f(z x_2) f(x_2)$
Inference valid for	Repeated sampling	Sample values of x_2
Classification rule	Discriminant rule	Logistic rule

^a y denotes the core, x the concomitant, and z the latent variables.

It can be shown that both classes of models assume the y - and x_2 - variables to be conditionally independent given z , due to the fact that the respective likelihoods factor in a similar way. The major difference between the two approaches is that in the concomitant variable model, inference is conditional upon the observed values of the x_2 -variables in the sample, which makes it more relevant to the data at hand; while for the standard mixture model, inferences are valid under repeated sampling due to the distribution assumptions made on the concomitant variables. The *standard mixture model* identifies groups of observations and, at the same time, discriminates between such groups akin to discriminant analysis, while the *concomitant variable mixture model* does so analogous to logistic regression. However, an important limitation of the models is that they are both based on the assumption of conditional independence of the y - and x_2 -variables, given the latent (z) variables. This assumption hampers a complete investigation of the nature of the relationships of identified classes and the concomitant variables.

2. Profiling Segments with Concomitant Variables

We assume that one has collected empirical data on both predictor and concomitant variables: $x = (x_1, x_2)$, where the first affects the basis/core y -variables, and the other the latent segment memberships (x_1 and x_2 are not necessarily mutually exclusive). In many Marketing applications, such a distinction between variable types is natural: the bases/core variables measure behavioural outcomes, such as the choice of or preference for services or products. The x_1 predictor variables can represent variables that affect the behavioural outcome including characteristics of the alternatives (brands) or their (perceived) benefits. The x_2 concomitant variables may present characteristics of the consumers such as general demographic descriptors, life-styles, needs, or attitudes. Assume a sample of N consumers, with the observations for consumer n contained in the $(J \times 1)$ vector y_n . In the sequel, the subscript n is omitted for convenience of notation. The consumers are assumed to come from a population that is a mixture of S unobserved segments, C_1, \dots, C_S , in proportions π_1, \dots, π_S . Segment membership for consumer n is contained in a $(S \times 1)$ vector z , with $z_s = 1$ if $y \in C_s$, and $z_s = 0$ if $y \notin C_s$. It is assumed that z is unobserved and has a multinomial distribution: $f(z|\pi) = \prod_s \pi_s^{z_s}$, with $\pi = (\pi_s)$. The prior probabilities π_s obey: $\sum_s \pi_s = 1$ and $\pi_s > 0$. Given $z_s = 1$, the observations on variable j are described by a probability density function in the exponential family, $f_{js}(y_j|\theta_{js}, \lambda_{js})$, with $\phi = (\phi_s)$, $\phi_s = (\theta_s, \lambda_s)$, θ_{js} is a canonical parameter for group s , and λ_{js} is the dispersion parameter for group s . Here, we assume the J measurements for each consumer are independent conditional upon s , so that: $f_s(y|\phi_s) = \prod_j f_{js}(y_j|\phi_s)$. The unconditional distribution of all observed variables is:

$$f(y, x_1, x_2|\Phi) = \sum_s \pi_s f_s(y|x_1, x_2; \Phi) f_s(x_1|x_2; \Phi) f_s(x_2|\Phi), \quad (1)$$

where $\Phi = \{\pi, \phi\}$. The assumption of conditional independence of the basis/core variables and descriptor or concomitant variables is relaxed in this model. The conditional

expectations, given segment s , of the basis/core variables and descriptor variables, can be written as:

$$E(y|x_1, x_2; s) = g_y^{-1}(\mu_s + x_1' \alpha_s + x_2' \beta_s), \quad (3)$$

$$E(x_1|x_2; s) = g_{x_1}^{-1}(v_s + x_2' \gamma_s), \quad (4)$$

$$E(x_2|s) = g_{x_2}^{-1}(\kappa_s), \quad (5)$$

where $g_y(\cdot)$ and $g_x(\cdot)$ are link functions.¹

The formulation above allows for a wide range of tests of dependencies. The conditional independence of the observed variables given the latent variable can be tested via:

	Assumption	Test
A1	$y \perp x_1 z$	$\alpha_s = 0, s = 1, \dots, S$
A2	$y \perp x_2 z$	$\beta_s = 0, s = 1, \dots, S$
A3	$x_2 \perp x_1 z$	$\gamma_s = 0, s = 1, \dots, S$

Starting from the saturated model, the three assumptions A1, A2 and A3 lead to eight possible models with different conditional independence relations between the variables. In addition, the independence of the descriptor and latent variables can be tested² via:

	Assumption	Test
B1	$x_1 \perp z$	$v_s = v; s = 1, \dots, S$
B2	$x_1 \perp z$	$\kappa_s = \kappa; s = 1, \dots, S$

Not all nested models seem particularly useful; for example, it only seems appropriate to test B1 if A3 holds. Table 2 provides an overview of the more useful models and their assumptions. A number of special cases seem worthwhile mentioning. For example, Model 8, M(8), in Table 2 is the standard mixture model, with all observed variables conditionally independent, i.e., A1 to A3 all hold. Model 20, M(20), in Table 2 arises if A2, A3, B1 and B2 hold, and is the standard mixture regression model (DeSarbo and Cron 1988). Model 15, M(15), in Table 2 is equivalent to the concomitant variable mixture regression model (Kamakura, Wedel and Agrawal 1994), and arises from assumptions A2, A3, and B2.

4. A Commercial Application

We illustrate the proposed modelling framework and testing procedure by applying it to a satisfaction study conducted by a financial service provider in Europe. We use a random sample of customers of the company who were included in a satisfaction survey. This company provided us with a moderately small, random sample of the database ($N = 166$),

Table 2. Overview of Models

M	Assumptions	Model	Independence Properties
1	–	$\sum_s \pi_s f_s(y x_1, x_2) f_s(x_1 x_2) f_s(x_2)$	–
2	A1	$\sum_s \pi_s f_s(y x_2) f_s(x_1 x_2) f_s(x_2)$	$y \perp x_1 z, x_2$
3	A2	$\sum_s \pi_s f_s(y x_1) f_s(x_1 x_2) f_s(x_2)$	$y \perp x_2 z$
4	A3	$\sum_s \pi_s f_s(y x_1, x_2) f_s(x_1) f_s(x_2)$	$x_1 \perp x_2 z$
5	A1A2	$\sum_s \pi_s f_s(y) f_s(x_1 x_2) f_s(x_2)$	$y \perp (x_1, x_2) z$
6	A1A3	$\sum_s \pi_s f_s(y x_2) f_s(x_1) f_s(x_2)$	$y \perp (x_1, x_2) z; x_1 \perp x_2 z$
7	A2A3	$\sum_s \pi_s f_s(y x_1) f_s(x_1) f_s(x_2)$	$(y, x_1) \perp x_2 z$
8	A1A2A3	$\sum_s \pi_s f_s(y) f_s(x_1) f_s(x_2)$	$y \perp (x_1, x_2, z); x_1 \perp x_2 z$
9	B1A3	$f(x_1) \sum_s \pi_s f_s(y x_1, x_2) f_s(x_2)$	$x_1 \perp (x_2, z)$
10	B1A1A3	$f(x_1) \sum_s \pi_s f_s(y x_2) f_s(x_2)$	$(y, x_2, z) \perp x_1$
11	B1A1A2A3	$f(x_1) \sum_s \pi_s f_s(y) f_s(x_2)$	$y \perp x_2 z; (y, x_2, z) \perp x_1$
12	B2A2	$f(x_2) \sum_s \pi_s f_s(y x_1) f_s(x_1 x_2)$	$y \perp x_2 (z, x_1); x_2 \perp z$
13	B2A3	$f(x_2) \sum_s \pi_s f_s(y x_1, x_2) f_s(x_1)$	$x_2 \perp (x_1, z)$
14	B2A1A3	$f(x_2) \sum_s \pi_s f_s(y x_2) f_s(x_1)$	$y \perp x_1 z; x_2 \perp z$
15	B2A2A3	$f(x_2) \sum_s \pi_s f_s(y x_1) f_s(x_1)$	$(y, x_1, z) \perp x_2$
16	B2A1A2	$f(x_2) \sum_s \pi_s f_s(y) f_s(x_1 x_2)$	$y \perp x_1 z; (y, z) \perp x_2$
17	B2A1A2A3	$f(x_2) \sum_s \pi_s f_s(y) f_s(x_1)$	$y \perp x_1 z; (y, x_1, z) \perp x_2$
18	B1B2A3	$f(x_1) f(x_2) \sum_s \pi_s f_s(y x_1, x_2)$	$(x_2, x_2) \perp z$
19	B1B2A1A3	$f(x_1) f(x_2) \sum_s \pi_s f_s(y x_2)$	$(y, x_2, z) \perp x_1; x_2 \perp z$
20	B1B2A2A3	$f(x_1) f(x_2) \sum_s \pi_s f_s(y x_1)$	$(y, x_1, z) \perp x_2; x_1 \perp z$
21	B1B2A1A2A3	$f(x_1) f(x_2) \sum_s \pi_s f_s(y)$	$(y, z) \perp (x_1, x_2)$

which suffices to illustrate our proposed methodology and nested model tests. Overall satisfaction ratings as well as ratings of satisfaction determinants (attribute ratings) were collected from the consumers of this financial service provider. The purpose of the company’s study was to derive market segments on the basis of the drivers of satisfaction with the company. Therefore, overall satisfaction was used as the dependent (y) variable in the subsequent analyses. Three satisfaction drivers were derived from previous analyses and interpreted as satisfaction with convenience of the branch office, with design of the branch office, and with counter service in the branch - they serve as the explanatory (x_1) variables. Further, the profitability of the customers was assessed with respect to savings and stocks, respectively, serving as the descriptor or concomitant (x_2) variables³.

We assume all variables to be approximately normally distributed in the analysis, based on checks of the empirical distribution functions, and skewness and kurtosis measures⁴. All variables were standardized prior to analysis. The respective means and other descriptive statistics are not provided for reasons of confidentiality.

We estimate all mixture models by maximizing the likelihood function using numerical algorithms. We use 10 random starting values for the parameters for each model to identify potential local optima. All models that we report are fully identified as evidenced by positive eigenvalues of the information matrix. We investigate the appropriateness of various numbers of segments S based on the full-saturated latent classification, M(1). The CAIC statistics (Bozdogan 1987) show that the $S=2$ segment solution has the smallest CAIC value ($S=1$: CAIC = 1744.13, $S=2$: CAIC = 1741.63; $S=3$: CAIC = 1768.29; $S=4$: CAIC = 1834.79), and thus provides the most parsimonious representation of this data set. Given the relatively small size of the sample ($N = 166$), $S=2$ seems a reasonable solution that we explore further to illustrate the various model tests.

Table 3 shows the likelihood ratio (LR) statistics for the various models investigated. We start from the saturated model without any conditional independence assumptions, M(1). Imposing the independence assumptions A1 and A3 result in a significant deterioration of fit as evidenced by the LR test statistic. However, imposing A2 does not, providing evidence that $y \perp x_2 | z$: overall satisfaction is conditionally independent of profitability, as reflected in M(3). Adding A1 or A3 to M(3) yields significant LR test statistics, so that models M(5) and M(7) cannot be selected over M(3). Thus, there is no evidence for conditional independence of overall satisfaction and the satisfaction indicators, nor of the satisfaction indicators and profitability. Note that the tests of M(5) against M(2), and M(7) against M(4), also reveal that A2, which assumes conditional independence of satisfaction and profitability indicators, cannot be rejected. Thus, M(3) seems to provide the best description of the data. Further, we test $x_2 \perp z$: independence of profitability and the latent segments, by testing M(12) against M(3), which does yield a highly significant LR value indicating that B2 does not hold. Thus, this test shows that the profitability indicators are useful as segment descriptors. M(15), which is the concomitant variable mixture model, arises from M(12) by assuming conditional independence of the profitability indicators and

Table 3. Model Tests for the Satisfaction Data for $S=2$

Model	Assumpt.	#Par	ln-L	M(1) ^a	M(2)	M(3)	M(4)	M(5)	M(6)	M(7)	M(12)
M(1)	–	37	–757.746								
M(2)	A1	31	–781.632	47.77*							
M(3)	A2	33	–759.545	3.60							
M(4)	A3	25	–784.957	54.42*							
M(5)	A1A2	27	–784.324		5.38	49.56*					
M(6)	A1A3	19	–793.777		24.29*		17.64*				
M(7)	A2A3	21	–786.960			54.83*	4.01				
M(8)	A1A2A3	15	–796.985					25.32*	6.42	20.05*	
M(12)	B2A2	31	–780.045			41.00*					
M(15)	B2A2A3	19	–783.051			47.01*				7.82*	6.01

^aCell entry is the LR test statistic, *denotes $p < 0.05$.

the satisfaction drivers ($x_1 \perp x_2 | z$). It does not provide a significantly worse fit than M(12), but it does fit significantly worse than M(7), from which it arises by imposing B2: $x_2 \perp z$, independence of the profitability variables and the latent segments. Thus, the standard mixture model M(8) and the concomitant variable mixture regression model M(15) are rejected in favor of model M(3). The latter model implies that the profitability descriptors do not directly relate to overall satisfaction, but that satisfaction and profitability are conditionally independent given the segments. Thus, the various model tests substantiate the appropriateness of profitability as a segment descriptor (rather than as an outcome) variable. The model reveals that at the aggregate level, there is a relation between satisfaction and profitability that disappears if one takes the segments into account. While there are significant differences in satisfaction and profitability among segments, within a segment these two sets of variables are independent. Thus, there is a set of common variables that drive both satisfaction and profitability, but conditioning on those makes satisfaction and profitability independent. However, overall satisfaction is affected by the drivers in each segment, and the their effects differ across the segments.

Table 4 provides the parameter estimates of model M(3). The size of the first segment is 0.185, that of the second segment 0.815. Table 4 shows that in Segment 1, overall satisfaction is significantly affected by branch design, while in Segment 2 it is influenced

Table 4. Parameter Estimates for the S=2 Solution of M(3)

	Segment 1			Segment 2		
	Estimate	A.S.E. ^a	P-value	Estimate	A.S.E.	P-value
<i>y</i> -model						
μ_1	0.037	0.385	0.461	0.067	0.088	0.222
α_1	-0.086	0.189	0.324	0.160	0.084	0.029
α_2	0.860	0.266	0.001	0.090	0.093	0.167
α_3	-0.425	0.394	0.141	0.455	0.095	0.000
β_1	0.000 ^b	-	-	0.000	-	-
β_2	0.000	-	-	0.000	-	-
<i>x</i> ₁ -model						
v_1	1.594	0.388	0.000	-0.150	0.090	0.048
γ_{11}	0.659	0.163	0.000	0.106	0.122	0.194
γ_{12}	0.395	0.405	0.165	0.164	0.082	0.022
v_2	1.696	0.379	0.000	-0.186	0.091	0.021
γ_{21}	0.904	0.174	0.000	0.052	0.130	0.344
γ_{22}	-0.213	0.445	0.316	0.171	0.084	0.021
v_3	2.307	0.406	0.000	-0.256	0.090	0.002
γ_{31}	0.506	0.178	0.002	0.177	0.120	0.069
γ_{32}	1.064	0.413	0.005	0.102	0.081	0.104
<i>x</i> ₂ -model						
κ_1	-1.373	0.161	0.000	0.312	0.084	0.000
κ_2	-0.755	0.132	0.000	0.172	0.081	0.017

^aA.S.E. = Asymptotic Standard Error.

^bConstrained to zero based on nested model tests.

by satisfaction with convenience and counter service. In neither of the two segments is there an association between overall satisfaction and profitability. However, the profitability indicators are associated with the satisfaction indicators in each segment. In Segment 1, satisfaction with convenience, design and counter service are all significantly and positively associated with profitability from savings: consumers from whom more profit is derived from savings are more satisfied with each of those three dimensions. In addition, satisfaction with counter service is positively affected by profitability of stock market investments. In Segment 2, however, the profitability of customer investment in stocks and bonds is positively related to satisfaction with convenience and design. No effect of profitability on satisfaction with counter service is found for this segment. Note from the intercepts that this segment is on average less satisfied with each of the three dimensions: the segment specific intercepts of the three satisfaction dimension are all negative. The profitability means in the two segments reveal that Segment 1, the smallest segment, provides the lowest profits from savings and stocks, while the profits derived from Segment 2 are substantially higher (note that the variables were standardized prior to analysis).

5. Conclusion

Profiling derived market segments with descriptor variables is instrumental in the implementation of marketing strategy since it makes segments accessible. In spite of several efforts in the marketing literature, profiling segments derived with finite mixture models using demographic and socioeconomic variables has often been unsuccessful. We have presented a new general approach for post-hoc market segmentation studies based on finite mixture models that addresses this problem. We show how several previously used finite mixture models can be seen as nested models with varying assumptions of the conditional independence amongst the x , y , and z random variables. We provide a maximum likelihood based framework for model testing that provides insight in the interrelationships between basis/core variables, predictor variables, descriptor variables, and latent segment membership. In addition to segment description, the concomitant variables can be used to classify new observations. For example, in the application provided, a satisfaction survey may have been conducted only for a restricted sample of the customers of this financial service provider. Based on the profitability indicators, other customers not included in the survey may be classified into the two segments that were identified on the basis of the satisfaction variables, and their satisfaction profile may be inferred.

Hierarchical Bayes' (HB) methods provide an alternative approach to deal with heterogeneity in consumer response, and may also include concomitant variables (e.g. Ainslie and Rossi 1998). HB models have proven to be powerful tools to model customer response and enable one to obtain individual level inferences. But, in this paper we focus on the substantive problem of market segmentation that is better accommodated through finite mixture models. Future research aimed at comparing the performance and usefulness of these competing approaches should provide meaningful insights as to the appropriateness and usefulness of each approach in modeling consumer heterogeneity.

Notes

1. Note that for this formulation to apply, the x_1 variables need to be assumed stochastic. If the variables are not stochastic and are fixed in the design of the study (e.g., as is the case in conjoint analysis), equation (4) vanishes.
2. Note that if, say, the predictor variables, x_1 , are independent of the latent variable z , the likelihood contains the distribution $f(x_1)$. In for example, standard mixture regression formulations one usually conditions on the observed values of x_1 rather than including their marginal distribution. However, both formulations lead to the same MLE's.
3. An alternative approach would be to model profitability as an outcome of satisfaction, rather than as a segment descriptor. Although it might be argued that customer profitability is driven by satisfaction, the company that provided the data was interested in the heterogeneity in the effect of satisfaction drivers and wanted to profile the derived segments with profitability. This makes sense in view of future applications of the results. For other customers, the bank has profitability information, but not satisfaction information. Thus, our model would enable the bank to predict the satisfaction segment of customers based on profitability. Therefore, after discussion with the client company that provided us with the data, we decided to use the profitability variables as segment descriptors.
4. Note that these assumptions render our model equivalent to a mixture of recursive simultaneous equation models (Jedidi, Jagpal and DeSarbo 1997).

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