MEASURING THE CONCENTRATION OF POWER IN THE INTERNATIONAL SYSTEM

James Lee Ray

and

J. David Singer

Department of Political Science
University of Michigan

ABSTRACT

The purpose of this paper is to examine some earlier efforts to measure the inequality of distribution within several different substantive contexts and to see how appropriate these different measures might be if they were applied to the distribution of "power potential" in the international system or any of its subsystems. We discuss several measures which we find would not be appropriate. We then present a measure which we find better suited for the purpose and go on to compare it to earlier measures of inequality. Several of these turn out to be very similar to the one we present, even though they were originally devised for quite different purposes, such as measuring the degree of economic differentiation in a society. We conclude with a demonstration of the essential similarities among most of the measures discussed.

AUTHORS' NOTE: We would like to acknowledge the helpful comments of our colleague Stuart Bremer, those of two anonymous readers for this journal, and the financial support of the National Science Foundation, under grant GS-28476X1.

SOCIOLOGICAL METHODS & RESEARCH
May 1973, Volume 1, Number 4

While notions of equality and distributive justice are dear to the hearts of most social scientists, the hard fact of life is that almost nothing is equally distributed. Whether the valued object is something as tangible as income, land, or votes, or so elusive a thing as status or influence, some individuals usually have more than others. The same holds for groups, be they social classes, labor unions, political parties or nations; in any social system, the chances are slim that each will possess the same fraction or share of valued objects. Nor is the problem restricted to the distribution of "goods." Such neutral distributions as age or hair color may be of concern, as might the distribution of such "bads" as the tax burden or the incidence of crime. This ubiquitous phenomenon is, furthermore, not merely a matter of idle curiosity or a simple question of social description. To the contrary, many theoretical arguments rest heavily on the predictive or explanatory power of a given set of distributions. From the primary group up through the international system, theoretical enlightenment often flows from an understanding of the pattern in which certain values or attributes are distributed among the component units.

Measuring such inequality in these varying contexts may seem, at first blush, to be a simple and straightforward matter. In a system having two component units, for example, we need merely measure the arithmetic difference or the ratio of their percentage shares in order to get an intuitively reasonable measure of the equality of a given distribution. A more difficult problem arises, however, when we try to quantify the inequality among *more than* two units. Even if the system at hand contains only three units, there are no longer such obvious ways to measure the inequality of a given distribution. Among three groups, for example, which of the following distribution patterns is most unequal: 70%, 20%, 10%; 70%, 30%, 0%; 70%, 15%, 15%? The answer is far from clear.

The purpose of this paper is to examine some earlier efforts to measure the inequality of distribution within several different substantive contexts, and to see how appropriate these different measures might be if they were applied to the distribution of "power potential" in the international system or any of its subsystems. In the course of our discussion, we will describe the fascinating, if sometimes laborious journey we have taken through a maze of indices reflecting an impressive variety of related—but not identical—concepts. Our hope will be to share with the reader some of that fascination, while saving him most of the labor involved in tracking these various indices back to their disparate origins. We shall conclude by describing an index we have found to be especially helpful if one is dealing with a system containing a relatively small, but variable number of units or categories.

There have been several illuminating discussions of measures of inequality in the last decade, among which the one by Alker and Russett (1964) is perhaps most familiar to political scientists. Others of importance are Alker (1965), Hall and Tideman (1967), Nutter (1968), Horvath (1970), Silberman (1967), and Singer (1968). But rather than summarize all of them here, we will introduce them in context as they become relevant to the issues at hand. We begin by delineating briefly some of the more important characteristics we expect to find in a useful measure of inequality or concentration.

We should admit, and indeed will even emphasize, that the criteria for selection of an index will vary as research purposes change. Generally speaking, however, we suggest that an index should have a range of zero to one; while not an essential requirement, such a range is readily accomplished in most cases, and it makes the measure easier to interpret, and index scores easier to compare. Second, its magnitude should *in*crease if there is an upward redistribution of shares from any lower-ranked unit to any higher-ranked unit, and vice versa. Third, it should reflect the shares of *all* the units in the system and not be largely or entirely determined by the shares of only a few of the component members. Finally, it is critical that the measure react to changes in system size in a manner which is appropriate to one's theoretical concerns. Such "appropriateness" is a major preoccupation in this paper, and we focus first on that particular problem.

SENSITIVITY TO SYSTEM SIZE

Most of the indices we will discuss here satisfy the first three criteria mentioned above. However, several of them are inadequate (for our purposes) in their response to changes in system size, especially when N (referring to the number of component units, groups, or categories, regardless of how many subgroups or individuals are in each) is small. Some existing indices tend to *in*crease in value when N increases, others *de*crease, and some are relatively insensitive to the size of the system, or, alternatively, to the number of categories used. Which of these relationships between an index of inequality and system size or category number is desirable? It turns out that there is no single answer to that question; rather, it depends very much on one's substantive concern. Consider the following examples.

If nation A possesses 90% of the military aircraft within a three-nation subsystem, and the rest of the aircraft are distributed evenly between the other two, the inequality in the subsystem is certainly high. But if two or more nations entered the subsystem, and nation A managed to increase production or imports enough to maintain its share at 90%, one might say the inequality within the subsystem is higher. That is, 90% of the hardware was originally "concentrated" in the hands of 33-1/3% of the actors (i.e., nation A), while, later on, that 90% is concentrated in the hands of a mere 20% of the units. The percentage controlled by A has remained the same, as has the percentage controlled by the rest of the subsystem members. Despite this basic similarity, from one point of view it seems obvious that an increase in N has led to a greater inequality or concentration within the subsystem. If one accepts this point of view, then one would want an index of inequality that *in*creases with N.

However, if one were interested in inequality of a different kind—e.g., industrial concentration—there are good reasons for wanting a different kind of index, even if one is dealing with exactly the same distributions. If an industry was formerly concentrated entirely in the hands of three firms, and two firms are added, the concentration of this industry has decreased, even if the largest firm manages to maintain its 90% share of the market, because the entire economic pie is now divided among five rather than three firms. This situation would suggest the need for an index that *de*creases when N increases.

Finally, there may be times when any kind of sensitivity to N will be inappropriate. Lieberson (1969), discussing a concept which is admittedly different from, but still related to, inequality—i.e., "religious diversity"—suggests such a case. Suppose we are comparing the diversity between two campus fraternities. Both fraternities have ten Jews and twenty Catholics, but the second fraternity also has one Protestant. Lieberson points out that a measure which increased with N (which in this case equals the number of categories) would give a significantly higher score to the second fraternity. This "radical difference" would be misleading, as would any significant difference that resulted from an index' sensitivity to the number of categories, Lieberson maintains, because the groups have nearly identical religious composition.

These examples make it clear that, depending on one's theoretical concerns and the particular concept at hand, one may want an inequality index which rises with N, falls when N rises, or which is not sensitive to changes in system size or category number. In a recent paper of ours (Singer et al., 1972), one of the predictor variables in the model was the changing distribution of power potential among the major powers during the 1816-1965 period.[1] During that epoch, the size of the major power subset ranged between 5 and 8. In such a longitudinal analysis, where one of the goals was to ascertain the effect of changing concentration of power upon variations in the incidence of major power war, the concentration measure has to be *comparable* from one observation to the next, despite fluctuations in size. That led us into a literature search in pursuit of such a measure, and that pursuit—plus the difficulty we had in finding such a measure—led to the paper at hand.

EVALUATING SOME PREVIOUS MEASURES

While the idea of concentration or unequal distribution seems unambiguous, attempts to make it operational, verbally or mathematically, reflect a remarkably wide range of interpretations. On the other hand, the diversity is not nearly as great as the number of alternative indices might lead one to suspect. Many of the indices of inequality (as well as indices of a variety of related concepts) turn out, despite surface differences, to be functionally equivalent. One set of measures is based on the sum of squares, or ΣP_i^2 approach, and the other is based on deviations from a line of perfect equality. And in the conclusion we will show that even these two types of indices are translatable into equivalent terms.

Sum of Squares Indices

The first category contains measures known variously as indices of concentration, fractionalization, diversity, or heterogeneity. Perhaps the best known of these is the Herfindahl-Hirschman index of industrial concentration, which simply equals the sum of the squares of each unit's percentage share, or ΣP_i^2 (Herfindahl, 1950). While this measure uses information about all the units in the system, its magnitude is heavily influenced by the scores of those with the largest shares of the market. That is, the squares of small percentages are very small indeed, and thus have a disproportionately modest impact on the final index score.

To illustrate, if the shares of the market (or other valued objects) are equally distributed in a five-unit system, HH = $(20\%)^2 + (20\%)^2 + (20\%)^2 + (20\%)^2 + (20\%)^2 = .20$. If ten new units are added to the system, and each controls .1%, leaving the original five with 19.8% each, that makes virtually no impression on HH, reducing it slightly to .196. This decrease in the index score occurs despite the fact that the addition of ten almost totally deprived units to the system—and the resulting predominant position of the original five units—has, at least from one point of view, substantially *increased* the inequality in the system.

However, we should remember that HH is a measure of a special kind of inequality (i.e., industrial concentration) and one can certainly argue, as we already have, that the addition of ten new firms is "deconcentrating" even if those firms only capture a very small share of the market. This kind of reasoning leads Hall and Tideman (1967) to argue that "a measure of concentration should be a decreasing function of N." The ten new firms in this example are "underprivileged" to a marked degree, but they still have succeeded in breaking into a market formerly dominated completely by five firms. The inequality added by the ten new firms exerts an upward pressure on HH, but that is more than offset by the increase in N–i.e., an increase in the number of firms which have managed to capture a share, no matter how small, of the market.

Such sensitivity to N may be appropriate when measuring industrial concentration, but its appropriateness is at least questionable when measuring the concentration of power potential in an international system. Consider these two simple systems:

I		II	
States	% shares	States	% shares
A	90	A	85
B	5	B	5
C	5	C	2.5
		D	2.5
		E	1
		F	1
		G	1
		H	1
		I	0.5
		J	0.5
HH = .82		HH = .73	

The HH index suggests that there is more inequality in system I than in system II. In some contexts, that judgment would be acceptable, but it should be noted that, in the first system, 90% of the power potential is in the hands of the upper third of the states, whereas in the second, 90% of the power potential is controlled by an even smaller minority—i.e., the upper fifth of the states.

The results in both examples are, of course, affected by the fact that HH does not have a range of zero to one. The lower limit of HH is 1/N, which means that HH can approach zero only as N approaches infinity. When N is as small as three, or even ten, the lower limits of HH are .33 and .10, respectively. In the example immediately above, HH "starts out" from a higher level in system I than in II, and the final index score is higher in that case, even though (to repeat) 90% of the power potential in II is concentrated in a smaller proportion of its component units.

A perusal of the literature reveals that there are several measures which are similar, or algebraically identical, to HH, even though some of them are not designed to measure industrial concentration, or even inequality per se. The discussion which follows should make obvious the similarity of such concepts as inequality, concentration, fractionalization, ethnicity, diversity, and heterogeneity. Greenberg (1956), for example, presents a measure of linguistic diversity, which is basically the sum of the probabilities that two individuals randomly chosen from a population will belong to the same linguistic group.[2] If one group makes up 50% of the society, then the probability that any two individuals chosen from the total population will be chosen from *that* particular group is .50 x .50, or .25. If .25 is added to the probability of randomly choosing two individuals from each of the remaining groups (whose number and size are of consequence only when *their* probability value is being computed), the result is Greenberg's index.

But this index turns out to be identical to ΣP_i^2, where P_i in this case is the percentage share of the total population held by the ith group, rather than the percentage share of the market controlled by the ith firm. The linguistic diversity index is functionally and arithmetically equal to the Herfindahl-Hirschman index of industrial concentration.

Similar to both these measures is the Rae and Taylor (1970) index of fragmentation. Their formula is

$$1 - \frac{1}{N(N-1)} \Sigma f_i(f_i - 1),$$

where f_i equals the number in the ith subgroup and N equals the number in the total group. A slight rearrangement of terms[3] reveals that this formula is equivalent to

$$1 - \sum \frac{(f_i)}{(N)} \frac{(f_i - 1)}{(N-1)} .$$

It then becomes obvious that f_i/N is a term representing the proportion of the whole made up by the ith subgroup, and that $(f_i - 1)/(N - 1)$ is only a very slight modification of that term. In short, this measure is equivalent to—or, more precisely, the complement of—ΣP_i^2.

However, we should point out that, if one is measuring fragmentation of a comparatively *small* body, such as a committee or legislature, the second term in Rae and Taylor's equation will not be identical to the first term, and the index will deviate perceptibly from the simple ΣP_i^2. To illustrate, if there were 100 representatives in a legislative body, the contribution of a group of 25 to the fragmentation index score would be 25/100 x 24/99, or .0606, whereas if ΣP_i^2 were the index, that group's contribution to the score would be .0625. Such differences, when summed over several groups, can lead to a noticeable difference between the Rae and Taylor index and ΣP_i^2.

Furthermore, the small difference will make conceptual sense. If we reason that the measure should reflect the sum of probabilities that two individuals randomly chosen from the legislative body will belong to the same party, then the smallness of the body makes it strictly incorrect to calculate those probabilities by simply squaring f_i/N. Replacement of individuals drawn from a population, and/or an infinitely large population should not be assumed; therefore f_i/N should be multiplied by $(f_i - 1)/(N - 1)$ in order to obtain those probabilities.

Yet another index which is based on the ΣP_i^2 is the Michaely (1962) concentration index, which does not measure industrial concentration, but "tendencies toward geographic concentration in transactions" (Puchala, 1970). It shows the extent to which an actor's transactions are widely distributed throughout the system (low score), focused on a small cluster of partners (high score), or shared with a single partner only (highest score). The index formula is $100\sqrt{\Sigma(X_{sj}/X_{.j})^2}$, where X_{sj} equals j's transactions with s, and $X_{.j}$ equals the total transactions for j; $X_{sj}/X_{.j}$ is a proportion, of course, and since it is squared, the similarity of this measure to ΣP_i^2 is obvious. Taking the square root of the summed squares changes the range within which the index is most sensitive. We used this same technique in constructing the measure to be described below.

This list of measures based on ΣP_i^2 can be continued almost indefinitely. Another important one is Lieberson's (1969) index of population diversity, as are several which he discusses: the Bachi (1956) and Simpson (1949) indices of diversity, the Bell (1954) index of ecological segregation, and the Gibbs and Martin (1962) measure of diversification in an industry.

Finally we would like to mention a measure of industrial concentration which "owes its intellectual parentage to the Herfindahl Summary Index" (Horvath, 1970), but which is sufficiently different to be worth considering if the ΣP_i^2 is not deemed satisfactory. Horvath's measure,

$$CCI = x_i + \sum_{j=2}^{n} (x_j)^2(1 + [1 - x_j]),$$

where I = 1, j = 2, 3, 4, . . . , n, and n = the number of firms in the industry, and x = the decimal fraction of assets (or sales, employment, or profit) belonging to each individual firm. A detailed discussion of this measure would be out of place here, since for our purposes this measure shares the disadvantages of its intellectual parent, ΣP_i^2. Suffice it to say that this measure focuses upon—and therefore reflects—both *absolute* concentration (having to do with the smallness of the number of firms in the industry) and *relative* concentration (having to do with comparisons of the sizes of the firms in the industry, regardless of the number of firms which exist).[4]

Deviation from Equality Indices

Our second category contains perhaps the best known measures of inequality in economics and political science—the Gini index[5] and the Schutz coefficient. Both are based upon the deviation of a Lorenz curve from the "line of perfect equality." The rationale behind these measures is indeed intuitively appealing. If one constructs a graph (see Figure 1a) indicating what percentage of a good is held by each percentage of a population, and if each 1% of the population possesses 1% of the good, the Lorenz curve will fall completely on the line of perfect equality, whose slope equals one. However, if the goods are not distributed evenly, the Lorenz curve will deviate from the line of perfect equality, as in examples in Figure 1b and 1c.

The Gini index and the Schutz coefficient measure these deviations in slightly different ways. According to Alker (1965), "the Gini index sums for each individual in the population, the difference between where he is on the Lorenz curve and where he would be expected to be in the case of democratic equality." The Schutz coefficient, on the other hand, sums "ratios of advantage" for each population percentile above *or* below the equal share point. It is based on the *slope* of the Lorenz curve, and in effect reflects how close the slope of the line below the equal share point is to zero, *or* how close the slope of the line *above* the equal share point is to infinity.

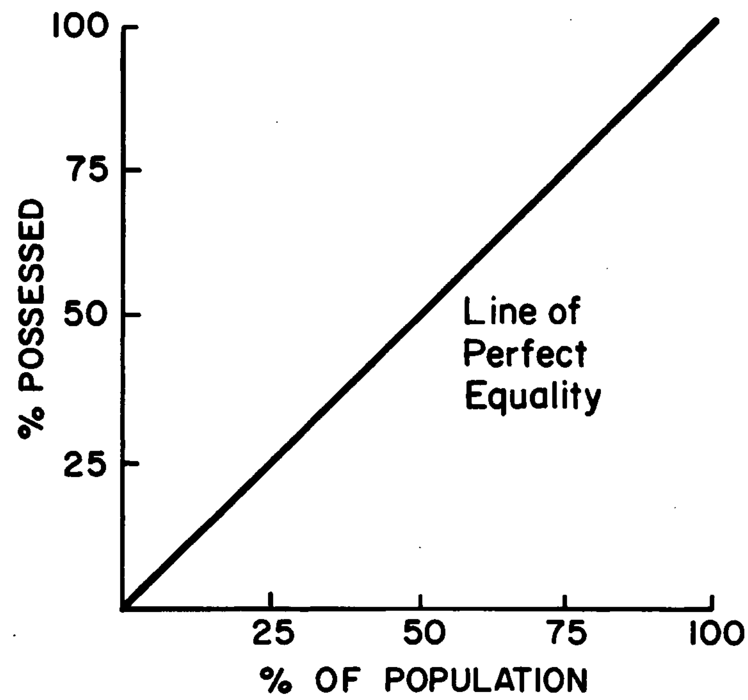


Figure 1a: LORENZ CURVE INDICATING PERFECT EQUALITY OF DISTRIBUTION

The Gini index appears to meet most of the basic requirements of a good measure of inequality. It utilizes the information available about all units, increases in response to exchanges of shares from lower- to higher-ranked units, and vice versa. However, if one is measuring the concentration of power potential in a rather small international subsystem, the Gini index has some properties which might *not* be desirable. Some of these are related to the fact that it has an upper limit of 1 – 1/N, because it was originally designed for continuous, rather than discrete, distributions. Figure 2–in which A holds all and B holds none–illustrates this problem. Even though the inequality of this situation is complete, only *half* of the "area of

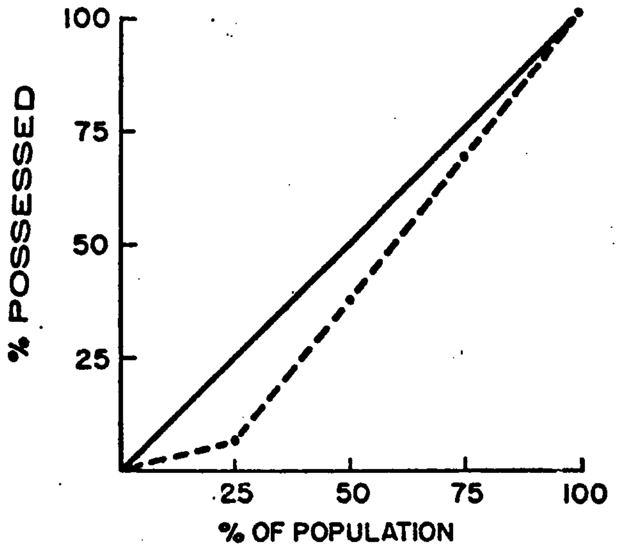


Figure 1b: LORENZ CURVE INDICATING 'SOME' INEQUALITY IN THE DISTRIBUTION

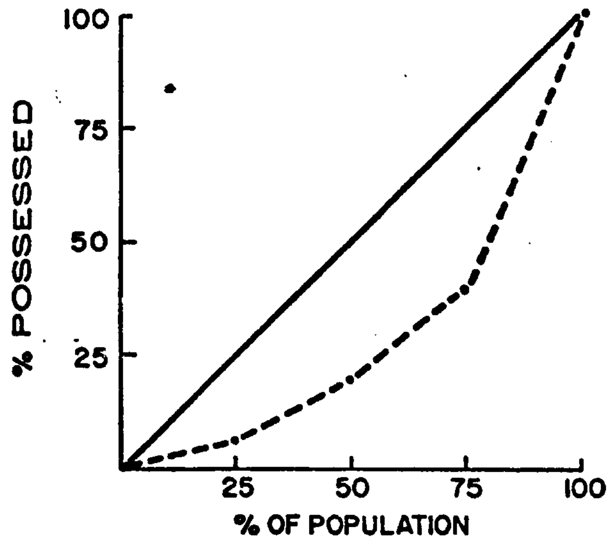
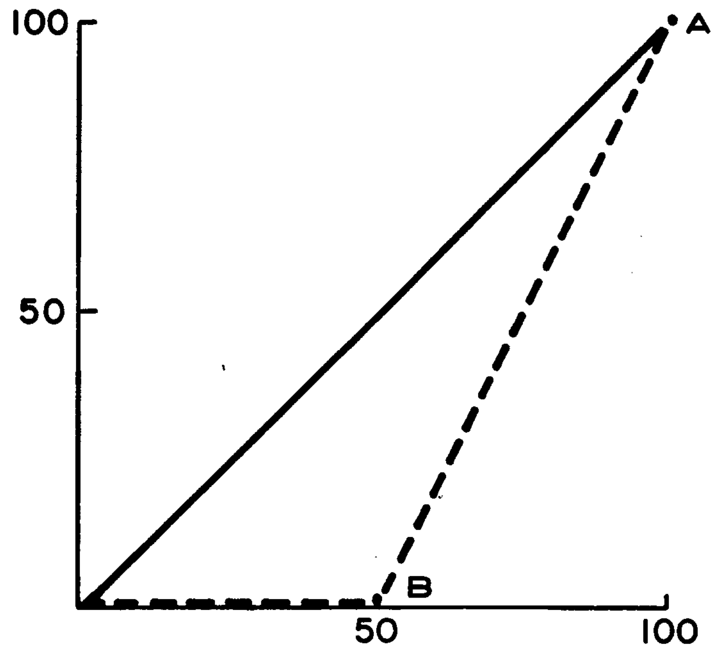


Figure 1c: LORENZ CURVE INDICATING 'GREATER' INEQUALITY IN THE DISTRIBUTION



UNITS	POSSESS
A	100%
B	0%

Gini index = $2\Sigma(X_i - Y_i)\Delta X_i$

$= 2[(.50 - 0).50 + (1.00 - 1.00).50 = .50$

Figure 2: LORENZ CURVE WITH ONE UNIT POSSESSING ALL, AND THE OTHER UNIT NONE

inequality" (the triangle below and to the right of the line of perfect equality) lies between the line of perfect equality and the Lorenz curve. Therefore, the Gini index—reflecting the percentage of the total area of inequality falling between the Lorenz curve and the line of perfect equality—equals .50, the upper limit in the two-unit case. In the three-unit case, of course, the upper limit would be .67.

This fluctuation in the upper limit of the Gini index, much like that of the lower limit of HH, can have an undesirable effect on index scores. Consider, for example, an attempt to measure the concentration of power potential at two points in time in a small international subsystem.

t_0		t_1	
States	% share	States	% share
A	90	A	70
B	6	B	15
C	4	C	10
		D	3
		E	2
Gini = .57		Gini = .59	

If the Gini index—calculated according to Alker's (1965) formula for approximations in case of discrete distributions—is used as one's measure of concentration of power potential, the subsystem appears to be marked by greater concentration at the second point in time.

In fact, however, the Gini index is slightly higher in the second case, *not* because the subsystem more nearly approaches the condition of perfect concentration (with one state so "unequal" as to control 100% of the power potential) in the latter case, but because the upper limit of the index has been increased by the addition of two states to the subsystem at that point in time. Although Gini has probably not been used in exactly this way, it would not be obviously unreasonable to make such a use of it. Our point here is that if it is used for this (or similar) purposes, it will be very sensitive to differences in system size when N is small, and the user should be sure that this sensitivity is appropriate for his purposes.

Gini Versus HH

As we have seen above, Gini has an upper limit of $1 - 1/N$, while HH (and the various similar measures) has a lower limit of $1/N$. There is at least one other striking difference which should be mentioned here. Hall and Tideman claim that if "a [system]

A has k times the number f units as [system] B, with $k > 1$, and the P_i's [percentage shares] in A are distributed such that corresponding to each P_i in B there are k units of size P_i/k, then the measure of concentration for A should be 1/k times the measure for B." The explanation is that "if each [unit] in a given [system] is divided into two [units] of equal size, the effect on a measure of concentration should be to reduce it by 1/2."

Although we agree that such a property may be desirable if industrial concentration is being measured, we are less certain that it is appropriate when measuring concentration of power potential, or other kinds of inequality. In any case, HH and the Gini index react very differently to situations such as that described above by Hall and Tideman. For example, consider the following two subsystems.

I		II	
States	% shares	States	% shares
A	40	A	20
B	30	B	20
C	20	C	15
D	10	D	15
		E	10
		F	10
		G	5
		H	5
HH = .30		HH = .15	
Gini = .25		Gini = .25	

It is readily seen that HH for system II equals 1/2 the HH score for system I, which, according to Hall and Tideman, is expected and desirable. But the Gini index remains unchanged, since in both system I and system II the upper 25% of the units control 40% of the goods, the next 25% control 30% of the goods, and so on. The "blindness" of the Gini index to the differences in these two cases is again attributable to the fact that it was not designed for discrete distributions. This difference between these indices cannot be resolved by "controlling for" N. If N is controlled for, HH = .20 in both cases, while Gini = .33 in the first case, and .29 in the second.

Another Gini "Blind Spot"

Kravis (1962) has pointed out that the Gini index assumes that "equal importance may be attached to equal absolute differences in income . . . even though one of the differences is taken between two low [value positions] and the other between two high ones." This means that very different distributions can and will generate Lorenz curves which deviate to the same extent from the line of perfect equality. The following very disparate distribution patterns, for example, both have a Gini value of .50.

I	II
70%	45%
7.5%	35%
7.5%	20%
7.5%	0%
7.5%	0%

If one is particularly interested in an inequality which tends toward monopoly (one unit controls 100%), then one would probably want system I to receive a higher score. On the other hand, if one's interest is in the extent to which the smaller units in a system are deprived, system II should receive a higher score. (Of course, no single measure can satisfy all theoretical needs, since one which would rank both of these cases "higher" is obviously not feasible.)[6]

The Schutz Coefficient

Shifting now to the Schutz coefficient, we find that it is similar to the Gini index in that its upper limit is also $1 - 1/N$. However, the reason for this may not be so intuitively clear.

The Schutz coefficient sums "ratios of advantage," and these ratios are comparisons of the holdings of the advantaged (or the disadvantaged) to the *average* percentage of holdings of all the units in the system. (Formally, the Schutz coefficient is

$$\sum_{v_i \geqq \bar{v}} \left(\frac{v_i}{\bar{v}} - 1 \right) \Delta X_i,$$

or

$$\sum_{v_i \leqq \bar{v}} \left(1 - \frac{v_i}{\bar{v}} \right) \Delta X_i,$$

where $\bar{v}$ = the average share of the units in the system; v_i = the share of the ith unit; and $\Delta X_i = X_{i+1} - X_i$, when $X_i = i/N$.) These average holdings become smaller with every increase in N. Again, in the extreme case, if the system has only two units, the *average* holding is 50% no matter how unequally the holdings may be distributed, and when N increases to 4, 5, or 10, the average holding decreases to 25%, 20%, and 10%, respectively. The larger N is, the larger is the ratio of advantage of the one unit which controls 100% of the goods, and the larger is the upper limit of the Schutz coefficient.

Perhaps the most unattractive characteristic of this measure is that it does not make use of all available information. Alker (1965) points out that Schutz can be calculated either by summing the ratios of advantage for those above the mean or by summing the ratios of *dis*advantage for those below the mean. Of course, this means that the coefficient cannot be sensitive to variations within that part of the distribution which is *not* included in the calculations. Furthermore, because the coefficient *sums* the ratios of advantage, and since different combinations of ratios can give the same sum, the coefficient can also be insensitive to variations in the distribution in the group which *is* included in the calculations. For example, the Schutz coefficient equals .50 in all three of the cases below.

I	II	III
70%	70%	70%
20%	7.5%	15%
10%	7.5%	15%
0%	7.5%	0%
0%	7.5%	0%

Before turning to our own proposed index, we should mention briefly several other measures of inequality, some of which may be excellent for certain purposes. One widely used measure of industrial concentration, CR, equals the fraction of the market held by the L largest firms, where L usually equals 4, 8, or 20. Another measure of industrial concentration, introduced by Hall and Tideman, is TH, defined as $1/(2\Sigma iP_i) - 1$, where i equals the rank of the ith largest unit, and P_i equals the percentage share of that unit. Several other measures are discussed by Alker and Russett (1964), including the ratio of the percentage controlled by the largest unit to the percentage controlled by the smallest unit, the Pareto coefficient, and the skewness of a distribution. Though it is possible that one of the measures might be ideal for some specialized purpose, we agree with Alker and Russett that in most cases they are not as generally applicable as the other measures discussed above.

THE PROPOSED INDEX OF CONCENTRATION

Before describing the index which we decided to use for our own purposes, let us restate briefly the criteria we hope to satisfy. First, we want our measure to have a range of zero to one, even at the lower end of the size spectrum, because we seek to measure concentration in both the full international system, with an N as large as 135, and in its smaller regional and functional subsystems—with Ns as low as 5. Also its magnitude should increase if there is an upward redistribution of shares from any lower-ranked to any higher-ranked unit, and vice versa. Furthermore, it should reflect the shares of *every* unit in the system, and not merely of those which fall, for example, above or below the mean. Finally, it is critical that the measure react to changes in system size in a manner which is appropriate to our theoretical concern.

With the few exceptions that we have mentioned explicitly above, most measures satisfy the first three criteria, but meeting the fourth criterion proves to be more problematical. As we have implied in the discussion above, because the measures we looked at had different *ranges* for different Ns, they appeared to us to be overly sensitive to changes in N. This led us to prefer a *standardized* measure which has the same range regardless of system size. The basic formula for our index of concentration (or CON) is: standard deviation of the percentage shares ÷ maximum possible standard deviation in a system of size N. Fortunately, this simplifies to

$$\mathrm{CON} = \sqrt{\frac{\Sigma P_i^2 - 1/N}{1 - 1/N}}$$

The maximum possible standard deviation of the percentage shares occurs when one unit controls 100% of the goods, while the rest of the units control none at all. And the minimum (zero) occurs when all the units control equal shares. Thus CON is equal to 1 whenever a single unit controls 100% of the goods, regardless of system size. It also has the virtue of taking into account information regarding all units, and it increases in value if there is a shift in shares from a lower-ranked to a higher-ranked unit, and vice versa.[7]

Now the fact that we opted for a standardized measure does not mean that we wanted or have a measure that is *totally* insensitive to system size. While the *range* of this measure is constant, and therefore insensitive, the *scores* on the index will rise and fall with system size, even when nothing else changes. For example, consider this subsystem at two points in time:

t_0		t_1	
States	% share	States	% share
A	90	A	90
B	6	B	6
C	4	C	4
		D	0
		E	0
CON = .85		CON = .88	

The only difference is that two units have been added at t_1; no shares have been shifted from some units to others. Yet CON is higher (because of the change in the standardizing denominator), as we believe it should be, since more power potential is concentrated in the hands of a smaller percentage of the subsystem's members. Had we wanted a measure that was insensitive to N (when nothing else changes), we could have used an *un*standardized measure such as HH. In the case at hand, HH would equal .82 on both occasions; the *range* of HH is sensitive to changes in N in such cases, but the *scores* are not. The advantage, we believe, of a standardized measure such as CON is that scores are comparable, in the sense that a score of .33, for example, indicates that the system shows one-third as much concentration as is possible, no matter what the size of the system. Unstandardized measures such as HH or Gini will reflect changes in N only because their ranges will change (and therefore may appear to be too sensitive to such changes) but not necessarily because the degree of concentration has changed. And in examples like the one above, the *scores* may not change when N changes, but those scores will not be comparable.

Now the price of this comparability as we have defined it is an insensitivity to changes in N at the extremes of the index. For example, if there are 5 units in a system and 1 unit controls 100%, CON = 1. If 7 units are added to the system, but 1 unit still controls 100%, CON still equals 1. This is so even though all the goods are concentrated in the hands of 20% of the units in the first case, but in the hands of an even smaller minority of

12.5% of the units in the second case. Such changes in N (when nothing else changes) will be reflected in CON as long as no one unit controls 100%. If one is dealing with that rare empirical domain in which such extreme cases occur, and one wants an index that will be sensitive to changes in N, then CON should not be used. In our case, these extreme distributions are not possible, and it seemed worthwhile to forego sensitivity to N in such cases for the sake of comparability across the much more common cases where there is some, but not absolute, inequality.

Perhaps the best way to illustrate the characteristics of the CON measure is to show how it varies vis-à-vis several alternative indices in a few simple examples. Assume that we are measuring the concentration of power potential among states in two different international subsystems containing three and five states, respectively.

I			II	
States	% shares		States	% shares
A	90		A	70
B	6		B	15
C	4		C	10
			D	3
			E	2
	.57	Gini	.59	
	.82	HH	.52	
	.85	CON	.64	

While HH and CON produce the intuitively reasonable higher index score for case I, the Gini scores are just the opposite, albeit not by much. This latter result is largely a function of N, since the upper limit of the Gini index is only .67 in case I, but .80 in case II. If we correct Gini for N (by dividing it by 1 – 1/N), it rises to .85 in I and only to .74 in II, making it more consonant with the HH and CON results. However, when Gini is corrected in this fashion, it has less discriminating power in this example (i.e., the sensitivity of the index as reflected in the *difference* between the scores in the two cases is diminished), a characteristic which also showed up when we used this modified Gini on our own historical data. One can enhance the discriminating power of some indices by taking the square root of the raw scores. But when we used that technique on our data with the Gini index, its discriminating power *decreased,* as it does in this example above (that is, the square root values of the Gini corrected for N are .92 and .86).

As we noted, the HH and CON scores seem to be in the "correct" order, but the HH pattern is largely a function of its lower limits, which are .33 and .20, respectively. And while such sensitivity to N happens to produce the appropriate scores in the cases at hand, it often will not. Consider the following pair of subsystems.

I			II	
States	% shares		States	% shares
A	33 1/3		A	32
B	33 1/3		B	32
C	33 1/3		C	32
			D	2
			E	2
	.33 1/3	HH	.31	
	.00	CON	.37	

HH indicates that the concentration of power potential in I is slightly higher, even though its equality of distribution is perfect. CON, on the other hand, reacts more appropriately in this empirical and theoretical context, since its upper and lower limits do not vary with the size of the system.

Let us consider one more example.

I			II	
States	% shares		States	% shares
A	40		A	20
B	30		B	20
C	20		C	15
D	10		D	15
			E	10
			F	10
			G	5
			H	5

I		II
.25	Gini	.25
.30	HH	.15
.26	CON	.17

While the HH results are appropriate according to the Hall and Tideman criterion, with its value in case I twice that produced for case II, the Gini index scores are identical. For our purposes, the face validity of both of these results leaves something to be desired. Conversely, the CON scores of .26 and .17 seem to us to reflect most appropriately the differences in the inequality of the distribution of power potential in the two systems. In any case, these illustrations demonstrate that—at least when one is measuring concentration or inequality among a rather small, but changing number of units—there are differences among such measures as HH, the Gini index, the Schutz coefficient, and CON which should be taken into account by any potential user.

SIMILARITY TO OTHER MEASURES

Over 15 years ago, in the conclusion of an article analyzing indices of segregation, Duncan and Duncan (1955) lamented that

> one lesson to be learned from the relatively unproductive experience with segregation indexes to date is that similar problems are often dealt with under different headings. Most of the issues which have

> come up in the literature on segregation indexes. . . . had already been encountered in the methodological work on measures of inequality, spatial distribution, and localization in geography and economics.

We have learned a similar lesson from our experience with indexes of inequality. The major problem we have discussed here—i.e., comparability in the face of a small but changing N—has been dealt with under different headings and solved in essentially the same manner. For example, Amemiya (1963) has developed an index of economic differentiation (IED) which equals

$$\sum_{i=1}^{n} \frac{n}{n-1} \left(\frac{P_i - 1}{n} \right)^2,$$

where n = the number of classifications of industry, and P_i is defined as the proportion of workers in the ith industry. If we say that n = N = the number of units in the system, and that P_i is the percentage controlled by the ith unit, we can see that IED is essentially the same as CON. That is,

$$\text{IED} = \sum_{i=1}^{N} N/N - 1(P_i - 1/N)^2 = N/N - 1\Sigma(P_i - 1/N)^2$$

$$= N/N - 1\Sigma(P_i^2 - 2P_i/N + 1/N^2) = N/N - 1\Sigma P_i^2$$

$$- 2/N\Sigma P_i + N[1/N]^2 = N/N - 1\Sigma P_i^2 - 2/N[1] + 1/N$$

$$= N/N - 1\Sigma P_i^2 - 1/N = \frac{\Sigma P_i^2 - 1/N}{N - 1/N} = \frac{\Sigma P_i^2 - 1/N}{1 - 1/N}$$

Thus, CON is equivalent to the square root of IED.

Similarly, Labovitz and Gibbs (1964) present a measure of the degree of division of labor (D) which equals

$$1 - \frac{(\Sigma X^2/[\Sigma X]^2)}{1 - 1/N},$$

where N equals the number of occupations, and X is the number of individuals in each occupational category. This is also essentially equal to CON. The term $\Sigma X^2/[\Sigma X]^2 = \Sigma P_i{}^2$, which means that $D = 1 - \Sigma P_i{}^2/1 - 1N$. Therefore,

$$\sqrt{1 - \frac{1 - \Sigma P_i^2}{1 - 1/N}} = \text{CON}.$$

In short, $\sqrt{1 - D} = \text{CON}$. It can further be shown that the index of qualitative variation (IQV), presented by Mueller et al. (1970), which equals

$$\frac{\Sigma n_i n_j}{\frac{k(k-1)}{2}(n/k)^2}$$

(where n_i = the number in the ith category, $i \neq j$, and k = the number of categories) is related to CON in the same way. That is, $\sqrt{1 - \text{IQV}} = \text{CON}$.

Aside from the fact that we use the index as a measure of concentration, rather than of economic differentiation, division of labor, qualitative diversity, and so on, the only substantial difference between CON and these other measures is that we use the square root of the same basic formula. We do this for several reasons. First, this procedure increases the discrimination of CON within the range of inequality in the small N systems that are of concern to us here. Second, we find the conceptual definition of CON as the standard deviation of the percentage shares divided by the maximum possible standard deviation in a system of size N an intuitively appealing one; this means that by definition, we should use the square root. Finally, one critic has already pointed out that, despite the fact that we criticize HH on the grounds that the squares of small percentages are very small, and that therefore they have a disproportionately small impact on the final index score, we turn around and use the squares of small percentages in our own index. But, by using the square root of the difference between these small squared percentages and 1/N, and even more importantly, by standardizing the measure the way we do, CON avoids this insensitivity to small percentages that HH displays, even though the squares of small percentages are used in both indices.

CONVERGENCE OF MEASURES

In examining the important differences among the available measures of concentration, we noted that they nevertheless fall into two basic classes. While retaining the distinction (sum of squares versus deviation from equality) for the sake of clarity, we now follow up the earlier suggestion that even this distinction is far from fundamental.

The mathematical convergence among these diverse measures begins to become evident when we note that all of them can be expressed in the same basic terms: P_i, i, and N, where P_i equals the percentage share of the ith unit, i equals the rank of the ith largest unit, and N equals the number of units in the system. This equivalence is further reflected by the fact that—with the exception of the Schutz coefficient—all the major measures are a function of either: (a) the sum of squared percentage shares, or (b) the sum of rank share products.

The most obvious similarity, already discussed above, is that between CON and HH, both of which are originally expressed in terms of P_i and N. If

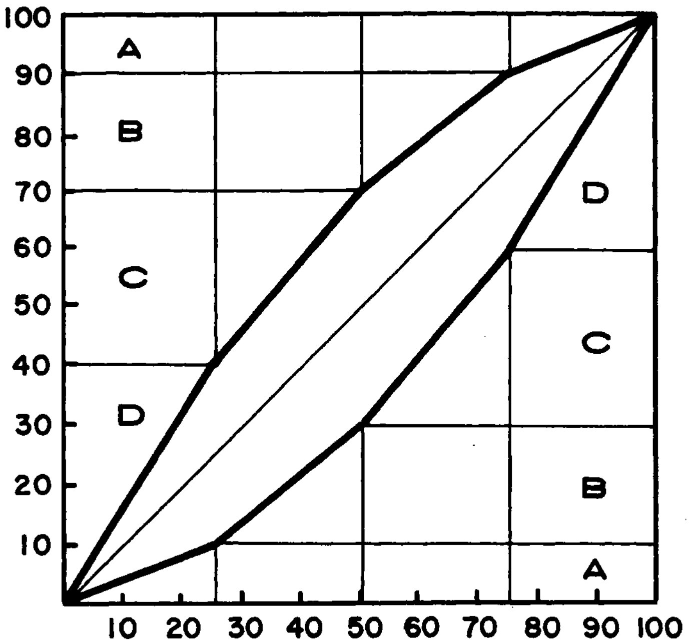
$$\mathrm{CON} = \sqrt{\frac{\Sigma P_i^2 - 1/N}{1 - 1/N}},$$

and HH = ΣP_i^2, we can also express CON as

$$\sqrt{\frac{HH - 1/N}{1 - 1/N}} \,.$$

Hence, it is clear that CON^2—and therefore CON itself—is a perfectly predictable function of HH, as long as we know the value of N. What is true of the relationship between CON and HH is also true, of course, of the relationship between CON and all those other measures discussed which are based on the sum of squared percentage shares, or some slight mutation thereof.

The Gini index can also be expressed in terms of P_i, i, and N, as may be seen by referring to Figure 3. What the Gini formula does, in effect, is to sum the area of the rectangles in the figure, and subtract that sum from one, the total "area of inequality" that lies above and below the line of perfect equality. The resulting difference is the proportion of the *total* area that lies between the Lorenz curve and the line of perfect equality; that proportion *is* the Gini index score. By referring to Figure 3, one can see that there are 2i − 1 rectangles associated with each group. For example, group A, which is the *fourth* largest group, has (2 x 4) − 1, or 7 rectangles associated with it. The area of each of the rectangles in the figure equals the product of 1/N (i.e., the proportion of the whole which each group constitutes, which is represented by the *length* of each rectangle as measured along the *horizontal* axis), and P_i (the proportion of the whole which each group controls, which is represented by the *width* of each rectangle as measured along the vertical axis), or P_i/N. Therefore, the area of the rectangles associated with group A is .25 x .10, or .10/4, which equals .025. There are 7 such rectangles, so group A's contribution is 7 x .025, or .1750. The contribution of all the other groups can be calculated in the same way. The Gini index is thus equal to $1 - \Sigma(2i - 1 [P_i/N])$. But this can be simplified to



GROUPS	% SHARES
A	10
B	20
C	30
D	40

Gini = .25

Figure 3: LORENZ CURVES ENCOMPASSING PROPORTION OF TOTAL AREA EQUAL TO THE GINI INDEX

$$1 - \frac{\Sigma(2i - 1)(P_i)}{N} = 1 - 1/N\Sigma(2i - 1)(P_i)$$

$$= 1 - 1/N\Sigma(2iP_i - P_i) = 1 - 1/N\Sigma 2iP_i - 1/N\Sigma P_i.$$

Since $\Sigma P_i = 1$, and the constant 2 can be brought outside the summation sign, this further simplifies to $1 - (2/N\Sigma iP_i) - 1/N$. Thus, it can be seen that the Gini index is a function of the rank share products, or ΣiP_i, as is TH, which, as we recall, equals $1/(2\Sigma iP_i) - 1$.

Turning to the Schutz coefficient, we find that it, too, can be expressed in terms of P_i and N. Recall that the original formula is

$$\sum_{v_i \geqq \bar{v}} (v_i/\bar{v} - 1)\Delta X_i.$$

However, v_i is equivalent to P_i, and both v and $\Delta X_i = 1/N$. (As long as each unit represents an equal percentage of the total number of units.) Thus for formula can be rewritten as:

$$\sum_{P_i \geqq 1/N} \frac{P_i}{1/N - 1} \frac{1}{N},$$

and this in turn simplifies to

$$\sum_{P_i \geqq 1/N} (P_i - 1/N).$$

Similarly, the alternative formula for the Schutz coefficient simplifies to

$$\sum_{P_i \leqq 1/N} (1/N - P_i).$$

SUMMARY

The measurement of inequality and related concepts is a pervasive problem in several social sciences. We have discussed various attempts to solve this problem and have found that the measures often differ in their reaction to changes in system size. In that discussion, we pointed out that some measures might not be appropriate if one were measuring the concentration of power potential in the international system or its subsystems. We have presented a measure that seems appropriate for such a purpose, and then examined its similarity to other measures which have been used for quite different purposes. Finally, we concluded, by focusing on the essential similarity of most of the measures we discussed, that they are all a function of P_i, N, and/or i, where P_i equals the percentage share of the ith unit, N equals the number of units in the system, and i equals the rank of the ith largest unit.

In closing, it might be appropriate to suggest that the establishment of a journal such as this was long overdue. In the literature of 5 disciplines, at least 25 scholars have utilized over 15 different journals to present their efforts to get at a central problem in the social sciences. Our fairly thorough search in the more likely sectors, as well as conversations with other political scientists, however, turned up only about half these efforts; until our first draft was circulated to specialists in sociology, the remaining sources remained beyond our ken. Even now, we suspect that economists and psychologists, for example, might lead us to additional papers on the same class of problem. Had this journal been available as a prominent solution, these independent "discoveries" and the attendant costly digressions might have been avoided.

Be that as it may, we trust that this effort represents a modest accretion to our methodological armamentarium. In addition to offering a small improvement on some existing indicators of concentration, it should help to put the problem into fuller context, offering as it does a synthesis and a codification of those which have gone before. At the least, its circulation among political scientists should serve as a reminder that crossing disciplinary boundaries may often be a salutary, sobering experience.

NOTES

1. The power—or more accurately, power potential—index reflects three sets of dimensions, each of which taps two phenomena; military (personnel and expenditures); industrial (energy consumption and iron/steel production); and demographic (total population and urban population). The 1816-1965 data base, plus the derived measures and their rationale, will appear in Singer and Small (forthcoming).

2. Greenberg also presents a more complex measure which is modified to reflect the dissimilarity of the languages involved.

3. Charles Taylor (1970) uses this formula to measure party concentration in legislatures.

4. A good discussion of the measure can be found in Horvath's article. For those who want to compare "within subset" concentration to "between subset" concentration, see Hexter and Snow (1970). And for a discussion of the special problem of inferring the concentration of an industry from the observation of a few firms, see Silberman (1967), and Hart and Prais (1956).

5. Several indices similar to the Gini index have been used to measure "segregation." For example, see the "nonwhite section index" and the "nonwhite ghetto index" (Jahn et al., 1947), the Cowgills' (1951) index of segregation, and the "reproducibility index" (Jahn, 1950). For a discussion of all these measures and their similarity to the Gini index and to each other, see Duncan and Duncan (1955), and Hornseth (1947).

6. It should be noted also that this example does not involve a change in the number of units (since we could have given the lowest two units in the second case a percentage only slightly larger than zero to meet the objection that system II "really" only contains three units); this is another case in which Gini cannot be made more "reasonable" simply by correcting it with a denominator of $1 - 1/N$.

7. It was interesting to discover while preparing this paper that Janda (1971) has come up with exactly the same formula to cope with the problem of measuring "party articulation."

REFERENCES

ADELMAN, M. A. (1959) "Differential rates and changes in concentration." Rev. of Economics and Statistics 41 (February): 68-69.

AITCHISON, J. and J. A. BROWN (1954) "On criteria for descriptions of income distribution." Metroeconomica 6 (December): 88-107.

ALKER, H. (1965) Mathematics and Politics. New York: Macmillan.

--- and B. M. RUSSETT (1964) "On measuring inequality." Behavioral Sci. 9 (July): 207-218.

AMEMIYA, E. C. (1963) "Measurement of economic differentiation." J. of Regional Sci. 5 (Summer): 84-87.

BACHI, R. (1956) "A statistical analysis of the revival of Hebrew in Israel," pp. 179-247 in R. Bachi (ed.) Scripta Hierosolymitana. Jerusalem: Magnus.

BELL, W. (1954) "A probability model for the measurement of ecological segregation." Social Forces 32 (May): 357-364.

BLAIR, J. M. (1956) "Statistical measures of concentration in business." Bull. of Oxford University Institute of Statistics 18 (November): 355-356.

BLALOCK, H. M., Jr. (1961) "Theory, measurement, and replication in the social sciences." Amer. J. of Sociology 66 (January): 342-347.

COWGILL, D. O. and M. S. COWGILL (1951) "An index of segregation based on block statistics." Amer. Soc. Rev. 16 (December): 825-831.

CUTRIGHT, P. (1967) "Inequality: a cross-national analysis." Amer. Soc. Rev. 32 (August): 562-578.

DUNCAN, O. D. and B. DUNCAN (1955) "A methodological analysis of segregation indexes." Amer. Soc. Rev. 20 (April): 210-217.

FINKELSTEIN, M. and R. M. FRIEDBERG (1967) "The application of an entropy theory of concentration to the Clayton Act." Yale Law J. (March): 671-717.

GIBBS, J. P. and H. L. BROWNING (1966) "The division of labor, technology, and the organization of production in twelve countries." Amer. Soc. Rev. 31 (February): 81-92.

GIBBS, J. P. and W. T. MARTIN (1962) "Urbanization, technology, and division of labor: international patterns." Amer. Soc. Rev. 27 (October): 667-677.

GINI, C. (1912) Variabilita e Mutabilita. Bologna.

GORT, M. (n.d.) "Analysis of stability and change in market shares." J. of Pol. Economy 71: 51-63.

GREENBERG, J. (1956) "The measurement of linguistic diversity." Language 32: 109-115.

HALL, M. and N. TIDEMAN (1967) "Measures of concentration." J. of Amer. Statistical Assn. 62 (March): 162-168.

HART, P. E. (1957) "On measuring concentration." Bull. of Oxford University Institute of Statistics 19 (August): 225.

--- and S. J. PRAIS (1956) "The analysis of business concentration: a statistical approach." J. of Royal Statistical Society 119: 150-181.

HERFINDAHL, O. C. (1950) "Concentration in the steel industry." Ph.D. dissertation. Columbia University.

HEXTER, J. L. and J. W. SNOW (1970) "Entropy measure of relative aggregate concentration." Southern Econ. J. 36 (January): 239-243.

HORNSETH, R. A. (1947) "A note on 'the measurement of ecological segregation' by Julius Jahn, Calvin F. Schmid, and Clarence Schrag." Amer. Soc. Rev. 12 (October): 603-604.

HORVATH, J. (1970) "Suggestion for a comprehensive measure of concentration." Southern Economic J. 36 (April): 446-452.

IJIRI, Y. and H. A. SIMON (1964) "Business firm growth and size." Amer. Econ. Rev. 54 (March): 77-89.

JAHN, J. A. (1950) "The measurement of ecological segregation: derivation of an index based on the criterion of reproducibility." Amer. Soc. Rev. 15 (February): 100-104.

--- C. F. SCHMID, and C. SCHRAG (1947) "The measurement of ecological segregation." Amer. Soc. Rev. 12 (June): 293-303.

JANDA, K. (1971) "Conceptual equivalence and multiple indicators in the cross-national analysis of political parties." Prepared for the ISS/UNESCO/ECPR Workshop on Indicators of National Development, August.
KRAVIS, I. B. (1962) The Structure of Income. Philadelphia: Univ. of Pennsylvania Press.
LABOVITZ, S. and J. P. GIBBS (1964) "Urbanization, technology and the division of labor: further evidence." Pacific Soc. Rev. 7 (Spring): 3-9.
LIEBERSON, S. (1969) "Measuring population diversity." Amer. Soc. Rev. 34 (December): 850-862.
--- (1964) "An extension of Greenberg's linguistic diversity measure." Language 40 (November): 526-531.
LORENZ, M. O. (1905) "Methods of measuring the concentration of wealth." Amer. Statistical Assn. J. 9 (June): 209-219.
MICHAELY, M. (1962) Concentration in International Trade. Amsterdam: North Holland.
MUELLER, J. H., K. F. SCHUESSLER, and H. L. COSTNER (1970) Statistical Reasoning in Sociology. Boston: Houghton Mifflin.
NELSON, R. L. (1963) Concentration in the Manufacturing Industries of the United States. New Haven, Conn.: Yale Univ. Press.
NUTTER, G. W. (1968) "Industrial concentration," pp. 218-222 in D. Sills (ed.) International Encyclopedia of the Social Sciences. New York: Free Press.
PRESTON, L. E. and N. R. COLLINS (1961) "The size structure of the largest industrial firms." Amer. Econ. Rev. 51 (December): 986-1011.
PUCHALA, D. (1970) "International transactions and regional integration." International Organization 24 (Autumn): 732-763.
QUANDT, R. E. (1966a) "Old and new methods of estimation and the Pareto distribution." Metrika 10: 55-82.
--- (1966b) "On the size distribution of firms." Amer. Econ. Rev. 56 (June): 416-432.
RAE, D. W. and M. TAYLOR (1970) The Analysis of Political Cleavages. New Haven, Conn.: Yale Univ. Press.
ROSENBLUTH, G. (1955) "Measures of concentration," in National Bureau of Economic Research (ed.) Business Concentration and Price Policy. Princeton: Princeton Univ. Press.
SILBERMAN, I. H. (1967) "On lognormality as a summary measure of concentration." Amer. Econ. Rev. 57: 807-831.
SIMON, H. A. and C. P. BONINI (1958) "The size distribution of business firms." Amer. Econ. Rev. 48 (September): 607-617.
SIMPSON, E. H. (1949) "Measurement and diversity." Nature 163 (April): 688.
SINGER, E. M. (1968) Antitrust Economics. Englewood Cliffs, N.J.: Prentice-Hall.
SINGER, J. D. and M. SMALL (forthcoming) The Strength of Nations: Comparative Capabilities Since Waterloo.
SINGER, J. D., S. BREMER, and J. E. STUCKEY (1972) "Capability distribution, uncertainty, and major power war, 1820-1965," in B. M. Russett (ed.) Peace, War, and Numbers. Beverly Hills: Sage Pubns.
TAYLOR, C. (1970) "Turmoil, economic development, and organized political opposition as predictors of irregular government change." Presented at the

Sixty-Sixth Annual Meeting of the American Political Science Association, Los Angeles, September 8-12.

THEIL, H. (1970) "On the estimation of relationships involving qualitative variables." Amer. J. of Sociology 76 (July): 103-154.

--- (1967) Economics and Information Theory. Chicago: Rand McNally.

WEISS, L. W. (1963) "Factors in changing concentration." Rev. of Economics and Statistics 44 (February): 70-77.

YNTEMA, D. (1933) "Measures of inequality in personal distribution of wealth or income." Amer. Statistical Assn. J. 28 (December): 423-433.