A97-37036

AIAA-97-3530

ADAPTIVE ASYMPTOTIC TRACKING OF SPACECRAFT ATTITUDE MOTION WITH INERTIA MATRIX IDENTIFICATION

by

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Abstract

The problem of a spacecraft tracking a desired trajectory is defined and addressed using adaptive feedback control. The control law, which has the form of a sixth-order dynamic compensator, does not require knowledge of the inertia of the spacecraft. A Lyapunov argument is used to show that tracking is achieved globally. A simple spin about the intermediate principal axis and a coning motion are commanded to illustrate the control algorithm. Finally, periodic commands are used to identify the inertia matrix of the spacecraft.

I. Introduction

The present generation of spacecraft require attitude control systems that provide rapid acquisition, tracking and pointing capabilities, while the equations that govern large angle maneuvers are coupled and nonlinear. As such, control system design must consider the nonlinear dynamics. Furthermore, since the mass properties of the spacecraft may be uncertain or may change due to fuel usage and articulation, it is necessary for the control system to be able to adapt changes in mass distribution. In this paper, we present a feedback control algorithm that achieves large angle tracking of velocity and attitude commands in spite of inertia uncertainty. Furthermore, we show how this tracking algorithm can be used to identify the spacecraft inertia matrix.

Many open-loop control strategies have been developed for large angle maneuvers[1,2]. However these open-loop schemes are generally sensitive to spacecraft parameter uncertainties, unexpected disturbances and initial attitude rates. In Refs. 3 and 4, global reorientation is achieved using body-fixed actuators without knowledge of the inertia of the spacecraft. The algorithms are based on decreasing the energy of the spacecraft until the desired orientation is achieved.

The attitude tracking problem has been considered in Ref. 5 using a locally convergent adaptive algorithm while adaptive feedback linearization is used in Ref. 6 to achieve tracking. However, the method of Ref. 6 requires measurements of the orientation and angular velocity of the spacecraft as well as angular acceleration. An adaptive tracking scheme has been developed that is globally valid except for a singularity[7]. A switching maneuver is used to avoid the singularity.

In the present paper, a spacecraft driven by reaction jets is commanded to track a desired trajectory using feedback control. The control algorithm requires no knowledge of the spacecraft inertia and re-

*Research supported in part by the Air Force Office of Scientific Research under grant F49620-95-1-0019 and the University of Michigan office of the Vice President for Research.

quires measurements of the orientation and angular velocity of the spacecraft. However, measurements of the angular acceleration are not required. The control law is globally valid, that is, free of singularities and has the form of a sixth-order dynamic compensator. The algorithm is adaptive in the sense of Ref. 8, p. 1 since it contains adjustable parameters and a mechanism for adjusting these parameters. Finally, the control algorithm is shown to be able to identify the spacecraft inertia matrix using a class of periodic commands.

We illustrate the tracking algorithm by commanding the spacecraft to perform a simple spin about its intermediate principal axis with the axis required to point in a given inertial direction. While the algorithm requires three torque inputs, we require no knowledge of the inertia except for the direction of the intermediate principal axis in the body frame. The problem of stabilizing a spacecraft about an intermediate principal axis without inertial pointing has been studied. For instance, in Refs. 9 and 10 the maneuver is performed using a single control torque. Next, a coning motion is commanded using the tracking algorithm. Finally, identification of the spacecraft inertia matrix is illustrated using a periodic command.

II. Equations of Motion

The spacecraft is modeled as a rigid body with actuators that provide body-fixed torques about three mutually perpendicular axes that define a body frame $\mathcal{B}$ with axes $X_{\mathcal{B}}$, $Y_{\mathcal{B}}$ and $Z_{\mathcal{B}}$. For each axis this assumption can be realized by employing a pair of actuators to produce equal and opposite forces perpendicular to the line joining the actuators. Note that the center of mass of the spacecraft does not need to be known. For $t \geq 0$, the equations of motion of the spacecraft are given by

$$J\dot{\Omega} = -\Omega^{\times} J\Omega + u, \tag{1}$$

$$\dot{\epsilon} = \frac{1}{2}\left(\epsilon^{\times}\Omega + \zeta\Omega\right), \tag{2}$$

$$\dot{\zeta} = -\frac{1}{2}\epsilon^{\mathrm{T}}\Omega, \tag{3}$$

where $\Omega = \Omega(t) \in \Re^3$ is the inertial angular velocity of the spacecraft with respect to an inertial frame $\mathcal{I}$,

$$J = \begin{bmatrix} J_{11} & J_{12} & J_{13} \\ J_{12} & J_{22} & J_{23} \\ J_{13} & J_{23} & J_{33} \end{bmatrix}$$

is the constant positive-definite inertia matrix of the spacecraft, both expressed in $\mathcal{B}$, $u = u(t) \in \Re^3$ is the vector of control torques, $(\epsilon, \zeta) = (\epsilon(t), \zeta(t)) \in \Re^3 \times \Re$ are the Euler parameters[11] representing the orientation of $\mathcal{B}$ with respect to an inertial frame $\mathcal{I}$ satisfying the constraint

$$\epsilon^{\mathrm{T}}\epsilon + \zeta^2 = 1,$$

and $(\dot{\ })$ denotes the derivative with respect to time t. The notation $a^{\times}$ for $a = [a_1\ a_2\ a_3]^{\mathrm{T}}$ denotes the skew-symmetric matrix

$$a^{\times} = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}.$$

The rotation matrix $B = B(\epsilon, \zeta) \in \mathrm{SO}(3)$ relating $\mathcal{B}$ to $\mathcal{I}$ is given by

$$B = (\zeta^2 - \epsilon^{\mathrm{T}}\epsilon)I_{3\times 3} + 2\epsilon\epsilon^{\mathrm{T}} - 2\zeta\epsilon^{\times},$$

where $I_{3\times 3}$ is the 3×3 identity matrix. We assume that (ϵ, ζ) and hence B are known for all time $t \geq 0$. However, J is assumed to be unknown.

Let the desired rotational motion of the spacecraft be described by the attitude motion of a frame $\mathcal{D}$ whose orientation with respect to $\mathcal{I}$ is specified by the Euler parameters $(\xi, \mu) = (\xi(t), \mu(t)) \in \Re^3 \times \Re$ satisfying

$$\xi^T \xi + \mu^2 = 1. \quad (4)$$

Let $D = D(\xi, \mu) \in SO(3)$ be the corresponding rotation matrix given by

$$D = (\mu^2 - \xi^T \xi) I_{3\times3} + 2\xi\xi^T - 2\mu\xi^\times, \quad (5)$$

and let $\nu = \nu(t) \in \Re^3$ denote the angular velocity of $\mathcal{D}$ with respect to $\mathcal{I}$ and expressed in $\mathcal{D}$ by

$$\nu = 2(\mu\dot{\xi} - \dot{\mu}\xi) - 2\xi^\times \dot{\xi}. \quad (6)$$

Let the time derivative of ν be denoted by $\dot{\nu} = \dot{\nu}(t) \in \Re^3$. Then $\dot{\nu}$ is given by

$$\dot{\nu} = 2(\mu\ddot{\xi} - \ddot{\mu}\xi) - 2\xi^\times \ddot{\xi}. \quad (7)$$

Let $(\varepsilon, \eta) = (\varepsilon(t), \eta(t)) \in \Re^3 \times \Re$ be the Euler parameters representing the orientation of frame $\mathcal{B}$ with respect to $\mathcal{D}$. Then (ε, η) satisfy

$$\varepsilon^T \varepsilon + \eta^2 = 1. \quad (8)$$

The Euler parameters (ε, η) are related to (ξ, μ) and (ϵ, ζ) by the quaternion multiplication rule (Ref. 11, p. 17)

$$\begin{aligned} \varepsilon &= \mu\epsilon - \zeta\xi + \epsilon^\times \xi, & (9)\\ \eta &= \mu\zeta + \xi^T \epsilon. & (10) \end{aligned}$$

The corresponding rotation matrix $C = C(\varepsilon, \eta) \in SO(3)$ is given by

$$C = (\eta^2 - \varepsilon^T \varepsilon) I_{3\times3} + 2\varepsilon\varepsilon^T - 2\eta\varepsilon^\times \quad (11)$$

and is related to B and D by

$$C = BD^T. \quad (12)$$

The angular velocity $\omega = \omega(t) \in \Re^3$ of $\mathcal{B}$ with respect to $\mathcal{D}$ and expressed in $\mathcal{B}$ is then

$$\omega = \Omega - C\nu. \quad (13)$$

We assume that the desired maneuver is specified in terms of (ξ, μ), which are assumed to be C^2 functions, and that measurements of Ω and the attitude of $\mathcal{B}$ with respect to $\mathcal{I}$ are available. Using (5), (6) and (7) the quantities ν, $\dot{\nu}$ and D can be computed from (ξ, μ). Furthermore, since (ϵ, ζ) and B can be calculated from the measurements of the attitude of $\mathcal{B}$, it follows that C, ε, and η can be determined using (9), (10) and (12). The angular velocity ω is then determined using (13). The following problem expresses the requirement that the attitude and angular velocity of frames $\mathcal{B}$ and $\mathcal{D}$ should coincide asymptotically.

Tracking Problem: Let $\xi : [0, \infty) \to \Re^3$ and $\mu : [0, \infty) \to \Re$ be given C^2 Euler parameters satisfying (4) for all $t \geq 0$. Find a dynamic feedback control law of the form

$$\begin{aligned} \dot{\alpha} &= f(\alpha, \omega, \varepsilon, \eta, \nu, \dot{\nu}), & (14)\\ u &= g(\alpha, \omega, \varepsilon, \eta, \nu, \dot{\nu}), & (15) \end{aligned}$$

such that $C \to I_{3\times3}$ and $\omega \to 0$ as $t \to \infty$.

Note that the attitude convergence condition $C \to I_{3\times3}$ does not necessarily imply the angular velocity convergence condition $\omega \to 0$.

Using (8) and (11) it follows that $\varepsilon \to 0$ if and only if $C \to I_{3\times3}$. Hence the tracking problem is solved if and only if $\omega \to 0$ and $\varepsilon \to 0$. Rewriting (1), (2) and (3) in terms of ω, ε and η we obtain

$$\begin{aligned} J\dot{\omega} &= -(\omega + C\nu)^\times J(\omega + C\nu) \\ &\quad + J(\omega^\times C\nu - C\dot{\nu}) + u, & (16)\\ \dot{\varepsilon} &= \frac{1}{2}\left(\varepsilon^\times \omega + \eta\omega\right), & (17)\\ \dot{\eta} &= -\frac{1}{2}\varepsilon^T \omega. & (18) \end{aligned}$$

These equations describe the motion of the spacecraft with respect to $\mathcal{D}$. We observe that the tracking problem has been converted into an asymptotic stabilization problem for ω and ε in (16), (17) and (18).

III. Adaptive Control Law

In this section, we present a feedback control law that asymptotically tracks a desired maneuver and thus satisfies the requirements of the tracking problem. The control law is global in the sense that asymptotic tracking is achieved for arbitrary initial conditions.

We observe that the inertia parameters J_{ij}, where $i,\ j = 1,\ 2,\ 3$, appear linearly in equation (16). To isolate these parameters, we define a linear operator $L : \Re^3 \rightarrow \Re^{3\times 6}$ acting on $a = [a_1\ a_2\ a_3]^{\mathrm{T}}$ by

$$L(a) = \begin{bmatrix} a_1 & 0 & 0 & 0 & a_3 & a_2 \\ 0 & a_2 & 0 & a_3 & 0 & a_1 \\ 0 & 0 & a_3 & a_2 & a_1 & 0 \end{bmatrix}. \tag{19}$$

Letting

$$\alpha \triangleq [J_{11}\ J_{22}\ J_{33}\ J_{23}\ J_{13}\ J_{12}]^{\mathrm{T}},$$

it follows that

$$Ja = L(a)\alpha.$$

Equation (16) can now be rewritten in the form

$$J\dot{\omega} = F(\omega, C, \nu, \dot{\nu})\alpha + u, \tag{20}$$

where $F : \Re^3 \times \Re^{3\times 3} \times \Re^3 \times \Re^3 \rightarrow \Re^{3\times 6}$ is defined by

$$\begin{aligned} F(\hat{\omega}, \hat{C}, \hat{\nu}, \hat{\rho}) \ &\triangleq\ -(\hat{\omega} + \hat{C}\hat{\nu})^{\times} L(\hat{\omega} + \hat{C}\hat{\nu}) \\ &\quad + L(\hat{\omega}^{\times}\hat{C}\hat{\nu} - \hat{C}\hat{\rho}). \end{aligned} \tag{21}$$

Theorem 1: Assume that ν and $\dot{\nu}$ are bounded and let $K_1 \in \Re^{3\times 3}$, $K_2 \in \Re^{3\times 3}$ and $Q \in \Re^{6\times 6}$ be positive definite. Then the control law

$$\begin{aligned} \dot{\hat{\alpha}} \ &=\ Q^{-1}[F(\omega, C, \nu, \dot{\nu}) \\ &\quad + G(\omega, \varepsilon, \eta)]^{\mathrm{T}}[\omega + K_1\varepsilon], \end{aligned} \tag{22}$$

$$\begin{aligned} u \ &=\ -[F(\omega, C, \nu, \dot{\nu}) + G(\omega, \varepsilon, \eta)]\hat{\alpha} \\ &\quad - (K_2K_1 + I_{3\times 3})\varepsilon - K_2\omega, \end{aligned} \tag{23}$$

where $G : \Re^3 \times \Re^3 \times \Re \rightarrow \Re^{3\times 6}$ is defined by

$$G(\hat{\omega}, \hat{\varepsilon}, \hat{\eta}) \triangleq -\frac{1}{2}L(K_1(\hat{\varepsilon}^{\times}\hat{\omega} + \hat{\eta}\hat{\omega})) \tag{24}$$

and F is given by (21), solves the tracking problem. Furthermore, $\hat{\alpha}$ is bounded for all $t \geq 0$ and $\dot{\hat{\alpha}} \rightarrow 0$ as $t \rightarrow \infty$.

Proof: Define σ, γ and β by

$$\sigma = \omega + K_1\varepsilon, \tag{25}$$

$$\gamma = \eta - 1, \tag{26}$$

$$\beta = \alpha - \hat{\alpha}. \tag{27}$$

Using (16), (17), (18), (22) and (23) we obtain the 13-dimensional system

$$J\dot{\sigma} = [H(\sigma, \varepsilon, \gamma, \nu, \dot{\nu}) + M(\sigma, \varepsilon, \gamma)]\,\alpha + u, \tag{28}$$

$$\begin{aligned} \dot{\varepsilon} \ &=\ \frac{1}{2}[\varepsilon^{\times}\sigma - \varepsilon^{\times}K_1\varepsilon \\ &\quad + (\gamma + 1)(\sigma - K_1\varepsilon)], \end{aligned} \tag{29}$$

$$\dot{\gamma} = -\frac{1}{2}\varepsilon^{\mathrm{T}}(\sigma - K_1\varepsilon), \tag{30}$$

$$Q\dot{\beta} = [H(\sigma, \varepsilon, \gamma, \nu, \dot{\nu}) + M(\sigma, \varepsilon, \gamma)]^{\mathrm{T}}\sigma, \tag{31}$$

where $H : \Re^3 \times \Re^3 \times \Re \times \Re^3 \times \Re^3 \rightarrow \Re^{3\times 6}$ is defined by

$$H(\hat{\sigma}, \hat{\varepsilon}, \hat{\gamma}, \hat{\nu}, \hat{\rho}) \triangleq F(\hat{\sigma} - K_1\hat{\varepsilon}, C(\hat{\varepsilon}, \hat{\gamma} + 1), \hat{\nu}, \hat{\rho}), \tag{32}$$

$M : \Re^3 \times \Re^3 \times \Re \rightarrow \Re^{3\times 6}$ is defined by

$$M(\hat{\sigma}, \hat{\varepsilon}, \hat{\gamma}) \triangleq G(\hat{\sigma} - K_1\hat{\varepsilon}, \hat{\varepsilon}, \hat{\gamma} + 1), \tag{33}$$

and u is given by the feedback law

$$u = h(\sigma, \varepsilon, \gamma, \nu, \dot{\nu}, \beta), \tag{34}$$

where $h : \Re^3 \times \Re^3 \times \Re \times \Re^3 \times \Re^3 \times \Re^6 \rightarrow \Re^3$ is defined by

$$\begin{aligned} h(\hat{\sigma}, \hat{\varepsilon}, \hat{\gamma}, \hat{\nu}, \hat{\rho}, \hat{\beta}) \ &\triangleq\ -K_2\hat{\sigma} - \hat{\varepsilon} - [H(\hat{\sigma}, \hat{\varepsilon}, \hat{\gamma}, \hat{\nu}, \hat{\rho}) \\ &\quad + M(\hat{\sigma}, \hat{\varepsilon}, \hat{\gamma})](\alpha - \hat{\beta}). \end{aligned} \tag{35}$$

It is observed that equations (28) - (31) are nonautonomous due to the presence of ν and $\dot{\nu}$. Under (34), we note that the origin $[\sigma^{\mathrm{T}}\ \varepsilon^{\mathrm{T}}\ \gamma^{\mathrm{T}}\ \beta^{\mathrm{T}}] = [0\ 0\ 0\ 0]$ is an equilibrium solution of the system (28) - (31).

Next, we show that under the control law (34), $\sigma \to 0$ and $\varepsilon \to 0$ as $t \to \infty$ for arbitrary initial conditions. To do this, consider the positive-definite candidate Lyapunov function $V : \Re^3 \times \Re^3 \times \Re^3 \times \Re^6 \to \Re$ defined by

$$V(\sigma,\varepsilon,\gamma,\beta) = \frac{1}{2}(\sigma^{\mathrm{T}} J\sigma + \beta^{\mathrm{T}} Q\beta) + \varepsilon^{\mathrm{T}}\varepsilon + \gamma^2. \quad (36)$$

Note that V is independent of time and is radially unbounded. The total time derivative of V along the trajectories of the system is given by

$$\begin{aligned}\dot{V}(\sigma,\varepsilon,\gamma,\beta) &= \Big[H(\sigma,\varepsilon,\gamma,\nu,\dot{\nu})^{\mathrm{T}}\sigma + M(\sigma,\varepsilon,\gamma)^{\mathrm{T}}\sigma \\ &\quad + Q\dot{\beta}\Big] - \sigma^{\mathrm{T}} K_2\sigma - \varepsilon^{\mathrm{T}} K_1\varepsilon.\end{aligned}$$

Using (31) we obtain the simplified expression

$$\dot{V}(\sigma,\varepsilon,\gamma,\beta) = -\sigma^{\mathrm{T}} K_2\sigma - \varepsilon^{\mathrm{T}} K_1\varepsilon, \quad (37)$$

which shows that $\dot{V}$ is negative semi-definite and not an explicit function of time. Since $V(\sigma(t),\varepsilon(t),\gamma(t),\beta(t)) \le V(\sigma(0),\varepsilon(0),\gamma(0),\beta(0))$ for all $t \ge 0$ and since V is radially unbounded, it follows that σ, ε, γ and β are bounded. Since by assumption ν and $\dot{\nu}$ are bounded and since α is constant it follows that $H(\sigma,\varepsilon,\gamma,\nu,\dot{\nu})$, $M(\sigma,\varepsilon,\gamma)$, $h(\sigma,\varepsilon,\gamma,\nu,\dot{\nu},\beta)$ and $\hat{\alpha}$ are bounded. The total time derivative of $\dot{V}$ along the trajectories of the system is given by

$$\begin{aligned}\ddot{V}(\sigma,\varepsilon,\gamma,\beta,t) &= -2\sigma^{\mathrm{T}} K_2 J^{-1}\left[H(\sigma,\varepsilon,\gamma,\nu,\dot{\nu})^{\mathrm{T}}\alpha\right. \\ &\quad + h(\beta,\sigma,\varepsilon,\gamma,\nu,\dot{\nu}) \\ &\quad \left. + M(\sigma,\varepsilon,\gamma)^{\mathrm{T}}\alpha\right] - 2\varepsilon^{\mathrm{T}} K_1\left[\varepsilon^{\times}\sigma\right. \\ &\quad \left. + (\gamma+1)(\sigma - K_1\varepsilon)\right].\end{aligned}$$

Since σ, ε, γ, β, $H(\sigma,\varepsilon,\gamma,\nu,\dot{\nu})$, $M(\sigma,\varepsilon,\gamma)$ and $h(\sigma,\varepsilon,\gamma,\nu,\dot{\nu},\beta)$ are bounded and α is constant, it follows that $\ddot{V}(\sigma(t),\varepsilon(t),\gamma(t),\beta(t),t)$ is bounded for all $t \ge 0$. Using Theorem 5.4 of Ref. 8 we conclude that $\sigma \to 0$ and $\varepsilon \to 0$. Furthermore, since ν and $\dot{\nu}$ are bounded by assumption and $\sigma \to 0$ and $\varepsilon \to 0$ it follows from (31) that $\dot{\beta} \to 0$ and thus $\dot{\hat{\alpha}} \to 0$.

Because $\sigma \to 0$ and $\varepsilon \to 0$, it follows from (25) that $\omega \to 0$. Hence we conclude that (22) and (23) solves the tracking problem. □

The system (28) - (31) has two equilibrium points, namely $[\sigma^{\mathrm{T}}\ \varepsilon^{\mathrm{T}}\ \gamma^{\mathrm{T}}\ \beta^{\mathrm{T}}] = [0\ 0\ 0\ 0]$ and $[\sigma^{\mathrm{T}}\ \varepsilon^{\mathrm{T}}\ \gamma^{\mathrm{T}}\ \beta^{\mathrm{T}}] = [0\ 0\ -2\ 0]$, both of which correspond to the same orientation. The equilibrium point $[\sigma^{\mathrm{T}}\ \varepsilon^{\mathrm{T}}\ \gamma^{\mathrm{T}}\ \beta^{\mathrm{T}}] = [0\ 0\ 0\ 0]$ is stabilized using the control law (22) and (23).

We observe that the control law (22) and (23) is global in the sense that, for arbitrary initial conditions, $\omega \to 0$ and $\varepsilon \to 0$ as $t \to \infty$ which ensures that tracking is achieved asymptotically. The control law given by (23) is a sixth-order dynamic compensator. It is noted that the control law requires knowledge only of ω, ε and η and not of the inertia of the spacecraft.

The control law (22) and (23) is adaptive in the sense of Ref. 8, p. 1. The state $\hat{\alpha}$ represents adjustable parameters that, under certain conditions (see Section IV), converges to α. Equation (22) represents the mechanism for adjusting these parameters. The state $\hat{\alpha}$ is consequently termed the ***adaptive parameter.*** Although the time derivative of the adaptive parameter converges to zero as $t \to \infty$, $\hat{\alpha}$ does not necessarily converge.

We now consider the convergence of $\hat{\alpha}$ for periodic command signals.

Theorem 2: Assume that ν is periodic and define $W : [0,\infty) \to \Re^{3\times 6}$ by

$$W(t) \stackrel{\triangle}{=} L(\dot{\nu}(t)) + \nu(t)^{\times} L(\nu(t)). \quad (38)$$

Under the control law given by (22) and (23), $\alpha - \hat{\alpha} \to \{\chi : W(t)\chi = 0 \text{ for all } t \geq 0\}$ as $t \to \infty$.

Proof: With σ, γ and β defined by (25), (26) and (27), respectively, we obtain the differential equations (28) - (31), where H and M are defined by (32) and (33), respectively, and $u = h(\sigma, \varepsilon, \gamma, \nu, \dot{\nu}, \beta)$, where h is defined by (35). Note that since ν is periodic and differentiable, $\dot{\nu}$ is periodic.

Consider the candidate Lyapunov function V defined by (36). Now V is C^1, positive definite, and radially unbounded, and $\dot{V}$ along the trajectories of the system is given by (37).

Let $x = \left[\sigma^{\mathrm{T}}\ \varepsilon^{\mathrm{T}}\ \gamma^{\mathrm{T}}\ \beta^{\mathrm{T}}\right]^{\mathrm{T}}$ and $E_0 = \{x : \dot{V}(x) = 0\}$. Let $x(t; x_0, t_0)$ denote the solution of the system (28) - (31) at time $t \geq t_0$ where $x(t_0; x_0, t_0) = x_0$. Let $L = \{(\hat{x}, \hat{t}) \in E_0 \times [0, \infty) : x(t; \hat{x}, \hat{t}) \in E_0 \text{ for all } t \geq \hat{t}\}$ and $N = \{x(t; \hat{x}, \hat{t}) : (\hat{x}, \hat{t}) \in L, t \geq \hat{t}\}$. Note that $N \subset E_0$. Using (8), (26) and (28) - (31) it follows that N is given by

$$N = \{(\sigma, \varepsilon, \gamma, \beta) : \sigma = 0, \varepsilon = 0, \gamma \in \{-2, 0\}, \beta \in G\},$$

where $G = \{\chi : W(t)\chi = 0 \text{ for all } t \geq 0\}$. It now folllows from Theorem 2.8 of Ref. 12 that $\alpha - \hat{\alpha} \to G$. □

The following corollary of Theorem 2 considers the special case in which ν is constant.

Corollary 1: Assume that ν is constant. Under the control law given by (22) and (23), $\alpha - \hat{\alpha} \to \{\chi : \nu^{\times} L(\nu)\chi = 0\}$ as $t \to \infty$.

Proof: Since ν is constant, Theorem 2 implies that $\alpha - \hat{\alpha} \to \{\chi : \nu^{\times} L(\nu)\chi = 0\}$. □

We now consider the case in which ν represents a constant spin about one of the principal axes which is equivalent to $\nu^{\times} L(\nu)\alpha = 0$. For such a command it is desirable that the control law satisfy $u \to 0$ as $t \to \infty$. The following result shows that the control law (22) and (23) has this property.

Corollary 2: Assume that ν is constant and satisfies $\nu^{\times} L(\nu)\alpha = 0$. Then, under the control law given by (22) and (23), $u \to 0$ as $t \to \infty$.

Proof: Under the control law given by (22) and (23), $\omega \to 0$ and $\varepsilon \to 0$ which implies that $u + \nu^{\times} L(\nu)\hat{\alpha} \to 0$ as $t \to \infty$. Since $\nu^{\times} L(\nu)\alpha = 0$, it follows from Corollary 1 that $\hat{\alpha} \to \{\kappa : \nu^{\times} L(\nu)\kappa = 0\}$. It thus follows that $u \to 0$ as $t \to \infty$. □

IV. Inertia Matrix Identification

In this section, we present a method for identifying the spacecraft inertia matrix. We first use Corollary 1 to identify the off-diagonal terms J_{12}, J_{23} and J_{31}.

Proposition 1: Let ν be constant. If $\nu = [0\ \nu_2\ 0]^{\mathrm{T}}$ where $\nu_2 \neq 0$ then, under the control law (22) and (23), $\hat{\alpha}_4 \to \alpha_4$ and $\hat{\alpha}_6 \to \alpha_6$ as $t \to \infty$. Furthermore, if $\nu = [0\ 0\ \nu_3]^{\mathrm{T}}$ where $\nu_3 \neq 0$ then, under the control law (22) and (23), $\hat{\alpha}_4 \to \alpha_4$ and $\hat{\alpha}_5 \to \alpha_5$ as $t \to \infty$.

Proof: From Corollary 1, under the control law (22) and (23), $\alpha - \hat{\alpha} \to \{\chi : \nu^{\times} L(\nu)\chi = 0\}$. Now $\nu^{\times} L(\nu)$ is computed to be

$$\nu^{\times} L(\nu) = \begin{bmatrix} 0 & 0 & 0 & \nu_2^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\nu_2^2 \end{bmatrix}.$$

It thus follows that $\nu^{\times} L(\nu)\chi = 0$ if and only if $\chi_4 = 0$ and $\chi_6 = 0$ where $\chi = [\chi_1\ \chi_2\ \chi_3\ \chi_4\ \chi_5\ \chi_6]^{\mathrm{T}}$. Thus $\hat{\alpha}_4 \to \alpha_4$ and $\hat{\alpha}_6 \to \alpha_6$ as $t \to \infty$. If $\nu = [0\ 0\ \nu_3]^{\mathrm{T}}$ where $\nu_3 \neq 0$ then $\nu^{\times} L(\nu)$ is computed to be

$$\nu^{\times} L(\nu) = \begin{bmatrix} 0 & 0 & 0 & -\nu_3^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \nu_3^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

It follows that $\hat{\alpha}_4 \to \alpha_4$ and $\hat{\alpha}_5 \to \alpha_5$ as $t \to \infty$. □

Hence the off-diagonal terms J_{12}, J_{13} and J_{23} can be identified by performing two constant tracking maneuvers. We now consider periodic maneuvers for identifying the entire inertia matrix.

Proposition 2: Let ν be periodic and let $W(t)$ be given by (38). Furthermore, let $0 \leq t_1 \leq t_2 \leq \ldots \leq t_n$ and suppose that

$$\operatorname{rank}\begin{bmatrix} W(t_1) \\ \vdots \\ W(t_n) \end{bmatrix} = 6. \tag{39}$$

Then, under the control law (22) and (23), $\hat{\alpha} \to \alpha$.

Proof: From Theorem 2 it follows that $\alpha - \hat{\alpha} \to \{\chi : W(t)\chi = 0 \text{ for all } t \geq 0\}$. However it follows from (39) that $\{\chi : W(t)\chi = 0 \text{ for all } t \geq 0\} = \{0\}$. Hence $\hat{\alpha} \to \alpha$. □

There are many signals that satisfy the conditions of Proposition 2. Consider, for example, the periodic signal

$$\nu(t) = [\sin t \ \ \sin 2t \ \ \sin 3t]^{\mathrm{T}}. \tag{40}$$

Then with $t_1 = 0$ and $t_2 = \pi/2$, we obtain

$$\begin{bmatrix} W(0) \\ W(\pi/2) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 3 & 2 \\ 0 & 2 & 0 & 3 & 0 & 1 \\ 0 & 0 & 3 & 2 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & -1 \\ -1 & -2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -3 & 0 & 1 \end{bmatrix}.$$

Since the determinant of $\begin{bmatrix} W(0) \\ W(\pi/2) \end{bmatrix}$ is -64, it follows that

$$\operatorname{rank}\begin{bmatrix} W(0) \\ W(\pi/2) \end{bmatrix} = 6.$$

Hence under the control law (22) and (23), $\hat{\alpha} \to \alpha$. Thus the inertia matrix can be identified using a single periodic command signal.

V. Numerical Simulations

In this section we present simulations to illustrate tracking and identification of the spacecraft inertia matrix. The two tracking maneuvers considered are a simple spin about the intermediate principal axis and a coning motion. The maneuver given by (40) is used to identify the spacecraft inertia. The same initial conditions are chosen for the three maneuvers. The initial angular velocity is $\Omega = [0.4 \ 0.2 \ -0.1]^{\mathrm{T}}$ rad/sec, the initial orientation of the spacecraft is given by $\epsilon = [-0.1 \ 0.1 \ -0.1]^{\mathrm{T}}$, $\zeta = 0.9849$, and the initial value of the adaptive parameter $\hat{\alpha}$ in kg·m^2 is $\hat{\alpha} = [22 \ 18 \ 13 \ 1.6 \ 1.0 \ 1.3]^{\mathrm{T}}$. The gains are chosen to be $K_1 = 20I_{3\times3}$, $K_2 = 5I_{3\times3}$ and $Q = I_{6\times6}$.

First we command the intermediate principal axis of the spacecraft to point in a given inertial direction and the spacecraft to perform a simple spin about the intermediate principal axis. To do this, we assume that the direction of the intermediate principal axis is known with respect to $\mathcal{B}$, and, without loss of generality, we assume that the intermediate principal axis coincides with the $Y_{\mathcal{B}}$ axis of $\mathcal{B}$. We assume no knowledge of either the moments of inertia or the directions of the other two principal axes.

The maneuver is performed using the control law given by Theorem 1. In the body frame $\mathcal{B}$, let

$$J = \begin{bmatrix} 20 & 0 & 0.9 \\ 0 & 17 & 0 \\ 0.9 & 0 & 15 \end{bmatrix},$$

so that $Y_{\mathcal{B}}$ is the intermediate principal axis, let $\nu = [0 \ 1 \ 0]^{\mathrm{T}}$ rad/sec, let the initial orientation be $\xi = [0 \ 0 \ 0]^{\mathrm{T}}$ and $\mu = 1$, and let the X, Y and Z axes constitute the inertial frame. Then the desired motion is a spin about the intermediate principal axis, with the intermediate principal axis aligned with the Y axis of the inertial frame.

Applying the control law given by Theorem 1, we observe from Figures 1 and 2 that tracking is achieved. Figures 3 and 4 indicate that while J_{12} and J_{23} are identified in accordance with Proposition 1, the remaining entries of the inertia matrix are

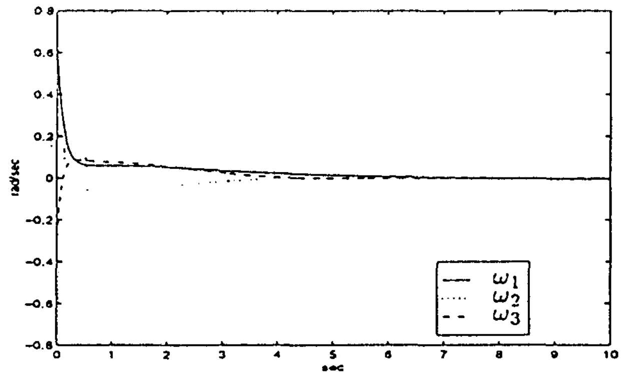

Figure 1: Relative angular velocity $\omega = [\omega_1\ \omega_2\ \omega_3]^{\mathrm{T}}$

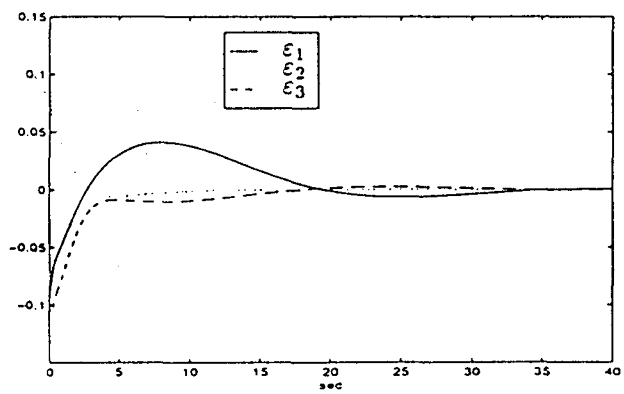

Figure 2: Relative Euler parameter $\varepsilon = [\varepsilon_1\ \varepsilon_2\ \varepsilon_3]^{\mathrm{T}}$

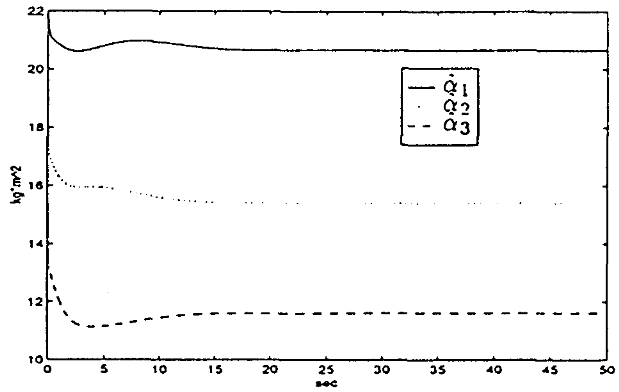

Figure 3: Adaptive parameters $\hat{\alpha}_1$, $\hat{\alpha}_2$ and $\hat{\alpha}_3$

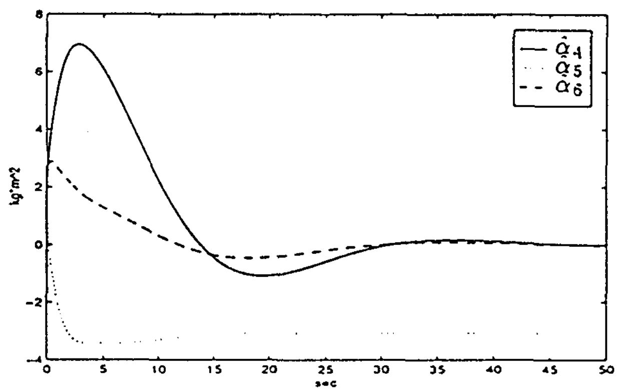

Figure 4: Adaptive parameters $\hat{\alpha}_4$, $\hat{\alpha}_5$ and $\hat{\alpha}_6$

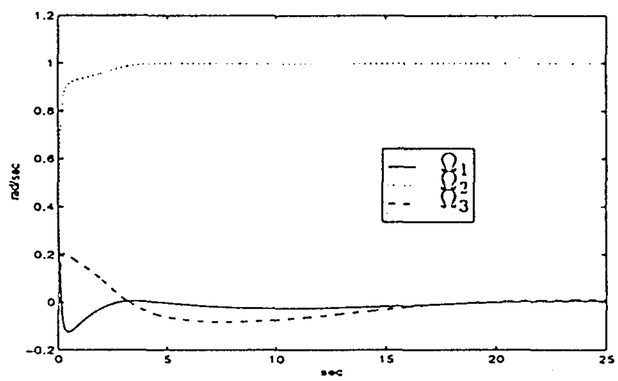

Figure 5: Angular Velocity of $\mathcal{B}$ with respect to Inertial $\Omega = [\Omega_1\ \Omega_2\ \Omega_3]^{\mathrm{T}}$

not. Figure 5 shows that Ω_2 converges to 1 rad/sec while Ω_1 and Ω_3 converge to 0 rad/sec. Thus the spacecraft approaches a simple spin about its intermediate axis.

Next we command the spacecraft to perform a specified coning motion. We use 3, 2, 1 Euler[13] angles ψ, θ and ϕ to represent the precession motion of the desired frame $\mathcal{D}$. Let $\psi(t) = 0.3t$ rad, $\theta(t) = -0.2778\pi$ rad and $\phi(t) = t$ rad. Then the desired motion is a coning maneuver with a precession rate of 0.3 rad/sec, a spin rate of 1 rad/sec and a coning angle of 40 degrees. The angular velocity ν

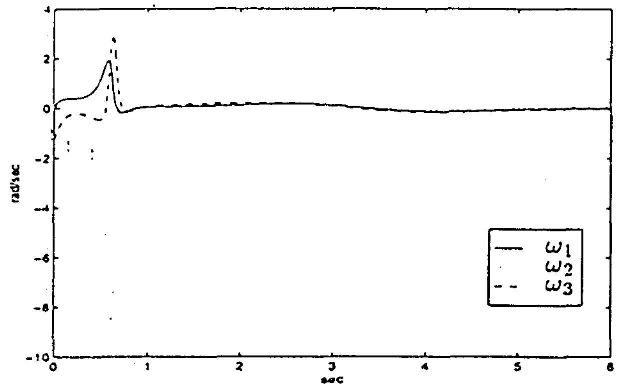

Figure 6: Relative angular velocity $\omega = [\omega_1\ \omega_2\ \omega_3]^T$

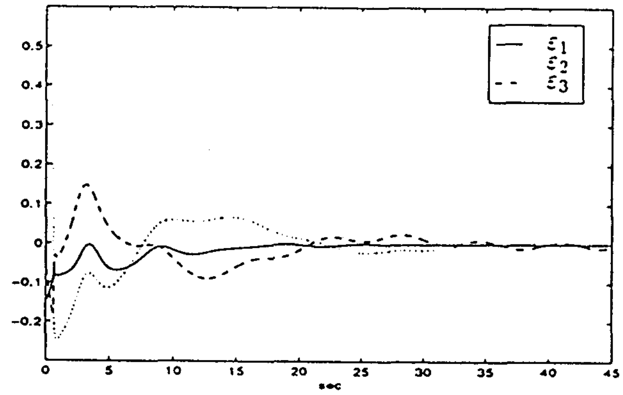

Figure 7: Relative Euler parameter $\varepsilon = [\varepsilon_1\ \varepsilon_2\ \varepsilon_3]^T$

and the initial values of the Euler parameters ξ and μ are computed from Ref. 11 using equation (24), p. 27, Table 2.1, p. 20 and equations (15) and (16), p. 18. The inertia matrix J of the spacecraft in kg·m^2 is given by

$$J = \begin{bmatrix} 20 & 1.2 & 0.9 \\ 1.2 & 17 & 1.4 \\ 0.9 & 1.4 & 15 \end{bmatrix}. \tag{41}$$

The controller given by Theorem 1 is used and it is observed from Figures 6 and 7 that $\omega \to 0$ and $\varepsilon \to 0$. However, note from Figures 8 and 9 that not all components of $\hat{\alpha}$ converge to α. Figure 5 shows the motion of the X_B axis of the spacecraft for a period of 100 seconds. The plots indicate that the desired coning motion is achieved.

To identify the inertia matrix, the maneuver $\nu(t) = [\sin t\ \sin 2t\ \sin 3t]^T$ rad/sec described in Section IV is chosen. The initial orientation of $\mathcal{D}$ is chosen to be $\xi = [0\ 0\ 0]^T$ and $\mu = 1$, and the inertia matrix is given by (41). Under the control law given by (22) and (23) we observe from Figures 11 and 12 that $\omega \to 0$ and $\varepsilon \to 0$. Furthermore, Figures 13 and 14 indicate that $\hat{\alpha} \to \alpha$ in accordance with Proposition 2. It is noted from the numerical sim-

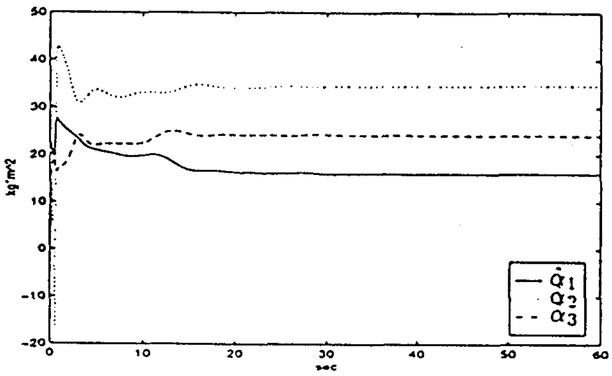

Figure 8: Adaptive parameters $\hat{\alpha}_1$, $\hat{\alpha}_2$ and $\hat{\alpha}_3$

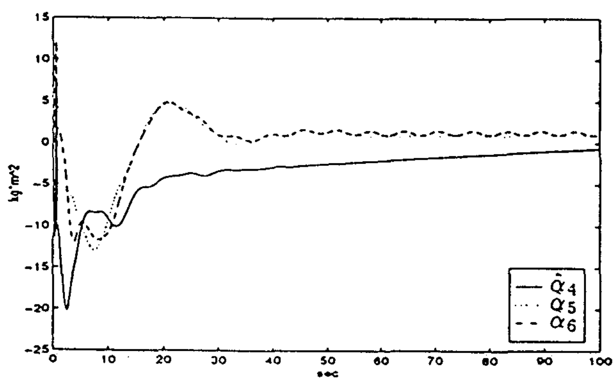

Figure 9: Adaptive parameters $\hat{\alpha}_4$, $\hat{\alpha}_5$ and $\hat{\alpha}_6$

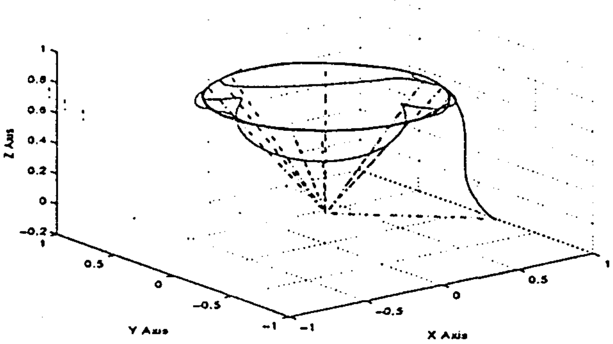

Figure 10: Motion of X_B axis

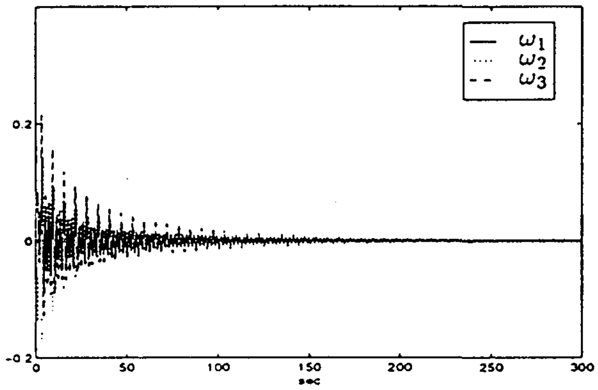

Figure 11: Relative angular velocity $\omega = [\omega_1\ \omega_2\ \omega_3]^T$

ulations that tracking is achieved rapidly whereas parameter identification takes much longer.

VI. Conclusions

An adaptive feedback control algorithm has been developed to provide global tracking of commanded spacecraft motion. The algorithm assumes no knowledge of the inertia of the spacecraft and is thus unconditionally robust with respect to this parametric uncertainty. It was shown using a

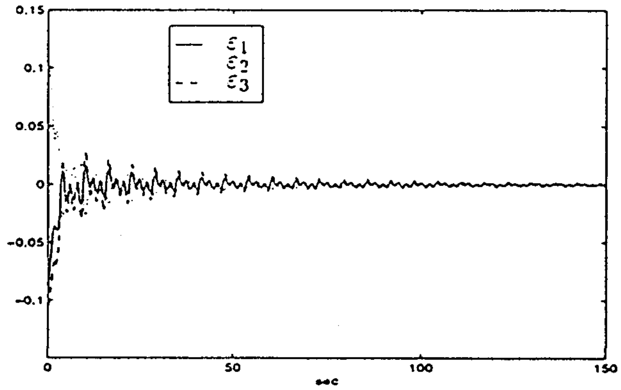

Figure 12: Relative Euler parameter $\varepsilon = [\varepsilon_1\ \varepsilon_2\ \varepsilon_3]^T$

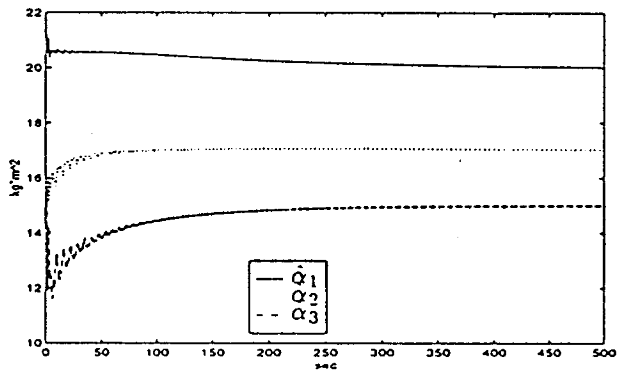

Figure 13: Adaptive parameters $\hat{\alpha}_1$, $\hat{\alpha}_2$ and $\hat{\alpha}_3$

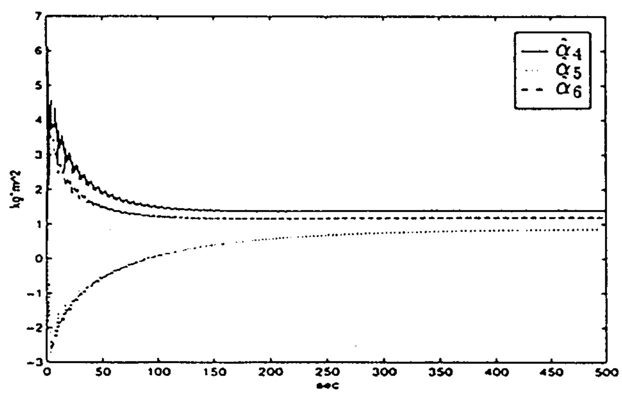

Figure 14: Adaptive parameters $\hat{\alpha}_4$, $\hat{\alpha}_5$ and $\hat{\alpha}_6$

Lyapunov argument that the attitude and angular velocity tracking error converge to zero. Furthermore, the control algorithm was used to identify the spacecraft inertia matrix. Numerical simulations illustrate tracking and identification of the inertia matrix.

References

[1]Junkins, J. L., and Turner, J. D., "Optimal Continuous Torque Attitude Maneuvers," *J. Guidance Contr.*, Vol. 3, no. 3, May-June 1980, pp 210-217.

[2]Skaar, S. B., and Kraige, L. G., "Large Angle Spacecraft Attitude Maneuvers Using an Optimal Reaction Wheel Power Criterion," *The Journal of Astronautical Sciences*, Vol. 32, no. 1, Jan.-Mar. 1984, pp. 47-61.

[3]Wie, B., Weiss, H., and Arapostathis, A., "Quaternion Feedback Regulator for Spacecraft Eigenaxis Rotations," *J. Guidance Contr.*, Vol. 8, no. 3, 1989, pp. 360-365.

[4]Joshi, S. M., Kelkar, A. G., and Wen, J. T.-Y., "Robust Attitude Stabilization of Spacecraft Using Nonlinear Quaternion Feedback," *IEEE Trans. Autom. Contr.*, Vol. 40, Oct. 1995, pp. 1800-1803.

[5]Singh, S. N., "Nonlinear Adaptive Attitude Control of Spacecraft," *IEEE Trans. Aerospace Elect. Sys.*, Vol. AES-23, No. 3, May 1987.

[6]Sheen, J. J., and Bishop, R. H., "Adaptive Nonlinear Control of Spacecraft," *Proc. Amer. Contr. Conf.*, Baltimore, MD, June 1994, pp. 2867-2871.

[7]Slotine, J. J. E., and Li, W., *Applied Nonlinear Control*, Prentice-Hall, NJ, 1989.

[8]Astrom, K. J. and Wittenmark, B., *Adaptive Control*, Addison-Wesley, Reading, MA, 2nd edition, 1995.

[9]Bloch, A. M., and Marsden, J. E., "Stabilization of Rigid Body Dynamics by the Energy-Casimir Method," *Systems & Control Letters*, 14, 1990, pp. 341-346.

[10]Aeyels, D., "On Stablization by Means of the Energy-Casimir Method," *Systems & Control Letters*, 18, 1992, pp. 325-328.

[11]Hughes, P. C., *Spacecraft Attitude Dynamics*, John Wiley & Sons, 1986.

[12]Narendra, K. S., and Annaswamy, A. M., *Stable Adaptive Systems*, Prentice Hall, 1989.

[13]Greenwood, D. T., *Principles of Dynamics*, Prentice-Hall, 1988.