Optical Holography with Partially Coherent Radiation

E. Leith

The University of Michigan, Ann Arbor, Mich. 48109

ABSTRACT

Holograms can be formed in light of little or no temporal coherence. The coherence requirements for Gabor in-line holography are examined. It is shown that, with achromatic interferometers formed from diffraction gratings, the coherence requirements for off-axis holography are equally low. Finally, two basic approaches to the making of holograms in completely white light are described.

INTRODUCTION

Nowadays we see holograms of remarkable quality viewed in white light. The holograms are bright, sharp, often large, with considerable depth and parallax. They belie the often-held supposition that holograms have to be viewed in highly coherent light--monochromatic light from a point source. These holograms which are the product much technical development a number of rather ingenious ideas, were developed in response to the realization that if display holography were to be of any widespread value, it had to be freed from the constraint of being viewed only with relatively expensive and inconvient sources such as lasers and mercury arcs.

It would be attractive, for various reasons, if holograms of such excellent quality could also be constructed in white light. The prevailing view among those somewhat knowledgeable, but not expert, in holography is that the making process is basically different, and even the holograms designed for white light viewing must still be made with highly coherent light, as for example from a laser.

On the other hand, if we consider that when a hologram is being viewed, the light passing through it is merely retracing the paths of the light used for making the hologram. And if we consider that the same laws of optics apply if we merely reverse the direction of the light rays, we might suspect that the coherence of the making and viewing processes might in a basic way be just the same. Thus, perhaps, holograms can be made in white light. There is considerable truth in this viewpoint, although the issue is by no means a simple one.

For the basic process of holography, as developed by its inventor Dennis Gabor, the truth of this supposition is quite simple. As we show in Fig. 1, a hologram is made of a small scattering "point" object O, of size Δx (we work in two dimensions instead of three, for simplicity). Light scattered by O interferes with the coherent background to form an interference pattern that is recorded as a hologram H. Each wavelength component of the source forms a zone plate pattern at H that is scaled differently, in inverse proportion to wavelength. The source wavelength spread, $\Delta\lambda$, must be limited to a value such that the outermost zones for the longest and shortest wavelength do not mismatch by more than, say, one half of a period. Simple considerations lead to the bandwidth limitation

$$\Delta\lambda/\lambda_o = 4(\Delta x)^2/\lambda_o z = 4/C \qquad (1)$$

where the various symbols are defined in Fig. 1. The quantity C we call the expansion ratio, which is the ratio of the width of a resolution element on the object and the linear spread of the zone plate it makes on the hologram. The introduction of this quantity leads to, as we see, an extremely simple expression for the allowable wavelength spread, just 4 times the reciprocal of the expansion ratio.

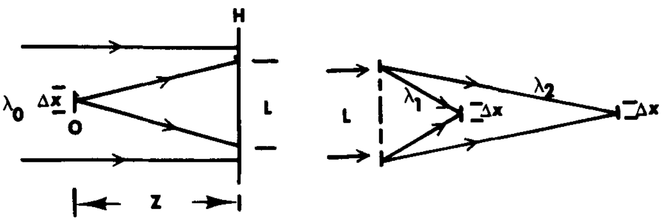


Fig. 1. Gabor hologram of a point object. left, construction; right, reconstruction

Similarly, on readout, the zone plate has a focal length inversely proportional to wavelength. If we view the hologram with a beam of polychromatic light and observe the plane where the image is in sharp focus at the midband wavelength, then other wavelengths will form images somewhat blurred in this plane. If we use the elementary laws of optics to calculate the allowable spatial bandwidth $\Delta\lambda$ before the image becomes noticeably blurred, we find

$$\Delta\lambda/\lambda_o = 4(\Delta x)^2/\lambda_o z = 4/C \qquad (2)$$

which is exactly the same expression as before. Here, we would call C the compression ratio, which is the ratio of the width of a zone plate on the hologram to the width Δx of the "point" image to which the zone plate focuses the light. Thus, for the simple, basic process of holography, as given by Gabor, our suspicions are confirmed. The making and the viewing processes have exactly the same monochromaticity requirements.

Moreover, these requirements can be very modest. For example, if we choose parameters such that the expansion ratio is 20 (i.e., a resolution cell on the object is spread into a zone plate response with an area 400 times greater), the allowable wavelength spread, for a midband of 5000 Angstroms, is 1000 Angstroms.

II. EXTENSION TO OFF-AXIS HOLOGRAPHY

Gabor in-line holograms suffer from poor signal to noise ratio. Higher quality is obtained from the so-called off-axis or carrier frequency holograms. There has been a widely held view that such holograms have greater coherence requirements for both the making and viewing steps, but this view is not necessarily true. Off-axis holography, as opposed to Gabor's original method, is a two-beam interferometric technique, with the object in one beam and with the other serving as the reference beam; this impinges on the recording plate at an angle with respect to the object beam, so as to produce a large number of fringes, typically several hundred to several hundred thousand. The chromaticity requirements are then a combination of those required for the holographic process and those required for the interference of the two beams. Now, there are many kinds of two-beam interferometers, and they differ greatly in their chromaticity requirements, so the total chromaticity requirement for off-axis holography depends on the kind of interferometer we use for introducing the reference beam.

It happens that there are achromatic interferometers that produce an unlimited number of very fine, high contrast fringes in perfectly white light. When such an interferometer is used for forming an off-axis hologram, the chromaticity requirement is no greater than for the basic process of Gabor in-line holography. Such interferometers use diffraction gratings as beam splitters. Similarly, a diffraction grating may be used in the hologram viewing process, so that the chromaticity requirements for viewing a hologram are also no greater than for the in-line case.

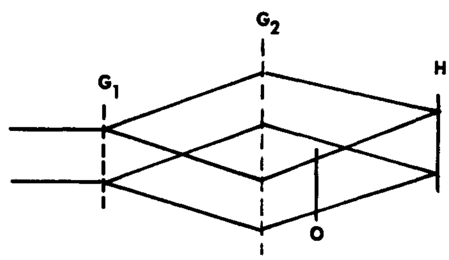


Fig. 2. A grating interferometer for making hologram.

Figure 2 shows an example of an interferometer formed from a pair of diffraction gratings. The first grating acts as a beam splitter, dividing the incident light into a zero and a first orders. If the grating is thick, so that the diffraction is of the Bragg type, there will be no other order. The second grating is a beam combiner, which brings the beams back together. The interferometer forms fringes whose spacing and position are independent of wavelength; thus, high contrast fringes are formed even for white light. The number of fringes and their spacing is just the number and spacing of the rulings on the first grating, and is thus independent of the wavelength spread.

It is easy to see why the fringe spacing is wavelength independent. The spacing of the fringes varies inversely as the angle between the two beams, and directly as the wavelength. However, longer wavelengths are diffracted by the gratings at greater angles and therefore, come back together at greater angles. The two factors, wavelength and combining angle, therefore just compensate, giving fringe spacing independent of wavelength. Analysis shows that the fringes for each wavelength will also be in perfect registry. Finally, analysis also shows that the fringes will form in a broad source. Thus, the fringe formation process requires neither temporal nor spatial coherence.[1,2]

We place an object transparency O in one of the beams, as indicated in Fig. 2. The fringe pattern will then, upon recording, become an off-axis hologram. Diffraction (i.e., scatter) by the object will cause the light in the object beam to be redistributed, thus somewhat upsetting the achromatic and broad source fringe formation process, so that now there will be some coherence requirements on the source. Analysis shows, however, that these requirements are exactly the very modest ones given by Eqs. 1 and 2 for basic holography.[1] Thus, by choosing the proper interferometer, off-axis holography can be done with the same coherence requirements as for basic, in-line holography.

III. MAKING HOLOGRAMS IN WHITE LIGHT

In recent years there has been considerable interest in making holograms in white light. As we have noted, holograms that can reconstruct in perfectly white light are now commonplace, and since we have argued that, for the basic holographic process, the chromaticity requirements for the making and viewing steps should basically be the same, we might now suspect that holograms can be formed in white light. And indeed they can. However, for reasons that are rather intricate, the white light hologram formation methods are really not counterparts of the previously-noted white light readout methods.

The white light methods fall into two categories. Both require fairly complex lens systems to achieve their results.

The first category is based on a procedure developed by Katyl[3], in which lens systems are used to achromatize a Fresnel or Fourier transformation process in a way analogous to the way lens systems are achromatized to form images.

The second category is based on the use of achromatic optical convolving systems which are integrated into achromatic interferometers.[4,5]

The first method has two salient characteristics:

1. All points on the hologram are formed simultaneously, i.e., in parallel.
2. The achromatization is only approximate.

The second method has these corresponding characteristics:

1. The holograms are formed one point at a time, i.e., sequentially, and the generation of the complete hologram requires a scanning process.
2. The achromatization is exact.

IV. KATYL'S METHOD

The achromatization method of Katyl has been used both in the viewing and in the making of holograms. It is a method analogous to the way a lens is achromatized --by putting simple lens elements together to form a compound lens. The lens elements are made from different kinds of glass, with different dispersion characteristics, so that the dispersion of one element compensates that of the other. Katyl's method differs in two respects.

First, instead of achromatizing so that the image is free from chromatic aberration the achromatization is done for some plane in the Fresnel field. For a two-dimensional object, the achromatization can be rather good. If the object is 3-D, the achromatization is done for only one plane of the object.

The second difference is that it is the diffraction process that is being achromatized rather than a lens system. The chromatic errors in diffraction processes are different from those of lenses and are severe; thus, highly dispersive correcting elements are needed. Katyl used glass dispersion lenses in some of this configurations, but for most he used a Fresnel zone plate, which enormously greater dispersion than glass lenses.

The correction of aberrations in an imaging system by the use of compensating lenses is a specialized process, and it is not appropriate to go here into what is basically a lens design problem. We note that Katyl shows experimental results of holograms made in white using the compensation systems and also shows for comparison results without the compensation system. The results are most impressive.

V. OPTICAL CONVOLVER METHOD

The optical convolving method consists of two parts (Fig. 3). First there is the system OP that achromatically generates a Fresnel diffraction pattern of an object transparency. This system is integrated into the second part, a grating interferometer that achromatically provides a reference beam.

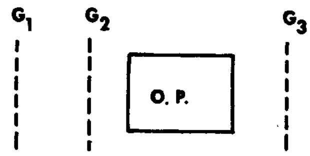


Fig. 3. The optical convolving method

The basic system for performing the former operation is shown in Fig. 4. The transparency O is illuminated with a polychromatic light source that is wavelength dispersed along the longitudinal, or z, axis. For wavelength λ_1, the source is a point a distance z_{s1} from the object; for a wavelength λ_2, the source is a different distance, z_{s2}, from the object. The source may be written as the function

$$S = \delta(z - z_s\lambda_0/\lambda), \qquad (3)$$

which indicates that the source point, as a function of wavelength, is located at $z = (\lambda_0/\lambda)z_s$, where z_s is the distance between source and object for wavelength λ_0. Such a source can be produce by illuminating a Fresnel zone plate with a point white light source and selecting one order while removing the others by appropriate spatial filtering.

The beam illuminating the object transparency is then $\exp(i\pi x^2/\lambda z)$ (where, as before, we drop the 3rd, or y dimension. Using Eq. 3, we write this as $\exp(i\pi x^2/\lambda_0 z_s)$. This form of the illuminating function is thus wavelength independent, a consequence of the axial dispersion. A lens L forms the Fourier transform of the product $O(x)\exp(i\pi x^2/\lambda_0 z_s)$, and we observe in the back focal plane the field distribution

$$u = \mathcal{F}[f(x - x')\exp(i\pi x^2/\lambda_0 z_s)], \qquad (4)$$

where $\mathcal{F}$ indicates Fourier transformation and x' describes movement of O(x) throught the optical system aperture. However, we confine our observation to a point on axis by placing at the back focal plane of the lens a mask containing a pinhole. The field in the pinhole represents the integral

$$u(x') = \int O(x - x')\exp(i\pi x^2/\lambda_0 z_s)dx = O*h. \qquad (5)$$

We have thus formed the convolution operation describing Fresnel diffraction, and it is significant to note that it has been formed achromatically, using a broad-spectrum source. Of course, it is generated one point at a time, and to generate the entire pattern, we must move the object transparency through the aperture.

This diffraction pattern can be recorded as a hologram by introducing a reference beam $u_0 = a_0\exp(i2\pi f_0 x')$, producing an irradiance $|u_0+u|^2$, which results in a hologram that produces the desired reconstructed wave. By using an achromatic interferometer to provide the two beams, object and reference, the recombination of the beams is accomplished achromatically, as we explain later.

Explicitly showing the zone plate which provides the dispersed source for the object leads to an alternative way (Fig. 5) of describing this holographic process. The zone plate, whose amplitude transmittance is $t_a = 1/2[1+\cos(\pi x^2/\lambda_0 z_s]$, imaged onto the transparency O. We suppose that in the process the zone plate is spatially filtered so that only the virtual image term reaches the transparency O. Thus, the light distribution impinging on the transparency is, to within a constant, $\exp(i\pi x^2/\lambda_0 z_s)$, which is the result given previously. A more complete description is given in Refs.

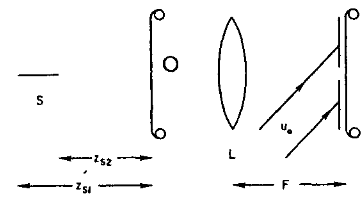


Fig. 4. Achromatic holographic configuration.

4 and 6. By formulating the operation in this manner, it is seen to be just the basic method of using a coherent optical system to convolve two functions by imaging one onto the other. The holographic process is then just the special case of choosing the convolving impulse response as a quadratic phase function. We could use any other convolving function; thus, the method we describe in terms of holography is indeed a very general method for using optical processing for performing the general operation g=f*h, where f and h are any functions.

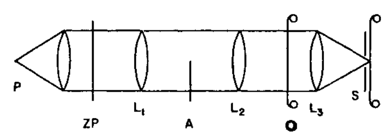


Fig. 5. Alternate description

To complete the achromatic holographic system of Fig. 4 (or 5) we must bring in a white light reference beam that is coherent with the object beam. This is done by means of the three-grating interferometer shown in Fig. 6. This interferometer is achromatic, just as

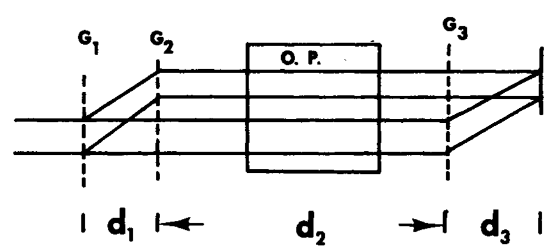


Fig. 6. The three grating interferometer

is the two-grating interferometer of Fig. 2. Using three gratings (all of the same spatial frequency) adds considerable flexibility, as explained in Ref. 5. One consequence is that the distance d_2 between gratings G_2 and G_3 is decoupled from the equation for fringe formation; the fringes form at a distance d_3 from the final grating regardless of the separation of gratings G_2 and G_3. We place the optical convolving system in the space between G_2 and G_3, in such a way that both object and reference beams tranverse the total optical system, otherwise the paths are not the same and coherence between the two beams is lost. The reference beam must of course also traverse the two plates containing the zone plate and the object O, however, it must go through portions of the plates that are clear, so that it will emerge as a uniform beam.

A system for making one-dimensionally dispersed holograms, such as the very popular multiplex holograms and Benton rainbow holograms, is shown in Fig. 7. The optical convolving system is identical with that of Fig. 5, except for the addition of a cylindrical lens L_3 (so

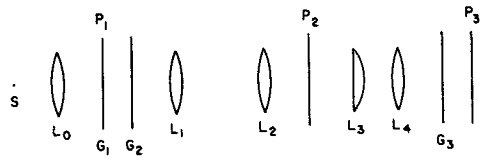


Fig. 7. The complete system

as to produce one-dimensional dispersion). G_2 and G_3 are identical gratings of spatial frequency f_1, and G_1 is an off-axis zone plate structure, with carrier f in the y direction and the zone plate structure in the x direction. In other words, we have a nice economy by having G_1 serve as both a cylindrical zone plate for one-dimensional convolution and also as the first grating of the interferometer. The zero and first orders of G_1 are selected, with the first order being modulated by the zone plate. G_2 demodulates the diffracted beam to zero spatial frequency in the y direction. Since G_2 affects the light distribution in the y direction only, G_2 can then be considered to be imaged in the x direction at object plane P_2, producing the required object distribution. The object is then moved through the aperture, as described earlier, while G_3 modulates the reference beam in order to produce the required fringes at the output plane.

A final comment: the grating G_3 can bring the reference beam to the recording plate at a very steep angle, so as to produce fringes of several hundred lines/mm, a spatial frequency far higher than the lens could pass. The fringes can be, in a special sense, considered as an image of the initial grating G_1, so we have imaged through the optical system a spatial frequency far in excess of what the modulation transfer function of the system would ordinarily allow, yet have suffered not even a slight loss of fringe contrast. It is done by the grating G_2 beating the reference beam to zero spatial frequency, i.e., making it travel along the optical system axis, and the gratings G_3 then restoring the reference beam to its original high spatial frequency, i.e., its steep angle.

An example of a hologram produced by Swanson in an achromatic system such as we have described is shown in Fig. 8.

To conclude, I point out some very recent work by my coworker G. Collins, who has advanced the Katyl method and combined it with the three grating interferometer method to produce a Fourier transform hologram in white light, without the need for a scanning process.

While the white light methods have advanced considerably over the past several years, I believe it is only a beginning, and there is considerable room for further advance.

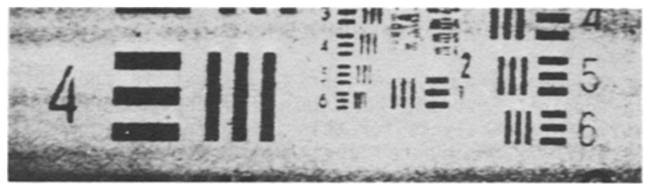


Fig. 8. Reconstruction from a hologram made in white light (courtesy G. Swanson).

REFERENCES

1. E. Leith and J. Upatnieks, J. Opt. Soc. Am., 57, 1123 975 (1967).

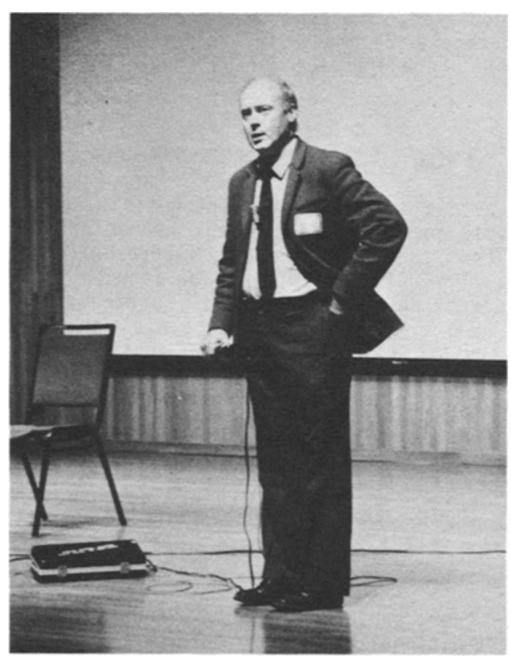
2. E. Leith and B. Chang, Appl. Opt. 12, 1957 (1973).

3. R. H. Katyl, Appl. Opt. 11, 1241, 1248, 1255 (1972).

4. E. Leith and J. Roth, Appl. Opt. 16 2565 (1977).

5. E. Leith and G. Swanson, Appl. Opt. 19 638 (1980).

6. E. Leith and J. Roth, Appl. Opt. 18, 2803 (1979).



Dr. Emmett Leith described optimum geometries for the formation of visible light holograms with sources of limited coherence properties. The extension to x-ray experiments was suggestive.