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Direct numerical simulation of transition in MHD duct flow

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Transition in the flow of electrically conducting fluid in a square duct with insulating walls is studied by direct numerical simulations. A uniform magnetic field is applied in the transverse direction. Moderate values of the Reynolds ($Re = 5000$) and Hartmann ($Ha = 0 \dots 30$) numbers are considered that correspond to the classical Hartmann & Lazarus [1] experiments. It is shown that the laminarization begins in the Hartmann layers, whereas the sidewall layers remain turbulent. Complete re-laminarization occurs in the range of $R = Re/Ha \approx 220$, which is in agreement with the H. & L. experiments.

The processes of flow re-laminarization in duct geometries were the first MHD phenomena studied experimentally by the pioneering work of Hartmann & Lazarus [1]. The experiments were performed with turbulent flow entering a straight duct with a length of several hundred hydraulic diameters, subjected to a uniform transverse magnetic field. The Reynolds numbers were $Re = 2000 \dots 5000$ and magnetic fields $\boldsymbol{B}$ corresponded to $Ha = 0 \dots 30$, and the pressure loss was measured in the test section. Complete flow laminarization was detected for $R \approx 220$, where $R \equiv Re/Ha$ is the Reynolds number based on the Hartmann layer thickness δ_{Ha}.

The essential feature of laminar MHD duct flow is a flat core and the characteristic boundary layers: the Hartmann layers of thickness scaling as $\sim Ha^{-1}$ at the walls perpendicular to the magnetic field and the side-wall (Shercliff) layers of thickness $\sim Ha^{-1/2}$ at the walls parallel to the magnetic field. In the previous linear studies of MHD duct flow (e.g., [2]) it was found that the optimal perturbations are localized in the side-wall layers, which are, therefore, expected to play an important role in transition. The same conclusions about the role of the side-wall layers have been made in the numerical study [3] for $Re \approx 2700$ and $Ha \approx 10 \dots 13$. Complete flow laminarization was also observed at $R \approx 220$. In the present work we would like to reproduce numerically the H. & L. experiments and extend the range of parameteric space towards experimental values.

We consider the flow of an incompressible, electrically conducting fluid in square duct subjected to a uniform magnetic field $\boldsymbol{B}_0 = B_0\boldsymbol{e}$, where $\boldsymbol{e}$ is the unit vector. The flow is driven by a pressure gradient $\partial P_0/\partial x$ in the streamwise x-direction. MHD effects are considered in the limit of low magnetic Reynolds number Re_m, i.e. the quasi-static approximation [4] is applied. The governing non-dimensional Navier-Stokes equations with the boundary conditions become:

$$\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v}\cdot\nabla)\boldsymbol{v} = -\nabla p + \frac{1}{Re}\triangle \boldsymbol{v} + N(\boldsymbol{j}\times\boldsymbol{e}), \quad (1)$$

$$\nabla\cdot\boldsymbol{v} = 0, \quad (2)$$

$$\boldsymbol{j} = (-\nabla\phi + \boldsymbol{v}\times\boldsymbol{e}), \quad (3)$$

$$\triangle\phi = \nabla\cdot(\boldsymbol{v}\times\boldsymbol{e}), \quad (4)$$

$$\boldsymbol{v} = \frac{\partial\phi}{\partial\boldsymbol{n}} = 0 \quad \text{on walls.} \quad (5)$$

The non-dimensional parameters are the Reynolds number $Re \equiv U_q L/\nu$ and the magnetic interaction parameter $N \equiv Ha^2/Re$, where $Ha \equiv BL\,(\sigma/\rho\nu)^{1/2}$ is the Hartmann number. Here U_q is the mean flux velocity, L is the half-height of the duct, σ is the electrical conductivity and the unit vector $\boldsymbol{e}$ denotes the direction of applied magnetic field.

The system (1–4) is solved numerically by our in-house DNS code developed for MHD flows in rectangular geometry. The solver is based on a finite-difference method with collocated grid arrangement. For spatial discretization we use a highly conservative scheme, proposed in [5] and extended in [6] to the case of low-Re_m MHD. The scheme is of the second order of approximation, the boundary conditions are periodic in the streamwise x-direction. The solver is hybrid-parallel with both MPI and Open MP interfaces for distributed and shared memory parallelization.

We performed a series of numerical simulations for duct flow at $Re = 5000$ and Hartmann numbers Ha varying from 0 to a certain maximum value at which the full flow laminarization was found. For each simulation a fully developed turbulent state at $Ha = 0$ was used as initial conditions. We also notice that the spatially evolving transtion from the H. & L. experiments cannot be directly studied here due to the periodic in-/out-flow conditions. To properly address this issue, the effect of domain size was studied too, so that the simulations were performed for three different lengths of periodicity L_x: 4π, 8π and 16π.

The time histories of the walls stresses τ_y (side-walls) and τ_z (Hartmann walls) are shown in figure 1 for short domain-length $L_x = 4\pi$ and Hartmann numbers $Ha = 0$ and 20. At $Ha = 20$ the Hartmann stresses τ_z exhibit barely fluctuating behavior, which can be associated with the Hartmann boundary layers approaching the laminarization threshold. The "large-scale" fluctuating character of the side-wall stresses τ_y can be viewed as turbulence suppression by the magnetic field and the

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transformation of turbulent eddies into large-scale streaky structures in the region of side walls. Finally, at $Ha = 25$ complete flow laminarization occurs. The flow transformation is shown in figure 2 by instanteneous velocity profiles for $Ha = 0$ and 20, visualized in a cross-section at $x = L_x/2$.

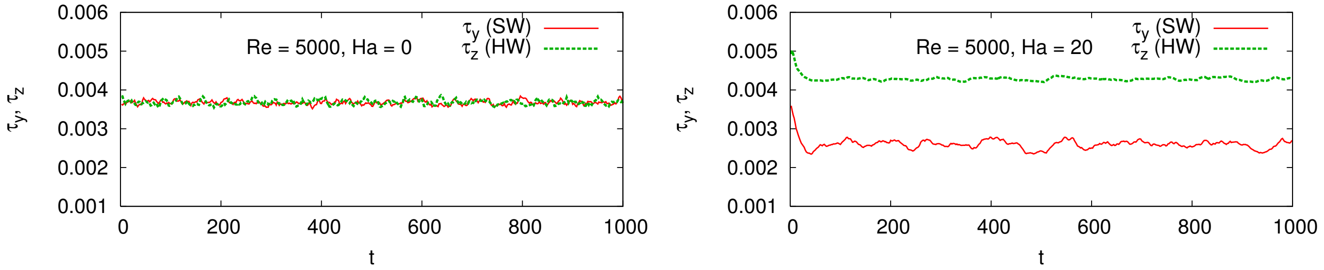

Fig. 1 Effect of the magnetic field on the wall stresses τ_y and τ_z for *Re* $= 5000$ and $Ha = 0$ and 20. Results of DNS for short box-size $4\pi \times 2 \times 2$ and numerical resolution $256(x) \times 128(y) \times 128(z)$ are shown. Time histories of the wall stresses $\tau_w = \tau_y + \tau_z$ are visualized separately as τ_y for side walls (SW) and τ_z for Hartmann walls (HW). The wall stresses are normalized by ρU_q^2.

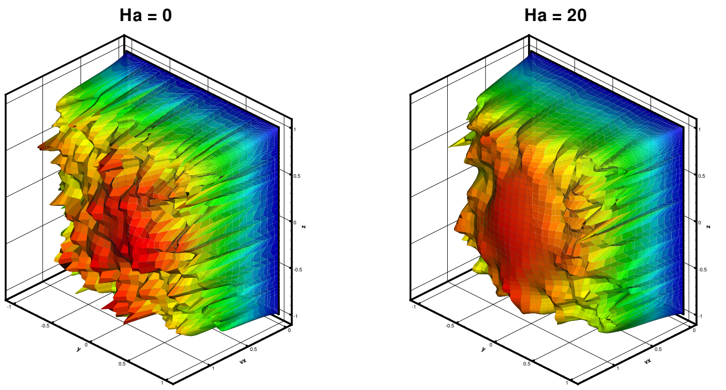

Fig. 2 Effect of the magnetic field on turbulence: instanteneous velocity profiles are shown for duct flow at $Ha = 0$ (left) and $Ha = 20$ (right). The magnetic field is imposed in the z-direction.

For better assessment of the transiton details, our further analysis focuses on the range of Hartmann numbers $Ha = 20 \ldots 25$. The procedure of numerical experiments was modified: in addition to the non-MHD turbulent state, fluctuating states at $Ha = 20$ and 22 were used as initial conditions too. Also, simulations were performed for middle ($L_x = 8\pi$) and large ($L_x = 16\pi$) domain sizes. The first observation concerns the effect of periodicity length: in longer domains unsteady regimes can be sustained at higher Ha, e.g., $Ha = 22$ for $L_x = 8\pi$ and $Ha = 25$ for 16π. Hence, the complete flow laminarization can be postponed. Secondly, it was found that different initial conditions can either evolve into laminar or fluctuating states. In particular, for the large domain-size $L_x = 16\pi$ three states can exist at $Ha = 25$: laminar state if switched directly from the initial regime at $Ha = 0$ and two fluctuating states, obtained from turbulent regimes at $Ha = 20$ and 22 correspondingly. Visual inspection of the flow structures as well as the analysis of the Reynolds stress tensor indicate that these regimes can be identified as weakly fluctuating large-scale streaks that remain apparent in either one or both side layers at $Ha \approx 22 \ldots 25$. At the same time, no fluctuating states beyond $Ha = 25$ were found.

We can conclude that the re-laminarization threshold cannot be identified in a clear-cut way. Instead, there is a range of Reynolds numbers $R = Re/Ha$ where the change of flow regime is expected. In our simulations this transitional range is identified as $R \sim 200 \ldots 220$, which is in very good agreement with the H. & L. experiments [1] and previous numerical study [3]. The major contribution to the wall stresses τ_w comes from the Hartmann layers, rather than from the side-wall layers. This effect becomes more pronounced approaching the limit of laminarization and, therefore, the complete laminarization may remain undetected in experimental studies. Further analysis should focus on the coherent structures in the side-wall layer that appear at the edge of laminarization.

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