Transverse Target Moments of SIDIS Vector Meson Production at HERMES

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Abstract. The on-going analysis of the transverse target (A_{UT}) moments of the non-collinear cross section for SIDIS dihadron production at HERMES is discussed. These moments access the transversity (h_1), pretzelocity ($h_{1T}^{\perp}$), and Sivers ($f_{1T}^{\perp}$) distribution functions convoluted with the unpolarized (D_1) and Collins (H_1) dihadron fragmentation functions. This measurement allows greater insight into the flavor decomposition and factorization of these functions. Additionally, the results will test the Lund/Artru fragmentation model, which predicts a sign change in the Collins function between pseudo-scalar mesons and certain partial waves of vector mesons. In preparation for this analysis, a new Monte Carlo generator has been written, and non-collinear fragmentation functions have been computed in a spectator model. Additionally, an alternate partial wave expansion is presented, providing a complementary interpretation of the fragmentation functions and allowing computation of the next-to-leading twist cross section.

1. Introduction and Motivation

Semi-inclusive deep inelastic scattering (SIDIS), the scattering of high energy leptons from nuclei, is one of the key processes used in probing the partonic spin structure. Many results for SIDIS production of single hadrons (particularly pseudo-scalar mesons) are available, including results from a variety of experiments.[1] However, less data is available about the SIDIS production of dihadrons, where a hadron pair is measured in the final state

$$l + N \rightarrow l' + h_1 + h_2 + X. \tag{1}$$

Dihadron production includes subprocesses where h_1 and h_2 come from the decay of a parent particle, such as a vector meson, as well as other subprocesses.[2]

Dihadron production involves an additional three variables over single hadron production. They are chosen to be M_h, the invariant mass of the dihadron; $\cos\vartheta$, the polar angle of one of the hadrons in the dihadron center of mass frame (identical to the polar angle used in exclusive vector meson production [3]); and ϕ_R, the azimuthal angle of the difference between the momenta of the two hadrons with respect to the lepton scattering plane, measured perpendicular to the momentum of the center of mass and in the virtual-photon nucleon center-of-mass frame [4]. The convention for dihadron production is such that if both both hadrons are charged, e.g. $\pi^+\pi^-$, the positively charged hadron is used in computing $\cos\vartheta$, and while if only one hadron

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[1] A nice review can be found in Ref. [1]
[2] For example, the contributing subprocesses in $\pi^+\pi^-$ dihadron production is detailed in Ref. [2].

is charged, e.g. $\pi^{\pm}\pi^0$, the charged hadron is used. Letting p_1, p_2 be the momenta of the two hadrons (p_1 corresponding to the hadron used in defining $\cos\vartheta$), the center of mass momentum is denoted P_h, and the difference between the two momenta is denoted $R = p_1 - p_2$. Defining $\boldsymbol{k}$ as the momenta of the incoming lepton and $\boldsymbol{q}$ as the momenta of the virtual photon, one can define ϕ_R as

$$\phi_R := \mathrm{signum}\left[(\boldsymbol{R} \times \boldsymbol{P}_h) \cdot \boldsymbol{n})\right] \arccos \frac{(\boldsymbol{q} \times \boldsymbol{k}) \cdot (\boldsymbol{P}_h \times \boldsymbol{R})}{|\boldsymbol{q} \times \boldsymbol{k}||\boldsymbol{P}_h \times \boldsymbol{R}|}, \tag{2}$$

with $\boldsymbol{n} = (\boldsymbol{q} \cdot \boldsymbol{P}_h)\boldsymbol{k} - (\boldsymbol{k} \cdot \boldsymbol{P}_h)\boldsymbol{q}$, and where all momenta are with respect to the virtual-photon nucleon center-of-mass frame.

The SIDIS process can be interpreted according to a factorization scheme [5, 6, 7], where moments of the cross section are sums of integrals of distribution functions (describing the partonic structure of the nucleon) and fragmentation functions (describing the quark hadronization process). Dihadron production accesses the same distribution functions as does pseudo-scalar production [4] and can be seen as a complementary access point. However, additional flavor combinations are also available in dihadron production, most notable the nearly pure access to the sum of strange and anti-strange quark flavors via the ϕ-meson. The Sivers function for ϕ-mesons is also related to gluon orbital angular momentum [8]. Additionally, dihadrons allow a collinear (transverse momentum integrated) access to transversity, which is not possible with pseudo-scalar production. The advantage of collinear access is that the evolution equations (Q^2 scaling) are known, while the evolution equations in the non-collinear case are not known. Another strong motivation for measuring both pseudo-scalar and dihadron production is that the Lund/Artru fragmentation model predicts a sign change of the Collins function between pseudo-scalar mesons and certain partial waves of vector mesons. A non-collinear analysis of both dihadrons and pseudo-scalars is necessary to test this model.

Previous results are available for one transverse target azimuthal moment from both HERMES and COMPASS for $\pi^+\pi^-$ pairs [9, 10]. However, a model prediction for COMPASS, based on the HERMES results, is a factor of three smaller than the actual COMPASS results. This may be due to differences in the $\cos\theta$ treatment, treatment of the depolarization factor, or possibly due to both experiments using the angle $\phi_{R\perp}$ instead of ϕ_R.[3] The angle $\phi_{R\perp}$ is computed by defining $\boldsymbol{R}_T$ as the projecting of $\boldsymbol{R}$ perpendicular to $\boldsymbol{P}_h$, and then considering the the azimuthal angle of $\boldsymbol{R}_T$ about the virtual photon direction, with respect to the lepton scattering plane, formally [9]

$$\phi_{R\perp} = \mathrm{signum}\left[(\boldsymbol{q} \times \boldsymbol{k}) \cdot \boldsymbol{R}_T\right] \arccos \frac{(\boldsymbol{q} \times \boldsymbol{k}) \cdot (\boldsymbol{q} \times \boldsymbol{R}_T)}{|\boldsymbol{q} \times \boldsymbol{k}||\boldsymbol{q} \times \boldsymbol{R}_T|}, \tag{3}$$

While the difference between ϕ_R and $\phi_{R\perp}$ is zero at leading twist, within the Q^2 range and acceptance of HERMES, the differences can be notable and show kinematic dependencies. COMPASS also integrates over $\cos\vartheta$, introducing an additional, unknown, scale factor. While the scale factor is presumably near one, actually fitting with respect to $\cos\vartheta$ would remove this possible source of discrepancy. Thus, to improve comparison of future measurements, all experiments should fit with respect to $\cos\vartheta$ and also use the angle ϕ_R rather than $\phi_{R\perp}$.

To test the Lund/Artru model and to gain additional information regarding flavor dependence, future plans for HERMES measurements include considering additional dihadrons as well as extracting moments of the non-collinear cross section. The most important additional dihadrons include $\pi^{\pm}\pi^0$, and K^+K^- (related to vector mesons $\rho^{\pm}$ and ϕ). As the asymmetry results for π^0 are considerably smaller than those for charged pions [11], the charged ρ-mesons are expected to yield a clearer test of the Lund/Artru model. The applicability of the leading analysis must be also be considered—requiring knowledge of the subleading twist cross section to test its effect. Several necessary items, not previously available for SIDIS dihadron production,

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[3] While COMPASS uses the quantity $\phi_{R\perp}$, they refer to it by the symbol ϕ_R.

have been prepared to accomplish these plans. These items include the non-collinear cross section, a Monte Carlo event generator, and models of non-collinear fragmentation functions.

2. Partial Wave Analysis

The Lund/Artru model of string fragmentation posits that a gluon flux tube is produced between the struck quark and the nucleon remnant. When the flux tube breaks, the produced quark, anti-quark pair are assumed to be in a system with quantum numbers equal to those of the vacuum. This requires the spins of the quark and anti-quark to be aligned and one unit of orbital angular momentum opposing these spins. Considering the struck quark to be transversely polarized, depending on whether the spins of the produced pair are parallel (states $|\frac{1}{2}, \pm\frac{1}{2}\rangle|\frac{1}{2}, \pm\frac{1}{2}\rangle$) or anti-parallel (states $|\frac{1}{2}, \pm\frac{1}{2}\rangle|\frac{1}{2}, \mp\frac{1}{2}\rangle$) with the spin of the struck quark, the orbital angular momentum will cause the produced meson to prefer either the right (parallel) or left (anti-parallel) relative to the struck quark spin. This reversal of the left-right asymmetry dependent on the alignment of the quark spins is manifest as a sign change in the Collins function.

However, the SIDIS amplitude is most clearly written with the spins in the direct sum basis, rather than the direct product basis, i.e. with reference to scalar and vector mesons, rather than mesons with quark spin aligned or anti-aligned. Thus, at the amplitude level, one would expect the Collins function of for pseudo-scalar mesons $|0,0\rangle$ and longitudinally polarized vector mesons $|1,0\rangle$ to have one sign, and transversely polarized vector mesons $|1,\pm1\rangle$ to have the opposite sign. However, since the cross section is product of the amplitude and the complex conjugate of the amplitude, the concept of Collins functions for specific polarization states of vector mesons can only be defined within a partial wave analysis.

Previous partial wave analyses of dihadron production have been in the direct product basis [4], i.e. specifying the spin state of both the dihadron in the amplitude and the dihadron in the conjugate of the amplitude. However, just as the direct sum basis (vector and pseudo-scalar mesons) is more convenient than the direct product basis (mesons with quarks aligned or anti-aligned) in describing the amplitude, the cross section can be more conveniently written in terms of the direct sum basis of the two-dihadron state. Equivalently, one can expand the fragmentation functions in terms of the partial waves of the two dihadron system.[4] This alternate partial wave expansion does not change the form of the cross section, but rather provides an alternate interpretation.

Advantages to this interpretation include having one symbol for each experimentally accessible partial wave. Partial waves $|2,\pm2\rangle = |1,\pm1\rangle|1,\pm1\rangle$ are exactly the square of the partial waves for transverse vector mesons, and thus the $|2,\pm2\rangle$ partial wave of the Collins function is expected to change sign with respect to the Collins function for pseudo-scalar production. There is no clear access to a Collins function for longitudinal vector mesons, as the state $|2,0\rangle$ includes both longitudinal vector mesons and interference between the two transverse polarizations of vector mesons. Additionally, the fragmentation function occurring in the collinear case, previously denoted $H_{1,UT}^{\sphericalangle\, sp}$, can be identified with the $|1,1\rangle$ partial wave of the Collins function, $H_1^{\perp|1,1\rangle}$. Thus we see that fragmentation function is not pure sp interference, but also includes interference between longitudinal and vector mesons. Additionally, the fragmentation functions existing in the collinear cross section are just integrals of certain partial waves of the non-collinear fragmentation functions. The specific partial wave which survives in the collinear case depends on the distribution function with which the fragmentation function is paired.

An additional advantage of this alternate partial wave expansion is that the dihadron cross section, before partial wave expansion, is identical to the pseudo-scalar cross section. This is expected, since both are cross sections for producing a integer-spin system. Alternately, one can identify the pseudo-scalar cross section with the $|0,0\rangle$ sector of the dihadron cross section. All

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[4] A rigorous, mathematical presentation is in preparation for publication. Until then, see Ref. [12].
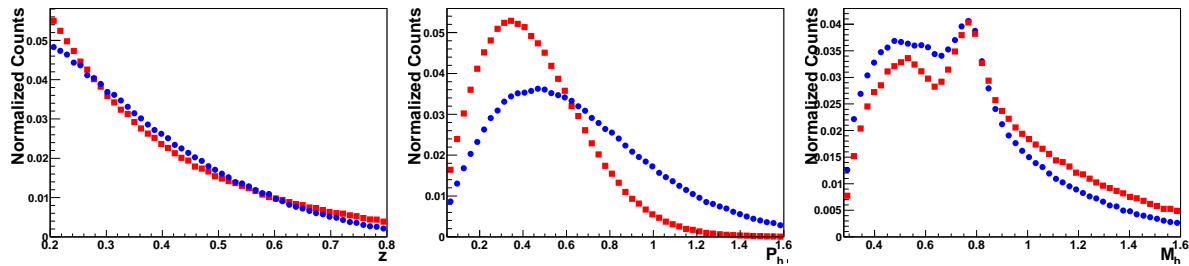
Figure 1. Comparison of data from `TMDGen` (blue circles) for a given choice of parameters vs `Pythia` (red squares) tuned to HERMES kinematics. The distributions are of the variables z (left panel), $P_{h\perp}$ (center panel), and M_h (right panel).

additional complications in the dihadron cross section are just additional partial waves of the same basic quantities. By noting this, one can then compute the dihadron cross section, at any twist, given the pseudo-scalar cross section at the corresponding twist. This is a tremendous computational savings over traditional methods [13]. A few terms of the next-to-leading twist cross section are given in Appendix A.

3. Monte Carlo Generator `TMDGen`

A new Monte Carlo generator has been developed. Though early versions were denoted `GMC_Trans`, the name has been changed to `TMDGen`. The generator allows one to select among available models for distribution and fragmentation functions, and throws events according to the SIDIS cross section, using either weights or acceptance/rejection. Both pseudo-scalar and dihadron production are currently included. Additionally, the intrinsic transverse momenta $\boldsymbol{p}_T$ and $\boldsymbol{k}_T$ are specifically modeled. This allows one to include any general $\boldsymbol{p}_T$ and $\boldsymbol{k}_T$ dependence, and to further study relations between the intrinsic transverse momenta and experimentally accessible variables. The generator is designed to be flexible and to accommodate the inclusion of additional models. Additionally, the generator is designed to link with any experiment's simulation chain as well as to work independent of any experiment.

A new non-collinear spectator model has been determined and implemented in the generator. The model is based on Ref. [2], which describes a collinear spectator model for $\pi^+\pi^-$ pairs. Both models replace the SIDIS average over all possible other final states (the X in Equation 1) with a single, on shell, spin-half spectator with mass $M_s \propto M_h$. The tree level fragmentation correlator can then be directly written, from which one can compute the fragmentation matrices by tracing the correlator with certain Dirac matrices. The non-collinear model computes these traces without the integration over the transverse momentum. Additionally, an extra z-dependent k_T-cutoff is needed, as the model only includes a cutoff with respect to k^2,

$$k^2 = \frac{z}{1-z}|\boldsymbol{k}_T|^2 + \frac{M_s^2}{1-z} + \frac{M_h^2}{z}. \tag{4}$$

Without the extra k_T cutoff, it is not possible to match both the M_h and $P_{h\perp}$ distributions, as shown in Figure 1. To consider $\pi^\pm\pi^0$ and K^+K^- pairs, one must also allow flavor dependence for the parameter sets and slightly modify the p-wave vertex function.

4. Conclusion

Three important items for the analysis of SIDIS dihadron production at HERMES have been discussed. The new partial wave analysis aids in the interpretation of these measurements and

allows the computation of the dihadron cross section from the pseudo-scalar cross section, at any twist. A new Monte Carlo generator, TMDGen, has been written and will later be released to the public. This generator will not only assist in the systematic studies for the HERMES analysis, but will also be useful for systematic studies for other experiments and in interpreting any SIDIS result in terms of fragmentation and distribution functions. Additionally, preliminary HERMES results for non-collinear dihadron production will be forthcoming.

Appendix A. Selected Cross Section Terms

By identifying the non-expanded dihadron cross section with the pseudo-scalar cross section, at any twist, one can compute the partial wave expanded dihadron cross section. The moments can be written in terms of structure functions, as was done for pseudo-scalar mesons in Ref. [7]. For example, the 45 transverse target, unpolarized beam moments, at next-to-leading twist, are

$$
\left(\frac{1}{S_T}\right)\frac{2\pi xyQ^2}{\alpha^2 M_h P_{h\perp}}\left(1+\frac{\gamma^2}{2x}\right)^{-1} d^9\sigma_{UT} = \left[\sum_{l=0}^{2}\sum_{m=-l}^{l}\right.
$$

$$
A(x,y)P_l(\cos\vartheta)\sin((1-m)\phi_h-\phi_S+m\phi_R)F_{UT,T}^{P_l(\cos\vartheta)\sin((1-m)\phi_h-\phi_S+m\phi_R)} \tag{A.1}
$$

$$
+ B(x,y)P_l(\cos\vartheta)\sin((1-m)\phi_h+\phi_S+m\phi_R)F_{UT}^{P_l(\cos\vartheta)\sin((1-m)\phi_h+\phi_S+m\phi_R)}
$$

$$
+ B(x,y)P_l(\cos\vartheta)\sin((3+m)\phi_h-\phi_S+m\phi_R)F_{UT}^{P_l(\cos\vartheta)\sin((3+m)\phi_h-\phi_S+m\phi_R)}
$$

$$
+ V(x,y)P_l(\cos\vartheta)\sin(m\phi_h+\phi_S+m\phi_R)F_{UT}^{P_l(\cos\vartheta)\sin(m\phi_h+\phi_S+m\phi_R)}
$$

$$
+ V(x,y)P_l(\cos\vartheta)\sin((2+m)\phi_h-\phi_S+m\phi_R)F_{UT}^{P_l(\cos\vartheta)\sin((2+m)\phi_h-\phi_S+m\phi_R)}.
$$

The subscripts UT identify these moments as unpolarized beam and transverse target moments, while the subscript UT,T additionally indicates the virtual photon is transversely polarized. The terms with depolarization factors $A(x,y)$ and $B(x,y)$ are twist-2, while those with $V(x,y)$ are twist-3. The depolarization factors are defined as

$$
A(x,y) = \frac{y^2}{2(1-\epsilon)}, \quad B(x,y) = \frac{y^2}{2(1-\epsilon)}\epsilon, \quad V(x,y) = \frac{y^2}{(1-\epsilon)}\sqrt{2\epsilon(1+\epsilon)}, \tag{A.2}
$$

consistent with Ref. [7]. These structure function can then be interpreted in terms of partial waves of the fragmentation functions. For example, the structure functions with transversity

and the Collins function are

$$F_{UT}^{P_l(\cos\vartheta)\sin((1-m)\phi_h+\phi_S+m\phi_R)} = \mathcal{C}\left[-\frac{|\boldsymbol{k}_T|}{M_h}\cos((m-1)(\phi_h-\phi_{k_T}))h_1 H_1^{\perp|l,m\rangle}\right], \qquad (A.3)$$

where $H_1^{\perp|l,m\rangle}$ denotes the $|l,m\rangle$ partial wave of the Collins function, and $\mathcal{C}[\cdot]$ denotes the convolution over intrinsic momentum, defined in Equation 4.1 of Ref. [7].

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