Physical improvements to the solar wind reconnection control function for the Earth's magnetosphere

Joseph E. Borovsky[1,2]

Received 27 September 2012; revised 10 December 2012; accepted 4 January 2013; published 21 May 2013.

[1] In a 2008 publication a first principles calculation of the dayside reconnection rate expressed in terms of upstream–solar wind parameters led to a rudimentary solar wind coupling function R_1 for the Earth's magnetosphere. Four improvements to that derivation are added in the present paper, resulting in a more correct solar wind control function describing the rate at which solar wind magnetic field lines connect into the Earth's magnetosphere. The first is the inclusion of the effect of β-dependent plasma compressibility on the reconnection rate. The second is a corrected calculation of the inflow of magnetic field lines into the reconnection X-line. The third is a more accurate estimate of the orientation of the reconnection X-line for asymmetric reconnection. The fourth correction accounts for the compression ratio of the Earth's bow shock for arbitrary orientation of the solar wind magnetic field. Two solar wind control functions result: one function, R_{2CS}, is based on the Cassak-Shay equation and another function, R_{2CSB}, is based on the Cassak-Shay-Birn equation. The control functions are tested using solar wind measurements and geomagnetic indices from 1980 to 2012, and some improved correlation coefficients over the rudimentary function R_1 are found. Simplified approximations to the new control functions are supplied: one is $R_{2CS\text{-approx}} = 1.68 \times 10^{-2} \sin^2(\theta/2) n_o^{1/2} v_o^2 M_A^{-0.3044} \exp(-[M_A/3.18]^{1/2})$, where the subscript "o" denotes the solar wind upstream of the bow shock and M_A is the solar wind Alfvén Mach number.

Citation: Borovsky, J. E. (2013), Physical improvements to the solar wind reconnection control function for the Earth's magnetosphere, *J. Geophys. Res. Space Physics*, *118*, 2113–2121, doi:10.1002/jgra.50110.

1. Introduction

[2] In this report, solar wind/magnetosphere coupling functions based on the rate of dayside reconnection are explored. The reconnection rate (amount of magnetic flux reconnection per unit time per unit length of the reconnection X-line) at the magnetopause is given by a magnetic field strength times a reconnection-inflow speed, with the inflow speed given by a hybrid Alfvén speed determined by the field strength B and plasma mass density ρ on both sides of the reconnection site: the magnetospheric side and the magnetosheath side. The magnetospheric magnetic field strength is determined by pressure balance with the ram pressure $\rho_o v_o^2$ of the upstream solar wind (subscript "o"). The mass density of the magnetosheath is given by the solar wind mass density ρ_o and the Mach number–dependent compression ratio of the bow shock. Applying pressure balance at the magnetopause, the magnetic field strength of the magnetosheath is given by the magnetic field strength of the magnetosphere and the plasma beta of the magnetosheath, which is Mach number–dependent. The dayside reconnection rate is given by a magnetic field strength times an Alfvén speed, which goes as $B^2/\rho^{1/2}$. With B^2 given by $\rho_o v_o^2$ and ρ given by ρ_o, the reconnection rate is $\rho_o^{1/2} v_o^2$, multiplied by Mach number–dependent terms that describe the compression ratio of the bow shock and the reduction of the magnetosheath magnetic field by finite β effects.

[3] *Borovsky* [2008] derived a dayside reconnection rate based on the Cassak-Shay equation [*Cassak and Shay*, 2007]. The Cassak-Shay equation expresses the reconnection rate as a function of the local plasma parameters on both sides of the reconnection site. In terms of magnetospheric (subscript "m") and magnetosheath (subscript "s") magnetic fields and mass densities, the Cassak-Shay equation for the rate of flux reconnection per unit length of the X-line can be written as

$$R_{CS} = \left(0.1/\pi^{1/2}\right) B_m^{3/2} B_s^{3/2} \Big/ \left\{ (B_m \rho_s + B_s \rho_m)^{1/2} (B_m + B_s)^{1/2} \right\} \quad (1)$$

[4] for antiparallel, asymmetric reconnection. The units of R are a magnetic field strength times velocity. In *Borovsky* [2008], this Cassak-Shay relation applied at the dayside magnetopause was written in terms of the upstream solar wind plasma parameters using the properties of the bow shock and the flow properties of the magnetosheath. This yielded an expression for the dayside reconnection rate in terms of solar wind parameters upstream of the bow shock. This expression was referred to as the "rudimentary solar wind control function" R. Here it will be denoted as R_1 to indicate the first version. Correlating this control function

[1]Space Science Institute, Boulder, Colorado, USA.
[2]University of Michigan, Ann Arbor, Michigan, USA.

Corresponding author: J. E. Borovsky, Space Science Institute, Boulder, Colorado, USA. (jborovsky@spacescience.org)

2169-9380/13/10.1002/jgra.50110

with geomagnetic activity indices showed it to be as accurate as the best of the tuned-variable solar wind driver functions.

[5] In *Borovsky* [2008], it was noted that the derivation of this reconnection control function had several approximations and that the function published was the first cut at what could be a more accurate derivation. In the present study, four improvements to that derivation are given, yielding a second version R_2 of the reconnection control function, which is an improvement over the first version. Using 1 h values of the solar wind parameters to calculate the control functions in the years January 1980 to May 2012, the improved control function is cross correlated with various geomagnetic indices to confirm the fact that it improves the description of geomagnetic activity in terms of solar wind parameters.

[6] The first and second versions of the solar wind control function are algebraically complicated. That being the case, simplified expressions for the control functions are provided that are approximately as accurate as the full version.

2. Improvements to the Derivation

[7] In this section, four improvements will be made to the derivation of the solar wind coupling function from the Cassak-Shay equation for the reconnection rate expressed in terms of local plasma parameters. The first will be a correction to the reconnection rate accounting for plasma compressibility (section 2.1). The second and third will be corrections to the clock-angle dependence of the rate of solar wind magnetic field line connection to the Earth (section 2.2). The fourth will be an improved estimate of the density compression ratio of the Earth's bow shock (section 2.3).

2.1. The Birn Compressibility Correction

[8] The *Borovsky* [2008] solar wind control function R_1 is based on the Cassak-Shay equation [*Cassak and Shay*, 2007] for the rate of asymmetric reconnection [expression (1)]. For asymmetric antiparallel reconnection, [*Birn et al*., 2010] rederived the Cassak-Shay equation accounting for the compressibility of the plasma flow near the reconnection site and accounting for the conversion of magnetic field energy into enthalpy (see also [*Soward and Priest*, 1982], [*Birn et al*. 2008, 2012], and [*Aunai et al*., 2011]. Antiparallel reconnection corresponds to a clock angle θ between the two magnetic fields of $\theta = 180°$. That rederivation produced a corrective factor $\mathcal{B}$ (equation (21) of *Birn et al*. [2010]) that multiplies the right-hand side of the Cassak-Shay equation:

$$\mathcal{B}(180^o) = \Gamma(\mathrm{B_m} + \mathrm{B_s})/(\lambda_s \mathrm{B_m} + \lambda_{\mathrm{m}} \mathrm{B_s}) \tag{2}$$

where $\Gamma \equiv \gamma/(\gamma-1) = 5/2$ for $\gamma = 5/3$ and where $\lambda_s \equiv (1+\Gamma\beta_s)/(1+\beta_s)$ and $\lambda_m \equiv (1+\Gamma\beta_m)/(1+\beta_m)$ with β being the plasma beta $\beta_s = 8\pi n_s k_B T_s/B_s^2$ and $\beta_m = 8\pi n_m k_B T_m/B_m^2$ with T being the sum of the electron and ion temperatures in the plasma.

[9] The compressibility correction acts at low β when the antiparallel-reconnecting plasma is compressible; at high β, the multiplicative correction factor is unity. As noted in [*Birn et al*. 2010, 2012] the presence of a guide field should reduce the compressibility of the reconnecting plasma. Hence, for non-antiparallel reconnection, the Birn compressibility correction as expressed by equation (2) should be weakened toward unity. Birn [Joachim Birn, private communication, 2012] has made a preliminary derivation of the multiplicative compressibility correction factor including a guide field that is valid for any value of the clock angle $0° \leq \theta \leq 180°$ (see Figure 1):

$$\mathcal{B}(\theta) = \left\{ \left(\lambda_{\mathrm{hybrid}}{}^2 + 4\Gamma \mathrm{A} \right)^{1/2} - \lambda_{\mathrm{hybrid}} \right\} / 2A \tag{3}$$

where

$$\lambda_{\mathrm{hybrid}} = (\Lambda_m \mathrm{B_s} + \Lambda_s \mathrm{B_m})/(\mathrm{B_m} + \mathrm{B_s}) \tag{4a}$$

$$\Lambda_{\mathrm{m}} = \left(3 + \Gamma\beta_{\mathrm{m}} - 2\,\mathrm{g}^2\right)/\left(2 + \beta_{\mathrm{m}} - \mathrm{g}^2\right) \tag{4b}$$

$$\Lambda_s = \left(3 + \Gamma\beta_{\mathrm{s}} - 2\,\mathrm{h}^2\right)/\left(2 + \beta_{\mathrm{s}} - \mathrm{h}^2\right) \tag{4c}$$

$$\mathrm{A} = (\Gamma - 2)\,\mathrm{a}/\left(2 + \beta_{\mathrm{s}} - \mathrm{h}^2\right) \tag{4d}$$

$$\mathrm{a} = \left[\left(1-\mathrm{g}^2\right)^{1/2} + \left(1-\mathrm{h}^2\right)^{1/2} \right]^2 / \left[1 + \mathrm{B_s}/\mathrm{B_m} \right]^2 \tag{4e}$$

and where $g = \sin(\alpha)$ and $h = \sin(\theta - \alpha)$, where θ is the clock angle between B_m and B_s, and α is the clock angle between B_m and the orientation of the reconnection X-line (see Figure 1 and section 2.2).

[10] The Alfvén Mach number dependence of the Birn correction factor is examined Figure 2 (top). The strongest correction occurs when there is no guide field, i.e., when $\theta = 180°$. Using the convenient notation $\chi \equiv B_m/B_s$, taking $\gamma = 5/3$, which gives $\Gamma = 5/2$, and approximating the plasma beta of the magnetosphere as $\beta_m \approx 0$ to give $\lambda_m \approx 1$, the Birn correction factor $\mathcal{B}(180°)$. Expression (2) can be written as

$$\mathcal{B}(180^o) = 5\left(\chi^2 + \chi\right)/\left(5\chi^2 + 2\chi - 3\right) \tag{5}$$

[11] This is the correction for $\theta = 180°$, where there is no guide field. Using the parameterization $\chi = B_m/B_s = (1+\beta_s)^{1/2}$ and $\beta_s = (M_A/6)^{1.92}$ [cf. equations (4), (5), and (7) of *Borovsky* (2008)], expression (5) is plotted as a function of M_A as the upper curve in Figure 2 (top): as can be seen,

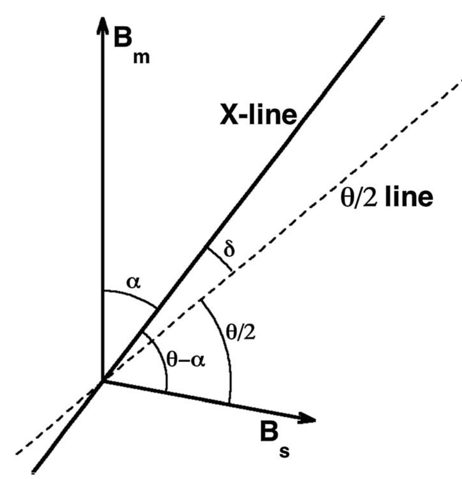

Figure 1. The angles associated with the orientation of the reconnection X-line when the magnetospheric magnetic field B_m and the magnetosheath magnetic field B_s form an angle of θ. α is the clock angle between B_m and the reconnection X-line. In this sketch, all of the vectors and lines lie in the plane of the dayside magnetopause.

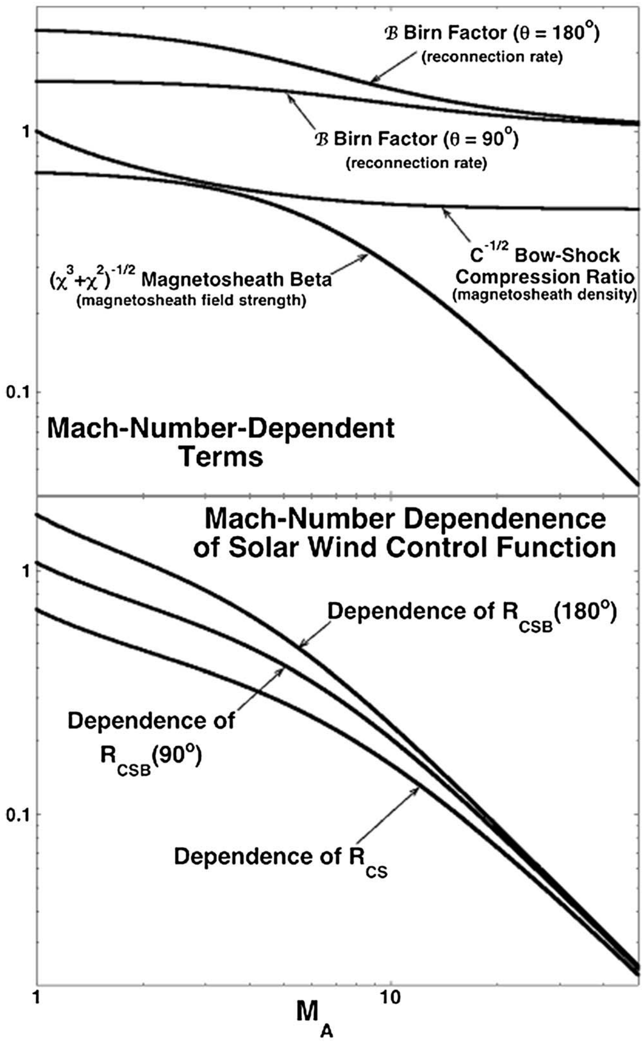

Figure 2. The Alfvén Mach number M_A dependences of (top) three factors entering into the Cassak-Shay-Birn equation and the Mach number dependence of the (bottom) reconnection control functions R_{2CS} and R_{2CSB} for the magnetosphere are plotted.

$\mathcal{B}(180°)$ varies from $\mathcal{B}(180°) = 2.5$ at $M_A = 1$ (low β_s) to $\mathcal{B}(180°) = 1$ as $M_A \to \infty$ (high β_s). The Mach number dependence of the compression correction factor $\mathcal{B}$ derived with a guide field is not as strong. The maximum guide field occurs for $\theta = 90°$; here, $g \approx h \approx \sin(45°) = 1/2^{1/2}$. At $\theta = 90°$, expression (3) can be approximated as

$$\mathcal{B}(90^o) \sim \left\{ \left(2\chi^2+1\right)(\chi+1)/4\chi^2 \right\} \left\{ \left([(4/3)+\Lambda_s\chi]^2 + \left[20\chi^2/\left(2\chi^2+1\right)\right] \right)^{1/2} - [(4/3)+\Lambda_s\chi] \right\} \quad (6)$$

where $\lambda_s \approx (5\chi^2 - 1)/(2\chi^2 + 1)$ at $\theta = 90°$. Again using $\chi = (1 + (M_A/6)^{1.92})^{1/2}$, $\mathcal{B}(90°)$ is plotted as a function of M_A as the second curve in Figure 2 (top). The shallower Mach number dependence of $\mathcal{B}(90°)$ is clearly seen.

[12] With this factor $\mathcal{B}$ given by expression (6) multiplying the right-hand side of the Cassak-Shay equation given by expression (1), the Cassak-Shay-Birn equation for the reconnection rate is

$$R_{CSB} = 0.05\pi^{-1/2} B_m^{3/2} B_s^{3/2} A^{-1} \left\{ \left(\lambda_{hybrid}^2 + 4\Gamma A\right)^{1/2} - \lambda_{hybrid} \right\} / \left\{ \left(B_m\rho_s + B_s\rho_m\right)^{1/2} \left(B_m + B_s\right)^{1/2} \right\} \quad (7)$$

where the factors A and λ_{hybrid} are defined in expression (4) and $\Gamma \equiv \gamma/(\gamma - 1)$.

2.2. The Clock-angle Dependence of the Number of Field Lines Reconnected

[13] The objective of the solar wind control function R is to calculate the rate at which magnetic field lines of the solar wind become connected to the Earth by dayside reconnection. The clock angle θ of the solar wind magnetic field, with respect to the direction of the Earths dipole field, is an important factor in the reconnection rate. It is taken to be the clock angle of the magnetic fields at the site of reconnection, but owing to field-line draping across the magnetopause, it may only be an approximation to the clock angle of the reconnecting fields. The control function R_1 of *Borovsky* [2008] had a $\sin(\theta/2)$ dependence, where θ is the clock angle of the magnetic field relative to the Earth's field at the dayside magnetopause ($\theta = 0°$ for a purely northward interplanetary magnetic field (IMF) and $\theta = 180°$ for a purely southward IMF). For symmetric reconnection, the X-line is oriented at $\theta/2$. That $\sin(\theta/2)$ dependence came from an argument that the rate of field line connection was given by the product of the magnetic field strength and the plasma inflow velocity, with the inflow velocity having a $\sin(\theta/2)$ dependence. In calculating the number of field lines connected, [*Borovsky* 2008] neglected to account for the projection of the inflowing magnetic field onto the reconnection X-line. As pointed out by *Sonnerup* [1974] for the reconnection of non-antiparallel magnetic fields in symmetric plasmas, one factor of $\sin(\theta/2)$ comes about in calculating the reconnection inflow speed and a second factor of $\sin(\theta/2)$ comes about from the projection angle of the inflowing magnetic field on the reconnection line, with the reconnection line tilting in response to the clock angle of the reconnecting magnetic fields. Hence, the number of field lines reconnected per unit length of the X-line varies as $\sin^2(\theta/2)$. If the reconnecting plasmas were symmetric and the X-line were oriented at $\theta/2$, that correction would be written as

$$\sin(\theta/2) \to \sin^2(\theta/2) \quad (8)$$

and in the Cassak-Shay expression [equation (1)], this correction would result in a $\sin^2(\theta/2)$ multiplicative factor.

[14] Note that *Sonnerup* [1974] argues that there is a critical clock angle $\theta_{crit} = \mathrm{invcos}(B_s/B_m)$ below which no reconnection will occur. In that case, the $\sin^2(\theta/2)$ clock angle term is replaced by

$$\begin{array}{ll} \sin^2(\theta/2) \to \sin^2\{(\theta - \theta_{crit})[90^o/(180^o - \theta_{crit})]\} & \text{for } \theta \geq \theta_{crit} \\ \sin^2(\theta/2) \to 0 & \text{for } \theta < \theta_{crit} \end{array} \quad (9)$$

At the magnetopause, pressure balance yields $B_s/B_m = (1 + \beta_s)^{-1/2}$ (cf. equations (4) and (5) of *Borovsky* [2008]), where β_s is the plasma beta of the magnetosheath near the magnetopause and where the beta of the magnetosphere has been approximated to be zero. The magnetosheath beta is parameterized as $\beta_s = (M_A/6)^{1.92}$ (cf. equation (7) of *Borovsky* [2008]); hence,

$$\theta_{crit} = \mathrm{invcos}\left(\left[1 + (M_A/6)^{1.92}\right]^{-1/2}\right) \tag{10}$$

[15] The Sonnerup θ_{crit} varies from 10.2° at $M_A = 1$ to 81° at $M_A = 40$. Attempts at implementing the Sonnerup critical clock angle into the solar wind control functions yield reduced correlation coefficients between solar wind parameters and various geomagnetic indices.

[16] *Swisdak and Drake* [2007] and *Schreier et al.* [2010] argue that this critical clock-angle effect [expressions (9) and (10)] does not prevent reconnection from occurring. Rather, they argue that the reconnection X-line tilts in a manner to maximize the outflow velocity from the reconnection site. Modifying the *Swisdak and Drake* [2007] argument, we examine the tilt of the reconnection X-line in a manner that maximizes the reconnection rate per unit length of the X-line (rate of magnetic flux transport into the reconnection X-line). For a plasma-physics problem in which reconnection leads to a lower energy state, maximizing the reconnection rate maximizes the rate of energy reduction. For this calculation, the geometry of the reconnection X-line is sketched in Figure 1. Denoting the clock angle between the magnetospheric magnetic field B_m direction and the magnetosheath magnetic field B_s direction as θ, the reconnection X-line will align itself with an angle α away from B_m, in which $\alpha \le \theta/2$. α can be thought of as the clock angle between the magnetospheric magnetic field B_m direction and reconnection X-line. For asymmetric, non-antiparallel reconnection, the reconnection outflow speed v_{out} can be written in terms of the components of the magnetic field normal to the orientation of the reconnection X-line as

$$4\pi v_{out}{}^2 = (B_{mn} + B_{sn})\,[(\rho_m/B_{mn}) + (\rho_s/B_{sn})]^{-1} \tag{11}$$

(cf. equation (7) of *Swisdak and Drake* [2007]) where the subscript "n" denotes the component normal to the X-line. Conferring with Figure 1, this is written in terms of the angles α and θ as

$$4\pi v_{out}{}^2 = (B_m g + B_s h)\,[(\rho_m/B_m g) + (\rho_s/B_s h)]^{-1} \tag{12}$$

(cf. equation (8) of *Swisdak and Drake* [2007]) where $g \equiv \sin(\alpha)$ and $h \equiv \sin(\theta - \alpha)$. For the Cassak-Shay equation, the reconnection rate is given by the outflow speed v_{out} times the hybrid magnetic field strength B_{hybrid} in the inflow: $R = 0.1\ v_{out}B_{hybrid}$. For the Cassak-Shay-Birn equation, the reconnection rate is given by $R = 0.1\ v_{out}B_{hybrid}\mathcal{B}$. The hybrid inflow magnetic field (cf. equation (3) of *Birn et al.* [2010]) is given in component form as

$$B_{hybrid} = 2B_{mn}B_{sn}/(B_{mn} + B_{sn}) = 2B_mB_sgh/(B_mg)B_sh \tag{13}$$

[17] The Birn compressibility factor $\mathcal{B}$ [expression (3)] is already written in component form. In this component notation, the Cassak-Shay equation $R = 0.1\ v_{out}B_{hybrid}$ becomes

$$R_{CS} = 0.1\pi^{-1/2}(B_mB_sgh)^{3/2}(\rho_mB_sh + \rho_sB_mg)^{-1/2}(B_mg + B_sh)^{-1/2} \tag{14}$$

and the Cassak-Shay-Birn equation $R = 0.1 v_{out}B_{hybrid}\mathcal{B}$ becomes

$$R_{CSB} = 0.05\pi^{-1/2}A^{-1}(B_mB_sgh)^{3/2}\left[\left(\lambda_{hybrid}{}^2 + 4\Gamma A\right)^{1/2} - \lambda_{hybrid}\right] \times (\rho_mB_sh + \rho_sB_mg)^{-1/2}(B_mg + B_sh)^{-1/2} \tag{15}$$

[18] Defining $\chi \equiv B_m/B_s$, the Cassak-Shay equation (14) in component form can be written conveniently as

$$R_{CS} = 0.1\pi^{-1/2}B_m{}^2\rho_s{}^{-1/2}\chi^{-1}(gh)^{3/2}(g + h\rho_m/\chi\rho_s)^{-1/2}(g\chi + h)^{-1/2} \tag{16}$$

and the Cassak-Shay-Birn equation (15) in component form can be written as

$$R_{CSB} = 0.05\pi^{-1/2}B_m^2\rho_s{}^{-1/2}A^{-1}\chi^{-1}(gh)^{3/2}\left[\left(\lambda_{hybrid}{}^2 + 4\Gamma A\right)^{1/2} - \lambda_{hybrid}\right] \times(g + h\rho_m/\chi\rho_s)^{-1/2}(g\chi + h)^{-1/2} \tag{17}$$

[19] The θ and α dependence of the Cassak-Shay equation (16) is contained in the function

$$f_{CS}(\theta, \rho_m/\rho_s, \chi, \alpha) = (gh)^{3/2}(g + h\rho_m/\chi\rho_s)^{-1/2}(g\chi + h)^{-1/2} \tag{18}$$

and the θ and α dependence of the Cassak-Shay-Birn equation (17) is contained in the function

$$f_{CSB}(\theta, \rho_m/\rho_s, \chi, \alpha) = A^{-1}\chi^{-1}(gh)^{3/2}\left[\left(\lambda_{hybrid}{}^2 + 4\Gamma A\right)^{1/2} - \lambda_{hybrid}\right]\times(g + h\rho_m/\chi\rho_s)^{-1/2}(g\chi + h)^{-1/2} \tag{19}$$

[20] For a given set of θ, ρ_m/ρ_s, and $\chi = B_m/B_s$ values, the reconnection rate of the Cassak-Shay equation is maximized by finding the maximum value of $f_{CS}(\theta,\rho_m/\rho_s,\chi,\alpha)$ [expression (18)] as a function of the X-line clock angle α. In Figure 3 (top), this value of $\alpha_{CS}(\theta)$, which maximizes f_{CS}, is plotted as a function of θ for various values of $\chi = B_m/B_s$, with ρ_m/ρ_s taken to be 1/6. The values of $\alpha_{CS}(\theta)$ are relatively insensitive to the value of ρ_m/ρ_s as long as $\rho_m/\rho_s << 1$. For $\rho_m << \rho_s$, the value of the angle α, which maximizes $f_{CS}(\theta,\rho_m/\rho_s,\chi,\alpha)$, is given approximately by

$$\alpha_{CS} \approx \theta/2 - 15^o\left(1 - (1 + \chi)^{-1/2}\right)\sin(\theta) \tag{20}$$

which is valid for $\chi \ge 1$. For a given set of θ, ρ_m/ρ_s, and $\chi = B_m/B_s$ values, the reconnection rate of the Cassak-Shay-Birn equation is maximized by finding the maximum value of $f_{CSB}(\theta,\rho_m/\rho_s,\chi,\alpha)$ [expression (19)] as a function of α. In Figure 3 (bottom), this value of $\alpha_{CSB}(\theta)$, which maximizes f_{CSB}, is plotted as a function of θ for various values of $\chi = B_m/B_s$, with ρ_m/ρ_s taken to be 1/6. Again, the values of $\alpha_{CSB}(\theta)$ are relatively insensitive to the value of ρ_m/ρ_s as long as $\rho_m/\rho_s << 1$. For $\rho_m << \rho_s$, the value of the angle α, which maximizes $f_{CSB}\ (\theta,\rho_m/\rho_s,\chi,\alpha)$, is given approximately by

$$\alpha_{CSB} \approx \theta/2 - 15^o\left(1 - 1.38\chi^{-0.9} + 0.619\chi^{-2}\right)\sin(\theta) \tag{21}$$

which is valid for $\chi \ge 1$.

[21] Note that for both Cassak-Shay and Cassak-Shay-Birn equations, the orientation of the X-line is less

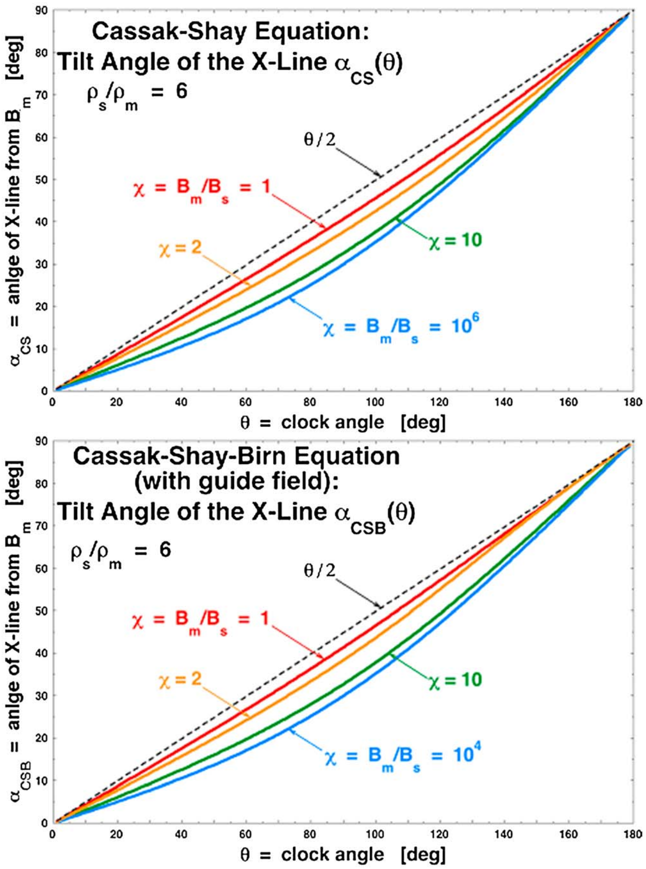

Figure 3. The clock angle α of the reconnection X-line away from the direction of the magnetospheric magnetic field is plotted as a function of the clock angle θ between the upstream–solar wind magnetic field and the Earth's dipole. The colors are for various values of the ratio of the magnetospheric to magnetosheath magnetic field strengths B_m/B_s.

than $\theta/2$ from the direction of the magnetospheric magnetic field direction, which is the high-Alfvén speed side of the reconnection site (cf. Figure 1). For asymmetric reconnection in which one plasma is labeled as the "fast" Alfvén speed side and the other plasma is labeled as the "slow" Alfvén speed side, the Cassak-Shay equation given by expression (1) can be written as

$$R \approx 0.2\, v_{\mathrm{Aslow}} \left(B_{\mathrm{fast}} B_{\mathrm{slow}}\right)^{1/2} \tag{22}$$

[22] Expression (22) points out that the slow-Alfvén speed side controls the reconnection rate. The X-line twists toward the magnetic field direction of the fast-Alfvén speed side (the magnetosphere) to increase the normal component of the magnetic field on the slow-Alfvén speed side (the magnetosheath side). This X-line twist is, in a fashion, to enhance the flux inflow from the lower Alfvén speed, which enhances the reconnection rate. As can be seen in Figure 3, the lower the field strength on that low-Alfvén speed side, the greater the twist, until a limit of about 15° away from the $\theta/2$ orientation is reached.

2.3. The Shock Compression Ratio Improvement

[23] In *Borovsky* [2008], the density compression ratio C of the bow shock at the nose was parameterized for a quasi-perpendicular bow shock. That is, it was parameterized for a solar wind magnetic field that is assumed to be perpendicular to the solar wind flow vector. Calling that parameterization of the compression ratio $C_\perp$, equation (10) of *Borovsky* [2008] yielded

$$C_\perp = \left\{2.44 \times 10^{-4} + \left[1 + 1.38\log_e(\mathrm{M_A})\right]^{-6}\right\}^{-1/6} \tag{23}$$

[24] A parameterization for the shock compression ratio is desirable for arbitrary orientations of the upstream solar wind magnetic field. Noting the angle θ_{Bn} of the upstream magnetic field relative to the Sun-Earth line

$$\theta_{\mathrm{Bn}} = \mathrm{invcos}(|\mathrm{B}_x|/|\mathrm{B}|) \tag{24}$$

the density compression ratio $C_\perp$ of *Borovsky* [2008] was for $\theta_{\mathrm{Bn}} = 90°$. Cubic equations for the density compression ratio of MHD (magnetohydrodynamic) shocks for arbitrary values of θ_{Bn} are given by equation (2.3) of *Kabin* [2001] or equation (12) of *Petrinec and Russell* [1997]; the solutions to those cubic equations were explored for a complete range of M_A ($= v_{sw}/v_A$), M_s ($= v_{sw}/C_s$), and θ_{Bn}. That exploration determined that the compression ratio C of the bow shock could be fit approximately as

$$\mathrm{C} = \mathrm{C}_\perp \sin^2\theta_{\mathrm{Bn}} + \left(1 - \sin^2\theta_{\mathrm{Bn}}\right) \mathrm{C}_{||} \tag{25}$$

where $C_{||}$ is the compression ratio at $\theta_{\mathrm{Bn}} = 0°$. An expression for $C_{||}$ can be found between equations (2.3) and (2.4) of [*Kabin* 2001] (for $\alpha = 0°$ and $\gamma = 5/3$):

$$C_{||} = 4/\left(1 + 3\,\mathrm{M}_s^{-2}\right) \tag{26}$$

[25] Using expressions (23) and (26), expression (25) yields the improved parameterization of the bow shock compression ratio

$$\mathrm{C} = \sin^2\theta_{\mathrm{Bn}}\left\{2.44 \times 10^{-4} + \left[1 + 1.38\log_e(\mathrm{M_A})\right]^{-6}\right\}^{-1/6} + \left(1 - \sin^2\theta_{\mathrm{Bn}}\right)\left\{4/\left(1 + 3\,\mathrm{M_s}^{-2}\right)\right\} \tag{27}$$

[26] For the OMNI2 1 h data set, the mean value of θ_{Bn} is 52° and using C instead of $C_\perp$ amounts to, on average, a 5% correction of the compression ratio.

3. The Improved Solar Wind Control Function

[27] Two new versions of the solar wind control function are derived: one from the Cassak-Shay equation [expression (14)] and one from the Cassak-Shay-Birn equation [expression (15)]. Using (a) the pressure-balance argument at the magnetopause

$$\mathrm{B_m} = (8\pi)^{1/2} \rho_o^{1/2} \mathrm{v_o} \left(1 + 0.5\,\mathrm{M_{ms}}^{-2}\right)^{1/2} \tag{28}$$

(cf. equation (4) of *Borovsky* [2008]) where M_{ms} is the magnetosonic Mach number of the solar wind and where the subscript "o" denotes the solar wind upstream of the bow shock, (b) the relation

$$\rho_s = \mathrm{C}\rho_o \tag{29}$$

cf. equation (9) of *Borovsky* [2008]) determined from MHD simulations of the magnetosphere, where C is the density compression ratio at the nose of the bow shock, (c) the notations $\chi = B_m/B_s$, $\rho_o = m_p n_o$, and $\rho_m = m_p \mu_m n_m$ with m_p being the proton mass and μ_m being the mean molecular weight of the magnetospheric ions, and (d) $\lambda_m \approx 1$ for β_m 0 and $\lambda_s = (5 - 2\chi^{-2})/2$ for $\gamma = 5/3$, the Cassak-Shay expression [equation (14)] is written as

$$R_{2CS} = 0.8\pi^{1/2} m_p^{1/2} n_o^{1/2} v_o^2 \left(1 + 0.5\, M_{ms}^{-2}\right) C^{-1/2} (gh)^{3/2} \Big/ \left\{\chi\left(g + h\mu_m n_m/\chi C n_o\right)^{1/2} (g\chi + h)^{1/2}\right\} \quad (30)$$

and the Cassak-Shay-Birn expression [equation (15)] is written as

$$R_{2CSB} = 0.4\pi^{1/2} m_p^{1/2} n_o^{1/2} v_o^2 \left(1 + 0.5\, M_{ms}^{-2}\right) C^{-1/2} A^{-1} (gh)^{3/2} \left\{\left(\lambda_{hybrid}^2 + 4\Gamma A\right)^{1/2} - \lambda_{hybrid}\right\} \Big/ \left\{\chi\left(g + h\mu_m n_m/\chi C n_o\right)^{1/2} (g\chi + h)^{1/2}\right\} \quad (31)$$

which are the two newly derived solar wind control functions describing the reconnection rate per unit length of the dayside X-line. The subscript "2" denotes the second version, with equation (12) of *Borovsky* [2008] being the first version R_1. In expressions (30) and (31), μ_m is the mean molecular weight of the magnetospheric ions and

$$\chi = (1 + \beta_s)^{1/2} \quad (32a)$$

$$\beta_s = (M_A/6)^{1.92} \quad (32b)$$

$$C = C_\perp \sin^2\theta_{Bn} + C_{||}\left(1 - \sin^2\theta_{Bn}\right) \quad (32c)$$

$$C_\perp = \left\{2.44 \times 10^{-4} + [1 + 1.38\log_e(M_A)]^{-6}\right\}^{-1/6} \quad (32d)$$

$$C_{||} = 4/\left(1 + 3\, M_s^{-2}\right) \quad (32e)$$

$$M_A = v_{sw}/v_A \quad (32f)$$

$$M_{ms} = v_{sw}/\left(v_A^2 + C_s^2\right) \quad (32g)$$

$$\lambda_{hybrid} = (\Lambda_m + \chi\Lambda_s)/(\chi + 1) \quad (32h)$$

$$\lambda_{hybrid} = (\Lambda_m + \chi\Lambda_s)/(\chi + 1) \quad (32i)$$

$$\Lambda_s = \left(3 + 2.5\beta_s - 2\,h^2\right)/\left(2 + \beta_s - h^2\right) \quad (32j)$$

$$A = 0.5\left[\left(1 - g^2\right)^{1/2} + \left(1 - h^2\right)^{1/2}\right]^2 \left(2 + \beta_s - h^2\right)^{-1} \left(1 + \chi^{-1}\right)^{-2} \quad (32k)$$

[28] where θ is the IMF clock angle with respect to the Earth's dipole, θ_{Bn} is the angle of the IMF relative to the Sun-Earth line, n_o and v_o are the upstream solar wind number density and velocity, n_m is the magnetospheric mass density just inside the dayside magnetopause, v_A is the Alfvén speed in the solar wind upstream of the bow shock, $C_s = (\gamma k_B(T_i + T_e)/m_p)^{1/2}$ is the sound speed in the solar wind upstream of the bow shock, $g = \sin(\alpha)$ and $h = \sin(\theta - \alpha)$, with the appropriate X-line tilt angle α given by expression (20) for the Cassak-Shay equation or by expression (21) for the Cassak-Shay-Birn equation.

[29] As outlined in the first paragraph of section 2, four corrections are in the derivation of the reconnection control function R_2 for the magnetosphere, making it an improved version of the rudimentary control function R_1 of *Borovsky* [2008].

4. A Simplified Expression for the Reconnection Control Function

[30] The rudimentary solar wind control function R_1 (equation (12) of *Borovsky* [2008]) is complicated and awkward to implement. The improved reconnection control functions R_{2CS} and R_{2CSB} given by expressions (30) and (31) are even more complicated and awkward to implement. The algebraic complexity of these control functions comes from the Mach number dependence through the variable $\chi = (1 + \beta_s)^{1/2} = (1 + (M_A/6)^{1.92})^{1/2}$. In Figure 2, that Mach number dependence is explored, under the simplifying assumption that $n_m << n_o$, $\theta_{Bn} = 90°$, and $\theta = 180°$ (where $g = h = 1$). In Figure 2 (top), three terms of the Cassak-Shay-Birn equation are plotted as a function of the Alfvén Mach number M_A of the solar wind. The three terms are (1) the $C^{-1/2}$ term associated with the compressibility of the bow shock [given by expression (32d)], which affects the mass density of the magnetosheath plasma at the reconnection site; (2) the Mach number dependence of the hybrid magnetic field strength for the reconnection site

$$\left(\chi^3 + \chi^3\right)^{-1/2} = \left[(1 + \beta_s)^{3/2} + (1 + \beta_s)\right]^{-1/2} = \left\{\left[1 + (M_A/6)^{1.92}\right]^{3/2} + \left[1 + (M_A/6)^{1.92}\right]\right\}^{-1/2} \quad (33)$$

which characterizes the reduction of the magnetosheath magnetic field strength from $8\pi\rho_o v_o^2$ owing to the nonzero β of the magnetosheath plasma; and (3) the Birn plasma-compressibility correction $\mathcal{B}$ [expression (3)] to the reconnection rate (which is plotted for $\theta = 180°$ and for $\theta = 90°$). The Cassak-Shay equation contains the first two of these terms: the Cassak-Shay-Birn equation contains all three. As can be seen in Figure 2 (top), the bow shock compressibility has a change in behavior at about $M_A = 3$ between low-Mach number weak compression and higher-Mach number compression approaching a factor of 4. The Mach number dependence of the magnetic field strength [expression (33)] has a change in behavior at about $M_A = 6$, with the magnetosheath plasma being low beta for $M_A << 6$ and the magnetosheath plasma being high beta for $M_A >> 6$. The Birn compressibility correction for a clock angle of $\theta = 180°$ makes a gradual transition from $\mathcal{B} \approx 2.5$ at $M_A = 1$ to $\mathcal{B} \approx 1$ at very-high Mach number; for $\theta < 180°$, the Birn compressibility correction is weaker because of the reduction of compressibility by the presence of the guide field.

[31] In Figure 2 (bottom), the full Mach number dependence of the solar wind control functions R_{2CS} and R_{2CSB} are plotted (holding $n_o^{1/2} v_o^2$ fixed). The Mach number dependence of R_{2CSB} is plotted twice, once for a clock angle $\theta = 180°$ and once for $\theta = 90°$. The reconnection control function exhibits a change in the Mach number dependence at about $M_A = 6$, reflecting the transition of the magnetosheath plasma from low-beta to high-beta as M_A increases across $M_A = 6$.

[32] To obtain simplified versions of the solar wind control functions R_{2CS} and R_{2CSB} given by expressions (30) and (31), the approximations $n_m = 0$ and $g = h = \sin(\theta/2)$ are taken, and the Alfvén Mach number–dependent portions of the control function that are plotted in Figure 2 (bottom) are fit as a function of M_A, and the term $(1 + 0.5\ M_{ms}^{-2})^{1/2}$ is approximated to be unity. Accordingly, the approximation to the Cassak-Shay reconnection control function for the magnetosphere R_{2CS} of expression (30) is

$$\mathrm{R}_{2\mathrm{CS-approx}} = 1.68 \times 10^{-2} \sin^2(\theta/2)\ n_o{}^{1/2} v_o{}^2 \mathrm{M_A}{}^{-0.3044} \times \exp\left(-[M_A/3.18]^{1/2}\right) \tag{34}$$

and the approximation to the Cassak-Shay-Birn reconnection control function for the magnetosphere R_{2CSB} of expression (31) is

$$\mathrm{R}_{2\mathrm{CSB-approx}} = 3.29 \times 10^{-2} \sin^2(\theta/2) n_o{}^{1/2} v_o{}^2 \mathrm{M_A}{}^{-0.18} \times \exp\left(-[\mathrm{M_A}/3.42]^{1/2}\right) \tag{35}$$

[33] The units of R in expressions (34) and (35) are gauss cm/s or, equivalently, nT km/s, where the solar wind number density n_o is in units of cm^{-3} and the solar wind speed v_o is in units of km/s. For the 198,000 hourly values available in the OMNI2 data set for 1980 to 2012, the linear correlation coefficient between $R_{2\text{CS-approx}}$ [expression (34)] and R_{2CS} [expression (30)] is 0.998, and the linear correlation coefficient between $R_{2\text{CSB-approx}}$ [expression (35)] and R_{2CSB} [expression (31)] is 0.990.

5. Correlations of the Reconnection Control Functions with Geomagnetic Indices

[34] For the hourly averaged solar wind parameters [*King and Papitashvili*, 2005] and geomagnetic indices for 1980 to 2012, the improved solar wind control functions R_{2CS} and R_{2CSB} are correlated with various geomagnetic activity indices and the value of Pearson's linear correlation coefficient r (equation (11.17) of *Bevington and Robinson* [1992]) is entered into Table 1. For every hour of solar wind data, a value of R_2 (n_o,v_o,M_A, M_{ms},θ,θ_{Bn}) is calculated and matched against the hourly averaged values of the geomagnetic indices. Approximately 198,000 hourly intervals are available with v_o, n_o, and T_p values to calculate R_2 (n_o,v_o,M_A,M_{ms},θ,θ_{Bn}).

[35] To evaluate the solar wind control functions, the magnetosonic Mach number M_{ms} is needed, but electron temperature measurements of the solar wind are often not available to calculate the ion sound speed. To evaluate M_{ms} from only proton-temperature measurements, the electron temperature T_e is approximated from the proton temperature T_p as follows. For the 1998 ACE electron moments of *Skoug et al*. [2000], a Gaussian fit to log(T_e) as a function of log (T_p) yields

$$\mathrm{T}_e = 7.21\ \mathrm{T}_p{}^{0.393} \tag{36}$$

[36] This expression is used to evaluate the sound speed $C_s = [\gamma k_B(T_p + T_e)/m_p]^{1/2}$ with $\gamma = 5/3$, which goes into the magnetosonic Mach number $M_{ms} = v_{sw}/(v_A^2 + C_s^2)^{1/2}$.

[37] In evaluating the reconnection control functions R_{2CS} and R_{2CSB}, the number density of the magnetospheric plasma is taken as $n_m = 0$.

[38] The correlation coefficients for R_{2CS}, R_{2CSB}, and other solar wind driver functions are also entered into Table 1. The columns are $-vB_z$ [*Rostoker et al*., 1972] (with B_z in GSM coordinates), vB_{south} [*Holzer and Slavin*, 1982] (where $B_{south} = -B_z$ for $B_z < 0$ and $B_{south} = 0$ for $B_z \geq 0$, again in GSM), Newell $v_o^{4/3} B_{o\perp}^{2/3} \sin^{8/3}(\theta/2)$ [*Newell et al*., 2007], R_1 [*Borovsky*, 2008], R_{2CS} [expression (30)], R_{2CSB} [expression (31)], $R_{2\text{CS-approx}}$ [expression (34)], and $R_{2\text{CSB-approx}}$ [expression (35)]. The geomagnetic indices used (rows of Table 1) are the 1 h lagged AE index, the 1 h lagged AU index, the 1 h lagged AL index, the northern polar cap index (Thule) with no time lag, the 1 h lagged midnight boundary index (MBI) [*Gussenhoven et al*., 1983], the Kp index with a 1 h time lag, and the pressure-corrected Dst index Dst* = Dst − 20.7 P_{ram} + 27.7 [*Borovsky and Denton*, 2010] with a 2 h time lag.

[39] In Table 1, the linear correlation coefficients between the various driver functions and the various geomagnetic indices can be seen. For the geomagnetic indices AE, AL, and PCI, the improved coupling functions R_{2CS} and R_{2CSB} yield higher correlation coefficients than obtained for the *Borovsky* [2008] rudimentary control functions R_1. For other indices, the rudimentary function R_1 yields higher correlation coefficients: those indices are AU, MBI, Kp, and Dst*.

[40] The bottom row of Table 1 contains a seven-index sum of the linear correlation coefficients for each driver function. As can be discerned from that sum, the Newell function and the R_1 and R_2 reconnection control functions perform superior jobs at correlating with geomagnetic

Table 1. Linear Correlation Coefficients between Various Solar Wind Driver Functions and Various Geomagnetic Indices[a]

	$-vB_z$	vB_{south}	Newell	R_1	R_{2CS}	R_{2CSB}	$R_{2\text{CS-approx}}$	$R_{2\text{CSB-approx}}$
AE 1 h lag	0.570	0.687	0.780	0.750	0.774	0.758	0.771	0.766
AU 1 h lag	0.445	0.542	0.650	0.676	0.674	0.665	0.669	0.663
−AL 1 h lag	0.573	0.689	0.764	0.710	0.744	0.727	0.744	0.738
PCI	0.576	0.653	0.757	0.735	0.756	0.744	0.752	0.750
−MBI 1 h lag	0.471	0.605	0.710	0.736	0.730	0.718	0.729	0.723
Kp 1 h lag	0.338	0.535	0.653	0.747	0.704	0.695	0.700	0.696
−Dst* 2 h lag	0.340	0.581	0.634	0.692	0.668	0.668	0.660	0.667
7-Index Sum	3.313	4.292	4.948	5.046	5.050	4.977	5.025	5.003

[a]The correlation coefficients are for the January 1980 to May 2012 1-h resolution OMNI2 data set. The Newell function is $v_o^{4/3} B_{o\perp}^{2/3} \sin^{8/3}(\theta/2)$, R_1 is given by equation (12) of *Borovsky* [2008], and R_{2CS}, R_{2CSB}, $R_{2\text{CS-approx}}$, and $R_{2\text{CSB-approx}}$ are given by expressions (30), (31), (34), and (35), respectively. The statistical errors $\pm 1/N^{1/2}$ (where N is the number of point pairs) for the correlation coefficients are ±0.002 for all geomagnetic indices except for MBI, in which the statistical error is ±0.003.

indices compared with the simpler functions $-vB_z$ and vB_{south}. It can be seen that the reconnection control function based on the Cassak-Shay-Birn equation R_{2CSB} does not perform quite as well as the simpler R_{2CSB} based on the Cassak-Shay equation.

[41] The correlation coefficients for the approximations $R_{2CS\text{-approx}}$ [expression (34)] and $R_{2CSB\text{-approx}}$ [expression (35)] in Table 1 are, on average, within a few percentage points of the coefficients for the full expressions R_{2CS} [expression (30)] and R_{2CSB} [expression (31)].

[42] Note in Table 1 that, in general, the inclusion of the Birn compressibility correction does not improve the correlation of the solar wind with the geomagnetic indices (i.e., compare R_{2CS} with R_{2CSB} and compare $R_{2CS\text{-approx}}$ with $R_{2CSB\text{-approx}}$).

6. Discussion

[43] This section contains discussions about (a) the role of the solar wind electric field in determining the magnitude of the driving of magnetospheric activity, (b) why the Birn compressibility correction as implemented does not improve the coupling functions for the magnetosphere, and (c) future work needed for further improvements to the reconnection control function for the magnetosphere.

6.1. Why Do *vB*-type (Electric Field) Driver Functions Do a Good Job of Describing Geomagnetic Activity?

[44] The quantity vB_z of the upstream solar wind is said to determine the electric field in the reconnection line at the dayside magnetopause [*Burton et al.*, 1975; *Kan and Lee*, 1979; *Axford*, 1984; *Grocott et al.*, 2009]. However, [*Borovsky et al.* 2008] showed that because of the diverging flow of the magnetosheath around the magnetosphere, the electric field at the dayside reconnection site is not equal to vB_z of the solar wind upstream of the bow shock [see also *Birn and Hesse*, 2007]. On the other hand, $-vB_z$ and other vB-type drivers do a good job of correlating with the level of geomagnetic activity (cf. Table 1). The question is: Why do they do so well? We see two possibilities as to why vB_z of the solar wind does such a good job of correlating with the level of geomagnetic activity: (1) vB_z coincidentally approximates the rate of reconnection of solar wind magnetic field lines with the Earth or (2) vB_z describes the strength of the solar wind MHD generator connected to the Earth's polar cap [*Goertz et al.*, 1993; *Borovsky et al.*, 2009] and it is describing the strength of coupling after the reconnection has occurred. The first point will be explored in this subsection.

[45] In point (1), the relationship of vB_z of the solar wind to the reconnection control functions R_{2CS} and R_{2CSB} is explored by looking at the Alfvén Mach number dependences of the solar wind coupling functions (30) and (31). In the approximations $\alpha = \theta/2$ and $g = h = \sin(\theta/2)$, these control functions can both be written in the form

$$R \propto \sin^2(\theta/2)\, n_o{}^{1/2} v_o{}^2 f(M_A), \tag{37}$$

where the Mach number dependences $f(M_A)$ are plotted in Figure 2 (bottom) for expressions (30) and (31). In the OMNI2 data set, more that 85% of the 1 h solar wind intervals have Alfvén Mach numbers in the range of $M_A = 4.5$ to 15. In that range, $4.5 < M_A < 15$, the curves in Figure 1 (bottom) can be fit by the power laws $f(M_A) = 4.14\ M_A^{-1.255}$ for the top curve R_{2CSB} and $f(M_A) = 1.33\ M_A^{-0.928}$ for the bottom curve R_{2CS}. Utilizing these fits in expression (37), and writing $M_A \propto n_o^{1/2} v_o / B_o$, the functional dependences of the solar wind control functions (30) and (31) in the range of $4.5 < M_A < 15$ become

$$R_{2CS} \propto \sin^2(\theta/2)\, n_o{}^{0.04} v_o{}^{1.07} B_o{}^{0.93} \tag{38a}$$

$$R_{2CSB} \propto \sin^2(\theta/2)\, n_o{}^{-0.13} v_o{}^{0.74} B_o{}^{1.26} \tag{38b}$$

[46] Expressions (38a) and (38b) have linear correlation coefficients with the 1 h lagged AE index of +0.756 and +0.707 for the 1980 to 2012 OMNI2 data set, so they do a good job of describing geomagnetic activity. Functions (38a) and (38b) are on the order of $\sin^2(\theta/2) v_o^1 B_o^1$ of the solar wind, which is related to the solar wind electric field. These are of a similar form to vB_z, vB_{south}, and Newell $v_o^{1.33} B_\perp^{0.66}$, but with $B_o = (B_x{}^2 + B_y{}^2 + B_z{}^2)^{1/2}$ instead of $B_\perp = (B_y{}^2 + B_z{}^2)^{1/2}$. Hence, it seems to be coincidental that the dayside reconnection rate has a function form that is similar to that of the solar wind electric field. Note that in the derivation of the reconnection rate in the present report and in [*Borovsky* 2008], the magnetic field B enters via a Mach number that determines the shock and sheath properties that reduce the reconnection field strength down from $(8\pi\rho_o v_o^2)^{1/2}$. No solar wind electric field appears in the derivation and no product of v times B appears in the derivation.

6.2. The Birn Compressibility Correction

[47] As can be seen in Table 1, R_{2CSB} for the Cassak-Shay-Birn picture has weaker correlations with geomagnetic activity than R_{2CS} for the Cassak-Shay picture does. Hence, including the Birn compressibility correction $\mathcal{B}$ did not improve the performance of the reconnection control function for the magnetosphere.

[48] As can be seen in Figure 2 (bottom), adding the Birn compressibility correction steepens the falloff of the coupling function with increasing values of the Alfvén Mach number M_A. This steepening of the M_A dependence lowers the correlations. In fact, if R_{2CS} [given by expression (30)] were to be multiplied by $M_A^{0.2}$ to make its Mach number dependence slightly shallower, noticeably improved correlation coefficients with the geomagnetic indices would be obtained.

[49] The Mach number dependence of R_{2CS} and R_{2CSB} are chiefly via the parameter $\chi = B_m/B_s = (1 + \beta_s)^{1/2}$ where $\beta_s = (M_A/6)^{1.92}$. The $\beta_s = (M_A/6)^{1.92}$ relation was obtained from fits to $\beta_s(M_A)$ in global MHD simulations of the solar wind flow around the magnetosphere [*Borovsky*, 2008]. These simulations had some idealizations like $B_x = 0$ in the solar wind and no dipole tilt. The inaccuracy of this $\beta_s(M_A)$ fit versus the actual values of β_s for the real magnetosphere could more than account for $M_A^{0.2}$ changes in R_{2CS} and R_{2CSB}.

6.3. Future Work

[50] Several future improvements of the derivation of the reconnection control function R_2 remain, including (1) more accurate parameterizations for the values of β_s and ρ_s as functions of the Alfvén Mach number M_A, (2) a parameterization of the magnetosheath magnetic field B_s and density ρ_s across the face of the magnetosphere and calculation of the

total reconnection rate integrated along the entire dayside reconnection X-line, and (3) accounting for the effects of velocity shear on the magnetopause reconnection rate.

[51] Avenues for the improvement of the correlations between solar wind parameters and the level of geomagnetic activity include (1) adding postreconnection MHD generator physics [e.g., *Goertz et al.*, 1993; *Lavraud and Borovsky*, 2008; *Borovsky et al.*, 2009] to the control function R_2, (2) adding a viscous interaction term to the control function [cf. *Oberc*, 1979; *Borovsky and Funsten*, 2003; *Newell et al.*, 2008], and (3) parameterizing the dayside-magnetospheric plasma mass density ρ_m and using $\rho_m \neq 0$ in the reconnection control function R_2.

7. Summary

[52] Following *Borovsky* [2008], improved physics was incorporated into a rederivation of the reconnection control function for the Earth's magnetosphere. This rederivation accounted for the *Birn et al.* [2010] compressibility correction to the reconnection rate, accounted for the tilt of the reconnection X-line, accounted for the projection of the inflow–magnetic field vector along the reconnection X-line, and relaxed the assumption of a quasi-perpendicular bow shock at the nose.

[53] New reconnection control functions for the magnetosphere were given, R_{2CS} and R_{2CSB}, based on the Cassak-Shay and the Cassak-Shay-Birn equations, respectively. These reconnection control functions express the magnetopause reconnection rate per unit length of the X-line in terms of the solar wind parameters n_o, v_o, θ, θ_{Bn}, M_A, and M_{ms} upstream of the bow shock, where n_o is the solar wind number density, v_o is the solar wind speed, θ is the magnetic field clock angle with respect to the dipole, θ_{Bn} is the angle of the solar wind magnetic field away from the Sun-Earth line, M_A is the Alfvén Mach number, and M_{ms} is the magnetosonic Mach number.

[54] Measured by the magnitude of the linear correlation coefficients with geomagnetic indices, R_{2CS} is a slight improvement over R_1 and R_{2CSB} is slightly inferior to R_1. It was pointed out that potential errors in the parameterization of the plasma-beta of the magnetosheath as a function of the solar wind Alfvén Mach number could produce errors in R_{2CSB} and R_{2CSB} that are larger than the Mach number difference between R_{2CSB} and R_{2CSB}.

[55] Arguments were presented as to why the Birn compressibility correction (derived for antiparallel reconnection) needs to be modified for non-antiparallel reconnection.

[56] Simplified expressions approximating the new reconnection control functions R_{2CS} and R_{2CSB} were given, with geomagnetic index correlation coefficients nearly as high as the full expressions.

[57] **Acknowledgments.** The author thanks Joachim Birn for his new derivation of the compressibility correction for the reconnection rate, Paul Cassak for his help, and Mick Denton for data. This work was supported at the Space Science Institute by the NASA CCMSM-24 Program, the NSF GEM Program, and the NASA SR&T Program and supported at the University of Michigan by the NASA Geospace SR&T Program.

References

Aunai, N., G. Belmont, and R. Smets (2011), Energy budgets in collisionless magnetic reconnection: Ion heating and bulk acceleration, *Phys. Plasmas*, *18*, 122901.

Axford, I. (1984), Driven and non-driven reconnection: Boundary conditions, in *Magnetic Reconnection in Space and Laboratory Plasmas*, (E. W. Hones, ed.), p. 360, American Geophysical Union, Washington, D.C.

Bevington, P. R., and D. K. Robinson (1992), *Data Reduction and Error Analysis for the Physical Sciences*, Second Edition, McGraw-Hill, New York.

Birn, J., and M. Hesse (2007), Reconnection rates in driven magnetic reconnection, *Phys. Plasmas*, *14*, 082306.

Birn, J., and J. E. Borovsky (2008), and M. Hesse, Properties of asymmetric magnetic reconnection, *Phys. Plasmas*, *15*, 032101.

Birn, J., J. E. Borovsky, M. Hesse, and K. Schindler (2010), Scaling of asymmetric reconnection in compressible plasmas, *Phys. Plasmas*, *17*, 052108.

Birn, J., J. E. Borovsky, and M. Hesse (2012), The role of compressibility in energy release by magnetic reconnection, *Phys. Plasmas*, *19*, 082109.

Borovsky, J. E. (2008), The rudiments of a theory of solar-wind/magnetosphere coupling derived from first principles, *J. Geophys. Res.*, *113*, A08228.

Borovsky, J. E., and M. H. Denton (2010), The magnetic field at geosynchronous orbit during high-speed-stream-driven storms: Connections to the solar wind, the plasma sheet, and the outer electron radiation belt, *J. Geophys. Res.*, *115*, A08217.

Borovsky, J. E., and H. O. Funsten (2003), Role of solar wind turbulence in the coupling of the solar wind to the earth's magnetosphere, *J. Geophys. Res.*, *108*, 1246.

Borovsky, J. E., M. Hesse, J. Birn, and M. M. Kuznetsova (2008), What determines the reconnection rate at the dayside magnetosphere?, *J. Geophys. Res.*, *113*, A07210.

Borovsky, J. E., B. Lavraud, and M. M. Kuznetsova (2009), Polar cap potential saturation, dayside reconnection, and changes to the magnetosphere, *J. Geophys. Res.*, *114*, A03224.

Burton, R. K., R. L. McPherron, and C. T. Russell (1975), The terrestrial magnetosphere: A half-wave rectifier of the interplanetary electric field, *Science*, *189*, 717.

Cassak, P. A., and M. A. Shay (2007), Scaling of asymmetric magnetic reconnection: General theory and collisional simulations, *Phys. Plasmas*, *14*, 102114.

Goertz, C. K., L.-H. Shan, and R. A. Smith (1993), Prediction of geomagnetic activity, *J. Geophys. Res.*, *98*, 7673.

Grocott, A., S. V. Badman, S. W. H. Cowley, S. E. Milan, J. D. Nichols, and T. K. Yeoman (2009), Magnetosonic Mach number dependence o the efficiency of reconnection between planetary and interplanetary magnetic fields, *J. Geophys. Res.*, *114*, A07219.

Gussenhoven, M. S., D. A. Hardy, and N. Heinemann (1983), Systematics of the equatorward diffuse auroral boundary, *J. Geophys. Res.*, *88*, 5692.

Holzer, R. E. and J. A. Slavin (1982), An evaluation of three predictors of geomagnetic activity, *J. Geophys. Res.*, *87*, 2558.

Kabin, K. (2001), a note on the compression ratio in MHD shocks, *J. Plasma Phys.*, *66*, 259.

Kan, J. R., and L. C. Lee (1979), Energy coupling function and solar wind-magnetosphere dynamo, *Geophys. Res. Lett.*, *6*, 577.

King, J. H., and N. E. Papitashvili (2005), Solar wind spatial scales in and comparisons of hourly Wind and ACE plasma and magnetic field data, *J. Geophys. Res.*, *110*, 2104.

Lavraud, B., and J. E. Borovsky (2008), Altered solar wind-magnetosphere interaction at low Mach numbers: Coronal mass ejections, *J. Geophys. Res.*, *113*, A00B08.

Newell, P. T., T. Sotirelis, K. Liou, C.-I. Meng, and F. J. Rich (2007), A nearly universal solar wind-magnetosphere coupling function inferred from 10 magnetospheric state variables, *J. Geophys. Res.*, *112*, A01206.

Newell, P. T., T. Sotirelis, K. Liou, and F. J. Rich (2008), Pairs of solar wind-magnetosphere coupling functions: Combining a merging term with a viscous term works best, *J. Geophys. Res.*, *113*, A04218.

Oberc, P. (1979), Magnetospheric tail dynamics and a concept of combined action of viscous drag and magnetic merging, *Planet. Space Sci.*, *27*, 1087.

Petrinec, S. M., and C. T. Russell (1997), Hydrodynamic and MHD equations across the bow shock and along the surfaces of planetary obstacles, *Space Sci. Rev.*, *79*, 757.

Rostoker, G.H.-L. lam, and W. D. Hume (1972), Response time of the magnetosphere to the interplanetary electric field, *Can. J. Phys.*, *50*, 544.

Schreier, R., M. Swiskak, J. F. Drake, and P. A. Cassak (2010), Three-dimensional simulations of the orientation and structure of reconnection X-lines, *Phys. Plasmas*, *17*, 110704.

Skoug, R. M., W. C. Feldman, J. T. Gosling, D. J. McComas, and C. W. Smith (2000), Solar wind electron characteristics inside and outside coronal mass ejections, *J. Geophys. Res.*, *105*, 23069.

Sonnerup, B. U. O. (1974), Magnetopause reconnection rate, *J. Geophys. Res.*, *79*, 1546.

Soward, A. M., and E. R. Priest (1982), Fast magnetic field-line reconnexion in a compressible fluid. Part 1. Coplanar field lines, *J. Plasma Phys.*, *28*, 335.

Swisdak, M., and J. F. Drake (2007), Orientation of the reconnection X-line, *Geophys. Res. Lett.*, *34*, L11106.