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Rationing Capacity in Advance Selling to Signal Quality

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We consider a seller who can sell her product over two periods, advance and spot. The seller has private information about the product quality, which is unknown to customers in advance and publicly revealed in spot. The question we consider is whether the seller has an incentive to signal quality in advance and, if so, how she can convey a credible signal of product quality.

We characterize the seller's signaling strategy and find that rationing of capacity in the advance period is an effective tool of signaling product quality. We find that the high-quality seller can distinguish herself by allocating less capacity than the low-quality seller in the advance period. We show that this signaling mechanism exists whenever advance selling would be optimal for both the high-quality and low-quality sellers if quality information was symmetric. We compare capacity rationing with other signaling tools, such as pricing and advertising, and show that capacity rationing is the preferred one.

Despite its capability of conveying quality information more efficiently than other tools, capacity rationing may still be very costly for the seller. When compared to the case when rationing was not allowed, the seller's ability to ration (rationing flexibility) sometimes makes the seller worse off, independently of her quality.

1. Introduction

Advance selling is used by service industries, including travel and entertainment, as well as in many retail areas, including toys, books, electronics, and media products. With advance selling, sellers offer customers an opportunity to purchase a product or service prior to the consumption time. This can benefit the seller as advance selling may increase total sales and profit. It can benefit the customers, too. Through buying in advance, customers can usually get guaranteed availability at discounted prices. However, because consumers buy the product before it is available, they are often uncertain about the quality of the product and their own value of the product at the time of consumption.

Consider advance selling of French wine, a practice known as *en primeur* ("wine futures" in French), which has been practiced by chateaus in Bordeaux for centuries as well as wine makers in other areas including Burgundy, Tuscany, and Rioja. Typically, six to twelve months after previous year's grape harvest, chateaus and vineyards offer customers opportunities to buy the new vintage. At the time of the *primeur* sales, the wine is not yet "finished" and still in barrels. It is a practice of advance selling, as the payment is due at the time of *en primeur*, but the delivery occurs after the wine is finished and bottled, usually twelve to eighteen months after the *en primeur* sales. Wineries may sell all or just a portion of their wines through the *en primeur* system and both wineries and negociants usually reserve some wine for later sales (2011 Bordeaux Futures Special Report[1]). Similar practices are offered by online and brick-and-mortar wine sellers in retail setting.

Wine quality is uncertain to buyers when the wines are sold during *en primeur*. Like many other experience goods (Su, 2009; Shulman et al., 2009), wines can truly be evaluated through the tasting of the actual products, which are unavailable during the advance sales. Even though tasting of young wines is sometimes offered and some information about the wine – such as the condition of harvested grapes, the total cases made, and how many cases are available for the *en primeur* sales - is available, the wines are six-month old, tannic, and still in the stage of malolactic fermentation. They often smell and taste quite unpleasantly and are vastly different from what they will be in one year or so when the production process is complete. Also, most wines are blends, while the *en primeur* samples can be either unblended or differently blended. In fact, Jancis Robinson, a notable wine expert, cites wine producers saying that "...none of the samples of 2000 being shown this week will be exactly the same blend as the final wine (a common complaint about these en primeur tastings of wines more than a year before final bottling)" (Robinson, 2001).

In contrast, chateaus, as the producers of the wines, have much more information about the grapes used for wine as well as the entire wine-making processes. For example, they have first-hand information on all of the important determinants of wine quality, such as climate, soil, viticultural and enological practices (Jackson, 2000). Furthermore, they know how representative the sample barrels are in terms of blending and overall quality. Consequently, the information on wine quality is asymmetric in advance (Hadj Ali and Nanges, 2006; Dubois and Nauges, 2006).

Such information asymmetry is difficult to resolve via information sharing or contracts. The seller (winery or chateau) has private information about what has happened so far to the wine, but with many factors (climate, soil, viticultural practices, etc.) playing a role, no single attribute can precisely determine the quality of wine. Thus, even if the seller is willing to share the information with the consumers during the advance sales (e.g., by showing the soil samples from the vineyards[2]), it is hard for consumers to integrate individual pieces of information to correctly predict the quality. Furthermore, it is also hard to verify the information (e.g., if the grapes used to make the wine indeed grew in the fields from where the soil samples were taken). Equally importantly, the wine-making process continues after the *en primeur* sales, and the final quality will be affected by the seller's various post-fermentation treatments, such as adjustments (to acidity, flavor, color, etc.), blending,

[1] http://www.jjbuckley.com/images/2011_BORDEAUX_REPORT.pdf

[2] We thank an anonymous referee for suggesting this.

stabilization, clarification, and bottling (Jackson, 2008). For blending alone, many wine-makers "may add to a blend up to 15 percent of wine from a vintage other than that cited on the label," a practice known as "inter-vintage blending" (Robinson, 2011). Many of the above steps will depend on available alternatives (e.g., which barrels to be mixed with), while others actually constitute some of the "trade secrets" of the wineries. It is not surprising that wineries do not post this type of information and would not want to contractually commit to it.

The information asymmetry about wine quality is gradually reduced over time and eventually resolved after the wines are bottled and released to the market. Customers can evaluate the quality of the finished wines by reading experts' wine reviews,[3] attending wine tasting events (e.g., Auffrey, 2008), ordering trial packs from online retailers (e.g.,theorganicwinecompany.com[4]), and tasting wines at social events by chance.

Event ticket is another example where the seller has significantly more information than the buyers about the products sold in advance. The advance ticket sales of the Ultra Music Festival in Miami, FL (a festival for electro music featuring some of the top DJs in the world) and the Bonnaroo Music Festival in Manchester, TN, both take place many months before the complete line-up and schedule are announced.[5] In fact, the advance sales of the 2014 Ultra Music Festival (www.ultramusicfestival.com) have begun without any announced line-up. Although the seller has private information about DJs headlining the festival, the complete line-up and schedules were announced after all advance tickets were sold.[6] Similar situations with asymmetric quality information include advance sales of new music albums, games, and electronics before their official release.

When advance selling is offered, the sellers can choose to sell either all or a portion of the products. We observe both practices. For example, some premier designer handbags (so called "it" bags in Kuksov and Wang, 2011) are sold out during pre-order and will never arrive to the store.[7] On the other hand, in the *en primeur* market, chateaus release a proportion, ranging from twenty percent to ninety percent, of their total production, thus intentionally limiting the wine availability in advance market. According to the New York Times (Prial, 1989), the price-setting Bordeaux chateaus sell their wines in stages, or "tranches." In the first stage, they usually release about 20 percent of their total production at the opening price. This rationing policy is well described by many wine-merchants and wine-experts, see Decanter article[8] and a newsletter from rarewineco.com.[9] For the 2013 Burning Man

[3] Reviews about the *en primeur* wines are not as reliable as the reports written after official release, largely due to uncertainties of wines in early stage.

[4] http://store.theorganicwinecompany.com/bordeaux-sampler-p47.aspx

[5] The advance tickets for the 2014 Ultra Music Festival (March 28-30, 2014) are already sold out in May 2013. http://www.ultramusicfestival.com/news

[6] One of the authors is an avid follower of electro dance music and actively follows the trend and events.

[7] http://www.fashionrules.com/2011/06/prada-fall-bags-and-shoes-to-pre-order/

[8] http://www.decanter.com/wine-learning/wine-advice/basics/495393/how-to-buy-en-primeur

[9] http://www.rarewineco.com/downloads/newsletter/archive/may801.pdf

Festival, exactly 3,000 tickets out of 58,000 tickets were allocated for the advance holiday sales that took place in December 20, 2012.[10] Limiting pre-orders of new products is also a common practice in electronic-device industries. Two best-known examples are Microsoft's Xbox 360, released in 2005 (Harford, 2005) and Sony PlayStation 3, released in 2006 (Sinclair, 2006). In both cases, retailers such as Gamestop accepted limited orders or limited the number of units that one consumer could order.

There are a number of reasons that can explain the sellers' practice of rationing capacity. Limiting the advance sales may create a hype and increase demand for new products (*Retailing Today* 2000; Dye 2000; Brown 2001) or it could be simply the reflection of capacity shortage. In this paper, we show that another reason for the seller's rationing is to signal product quality.

Offering advance sales when the asymmetry in quality information exists can work for or against the sellers. On one hand, it is possible for sellers of low-quality products to hide the inferior quality in advance and to boost their sales by locking many customers who would not have made the purchase if quality were known. On the other hand, the sellers who are unable to prove their quality, may need to give a considerable price discount to induce customers to buy early, as customers always have an option to delay purchase until quality is fully revealed. Given these two opposite drivers, it is not clear whether and when the seller should offer advance selling, and if so, whether it is possible to convey some of the information about the product quality to buyers through the terms of advance selling (such as price or limited quantities offered for sale). Some key questions we like to address are as follows. How does asymmetric quality information affect seller's profit, and how much can sellers gain from offering advance selling? Can the seller of high-quality products credibly signal her quality? When is it beneficial to engage in actions that signal the quality level and, if so, which signals will be most effective? We study these questions in this paper and, in particular, we examine the role of *capacity rationing* as a signal of quality. We show that capacity rationing (i.e., limiting supply in the advance period and choosing to satisfy a portion of advance demand) can be an efficient way to convey the information about the product quality when compared to signaling through an advertisement. We show that as long as both types of sellers would offer advance selling when the quality information was symmetric, there exists an equilibrium in which the seller of high-quality products allocates less capacity in advance than the seller of low-quality products to distinguish herself (signal her type).

Literature Review

Our work is closely related to the literature on signaling quality. Several different forms of signals of quality have been examined in existing literature, including advertising (Kihlstrom and Riordan 1984; Milgrom and Roberts 1986), pricing (Bagwell and Riordan, 1991), warranties (Lutz, 1989), money-back guarantee (Moorthy and Srinivasan, 1995), umbrella branding (Wernerfelt, 1988), and scarcity

[10] Associated Press, http://www.pressdemocrat.com/article/20130106/WIRE/130109747

(Stock and Balachander, 2005). Our paper contributes to the literature in two ways. We show that capacity rationing in advance selling can be used to signal quality and we also evaluate how efficient signaling through rationing is, compared to other signaling tools such as pricing and advertising.

Among the signaling literature, Stock and Balachander (2005) is closest to our paper. It considers scarcity as a signal of quality. They show that a seller who has sufficient capacity to meet all demand may intentionally dispose some of the capacity to create scarcity for uninformed customers ("followers"). Thus, a high-quality seller signals quality by making product scarce for followers and charging full-information price for all customers. This strategy is optimal under two conditions: informed customers make purchase first (before followers) and price is constant over two periods. Even though both models use sales quantity as a signal of quality, our model is substantially different. First, we assume that quality uncertainty exists before the product is released (e.g., wine *en primeur* or pre-sale of a video game), and is resolved at the product release (e.g., when wines are released to consumers or the game hits the store). Stock and Balachander (2005) assume the exact opposite: quality is perfectly known in advance, but only to advance customers ("innovators"). Second, in our setting, the seller can dynamically change price over time and the advance customers can strategically choose when to buy, i.e., whether to buy in advance under imperfect quality information or wait till information is publicly revealed in spot. Such strategic customer behavior is supported by many empirical evidences (Su, 2007; Aviv and Pazgal, 2008) and is not captured in Stock and Balachander (2005). Third, while their model assumes that total capacity can be adjusted, all of the capacity must be available in advance and the seller cannot limit the quantity sold in advance. In contrast, we consider allocation of capacity between advance (before the product is available) and spot periods (after it is available).

Our paper is also related to the advance selling literature, especially papers considering consumer's uncertain valuations (e.g., Xie and Shugan, 2001; Gallego and Sahin, 2010; Prasad et al., 2010; Yu et al., 2010; Chu and Zhang, 2011). All of the papers in this stream, except Chu and Zhang (2011), assume that all information about the product is publicly available, i.e. sellers do not have any private information. In contrast, our paper considers both customer uncertain valuation and seller's private information about quality. The impact of asymmetric quality information on the seller's strategy and profit from advance selling is, in fact, our focus. Among the above papers, Chu and Zhang (2011) is the only work that considers sellers' private information about quality. In their model the seller decides the amount of quality information to release in advance. The paper shows how this decision affects customers' valuation of the product. In contrast, the seller in our model cannot and does not directly control the information released in advance. Quality information can only be *inferred* through the seller's selling strategies (e.g., pricing and capacity rationing).

There are other papers that examine capacity rationing, but not in the context of advance selling. Among these, Liu and van Ryzin (2008) show that capacity rationing can induce risk-averse customers to buy early at the regular price instead of waiting for a clearance price. Zhang and Cooper (2006) evaluate the benefit of rationing with both fixed and flexible pricing. Gilbert and Klemperer (2000) find that rationing is preferred to market-clearing price when customers incur seller-specific sunk cost. These papers, however, all ignore the signaling effect of rationing.

The rest of the paper is organized as follows. We define the problem and equilibrium concept in §2 and provide some preliminary results in §3. In §4, the seller's optimal (equilibrium) strategy when there is no quality uncertainty (full-information case) is presented. §5 and §6 are the main thrusts of the paper. In §5, we present the equilibrium strategy and outcome when quality is uncertain and the seller has the option of rationing capacity. In §6, we evaluate the value of rationing and characterize the conditions under which the seller prefers signaling through rationing. In §7, we compare rationing with advertising, which is another signaling tool that has been extensively studied. We discuss several extensions of our model and conclude the paper in §8. All proofs are presented in the appendix.

2. The Model

We consider a risk-neutral seller offering a product to risk-neutral customers over two periods, advance period and spot period. While the seller knows the quality of the product in advance, the quality is not observable by customers until the spot period. The seller decides the price and quantity that she will offer in the advance period, and then later the price for the spot period. We assume that customers are strategic and choose whether and when to buy the product. In what follows, we describe the seller's and the customer's problems in detail, and then define the sequence of the events.

2.1. The Seller

The seller's product can be of either high (H) or low (L) quality (with $H > L$). We assume that the seller's total capacity is T and the marginal production cost is c, both of which are common knowledge.[11] It should be noted that our analysis can be extended to the case where high-quality products are more costly to produce and all of our results continue to hold. We follow the signaling literature (see a comprehensive review in Sobel, 2007 or Kirmani and Rao, 2000) and assume that the quality of the seller's product is exogenously given and cannot be chosen by the seller.

To isolate the effect of rationing as a signal, we assume that the seller's capacity is exogenously determined and that the seller does not have freedom to change her capacity. There are a number of examples supporting this. For instance, only the grapes harvested from a certain lot can become Premier Cru, thus the total capacity cannot be changed freely by the seller. For the event ticket selling, exactly 58,000 tickets are available for the 2013 Burning Man Festival, because the number of attendants is regulated by Pershing County, Nevada, where the festival takes place. Similarly, the total number of tickets for the 2013 Ultra Music Festival requires approval from the Miami-Dade County.

[11] We follow Stock and Balachander (2005) and consider the case when the seller's marginal cost is independent of quality: high-quality and low-quality sellers have the same constant marginal cost of production. Walton (1986) and Srinivasan and Lovejoy (1997) show that high-quality products are not necessarily more costly to produce. For instance, the prices of some wines are largely distanced from their production costs, see http://www.wineanorak.com/whydoeswinecost.htm.

2.2. The Customers

Customers are strategic and risk-neutral: they choose the option that maximizes their expected utility when facing multiple purchasing opportunities. If customers do not buy the product, their reservation utility is zero. Otherwise, a customer's net utility is $U = t + \alpha - p$, where p is the price of the product, $t \in \{H, L\}$ represents the product quality (H or L), and α is the customer's private valuation that captures the heterogeneity in individual customer's willingness-to-pay. In particular, α corresponds to the combined effect of all idiosyncratic factors, such as individual preferences about the flavor of the wines or customers' mood at the time of consumption (Hauser and Wernerfelt, 1990). Without loss of generality, we assume that the sum of quality and individual valuation, $t + \alpha$, is nonnegative for all realizations. We note that all derived results also hold for a multiplicative utility function (i.e., $U = \alpha t - p$).

In advance, customers are uncertain about both quality t and their individual valuations α's, both of which are resolved in the spot period. For example, in the *primeur* market, customers are uncertain not only about wine quality, but also about their individual valuation of consumption, because the wine is still in developing stage and only available (if any) for barrel tasting of work-in-progress. Similarly, customers prepaying for festivals are unsure about both the quality of the event (which depends on, e.g., which DJs will perform at the Ultra Music Festival), and their personal states (e.g., mood, health, scheduling conflicts) on the event date.

Specifically, let q to be the probability that the product quality is high (H), a common prior for all customers in the advance period. While individual valuations are different for different customers in spot, in advance they all follow the same prior distribution with *cdf* $G(\cdot)$ and *pdf* $g(\cdot)$. This approach is taken by several papers on advance selling including Xie and Shugan (2001), Gallego and Sahin (2010), Prasad et al. (2010), and Nasiry and Popescu (2012). Throughout the paper, we impose the following assumptions on the distribution function $G(\cdot)$:

(1) $G(\cdot)$ is twice continuously differentiable and has a finite support $[\underline{\alpha}, \overline{\alpha}]$.

(2) $g(\cdot) = G'(\cdot) > 0$ on $(\underline{\alpha}, \overline{\alpha})$ and is log-concave.

(3) For any $k \in [0,1]$, $\left(\frac{G(x)\overline{G}(x)}{g(x)}\right)' + \overline{G}(x) - k$ crosses zero at most once and from above.[12]

These assumptions are not very restrictive and cover many distributions and their truncated versions, including uniform, exponential, logistic, normal, extreme-value, power, Weibull, beta, gamma, and χ (χ^2) with most parameter values. Note that condition (2) implies that the distribution has an increasing failure rate (IFR) and that the tail distribution $\overline{G}(\cdot) = 1 - G(\cdot)$ has an inverse on $[0,1]$, which we denote by $(\overline{G})^{-1}(\cdot)$.

Following the standard approach in the advance-selling literature (see Xie and Shugan 2001, Gallego and Sahin 2010, Yu et al. 2010, to name a few), we use the fluid model, where the proportion of customers with an individual valuation less than or equal to x is $G(x)$.

[12] A function $a(\cdot)$ crosses zero at most once and from above, if and only if $a(x_0) < 0$ implies $a(x) < 0$ for all $x > x_0$.

2.3. The Game

The sequence of events is illustrated in Figure 1. N_1 customers who are uncertain about both product quality, t, and individual valuation, α_i, arrive in the advance period. The seller decides whether to offer advance selling and, if so, announces the advance price, p_1, and the rationing decision, S, which is maximum the seller is willing to sell in advance.

Upon observing the seller's offer, customers update their belief about the product quality. Let $b(p_1, S)$ denote customers' posterior belief about the probability that the seller offers high-quality products. Based on this updated belief, customers decide whether to buy in advance or to wait and postpone the purchasing decision till the spot period.

In the spot period, another cohort of N_2 customers arrive and the seller sells the remaining quantity at price p_2. At this time, the product is available for consumption and the product quality and customers' individual valuations become known to themselves. All the remaining customers, including those who have not bought in advance and those who arrived in spot, decide whether to buy in spot with the full information. At the end of the spot period, consumption takes place and there is no salvage value for any unsold capacity. Note that having two separate streams of arrivals, N_1 and N_2, allows us to capture situations in which not every customer is aware of the product in the advance period. For example, while some consumers may become aware of the release of a new game and place a pre-order, others (who may have the same willingness to pay) may not realize or be aware of the product release until the game actually hits the shelf, see Prasad et al. (2008) and Stock and Balanchander (2005) for more examples and discussion. Furthermore, all of our results hold for the case $N_2 = 0$: a single stream of buyers arriving in the advance period.

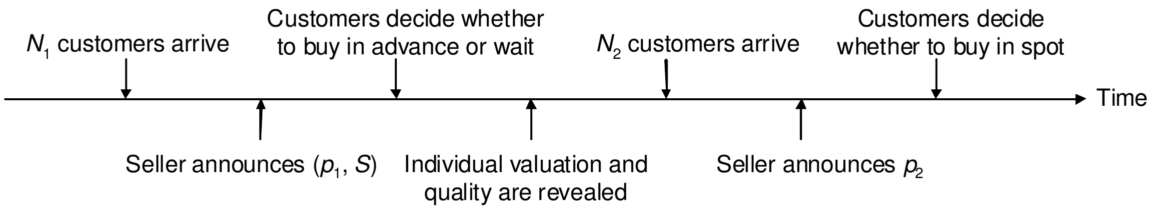

Figure 1 Sequence of events

In our model, the seller's rationing decision is observable to the customers. As reported in trade articles (e.g., Prial, 1989), chateaus release a portion of wines for sales during *en primeur*. Some vineyards announce the total quantity and the amount they sell in advance up front. For instance, Midsummer Cellar, a Californian winery located in St. Helena, announced that, for their 2010 Canon Creek Cabernet Sauvignon, only 100 cases (1,200 bottles) out of 300 cases would be available during the advance purchaseperiod.[13] Similarly, the organizers of the 2013 Burning Man Festival pre-announced

[13] http://www.midsummercellars.com/Futures.htm

at the beginning of a selling season how many tickets would be available at each of the sales dates and committed to this specific plan (e.g., 3,000 in December 2012, 40,000 in February 2013, and 1,000 in August 2013). The holiday sales took place on December 20, 2012, and in fact, all 3,000 allocated tickets were sold in a matter of hours[14] and the press repeatedly reported the sales of the tickets.[15] The commitment to the quantity sold in advance can be verified – if in dispute, a judge can order *ex post* to audit sales data to verify that the announced quantity was indeed sold. The credibility of commitment is tightly linked to the reputation of the seller as well. Su and Zhang (2009) say “when stocking quantities and service levels are verifiable ex post, the seller may be averse to misrepresentation due to reputation concerns.”

We assume that the seller determines and announces the spot price at the beginning of the spot period. If the seller can commit to a spot price during advance selling, then the spot price itself may be used as a signal of quality. Such signaling role of price has been studied in a static model (one-shot sales) by Bagwell and Riordan (1991). However, in some situations it is difficult to commit to a certain spot price. For example, the spot price of a wine (often 2-3 years after the en primeur) is influenced by the seller’s production process that has not yet taken place.

2.4. Equilibrium Concepts

Our signaling game is a sequential game with incomplete information and the equilibrium concept we employ is perfect Bayesian equilibrium (PBE) (Fudenberg and Tirole, 1991; Katok et al., 2012). Typically, two classes of PBE, separating and pooling equilibria, exist for a signaling game. In a separating equilibrium, a high-quality seller can successfully distinguish himself from a low-quality seller during the advance sales. This can be done by choosing a strategy that the low-quality seller does not have an incentive to mimic. Consequently, customers can perfectly infer the seller’s type. In contrast, in a pooling equilibrium, the high-quality seller cannot economically differentiate himself and both types of sellers adopt the same strategy in advance. Resultantly, customers cannot infer any information about the quality of the seller in equilibrium.

In our game, the seller has two potential tools to signal quality to customers in the advance period: price and capacity ration. Hence, we define a separating equilibrium as an equilibrium in which either only one of the two types of sellers offer advance selling, or both types sell in advance but differ in their advance prices or/and capacity rations. On the other hand, we define a pooling equilibrium as one in which either both types of sellers only sell in spot, or both types sell in advance and use the same

[14] Associated Press, http://www.pressdemocrat.com/article/20130106/WIRE/130109747

[15] The official website of the burning man festival: http://blog.burningman.com/2013/01/news/burning-man-2013-ticket-sales/,
Associated Press, http://www.pressdemocrat.com/article/20130106/WIRE/130109747,
Burning Man announces ticket plan with $380 set price, Reno Cazette-Journal,
http://www.rgj.com/article/20130105/EVENTS04/301050046/Burning-Man-announces-ticket-plan-380-set-price

advance price and ration. To avoid trivialities, we focus on the set of *participating* equilibria where, if an advance offer is made in equilibrium, the customers' response in equilibrium is to accept the offer and buy in advance. If a seller makes an advance offer which is rejected by customers in equilibrium, then it is equivalent to the seller not offering any advance selling. The concept of "participating equilibrium" is also used in economics and finance literature, e.g., Janssen et al. (2005) and Easley and O'Hara (2009).

In addition, to limit the number of equilibria in a signaling game, we apply the *intuitive criterion* by Cho and Kreps (1987) for customers' beliefs on off-equilibrium paths. The intuitive criterion requires that, if an off-equilibrium strategy makes only one type of the seller strictly better off than the same seller following the equilibrium strategy, then observing this specific strategy enables the customers to correctly identify the seller's type. In addition to the intuitive criterion, we also impose the Pareto-dominance: if multiple equilibria exist, we will focus on the equilibrium that Pareto dominates all the other ones from the seller's point of view, i.e., the equilibrium where both types of the seller obtain (weakly) higher profits than they do in any other equilibrium. Such an equilibrium is *a focal equilibrium* and is supported by evidence from behavioral experiments (Schelling, 1960). Also, since in our model the seller moves first, the seller can always choose the equilibrium most appealing to himself and expects customers to foresee his choice. If there are multiple equilibria and both types of sellers are indifferent in choosing any of them, we will focus on the equilibrium at which the advance sales is the largest (lexicographically largest, similar to Federgruen and Heching 1999).

In the next section, we formally formulate the problem and provide preliminary results. In the following sections, we consider a baseline case, where advance customers know the true quality (full-information setting) and then proceed to the focus of the paper: when advance customers are uncertain about quality (asymmetric-information setting).

3. Formulation and Preliminary Results

Following backward induction, we first examine the seller's decision in the spot period, when quality information is fully revealed.

3.1. Spot Period

Consider a subgame where the seller sold S units in the advance period. Since the seller's total capacity is T and the number of advance customers is N_1, clearly $S \in [0, \min(T, N_1)]$, the remaining capacity is $T - S$, and there are $N_1 + N_2 - S$ customers in the spot period. Since the customers know the product quality and their individual valuation, a customer with valuation α will buy the product of quality t, $t = H, L$, in spot if and only if her utility is nonnegative, i.e., $U = t + \alpha - p_2 \geq 0$ or $\alpha \geq p_2 - t$. Hence, the number of customers who want to buy the product in spot is $(N_1 + N_2 - S)\overline{G}(p_2 - t)$.

As the sales quantity in the spot period is the smaller of the seller's remaining capacity and the spot demand, for given remaining capacity, $T-S$, the type-t seller chooses a spot price p_2 to maximize the expected spot profit $\pi_{2t}(p_2, S)$.

$$\pi_{2t}(p_2, S) = p_2 \min\{T-S, (N_1+N_2-S)\overline{G}(p_2-t)\} \tag{1}$$

Let the optimal spot price be $p^*_{2t}(S)$ and the corresponding spot profit $\pi^*_{2t}(S) = \pi_{2t}(p^*_{2t}(S), S)$. The following lemma characterizes $p^*_{2t}(S)$.

Lemma 1

$$p^*_{2t}(S) = \max\left(p^U_{2t}, p^B_{2t}(S)\right) = \begin{cases} p^U_{2t} & \textit{if } \frac{T-S}{N_1+N_2-S} \geq \overline{G}(p^U_{2t}-t) \\ p^B_{2t}(S) & \textit{otherwise} \end{cases} \tag{2}$$

where p^U_{2t} *maximizes the unconstrained profit* $p_2\overline{G}(p_2-t)$ *and* $p^B_{2t}(S)$ *is the market-clearing price. Specifically,*

$$p^U_{2t} \begin{cases} \in (t+\underline{\alpha}, t+\overline{\alpha}) \textit{ and is a solution to } p^U_{2t} = \frac{\overline{G}(p^U_{2t}-t)}{g(p^U_{2t}-t)} & \textit{if } t < \bar{t} = \frac{1}{g(\underline{\alpha})} - \underline{\alpha} \\ = t+\underline{\alpha} & \textit{if } t \geq \bar{t} \end{cases} \tag{3}$$

and $\overline{G}(p^B_{2t}(S)-t) = \min\left(1, \frac{T-S}{N_1+N_2-S}\right)$.

Lemma 1 shows that product quality, valuation uncertainty, and the seller's remaining capacity all play a role in the optimal spot price. When the product quality is high enough to dominate valuation uncertainty (i.e., $t > \bar{t}$) and the seller has sufficient capacity, the seller finds it optimal to sell to all remaining customers by setting the spot price to $t+\underline{\alpha}$. With lower quality ($t \leq \bar{t}$), the seller with sufficient capacity will charge an interior spot price. Thus, even when the seller has ample capacity, the optimal price changes in product quality. The threshold quality, $\bar{t}$, will play a role both in full information and asymmetric information cases.

On the other hand, if the seller's capacity is tight so that spot demand at p^U_{2t} exceeds the remaining capacity $T-S$, it is optimal to charge the capacity clearing price, $p^B_{2t}(S)$. Clearly, when the spot price is raised from p^U_{2t} to $p^B_{2t}(S)$, the profit is increased while the sales remain equal to the remaining capacity. This also means that no shortage can take place in the spot period, which is summarized in the following corollary.

Corollary 1 *For* $S \in [0, \min(T, N_1)]$, $T-S \geq (N_1+N_2-S)\overline{G}(p^*_{2t}(S)-t)$.

The fact that shortage in supply never occurs in spot will affect the customers' purchasing decision in advance, and, consequently, the seller's decision.

3.2. Advance Period

If the seller decides not to sell in advance, setting advance ration S to zero, all N_1 customers must wait until the spot period and the seller's total profit over the two periods is simply $\pi^*_{2t}(0)$. If, however, the seller offers advance selling with advance price p_1 and *positive* capacity ration S, advance customers update their belief about the probability of high quality to $b(p_1, S)$. Based on this belief, customers choose to buy in advance or wait and delay the purchasing decision to spot, by comparing the expected utilities. When purchasing in advance, a customer's expected utility is $U_A(p_1, S) = \mathrm{E}_{\alpha,t}[t + \alpha - p_1]$. If she decides to wait, she will buy only if the revealed quality and individual valuation are high enough. From Lemma 1 and Corollary 1, the expected utility from waiting to spot is $U_D(p_1, S) = \mathrm{E}_{\alpha,t}[\max(t+\alpha-p^*_{2t}(S), 0)]$. Since $U_D \geq 0$, buying in the advance period is optimal for the customer if and only if $U_A(p_1, S) \geq U_D(p_1, S)$, which is equivalent to

$$p_1 \leq \mathrm{E}_{\alpha,t}\left[\min(p^*_{2t}(S), t+\alpha)\right] \tag{4}$$

Notice that the right-hand side of equation (4) provides the customer's maximum willingness to pay in advance. Intuitively, a customer would not pay any advance price higher than the expectation of the minimum of spot price and total product value (the sum of quality and individual valuation), since the customer always has the option of waiting to spot.

Although the customers will eventually realize valuations at the time of consumption, their decision in the advance period is based on the expected utility. Because customers share the same *ex ante* distribution about individual valuation, α, and the same updated belief about quality, t, they have the same maximum willingness-to-pay, as expressed in equation (4).Consequently, for given price p_1 and ration $S > 0$, either all or none of them buy in advance, resulting in the seller selling out all of the S units or making no sales at all. Thus, for any ration $S > 0$, the seller can choose a price leading to the sales of all S units.

After explicitly including the posterior belief $b(p_1, S)$ in equation (4) we have

$$p_1 \leq b(p_1, S)p^*_{1H}(S) + (1 - b(p_1, S))p^*_{1L}(S) \tag{5}$$

where $p^*_{1t}(S) = \mathrm{E}_\alpha[\min(p^*_{2t}(S), t+\alpha)]$ represents advance customers' maximum willingness-to-pay when they believe the seller is of type t.

Let $\pi^a_t(p_1, S, b)$ denote the expected total profit for a type-t seller who sets an advance price p_1 and rations capacity S to advance customers, who believe the seller to be of high type with probability $b = b(p_1, S)$. Superscript a stands for *asymmetric information*. If the condition in equation (5) is satisfied, all customers choose to buy in advance and the seller sells all S units. Thus, we have

$$\pi^a_t(p_1, S, b) = p_1 S + \pi^*_{2t}(S) \tag{6}$$

For p_1 not satisfying equation (5), none of the customers buys in advance and the seller's total profit $\pi_t^a(p_1, S, b) = \pi_{2t}^*(0)$. Thus, in correspondence to a set of customers' beliefs $\{b(p_1, S)\}$, the type-t seller chooses the price-ration pair (p_1, S) to maximize his expected total profit $\pi_t^a(p_1, S, b)$.

Recall that we consider only participating equilibria, where the seller's advance offer induces all customers to buy in advance, i.e., equation (5) is satisfied. Since $b(p_1, S) \in [0, 1]$, customers would never buy at any price higher than $p_{1H}^*(S)$, and yet would always do so at a price lower than $p_{1L}^*(S)$. For the remainder of the paper, when identifying participating equilibria, it suffices to consider only *feasible* strategies: $S = 0$ (when p_1 is irrelevant), or $0 < S \le \min(T, N_1)$ and $p_1 \in [p_{1L}^*(S), p_{1H}^*(S)]$.

Our objective is to evaluate the impact of asymmetric information about quality. To do this, we first analyze a benchmark case in which customers know the true quality of the product in advance.

4. Base Case: Full Information about Quality

When customers know the true quality t in advance, for given advance ration S, their maximum willingness-to-pay is simply $p_{1t}^*(S)$. Thus, $p_{1t}^*(S)$ is exactly the advance price the seller will quote: any lower price is strictly dominated and any higher price is rejected by customers. With the optimal spot and advance prices characterized as functions of S, the seller chooses a capacity ration S to maximize her total expected profit, $\pi_t^f(S)$:

$$\max_{S \in [0, \min(T, N_1)]} \pi_t^f(S) = p_{1t}^*(S)S + \pi_{2t}^*(S) \tag{7}$$

Denote the optimal ration under full information by S_t^f and the corresponding optimal advance price by p_{1t}^f, where superscript f stands for the *full information* case. The following theorem characterizes when it is optimal to use advance selling. Later we describe price p_{1t}^f and ration S_{1t}^f for each type.

Theorem 1

(a) There exist two critical numbers, T_1 and T_D, $0 \le T_1 \le T_D \le N_1 + N_2$, such that

- *if $T \le T_1$, then $S_t^f = 0$* [**no advance selling**],
- *if $T \in (T_1, T_D)$, then $0 < S_t^f < \min(T, N_1)$* [**limited advance selling**],
- *If $T_D \le T < N_1 + N_2$ or $T \ge N_1 + N_2$ and $t < \bar{t}$, then $S_t^f = \min(T, N_1)$* [**full advance selling**],
- *If $T \ge N_1 + N_2$ and $t \ge \bar{t}$, then S_t^f is any value between zero and $\min(T, N_1)$* [**advance selling and spot-only selling are equivalent**].

(b) T_1 and T_D are independent of quality level.

In case when advance selling and spot-only selling are equivalent (the last bullet in part (a)), we assume that the seller offers advance selling and ration $S = \min(T, N_1)$ since the seller can accrue the revenue early.

Advance selling allows the seller to take advantage of customers' uncertainty about their individual valuations. By offering a discount in advance, customers are willing to buy before their valuations are revealed. This discount increases the sales as well as the profit. Notice from Theorem 1 that the total capacity plays a significant role: The seller prefers to offer advance selling (or at least is indifferent between spot-only or advance selling) when the capacity is large. On the other hand, the seller rations capacity in advance ($S < \min(T, N_1)$) when capacity is at intermediate level ($T \in (T_1, T_D)$). To see why, note that the seller would be able to sell all of his capacity. However, for the case of full advance selling, $S = \min(T, N_1)$, a significant portion of customers would buy at a (possibly heavily) discounted advance price, which erodes the seller's ability to price discriminate. If, on the other hand, the seller does not sell in advance, some capacity will never be used. Offering some quantity in advance not only increases the sales quantity but also raises the spot price. The product quality affects the seller's policy, but only when the seller has large capacity ($T \geq N_1 + N_2$). If the product quality is very high ($t \geq \bar{t}$), it is optimal for the seller to set the price low enough so that all customers want to buy and the seller becomes indifferent in selling the product between the two periods.

The obvious next questions are which of the two sellers – high-type or low-type – quotes a higher price and which sells a bigger quantity in advance. The following theorem answers these questions.

Theorem 2 *Consider full-information case.*
(i) The two types of sellers always ration the same amount of capacity in advance, i.e., $S_H^f = S_L^f$.
(ii) When both types sell in advance ($S_H^f > 0$)*, the high-type seller charges a strictly higher advance price,* $p_{1H}^f > p_{1L}^f$.

As expected, the high-type seller charges a strictly higher price in advance. This is intuitive as customers who are aware in advance of quality are willing to pay more for high quality. However, the two types of sellers always ration the same amount of capacity in advance. This is because, the difference in quality is perfectly captured by the price difference. Thus, in the full-information setting, quality difference is only reflected in price. We shall see that the result is drastically different when customers are uncertain about the quality.

5. Asymmetric Information about Quality

We now study the case when customers in the advance period are not sure about the product quality. We examine if the seller benefits by signaling product quality through the terms of advance selling (i.e., price and quantity) and, when pricing and rationing are both available to signal quality, we characterize which of the two levers is more effective for the seller. We will show that, unlike the full-information case where quality can be conveyed by price, the quality uncertainty forces a seller to distort both the price and rationing to signal the product quality. We first characterize the properties of a separating equilibrium, where the terms of advance selling perfectly communicate the quality information to customers. We then examine pooling equilibria, where advance selling is uninformative about the quality.

In preparation for equilibrium analysis, it is useful to examine the seller's profit in a special case, where all consumers believe that the product quality is high, that is, $b(p_1, S) = 1$. We will show the "single-crossing" property (Athey, 2001) holds in this case, which will be used later for the general setting. When $b(p_1, S) = 1$, customers believe that the true quality is high and would accept any feasible advance price, $p_1 \le p^*_{1H}(S)$. Hence, the type-t seller's total profit is

$$\pi^a_t(p_1, S, 1) = p_1 S + \pi^*_{2t}(S) \tag{8}$$

where superscript a stands for asymmetric information. The following lemma shows that iso-profit curves of the two types of seller satisfy the single-crossing property.

Lemma 2

Consider $S > 0$ and $b = 1$. Iso-profit curves for two types of sellers (low and high) cross at most once. When they cross, the high type's iso-profit curve crosses the low type's from below, i.e., the high type's iso-profit curve is below the low type's on the left of the crossing point and is above on the right.

Figure 2 illustrates Lemma 2. A point, (p_1, S), represents the seller's strategy – its advance price and rationed quantity, respectively. The two increasing curves, $p^*_{1L}(S)$ and $p^*_{1H}(S)$, represent the lower and upper bounds of the feasible price for given rationing quantity, $S > 0$. Inside the feasible region, we show iso-profit curves, one for high-type seller and one for low-type seller. For each type of the seller, the total profit remains constant on the corresponding iso-profit curve. This *single-crossing property* is important in our analysis. It is used to show that the high-type seller will decrease its rationing quantity to signal its quality during advance selling (Theorem 3). It is also used to show that certain pooling equilibria will be ruled out by the intuitive criterion (Theorem 4).

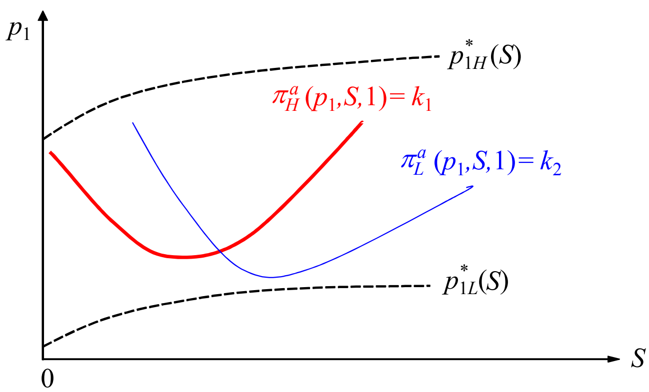

Figure 2 **Single crossing of iso-profit curves for $b = 1$ (k_1, k_2 are constants)**

5.1. Separating Equilibrium

In a separating equilibrium, the seller's quality will be fully revealed in the advance period since low- and high-type sellers use different strategies. In the context of our game, a separating equilibrium must follow one of the following cases: (1) either only one type of seller offers advance selling, or (2) both do, but they are different in price, in quantity rationed, or in both. Denote type-t seller's equilibrium strategy pair by (p^a_{1t}, S^a_t), where $t = H$ or L. In any separating equilibrium, customers can infer the true quality: $b(p^a_{1H}, S^a_H) = 1$ and $b(p^a_{1L}, S^a_L) = 0$.

Following standard argument, it is straightforward to show that, in any separating equilibrium, the low-type seller's strategy is always the same as under the full information, i.e., $p^a_{1L} = p^f_{1L}$ and $S^a_L = S^f_L$. This can be shown by following the standard arguments as in, e.g., Lutz (1989) and Sobel (2007): since the low-type seller is always perfectly discerned in a separating equilibrium, the above strategy will yield the highest payoff for her. On the other hand, the problem that the high-type seller needs to solve is as follows:

$$(p^a_{1H}, S^a_H) = \arg\max_{p_1,S} \pi^a_H(p_1, S, 1) = p_1 S + \pi^*_{2H}(S)$$

$$\text{subject to:} \qquad S = 0 \text{ or } \{S \in (0, \min(T, N_1)] \text{ and } p_1 \in [p^*_{1L}(S), p^*_{1H}(S)]\} \tag{9}$$

$$\pi^a_L(p_1, S, 1) \le \pi^a_L\left(p^f_{1L}, S^f_L, 0\right) \tag{10}$$

$$\pi^a_H(p_1, S, 1) \ge \pi^a_H\left(p^f_{1L}, S^f_L, 0\right) \tag{11}$$

$$(p_1, S) \ne \left(p^f_{1L}, S^f_L\right) \tag{12}$$

In the above formulation, the high-type seller maximizes his own profit, subject to the following four constraints: first, the feasibility condition defined in §3.2; second and third, the low type prefers to be perceived as a low type, rather than imitate the high type's strategy and the high type does not have an incentive to imitate the low type (incentive compatible); and last, the high type's strategy is not identical to the low type's.

Figure 3 shows two iso-profit curves where the profit of each seller equals to the profit when both sellers use the low type's equilibrium strategy. Clearly these two curves contain (and thus cross at) the strategy point (p^f_{1L}, S^f_L). Note that the seller's profit will increase if she manages to sell all the S units at a higher advance price p_1. Hence, the points above a given iso-profit curve correspond to higher profits. By the single-crossing property (Lemma 2) and the incentive-compatibility constraints, only the points inside the shaded area are feasible strategies for the high-type seller. From the graph, any feasible point satisfies $S < S^f_L$, that is, to differentiate himself from the low-quality seller, the high-quality seller should ration strictly lower capacity S during the advance sales. The next theorem formally proves this result and shows that this separating equilibrium exists if and only if both sellers are going to sell in the advance under the full information case.

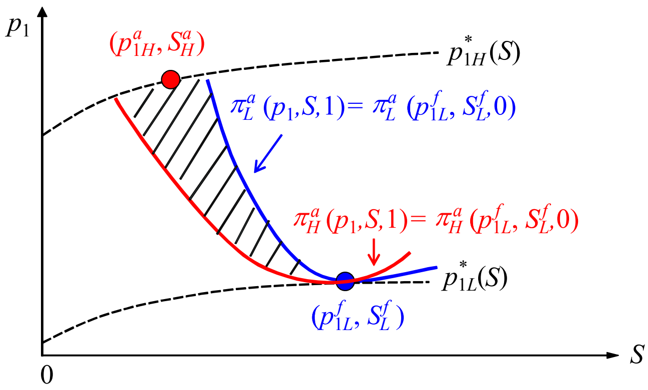

Figure 3 **A separating equilibrium**

Theorem 3

(i) A separating equilibrium exists if and only if $T > T_1$, in other words, if and only if the capacity is sufficiently large so that advance selling is optimal for both types under the full-information case.

(ii) In a separating equilibrium, the low-type seller follows her full-information strategy (p^f_{1L}, S^f_L).

(iii) In a separating equilibrium, the following characterize the H-type seller's rationing:

(a) the high-type seller rations strictly less than the low-type seller, $S^a_H < S^a_L$.

(b) the high-type seller rations strictly less than he would under the full information, $S^a_H < S^f_H$.

(c) When the high-type seller offers advance selling ($S^a_H > 0$), his price is lower than the price he would charge under the full information: $p^a_{1H} \le p^f_{1H}$. This price, however, is the same as the price that he would use when the same quantity S^a_H is sold in the full information case, i.e., $p^a_{1H} = p^*_{1H}(S^a_H)$.

(d) For given low quality L, the high-type seller's equilibrium ration S^a_H is nonincreasing in his quality H.

Theorem 3(i) implies that a separating equilibrium arises whenever capacity is not too limited ($T > T_1$) so that both sellers offer advance selling under the full information case (see Theorem 1). Notice that no additional condition is required for the existence of a separating equilibrium. It implies that under fairly general conditions, the high-type seller can distinguish from the low-type with the terms of advance selling, (p_1, S). On the other hand, if the capacity is tight ($T \le T_1$), this separating equilibrium breaks apart as neither type wants to sell in advance under the full information.

Parts (ii) and (iii) of Theorem 3 further characterize this separating equilibrium. The information asymmetry only affects the high-type seller as the low-type seller follows her strategy in the full information case. The high-type seller needs to distort his strategy from the full-information levels and differentiate himself from the low-type. What is interesting is how the high-type seller accomplishes it. If both sellers offered the same quantity during the advance sales, the high-type's advance price would be higher and, consequently, low-type seller could increase her profit by matching the high-type seller's advance price, without any consequence on low-type's profits in spot. To signal his type, the high-type seller needs to change the ration for advance sales. Part (iii-a) shows the high-type seller will decrease the quantity available during the advance sales to the extent that cannot be economically mimicked by low type. Note that for a given advance price, reducing the capacity ration will decrease both types' advance profits by the same amount. However, in the spot period where the quality information is revealed, the high-type seller is able to charge a higher spot price than a low-type seller. Consequently, decreasing the advance sales hurts the low-type seller more than the high-type seller. Part (iii-b) immediately follows from Theorem 2, $S_L^f = S_H^f$, and part (ii), $S_L^a = S_L^f$. Reducing the ration signals the product quality in the following way. Customers, upon observing that only a small portion of the total capacity is offered in advance, infer that the seller is very confident about her quality and reserves a lot to sell in the spot period. In contrast, a large ration in advance will be associated with low quality, as the low-type seller expects a weak spot market and has a strong incentive to sell a lot in advance. Part (iii-d) reinforces this intuition. As the quality difference increases, the high-type seller will reserve more capacity to sell in the spot market.

Part (iii-c) explains how the high-type seller changes the advance selling price. One may think that the high-type seller should increase the advance price as he reduces the advance ration. However, the effect is exactly opposite. As the capacity ration decreases, more capacity is available in spot and spot price decreases. As spot price decreases, customers are willing to pay less in advance. Nevertheless, it should be noted that this price change is simply a consequence of the change in the seller's rationing rather than the seller's deliberate attempt to use price as a signal. To see this, notice that the high-type seller offers the same price as he would when he sells the quantity S_H^a under the full information case, implying that there is no additional price distortion due to information asymmetry.

It should be noted that the high-type seller uses the exactly opposite strategy to what he would do under the full information case. In the full information case (see §4), price is the signal, and rationing is not: both sellers use the same rationing policy, but a high-type seller charges a higher price to differentiate from a low-type seller. In the asymmetric information case, price cannot signal quality, while rationing can.

5.2. Pooling Equilibrium

In a pooling equilibrium, both sellers follow the same strategy during the advance sales – i.e., either both types sell in advance with the same price and ration, or neither sells in advance, thus the quality information will not be revealed. We show that applying the intuitive criterion eliminates a pooling equilibrium in which both sellers offer advance selling.

Theorem 4 *By the Intuitive Criterion, a pooling equilibrium where both types offer the same quantity during advance sales cannot be sustained.*

Figure 4 illustrates why this is the case. Suppose a pooling equilibrium (p_1^E, S^E) exists with $S^E > 0$. Such strategy (a point in Figure 4) must be in the interior of the feasible region. Customers would not pay any advance price greater than or equal to p_{1H}^* when they believe that the product might be sold by a low-quality seller. On the other hand, any price less than or equal to p_{1L}^* is Pareto-dominated from the sellers' perspectives. From the single-crossing property (Lemma 2), the high-type seller's iso-profit curve must lie below the low-type's curve for $S \in (0, S^E)$, and the two curves cross at (p_1^E, S^E). Hence, if the high-type seller unilaterally reduces the advance ration and deviates to (p_1', S') for some $0 < S' < S^E$, this move will make the high-type seller strictly better off and the low-type strictly worse off. From the intuitive criterion, customers will believe that the seller is the high-type. This supports the high-type's unilateral deviation and breaks the pooling equilibrium.

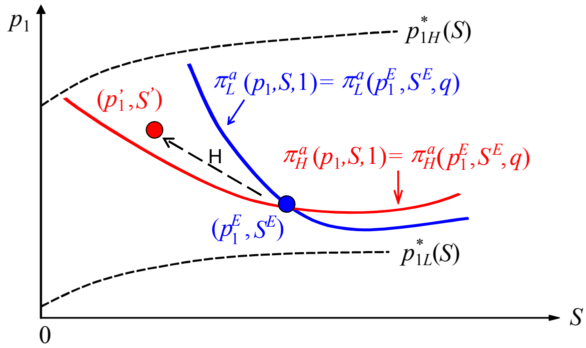

Figure 4 **Intuitive criterion eliminates a pooling equilibrium**

Theorem 4 further highlights why rationing can be effectively used to signal quality. As long as advance selling is desirable, the high-type seller would never pool with the low-type seller during advance sales, since he can gain more from revealing himself to customers by unilaterally lowering the capacity ration in advance.

5.3. Structure of Equilibrium

Following Theorems 3 and 4, we further characterize the seller's equilibrium strategies as functions of capacity T and quality levels L and H. Theorem 5 summarizes the result.

Theorem 5 *When quality information is asymmetric in advance, equilibrium strategies are as follows:*

i) If $T \le T_1$, neither type offers advance selling.

ii) If $T \in (T_1, T_D)$, both types offer limited advance selling, and the high-type seller rations less in advance than the low-type seller.

iii) If $T \in [T_D, N_1 + N_2)$, both types offer advance selling, and the high-type seller limits the advance sales while the low-type does not.

iv) If $T \ge N_1 + N_2$, then the equilibrium depends on the values of L and H. Specifically, *a. if $L < \bar{t}$, the high-type offers limited advance selling and the low-type offers full advance selling.*

b. if $L \geq \bar{t}$, the high-type only sells in spot while the low-type is indifferent between offering advance selling and selling only in spot.

Theorem 5 implies that the quality uncertainty does not affect the strategy and profit of the low-quality seller. However, the high-type seller has to sacrifice a portion of his profit by limiting the amount sold in advance (and possibly also lowering the price quoted in advance) to differentiate his type during advance sales. One may expect that in such a case, the high-quality seller is less likely to sell in advance. Interestingly, as long as advance selling is strictly preferred by the high type under the full-information setting (parts ii-iv.a of Theorem 5), the high-quality seller continues to offer advance selling. Although information asymmetry reduces the profit gain achieved by advance selling, its effect does not distort the seller's strategy enough to abandon advance selling. Reducing rationing quantity is sufficient to differentiate his type from the low-quality seller. The only exception to this rule is the case iv-b, when the seller has large capacity, $T \geq N_1 + N_2$, and the quality of the low-type product exceeds the threshold, $\bar{t}$. In this situation, the seller has so much capacity and finds it optimal to sell to all customers. Hence, in the full information case, the seller charges the same price in both advance and spot period (see Theorem 1). But, when the quality is uncertain and the high-type seller offers the same rationing quantity as the low-type seller, the low-type seller can easily mimic the high-type and remove the ability to signal. Consequently, the high-type seller is better off selling only in spot.

6. Value of Rationing

Our previous results show that in the presence of quality uncertainty, sellers can signal quality by limiting the amount sold in advance. One interesting question that follows is how big the benefit from such signaling is. In the full information case, an option to ration capacity never hurts the seller since it gives the seller more choices in allocating capacity: instead of 0 or $\min(T, N_1)$, S could be any quantity $S \in [0, \min(T, N_1)]$. In fact, as shown in part (a) of Theorem 1, rationing makes the seller strictly better off when his capacity is at an intermediate level. The value of capacity rationing, however, becomes less evident when quality is uncertain. This is because, while the ability to ration helps the high-type seller to differentiate from the low-type seller, signaling by reducing the ration in the advance sales will decrease the profit.

To examine whether the seller always benefits from rationing, we first examine the equilibrium outcome when sellers cannot ration their capacity during the advance selling. That is, if the seller offers advance selling, he needs to accept all demand up to his total capacity: $S = \min(N_1, T)$. A few observations immediately follow under the no-rationing case. If both sellers set to offer advance selling, the quantity will be the same, and the low-type seller has an incentive to mimic the high-type seller's price. Consequently, the high-type seller can never differentiate himself by advance selling. Theorem 6 characterizes the equilibrium for the no-rationing case.

Theorem 6 *For given H, let $\delta = H - L$ denote the difference in quality levels. When rationing is not allowed (i.e., $S = \min(T, N_1)$ or 0), there exist a function $t^D(T)$ such that the following holds in equilibrium,*

i) If $L > t^D(T)$, neither type sells in advance,

ii) If $L \leq t^D(T) < H$, only the low-type seller offers advance selling.

iii) If $H \leq t^D(T)$, there exists a threshold $\bar{\delta} \geq 0$ and a function $\bar{q}(\delta) \in [0,1]$ for $\delta > 0$ such that

a. if $\delta > \bar{\delta}$ and $q < \bar{q}(\delta)$, only the low-type sells in advance,

b. otherwise (i.e., $\delta \leq \bar{\delta}$ or $q \geq \bar{q}(\delta)$), both types sell in advance with a common advance price.

Part i) of Theorem 6 implies that, when sellers cannot ration, neither seller offers advance selling when the qualities of both types are sufficiently high ($L > t^D(T)$). When this happens, neither seller benefits from advance selling at a discounted price. When the quality of the low-type product falls below the threshold, $t^D(T)$, at least one of the two sellers offers advance selling in equilibrium. The low-type seller has incentive to sell as much as possible in advance before the quality information is revealed. The high-type seller chooses to sell in advance only if customers strongly believe that the product quality (the probability that the seller is a high-type) is high, $q \geq \bar{q}(\delta)$, or the quality difference is sufficiently small, $\delta \leq \bar{\delta}$. In such cases, a pooling equilibrium where both types charge a sufficiently high price in advance emerges, and both sellers benefit from selling in advance (compared to selling only in spot).

We now compare the equilibria under rationing (Theorem 5) and no rationing (Theorem 6) cases. We find that having an operational flexibility to ration does not necessarily benefit the seller. In other words, both sellers can be better off when they do not have the ability to ration. To see why this is the case, notice that the high-quality seller always uses rationing to signal his quality as long as advance selling is desirable. Thus, in the no-rationing pooling equilibrium, the low-type seller may be able to hide her inferior quality and quote a higher advance price compared to a separating equilibrium that arises in the rationing case. Thus, the low-type seller will be better off without the ability to ration. Interestingly, the inability to ration can benefit the high-type seller too. While rationing enables the seller to signal its type, this signaling can be costly as the high-type seller is forced to reduce the amount sold in advance and to lower both spot and advance prices. The cost associated with signaling can outweigh any profit incurred by the pooling equilibrium in the no-rationing case.

Figure 5 illustrates in more details how both sellers can be strictly better off in the no-rationing equilibrium. According to Theorem 6, an outcome where both sellers offer the full advance selling at the same price (point **NR**) is a pooling equilibrium when rationing is disallowed. Suppose now that both sellers can ration their capacity in advance. First note that from Theorem 4, no pooling equilibrium where both types sell in advance can be sustained, thus **NR** cannot be an equilibrium and a deviation must occur. In this situation, the low-type seller will follow its full-information strategy

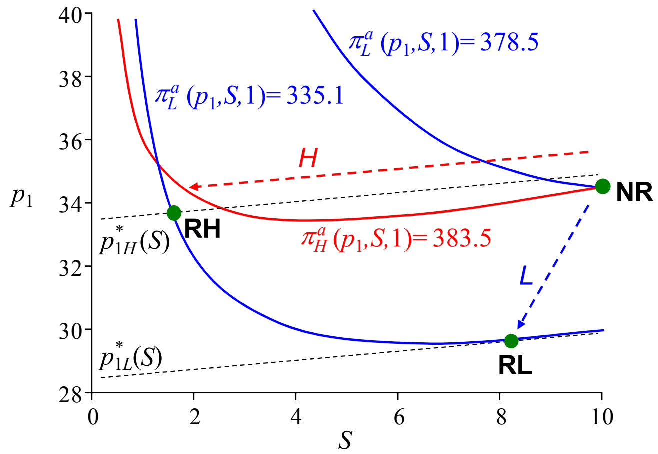

Figure 5 **An example where both types are strictly worse off by capacity rationing:** $q = 0.9$, $N_1 = 10$, $N_2 = 10$, $\alpha \sim$ **Uniform**$[-5,5]$, $T = 11$, $H = 35$, $L = 30$. **For the labelled points, the corresponding strategy pair and profits for type-**t **seller,** $(p_1, S; \pi_L^a(p_1,S,1), \pi_H^a(p_1,S,1))$**, are as follows: NR**$(34.45, 10; 378.5, 383.5)$, **RL**$(29.75, 8.39; 335.1, 348.1)$, **RH**$(33.69777, 1.62; 335.1, 381.9)$.

(point **RL**) (see Theorem 3). Consider the iso-profit curve for the low-type that runs through **RL**. Any point above this curve cannot be chosen by the high-type, as it can be mimicked profitably by the low-type seller (this is formally written in the constraint (10)). To prevent the low-type from mimicking, the high-type needs to lower ration to a point where low-type is indifferent between mimicking and following her full-information strategy. But, doing so will also lower both advance and spot prices and erode the seller's profit (point **RH**). At this point, both sellers earn strictly lower profits than in the no-rationing situation. In short, the high-type seller is worse off because signaling costs too much and the low-type seller is worse off as it cannot pool with the high type. Hence, both sellers would prefer not having the ability to ration.

An immediate question is why both sellers are not able to follow a pooling equilibrium when they have the flexibility to ration. The answer is that, similarly to the prisoner's dilemma, the no-rationing equilibrium, while achieving the Pareto-dominant outcome, cannot be enforced once rationing becomes feasible. The high-type seller, induced by short-term increase of profit, cannot resist the temptation to deviate to reduce the rationing in advance. This incentive of rationing, however, triggers a downward spiral, leading to the outcome where both parties lose.

It should be noted that the phenomenon that pooling is better for both sellers occurs only when customers strongly believe that the product quality is high (i.e., q is high). In fact, it can be shown that, *ceteris paribus*, the value of rationing decreases in the prior belief, q. As q increases, customers are more optimistic about high quality and are willing to pay a higher price. On the other hand, as Theorem 5 illustrates, the rationing equilibrium is independent of the prior belief q. Hence, as q increases, the increase in the pooling price makes the no-rationing pooling equilibrium more appealing and thus the rationing option becomes less desirable. Our result shows that while capacity rationing can signal quality, it is sometimes costly and makes both sellers worse off compared to the no-rationing case. As the prior belief of high quality decreases, the value of rationing increases.

7. Signaling: Rationing versus Advertising

So far we have shown that a seller can use capacity rationing to signal product quality during the advance sales. We now elaborate how capacity rationing differs from advertising, another signaling tool that has been studied extensively in the literature. Although advertisement can help the seller in many different ways (e.g., raising the willingness to pay, increasing the market size), we consider a pure signaling role of advertisement. Thus, we assume that advertising affects neither valuation distribution nor market size. Instead, we consider the case where advertising is a pure dissipative cost that the seller incurs to its customers for signaling. Such uninformative advertising has been considered in the literature, see Kihlstrom and Riordan (1984), Milgrom and Roberts (1986), Bagwell and Ramey (1988), Stock and Balachander (2005), and the references therein.

If customers were perfectly informed about product quality in advance, advertising would not be used in our settings. With unknown quality, advertising may enable the seller to convince customers about quality of the product and charge a higher price during the advance sales. Clearly, there is no benefit of advertising if the seller sells only in the spot period, when quality is already revealed.

To evaluate effectiveness of advertising in our setting, consider a case when seller uses adverting in the advance period instead of rationing. That is, S must be either zero (no advance sales) or $\min(T, N_1)$, and advertising may be used in advance. All other settings, including the sequence of events, are the same as before. In the first stage, the seller decides whether to offer advance selling or not, i.e., $S = 0$ or $\min(T, N_1)$. If he does ($S = \min(T, N_1)$), then the seller chooses the advance price p_1 and the advertising expenditure A. Having observed the price and advertising expense, customers form a posterior belief about the product being high quality, denoted by $b(p_1, S, A)$. For given posterior belief, the maximum price that customers will accept during the advance period is $b(p_1, S, A)p^*_{1H}(S) + (1 - b(p_1, S, A))p^*_{1L}(S)$. Hence, the seller offers an advance selling if $p_1 \leq b(p_1, S, A)p^*_{1H}(S) + (1 - b(p_1, S, A))p^*_{1L}(S)$. The seller's expected profit over the two periods is

$$\pi_t^{a,AD}(p_1, S, A, b) = p_1 S + \pi^*_{2t}(S) - A.$$

Otherwise, the seller's profit is $\pi_t^{a,AD}(p_1, S, A, b) = \pi^*_{2t}(0)$. The next result characterizes the equilibrium.

Theorem 7 *(i) If $H \leq t^D(T)$ and $\delta \leq \overline{\delta}$, there exists a separating equilibrium under which both types offer full advance selling, but only the high-type seller advertises. In all other cases, no seller advertises. (ii) The separating equilibrium in part (i) is Pareto-dominated by a no-advertising pooling equilibrium.*

Under the separating equilibrium described in part (i), the high-type seller proves his type through advertising that the low-type seller cannot afford. This equilibrium can be sustained only when both types find it optimal to sell in advance under the full information case ($H \leq t^D(T)$) and the quality difference is sufficiently small ($\delta \leq \bar{\delta}$). The first condition is straight-forward because, if a seller prefers spot-only selling under the full information case, his preference would remain the same in the asymmetric information case, since advertising only decreases the seller's profit from advance selling. For the second condition, notice that the difference in the advance prices between the two types of seller will be large when the quality difference is large. Consequently, it is very attractive for the low-type to mimic the high-type. To separate from the low-type, the high-type needs to incur a high advertising cost, which makes advance selling less appealing. When the quality difference is sufficiently large, the high-type is better off selling only in spot.

Even when the separating equilibrium in part (i) can be sustained, the high type has to spend significantly on advertising. In fact, the expected profit that the high-type seller earns from sales in the advance period will be the same as that of the low-type seller. To see why, first recall that since the true quality will be revealed in spot, the advertisement intends to influence the advance sales and profit. In the separating equilibrium where both offer full advance selling, although the high type charges a higher advance price than the low type, any extra revenue that the high-type earns will be fully offset by the advertising cost - otherwise it will incentivize the low type to mimic. Under this situation, there is another equilibrium that is better for both types of seller: pooling with no advertisement. The low type earns more than under the separating equilibrium since the pooling price is higher than her separating price, and the high-type is better off pooling as he does not incur advertising cost. It can be shown that this pooling equilibrium Pareto dominates the separating equilibrium. Obviously, our analysis assumes a very limited role of advertising. Advertising may increase the valuation distribution or market size, factors we do not study in this paper.

Now, consider the case that the seller is allowed to ration, i.e., S can be any value between zero and $\min(T, N_1)$. Applying similar analysis, it can be shown that the intuitive criterion used in Theorem 4 rules out any pooling equilibrium in which both types sell in advance. Also, in any separating equilibrium, only the high-type seller will signal its quality as the low-type seller will follow his full-information strategy $\left(p_{1L}^f, S_L^f, 0\right)$ and place no advertisement. It turns out that the high-type seller prefers to use rationing over advertising as a signal. The result is summarized below.

Theorem 8 *Neither of the two sellers invests in advertising in any separating equilibrium.*

Theorem 8 shows that signaling by rationing is more efficient than by advertising. To understand this, compare these two signaling levers. Advertising is a pure cost to the seller: in other words, the seller will not advertise if the product quality is publicly known. On the other hand, advance selling (and capacity rationing) can increase seller's profit even when there is no quality uncertainty (Theorem 1). Allowing the seller to choose its rationing quantity helps the seller (at least partially) offset the signaling cost, as the seller can adjust availability and price in the spot period according to the cost of signal.

8. Conclusion

As advance selling has been rapidly adopted in practice, academic research has examined a number of reasons why firms should offer advance selling. These reasons include consumer's risk aversion (Png, 1989), advance demand information (Tang et al., 2004; Li and Zhang, 2012), and consumer's valuation uncertainty (Xie and Shugan, 2001; Gallego and Sahin, 2010). To the best of our knowledge, this paper is the first one that describes and analyzes the role of advance selling as a signal of product quality. When consumers do not have perfect information about product quality, we show that the seller can signal the product quality by rationing capacity available during advance sales.

We show that consumer's uncertainty about product quality always worsens the profit of a seller offering high-quality product, since his product cannot be fully appreciated during advance sales, with consumers being doubtful about product quality. Pricing alone cannot be a signal for quality as the low-type seller can easily mimic high-type seller. In order to differentiate herself from the low-quality seller, the high-type seller sacrifices some profit by reducing its capacity ration during advance sales, compared to the optimal level with known quality. While it may seem optimal for the high-quality seller to bypass advance selling and sell only after the quality information is released, selling portion of capacity in advance is typically better for the high-quality seller except for two special cases. The first case is when the total capacity is very tight. In this case, the seller can clear the capacity at a very high spot price and neither seller wants to sell in advance. Thus, the presence of low-quality seller does not reduce the high-type seller's profit. The second case is when the seller has a large capacity and would like to sell to all customers by pricing the product very low. Instead of sending a costly signal, the high-type seller sells only in the spot period while the low-type seller is indifferent between advance selling and spot only. In all other cases, the high-quality seller uses reduced ration as a primary signal device. Interestingly, as long as both types of sellers offer advance selling in the full-information case, then the high-quality seller can use rationing to distinguish himself. Our finding on rationing capacity to signal quality is consistent with several examples in practice. One of such examples is the premium French wine's advance (en primeur) market, where chateaux intentionally limit the availability of wine sold in *en primeur* to convey high quality (Prial, 1989; Stimpfig, 2012).

Although rationing can be a very effective tool for signaling and for increasing profits, we show that the seller does not always benefit from having operational flexibility to ration. When compared to the case where rationing is not allowed, rationing flexibility can make both high-quality and low-quality sellers strictly worse off. This happens when customers are optimistic about the product's high quality and the seller's capacity is not too tight. While rationing enables the seller to signal its type, reducing the amount that he sells in advance (and consequently lowering both spot and advance prices) can outweigh any profit incurred by simply pooling with the low-type seller (which is the outcome when rationing is not allowed). Since consumers are optimistic about product quality, the high-type seller does not lose too much by pooling with the low-type. We also compare the benefits of signaling through rationing with uninformative advertising and show that rationing is more efficient. All major results and insights carry through when the marginal cost of a high-quality product (c_H) is different from that of a low-quality seller (c_L). All results also hold when the utility of a consumer is a multiplicative function ($U = \alpha t - p$) instead of an additive function ($U = \alpha + t - p$). The proofs for these two variations are available in a supplementary document.

References

Athey, S., 2001. Single Crossing Properties and the Existence of Pure Strategy Equilibria in Games of Incomplete Information, *Econometrica*, 69(4), 861-89.

Auffrey, R., 2008. URL: passionatefoodie.blogspot.hk/2008/04/2005-bordeaux-tasting-saturday.html. Accessed on July 18, 2012.

Aviv, Y., A. Pazgal, 2008. Optimal Pricing of Seasonal Products in the Presence of Forward-Looking Consumers. *M&SOM*, 10(3), 339-359.

Bagwell, K., G. Ramey, 1988. Advertising and Limit Pricing. *RAND J. of Econom.*, 19(1), 59-71.

Bagwell, K., M. Riordan, 1991. High and Declining Prices Signal Product Quality. *Amer. Econom. Rev.*, 81(1), 224-239.

Brown, S., 2001. Torment Your Customers (They will Love It). *Harvard Business Rev.*, 79(9), 82-88.

Cho In-K., D.M., Kreps, 1987. Signaling Games and Stable Equilibria. *Quart. J. Econom.*, 102(2), 179-221.

Chu, L.Y., H. Zhang, 2011. Optimal Preorder Strategy with Endogenous Information Control. *Management Sci.*, 57(6), 1055-1077.

Dubois, P., C. Nauges, 2006. Identifying the Effect of Unobserved Quality and Expert Reviews in the Pricing of Experience Goods: Empirical Application on Bordeaux Wine, *Working paper, University of Toulouse.*

Dye, R., 2000. The Buzz on Buzz. *Harvard Business Rev.*. 78(6), 139-146.

Easley, D., M. O'Hara, 2009. Ambiguity and Nonparticipation: The Role of Regulation. *Rev. Financ. Stud.*, 22(5), 1817-1843.

Federgruen, A., A. Heching, 1999. Combined Pricing and Inventory Control under Uncertainty. *Oper. Res.*, 47(3), 454-475.

Gallego, G., O. Sahin, 2010. Revenue Management with Partially Refundable Fares. *Oper. Res.*, 58(4), 817-833.

Gilbert, R.J., P. Klemperer, 2000. An Equilibrium Theory of Rationing. *RAND J. Econom.*, 31(1), 1-21.

Hadj Ali H., C. Nauges, 2006. The Pricing of Experience Goods: The Example of "en primeur" Wine. *Amer. J. Agri. Econom.*, 89(1), 91-103.

Healy, R., 2010. The Dark Side of Advance Selling. URL: www.ryanhealy.com/dark-side-of-advance-selling. Accessed on July 18, 2012.

Harford, T., 2005. The Great Xbox Shortage of 2005. URL: www.slate.com/articles/arts/everyday_economics /2005/12/the_great_xbox_shortage_of_2005.html. Accessed on July 18, 2012.

Jackson, R.S., 2000. Wine Science: Principles, Practice, Perception. Academic Press.

Janssen, M., Moraga-Gonzalez J. L., M. R. Wildenbeest, 2005. Truly Costly Sequential Search and Oligopolistic Pricing. *Int. J. Ind. Org.*, 23, 451-466.

Katok, E., Olsen T., V. Pavlov, 2012. Wholesale Pricing under Mild and Privately Known Concerns for Fairness. Forthcoming in *Production Oper. Management*.

Kihlstrom, R.E., M.H. Riordan, 1984. Advertising as a Signal. *J. of Political Econom.*, 92(June), 427-450.

Kirmani, A., A.R. Rao, 2000. No Pain, No Gain: A Critical Review of the Literature on Signaling Unobservable Product Quality. *J. Marketing*, 64(2), 66-79.

Kuksov, D., K. Wang, 2011. A Model of the "It" Products in Fashion. Working paper, Washington University in St. Louis.

Li, C., F. Zhang, 2012. Advance Demand Information, Price Discrimination, and Pre-order Strategies. Forthcoming in *M&SOM*.

Lim, W.S., 2009. Overselling in a Competitive Environment: Boon or Bane? *Marketing Sci.*, 28(6), 1129-1143.

Liu, Q., G. J. van Ryzin, 2008. Strategic Capacity Rationing to Induce Early Purchases. *Management Sci.*, 54(6), 1115-1131.

Lutz, N.A., 1989. Warranties as Signals under Consumer Moral Hazard. *RAND J. Econom.*, 20(2), 239-255.

Milgrom, P., J. Roberts, 1986. Price and Advertising Signals of Product Quality. *J. of Political Econom.*, 94(August), 796-821.

Moorthy, S., K. Srinivasan, 1995. Signaling Quality with a Money-Back Guarantee: The Role of Transaction Costs. *Marketing Sci.*, 14(4), 442-466.

Nasiry, J., I. Popescu, 2012. Advance Selling When Consumers Regret. *Management Sci.*, 58(6), 1160-1177.

Prasad A., K.E. Stecke, X.Y. Zhao, 2010. Advance Selling by a Newsvendor Retailer. *Production Oper. Management*, 20, 129-142.

Prial F.J., 1989. Wine Talk. *New York Times*, March 22. URL: www.nytimes.com/1989/03/22/garden/ wine-talk-898389.html. Accessed on July 18, 2012.

Retailing Today, 2000. What's hot: Playstation2. (Nov. 20).

Robinson, J. 2001, Diary of an en primeur wine-taster. URL: www.jancisrobinson.com/articles/jr7133.html. Accessed on July 18, 2012.

Robinson, J., 2011. Primed and en primeur. *Financial Times*, April 8, 2011.

Schelling, T. C., 1960. The Strategy of Conflict. Cambridge, Massachusetts: Harvard University Press.

Shugan, S.M., J.H. Xie, 2000. Advance Pricing of Services and Other Implications of Seperating Purchase and Consumption. *J. Service Res.*, 2(February), 227-239.

Shulman, J.D., Coughlan, A.T., R.C. Savaskan, 2009. Optimal Restocking Fees and Information Provision in an Integrated Demand-Supply Model of Product Returns. *M&SOM*, 11(4), 577-594.

Sinclair, B., 2006. Is a PS3 Shortage in the Offing? URL: news.cnet.com/Is-a-PS3-shortage-in-the-offing/2100-1043_3-6124297.html. Accessed on July 18, 2012.

Sobel, J, 2007. Signaling games. *University of California in San Diego.*

Stimpfig, J. 2012. En primeur: 2011 proves a hard sell. *Finantial Times*, June 19.

Stock, A., S. Balachander, 2005. The Making of a "Hot Product": A signaling Explanation of Marketers' Scarcity Strategy. *Management Sci.*, 51(8), 1181-1192.

Su, X., 2007. Intertemporal Pricing with Strategic Customer Behavior.*Management Sci.*, 53(5), 726-741.

Su, X., 2009. Consumer Return Policies and Supply Chain Performance.*M&SOM*, 11(4), 595-612.

Tang, C.S., K. Rajaram, A. Alptekinoğlu, J. Ou, 2004. The Benefits of Advance Booking Discount Programs: Model and Analysis. *Management Sci.*, 50(4), 465-478.

Xie, J.H., S.M. Shugan, 2001. Electronic Tickets, Smart Cards, and Online Prepayment: When and How to Advance Sell. *Marketing Sci.*, 20(3), 219-243.

Yu, M., Kapuscinski, R., H. Ahn, 2010. Advance Selling: Effects of Interdependent Customer Valuations and Seller's Capacity. *Working Paper, Ross School of Business, University of Michigan.*

Zhang, D., W.L. Cooper, 2008. Managing Clearance Sales in the Presence of Strategic Customers. *Production Oper. Management*, 17(4), 416-431.

Appendix A: Proof of Lemma 1 and Corollary 1

The proof, directly adapted from Yu et al. (2010), is provided in the supplementary document.

Appendix B: Proof of Theorem 1

(a) The proof, directly adapted from Yu et al. (2010), is provided in the supplementary document.

(b) First recall the definitions of T_1 and T_D in Yu et al. (2010): for $T < N_1 + N_2$ and $S \in [0, \min(T, N_1)]$, let $f_t^B(S) = \mathrm{E}[\min(t+\alpha, p_{2t}^B(S))]S + p_{2t}^B(S)(T-S)$. T_1 is the largest T such that $f_t^B(S)$ is maximized at $S=0$ and T^D is the smallest T such that $f_t^B(S)$ is maximized at $S = \min(T, N_1)$.

To show that T_1 and T_D are independent of the quality level t, it then suffices to show that $df_t^B(S)/dS$ is independent of t. To this end, note that by Lemma 1, $p_{2t}^B(S) = t + (\overline{G})^{-1}(\frac{T-S}{N_1+N_2-S})$. Substituting this in the expression of $f_t^B(S)$, we have $f_t^B(S) = \mathrm{E}[\min(\alpha, (\overline{G})^{-1}(\frac{T-S}{N_1+N_2-S}))]S + (\overline{G})^{-1}(\frac{T-S}{N_1+N_2-S})(T-S) + tT$, which depends on t only through the term tT. Clearly, $df_t^B(S)/dS$ is independent of t, and hence, so are T_1 and T_D.

Appendix C: Proof of Theorem 2

The proof uses the following lemma. The proof of all the lemmas are available in the supplementary document.

Lemma C.1 *$p_{2H}^*(S) \geq p_{2L}^*(S)$, $p_{1H}^*(S) > p_{1L}^*(S)$.*

By Lemma C.1, to show $p_{1H}^f > p_{1L}^f$, it suffices to show $S_H^f = S_L^f$. Meanwhile, by Theorem 1, to show $S_H^f = S_L^f$, it suffices to show that S_t^f is independent of t for $T \in (T_1, T_D)$. By Yu et al. (2010), for $T \in (T_1, T_D)$, S_t^f satisfies $df_t^B(S)/dS = 0$, where $f_t^B(S)$ is as defined in the proof of Theorem 1 (b). Since $df_t^B(S)/dS$ is independent of t (shown in proof of Theorem 1 (b)), so is S_t^f for $T \in (T_1, T_D)$.

Appendix D: Proof of Lemma 2

The proof uses the following lemma.

Lemma D.1 $\pi^*_{2H}(S) - \pi^*_{2L}(S)$ *strictly decreases in* S.

For $b = 1$, $S > 0$, and constants k_1 and k_2, let the iso-profit curves of the high type for profit k_1 and of the low type for profit k_2 be $\{(S, p_1) : \pi^a_H(p_1, S, 1) = k_1, (p_1, S) \text{ is feasible}\}$ and $\{(S, p_1) : \pi^a_L(p_1, S, 1) = k_2, (p_1, S) \text{ is feasible}\}$, respectively. We prove by contradiction that (i) they cross at most once and (ii) when they do cross, the former crosses the latter from below.

(i) Suppose that the two curves cross at two *distinct* points (p'_1, S') and (p''_1, S'') with $0 < S'' \le S'$. By definition of the iso-profit curve, $\pi^a_H(p'_1, S', 1) = \pi^a_H(p''_1, S'', 1) = k_1$ and $\pi^a_L(p'_1, S', 1) = \pi^a_L(p''_1, S'', 1) = k_2$. From the expression of $\pi^a_t(p_1, S, 1)$ in equation (8), we get

$$p'_1 S' + \pi^*_{2H}(S') = p''_1 S'' + \pi^*_{2H}(S'') = k_1 \tag{13}$$

$$p'_1 S' + \pi^*_{2L}(S') = p''_1 S'' + \pi^*_{2L}(S'') = k_2 \tag{14}$$

Subtracting equation (14) from equation (13), we get $\pi^*_{2H}(S') - \pi^*_{2L}(S') = \pi^*_{2H}(S'') - \pi^*_{2L}(S'')$. Since $\pi^*_{2H}(S) - \pi^*_{2L}(S)$ strictly decreases in S (Lemma D.1), we immediately have $S' = S''$. This result, together with equation (13) and the fact $S' > 0$, further implies $p'_1 = p''_1$. This, however, contradicts with the assumption that (p'_1, S') and (p''_1, S'') are two distinct points.

(ii) To show that the high type's curve crosses the low type's from below, we will prove that the high type's curve is below the low type's on the left of the crossing point. The result for the other side of the crossing point can be shown in a same way.

Suppose that the two curves cross at point (p'_1, S') with $S' > 0$ and that the high type's curve is above the low type's on the left of the crossing point. Hence, there must exist two points (p''_{1H}, S'') and (p''_{1L}, S'') on the high type's and low type's curves, respectively, such that $0 < S'' < S'$ and $p''_{1H} > p''_{1L}$. By definition of the iso-profit curve and equation (8), we have

$$p'_1 S' + \pi^*_{2H}(S') = p''_{1H} S'' + \pi^*_{2H}(S'') = k_1 \tag{15}$$

$$p'_1 S' + \pi^*_{2L}(S') = p''_{1L} S'' + \pi^*_{2L}(S'') = k_2 \tag{16}$$

Subtracting equation (16) from equation (15), we get $(p''_{1H} - p''_{1L})S'' + \pi^*_{2H}(S'') - \pi^*_{2L}(S'') = \pi^*_{2H}(S') - \pi^*_{2L}(S')$. Since $p''_{1H} > p''_{1L}$ and $S'' > 0$, we have $\pi^*_{2H}(S'') - \pi^*_{2L}(S'') < \pi^*_{2H}(S') - \pi^*_{2L}(S')$. This, however, contradicts with the fact $S'' < S'$ and Lemma D.1.

Appendix E: Proof of Theorem 3

(i) First note that by Theorem 1, $S^f_L > 0$ if and only if $T > T_1$. Hence, it suffices to show that a separating equilibrium exists if and only if $S^f_L > 0$.

($\Rightarrow$) Prove by contradiction. Suppose $S^f_L = 0$ and a separating equilibrium exists. First note that equations (10) and (11) jointly imply $\pi^*_{2H}(S^a_H) - \pi^*_{2L}(S^a_H) \ge \pi^*_{2H}(S^f_L) - \pi^*_{2L}(S^f_L)$, which further implies $S^a_H \le S^f_L$ by Lemma D.1. Since $S^f_L = 0$ and $S^a_H \ge 0$ (constraint (9)), we immediately have $S^a_H = 0$. In other words, both types of sellers sell only in spot in the separating equilibrium. This, however, contradicts with the definition of a separating equilibrium.

($\Leftarrow$) It suffices to show that when $S_L^f > 0$, there exists a solution for the high-quality seller's problem (equations (9) through (12)). To this end, define a function of S for $S \in [0, \min(T, N_1)]$:

$$M(S) = \pi_L^a(p_{1H}^*(S), S, 1) - \pi_L^a\left(p_{1L}^f, S_L^f, 0\right) = p_{1H}^*(S)S + \pi_{2L}^*(S) - [p_{1L}^f S_L^f + \pi_{2L}^*(S_L^f)].$$

Clearly $M(S)$ is continuous in S. Also recall the following facts: $p_{1L}^f = p_{1L}^*(S_L^f)$, S_L^f maximizes $\pi_L^a\left(p_{1L}^*(S), S, 0\right)$, and $p_{1H}^*(S) > p_{1L}^*(S)$ (Lemma C.1). These facts imply the values of $M(S)$ at two boundary points, 0 and S_L^f:

$$M(0) = \pi_{2L}^*(0) - \pi_L^a\left(p_{1L}^*(S_L^f), S_L^f, 0\right) \leq 0$$
$$M(S_L^f) = \left(p_{1H}^*(S_L^f) - p_{1L}^f\right) S_L^f = \left(p_{1H}^*(S_L^f) - p_{1L}^*(S_L^f)\right) S_L^f > 0 \tag{17}$$

Hence, there exists a point $S \in [0, S_L^f)$ satisfying $M(S) = 0$. Let $\underline{S} = \min\{S \in [0, S_L^f) : M(S) = 0\}$. Below we show that $(p_{1H}^*(\underline{S}), \underline{S})$ satisfies all the constraints in the high-type seller's problem:

- Constraint (9): by definition of $\underline{S}$, $0 \leq \underline{S} < S_L^f \leq \min(T, N_1)$. Also, when $\underline{S} > 0$, clearly $p_1 = p_{1H}^*(\underline{S})$ satisfies $p_1 \in [p_{1L}^*(\underline{S}), p_{1H}^*(\underline{S})]$.
- Constraint (10): by definition of $\underline{S}$, $\pi_L^a(p_{1H}^*(\underline{S}), \underline{S}, 1) = \pi_L^a(p_{1L}^f, S_L^f, 0)$.
- Constraint (11): by Lemma D.1 and the fact $\underline{S} < S_L^f$, $\pi_H^a(p_{1H}^*(\underline{S}), \underline{S}, 1) - \pi_H^a(p_{1L}^f, S_L^f, 0) > \pi_L^a(p_{1H}^*(\underline{S}), \underline{S}, 1) - \pi_L^a(p_{1L}^f, S_L^f, 0) = 0$.
- Constraint (12): by definition of $\underline{S}$, $\underline{S} < S_L^f$. Hence, $(p_{1H}^*(\underline{S}), \underline{S}) \neq (p_{1L}^f, S_L^f)$.

(ii) First note that if L type sells in advance in the separating equilibrium (i.e., $S_L^a > 0$), then his advance price must equal to $p_{1L}^*(S_L^a)$. That is because any price lower than $p_{1L}^*(S_L^a)$ is strictly dominated and any price higher will not be accepted by advance customers, as L type is perfectly discerned in the separating equilibrium. If, however, L type does not sell in advance (i.e., $S_L^a = 0$), his profit in equilibrium is simply $\pi_{2L}^*(0)$. In both cases, the low-type seller's profit in the separating equilibrium is $p_{1L}^*(S_L^a)S_L^a + \pi_{2L}^*(S_L^a)$. Furthermore, since S_L^f maximizes $p_{1L}^*(S)S + \pi_{2L}^*(S)$ for $S \in [0, \min(T, N_1)]$ (ref. equation (7)), the *highest* profit that the low type can make in the equilibrium is $p_{1L}^*(S_L^f)S_L^f + \pi_{2L}^*(S_L^f)$.

On the other hand, if L type follows his full-information strategy $(p_{1L}^*(S_L^f), S_L^f)$ and $S_L^f > 0$, customers will always buy in advance regardless of their posterior belief b, because their maximum willingness-to-pay is at least $p_{1L}^*(S_L^f)$ (ref equation (5)). Consequently, the *lowest* profit that the low type can guarantee to make in a separating equilibrium is also $p_{1L}^*(S_L^f)S_L^f + \pi_{2L}^*(S_L^f)$.

From the two facts above, L type always follows his full-information strategy in a separating equilibrium, i.e., $p_{1L}^a = p_{1L}^*(S_L^f) = p_{1L}^f$ and $S_L^a = S_L^f$.

(iii) We first show the first half of (iii-c): if $S_H^a > 0$, $p_{1H}^a = p_{1H}^*(S_H^a)$, and then use it to prove the other results.
<u>(iii-c) if $S_H^a > 0$, $p_{1H}^a = p_{1H}^*(S_H^a)$</u>: Prove by contradiction. Suppose $S_H^a > 0$ and $p_{1H}^a \neq p_{1H}^*(S_H^a)$. By constraint (9), we immediately have $p_{1H}^a < p_{1H}^*(S_H^a)$. To reach a contradiction, it suffices to show that, compared to (p_{1H}^a, S_H^a), the feasible strategy $(p_{1H}^*(\underline{S}), \underline{S})$ identified in part (i) strictly improves H type's profit.

To this end, first note that by definition of $\underline{S}$ and the fact $M(0) \leq 0$, we have $M(S) \leq 0$ for all $S \leq \underline{S}$. Meanwhile, since $p_{1H}^a < p_{1H}^*(S_H^a)$, constraint (10) must be binding with (p_{1H}^a, S_H^a), otherwise the high-quality seller can strictly increase the total profit by slightly raising p_{1H}^a while fixing S_H^a. The binding constraint (10) and $p_{1H}^a < p_{1H}^*(S_H^a)$ jointly imply $\pi_L^a\left(p_{1H}^*(S_H^a), S_H^a, 1\right) > \pi_L^a\left(p_{1H}^a, S_H^a, 1\right) = \pi_L^a\left(p_{1L}^f, S_L^f, 0\right)$, i.e., $M(S_H^a) > 0$. All of these results jointly imply $S_H^a > \underline{S}$.

Next we prove that $(p_{1H}^*(\underline{S}), \underline{S})$ dominates (p_{1H}^a, S_H^a). By definition of $\underline{S}$, the binding constraint (10), Lemma D.1, and the fact $S_H^a > \underline{S}$, we have

$$\begin{aligned}
&\pi_H^a(p_{1H}^*(\underline{S}), \underline{S}, 1) - \pi_H^a\left(p_{1H}^a, S_H^a, 1\right) \\
&= \pi_L^a\left(p_{1H}^*(\underline{S}), \underline{S}, 1\right) - \pi_{2L}^*(\underline{S}) + \pi_{2H}^*(\underline{S}) - \left[\pi_L^a\left(p_{1H}^a, S_H^a, 1\right) - \pi_{2L}^*\left(S_H^a\right) + \pi_{2H}^*\left(S_H^a\right)\right] \\
&= \pi_L^a\left(p_{1L}^f, S_L^f, 0\right) - \pi_{2L}^*(\underline{S}) + \pi_{2H}^*(\underline{S}) - \left[\pi_L^a\left(p_{1L}^f, S_L^f, 0\right) - \pi_{2L}^*\left(S_H^a\right) + \pi_{2H}^*\left(S_H^a\right)\right] \\
&= \pi_{2H}^*(\underline{S}) - \pi_{2L}^*(\underline{S}) - \left[\pi_{2H}^*\left(S_H^a\right) - \pi_{2L}^*\left(S_H^a\right)\right] > 0.
\end{aligned}$$

<u>(iii-a)</u> Prove by contradiction. Suppose $S_H^a \geq S_L^a$. Since $S_L^a = S_L^f$ by part (ii), we have $S_H^a \geq S_L^f$. Meanwhile, as shown in part (i), $S_H^a \leq S_L^f$. These facts jointly imply $S_H^a = S_L^f$. Also note that in an equilibrium, $S_L^f > 0$ by part (i). Hence, $p_{1H}^a = p_{1H}^*(S_H^a) = p_{1H}^*(S_L^f)$ by part (iii-c). However, from equation (17), $(p_{1H}^*(S_L^f), S_L^f)$ violates constraint (10) and hence cannot be H type's equilibrium strategy.
<u>(iii-b)</u> Follows immediately from parts (ii) and (iii-a), as well as Theorem 2.
<u>(iii-c) if $S_H^a > 0$, $p_{1H}^a \leq p_{1H}^f$:</u> $p_{1H}^a = p_{1H}^*(S_H^a)$ by part (iii-c) and $p_{1H}^f = p_{1H}^*(S_H^f)$ by definition of p_{1H}^f. The result then follows from the facts that $p_{1H}^*(S)$ is nondecreasing in S and $S_H^a < S_H^f$ (shown above).
<u>(iii-d)</u> By parts (iii-a) through (iii-c), the high type's problem is equivalent to $p_{1H}^a = p_{1H}^*(S_H^a)$ if $S_H^a > 0$ and

$$\begin{aligned}
S_H^a = \arg \max_{S \in [0, S_H^f)} \ & \pi_H^a(p_{1H}^*(S), S, 1) = p_{1H}^*(S)S + \pi_{2H}^*(S) \\
\text{subject to:} \quad & \pi_L^a(p_{1H}^*(S), S, 1) \leq \pi_L^a\left(p_{1L}^f, S_L^f, 0\right) \qquad (18) \\
& \pi_H^a(p_{1H}^*(S), S, 1) \geq \pi_H^a\left(p_{1L}^f, S_L^f, 0\right) \qquad (19)
\end{aligned}$$

Note that $\pi_t^a(p_{1t}^*(S), S, 1) = \pi_t^f(S)$ for $t = H$ or L. Based on the property of $\pi_t^f(S)$ characterized in Yu et al. (2010), below we prove by considering three cases:

- $T_1 < T < N_1 + N_2$: By Yu et al. (2010), both $\pi_H^a(p_{1H}^*(S), S, 1)$ and $\pi_L^a(p_{1L}^*(S), S, 1)$ strictly increase in $S \in [0, S_H^f)$. It is easy to show that $(p_{1H}^*(S) - p_{1L}^*(S))S$ is nondecreasing in S, and hence $\pi_L^a(p_{1H}^*(S), S, 1) = \pi_L^a(p_{1L}^*(S), S, 1) + (p_{1H}^*(S) - p_{1L}^*(S))S$ strictly increases in $S \in [0, S_H^f)$. As a result, constraint (18) must be binding when $S = S_H^a$. Fixing L and S, we have $\pi_L^a(p_{1H}^*(S), S, 1)$ strictly increases in $H > L$, while $\pi_L^a\left(p_{1L}^f, S_L^f, 0\right)$ remains constant. Hence, S_H^a is nonincreasing in $H > L$. (It is also easy to see $S_H^a > 0$, as the inequality in constraint (18) is strict when $S = 0$. We will use this result in later proof.)
- $T \geq N_1 + N_2$ and $L \geq \bar{t}$: By Yu et al. (2010), both $\pi_H^a(p_{1H}^*(S), S, 1)$ and $\pi_L^a(p_{1L}^*(S), S, 1)$ are independent of S for $S \in [0, \min(T, N_1)]$. Meanwhile, since $p_{1H}^*(S) > p_{1L}^*(S)$, constraint (18) is satisfied only when $S = 0$. It is easy to show that $S = 0$ also satisfies constraint (19). Hence, $S_H^a = 0$ for all $H > L$.
- $T \geq N_1 + N_2$ and $L < \bar{t}$: By Yu et al. (2010), $\pi_L^a(p_{1L}^*(S), S, 1)$ strictly increases in $S \in [0, S_H^f)$. If $H < \bar{t}$, then the same monotonicity applies to $\pi_H^a(p_{1H}^*(S), S, 1)$. Following the same logic as in the first bullet, we can show that S_H^a is nonincreasing in $H > L$ (and $S_H^a \in (0, S_H^f)$). If, however, $H \geq \bar{t}$, then $\pi_H^a(p_{1H}^*(S), S, 1)$ is independent of S. Since $\pi_L^a(p_{1H}^*(S), S, 1)$ strictly increases in S, there exists an interval $[0, \bar{S}]$ for some $\bar{S} \in (0, S_H^f)$ such that constraint (18) is binding at $S = \bar{S}$ and the high type is indifferent in choosing any $S \in [0, \bar{S}]$. Hence, $S_H^a = \bar{S}$ as it leads to the lexicographically largest equilibrium, and it decreases in $H > L$ since $\pi_L^a(p_{1H}^*(S), S, 1)$ strictly increases in H.

Appendix F: Proof of Theorem 4

Suppose a pooling equilibrium exists in which both types of sellers offer advance selling at (p_1^E, S^E). Per definition, $S^E > 0$ and customers' posterior belief $b(p_1^E, S^E)$ is the same as the prior belief q. Furthermore, since customers would buy in advance in the equilibrium (ref. participating equilibrium), we have $p_1^E \le qp_{1H}^*(S^E) + (1-q)p_{1L}^*(S^E)$, further implying $p_1^E < p_{1H}^*(S^E)$ by Lemma C.1. To show that Intuitive Criterion always eliminates such a pooling equilibrium, it suffices to show that there always exists a strategy pair (p_1', S') such that the high-quality seller strictly prefers choosing (p_1', S') and being perceived as a high-quality seller than pooling at (p_1^E, S^E), while the low-quality seller has the opposite preference.

To this end, we first define two functions of S for $S \in [0, \min(T, N_1)]$:

$$D_H(S) = \pi_H^a(p_{1H}^*(S), S, 1) - \pi_H^a(p_1^E, S^E, q) = p_{1H}^*(S)S + \pi_{2H}^*(S) - [p_1^E S^E + \pi_{2H}^*(S^E)],$$
$$D_L(S) = \pi_L^a(p_{1H}^*(S), S, 1) - \pi_L^a(p_1^E, S^E, q) = p_{1H}^*(S)S + \pi_{2L}^*(S) - [p_1^E S^E + \pi_{2L}^*(S^E)].$$

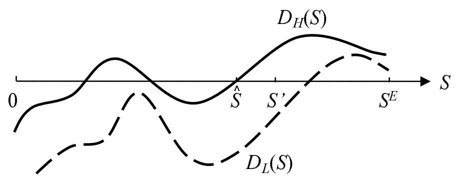

Figure 6 **Illustration of** $D_H(S)$ **and** $D_L(S)$

Clearly, both $D_H(S)$ and $D_L(S)$ are continuous in S. Furthermore, note that $D_H(S^E) = (p_{1H}^*(S^E) - p_1^E)S^E > 0$ since $p_1^E < p_{1H}^*(S^E)$ and $S^E > 0$, and that $D_H(0) \le 0$ since otherwise the high-quality seller would strictly prefer selling only in spot to advance selling with $(p_1^E,\ S^E)$. Hence, there exists at least a point $S \in [0, S^E)$ satisfying $D_H(S) = 0$. Let $\hat{S} = \max\{S : S \in [0, S^E), D_H(S) = 0\}$. Clearly, $D_H(S) > 0$ for all $S \in (\bar{S}, S^E]$. Meanwhile, by Lemma D.1, for $S < S^E$, $D_H(S) - D_L(S) = \pi_{2H}^*(S) - \pi_{2L}^*(S) - (\pi_{2H}^*(S^E) - \pi_{2L}^*(S^E)) > 0$, implying $D_L(\hat{S}) < D_H(\hat{S}) = 0$. Since $D_L(S)$ is continuous in S, there must exist a S' in the right neighborhood of $\hat{S}$ such that $D_H(S') > 0$ and $D_L(S') < 0$. Let $p_1' = p_{1H}^*(S')$. Clearly, (p_1', S') satisfies $\pi_H^a(p_1', S', 1) > \pi_H^a(p_1^E, S^E, q)$ and $\pi_L^a(p_1', S', 1) < \pi_L^a(p_1^E, S^E, q)$.

Appendix G: Proof of Theorem 5

By Theorem 4, only two classes of equilibria are possible: separating equilibrium and pooling equilibrium where neither type sells in advance. Below we prove by considering the following four cases:

• i) $T \le T_1$: By Theorem 3 (i), there does not exist a separating equilibrium. Furthermore, by Theorem 1, neither type sells in advance under full information. Hence, under asymmetric information, neither type would deviate from selling only in spot.

• ii) and iii) $T \in (T_1, N_1 + N_2)$: First note that neither type selling in advance cannot be sustained as an equilibrium, since otherwise L type always has an incentive to deviate to sell in advance with his full-information strategy. On the other hand, by Theorem 3 (i) to (ii-b), a separating equilibrium always exists with $S_L^a = S_L^f$ and $S_H^a < S_L^a$. Furthermore, by Theorem 1, both types strictly prefer advance selling to selling only in spot under full-information setting. Thus, $S_H^a = 0$ is always dominated by some $S_H^a > 0$. Hence, by Theorem 1, $S_H^a \in (0, \min(T, N_1))$ and if $T \in (T_1, T_D)$, $S_L^a \in (0, \min(T, N_1))$ and if $T \in [T_D, N_1 + N_2)$, $S_L^a = \min(T, N_1)$.

• iv.a) $T \geq N_1 + N_2$ and $L < \bar{t}$: Similar to the second bullet, only a separating equilibrium exists. By Theorems 1 and 3, $S_L^a = \min(T, N_1)$ and the low type strictly prefers advance selling to selling only in spot. Meanwhile, by the proof of Theorem 3 part (iii-d), $S_H^a \in (0, S_H^f) \subset (0, \min(T, N_1))$.

• iv.b) $T \geq N_1 + N_2$ and $L \geq \bar{t}$: As shown in the proof of Theorem 3 part (iii-d), $S_H^a = 0$. Meanwhile, by Theorem 1, the low type is indifferent between (full or limited) advance selling and selling only in spot. Hence, both kinds of equilibria can be sustained: either a separating equilibrium where H type sells only in spot and L type offers (full or limited) advance selling, or a pooling equilibrium where neither type sells in advance.

Appendix H: Proof of Theorem 6

The proof uses the following three lemmas, where Lemma H.1 shows the seller's optimal no-rationing strategy in the full-information case, Lemmas H.2 and H.3 characterize the separating equilibrium and pooling equilibrium for the no-rationing model, respectively.

Lemma H.1 *When capacity rationing is not allowed and customers in advance are perfectly informed of quality, there exists a function $t^D(T)$ for $T > 0$ such that if $t \leq t^D(T)$, the seller should offer full advance selling; otherwise, the seller should sell only in spot.*

Lemma H.2 *When capacity rationing is not allowed, in any separating equilibrium, L type offers full advance selling at price $p_1^L(\min(T, N_1))$, while H type sells only in spot.*

Lemma H.3 *(i) In the focal pooling equilibrium where both types of sellers sell in advance, the equilibrium price is $p_1^E = qp_{1H}^*(\min(T, N_1)) + (1-q)p_{1L}^*(\min(T, N_1))$. (ii) The focal pooling equilibrium is sustained only if $H \leq t^D(T)$. (iii) When $H \leq t^D(T)$, there exist a threshold $\bar{\delta} \geq 0$ and a function $\bar{q}(\delta) \in [0,1]$ for $\delta > 0$ such that the focal pooling equilibrium is sustained if either $\delta \leq \bar{\delta}$ or $q \geq \bar{q}(\delta)$.*

The proof of Theorem 6 is naturally divided into the following three cases:

• $L > t^D(T)$: By Lemma H.1, neither type sells in advance under full information. Hence, under asymmetric information, neither type would deviate from selling only in spot.

• $L \leq t^D(T) < H$: By Lemma H.1, only L type sells in advance under full information. Hence, under asymmetric information, H type does not have incentive to sell in advance, as the maximum profit he can get from selling in advance is $\pi_H^f(\min(T, N_1))$, which does not exceed $\pi_H^f(0)$ by Lemma H.1. Similarly, L type does not have an incentive to sell only in spot, as $\pi_L^f(\min(T, N_1)) \geq \pi_L^f(0)$ by Lemma H.1.

• $H \leq t^D(T)$: By Lemma H.1, both types sell in advance under full information. Under asymmetric information, both types selling in advance occurs only in a pooling equilibrium (by Lemma H.2) and the focal pooling equilibrium is sustained if either $\delta \leq \bar{\delta}$ or $q \geq \bar{q}(\delta)$ (by Lemma H.3). If, however, $\delta > \bar{\delta}$ and $q < \bar{q}(\delta)$, pooling with L type in advance makes H type worse off compared to selling only in spot. Meanwhile, L type would rather sell in advance alone than selling only in spot (by Lemma H.1). Hence, a separating equilibrium is sustained.

Appendix I: Proof of Theorem 7

We follow three steps to prove the result: (i-a) positive advertising expenditure can be sustained only in a separating equilibrium where both types sell in advance and only high type advertises; (i-b) the separating equilibrium in point (i-a) is sustained if and only if $L < H \leq t^D(T)$ and $\delta \leq \bar{\delta}$; (ii) whenever the separating equilibrium in point (i-a) is sustained, it is pareto dominated by a focal pooling equilibrium where both types sell in advance and neither advertises.

(i-a) First, similarly to the proof of Lemma H.2, it can be shown that no separating equilibrium exists where only high type sells in advance. Likewise, no separating equilibrium exists where both types sell in advance and advertise, or only low type does so. In either case, the low type would be better off deviating to no advertising at all, since advertising can neither increase sales nor improve margin for her. Furthermore, any pooling equilibrium where both types sell and advertise in advance is pareto dominated by the focal pooling equilibrium where both sell in advance and yet neither advertises. Thus, the only possible scenario to sustain a positive advertising spending is a separating equilibrium where both types offer advance selling and yet only high type advertises.

(i-b) A separating equilibrium where both types sell in advance and only high type advertises exists if and only if there exists some $A > 0$ satisfying the following four conditions:

$$p^*_{1H}(\min(T,N_1))\min(T,N_1) + \pi^*_{2H}(\min(T,N_1)) - A \geq \pi^*_{2H}(0)$$
$$p^*_{1L}(\min(T,N_1))\min(T,N_1) + \pi^*_{2L}(\min(T,N_1)) \geq \pi^*_{2L}(0)$$
$$p^*_{1H}(\min(T,N_1))\min(T,N_1) + \pi^*_{2L}(\min(T,N_1)) - A \leq p^*_{1L}(\min(T,N_1))\min(T,N_1) + \pi^*_{2L}(\min(T,N_1))$$
$$p^*_{1H}(\min(T,N_1))\min(T,N_1) + \pi^*_{2H}(\min(T,N_1)) - A \geq p^*_{1L}(\min(T,N_1))\min(T,N_1) + \pi^*_{2H}(\min(T,N_1))$$

where the first two inequalities ensure that both types prefer selling in advance to selling only in spot, and the last two inequalities guarantee that neither type has an incentive to mimic the other type. After simplifying, these conditions are equivalent to

$$p^*_{1H}(\min(T,N_1))\min(T,N_1) + \pi^*_{2H}(\min(T,N_1)) - \pi^*_{2H}(0) \geq A = [p^*_{1H}(\min(T,N_1)) - p^*_{1L}(\min(T,N_1))]\min(T,N_1)$$
$$p^*_{1L}(\min(T,N_1))\min(T,N_1) + \pi^*_{2L}(\min(T,N_1)) \geq \pi^*_{2L}(0)$$

By the proof of Lemma H.3, these conditions are satisfied if and only if $H \leq t^D(T)$ and $\delta \leq \overline{\delta}$.

(ii) From the equilibrium conditions in point (i-b), it is easy to see that the type-t seller's profit in the separating equilibrium equals to $p^*_{1L}(\min(T,N_1))\min(T,N_1) + \pi^*_{2t}(\min(T,N_1))$. However, when $H \leq t^D(T)$ and $\delta \leq \overline{\delta}$, by Lemma H.3, a no-advertising pooling equilibrium also exists, where the type-t seller's equilibrium profit equals to $p^E_1 \min(T,N_1) + \pi^*_{2t}(\min(T,N_1))$. Since $p^E_1 \geq p^*_{1L}(\min(T,N_1))$, the separating equilibrium is pareto dominated by the pooling equilibrium.

Appendix J: Proof of Theorem 8

First note that L type never invests in advertising in any separating equilibrium, where he always follows his full-information strategy. Denote H type's equilibrium strategy by (p^*_1, S^*, A^*). Per definition of a separating equilibrium, (p^*_1, S^*, A^*) is a solution to the following problem:

$$\max_{p_1,S,Q} \pi^{a,AD}_H(p_1,S,Q,1) = p_1 S + \pi^*_{2H}(S) - Q$$
$$\text{subject to} \quad S = 0 \text{ or } S \in (0, \min(T,N_1)] \text{ and } p_1 \in [p^*_{1L}(S), p^*_{1H}(S)], Q \geq 0 \quad (20)$$
$$\pi^{a,AD}_L(p_1,S,Q,1) \leq \pi^{a,AD}_L\left(p^f_{1L}, S^f_L, 0, 0\right) \quad (21)$$
$$\pi^{a,AD}_H(p_1,S,Q,1) \geq \pi^{a,AD}_H\left(p^f_{1L}, S^f_L, 0, 0\right) \quad (22)$$
$$(p_1,S,Q) \neq \left(p^f_{1L}, S^f_L, 0\right) \quad (23)$$

We prove $A^*=0$ by contradiction. Suppose $A^*>0$. To reach a contradiction, it suffices to show that there exists a feasible strategy $(p_1', S', 0)$ which strictly improves H type's profit from what he can get by following strategy (p_1^*, S^*, A^*).

To this end, first note that when $A^*>0$, H type should sell in advance in equilibrium (i.e., $S^*>0$), since otherwise he could not enjoy any benefit from the advertising. Meanwhile, constraint (21) must hold as equality at (p_1^*, S^*, A^*), since otherwise A^* can be decreased by a small amount such that all the constraints are satisfied and H type's total profit is strictly improved. That is,

$$\pi_L^{a,AD}\left(p_1^*, S^*, A^*, 1\right) = \pi_L^{a,AD}\left(p_{1L}^f, S_L^f, 0, 0\right). \tag{24}$$

Subtracting equation (24) from equation (22), we get

$$\pi_H^{a,AD}\left(p_1^*, S^*, A^*, 1\right) - \pi_L^{a,AD}\left(p_1^*, S^*, A^*, 1\right) \geq \pi_H^{a,AD}\left(p_{1L}^f, S_L^f, 0, 0\right) - \pi_L^{a,AD}\left(p_{1L}^f, S_L^f, 0, 0\right),$$

which further implies,

$$\pi_{2H}^*(S^*) - \pi_{2L}^*(S^*) \geq \pi_{2H}^*(S_L^f) - \pi_{2L}^*(S_L^f) \tag{25}$$

By Lemma D.1, equation (25) implies $S_L^f \geq S^* > 0$.

Now, define a function of S for $S \in [0, \min(T, N_1)]$: $M(S) = \pi_L^{a,AD}\left(p_{1H}^*(S), S, 0, 1\right) - \pi_L^{a,AD}\left(p_{1L}^f, S_L^f, 0, 0\right)$. Clearly $M(S)$ is continuous in S. Furthermore, $M(0) \leq 0$ since $\left(p_{1L}^f, S_L^f, 0\right)$ is L type's full-information strategy. Meanwhile, by equations (20) and (24),

$$\begin{aligned}
M(S^*) &= \pi_L^{a,AD}\left(p_{1H}^*(S^*), S^*, 0, 1\right) - \pi_L^{a,AD}\left(p_{1L}^f, S_L^f, 0, 0\right) \\
&\geq \pi_L^{a,AD}\left(p_1^*, S^*, 0, 1\right) - \pi_L^{a,AD}\left(p_{1L}^f, S_L^f, 0, 0\right) \\
&= \pi_L^{a,AD}\left(p_1^*, S^*, A^*, 1\right) + A^* - \pi_L^{a,AD}\left(p_{1L}^f, S_L^f, 0, 0\right)\big) = A^* > 0
\end{aligned}$$

Hence, there exists a $S' \in [0, S^*)$ such that $M(S') = 0$. That is,

$$\pi_L^{a,AD}\left(p_{1H}^*(S'), S', 0, 1\right) = \pi_L^{a,AD}\left(p_{1L}^f, S_L^f, 0, 0\right) = \pi_L^{a,AD}\left(p_1^*, S^*, A^*, 1\right) \tag{26}$$

Since $S' < S^* \leq S_L^f$, it is easy to check that $(p_{1H}^*(S'), S', 0)$ satisfies all constraints. Furthermore, noting $S' < S^*$, by equation (26) and Lemma D.1, we have

$$\begin{aligned}
&\pi_H^{a,AD}\left(p_{1H}^*(S'), S', 0, 1\right) - \pi_H^{a,AD}(p_1^*, S^*, A^*, 1) \\
&= \pi_L^{a,AD}\left(p_{1H}^*(S'), S', 0, 1\right) + \pi_{2H}^*(S') - \pi_{2L}^*(S') - \left[\pi_L^{a,AD}(p_1^*, S^*, A^*, 1) + \pi_{2H}^*(S^*) - \pi_{2L}^*(S^*)\right] \\
&= \pi_{2H}^*(S') - \pi_{2L}^*(S') - \left[\pi_{2H}^*(S^*) - \pi_{2L}^*(S^*)\right] > 0
\end{aligned}$$

That is, compared to (p_1^*, S^*, A^*), $(p_{1H}^*(S'), S', 0)$ strictly improves H type's profit. This is a contradiction with the optimality of (p_1^*, S^*, A^*). Hence, $A^*=0$.