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Implementation of Rational Expectations Equilibrium With Strategic Behavior

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1. Introduction

In economies with uniform or perfect information, Walrasian allocations are implementable by Nash equilibria. Hurwicz [1979] shows that it is possible to design a mechanism for allocating commodity bundles to agents such that, for all classical specifications of preferences, Nash equilibria are allocations that can be supported by competitive prices at the agent's initial endowments. We show that a similar result is not possible for Rational Expectations Equilibrium (REE) allocations.

The implementation problem for economies with uniform information was modeled as a game with complete information, and so the concept of Nash equilibrium was used. However, REE are of interest only when there is differential information. The implementation problem for REE therefore requires a game with incomplete information. An appropriate generalization of the concept of Nash equilibrium to such games is the concept of Bayes Nash equilibrium. We use this concept of equilibrium in our analysis.

We provide a condition on information structures that is necessary and sufficient for REE allocations to be implementable for all classical environments. This condition essentially requires that it be possible to predict any trader's information, given the information of all other traders. If this condition is met, traders who lie about their information can be detected and punished. Thus, REE allocations can be implemented by a mechanism which provides the REE allocation for truth-telling and provides a punishment allocation otherwise. If our information condition is violated, then, for some trader, some lie about his information cannot be detected. So for some preferences, the trader will choose to lie in order to obtain the REE allocation associated with his lie.

For example, suppose that some trader's preferences do not depend on his non-publicaly predictable information, but the preferences of other traders do depend on his information. In this case, as a lie cannot be detected, there is no cost to the trader from misrepresenting his information, and he will do so whenever misrepresentation manipulates the allocation to his advantage. This ability of traders to affect the preferences of others is responsible for the difference between our results and the results for economies without differential information about payoff-relevant events. This ability makes it impossible to design a mechanism to implement REE allocations in some environments.

In market games where there is no private information, but where traders can misrepresent their demands, Roberts and Postlewaite [1976] have shown that the gain from noncompetitive behavior approaches zero as the number of traders increases. We provide a necessary condition for a result of this type to hold in economies with differential information. This condition requires that as the number of traders grows large each trader must become informationally small in the sense that the effect of his information on beliefs declines to zero.

Our results can be interpreted as belonging to the literature on implementation of REE (see Beja [1976], Jordan [1979], [1980], and Kobayashi [1977]). In these papers, however, strategic behavior was not permitted. More recently, Krishina [1981] has considered a model with strategic behavior, but the concept of equilibrium in his model was defined to correspond to REE. A model with a continuum of consumers who receive independent signals about a payoff relevant vector of parameters has been studied by Laffont [1983]. Laffont shows that REE are implementable. This occurs because given the information of all other consumers, no individual consumer's information is valuable. This information

2. The Model

Following Myerson's [1983] terminology, a *Bayesian collective choice problem* is a k-tuple

$$\Gamma = (A, S_1,\ldots,S_n,\mu_1,\ldots,\mu_n,u_1,\ldots,u_n,\omega_1,\ldots,\omega_n) .$$

The set $A \subset R^{\ell n}$ is the set of feasible allocations of the ℓ consumption goods, and $B(A)$ is its Borel sigma-field. The set S_i is trader i's finite signal set. Let $S = \Pi_1^n S_i$, and $S_{-i} = \Pi_{j \neq i} S_j$. Then $\mu_i(t_{-i}|s_i)$ is the probability that trader i assigns to the event that the other traders have observed the joint signal vector $t_{-i} \in S_{-i}$ when he observes signal $s_i \in S_i$. The function $u_i : R_+^{\ell n} \times S \to R$ is trader i's expected utility function, which gives the expected utility of trader i of any allocation $a \in A$ conditional upon the joint signal s. Let U_S denote the space of all n-tuples of utility functions, each defined on $R_+^{\ell n} \times S$, such that each trader i's utility, u_i, satisfies the following conditions:

i) u_i depends only on i's allocation, x_i.

ii) For each $s \in S$; u_i is C^2, strictly concave and strictly monotone in x_i.

iii) For each $s \in S$, the closures in R^ℓ of the level sets of u_i do not intersect the boundary of R_{++}^ℓ.

Endow U_S with the topology of uniform C^2 convergence on compact subsets of R_{++}^ℓ for each $s \in S$. We assume that $u = (u_1,\ldots,u_n) \in U_S$. The vector $\omega_i \in R_{++}^\ell$ is trader i's endowment. Define $\omega = (\omega_i)_1^n$. The set of feasible allocations is thus

$$A = \{(x_1,\ldots,x_n) \in \Pi_1^n R_+^\ell \mid \sum_i (x_i - \omega_i) = 0\} .$$

of full information equilibria. (See Radner [1979], Allen [1981], and Jordan and Radner [1982].)

A *direct mechanism* for Γ is a stochastic kernel

$$m: B(A) \times S \to R ,$$

which is to say

i) $m(X|\cdot)$ *is measurable for all* $X \in B(A)$,

ii) $m(\cdot|s)$ *is a probability distribution for all* $s \in S$.

A direct mechanism is a device for converting signals to allocations. Each player announces a signal to the center. Upon observing the reported joint signal s , the center chooses an allocation from the probability distribution $m(\cdot|s)$. Thus each direct mechanism induces a game of incomplete information. Strategies for player i are conditional distributions $\sigma_i: S_i \times S_i \to R$, where $\sigma_i(t_i|s_i)$ is the probability that player i will announce signal t_i when his true singal is s_i . The payoff to player i of announcing signal t_i when his true signal is s_i and the other players are playing the vector of strategies σ_{-i} is

$$\hat{U}_i(t_i,\sigma_{-i}|s_i) = \sum_{\{s_{-i} \in S_{-i}\}} \sum_{\{t_{-i} \in S_{-i}\}} \int u_i(x|t_{-i},s_i)m(d\,x|t_{-i})$$

$$\sigma_{-i}(t_{-i}|s_{-i})\mu_i(s_{-i}|s_i) .$$

Thus the payoff to player i from playing strategy σ_i , when his true signal is s_i and the other players are playing the vector of strategies σ_{-i} is

$$U_i(\sigma_i,\sigma_{-i}|s_i) = \sum_{\{t_i \in S_i\}} \hat{U}_i(t_i,\sigma_{-i}|s_i)\sigma_i(t_i|s_i) .$$

3. Implementation of REE

In this paper we are concerned with information structures. An environment is a vector of characteristics $c = (\omega,u)$ and a joint distribution of signals μ. We characterize those μ such that the REE social outcome correspondence is truthfully implementable on $D(\mu) = \{(c,v) \in E_S \mid v = \mu\}$. Our goal is to identify those joint distributions for which the REE social outcome correspondence can be truthfully implemented for every environment with that joint distribution and for which the correspondence is not empty.

The set of joint distributions μ on any S such that the REE correspondence F can be truthfully implemented on $D(\mu)$ is small. Essentially, it is those joint distributions which admit no ultimately private information.

Definition. A distribution $\mu \in P(S)$ has only *Publically Predicable Information* (PPI) if for all $s_i \in S_i$, $t_{-i} \in S_{-i}$, $\mu(s_i | t_{-i})$ is either 0 or 1.

This is to say that μ satisfies PPI if and only if the signal observed by any one trader can be identified almost surely if the signals observed by all the other traders are known. Versions of this condition have appeared before in the literature (Harris and Townsend [1981], Postlewaite and Schmeidler [1983]). An environment $e = (\omega,u,\mu)$ is said to satisfy PPI if μ satisfies PPI.

We will demonstrate that the REE correspondence F is truthfully implementable on those environments $e \in E_S$ satisfying PPI constructively by giving a class of mechanisms which truthfully implements F. For what will turn out to be obvious reasons, we call these mechanisms *punishment mechanisms*. Let f be a measurable selection from F. (The existence of a measurable selection from F follows from the standard results on full information economies. See Hildenbrand [1974].)

If PPI is not satisfied, there exists a trader, say trader 1, two signals s_1^1, $s_1^2 \in S_1$, and nonempty set $A_{-1} \subset S_{-1}$ such that $(s_1^j, a_{-1}) \in \operatorname{supp} \mu$ for all $a_{-1} \in A_{-1}$ and $j = 1, 2$. Let $A = \{(s_1^j, a_{-1}) \mid j = 1, 2, a_{-1} \in A_{-1}\}$.

We specify initial endowments as follows: For trader 1,

$$\omega_1 = (\gamma_1, \ldots \gamma_1) .$$

For trader i, $i \neq 1$,

$$\omega_i = (\gamma_2, \ldots \gamma_2) .$$

Preferences are Cobb-Douglas. We have

$$u_i(x,s) = \Pi_j x_j^{\alpha(s)_{ij}}.$$

For trader 1, for all $s_{-1} \in S_{-1}$,

$$\alpha(s)_{11} = \begin{cases} 1 - \varepsilon_1 \text{ if } s_1 = s_1^1 \\ 1 - \varepsilon_2 \text{ if } s_1 = s_1^2 \end{cases}, \text{ and}$$

for $j \neq 1$,

$$\alpha(s)_{ij} = \begin{cases} \varepsilon_1/(\ell-1) & \text{if } s_1 = s_1^1 \\ \varepsilon_2/(\ell-1) & \text{if } s_1 = s_1^2 . \end{cases}$$

For trader $i \neq 1$,

$$\alpha(s)_{i1} = \begin{cases} 1 - \varepsilon_1 & \text{if } s \in A,\ s_1 = s_1^1 \\ \varepsilon_2 & \text{if } s \in A,\ s_1 = s_1^2 \end{cases}, \text{ and}$$

for $j \neq 1$,

$$\alpha(s)_{ij} = \begin{cases} \varepsilon_1/(\ell-1) & \text{if } s \in A,\ s_1 = s_1^1 \\ (1-\varepsilon_2)/(\ell-1) & \text{if } s \in A,\ s_1 = s_1^2 . \end{cases}$$

Finally, for $i \neq 1$, $\alpha(s)_{ij}$ is specified in some arbitrary way for $s \notin A$.

For an open and dense set of $0 < \varepsilon_1, \varepsilon_2 < 1$ and specifications for $s \notin A$, REE's exist and are full information equilibria. (The equilibrium will not be fully revealing, because since the payoff to each trader of any joint signal vector in A depends only on s_1, the a_{-1} will not be revealed. However, the

$\varepsilon_1, \varepsilon_2 > 0$ sufficiently small, the lower bound will exceed γ_1 whenever

$$\gamma_2 > \frac{p\gamma_1}{(n - 1)(1 - p)}.$$

This provides a counter-example, and thus completes the proof. Q.E.D.

The counter-example makes it clear that the impossibility of truthful implementability is not an isolated occurrance. If the space of utilities is provided with the topology of C^2 convergence on compact sets in x for each $s \varepsilon S$ if endowments are topologized with the usual topology on $R_+^{\ell n}$; and if the space of characteristics is then given the product topology, it is easy to see that truthful implementation of REE's will fail for an open set of characteristics in $D(\mu)$.

The counter-example exhibits the peculiar property that much information is not payoff-relevant. But note that for an open and dense set of parameters the REE allocations are also equilibruim allocations for the full information economy. Since utilities are all Cobb-Douglas, the allocations are continious in the parameters. Hence, by sufficiently small perturbations in the parameters one can find counter-examples where all the information is payoff-relevant, and where the REE is fully revealing.

The revelation principle and Theorem 2 together imply the most important result of this paper--that the REE social outcome correspondence is not implementable if the joint distribution of signals μ does not satisfy PPI.

Theorem 3. If μ does not satisfy PPI, then the REE social outcome correspondence is not implementable on $D(\mu)$.

Proof. Suppose that the REE social outcome correspondence F were implementable by some mechanism on $D(\mu)$. Then the revelation principle implies that there exists a direct mechanism which truthfully implements F on $D(\mu)$. But Theorem 2 states that this cannot happen unless μ satisfies PPI. Q.E.D.

Trader i's utility is now represented as $u_i(x_i,\theta)$ for x_i an allocation to i and $\theta \in \Theta$. For each $\theta \in \Theta$, u_i is assumed to satisfy the analogues of conditions (ii) and (iii) of Section 2. Let U_Θ^I denote the space of all I-tuples of such functions. Define A^I, $E'_{S^I} = R_+^{\ell I} \times U_\Theta^I \times P(\Theta \times S^I)$, $E_{S^I} \subset E'_{S^I}$ and the REE social outcome correspondence F^I: $E_{S^I} \times S^I \rightarrow A^I$ in the obvious ways.

A direct mechanism is now a sequence of functions

$$\{m(I,.,)\}_{I=1}^{\infty}$$

such that for each $I \in \{1,\ldots\}$, $m(I,.,.)$ is a stochastic kernel on $B(A^I) \times S^I$. Given a direct mechanism, m, the expected payoff to trader i of announcing signal t_i when he observes signal s_i is

$$U_i^I(t_i,s_i) = \sum_{\{s_{-i}^I \in \underset{j \neq i}{\Pi^I} S_j\}} \int u_i(x_i,\theta)[m(dx_i|I,s_{-i}^I,t_i) \times \mu(\theta|s_{-i}^I,s_i)]\, \hat{\mu}(s_{-i}^I|s_i).$$

The maximal gain to trader i from misrepresentation when he observes s_i is thus

$$G_i^I(s_i) = \underset{t_i \in S_i}{\text{MAX}} [U_i^I(t_i,s_i) - U_i^I(s_i,s_i)].$$

Theorems 2 and 3 can be reinterpreted to say that PPI is a necessary condition for the gains from misrepresentation to be non-positive under any mechanism that implements the REE social outcome correspondence. In this section we identify a necessary condition for the gains from misrepresentation to converge to zero as the economy grows large. In market games where there is no private information, but where traders can misrepresent their demands, the gain to any trader from misrepresentation approaches zero as the number of traders increases (see Roberts and

$$u_i(x_i, \theta) = \sum_{j=1}^{\ell} \alpha_{ij}(\theta)\ \ell n(x_{ij}), \text{ for } i = 2,\ldots$$

and

$$u_1(x_1, \theta) = \sum_{j=1}^{\ell} \alpha_{ij}(\theta)\ \ell n\ (1 + x_{1j}).$$

Suppose for all s_{-1} such that (s_1^1, s_{-1}), $(s_1^2, s_{-1}) \in \text{supp}\ \hat{\mu}$: (1)

For trader 1,

$$E[\alpha_{11}(\theta)|s_1,s_{-1}] = \begin{bmatrix} 1 - \varepsilon_1 & \text{if } s_1 = s_1^1 \\ 1 - \varepsilon_2 & \text{if } s_1 = s_1^2, \text{ and} \end{bmatrix}$$

for $j \neq 1$,

$$E[\alpha_{ij}(\theta)|s_1,s_{-1}] = \begin{bmatrix} \varepsilon_1/(\ell-1) & \text{if } s_1 = s_1^1 \\ \varepsilon_2/(\ell-1) & \text{if } s_1 = s_1^2, \end{bmatrix}$$

where $0 < \varepsilon_1, \varepsilon_2 < 1$ and $\varepsilon_1 \neq \varepsilon_2$.

For all traders $i \neq 1$,

$$E[\alpha_{i1}(\theta)|s_1,s_{-1}] = \begin{bmatrix} 1 - \varepsilon_1 & \text{if } s_1 = s_1^1 \\ \varepsilon_2 & \text{if } s_1 = s_1^2, \text{ and} \end{bmatrix}$$

for $j \neq 1$,

$$E[\alpha_{ij}(\theta)|s_1,s_{-1}] = \begin{bmatrix} \varepsilon_1/(\ell-1) & \text{if } s_1 = s_1^1 \\ (1-\varepsilon_2)/(\ell-1) & \text{if } s_1 = s_1^2. \end{bmatrix}$$

Simple calculations give the following equilibrium consumptions for trader 1. If $s_1 = s_1^1$,

$$x_{1j}(s_1^1) = 1 \text{ for all } j.$$

If all $s_1 = s_1^2$,

$$x_{11}(s_1^2) = \frac{(1-\varepsilon_2)[1 + \gamma(I-1)]}{(1-\varepsilon_2) + (I-1)\gamma\ \varepsilon_2}$$

$$x_{1j}(s_1^2) = \frac{\varepsilon_2[1 + \gamma(I-1)]}{\varepsilon_2 + (I-1)\gamma(1 - \varepsilon_2)}, \text{ for } j =/ 1.$$

5. Conclusions

We view the implementation question for REE's as a test of the reasonableness of the behavorial assumptions and the equilibrium concept underlying the REE social outcome correspondence. If REE's cannot be implemented, or if necessary conditions for implmentation are very restrictive, then the ability of models based on the REE concept to predict behavior and equilibrium outcomes should be questioned. Alternatively, if sufficient conditions for implementation are discovered then models based on the REE concept may provide useful predictions. A positive answer to the implementation question yeilds only a potential for useful predictions, as what is shown is that there exists some artificial, intermediate mechanism that implements REE's. It does not imply that market institutions actually do yield REE's.

As a result of the stringency of our necessary conditions for implementation we view our results as being negative. Our conditions of publically predictable information, or informational smallness, may be acceptable in markets where traders exogenously receive information, but they seem unlikely to be satisfied if the choice of information is made endogenous. Results on information choice in two different market games can be found in Dubey, Geanakoplos and Shubick [1982] and Blume and Easley [1983].

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