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On the Game Theoretic Foundations of Market Equilibrium with Asymmetric Information

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The assumption that consumers and producers are "price takers" is crucial for the perfectly competitive model. This assumption has been justified in markets with many small, identically informed traders by studying the limit equilibria of large Cournot-Nash market games. In this paper we take a first step to an analysis of competitive behavior in markets with differentially informed traders.

We study a two period Cournot model in which traders are uncertain about production costs. Some traders have observed an informative signal while others have not. Information revealed by equilibrium prices in the first stage is payoff-relevant for the second period decision problem. We identify various perfect Bayes-Nash equilibria for this two-period model when the number of traders is large. Revealing equilibria are always possible. When the number of informed traders is small, non-revealing equilibria can also occur.

In the last section of the paper we let traders choose whether or not to acquire information at some cost. As the number of traders grows large, the equilibrium number of informed traders remains small. When the information acquisition decision is endogenous, non-revealing equilibria may occur.

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1. Introduction

The assumption that consumers and producers are "price-takers" is crucial for the perfectly competitive model. This assumption has been justified in markets with many small, symmetrically informed traders by studying the limit equilibria of large Cournot-Nash market games. (See the 1980 JET symposium.) A similar analysis for markets with asymmetrically informed traders is needed in order to examine the sensibility of potential market equilibrium concepts and to discover how privately held information is used in market settings. In this paper we take a first step to an analysis of competitive behavior in markets with asymmetrically informed traders.

Our study is partly motivated by some of the issues surrounding rational expectations equilibria (REE's); for a more complete discussion of these issues see Jordan and Radner [1982]. Much REE analysis begs the question of how traders' private information finds its way into equilibrium prices. This is precisely the sort of question upon which a game-theoretic analysis sheds light. A second question arising from REE's has to do with the incentives for traders to seek out and acquire possibly costly information. We discuss this problem as well.

Several authors have previously studied Cournot-Nash equilibrium in market games with incomplete and asymmetric information (Dubey, Geanakoplos and Shubik [1982], Gal-Or [1982], Kihlstrom and Postlewaite [1983], Novshek and Sonnenschein [1982], and Palfrey [1982]). There are two serious limitations to the analyses presented in these papers. First, several of the models are single period. Information revealed by informed traders through current actions cannot be taken advantage of by uninformed traders (in Novshek and Sonnenschein, and Palfrey), so some of the incentives for strategic use of information are missing. Traders do not observe equilibrium prices until *after* they have put their output on the market, so the information contained in the equilibrium price can have no effect on the output decisions. There are two ways around this difficulty. Strategies can be viewed as functions from outcomes of the game to individual actions. The game already has a function which aggregates individual actions into outcomes, but for this approach a mechanism must be specified to select fixed points from the composite map. For example, every player in a market game could hand in excess demand functions, but a mechanism must be specified for selecting a particular market clearing price. (Kihlstrom and Postlewaite take this approach, but their's is a model of an informed monopolist, and therefore not suited to the study of large markets.) Alternatively, one can study repeated games rather than single stage games. Information revealed in the first stage can have an impact on second period profits. This approach is consistent with the sequential approach to REE's taken by Hellwig [1982] and Blume and Easley [1983]. In this paper we study a two stage game. A subsequent paper will present some examples of the first type.

Second, all of these papers have informed traders committing themselves to a strategy–a statement of how they will act given what they have observed–before the observation is made. Traders act so as to maximize ex ante expected utility. This is not a problem in the aforementioned papers where there are no incentives to masquerade, but when the information revealed by a trader's actions has payoff relevant consequences for him a moral hazard problem appears. We do not allow precommitment, and we model the moral hazard problem by searching for subgame perfect equilibria.

Our model features linear demand curves and constant but unknown marginal cost. We look for subgame perfect Bayes Nash equilibrium in the multistage game. We find that the single stage equilibrium strategies are also equilibrium strategies in the multistage game, and these strategies reveal the informed traders' information. However, other equilibria, in which information is not revealed, also exist. In order to make these equilibria disappear, informed traders must be small not just in terms of resources, but also in terms of information. If the information available to one trader is also shared by other traders, the gains from using that information strategically shrink. We also study an example wherein signals can be purchased by players, and we see how the number of informed players and equilibrium prices vary with such parameters of the game as the cost of information and the information content of the signal.

This paper consists of six sections. In the next section our two stage market model is described. In section 3 we consider equilibria with many uninformed firms but few informed firms. In section 4 we study equilibria that arise with large numbers of both informed and uninformed firms. Endogenous acquisition of information is treated in section 5. Conclusions are drawn in section 6.

2. The Model

We model the interaction between imperfectly competitive and asymmetrically informed firms as a repeated incomplete information game. To demonstrate the phenomena we are looking for, it suffices to consider two periods, which are indexed by the subscript t. Firms are indexed by $i \in \{1, ..., I\}$. In each period firms move simultaneously. A move by firm i in period t is a quantity of output $q_{it} \geq 0$. Let $Q_t = \Sigma_i q_{it}$. The price that each firm receives for its output is

$$p_t = p(Q_t) = a_t - Q_t.$$

All firms have the same constant marginal cost of production. This cost is unknown, but it can take on only one of the two values θ_L^* and θ_H^*. We assume

$$\theta_H^* > \theta_L^*.$$

(Our results are qualitatively similar to those obtainable in a model with uncertainty over the intercept term of the demand curve.) The prior probability that each firm places on θ_L^* is $\delta > 0$. Previous to first stage play, firms 1 through n observe in common a signal correlated with θ^*. These firms are the *informed firms*. The remaining $m = I - n$ *uninformed firms* receive no signal. The signal s is drawn from the set $\{s_L, s_H\}$. The conditional probability of observing signal s_L given true cost θ_L^* equals that of s_H given θ_H^*, and is denoted by β, which is greater than $1/2$. The prior probability of observing signal s_L can be computed from δ and β, and is denoted by γ_1. Thus

$$\gamma_1 = \beta\delta + (1-\beta)(1-\delta).$$

Again we emphasize that all informed traders observe the same value of s.

Uninformed firms use their observations of first period price to make an inference about the signal received by the informed. (Price contains no information for the informed firms.) We assume that during the play of the game no firm receives any information about costs other than the pre-play signal s for the informed firms, and first period price for the uninformed. In particular, we assume that the first period production experience generates no information about marginal cost which can be used before the second period production decision must be made. Information is generated only with a lag.

The objective of each firm is to select a strategy that will maximize its total expected profits summed over both stages of the game. The different types of firms have different strategy sets. For uninformed firms, a strategy is a pair

$$\pi_i = (\pi_{i1}, \pi_{i2}(\cdot)),$$

where the first element is a quantity to play in stage 1, and the second element is a Borel-measurable function from prices to quantities produced in the second stage. (In this paper we will study only pure strategy equilibria, and so we need not worry about the definition of mixed strategies.) For informed firms, a strategy is a pair of Borel-measurable functions:

$$\pi_i = (\pi_{i1}(\quad), \pi_{i2}(\quad))$$

from the set of signals to quantities for the first stage, and from signals cross first stage prices to quantities in the second stage.

Our analysis will be restricted to symmetric equilibria–wherein all firms of the same type have the same strategy. Thus we talk about the uninformed firms' strategy π_U and the informed firms' strategy π_I. Denote by S_I and S_U the strategy sets of the informed and uninformed firms, respectively.

The game of incomplete information is not yet specified. Uninformed firms will condition their beliefs about second stage play on the outcome of first stage play. The inference structure for this learning must be specified. The specification of subjective beliefs of the uninformed firms conditioned on the first stage price they observe is crucial for the determination of equilibrium. (We will look for subgame perfect Bayes Nash equilibria.) In a pure strategy equilibrium there will be two (possibly identical) prices, one associated with each signal. The specification of beliefs given the observation of these prices must be determined in accordance with Bayes rule, but since prices other than these two occur with probability zero, Bayes rule leaves indeterminate the specification of beliefs conditional on these observations. But specification of beliefs on this set determine the equilibrium. When an informed firm considers deviating from the proposed equilibrium strategy in the first stage, he knows that his deviation will change prices. The profit he will make from this deviation depends upon how the uninformed firms will react in the second stage, and their reaction is determined by the beliefs they will carry forward to the second stage due to the newly observed price. Thus, to achieve equilibrium, not only must expectations be statistically correct on the set of possibly observable equilibrium prices, but they must be specified off of this set of prices in such a way that no informed firm has any incentive to change the equilibrium prices. Expectations are given by a function:

$$\gamma_2 : \mathbf{R}_+ \rightarrow [0,1],$$

where $\gamma_2(p)$ is the conditional probability attached by uninformed firms to the event that the signal was s_L.

Now we can formally define an equilibrium for this game. To do this we need some additional notation. Define:

$$\pi_{-It} = (n-1)\pi_{It} + m\pi_{Ut},$$

$$\pi_{-Ut} = n\pi_{It} + (m-1)\pi_{Ut},$$

$$p_{I1}(s_j; \pi_I') = p(\pi_{-I1}(s_j) + \pi_{I1}'(s_j)),$$

$$p_{I2}(s_j, p_1; \pi_I') = p(\pi_{-I2}(s_j, p_1) + \pi_{I2}'(s_j, p_1)),$$

$$p_{U1}(s_j; \pi_U') = p(\pi_{U1}(s_j) + \pi_{U1}'),$$

$$p_{U2}(s_j, p_1; \pi_U') = p(\pi_{-U2}(s_j, p_1) + \pi_{U2}'(p_1)),$$

$$p_1(s_j) = p(\pi_{I1}(s_j) + \pi_{U1}).$$

Definition 1. A triple (π_I, π_U, γ_2) is a symmetric sequential equilibrium for the two stage game if:

1. Profit maximization for the informed: For $j = L, H$; π_I maximizes over all $\pi_I' \in S_I$ the expression

$$E\{(p_{I1}(s_j; \pi_I') - \theta)\pi_{I1}'(s_j) : s_j\} + E\{(p_{I2}(s_j, p_{I1}(s_j; \pi') : \pi') - \theta)\pi_{I2}'(s_j) : s_j\}.$$

2. Profit maximization for the uninformed: π_U maximizes over all $\pi_U' \in S_U$ the expression

$$E\{(p_{U1}(s; \pi_U') - \theta)\pi_{U1}'\} + E_\gamma\{(p_{U2}(s, p_{U1}(s; \pi_U') : \pi_U') - \theta)\pi_{U2}'(p_1) : p_1\}.$$

3. Consistency of expectations with Bayes rule at equilibrium:

$$\gamma_2(p_1(s_L)) = 1 \quad \textit{if} \quad p_1(s_L) \neq p_1(s_H),$$

$$\gamma_2(p_1(s_L)) = \gamma_1 \quad \textit{if} \quad p_1(s_L) = p_1(s_H),$$

and

$$\gamma_2(p_1(s_H)) = 0 \quad \textit{if} \quad p_1(s_L) \neq p_1(s_H),$$

$$\gamma_2(p_1(s_H)) = 1 - \gamma_1 \quad \textit{if} \quad p_1(s_L) = p_1(s_H).$$

There are two things to remark about this definition of equilibrium. First, conditions 1 and 2 imply that given all relevant beliefs at the end of the first stage, second stage plays are chosen to maximize second stage profits. Condition 3 requires that expectations at the beginning of stage two be consistent with Bayes rule and first stage play. Thus this equilibrium is a sequential equilibrium. Second, within this game is a game played by informed firms against themselves. Each informed firm plays a game between his two possible types. It will become apparent that informed firms, when they observe a "low" signal, would like uninformed firms to think they have observed a "high" signal, while when they observe a "high" signal, they want this fact known. The tradeoff between the costs and benefits for "low" observers in covering up their signal is one of our main interests in the remainder of the paper. Will equilibrium strategies reveal the informed firms' signal to the uninformed, or can "low" observers successfully masquerade as "high" types? Note that this question never arises in one stage models, because all informed firms' costs of information revelation arise from its use by uninformed firms in the second stage.

Our benchmark strategies are that which would occur in equilibrium if, in the first stage, informed firms neglected to include the effect on second stage profits of first stage information revelation. We call these the ***single stage nash equilibrium*** (SSN) strategies. These would be equilibrium strategies if all firms were informed ($m = 0$), since then no additional information would be revealed to any firm during play of the game. We will investigate the robustness of this equilibrium to the presence in the game of uninformed traders.

In order to define the SSN strategies we need some additional notation. Define:

$$\theta_j = E\{\theta : s_j\} \quad for \quad j = L, H; \quad and$$
$$\theta_t = \gamma_t\theta_L + (1-\gamma_t)\theta_H \quad for \quad t = 1, 2;$$

The informed firms' expectation of θ given their information s_j is θ_j. The uninformed firms' expectation of θ given their beliefs (γ_t) is θ_t.

The SSN strategies depend on s and γ_t for the informed, and on γ_t for the uninformed. The SSN strategies, denoted by ${}_s\pi_I$ and ${}_s\pi_U$, are:

1. If $(n+1)a + m\theta_t - (m+n+1)\theta_H > 0$:

$$ {}_s\pi_I(s_j, \gamma_t) = \frac{(m+n+1)(a-\theta_j) - m(a-\theta_t)}{(n+1)(m+n+1)},$$
$$ {}_s\pi_U(\gamma_t) = \frac{a-\theta_t}{m+n+1}.$$

2. If $(n+1)a + m\theta_t - (m+n+1)\theta_H \leq 0$:

$$ {}_s\pi_I(s_L, \gamma_t) = \frac{a-\theta_L + m(1-\gamma_t)(\theta_H-\theta_L)}{(m+1)(n+1) - mn\gamma_t},$$
$$ {}_s\pi_I(s_H, \gamma_t) = 0,$$
$$ {}_s\pi_U(\gamma_t) = \frac{(n+1)(a-\theta_t) - n\gamma_t(a-\theta_L)}{(m+1)(n+1) - mn\gamma_t}.$$

The strategies are divided into these two cases according to whether or not the non-negativity constraint is binding for informed firms when they observe a "high" signal (indicative of high costs).

3. Large Numbers of Uninformed Firms

We begin our analysis of the model presented in section 2 by looking for equilibria in the case where $m = \infty$ and $n < \infty$. We show that the SSN strategies give an equilibrium in which first stage play reveals the informed firms' signal. However, we also find some non-revealing equilibria.

Since $\theta_L < \theta_H$, the relevant SSN strategies in this case are those in case 2. The prices that would obtain in the first period as a result of SSN play are:

$$p(s_H) = p_H = \frac{\theta_1 + n(1-\gamma_1)\theta_H}{n(1-\gamma_1)+1} \qquad \textit{if} \quad s = s_H, \quad \textit{and}$$

$$p(s_L) = p_L = \frac{\theta_1 + n(1-\gamma_1)\theta_L}{n(1-\gamma_1)+1} \qquad \textit{if} \quad s = s_L.$$

Note that $p_L < p_H$, and so the signal will be revealed by the first stage equilibrium price.

Our first result is that in the limit economy with large numbers of uninformed, the SSN strategies are part of an equilibrium.

Theorem 1. In the market with $m = \infty$ and $\infty > n > 1$, there are expectations γ_2 such that $({}_s\pi_I, {}_s\pi_U, \gamma_2)$ is an equilibrium.

Proof. Choose $p' \in (p_H - (n-1){}_s\pi_I(s_L, \gamma_1), p_H]$. Note that $p' > p_L$. Let

$$\gamma_2(p) = 1 \qquad \textit{if} \quad p < p'$$

$$\gamma_2(p) = 0 \qquad \textit{if} \quad p \geq p'.$$

It is clear that any uninformed firm's best response to $({}_s\pi_I, {}_s\pi_U, \gamma_2)$ is ${}_s\pi_U$ and that any informed firm's best second stage response is ${}_s\pi_{I2}$. Thus we need only show that ${}_s\pi_{I1}$ is the best first stage response for informed firms for both s_L and s_H.

Note that any deviation from ${}_s\pi_{I1}$ that does not change the value of γ_2 must, from the construction of SSN strategies, lower profits. Changes in the value of γ_2 affect only second stage profits. A simple calculation shows that only reductions of γ_2 increase second stage profits for informed firms. Thus deviations from SSN strategies in stage one are profitable for informed firms only if they result in lower values for γ_2.

Suppose $s = s_H$. Then $\gamma_2(p_H) = 0$, so no deviations from SSN strategies can lower γ_2. Suppose $s = s_L$. Then $\gamma_2 = 1$, and an informed firm can affect γ_2 only if it can select a quantity q such that, given ${}_*\pi_{-I1}(s_L, \gamma_1), p_1 > p'$. For $n > 1$, p' is constructed to exceed $p(0 +_* \pi_{-I1}(s_L, \gamma_1))$. So it is impossible for informed firms to raise price sufficiently to affect the value of γ_2. Hence ${}_*\pi_{I1}$ is a best response to SSN strategies when expectations are given by γ_2. Note that with SSN strategies γ_2 is correct. Thus ths SSN strategies together with γ_2 is a Nash equilibrium of the two stage game.[]

Let $p(s_j)$ be a subgame-perfect Bayes Nash equilibrium price function for stage 1. We say that an equilibrium is *revealing* if $\gamma_2(p(s_L)) = 1$ and $\gamma_2(p(s_H)) = 0$. If these two probabilities are equal, equilibrium price carries no information about signals and we call the equilibrium *non-revealing*. It is not hard to show that in this model equilibria will be either revealing or non-revealing. Partial revelation requires mixed strategies, and so it does not occur in pure strategy equilibria. For example, a SSN equilibrium is revealing. But if in any equilibrium prices were such that $p(s_L) = p(s_H)$, then the equilibrium would be non-revealing. For certain values of n and γ_1 (equivalently n, β and δ) non-revealing equilibria exist.

Theorem 2. In the market with $m = \infty$ and $n < \infty$, there are non-revealing equilibria if and only if $1/n \geq 1 - \gamma_1$. In these equilibria, for all j, $p(s_j) = \theta_1$.

Proof. Consider the following strategies:

$$\pi_{I1}(s_j, \gamma_1) = 0 \quad for \quad all \quad j,$$

$$m\pi_{U1}(\gamma_1) = a - \theta_1,$$

$$\pi_{I2} =_* \pi_{I2},$$

$$\pi_{U2} =_* \pi_{U2}.$$

Let

$$\gamma_2(p) = \gamma_1 \qquad if \quad p = \theta_1,$$

$$\gamma_2(p) = 1 \qquad if \quad p \neq \theta_1.$$

Simple calculations show that the strategies of the uninformed and the second stage strategies of the informed firms are best responses given these strategies and expectations. With $m = \infty$ an informed firm which observes $s = s_H$ finds $p_{I1}(s_H; 0) < \theta_H$, so $\pi_{I1}(s_H, \gamma_1) = 0$. Thus we need only consider $\pi_{I1}(s_L, \gamma_1)$.

Given the strategies above, the first stage profit expected by an informed firm which observes $s = s_L$ is 0, and the second stage profit is:

$$\frac{(1-\gamma_1)^2(\theta_H - \theta_L)^2}{[n(1-\gamma_1) - 1]^2}$$

If the informed firm in question deviates from $\pi_{I1} = 0$ in stage one, the uninformed firms will observe $p_1 \neq \theta_1$, and so γ_2 will be 1.

At $\gamma_2 = 1$ a simple calculation shows that each informed firm's stage 2 profits are 0. Thus if an informed firm deviates in stage one, it will choose the deviation so as to maximize first stage profits alone. The maximal profits achievable for an informed firm contemplating deviation are:

$$\frac{(1-\gamma_1)^2(\theta_H - \theta_L)^2}{4}.$$

Thus the strategies given above are equilibrium strategies (given γ_2) if and only if:

$$\frac{(1-\gamma_1)^2(\theta_H - \theta_L)^2}{[n(1-\gamma_1)+1]^2} \geq (1-\gamma_1)^2(\theta_H - \theta_L)^2/4.$$

This condition is equivalent to $1/n \geq 1 - \gamma_1$.

It is easy to show that if $1/n < 1-\gamma_1$ then non-revealing equilibria are not possible. This can be seen by observing that $\pi_{I1}(s_H, \gamma_1) = 0$ and that γ_2 of the construction just given was chosen to make deviations from $\pi_{I1}(s_L, \gamma_1) = 0$ as unprofitable as possible. Since deviations are profitable for this γ_2 when n is sufficiently large, they will also be profitable for any other γ_2 which supports a non-revealing equilibrium for small n. []

Note that if $n = 1$ the condition $1/n \geq 1 - \gamma_1$ in theorem 2 is met for all γ_1. Thus if there is only one informed firm among a large number of uninformed firms, there is always a non-revealing equilibrium.

The existence condition for non-revealing equilibria in theorem 2 is a joint condition on on priors δ, the informativeness β of the signal, and the number n of informed firms. When $\delta \leq 1/2$ the condition can be satisfied only if $n \leq 2$. When $\delta > 1/2$ the condition for the existence of non-revealing equilibria requires $\beta \geq (\delta - 1/n)/(2\delta - 1)$. For $n = 2$ this condition is always satisfied when $\delta > 1/2$. (Recall that $\beta \geq 1/2$.) For $n > 2$ the set of (δ, β) pairs satisfying this condition is shown in figure 1. For $n > 2$ non-revealing equilibria will exist only if the informativeness β of the signal is large relative to $1 - 1/n$. Alternatively, for equilibrium to be guaranteed revealing the signal must be sufficiently uninformative. As n gets large, the upper bound on the informativeness of the signal required to rule out non-revealing equilibria grows larger. Thus there are two ways to rule out non-revealing equilibria. The signal may be uninformative, in which case the effects of information revelation are small, or there may be a large number of informed firms, in which case the rents from being informed are small. We call informed firms in either one of these circumstances "informationally small."

It is important to note that in our example, in the case where both SSN (revealing) and non-revealing equilibria exist, the ex-ante expected profits to each player before the signal is observed are identical. This is merely an artifact of our specification, but we point it out to show that there is no principle which will allow us to discard the non-revealing equilibrium in favor of the SSN equilibrium. In particular, the two equilibria are ex ante Pareto indifferent.

4. Large Numbers of Informed and Uninformed Firms

In this section we characterize the equilibria that result with large numbers of both types of firms. Imagine the number of market participants getting larger, with both m and n going to ∞. In this section we study the limit equilibria of the economy. The results of this section can be justified as limits of equilibria in the case where both m and n are finite. In the finite case there are additional equilibria not present in the limit, but these equilibria converge to the equilibria we describe here.

The analysis of the limit economy still depends upon the relative sizes of the informed and uninformed groups of traders. We think of the economy growing in such a way that n/m, the ratio of informed firms to uninformed firms, converges to a number $K > 0$. In this section we show that the potential for strategic behavior with respect to information revelation disappears, and all equilibria are revealing. The details of the analysis, however, depend upon the magnitude of K.

Theorem 3. In the limit market where n and m converge to ∞ such that n/m converges to $K > 0$:

i) There are expectations γ_2 such that $({}_{*}\pi_I, {}_{*}\pi_U, \gamma_2)$ is an equilibrium,

ii) There are no non-revealing equilibria.

Proof. Let

$$\gamma_2(p) = 1 \qquad \text{if} \quad p < p_H,$$
$$\gamma_2(p) = 0 \qquad \text{if} \quad p \geq p_H.$$

We show that with this γ_2, the SSN strategies form an equilibrium. Suppose first that

$$K > \gamma_1(\theta_H - \theta_L)/(\beta - \theta_H).$$

By an argument similar to that in the proof of theorem 1, we know that for an informed firm observing s_H, ${}_{*}\pi_I$ gives a best response to the SSN strategies since any deviation reduces first stage profits without lowering γ_2. In the case when the informed firms observe s_L, any deviation from ${}_{*}\pi_{I1}$ that does not result in $p(s_L) \geq p_H$ reduces first stage profits without affecting γ_2. In order to have $p(s_L) \geq p_H$ an informed firm must produce no more than $\pi' = {}_{*}\pi_{I1}(s_L, \gamma_1) - (p_H - p_L)$. In the limit, $\pi' = \theta_L - \theta_H < 0$, which is not feasible. Hence ${}_{*}\pi_{I1}$ is a best response to SSN strategies given γ_2. It is easy to see that all the other components of all the other strategies are best responses to SSN play. In this case γ_2 is correct, proving that $(,\pi_I,, \pi_U, \gamma_2)$ is an equilibrium.

The analysis for K small is essentially the same. If firms observe s_H they cannot profit from any deviation, and if they observe s_L the necessary deviation is not feasible.

We next show that, in the limit, there are no non-revealing equilibria. Note that in the second stage all firms play SSN strategies and so, regardless of γ_2, $p_2(s_H) = \theta_H$ and $p_2(s_L) = \theta_L$. Thus in the second stage all informed firms expect zero profits regardless of γ_2. Suppose there is a non-revealing equilibrium. Then there is a price $p^* = p_1(s_H) = p_1(s_L)$, and so p^* is not equal to at least one of the θ_j. Suppose that $p^* > \theta_L$. Calculation shows that *each* informed firm will produce the amount $p^* - \theta_L > 0$. But then total output is infinite, which leads to a contradiction. Hence $p^* \leq \theta_L$. This implies that $p^* < \theta_H$, so $\pi_{I1}(s_H) = 0$. With only the uninformed firms producing, we have $p(s_H) = \theta_1$, so $p^* > \theta_L$, a contradiction. []

5. Endogenous Information Acquisition

In this section we ask what happens when being informed is an endogenous decision. We suppose that, before the commodity market opens, there is an opportunity for information acquisition. We demonstrate in this section that endogenous information acquisition yields only a finite number of informed firms, and can lead to non-revealing equilibria.

Assume that the number of firms is is countable. Firm i has a cost, c_i, of acquiring information. Firms are ordered by their cost, with $i < j$ implying that $c_i < c_j$. Assume that:

$$inf\{c_i\} = c' > 0.$$

We restrict attention to pure strategy equilibria. With pure strategies for information acquisition, each firm will know at the end of the acquisition stage how many firms have chosen to become informed. This information available at the beginning of the second stage, together with the usual common knowledge assumptions about priors of uninformed firms and the demand function, is sufficient to determine equilibrium payoffs in the subsequent stages. Once firms have made their acquisition decision, so that the number of informed firms, n, is known, the analysis of the subsequent stages is exactly that given in section 3 if n is finite, and section 4 if n is infinite.

Our purpose is to illustrate the compatibility of non-revealing equilibria with endogenous information acquisition. Thus we assume that if n is small enough so that the non-revealing equilibrium exists, then the non revealing equilibrium will prevail in the market subgame. Otherwise, the SSN equilibrium, which always exists, will prevail. With this convention, define $V_I(n)$ to be the profits from the market subgame that accrue to any informed firm if there are n informed players. Let $V_U(n)$

denote the profits from the market subgame that accrue to any uninformed firm when there are n informed firms.

Consider firm i, and suppose that n firms other than firm i are acquiring information. The value to firm i of remaining uninformed is $V_U(n)$. The value to firm i of becoming informed is the profits he will make with $n+1$ informed firms less his cost of becoming informed. This is $V_I(n+1)-c_i$. Clearly a firm will choose to become informed only if the return from becoming informed exceeds the return from remaining uninformed. With this in mind it is easy to compute an equilibrium. Let n^* denote the number of informed firms in a pure strategy equilibrium. For acquirers of information, it must be the case that:

$$V_I(n^*) - c_i > V_U(n^* - 1).$$

For firms who choose to remain uninformed, it must be that:

$$V_I(n^* + 1) - c_i \leq V_U(n^*).$$

First observe that there can be no equilibrium with infinite n^*. This follows because calculations show that as n gets large, $V_I(n)$ converges from above to 0. Thus for n sufficiently large, $V_I(n)-c' < 0$, and so no firm i can satisfy the acquirer condition.

Since n^* is finite, the equilibrium conditions simplify. In any equilibrium there will be an infinite number of uninformed firms, and so $V_U(n^*) = 0$. Thus firm i will choose to acquire information if and only if the value of information exceeds the cost of becoming informed. For informed firms, we thus have $V_I(n^*) > c_i$. For uninformed firms we get $V_I(n^* + 1) < c_i$.

The natural equilibria to look for are those where the n^* lowest cost firms purchase information. We call these equilibria *monotonic equilibria*. They exist, but unfortunately there are others as well. To see this, note that it is possible to have

$$V_I(n) > c_{i+1} > c_i > V_I(n+1),$$

so it is possible to have firm i−1 acquire information while firm i remains uninformed. However, we will study equilibria where only those firms with the lowest cost of acquisition choose to become informed.

To see that monotonic equilibria exist, note that $V_I(n)$ is decreasing in n, while c_i increases. Let n^* be the first n such that $V_I(n) > c_n$ and $V_I(n+1) \leq c_{n+1}$. If no such n exists, let $n^* = 0$. Let all firms i with $i \leq n$ become informed, while all other firms remain uninformed. It follows form the monotonicity of the c_i and the $V_I(n)$ schedules that the aquisition condition is satisfied for all i less that or equal to n^*, and the condition for remaining uninformed is satisfied for all other firms. Thus this is an equilibrium.

Some simple comparative statics results for monotonic equilibrium are easy to obtain. First, as $\theta_H^* - \theta_L^*$ increases, so does n^*. Second, perform the experiment of increasing the informativeness of the signal while changing the prior beliefs on the true θ's in such a way that the prior beliefs about observing a low signal, γ_1, remain the same. Then n^* increases. Finally, increase each firm's cost of acquiring information. Then n^* decreases. In fact, if $c' \geq V_I(1)$, then $n^* = 0$. The first two results can be obtained from looking at the $V_I(n)$ function, and the last result follows from the monotonicity of the V_I and c_i schedules.

The first result is intuitively clear. The more critical the cost difference, the more firms are willing to pay to become informed. Thus, the more critical the cost difference, the more firms will choose to become informed. The second experiment changes the informativeness of the signal without changing the posterior probability distribution of observations. It provides a way of measuring a "pure informativeness effect."[7] Increases in informativeness also induce more firms to become informed–the value of the signal is higher. Finally, when information acquisition becomes more expensive, fewer firms will choose to become informed, which again is what intuition would suggest. The last result is important. Fix all the parameters for the market subgame. Then just by varying the acquisition cost structure, the number of informed firms can be varied from zero to many. Since the nature of market equilibrium for a given number of firms n–whether it is revealing or non-revealing–is independent of the acquisition cost structure, this shows that by varying the c_i both revealing and non-revealing equilibria can be obtained.

6. Summary and Conclusions

In this paper we have studied equilibria of a market game with incomplete information. Market outcomes are affected by players desires to reveal or cover up payoff-relevant information which can be utilized by uninformed players in a subsequent market period. We examined limit economies with an infinite number of uninformed players and found that both revealing and non-revealing equilibria are possible. Non-revealing equilibria can be ruled out in two circumstances: when the number of informed firms is sufficiently large that the rents to the informational advantage are competed away, and when the informativeness of the signal is sufficiently low that the returns from information are small.

We have examined only limit economies in this paper, but we have also studied finite economies sufficiently to know that the results of section three and four can be justified as asymptotic results. In finite economies there are revealing equilibria other than the SSN equilibria, but as the economy grows large the equilibrium strategies converge to the SSN strategies.

Perhaps the main innovation this paper has to offer is the analysis of endogenous information acquisition. Our conclusion is that in large markets the number of firms who choose to acquire costly information is small. Thus, even in large markets the return to information is positive. It is possible that the number of firms choosing to acquire information is sufficiently small that it pays to play strategically with respect to information, and so information is transmitted inefficiently.

Unfortunately the information structure of our model is not a natural one. The perfect correlation of all firms' signals make information revelation an all or nothing event. In a market game with imperfectly correlated signals the incentives to not reveal are probably less, and so non-revealing equilibria may be harder to find. We hope to explore this in a future paper.

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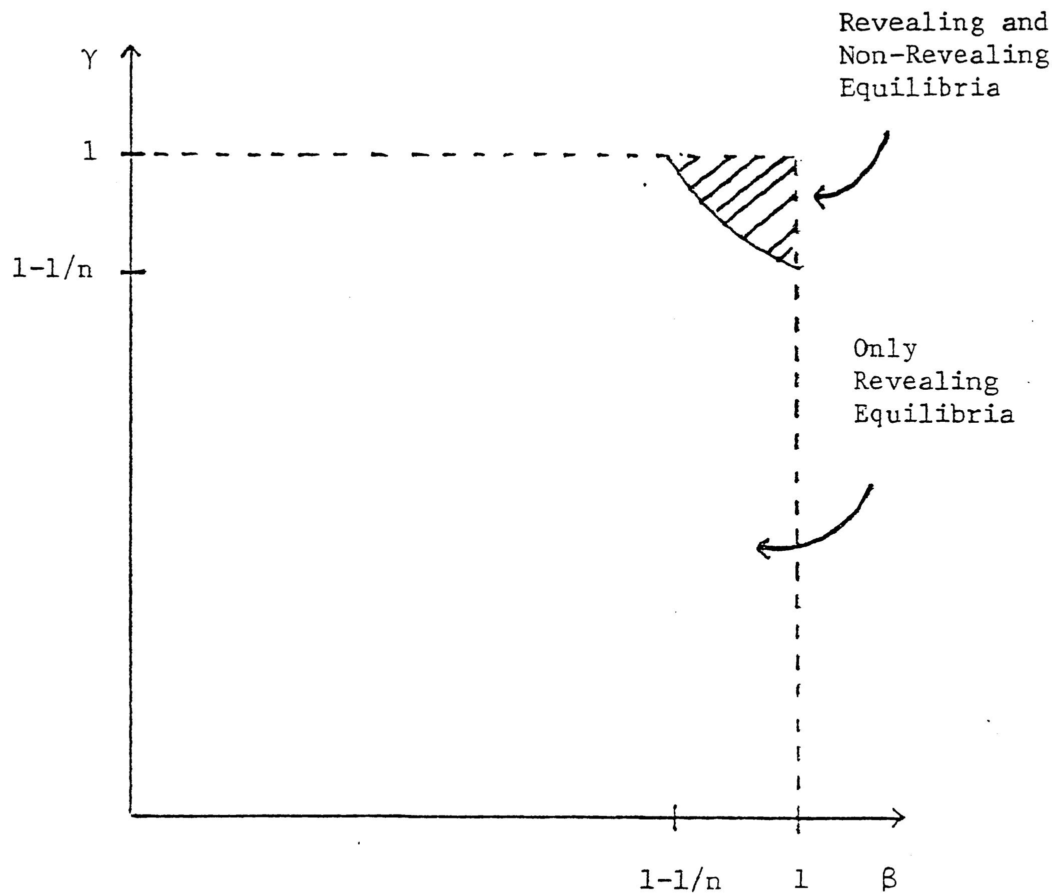
Revealing and
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FIGURE 1

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