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THE TAKING OF LAND: WHEN SHOULD COMPENSATION BE PAID?

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THE "TAKING" OF LAND: WHEN SHOULD COMPENSATION BE PAID?[1]

After guaranteeing all citizens a fair trial which must be preceded by a Grand Jury indictment, and protecting them against double jeopardy and self-incrimination, the Fifth Amendment to the U.S. Constitution concludes with the clause "... nor shall private property be taken for public use, without just compensation."[2]

As the case law has developed, the set of activities of government which can be categorized as a "taking" and for which compensation must be paid, has expanded in scope beyond those instances in which the government actually takes physical control of the land.[3] Since governmental activities not involving explicit physical taking of the land can and often do have substantial impacts (positive as well as negative) on land values, one can imagine a very large set of activities that might be considered to involve a "taking." In fact, the normative question of where the line between taking and not taking ought to be drawn has been treated seriously by a large number of legal commentators.[4] It is not our purpose here to attack this important, but difficult problem involving problems of equity as well as efficiency. Rather we will respond to a more narrow problem that is posed, at least implicitly, by some of the legal commentators. Specifically, we ask under what conditions the compensation of landowners for losses caused by a government activity will be economically efficient in the Pareto sense. We stress that while our conclusions about efficiency of compensation ought to provide useful input to the continuing "takings" controversy, the narrow focus can't possibly allow us to reach a definitive conclusion about the socially appropriate policy. We also see our model and its outcomes as providing useful information about the advantages and disadvantages of a governmental policy of providing ex post relief or compensation for private individuals who ex ante fail to purchase, or who cannot purchase because of market failure, private insurance against certain risks.

The concept of economic efficiency has been utilized by a number of commentators to provide support for whatever policy is being argued. Since the efficiency arguments are often confusing, if not confused, a brief mention of some of these instances may provide a useful background for the more technical analysis which follows.

As put by Ellickson (1973) society will utilize compensation efficiently when the presence of compensation will result in smaller resource costs (including the nuisance associated with a private use of the land, the costs of preventing that external effect to occur, and the administrative costs of obtaining information, etc.) than would result were compensation to be absent. The basic concept of efficiency is used by Baxter and Altree (1972) to argue that a first-in-time rule which pays compensation only to those land uses in effect prior to a government investment (in fact, prior to any discussion of such an investment, such as a new airport) will be efficient.

Johnson (1977) as well as Baxter and Altree (1972) argue that only if the government is forced to bear the decreased value of neighboring lands as well as paying for land actually taken will their decision to take the land be an efficient one. The argument assumes, of course, that governments must actually pay compensation to correctly perceive social costs.

Even given such an assumption, this argument is troublesome for two reasons. First, it doesn't consider the possibility that compensation may encourage suboptimal land use development in the future, i.e., that any payment of compensation will give incorrect incentives to future entrants to the land market. Second, even if government faces some sort of fiscal illusion so that costs not actually paid are discounted in its decision-making process, it doesn't follow that full payment will guarantee efficiency. Whether or not the outcome will be efficient is likely to depend much more crucially on the nature of the government's fiscal illusion.

The same efficiency notion is employed by Michelman (1968) to provide support for his quite different distinction between cases in which compensation should and should not be paid. One of Michelman's arguments concerns the issue of why compensation ought to be paid when the government intervenes in the market, but not when capital gains or losses are made as a result of the whims of the private, rather than the public sector. His distinction focuses on the fact that the risk of public action involves the possibility of risk of a strategic and non-random action affecting an individual in a manner quite distinct from what might happen in the private sector. Michelman argues that substantial resources might have to be utilized to fend off such a risk, with an implied inefficiency resulting. Whether or not Michelman's argument is valid clearly depends on the nature of individual risk aversion as well as the nature of the risk involved. The question not posed by Michelman, which would do much to clarify his analysis, relates to the conditions under which a private insurance market for risk might develop. If his argument is to be valid, Michelman would have to provide convincing evidence that an insurance market for private risk does exist or could develop without much cost, whereas there are serious roadblocks to the development of a market for insurance against governmental risk.

As these two examples suggest, the question of whether or not compensation is efficient is an unsettled one. In fact, the reaction of many economists to the thought that compensation could ever be efficient may be one of surprise. This paper is an attempt to eliminate some of the confusion about the efficiency of compensation by posing the issue within a very specific framework -- one in which the government may or may not take a particular parcel of land for public use, depending on the state of the world, and depending on its own cost-benefit calculation about the worthiness of the public project given that state of the world. The outline of the paper is as follows.

In Section II we describe the basic model and discuss the process by which an efficient market outcome may occur without the explicit payment of compensation. Using this discussion as background we prove a theorem which states a set of sufficient conditions under which no compensation will lead to an efficient outcome. One important assumption states that the government decision to take the land is independent of the current use value of the land.

In Section III we relax this assumption and prove that the no compensation market outcome may be inefficient, and that in certain cases the payment of compensation will improve social welfare. However, in general, the no compensation scheme can be fully efficient. We then treat in Section IV the issue of whether compensation can improve welfare in a world in which individuals are risk-averse and private insurance is not available.

In Section V we drop another assumption from the model of Section II, namely that the government accounts for all social costs in making its project decision, whether or not the cost involves a budgetary outlay. We show that in the presence of budgetary fiscal illusion, the no compensation outcome will be inefficient, but that the requirement of full compensation will not necessarily improve social welfare. In fact, we show explicitly a case in which the full compensation outcome is worse than the no compensation result.

Section VI contains some conclusions, qualifications, and suggests further research. Because the complete proof of each of our results involves some amount of detail, we've chosen to provide the proofs in the appendix to the paper. Throughout the text we've chosen to present the results heuristically relying heavily on the use of graphs. The graphs add substantial intuition to the analysis -- but we view the full proofs of the theorems as an integral part of the paper.

II. The Basic Model -- Project Choice Independent of Land Use

The analysis of the taking problem can be discussed in the context of a relatively simple model of land use regulation. Suppose that there are two types of land: type 1, highland and type 2, river valley.[5] Both land areas are initially undeveloped, and development is assumed to involve the allocation of a fixed amount of total capital (normalized equal to 1) to the valley and highland. We denote the valley allocation of capital used per unit of time $(0 \le x \le 1)$ x, and the highland allocation 1-x. If a competitive market determined the amount of capital allocated to each type of land, the following profit function would be maximized:

$$R(x) = f_1(1-x) + f_2(x) - 1 \tag{2.1}$$

where f_1 is the production function for output produced in the highland, and f_2 is the production function for the same output produced in the valley. The price of the good produced is normalized at 1, and is assumed to be unaffected by the capital allocation decision. In addition we assume that each f_i $(i=1,2)$ is continuous with continuous first and second derivatives, with $f_i'(x)>0$ and $f_i''(x)<0$. Finally we assume that $f_i'(1)>1$, and $\lim_{x\to 0} f_i'(x) = +\infty$. These assumptions will be sufficient to guarantee that all capital will be allocated among the two sites, with a positive amount to each site. The values $f_1(1-x) - (1-x)$ and $f_2(x) - x$ are the rental values of the highland and valley land, respectively. It is easy to see that private market optimization would allocate resources in such a way that the marginal product of capital is the same on both types of land, i.e.,

$$f_1'(1-x) = f_2'(x) \tag{2.2}$$

In the absence of externalities, an efficient allocation will satisfy (2.2) as well.

We now assume that there is a zero probability that the government will want type 1 land for any public project. However, with probability α, $0 \leq \alpha \leq 1$ type 2 land will yield a net social benefit B if the land is utilized as part of a government project.[6] α represents the investor's uncertainty about future states of the world. However, we assume that at the time at which the government makes its decision about whether to undertake the project the state of the world is known. In addition, we assume that the government's decision to undertake the project is independent of the current state of land use on either type 1 or type 2 land. As an example, suppose that if the price of energy becomes sufficiently high in the future, society will wish to build a hydroelectric project that will flood the valley. Then α might represent the probability that a certain event making the project desirable (e.g., the price of oil exceeding a certain price) will occur. To clarify the discussion we will associate state 2 with the occurrence of such an event, and state 1 with its non-occurrence. B would then represent the net benefits after construction costs from the hydroelectric project.

There are a number of reasonable decision rules that might be employed by society to determine whether or not to undertake the project. The simplest -- and by no means the best -- would be an ex ante rule to undertake the project if the price of oil exceeds a certain value, no matter the value of the land to be flooded. Along with this decision rule will be some rule governing the compensation to be paid those whose land is flooded (taken). A number of compensation schemes are possible:

among them are no compensation; compensation for the value of the land ($f_2(x)-x$), compensation for capital utilized (x), and full compensation for both the value of the land and the capital invested on it, $f_2(x)$.

If the project is to be undertaken when state 2 occurs, the expected value of social welfare is

$$S(x) = f_1(1-x) + (1-\alpha)\ f_2(x) + \alpha B - 1 \qquad (2.3)$$

Here $(1-\alpha)\ f_2(x)$ is the expected value of output on type 2 land, while αB is the expected return from the government project. It is clear that a necessary and sufficient condition for the maximization of $S(x)$ is that the expected marginal product of resources invested in the valley equal the marginal product of resources invested in the highlands.

$$f_1'(1-x) = (1-\alpha)f_1'(x) \qquad (2.4)$$

Expected private benefits will depend in general upon the compensation scheme used by the government as well as the value of the benefits of the hydroelectric project going to the private investors. For this initial discussion we will eliminate any income effects associated with the project by assuming that project benefits are not received by investors in the land or that any benefits that are received are taxed away lump sum by the government.

Under these assumptions expected private benefits when compensation is paid are

$$P(x) = f_1(1-x) + (1-\alpha)f_2(x) + \alpha[\delta(f_2(x)-x)+\gamma x+C] - 1 \qquad (2.5)$$

where δ is the fraction of land rent value paid as compensation by the government when the project (a dam which results in the flooding of the river valley) is undertaken. In addition, γ is the fraction of capital costs paid, and C represents the amount of lump-sum compensation paid if the project is undertaken.

Equation (2.5) can be used to derive the first important result, that full compensation is not optimal.[7] One interpretation of the Constitutional prohibition against taking is that it mandates δ and γ values of one (full compensation for both land value and structure). The implementation of such a policy yields the same private profit function as in (2.1) and a competitive equilibrium allocation satisfying (2.2). The concavity of f_2 means a competitive equilibrium value of x larger than is socially optimal. This case is illustrated in Figure 1 where the social welfare function is labeled S(x) and achieves a maximum at x^* and the private profit function with full compensation is labeled R(x) and achieves a maximum at $\tilde{x}>x^*$.

Further reference to equation (2.5) completes this result by considering the case of a taking without compensation ($\delta=\gamma=C=0$). In this case the private profit function is

$$P(x) = f_1(1-x) + (1-\alpha)f_2(x) - 1 \tag{2.6}$$

This function, illustrated by the P(x) curve in Figure 1, is an image of S(x) shifted down by αB. It therefore achieves a maximum at the same socially optimal $x=x^*$ as does the social welfare function. In other words, the absence of compensation -- at least compensation proportional to land values and capital -- forces the private investors to account for the probability that there will be a socially preferred use of the land. In this respect Sax (1971) was correct to suggest that "...a system which compels compensation in the event of severe diminution in value ignores the possible incentive function of leaving costs on private resource users."

Since B is independent of x, a comparison of (2.3) and (2.5) makes it clear that taking without compensation is economically efficient.[8]

In fact, any lump-sum compensation scheme is efficient. This isn't surprising, of course, since we've implicitly assumed that all other investments in the economy are optimally chosen.

III. Optimal Compensation When Project Choice Is Dependent upon Current Land Use

We have considered so far the case in which the decision to undertake the project and take the land is independent of what private owners do. Such an assumption is not realistic in many cases because public works projects are rarely undertaken on the most expensive land or on land which has valuable structures or has been recently improved. Highway planners, for instance, often choose to undertake slum clearance as a side benefit of their route choices. Therefore, in this section we incorporate the possibility that the taking decision depends on the private investment decision.

To do so, let there be an $\bar{x}$, $0 \leq \bar{x} \leq 1$ such that $B = f_2(\bar{x})$. Because $f_2'(x) > 0$ for $0 \leq x \leq 1$ this implies that for $x > \bar{x}$ the benefits of the project are smaller than the value of the output lost by flooding the valley. Therefore, if $x > \bar{x}$, it will never be socially beneficial for the government to undertake the project.

Under this condition the social welfare function is

$$S(x) = \begin{cases} f_1(1-x) + (1-\alpha) f_2(x) + \alpha B - 1 & x \leq \bar{x} \\ f_1(1-x) + f_2(x) - 1 & x > \bar{x} \end{cases} \tag{3.1}$$

and the private benefit function is

$$P(x) = \begin{cases} f_1(1-x) + (1-\alpha) f_2(x) - 1 + \alpha(\delta(f_2(x) - x) + \gamma x + C) & x \leq \bar{x} \\ f_1(1-x) + f_2(x) - 1 & x > \bar{x} \end{cases} \tag{3.2}$$

With this description of social and private welfare, it is useful to consider two possibilities, each of which call for a different policy prescription.

One possible outcome is that $f_1(1-x)+(1-\alpha)f_2(x)+B-1$ achieves a maximum for an $x^*>\bar{x}$, satisfying

$$f_1'(1-x^*) + (1-\alpha)f_2'(x^*) = 0 \tag{3.3}$$

This implies that at its maximum value, the benefit B of the project is smaller than the value of output lost, $f_2(x^*)$, from undertaking it. Therefore, the project should never be undertaken and a first best optimum is achieved when a no compensation ($\delta=\gamma=C=0$) policy is adopted. However, no compensation is not the only optimum plan. Another which will achieve a first best optimum offers a lump-sum compensation C that is small enough that there is no $x\leq\bar{x}$ such that

$$f_1(1-x) + f_2(x) > f_1(1-x) + (1-\alpha)f_2(x) + \alpha C$$

A third plan that induces a first best optimum is one that requires full proportional compensation, i.e., $\delta=\gamma=1$ and no lump-sum compensation C=0. To see this, consider what happens to P(x) in (3.2) if the plan is adopted

$$P(x) = f_1(1-x) + f_2(x) - 1 = R(x) \tag{3.4}$$

Thus, under the full compensation plan, the private investors will select a socially optimal allocation of capital by ignoring the probability that the government will want to use the valley. We stress, however, that the offer of full compensation is a hollow offer in this first-best case, since, by assumption, the government will never take the land, and no compensation will ever be paid.[9]

The second case to consider under (3.1) and (3.2) is that in which the social benefit is maximized at $x^*\leq x$. This occurs when $S(x^*)\geq\sup\{S(x): x>\bar{x}\}$. In this case, which is illustrated in Figure 3, it is socially desirable for the government to undertake the project when private resources are optimally allocated. In such a case, it might be beneficial for the private sector to invest more than is socially optimal in type 2 land in order to affect the probability that the government project will be undertaken. In Figure 3, $S(x)$ is greater than $P(x)$ when the project will be undertaken with probability α, i.e., when $x \leq \bar{x}$. However, for $x > \bar{x}$, the project will not be undertaken and $S(x) = P(x)$. Clearly, since $\sup\{P(x), x > \bar{x}\} > \sup\{P(x) = x \leq \bar{x}\}$, private investors will be encouraged to overcapitalize the type 2 river valley land sufficiently to discourage the government project from occurring.

To analyze the case further a number of definitions are useful. First let $\pi = \sup\{P(x) = x \geq x\}$. Also, let x^* be the socially optimal value of x. Notice that for $x < \bar{x}$ both $S(x)$ and $P(x)$ achieve their maximum value at x^*. Let $C^* = \pi - P(x^*)$ be the difference between the maximal value of P and its value at the socially optimal level of investment. Finally, let x_0 and x_1 be defined such that $S(x_0)$ and $S(x_1)$ $= \pi$. Since $S(\ )$ is globally concave and achieves a unique maximum for $x^* < \bar{x}$, there will be at most two values x_0 and x_1 where $x_0 < x_1$.

One way to achieve a Pareto optimal investment, x^*, in the valley (on type 2 land) is to offer a lump-sum compensation $C = C^*/\alpha$ (or higher).[10] From (3.2), and Figure 3, we can see that using only lump-sum compensation at this level the value of private benefits at x^* is equal to π. The private entrepreneur will achieve the same expected profit by investing in a socially optimal manner. Furthermore, x^* maximizes $P(x)$ for $x < \bar{x}$ because lump-sum compensation leaves all marginal conditions unchanged.

It is not surprising that a first-best optimum can be achieved with a lump-sum compensation plan. It is not obvious, however, whether one can achieve the same optimal allocation of capital through a proportional or any other δ-γ compensation scheme. From (3.2) it can be seen that any Pareto optimal (δ-γ) scheme must satisfy the following two conditions

$$\begin{aligned} &\delta(f_2(x^*)-x^*) + \gamma x^* \leq C^*/\alpha \\ &\delta(f_2'(x^*)-1) + \gamma = 0 \end{aligned} \tag{3.5}$$

The first condition insures that expected profits at x^* are at least as large as they are at x (the full private optimum) and the second condition insures that x^* maximizes P(x) for $x \leq x^*$. Using the strict equality case, these two equations can be solved for δ and γ to yield

$$\begin{aligned} &\delta = -C^*/[\alpha(f_2'(x^*)x^*-f_2(x^*))] \\ &\gamma = [(f_2'(x^*)-1)C]/\alpha(f_2'(x^*)x^*-f_2(x^*)] \end{aligned} \tag{3.6}$$

The strict concavity of f implies that the denominator of both expressions is negative. Therefore any Pareto optimal (δ-γ) plan must involve a subsidy to land value at rate δ and a tax on capital at rate γ. In addition δ must be greater than $|\gamma|$, with the degree to which the tax and subsidy rate differ depending upon the value of $f_2'(x^*)-1$.[11] A careful examination of this policy makes its effectiveness quite clear. Figure 4 shows that the marginal tax on capital is independent of land use, while the marginal value of land compensation diminishes with land utilization. The two are equal at the optimal land use x^*. Thus, at x^* both marginal incentives exactly cancel so that there are no distortions in land use. At the same time, total compensation eliminates any non-marginal distortions. To put the matter somewhat differently, the tax on capital is a necessary component of the first-best policy because private investors must be discouraged from overinvesting in type 2 land. While the capital tax corrects the decision on the intensive margin about capital utilization, it does nothing to correct the problem on the extensive margin. The land value subsidy is used to compensate individual investors sufficiently so that private profits at the social optimum are at least as great as profits when the government project isn't undertaken. And, while the land value subsidy itself distorts private investment, this distortion can be corrected by the capital tax.

To pursue the matter further, we consider some special cases involving a limited δ-γ compensation rule. Suppose our policy alternatives only allowed a compensation for rents, i.e., $\delta>0$ and $\gamma=0$. It is easy to see that no such compensation scheme could be Pareto optimal because the first equation in (3.5) requires $\delta>0$ and the second equation requires $\delta=0$. For a similar reason no plan which requires $\delta=\gamma$ can be Pareto optimal.[12]

Given the difficulties of applying the first-best δ-γ scheme, a reasonable question to ask is whether there exists a proportional compensation plan ($\delta=\gamma$) which yields a second best optimum -- namely, one that induces a capital allocation $x^0 \varepsilon [x_0, x_1]$ for which $S(x^0) \geq \pi$. We show in the appendix (Theorem 4) that there is no proportional compensation plan that yields either a first or second best allocation of capital.

It shouldn't be surprising that all plans in which $\delta=\gamma$ cannot induce a better allocation of capital than no compensation under the assumptions that we've made. In the model, the ability to alter project choice causes the privately chosen investment in type 2 land to be greater than is socially optional. Since, the $\delta=\gamma$ scheme requires that capital investment in the valley as well as land values be subsidized, the marginal incentives are further distorted.[13]

While proportional compensation will not yield even a second-best allocation when the private "certain" profit function

$$R(x) = f_1(1-x) + f_2(x) - 1 \tag{3.7}$$

achieves a maximum at $\tilde{x}>\bar{x}$, it will yield a second best allocation if it achieves a maximum for $\tilde{x}<\bar{x}$.[14] This case is illustrated in Figure 5. As we have shown earlier, full proportional compensation implies that the private market is equivalent to the profit function (3.7). Suppose, as in Figure 5 the maximum value of private profits when there is no compensation, is smaller than $R(\bar{x})$, i.e.,

$$P(x^*) < R(\bar{x}) \tag{3.8}$$

Then without compensation the private market will allocate $\bar{x}$ to type 2 land, while with full compensation it will allocate $\tilde{x}<\bar{x}$. While $\tilde{x}$ is not a first best allocation, it is superior to the one induced when no compensation is available.

IV. The Effects of Risk Aversion

Siegan (1977, p. 26) has argued that taking without compensation is inefficient because it produced unnecessary uncertainties in the real estate market. The possibility of a taking is seen as a political risk which because "...[p]urchasing land for development requires the expenditure of huge sums of money....it is likely therefore that as political risks increase, more potential investors, lenders, builders, and developers will drop out. The obvious conclusion is that compensation by government will improve social welfare.

In order to analyze Siegan's conjecture the previous analysis must be modified. Under our current assumption that all agents are risk neutral, the presence of political risk is nondistorting. In fact, if there is no private investment plan that can affect the probability of a taking, no compensation leads to an optimal allocation of capital, because private investors accurately account for the risk of a taking in their investment decision. Thus, as Siegan mentions, the argument for compensation is incorrect when investors are not risk averse. In order to examine Siegan's assertion in detail, we start by assuming that private investors are risk averse, but that the government still acts as a risk neutral social optimizer. Arrow and Lind (1970) have argued that such an assumption is plausible under the assumption that there are a large number of investment choices whose benefits are uncorrelated with one another. In such a case the optimal outcome occurs when the government behaves in a risk-neutral manner. This is, of course, essentially the same argument used by those economists who advocate the use of a social rate of discount for project evaluation which is less than the private opportunity cost of capital. We are not fully satisfied that the assumptions of the

Arrow model apply here since investment benefits may not be independent. We do note, however, that if the government is also risk averse, then our earlier results about the suboptimality of full compensation apply. Clearly, the case most favorable to the Siegen line of argument is the risk-neutral government case.

For the purpose of devising an optimal policy we will confine our investigation to the case in which the decision to take the land is independent of the capital invested in it. The social welfare function is the same as in (2.4) because the government is assumed to be risk neutral, i.e.,

$$S(x) = f_1(1-x) + (1-\alpha)f_2(x) + \alpha B - 1 \tag{4.1}$$

However, the private objective function is different than in the previous sections to account for risk aversion. In particular,

$$P(x) = (1-\alpha)U(f_1(1-x)+f_2(x)-1)+\alpha U(f_1(1-x)-1+\delta(f_2(x)-x)+\gamma x+C)) \tag{4.2}$$

where $U(\quad)$ is strictly concave differentiable function with $U'(\quad)>0$ and $U''(\quad)<0$.

In Section II we showed that no compensation $\delta=\gamma=C=0$ will yield a first best capital allocation. If these plans were pursued with risk-averse investors (4.2) would yield a private optimizing condition

$$f_1'(1-x) = \frac{(1-\alpha)U'(f_1(1-x) + f_2(x)-1)}{(1-\alpha)U'(f_1(1-x)+f_2(x)-1) + \alpha U'(f_1(1-x)-1)} \tag{4.3}$$

This condition implies that at the competitive allocation, $\tilde{x}$, $f_1'(1-\tilde{x}) \leq (1-\alpha)f_1'(x)$ with equality if and only if $U'(f_1(1-\tilde{x})+f_2(x)-1) = U'(f_1(1-\tilde{x})-1)$. But the equality, which is necessary for social optimality, cannot be met if investors are risk averse. Thus, by the concavity of $f(\ )$, (4.3) implies that $\tilde{x}<x^*$. In other words Siegan is partially correct: without compensation too little capital will be put onto the river valley. But it can be shown that fully proportional compensation is not optimal.[15]

A full proportional compensation plan is one in which $\delta=\gamma=1$ and $C=0$. Under this plan a private optimum is achieved for an $\tilde{x}$ such that

$$f_1'(1-\tilde{x}) = f_2'(\tilde{x})>(1-\alpha)f_2'(\tilde{x}) \tag{4.4}$$

Since with full proportional compensation private investors are insured by the government, they are induced to put too much capital on type 2 land, $\tilde{x}>x^*$. Therefore, we see that full compensation, for both land and capital does not induce a social optimum.[16] We should note, however, that in general there will exist a partial compensation scheme which is first-best. We also note in conclusion that a first-best optimum can be achieved when the compensation is set equal to the amount that would have been given at the optimum x^* under full proportional compensation.

V. Compensation When the Government Project Decision Is Suboptimal

One argument for government compensation presumes that the government suffers from a kind of fiscal illusion when the project decision is made. In the extreme case, only dollars actually paid enter as costs in the cost-benefit calculation. In this case, and in the more general case in which costs not on the budget are partially discounted, Berger (1974), Johnson (1977) and others argue that the payment of full compensation will lead to a first-best social outcome. In this section, we modify our earlier analysis by assuming that the government consistently discounts the lost revenues of the river land investor when a taking occurs. We also assume that private investors are aware of the decision-making criterion used by the government, and can (in some cases) alter their capital allocation decision so as to influence the project evaluation. We will show that in such an environment, no compensation is sub-optimal, but so is the payment of full compensation. In fact, full compensation may be less desirable than no compensation. Finally, there will, in general, exist a lump sum compensation plan which is first best. Since there are a number of distinct cases that can occur, we have chosen to examine two cases carefully. Our analysis is not meant to fully cover all possible occurrences.[17]

The social welfare function remains unchanged, but the government unit making the project decision is assumed to have a criterion which chooses the project a) when state 2 occurs; and b) when

$$B-C>\theta(f_2(x)-C) \tag{5.1}$$

where C is any compensation that is paid when the taking occurs and $0\leq\theta\leq1$ represents a measure of the fiscal illusion of the project decision-making unit.

The project choice criterion given in (5.1) allows us to consider a range of alternative assumptions about the form of the fiscal illusion. When $\theta=1$, the choice criterion becomes $B=f_2(x)$ as before, and no fiscal illusion is present. Therefore, we restrict our consideration to the case in which θ is strictly less than 1. Note that when $\theta=0$, the criterion becomes $B>C$, the extreme case in which the government counts as costs actual compensation paid, but takes no account of the lost value of output on type 2 land. In the intermediate case, compensation is included as a full budgetary cost when paid, but is discounted when it is perceived as reducing the net losses to private investors when the land is taken. The same story can be restated in a somewhat different light by rewriting (5.1) as follows:

$$B-C(1-\theta)>\theta f_2(x) \tag{5.2}$$

Here compensation paid out is valued at rate $(1-\theta)$, whereas private investments loss are valued at rate θ.

Note that if C is chosen equal to $f_2(x)$ (a full proportional compensation plan), then (5.2) becomes $B>f_2(x)$, and the analysis of Section III of the paper applies. This choice of C yields the same outcome as the full proportional compensation plan. It is after all equivalent to it. The only difference is the decision rule used by the government, but the outcomes are the same. If the production function is $f_2(x)$ as illustrated in Figure 1, the private market will not be able to choose an x large enough to assure that the project is not undertaken. But since compensation will be paid for both lost land and capital value, too much capital will be used on the river valley. If the production function is represented by $f_2'(x)$, the private market can block the project by putting at least $\bar{x}$ capital on the valley land. This would lead to overinvestment on type 2 land.

It is interesting to notice that if there is pure fiscal illusion $\theta=0$ and thus government is allowed to take without compensation, the socially optimal level of x will occur in the cases illustrated in Figures 1 and 3. The explanation is that the probability that the land is taken is independent of the capital invested in it. Thus private investors will suffer the entire capital loss in the event of a taking and the possible loss will induce them to choose the socially optimal distribution of capital. The only time that no compensation yields the incorrect outcome in the case of pure fiscal illusion is the one illustrated in Figure 2 -- when the competitive equilibrium capital allocation is optimal.

VI. Conclusions

We've intentionally limited our analysis to the problem of whether the payment of compensation for land taken by eminent domain is efficient. We've seen that when the decision to take the land is independent of current land use that no compensation is efficient and full compensation for the value of the land and structures will be inefficient. However, as our analysis proceeded, the conclusions changed as the government project choice criterion changed. When the project decision rule was to undertake when net benefits were greater than the value of lost output on the land, we saw that two types of externalities created a suboptimal outcome.[18] First, the presence of compensation can distort marginal private investment decisions by changing the relative return on capital on different types of investments. Second, the project decision rule creates an externality on the extensive margin, because private investors may be able to overcapitalize their land sufficiently to cause a socially optimal project not to be undertaken.

We then considered the possibility that full compensation might be efficient in a world with risk averse private investors and a risk neutral government. In that case we found that neither full compensation nor no compensation was efficient in general. Finally, we found that when the project decision rule involved a form of fiscal illusion in which the lost output on the land was partially or fully discounted, that once again neither no compensation nor full compensation was efficient. Thus, our analysis shows quite clearly that under a wide variety of assumptions about the government's decision-making criterion the payment of full compensation is inefficient.

Our analysis has implications beyond the taking question, however. To see this consider the converse of our analysis. Rather than looking for an optimal compensation rule, give a particular project choice criterion, we might ask what the appropriate decision-rule is given a particular compensation rule. A little thought suggests the first-best result -- to the extent that compensation is given (it need not) the compensation should be equal to (or more generally a function of) the value of lost output on the land when the land is optimally utilized (i.e., $f_2(x^*)$). The appropriate project criterion is to do the project if net benefits outweigh the value of the optimally lost output. In such a case, private investors cannot distort project choice, nor does compensation affect marginal investment decisions.

This line of thought has direct and interesting implications for the use of cost-benefit analysis of projects in which land is purchased or taken. Under our assumptions the current value of the land is an inappropriate measure of cost, and suboptimal decisions will be made when current value is used. The problem of determining the appropriate value of land is, of course, a difficult one, given the dynamics of land markets and of cost-benefit analysis. The problem arises because current land value is a function of expected future project choices, and future project choices are a function of current land values. The evaluationof cost-benefit analysis as a project criterion in a dynamic model of this sort under varying assumptions about the availability of information and about the formation of investor expectations appears to us to be a fruitful topic for further research.

Footnotes

[1]The authors wish to thank Peter Diamond for bringing the topic of the paper to their attention. In a number of conversations, Ted Bergstrom foresaw many of the results of this paper. In addition, Glen Loury and members of the Michigan Public Finance seminar provided helpful discussion and comments on our work. The authors assume full responsibility for any errors remaining in the p

[2]The "taking" amendment is applied to the individual states by the 14th amendment. See, for example, D. Hagman (1971, Chapter 14).

[3]The case law is an extensive one, and not worth separate treatment here. However, any of a substantial set of legal commentaries provide a useful overview of the case law development. See, for example, F. Bosselman, D. Callies and J. Banta (1973); Michelman (1978); Plater (1974); Sax (1964, 1971); Berger (1974); Baxter and Altree (1972); and Siegan (1977).

[4]See the references in note 2.

[5]The river valley-highland case is only an example. Our analysis would also apply to the taking of urban land in which land in one location is much more likely to be taken than land in another location.

[6]We could, of course, make B stochastic and α a function of the realization of B. However, this would substantially complicate the analysis.

[7]See Corollary 2 to Theorem 1 in the appendix.

[8]See Theorem 1 in the appendix.

[9]See Theorem 2 in the appendix.

[10]See Theorem 3 in the appendix.

[11]The second derivative of (3.2) with respect to x for any constant $\delta \geq 0$ and δ is $f_1''(1-x) + (1-\alpha)f_2'' + \alpha\delta f_2''(x) \leq 0$ for all x. Therefore, satisfaction of the first-order condition in (3.4) is both necessary and sufficient for a maximum.

[12]See Theorem 4 in the appendix.

[13]The same difficulty arises when land values are subsidized, but capital usage is not. We leave the details to the reader.

[14]See Theorem 5 in the appendix.

[15]See Theorem 6 in the appendix for complete details.

[16]See Theorem 6 in the appendix.

[17]See Theorem 7 in the appendix.

[18]Other authors have discussed these two sources of suboptimality in other contexts. See Carlton and Loury (forthcoming) and Polinsky (forthcoming).

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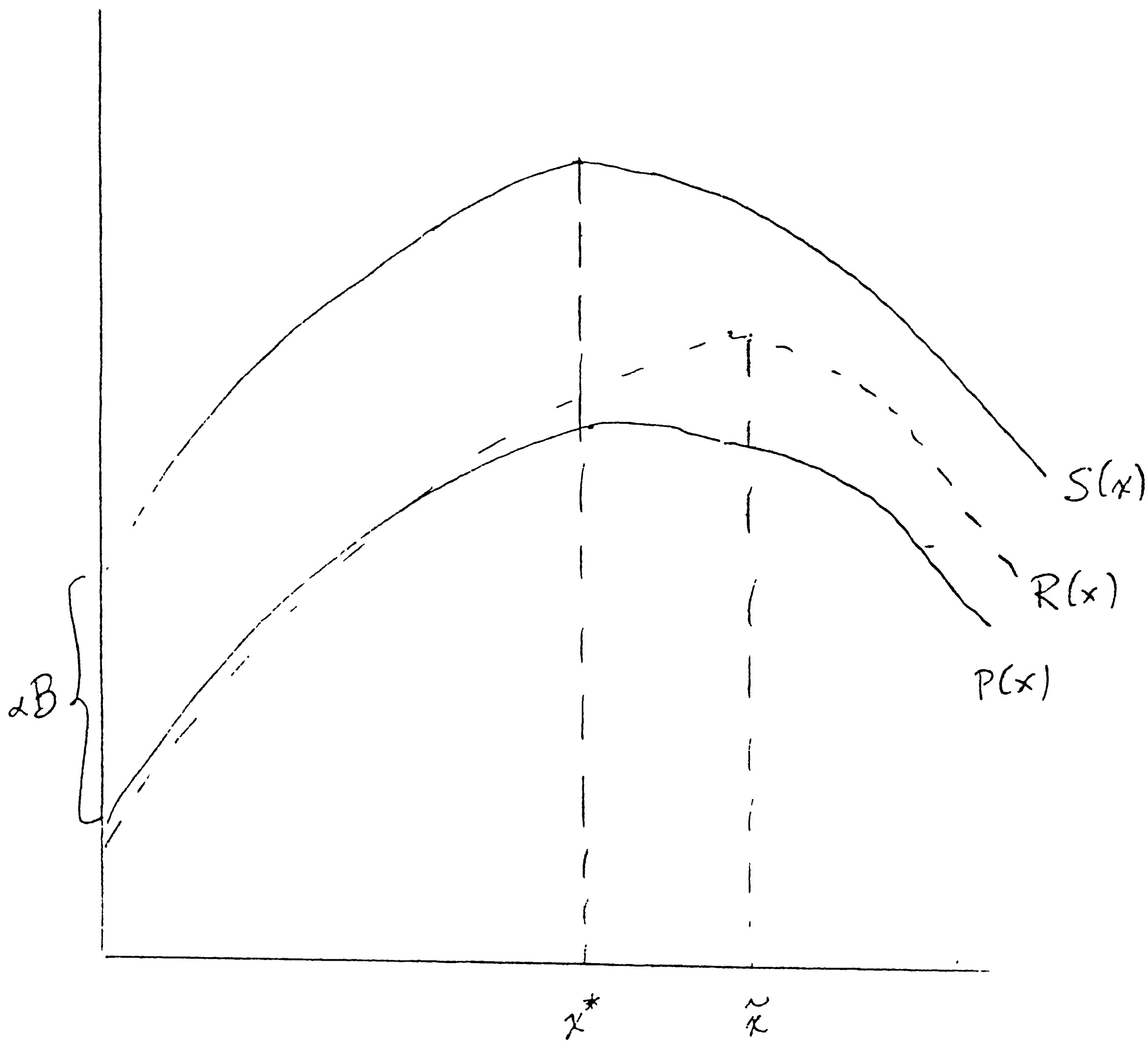

Figure 1

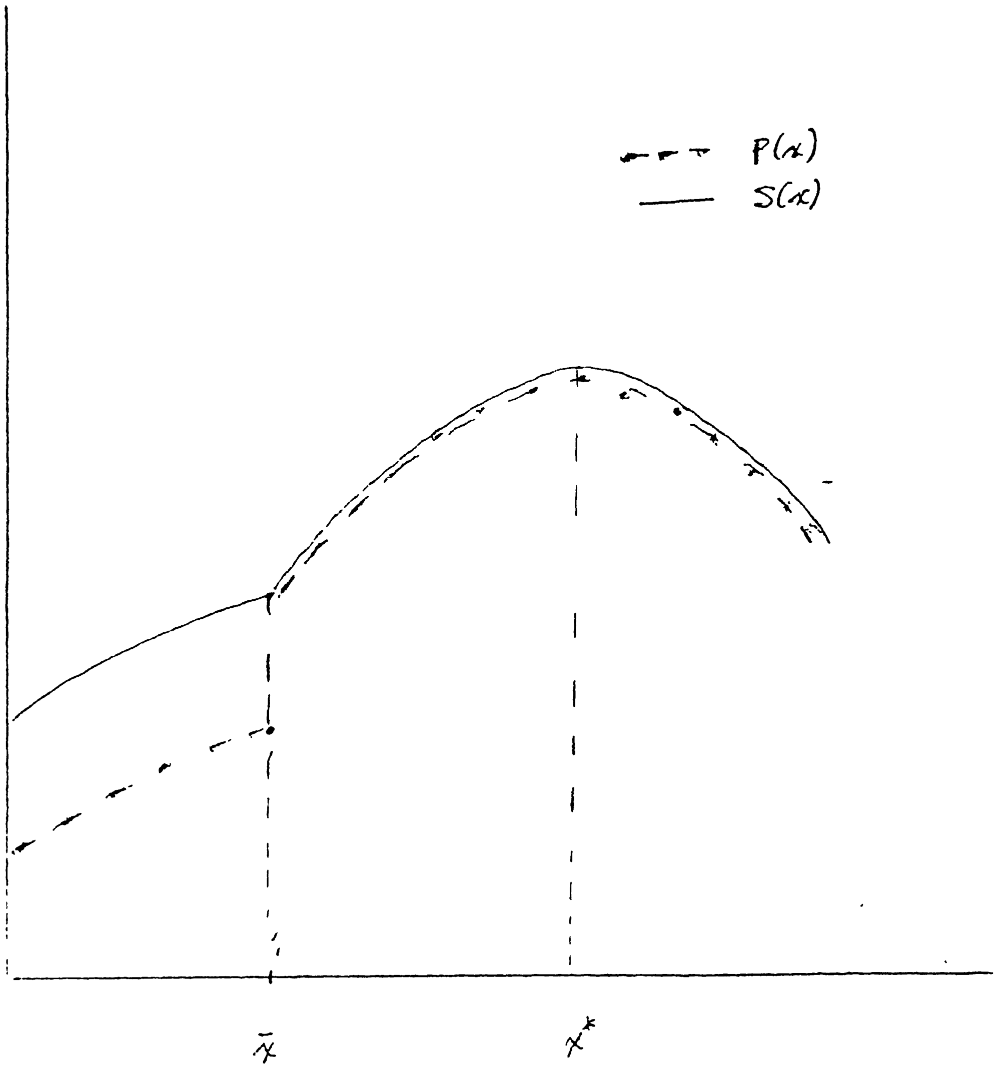

Figure 2

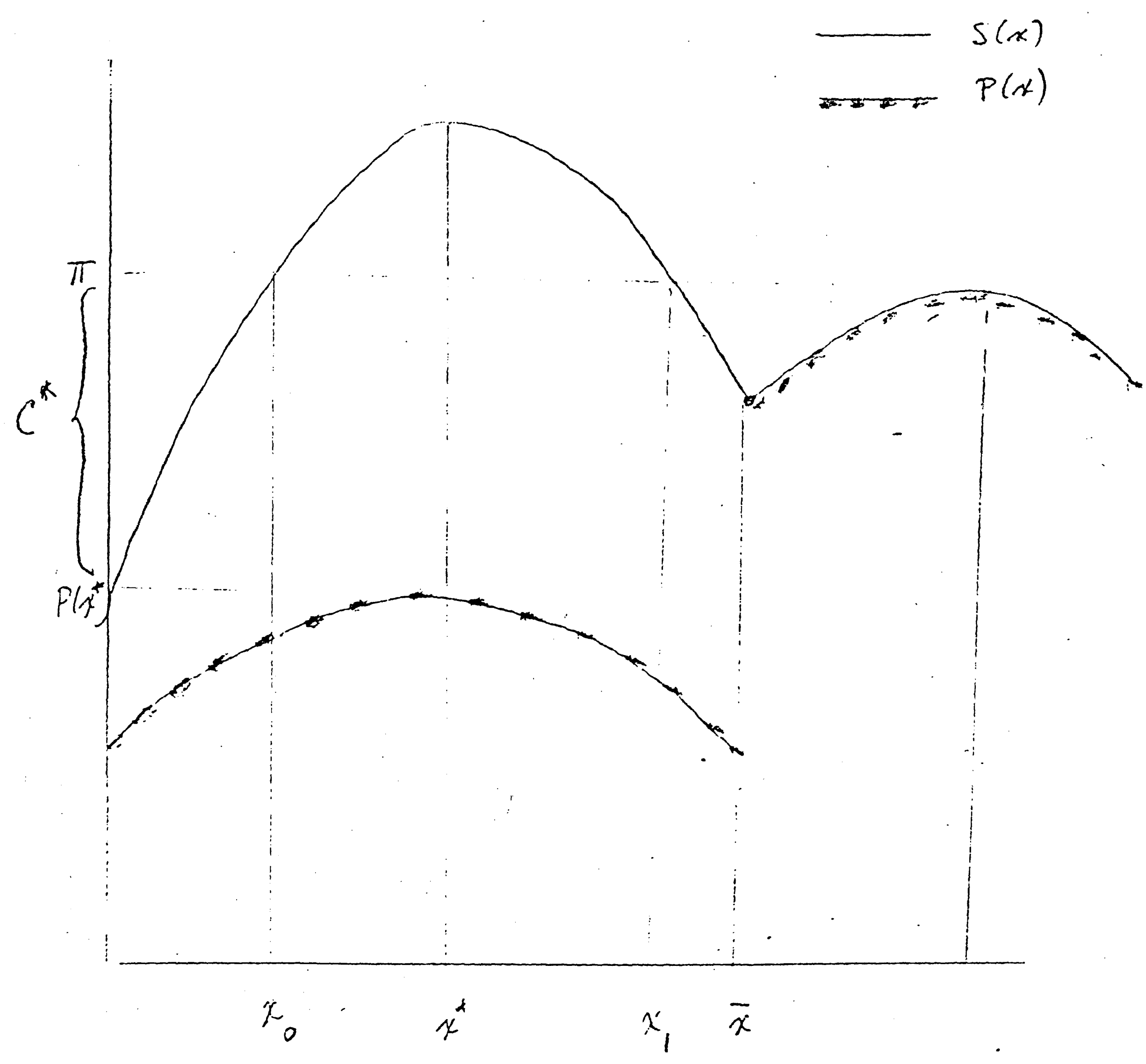

Figure 3

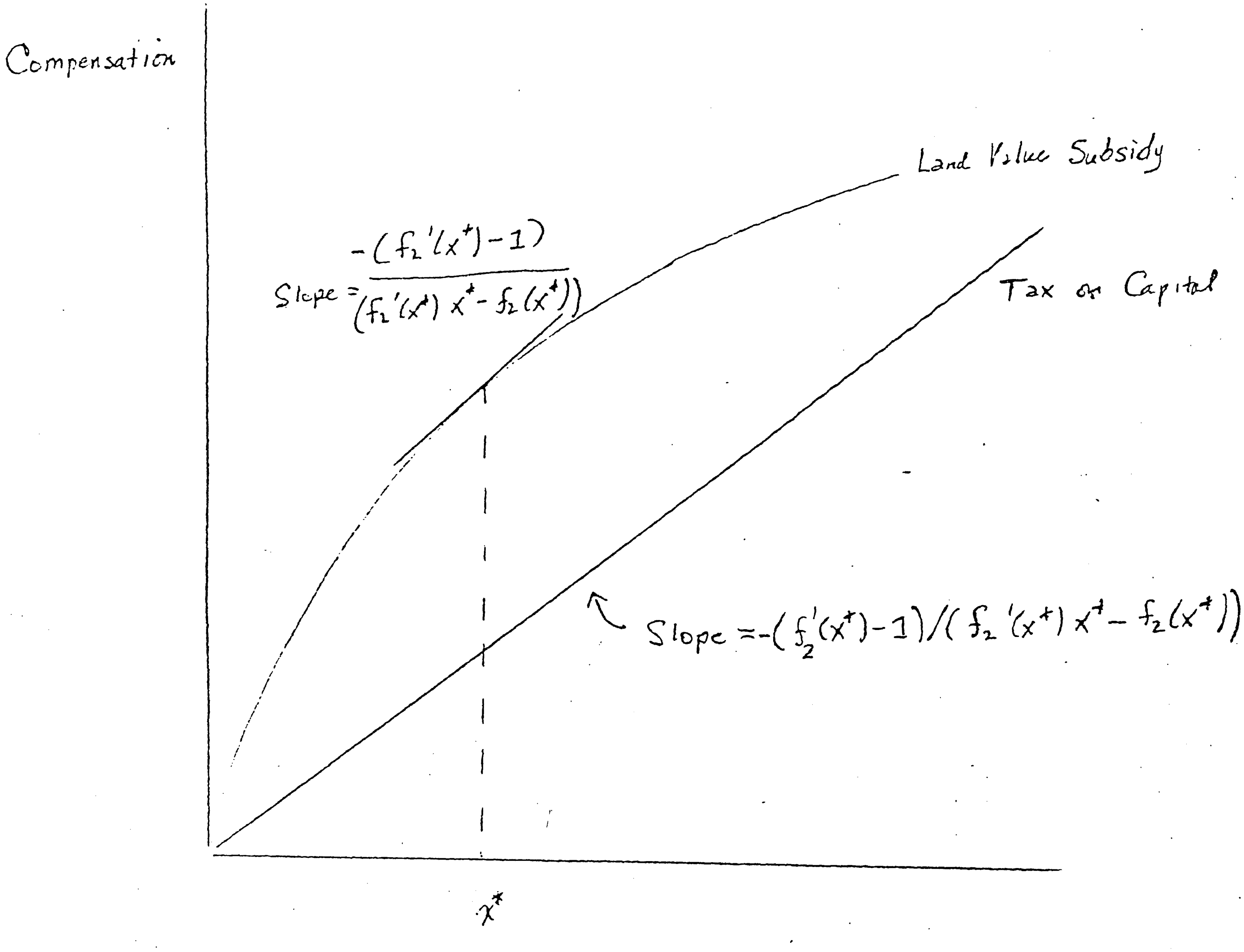
Compensation
Land Value Subsidy
Tax on Capital
Slope $= \frac{-(f_2'(x^*)-1)}{(f_2'(x^*)\,x^* - f_2(x^*))}$
Slope $= -(f_2'(x^*)-1)/(f_2'(x^*)\,x^* - f_2(x^*))$
x^*

Figure 4

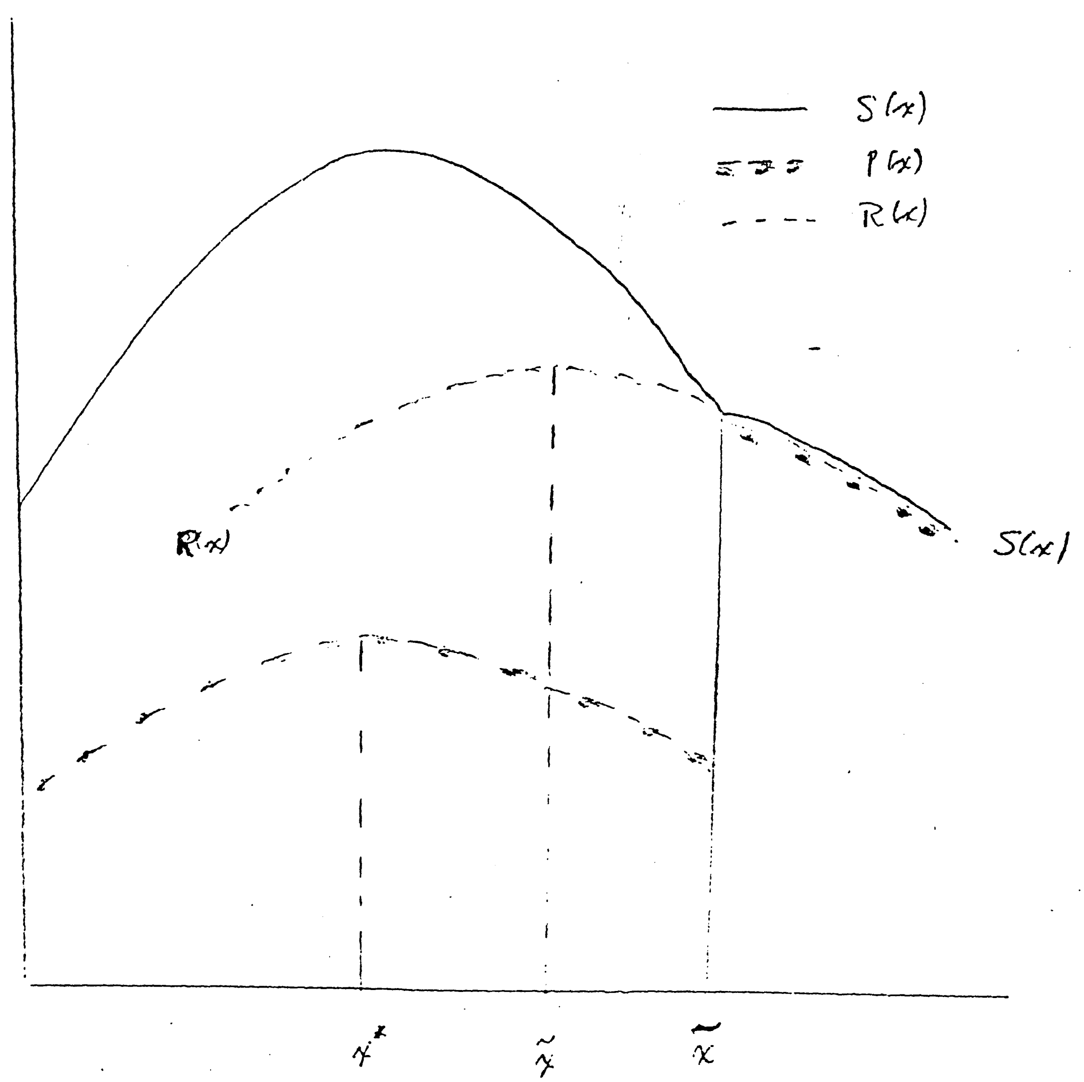

Figure 5

Appendix

Assume that there are two types of land; type 1 and type 2. Total revenue from productive activities on type i land with k units of capital utilized is $f_i(k)$. We also assume:

A.1. $f_i: \mathbb{R}_+ \to \mathbb{R}$ is twice continuously differentiable on $\mathbb{R}_{++}$. $f_i(0)=0$.

A.2. $f_i' > 0$, $f_i'(1) > 1$, $\lim_{k\to 0} f_i'(k) = +\infty$.

A.3. $f_i'' < 0$.

There is one unit of capital to allocate among the two types of land, and we consider the problem of how to optimally allocate this capital.

There are two states of nature. In state b there is a public sector project that would yield a net public benefit of B. In state a, all public benefits from any public sector project are 0. State a occurs with probability $1-\alpha$, and state b occurs with probability α. If state b occurs, the government decides whether or not to undertake the public sector project. At the outset we assume that the government acts so as to maximize expected social welfare.

Let x denote investment on <u>type 2</u> land. Then 1-x is <u>type 1</u> investment. (It will be seen that assumption A.2 implies that all capital will be invested.) The expected social welfare function is

$$S(x) = \begin{cases} f_1(1-x) + f_2(x) - 1 & f_2(x) > B \\ f_1(1-x) + (1-\alpha)f_2(x) + \alpha B - 1 & f_2(x) \leq B. \end{cases}$$

Private profits depend upon how and how much compensation is paid in the event that type 2 land is used for the public sector profits. We consider compensation schemes that are the sum of a lump sum payment, a δ per dollar compensation for foregone rent, and a γ per dollar compensation for lost capital. If land markets are competitive, then rents on type 2 land are $f_2(x) - x$. Compensation paid is:

$$C + \delta f_2(x) - \delta x + \gamma x.$$

In this case, the private profit function is

$$P(x) = \begin{cases} f_1(1-x) + f_2(x) & f_2(x) > B \\ f_1(1-x) + (1-\alpha)f_2(x) + \alpha[C+\delta(f_2(x)-x)+\gamma x] - 1 & f_2(x) \leq B \end{cases}$$

First, we consider the case where $f_2(1) \leq B$. The public project will or will not be undertaken independent of the value of x.

Theorem 1. Suppose $f_2(1) \leq B$, and that $\delta > (\alpha-1)/\alpha$

i) If x^* maximizes $S(x)$ and

either a) $\delta = \gamma = 0$,

or b) $f_2'(x^*) = 1 - \gamma/\delta$,

then x^* maximizes private profits.

ii) If x^* maximizes $P(x)$ and a) or b) holds, then x^* maximizes $S(x)$.

Proof. i) Suppose that x^* maximizes $S(x)$. Since $f_2(x) \leq f_2(1) \leq B$ for all $0 \leq x \leq 1$, $S'(x) = (1-\alpha)f_2'(x) - f_1'(1-x)$. A.2 implies that $S'(x) > 0$ for x near 0, and $S'(x) < 0$ for x near 1. A.3 implies that $S''(x) < 0$ for all x, $0 < x < 1$, so $0 < x^* < 1$ and $S'(x^*) = (1-\alpha)f_2'(x^*) - \alpha f_1'(1-x^*) = 0$. Then at x^*, $P'(x^*) = (1-\alpha)f_2'(x^*) - f_1'(1-x^*) + \alpha[\delta f_2'(x^*)+\gamma-\delta]$ $= \alpha[\delta f_2'(x^*)+\gamma-\delta]$. If either a) or b) holds, then $P'(x^*) = 0$. If $\delta > (\alpha-1)/\alpha$, then $P(x)$ is concave, so x^* maximizes $P(x)$.

ii) If x^* maximizes $P(x)$, $\delta > (\alpha-1)/\alpha$ implies that $x^* > 0$, and A.2 implies that $x^* < 1$. Thus $P'(x^*) = 0$. But if either a) or b) holds, then $(1-\alpha)f_2'(x^*) - \alpha f_1'(1-x^*) = 0$, and x^* maximizes $S(x)$.

Q.E.D.

Corollary 1. Any purely lump sum scheme is optimal.

Corollary 2. No non-trivial scheme that equally compensates rent and capital, or compensates capital more than rent, is optimal.

Proof. $f_2'(x^*) = 1 - \gamma/\delta \leq 0$, which contradicts A.2.

Q.E.D.

Now consider the case where $f_2(1) > B$. Then there exists an $\bar{x}$ such that $f_2(\bar{x}) = B$. If $x \leq \bar{x}$, then in state b the government will undertake the project. If $x > \bar{x}$, the government will not undertake the project.

Theorem 2. If $\sup\{S(x):x\leq\bar{x}\} < \sup\{S(x):x>\bar{x}\}$ then $C=\gamma=\delta=0$ and $\gamma=\delta\leq 1$, $C=0$ are efficient.

Proof. i) $C=\gamma=\delta=0$. Note that $P(x) = S(x)$ for $x > \bar{x}$ and when there is no compensation $P(x) = S(x) - \alpha B$ for $x \leq \bar{x}$. Thus $\sup\{P(x):x>\bar{x}\}$ $= \sup\{S(x):x>\bar{x}\} > \sup\{S(x):x\leq\bar{x}\} > \sup\{P(x):x\leq\bar{x}\}$. Thus investors will choose $x > \bar{x}$, and since in this event $P(x) = S(x)$, the private and social optima coincide.

ii) $C = 0$, $\gamma=\delta\leq 1$. It suffices to consider the case $\gamma=\delta=1$. In this event $P(x) = f_1(1-x)+(1-\alpha)f_2(x)-1+\alpha f_2(x)-\alpha x+\alpha x = f_1(1-x)+f_2(x)-1$ on $x \leq \bar{x}$. Thus $P(x) = f_1(1-x) + f_2(x) - 1$ for all x. But $f_1(1-x)+f_2(x)-1 \leq \sup\{S(x):x\leq\bar{x}\}$ for $x\leq\bar{x}$, by the definition of $\bar{x}$. Thus $\sup\{f_1(1-x) + f_2(x) - 1:x\leq\bar{x}\} \leq \sup\{S(x):x\leq\bar{x}\} < \sup\{S(x):x>\bar{x}\} = \sup\{P(x):x>\bar{x}\}$, so the private optimum has $x>\bar{x}$. Again, the social and private optima coincide.

Q.E.D.

In the case of theorem 2, there is no incentive to overinvest in land so as to manipulate the government decision. Now consider the case where the investors will have incentive to overinvest on type 2 land.

Let $\eta = \sup\{P(x):x\leq\bar{x}\}$. Let $\pi = \sup\{P(x):x>\bar{x}\}$. Let $S(x^*) = \sup\{S(x):0\leq x\leq\bar{x}\}$. Note that $\pi = \sup\{S(x):x>\bar{x}\}$.

Theorem 3. Suppose that $S(x^*) > \pi > \eta$. Then

i) $C=\delta=\gamma=0$ is inefficient.

If $\delta > (\alpha-1)/\alpha$, then

ii) $C \geq \frac{\pi-\eta}{\alpha}$, $\gamma=\delta=0$ is efficient

iii) there exist efficient compensation schemes with $C=0$, but every such scheme has $\delta>0$, $\gamma<0$.

Proof. i) is self evident, since the social optimum is at $0\leq x^*\leq\bar{x}$, but there exists $\hat{x}>\bar{x}$ with $P(\hat{x}) > \eta$.

ii) Suppose that $C \geq \frac{\pi-\eta}{\alpha}$, $\delta=\gamma=0$. Then $P(x) = f_1(1-x)+(1-\alpha)f_2(x)+\alpha C$, so $\sup\{P(x):x\leq\bar{x}\} = \eta+\alpha C \geq \eta + \alpha(\pi-\eta)/\alpha = \pi$.

iii) It can be shown that $0<x^*<\bar{x}$. Thus if $C=0$, any external compensation scheme must satisfy:

$$\delta(f_2(x^*)-x^*) + \gamma x^* = K \geq (\pi-\eta)/\alpha,$$

$$\delta(f_2'(x^*)-1) + \gamma = 0.$$

Solving, $\delta = K/(f_2(x^*) - x^*f_2'(x^*))$

$$\gamma = (1-f_2'(x^*))K/(f_2(x^*) - x^*f_2'(x^*)).$$

Since f_2 is concave (A.3), $f_2(0) - f_2(x^*) \leq f_2'(x^*)x^*$. Since $f_2(0) = 0$ (A.1), $x^*f_2'(x^*) \leq f_2(x^*)$, and the denominator is positive. $K \geq (\pi-\eta)/\alpha > 0$ so $\delta>0$. A.2 and A.3 together imply that $f_2'(x) > 1$ for all $0\leq x\leq 1$, so $\gamma<0$. Then $\delta > (\alpha-1)/\alpha$ implies that the first order conditions are sufficient, as $P(x)$ is concave on $x\leq\bar{x}$.

Q.E.D.

Theorem 4. Suppose that π is attained at $\tilde{x}>\bar{x}$. Then no proportional compensation scheme ($\gamma=\delta$) can lead to a capital allocation Pareto superior to that achieved without compensation.

Proof. Since $\pi \geq S(\bar{x})$ by the continuity of $S(x)$, $S(x) > \pi$ implies $x^*<\bar{x}$. Thus necessary conditions for a second best optimum x^0 to exist with $\delta=\gamma$ are

$$(*) \quad f_1(1-x^0) + (1-\alpha)f_2(x^0) + \alpha\delta f_2(x^0) \geq \pi + 1.$$

$$(**) \quad -f_1'(1-x^0) + (1-\alpha)f_2'(x^0) + \alpha\delta f_2'(x^0) = 0.$$

Solving (**) for δ, $\delta = \dfrac{f_1'(1-x^0) - (1-\alpha)f_2'(x^0)}{\alpha f_2'(x^0)}$.

Substituting into (*)

$$f_1(1-x^0) + \frac{f_2(x^0)}{f_2'(x^0)} f_1'(1-x^0) \geq \pi+1.$$

Denote the left-hand side of this inequality by $\psi(x^0)$. We show that this inequality is true for no $x \leq \bar{x}$. Note that for all $x = [0,1]$,

$$\psi'(x) = \frac{-f_1(1-x)f_2(x)}{f_2'(x)} \left(\frac{f_2''(x)}{f_2'(x)} + \frac{f_1''(1-x)}{f_1'(x)} \right) > 0$$

At $\hat{x}$, $f_1'(1-\hat{x}) = f_2'(x)$. Thus $\pi + 1 = \psi(\hat{x})$. Hence $\psi(x) < \psi(\hat{x}) = \pi + 1$ for all $x<\hat{x}$, and, in particular, for all $\hat{x}\leq\bar{x}$.

Q.E.D.

Theorem 5. There exist second best compensation schemes with $C=\gamma=0$.

Proof. Choose $\delta>0$ such that $\delta(f_2(x^*)-x^*) \geq (\pi-\eta)/\alpha$. Then the private profit function will be

$$\hat{P}(x) = f_1(1-x) + (1-\alpha)f_2(x) + \alpha\delta(f_2(x)-x) \text{ and } P(x^*) \geq \pi.$$

Furthermore, $\delta>0$ implies that $P''(x^*)<0$.

Q.E.D.

Now we relax the assumptions of individual and social risk aversion. We consider the case where individuals are risk averse but society is not, and individual actions cannot affect the government decision. Thus $S(x)$ is as before. But now

$$P(x) = (1-\alpha)U(f_1(1-x)+f_2(x)-1)+\alpha U(f_1(1-x)-1+\delta(f_2(x)-x)+\gamma x+C)$$

Assume:

A.4 $U' > 0$, $U'' < 0$

For this problem the following efficiency results obtain.

Theorem 6. Suppose $f_2(1) < B$. Then

i) $\delta = \gamma = C = 0$ is not efficient, as type 1 land is overutilized,

ii) $\delta = \gamma = 1$, $C = 0$ is not efficient, as type 2 land is overutilized,

iii) $\delta = \gamma = 0$; $C = f_2(x^*)$ is efficient.

Proof. Let $\pi_1(x) = f_1(1-x) + f_2(x) - 1$ and $\pi_2(x) = f_1(1-x) + \delta f_2(x) + (\gamma-\delta)x + C - 1$. Then $\pi_1' = -f_1' + f_2'$, and $\pi_2' = f_1' + \delta f_2' + (\gamma-\delta)$. Note that for γ, δ, $C \geq 0$, $P''(x) < 0$.

i) It suffices to show that $P'(x^*) < 0$.

$$\begin{aligned} P'(x^*) &= (1-\alpha)U'(\pi_1)\pi_1' + \alpha U'(\pi_2)\pi_2'. \\ &= (1-\alpha)U'(\pi_1(x^*))(-f_1'(1-x^*)+f_2(x^*))+\alpha U'(\pi_2(x^*))(-f_1'(x^*)) \\ &= \alpha U'(\pi_1)f_1'(1-x^*)-\alpha U'(\pi_2)f_1'(1-x^*) \text{ since } f_1'(1-x^*) = (1-\alpha)f_2'(x^*). \end{aligned}$$

But $\pi_1 > \pi_2$, so A.4 implies $P'(x^*) < 0$.

ii) It suffices to show that $P'(x^*) > 0$.

$\pi_2' = f_1' + f_2'$, so

$$\begin{aligned} P'(x^*) &= (1-\alpha)U(\pi_1)(-f_1'(1-x^*)+f_2'(x^*))+\alpha U'(\pi_2)(-f_1'(1-x^*)+f_2'(x^*)) \\ &= \alpha f_2'(x^*)((1-\alpha)U'(\pi_1)+ U'(\pi_2)) > 0, \text{ since } f_1'(1-x^*) = (1-\alpha)f_2'(x^*). \end{aligned}$$

iii) It suffices to show that $P'(x^*) = 0$.

Note that $\pi_1(x^*) = \pi_2(x^*)$. Thus

$P'(x^*) = U'(\pi_1)(-f_1'(1-x^*) + (1-\alpha)f_2'(x^*)) = 0$ since $f_1'(1-x^*) = (1-\alpha)f_2'(x^*)$.

Q.E.D.

Corollary 3. There exists a scheme with $1 > \delta = \gamma > 0$, $C = 0$ which is efficient.

Proof. Denote by $P(x;\theta)$ the expected profile under the scheme $\delta = \gamma = \theta$, $C = 0$. Note that $P'(x;\theta) = \frac{d}{dx} P(x;\theta)$ is continuous in θ. Now $P'(x^*,1) > 0 > P'(x^*,0)$, so there exists a θ, $0 < \theta < 1$, such that $P'(x^*,\theta) = 0$.

Q.E.D.

Now we consider the case in which both society and individuals are risk neutral. We assume that the government discounts at rate $0 \leq \theta \leq 1$ all net benefits occurring to investors on type 2 land. The government then undertakes the project in the event that state 2 occurs so long as

$$B - D > \theta(f_2(x)-D),$$

where D is the total compensation paid: $\delta(f_2(x)-x) + \gamma x + C$. Denote by $\bar{x}_\theta$ the solution to

$$B - D(\bar{x}_\theta) = \theta(f_2(\bar{x}_\theta) - D(\bar{x}_\theta)).$$

Thus if $x \leq \bar{x}_\theta$, type 2 land will be taken with probability α; if $x > \bar{x}_\theta$, type 2 land will not be taken. Our concern is with how compensation schemes can be used to manipulate the government decision. Thus we consider the case in which it is non-optimal to take the land and yet the government would take it were no compensation to be paid. Denote by x^* the social optimum.

Theorem 7. Suppose that $x^* > \bar{x}_1$ and $B > \theta f_2(x^*)$. Then any lump-sum compensation scheme C satisfying $\alpha B > C > (1-\theta)^{-1}(B-\theta f_2(x^*))$ is efficient.

<u>Proof</u>. Rewriting the right inequality, $(1-\theta)C > B - \theta f_2(x^*)$ so $B - C < \theta(f_2(x^*) - C)$, so the land will not be taken if investors invest x^* on type 2 land. Since $C < \alpha B$, $\sup\{P(x): x \leq \bar{x}_1\} \leq \sup\{S(x): x \leq \bar{x}_1\} < S(x^*) = P(x^*)$, and so investors will choose $x > \bar{x}_1$. However, on this set $P(x)$ is maximized at x^*, and so investors will invest the socially optimal amount of capital on type 2 land.

Q.E.D.