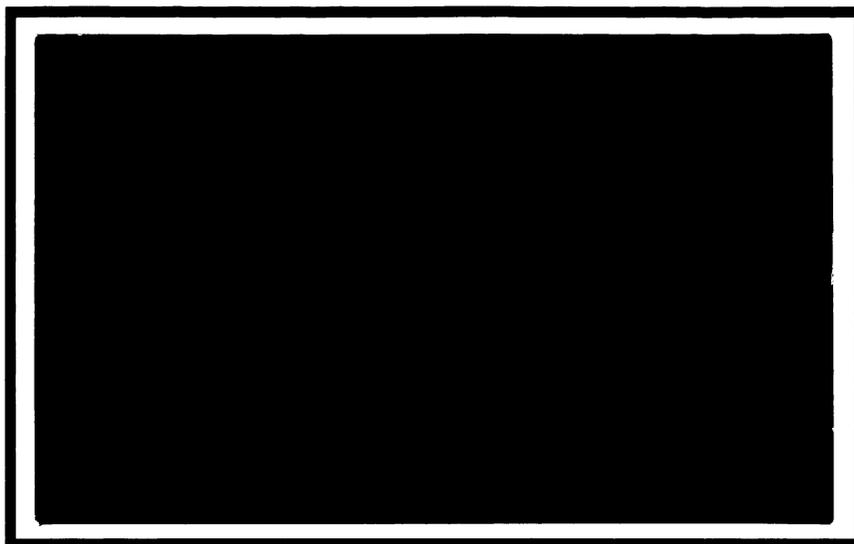


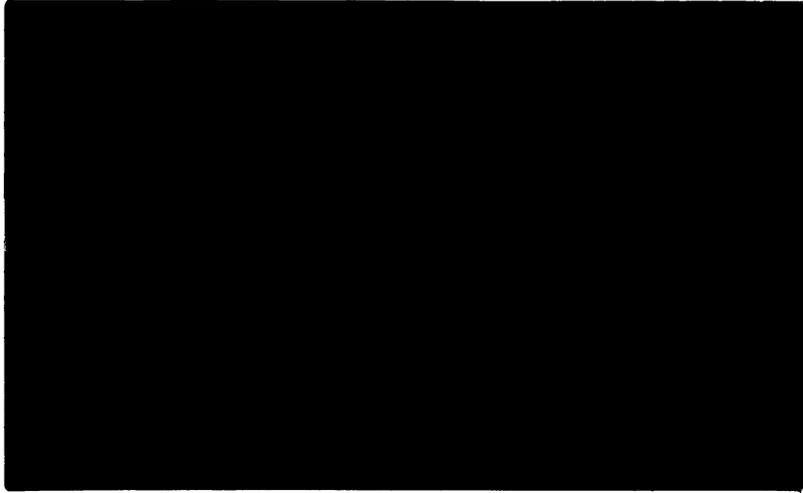
C-54

**Center for Research on Economic and Social Theory  
Research Seminar in Quantitative Economics**

**Discussion Paper**



DEPARTMENT OF ECONOMICS  
**University of Michigan**  
Ann Arbor, Michigan 48109



A DYNAMIC MODEL OF ADVERTISING

Mark Bagnoli  
Department of Economics  
The University of Michigan  
Ann Arbor, Michigan 48109

C-54

revised: November 1983



## ABSTRACT

This paper analyses the incentives for firms to advertise the price they charge for a product to imperfectly informed consumers. I find that firms do have an incentive to advertise their price. Furthermore, when the consumers are sufficiently flexible in their timing of purchases, the resultant equilibrium is a contestable equilibrium. This continues to be true even if there are such costs, so long as these costs are smaller than the cost of advertising.

## Introduction

This paper models firms' incentives to transmit information about prices to imperfectly informed consumers. The purpose of this exercise is to understand market outcomes when a firm can advertise the price it charges.

A great deal of attention has been focused on markets which have imperfectly informed consumers.<sup>1</sup> Virtually all of this work has focused upon the consumers' incentives to obtain information on prices. From a specification of these costs of obtaining information, the market outcomes have been determined and the properties of these outcomes studied.

The two central questions that this paper addresses are: Do firms have an incentive to inform consumers of their prices? What is the equilibrium when firms can transmit information?

To address these questions, I have made certain assumptions that allow me to focus on the transmission of information by firms. I construct a model in which the consumers never acquire information privately through search. Also, I employ assumptions that make it impossible for a firm to build a reputation among the consumers. Within this structure, the firms do have an incentive to transmit information. Equilibrium has the firms advertising the price they have chosen to charge. This result holds so long as the costs of advertising are not prohibitive.

To explore the equilibria generated in this market, an assumption concerning the distribution of costs to consumers of obtaining the information contained in an advertisement must be made. Initially the model is solved under the assumption that these costs are zero for every consumer.

Under this assumption the outcome has every active firm choosing to sell at the contestable price and to advertise this price weekly. This price is equal to the firm's marginal costs of production and equal to the firm's average costs including its advertising expenditure. In equilibrium, the firms all earn zero profits; the advertising costs have been incorporated in the price the consumers pay. The number of units traded is less than the number that would have been traded, had the consumers been informed. Therefore, if the consumers could be costlessly informed (say by the government), the market outcome is not Pareto efficient. On the other hand, if information transmission is costly to all, the government only has access to the same transmission media and must incur the same costs as do firms, then the outcome is Pareto efficient.

The basic model is extended in a number of directions: sunk costs in production are included, different distributions of costs to consumers of obtaining information are considered and the use of advertising as a barrier to entry is examined.

I find that introducing sunk costs into the analysis has no effect so long as they are small relative to advertising expenditures. Sunk costs are a barrier to entry and may partially insulate incumbents from competition through entry. If sunk costs are small enough so that competition from entry is still viable, the existence of sunk costs do not affect the aforementioned market outcome. When they are large enough to seriously affect the discipline imposed upon the incumbents by potential entry, the model breaks down. One reason for this breakdown is the assumption of simple strategy spaces for the firms. The work of Green

and Porter (1981) suggests that enlarging the strategy spaces of the firms would result in some sort of a "collusive" outcome in this non-cooperative game which is unattainable when the firms can only choose a price and whether or not to advertise it.

An examination is conducted for a different distribution of the consumers' costs of obtaining the advertised information. I divide the consumers into two distinct groups. The uninformed are those consumers whose costs of obtaining the information are prohibitive, and the informed are those who have zero costs of obtaining information. If one thinks of the advertising medium as a newspaper, one group never reads the paper and the other obtains at no cost the information contained in advertisements in the newspaper.

The equilibrium in this version of the model depends upon the number of uninformed consumers. If there are not too many, then they do not congest the positive externality provided by the informed consumers. In this case, the existence of the uninformed consumers does not affect the market outcome. When there are too many uninformed consumers,<sup>2</sup> they do congest the externality, and this affects the market outcome. The result is a two price equilibrium. The high price is the monopolistically competitive price and the low price is the "competitive" price (equal to the firms' marginal costs of production and equal to the firms' average costs including its advertising expenditure). In other words, in a dynamic model with an endogenous information source, I replicate the results of Salop and Stiglitz (1977).

Most of the literature that deals with market equilibria, when consumers are imperfectly informed about prices, has examined how the equilibria

are affected by different information acquisition costs. Essentially, these efforts adopt one of two viewpoints. Either the consumers have a given search strategy or they can acquire (buy) full information. The former view is best represented in Wilde and Schwartz (1979). The latter view has been adopted in Salop and Stiglitz (1977), Varian (1980) and others. This latter view simply assumes the existence of the availability of full information.

However, this paper inquires into the incentives for the firms to tell the consumers of their prices. In other words, instead of simply assuming that full information is available, I seek to determine if the firms would willingly provide it. They do this by advertising prices.

Finally, let me briefly mention an outstanding paper by Gerald Butters (1977). In that paper, he considered the advertising of prices by firms via leaflets. The firm sent a letter to a consumer telling customers what price the firm was charging. Butters' price distribution results from his assumption that the firms cannot direct their advertisements. They cannot determine to whom they are mailing the information. When this assumption is dropped, as in Bagnoli (1982) very different results obtain. The firms never advertise to a consumer receiving another firm's advertisement. Thus, each consumer ends up knowing just one price. Obviously, this suggests that the medium used, "leaflets" or "newspapers", has a relatively strong influence on the market outcomes. The logical next step in understanding markets in which firms transmit prices is to provide multiple advertising media to the firms.

Section 2 of this paper presents the basic model that will be employed. The equilibrium of the model is derived in Section 3. Included in this section is the proof that firms do have an incentive to transmit information and an analysis of the introduction of sunk costs. Section 4 explores the effects of altering the assumption on the distribution of consumers' costs of obtaining the information advertised by the firm. Section 5 discusses the use of advertising of prices as a barrier to entry and it is followed by my conclusions.

## Section 2

Let the time intervals be indexed by  $T=1, 2, \dots$  and let each interval  $T$  be divided into  $\tau$  periods indexed by  $t=1, 2, \dots, \tau$ . There are  $L$  potential buyers in each interval and they choose a period in which to shop. Throughout, the interval  $\tau$  is assumed short, so that discounting is not done within it. This structure will be described in the following language: Each week there are  $L$  potential consumers choosing a day on which to shop.

Each consumer has the same demand curve  $d(p)$ , with  $d'(p) < 0$ . I assume that the consumers know at each date  $t$  which stores are open (as well as their locations), costlessly. There is a newspaper published in each period  $t$  and it may contain advertised prices. Every consumer reads the newspaper<sup>3</sup> and if it contains ads, the consumer obtains this information on prices costlessly. All other means of information acquisition are assumed to be prohibitively expensive.

Finally, to avoid the complications associated with the potential for firms to develop reputations, a form of memorylessness is imposed. The  $L$  consumers read the newspaper and choose a day to shop. Once they have shopped, they begin collecting information again but they discard (forget) any information collected prior to their shopping day.

There are an infinite number of identical potential firms,  $N$ , large enough so that there is an always inactive firm in every equilibrium. They each have costs of production  $C(q)=F+c(q)$  which are represented by a U-shaped average cost curve. A portion of  $F$ ,  $S(F) \geq 0$  is a sunk cost incurred if the firm is ever active.

The firms, unlike the consumers, are perfectly informed: they "play" a game of complete information. A firm can choose to advertise its price in the newspaper. An advertisement costs  $A$  dollars per day to run and the firm may specify, in the ad, the length of time an advertised price is valid<sup>4</sup>. Finally, the firm is legally obligated to honor its advertisements.

This last assumption is necessary because the consumers shop but once a week and choose a store at which to shop based on the information available. In conjunction with the assumption that the consumers are memoryless; this implies that a firm always has an incentive to advertise as low a price as possible so as to maximize the number of consumers that choose to shop at its store. Unfortunately, it does not have any incentive to sell at the advertised price. If the firm charges the monopoly price, it does not affect the current number of sales and charging the monopoly price while advertising a low price does not affect future sales because consumers are memoryless. As a result, honesty is not an inherent feature of advertising in this model but since firms are, in fact, legally bound to advertise truthfully it seems relatively harmless to construct the model as I have.

My assumptions have two implications which I would like to note. First, the firms choose a lifetime for an ad which is not longer than  $\tau$ . Because the consumers are memoryless, there are no consumers who recall seeing the ad  $\tau+1$  periods after the advertisement was run. Second, the consumers believe that any firm that chooses not to advertise charges the monopoly price. Since the consumers are memoryless, the firm cannot affect its number of shoppers unless it advertises. Because the number of shoppers entering the store is independent of the price charged by a firm

that does not advertise, the firm maximizes profits by charging the monopoly price. With the very limited information available to the consumers, there is no other rational belief on the part of the consumers.

Section 3

Assume, initially, that advertising is prohibited and that there are no sunk costs. The consumers only know which stores are open; they have no information on the prices being charged by the active firms. As was stated above, the consumers will act as if each firm were charging  $r$  in each period. Therefore, the consumers are indifferent about their shopping day in any week. Consequently, I assume that they divide themselves evenly among the active firms, and each firm sees the same number of shoppers as any other firm in each period.

The prohibition on advertising restricts the firms' strategy sets to the set of possible prices; a subset of  $\mathbb{R}_+$ . A firm's strategy choice is a selection of a price to charge. The prohibition on advertising leaves the firm no means by which to affect the number of consumers shopping in its store. Thus, firm  $i$ 's profits in period  $t$  are

$$\pi_t^i = p d(p) \ell/n - C(d(p) \ell/n)$$

if  $n$  firms are active in period  $t$  and  $i$  charges the price  $p$ . Recall that  $i$ 's profits from inactivity are zero,  $\pi(\emptyset) = 0$ . Define  $p_m(n)$  by

$$p_m(n) = \operatorname{argmax}_p p d(p) (\ell/n) - C(d(p) \ell/n),$$

define  $n_1$  as the largest  $n^5$  such that

$$0 = p_m(n_1) d(p_m(n_1)) (\ell/n_1) - C(d(p_m(n_1)) \ell/n_1),$$

and let  $r = p_m(n_1)$ .

Theorem 1: If advertising is prohibited,  $n_1$  firms charging  $r$  in each period  $t$  is a market outcome.

Before proving this theorem, note that a set of strategies that constitute a Nash equilibrium have not been specified. Instead, I provide a

description of the market in each period. I take this approach because there is a large set of strategy vectors each of which result in the same market outcome:  $n_1$  firms charging  $r$  in each period, and each of them constitutes a Nash equilibrium. This means that there are multiple Nash equilibria but, as I will show in theorem 2, all equilibria result in the same market outcome. I am unable to specify which subset of the  $N$  firms are active in each period but I can determine which market outcome arises.

Proof: Consider a candidate Nash equilibrium in which firms 1 through  $n_1$  charge  $r$  and the  $N-n_1$  firms choose inactivity in each period. Let this set of strategies be labelled  $s^*$ . By the definition of  $n_1$ ,

$$\pi_t^i(s^*) = 0 \quad i = 1, 2, \dots, n_1; \forall t.$$

By assumption, there are no costs to choosing to be always inactive so that

$$\pi_t^i(s^*) = 0 \quad i > n_1; \forall t.$$

To show that  $s^*$  constitutes a Nash equilibrium, I must show that no firm  $i$ , has a feasible strategy which yields larger profits than  $s_i^*$ .

Firm  $i < n_1$  earns

$$\pi^i(s^*) = \sum_T \beta^T \sum_{t=1}^T \pi_t^i(s^*) = 0$$

where  $\beta$  is the discount rate. Consider an alternative strategy  $s_i$  which is a specification of a sequence of prices  $\{p_j\}_{j=1}^{\infty}$ , which for some  $j$  has  $p_j \neq r$ .

If firm  $i$  uses  $\{p_j\}_{j=1}^{\infty}$ , then in any period  $j$  in which  $p_j \neq r$ ,

$$\pi_j^i(s_{(i)}^*, s_i) = pd(p)^{j/n_1} - C(d(p)^{j/n_1})$$

which is the negative by the definition of  $n_1$ ,

Therefore,

$$\pi^i(s^*, s_i) = \sum_T \beta^T \sum_{t=1}^T \pi_t^i(s_i^*, s_i) \leq 0,$$

and I conclude that firm  $i \leq n_1$  has no alternative strategy which earns larger profits.

Now, consider a currently inactive firm  $i > n_1$ . Recall that it earns zero profits. If it adopts an alternative strategy, it must choose to be active in some period  $j$ . Thus

$$\pi_j^i(s_{(i)}^*, s_i) = p_j d(p_j) \ell / (n_1 + 1) - C(d(p_j) \ell / (n_1 + 1))$$

By the definition of  $n_1$ ,

$$\pi_j^i(s_{(i)}^*, s_i) < 0.$$

This means that if  $i$  adopts  $s_i$  (an alternative to being inactive in each period),

$$\pi_j^i(s_{(i)}^*, s_i) \sum_{\tau} \beta^{\tau} \sum_{t=1}^T \pi_t^i(s_{(i)}^*, s_i) < 0.$$

As a result,  $s^*$  is shown to be a Nash equilibrium, and therefore  $n_1$  firms charging  $r$  in each period is a market outcome. □

Notice that  $s^*$  leaves the active firms indifferent between being active and being inactive. Since the consumers know costlessly which firms are open and since there are no sunk costs, any firm finds all sequences of activity and inactivity equally profitable so long as the number of active firms in any period is  $n_1$ . Thus, any set of sequences of activity and inactivity by the  $N$  firms that results in  $n_1$  firms charging  $r$  in each period will constitute a Nash equilibrium.

As I stated before a multiplicity of Nash equilibria is generally a defect in a model. However, it ought not to be, so long as every Nash equilibrium generates the same market outcome. Theorem 2 shows that this is the situation here.

Theorem 2: If advertising is prohibited,  $n_1$  firms charging  $r$  in each period is the unique market outcome.

Proof: Assume that there is a Nash equilibrium  $\tilde{s}$  which does not have  $n_1$  firms charging  $r$  in each period.

Suppose that  $\tilde{s}$  has  $n < n_1$  firms active in period  $t$ . Consider the always inactive firm  $N$ . If  $s_N = \{\phi\}_{j=1}^{\infty}$  then  $\pi^N(s_{(N)}, s_N) = 0$  for all  $s_{(N)}$ . Since  $n < n_1$  in period  $t$ , if  $N$  chose to enter and charge  $p_m(n+1)$ , it would earn positive profits in period  $t$ . Because there are no sunk costs, this strategy yields larger profits than  $s_N = \{\phi\}_{j=1}^{\infty}$  which means that any  $\tilde{s}$  that causes  $n < n_1$  active firms in some period is not a Nash equilibrium.

If  $n > n_1$  in some period  $t$  then by the definition of  $n_1$  every active firm in that period must earn negative profits. Furthermore, if

$$\pi^i(s) \equiv \sum_T \beta^{T-t} \sum_{t=1}^T \pi_t^i(\tilde{s}) < 0 \text{ for some } i,$$

$i$  could earn larger profits by choosing inactivity. Thus, if in some period  $t$ ,  $n > n_1$ , and  $\tilde{s}$  is a Nash Equilibrium, then there is some period  $j$  such that  $\pi_j^i(\tilde{s}) > 0$  for each active firm in  $t$ . Otherwise  $\pi^i(\tilde{s}) < 0$  for each active firm in  $t$ .

For  $\pi_j^i(\tilde{s}) > 0$  in some period  $j$ , there must be  $n < n_1$  firms active in  $j$ . I have already shown above, that if  $\tilde{s}$  implies  $n < n_1$  in some period then  $\tilde{s}$  cannot be a Nash equilibrium. Thus, if  $\tilde{s}$  implies  $n > n_1$  in some period, it cannot be a Nash equilibrium because it must also have  $n < n_1$  in some other period.

Finally if  $n = n_1$  in  $\tilde{s}$  and  $\tilde{s}$  does not have  $n_1$  firms charging  $r$  in each period then  $\tilde{s}$  has at least one firm charging  $p \neq r$ . If firm  $i$  charges  $p \neq r$  then it earns profits

$$\pi_t^i(\tilde{s}) = pd(p)\ell/n_1 - C(d(p)\ell/n_1) < 0.$$

Thus, all Nash equilibria result in  $n_1$  firms charging  $r$  in each period.  $\square$

If an advertising medium is now available, the firm's strategy space is enlarged. In any period  $t$ , it may now choose to advertise the price it wishes to charge. If the firm chooses to advertise, it must determine the lifetime of the ad and must decide whether to place restrictions on the buyers. In other words, the firm must specify how long the advertised price will be adhered to and whether it will restrict the number of units it has for sale or will stand ready to sell to all customers.

Suppose that a firm has decided to charge  $p$  in every period and that it will inform the consumers via advertisements. The firm, under these assumptions, advertises in the first period and pays  $A$  dollars to do so. That advertisement informs every potential consumer in the first  $\tau$  periods. If the firm chose to run the ad again in period  $t < \tau$ , it would inform some consumers (those who had shopped prior to  $t$ ) but would be providing to some consumers information they already had. In other words, to minimize the costs of disseminating information about  $p$ , this firm advertises once a week and states in the ad that  $p$  will be in effect for the whole week. The firm need not choose a longer lifetime because no one will remember it, but it must choose to advertise at least once a week if it wishes to inform the shoppers of the price it has chosen for that week. Under this plan, the firm spends  $A$  dollars per week to advertise.

Define  $q^*$  as the quantity that minimizes this firm's daily average costs and  $p^*$  as that associated level of average costs.

$$q^* \equiv \operatorname{argmin} \left\{ \left[ C(q) + \frac{1}{\tau} A \right] / q \right\}$$

$$\text{and } p^* = \left[ C(q^*) + \frac{1}{\tau} A \right] / q^*$$

Further, let  $d(p^*)\ell/n^* = q^*$ ,<sup>6</sup> and assume that  $r > p^*$ .

Suppose that  $n^*$  firms choose to charge  $p^*$  in every period. They also choose to have each period's shoppers informed of their prices. Thus, they advertise  $p^*$  once a week and the ad guarantees that the price will be in effect for the whole week. Let  $a^*$  represent this strategy of adopting both this advertising strategy and the price  $p^*$ . If  $n^*$  firms do adopt  $a^*$ , a representative firm  $i$  earns

$$\sum_{t=1}^{\tau} [p^*d(p^*)\ell/n^* - C(d(p^*)\ell/n^*)] - A$$

each week. Rewriting yields,

$$W_T^i = \sum_{t=1}^{\tau} [p^*d(p^*)\ell/n^* - C(d(p^*)\ell/n^*) - \frac{1}{\tau} A]$$

which, using the definitions of  $p^*$ ,  $n^*$  and  $q^*$  is equal to zero. Note that it is appropriate to set the total number of shoppers in  $t$  to  $\ell$  because  $\ell$  consumers shopping each period is consistent with utility maximizing behavior when the consumers shop once each week. With  $\ell$  consumers shopping in each period, all  $L$  consumers obtain the unit they desire at  $p^*$ . If the shopping pattern were different, the consumers could never do better but could do worse.<sup>7</sup>

Obviously, if  $W_T^i = 0$ ,  $i$ 's profits from adopting  $a^*$  when  $n^* - 1$  others do adopt  $a^*$  and the remaining firms choose to be always inactive, are

$$\pi^i = \sum_T \beta^T W_T^i = 0.$$

The modelling difficulties posed by the availability of an advertising medium are the complications that may arise when the consumers choose a shopping pattern. To see this clearly, consider the consumers' problem when  $n^* - 1$  firms adopt  $a^*$ , one firm chooses to be open but not advertise and the remaining firms choose inactivity in  $t$ . Profit maximizing behavior by the  $n^* - 1$  firms results in a restriction of sales to  $q^*$  units per firm, per period. In period  $t$ , the consumers know that for the week,  $n^* - 1$  firms

will sell at  $p^*$ ,  $(n^* - 1)q^*$  units per day. The consumers expect the firm that has chosen to be open but not to advertise, to charge  $p_m(n^*)$ . This results in the consumers' being forced to determine a shopping pattern without knowing their future prospects. They do know that the set of prices available in the future (for the remainder of the week) can not be any worse than the prices they face in  $t$ . Unfortunately, knowing this is not enough to completely characterize their behavior. One must also specify the means by which the  $(n^*-1)q^*$  units are allocated when more than  $(n^*-1)q^*$  consumers shop at the  $n^*-1$  firms. The firms must sell the units at  $p^*$  as promised by their advertisement, thus some mechanism other than price must be used to allocate the units.

Also, the consumers' ability to understand the market must be specified. In particular, one must specify their ability to understand the entry and exit decisions of the firms. If, in the present circumstances, the consumers acted believing that a firm would enter using  $a^*$  next period, their shopping decision would be different from the one that arises if they do not expect entry during the week.

Since rationing is not the primary consideration, I will assume that an "efficient" rationing mechanism is used. If  $\mu$  is the number of consumers that choose to shop in  $t$ , then the rationing mechanism,  $M$ , allocates the  $(n^*-1)q^*$  units so that no additional consumer surplus is foregone as a result of rationing. This implies that the residual demand curve is simply

$$R(p) = d(p)\mu - q^*(n^*-1)$$

Obviously, utility maximization given the rationing mechanism in conjunction with the consumers' beliefs about entry determine  $\mu$ . Also,  $\mu$  is such that every consumer is indifferent between shopping on  $t$  or any other day.

Suppose that the consumers expect rapid entry at  $p^*$  in  $t+1$ , and that

they choose not to buy any units from the firm that is not advertising. The total number of units available for purchase at  $p^*$  during the week is  $(n^*-1)q^*\tau + (\tau-t)q^*$  which, by the definition of  $n^*$  equals  $L-q^*$ . In other words, the consumers find that there is excess demand if they do not purchase from the firm that is not advertising. Thus, even if entry occurs rapidly, if  $n^*-1$  firms using  $a^*$  and one active firm does not advertise in  $t$ , rationing must take place.

If one refers to the outcome of  $M$  as the cost to the consumers of entering the mechanism,<sup>8</sup> and one makes the assumption that these costs are non-decreasing in excess demand<sup>9</sup> then utility maximization guarantees that  $\mu > (n^*-1)q^*$  in period  $t$ . Rationing does take place in period  $t$ . In general,  $\mu$  depends on the particular rationing mechanism and the vector of prices the consumers expect to see during the week

$$\mu = \mu(M, p^t, p^{t+1}, \dots, p^{t+\tau}).$$

From theorem 3, we will see that two conditions must be met if enough discipline is to be placed upon the active firms by the potential for future entry. Let  $\gamma$  be the number of consumers shopping at a high price store in period  $t$ . Obviously,  $\gamma$  is derived from  $M$  and  $\mu(M, p^t, p^{t+1}, \dots, p^{t+\tau})$  such that the  $\gamma$  consumers are just indifferent between shopping in  $t$  and waiting; also, they are just indifferent between shopping at the high price store and incurring the costs of the rationing mechanism associated with an attempt to purchase at a lower price. Thus, I write  $\gamma$  as

$$\gamma = \gamma(M, p^t, p^{t+1}, \dots, p^{t+\tau}).$$

Let  $v$  stand for the  $n^*-1$  vector of  $p^*$ 's:  $v = (p^*, p^*, \dots, p^*) \in \mathbb{R}_{++}^{n^*-1}$ .

Finally, when  $n^*-1$  firms have adopted  $a^*$ , let  $(v, \rho)$  represent the consumers' beliefs about their future alternatives. With these definitions, the two conditions referred to above are:

$$C1: \gamma(M, (v, r), (v, \rho), \dots, (v, \rho)) \leq \ell/n_1$$

$$C2: pd(p)\gamma(M, (v, p), (v, \rho), \dots, (v, \rho)) - C(d(p)\gamma) - A \leq 0 \quad \forall p.$$

These two conditions should be interpreted as putting limits on the number of captive consumers, those "forced" to buy from a high price store in  $t$ . They require that the consumers perceive enough flexibility in their alternatives so that the equilibrium shopping pattern does not permit a firm to profitably specialize in those consumers who choose to shop at a high price store in  $t$ .

The second condition should be interpreted as accomplishing this by requiring a rapid response by entrants to situations that induce entry. In this model, entry occurs in the future and more importantly, there is but one advertising medium. This means that the entrant's ability to disseminate information is constrained by the frequency with which the paper is published. With a potential abuse of terminology, the information transmission mechanism acts as a potential barrier to entry.

To see this clearly, consider the effect of doubling the frequency of advertising. Think of it as a situation in which morning and afternoon editions of the newspaper are now published each day. The number of shopping periods rises from  $\tau$  per week to  $2\tau$  per week. It immediately implies that there are fewer shoppers in each shopping period, and it implies that the benefits from shopping at a low price store increase. This latter effect follows from the definition of  $p^*$ :

$$p^* = [C(q^*) + \frac{1}{\tau} A]/q^*.$$

If there are now  $2\tau$  periods in a week, the low price stores charge  $p^*(2\tau)$  not  $p^*(\tau)$  and  $p^*(2\tau) < p^*(\tau)$ . These two effects combine to reduce  $\gamma$ , the number of captive consumers in  $t$ .

Before proceeding to theorem 3, one can illustrate the constraints imposed by conditions C1 and C2. Condition C1 requires that the residual

demand curve  $R(p)$  is as drawn in figure 1.

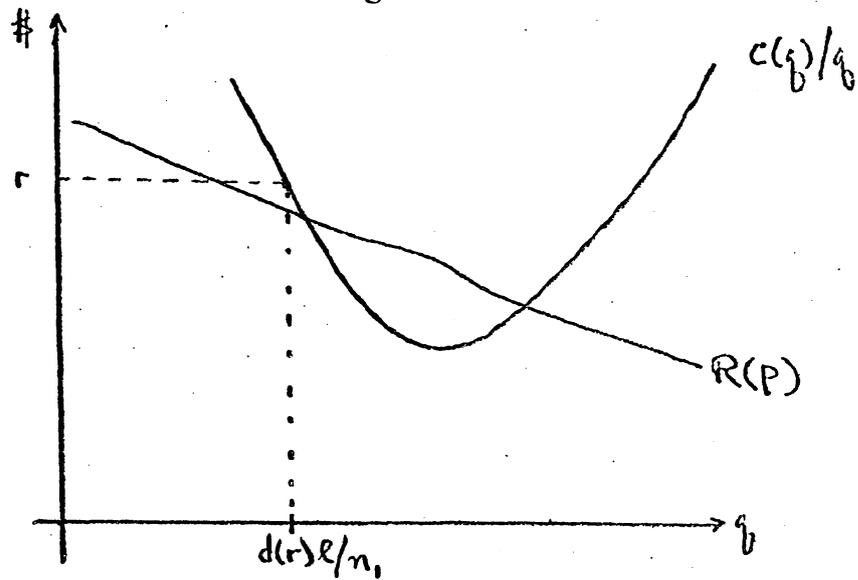


Figure 1

Condition C2 requires that  $R(p)$  is as drawn in figure 2, also.

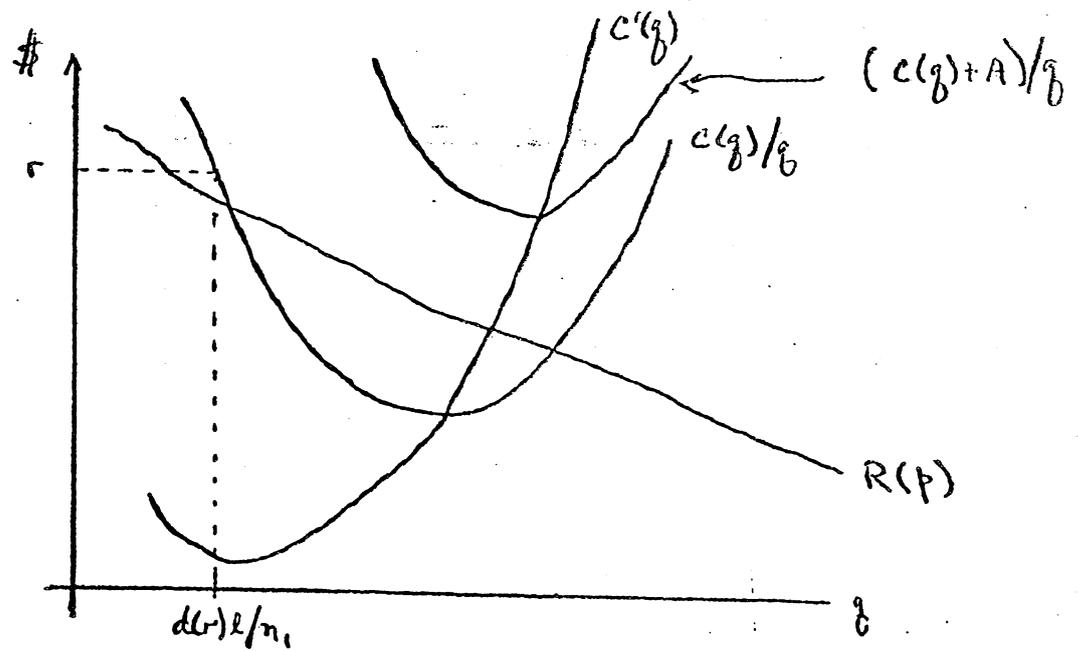


Figure 2

The conditions taken together imply:

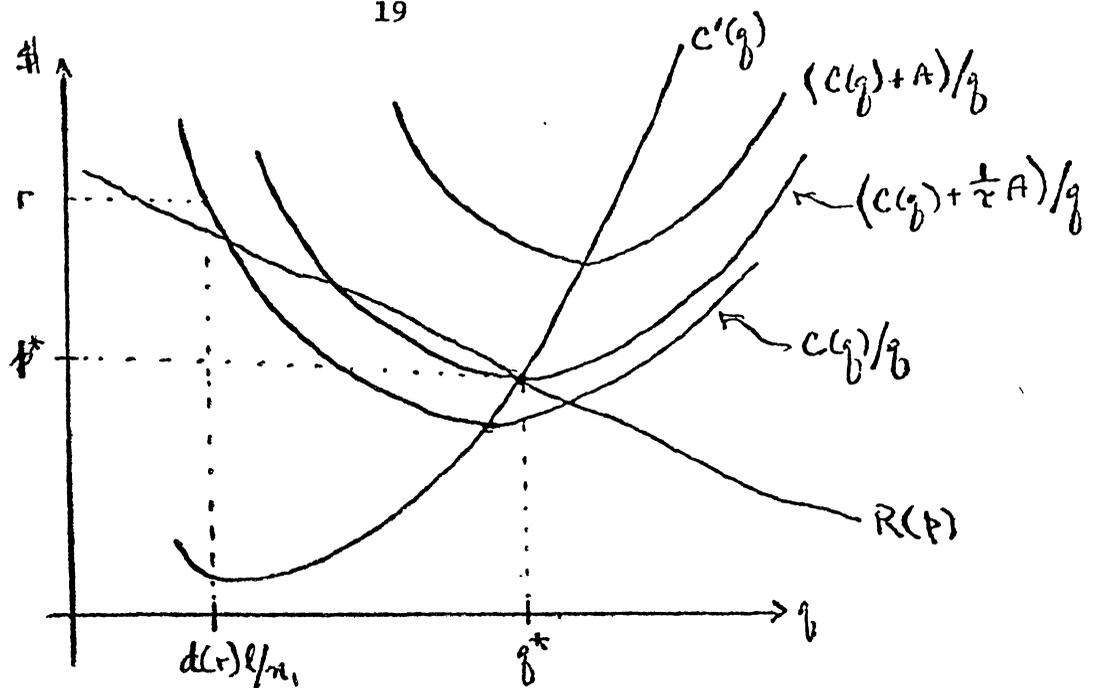


Figure 3

Theorem 3: If C1 and C2, then  $n^*$  firms advertising  $p^*$  once a week is a market outcome.

Proof: Let firms 1 through  $n^*$  adopt  $a^*$  and let the remaining firms adopt  $b^*$ . Define  $b^*$  as a strategy which has the firm inactive so long as entry will result in the firm earning losses. If the history shows that  $n^*$  firms have not advertised  $p^*$ , i.e. entry will yield non-negative profits, then  $b^*$  results in entry using the advertising strategy  $a^*$ . Let this vector of strategies be represented by  $s^*$ , then

$$\sum_T \beta^T \sum_{t=1}^T \pi_t^i(s^*) = 0 \quad i = 1, 2, \dots, N.$$

This is 0 for firms 1 through  $n^*$  by the definition of  $p^*$  and  $n^*$  and it is 0 for the remaining firms because  $\pi_t(\phi) = 0$ .

Consider a firm  $i > n^*$ . To be active, it must advertise since  $n^*$  firms are selling  $q^*$  units at  $p^*$  each period. Since  $n^*$  firms are using  $a^*$ , advertising  $p > p^*$  yields profits of  $-A$  because no one shops in  $i$ 's store. Advertising  $p = p^*$  for a week yields

$$\sum_{t=1}^{\tau} [p^*d(p^*)\ell/(n^*+1) - C(d(p^*)\ell/(n^*+1))] - \frac{1}{\tau} A]$$

which is negative by the definitions of  $n^*$  and  $p^*$ . Furthermore, advertising  $p^*$  for any shorter length of time yields negative profits. Thus, for any firm  $i > n^*$ , there is no strategy that yields larger profits than choosing  $b^*$ .

Now consider an active firm  $i \leq n^*$ . It is earning zero profits using  $a^*$  and it would earn zero profits if it chose inactivity. If it chooses not to advertise, it earns

$$pd(p)\gamma(M, (v,r), (v,\rho), \dots, (v,\rho)) - C(d(p)\gamma) \leq 0 \quad \forall p,$$

by C1. If the firm chose to advertise  $p > p^*$ , it earns

$$pd(p)\gamma(M, (v,p), (v,\rho), \dots, (v,\rho)) - C(d(p)\gamma) - A.$$

Since entry would occur in the next period, the firm must cover its entire advertising expenditure in  $t$ . By C2, choosing to advertise  $p > p^*$  also yields non-positive profits.

Thus,  $s^*$  is a Nash equilibrium.  $\square$

Combining this theorem with the following lemma, one concludes that firms do have an incentive to transmit information to the otherwise uninformed consumers.<sup>10</sup>

Lemma 1: All Nash equilibria involve firms that advertise their price.

Proof. Recall from theorems 1 and 2 that the unique market outcome has  $n_1$  firms charging  $r$  when advertising is prohibited. To see that this is not supportable as a Nash equilibrium when advertising is permitted, consider the result of a firm, active in  $t$ , choosing to advertise  $p < r$ . This results in rationing because all consumers would rather buy at  $p$  than  $r$ . The firm choosing to advertise can sell all that it wishes. This is the largest quantity such that the firm's marginal costs equal  $p$ . Since  $r > p^*$ , there

is a  $p$ , ( $r > p > p^*$ ) such that the firm earns positive profits. Therefore, there is no Nash equilibrium when no advertising is done.  $\square$

Before extending the analysis to encompass the existence of sunk costs in production, I would like to consider briefly the institution of rainchecks. In static models of firms that choose prices and have U-shaped average costs of production the institution of rainchecks is thought to guarantee an equilibrium in which each firm charges a price equal to marginal and average cost.<sup>11</sup>

The idea is that the price equal to marginal and average costs fails to be a Nash equilibrium without rainchecks because each active firm has an incentive to raise its price and earn positive profits.<sup>12</sup> It earns positive profits because its consumers have nowhere else to go. Rainchecks are thought of as a way to provide the consumers with a means of obtaining the product at the low price when the firm they would have frequented raises its price. This firm would no longer obtain positive profits because it no longer receives any customers and, as a result, a Nash equilibrium exists in which the active firms chose a price equal to marginal and average costs and no rainchecks are issued.

In a static model, a promise to provide the product tomorrow at today's price (a raincheck) is inconsistent. In my explicitly dynamic model, rainchecks make more sense but fail to accomplish the desired objective.

To see this, suppose that the first  $n^*-1$  firms use  $a^*$  but offer rainchecks too. Let firm  $n^*$  choose to advertise a higher price, and suppose that the remaining  $N-n^*$  choose  $b^*$ .<sup>13</sup> The result in this scenario is to leave firm  $n^*$  with no shoppers. The consumers can shop at another store and receive a raincheck entitling the bearer to a unit of the product at  $p^*$ . In the next period, one firm enters using  $a^*$ . However, there will be excess demand in this period too. The number of units for sale is  $n^*q^*$ . There are  $\ell = n^*q^*$  consumers opting to shop in this period

but there are also  $\ell/n^*$  rainchecks outstanding. In other words, in every period after the rainchecks are issued, there is excess demand. This means that issuing rainchecks simply implements a particular rationing mechanism. It is equivalent to a mechanism that randomly selects  $\ell/n^*$  customers who will fail to obtain the product each week.<sup>14</sup> Thus, it is not the case that firm  $n^*$  receives no customers. It receives  $\gamma(M^I, (v, p_{n^*}), (v, p^*), \dots, (v, p^*))$  which, as was indicated, must satisfy C2 for an equilibrium to exist.

Extending the model (with its "efficient" rationing mechanism) to handle the existence of sunk production costs is reasonably straightforward. Let  $S(F)$  represent the sunk part of the firm's fixed costs: the costs incurred by a firm when exiting the market. When advertising is prohibited, the existence of these sunk costs simply restricts the set of Nash equilibria; they eliminate those Nash equilibria in which firms change from active to inactive. This follows from the observation that the equilibria involve earning zero profits. Thus, any switch from activity to inactivity leaves that firm earning negative profits and thus can be dominated if the firm chooses never to be active.

When advertising is permitted, sunk costs restrict the scope of discipline placed on active firms by future potential entry. Sunk costs act as a deterrent to entry because they impose a cost to exiting. Thus, theorem 3 must be modified to account for this change in the relative strengths of the incumbents and potential entrants.

An outcome equivalent to theorem 3 results whenever it is in the best interest of the incumbent to exit if entry were induced by the incumbent's strategy. In this case, the incumbent must find that the cost of advertising and selling at  $p^*$  for a week when  $n^*-1$  other firms are using  $a^*$ , less its short term profits exceed the sunk costs of production. The cost of competing after entry is  $D$ :

$$D = \sum_{t=1}^{\tau} [p^*d(p^*)\ell/(n^*+1) - C(d(p^*)\ell/(n^*+1))] - A.$$

Given C1, it is obvious that no advertising and then competing with the entrant results in losses no smaller than D because the firm's short term profits are

$$pd(p)\gamma(M, (v,r), (v,\rho), \dots, (v,\rho)) - C(d(p)\gamma) \leq 0.$$

Let  $\tilde{p}$  represent the profit maximizing price to advertise in t when  $n^* - 1$  firms advertise  $p^*$  for a week, and the remaining firms choose to be inactive in t. The firm choosing to advertise  $\tilde{p}$  earns

$$\tilde{p}d(\tilde{p})\gamma(M, (v,\tilde{p}), (v,\rho), \dots, (v,\rho)) - C(d(\tilde{p})\gamma) - A$$

which by C2 is not positive. Therefore, a sufficient condition for potential entry to place enough discipline on the currently active firms is

$$S(F) \leq D,$$

because the firm's short term profits are always non-positive. If the sunk costs of production are less than the costs of competing after entry, the incumbent will choose to exit rather than compete if entry occurs. As a result, if  $S(F) \leq D$  and C1 and C2 hold then a market outcome has  $n^*$  firms advertising  $p^*$  once a week. However, the set of Nash equilibria has been reduced by the introduction of sunk costs. All of the equilibria that involved firms changing from being active to being inactive are eliminated, just as in the case when advertising was prohibited.

The purpose of analyzing the model with positive sunk costs was to show that the model was not very sensitive to the assumption that sunk costs are zero. The discussion, above, verifies that this is the case.

However, a more interesting observation can be made. This is an example of a contestable equilibrium when sunk costs are positive. As is well known, sunk costs put entrants at a competitive disadvantage because the incumbent

always stands ready to accept operating losses not larger than its sunk costs. The entrant can always choose not to enter and earn zero profits. Consequently, it does. In this model, it is not that simple. The incumbent retains the competitive advantage derived from the existence of positive sunk costs but it now has a (potentially) offsetting disadvantage. Because it is the incumbent, it must choose an advertising strategy knowing that entrants will then be able to respond to it. The incumbent is free to respond to the entrants but to do so requires additional advertising. In each stage of this: one chooses, the other responds, the first responds, etc., the previous actor is required to expend resources advertising that are lost if the actor's opponent chooses not to leave the market. Thus, while sunk costs give the incumbent a competitive advantage vis à vis the potential entrant, the necessity to commit to an advertised price first may constrain the incumbent's choice. The incumbent becomes less flexible in its ability to compete with entrants. The requirement that  $D \geq S(F)$  is simply a sufficient condition that ensures that the incumbents flexibility is diminished enough to negate the advantage given it by the existence of positive sunk costs.

Section 4:

In section 3, I assumed that every consumer learned of the firm's advertisements. In this section, I analyse the effects of relaxing this assumption. Instead of assuming that all  $L$  consumers "read the newspaper", I assume that  $\tau U < L$  of them do not.<sup>16</sup> One way to generate this situation is to assume that  $L - \tau U$  consumers have zero cost of reading the newspaper and the  $\tau U$  consumers have costs of reading the newspaper which are so large that they never choose to. If each of the  $U$  consumers faced a cost  $c_i$ ,  $i \in \tau U$  such that  $c_i > r - p^*$ , then they would never choose to read the paper. Thus, one justification for the assumption that  $\tau U$  consumers do not read the paper but  $L - \tau U$  do is that the distribution of costs of becoming informed is of the type just described.<sup>17</sup>

Since these uninformed consumers face the same problem as do any consumers when no prices are advertised, I assume that they act in the same way. Knowing only the location of the active firms, the uninformed consumers divide themselves evenly among the active firms. This means that each of  $n$  active firms finds that  $U/n$  of the uninformed consumers shop at its store.

Let  $u^*$  solve  $rd(r)u^* - C(d(r)u^*) = 0$ , and assume that there are no sunk costs in production. Interpret  $\gamma$  as the number of informed shoppers who do not shop at a low price store when rationing is present. If  $\alpha$  is the firm's share of the uninformed consumers then C1 and C2 become

$$C3: \gamma(M, (v, r), (v, \rho), \dots, (v, \rho)) + \alpha \leq \ell/n_1$$

$$C4: pd(p)[\gamma(M, (v, p), (v, \rho), \dots, (v, \rho)) + \alpha] - C(d(p)(\gamma + \alpha)) - A \leq 0 \quad \forall p.$$

Where the adjustment in the conditions simply accounts for the existence of the uninformed consumers.

Theorem 4: if C3, C4 and  $U/n^* \leq u^*$  then  $n^*$  firms advertising  $p^*$  each week is a market outcome.

Proof: Since  $U/n^* \leq u^*$ ,  $rd(r)U/n^* - C(d(r)U/n^*) \leq 0$ . This implies that it is unprofitable to specialize in uninformed consumers. As a result, by the proof of theorem 3,  $n^*$  firms advertising  $p^*$  each week is a market outcome.  $\square$

Theorem 4 simply confirms that if there are not too many uninformed consumers ( $U/n^* \leq u^*$ ) then they do not congest the externality provided by the consumers who do read the newspaper. Consequently, their existence has no effect on the market outcome analysed previously.

Theorem 5: if C3, C4 and  $U/n^* > u^*$  then the market outcome has  $n_2 - k^*$  firms charging  $r$  and not advertising and  $k^*$  firms advertising  $p^*$  each week.

Proof: Define  $n_2$  implicitly as the largest  $n$  such that  $rd(r)U/n_2 - C(d(r)U/n_2) = 0$ , and  $k^*$  by  $p^*d(p^*)[\ell/k^* + U/n_2] - C(d(p^*)(\ell/k^* + U/n_2)) - \frac{1}{\tau}A = 0$ ,<sup>18</sup> where  $\ell/k^*$  is a firm's share of the "informed" consumers. Let  $s^*$  represent a set of strategies which has firms 1 through  $k^*$  using  $a^*$ , firms  $k^* + 1$  through  $n_2$  charging  $r$  each period and not advertising, and the remaining firms using  $b^*$ . To show that this candidate ( $s^*$ ) is a Nash equilibrium, I must show that no firm has available strategy whose use results in larger profits.

Consider first, one of the inactive firms. By the definition of  $n_2$ , entering without advertising earns negative profits because

$$rd(r)U/(n_2+1) - C(d(r)U/(n_2+1)) < 0.$$

Entering and advertising  $p^*$  earns

$$\sum_{t=1}^{\tau} [p^*d(p^*)[\ell/(k^*+1) + U/(n_2+1)] - C(d(p^*)(\ell/(k^*+1) + U/(n_2+1)))] - A$$

which is negative by the definition of  $k^*$ . Entering and advertising  $p > p^*$  earns non-positive profits by C4. Thus, no inactive firm has an alternative strategy that earns larger profits than inactivity.

Consider, now, one of the firms specializing in the uninformed consumers.

Opting for inactivity earns zero profits. If it chooses to advertise  $p = p^*$ , it earns

$$\sum_{t=1}^{\tau} [p^*d(p^*)[\ell/(k^*+1) + U/n_2] - C(d(p^*)(\ell/(k^*+1) + U/n_2))] - A < 0$$

by the definition of  $k^*$ . By C4, this firm earns non-positive profits from advertising  $p > p^*$ .

Finally, consider one of the first  $k^*$  firms. It too earns zero profits from inactivity. From C3, choosing not to advertise and to charge  $r$  results in non-positive profits and from C4, choosing to advertise  $p > p^*$  earns non-positive profits.

Thus,  $s^*$  is a Nash equilibrium and  $k^*$  firms advertising  $p^*$  weekly,  $n_2 - k^*$  firms charging  $r$  daily is a market outcome.  $\square$

These two theorems indicate the relationship between this model and more traditional models of imperfectly informed consumers. Under the special assumption I have employed on the costs of reading the newspaper, these theorems replicate the results of Salop and Stiglitz (1977). However, this formulation differs in two respects. First, the existence of "advertised" prices is not assumed as it was in Salop and Stiglitz. The prices are advertised by the firms as part of their profit maximizing strategy. Second, equilibrium exists with U-shaped average costs when there are a finite number of active firms.<sup>19</sup>

Before leaving this subject, let me note that throughout, I have referred to the advertising medium as a newspaper. This might give the impression that the assumption that the informed incur no costs of "reading" the newspaper is unwarranted. Two comments are in order. First, the particular advertising medium has been left open. I do refer to it as a newspaper for convenience, but any medium will do. Secondly, a newspaper, as well as most media that come to mind, are really bundled commodities.<sup>20</sup>

As a result the assumption that some consumers are informed at zero cost may, in fact, be satisfied.

Section 5:

In the previous discussion, the set of advertising strategies available to a firm was restricted. The firm was free to choose the price it wished to advertise, as well as how long the advertisement was to be effective. The firm was barred from using an advertising strategy which announced at  $t$  that today it would charge one price, tomorrow a different price.

There are two reasons for postponing a consideration of this type of advertising strategy. First, this type of advertising strategy may cause advertising to act as a sunk cost. Second, it may provide a means by which an incumbent can "commit" to a future action. These combine to weaken the discipline potential entry places on current actions.

Return to the simple model in which there are no sunk costs in production, everyone reads the newspaper, and conditions C1 and C2 hold. With the restricted set of advertising strategies, theorem 3 indicates that there is a Nash equilibrium in which the first  $n^*$  firms use  $a^*$  and the remaining firms use  $b^*$ . When the set of strategies is enlarged, to encompass strategies of the type mentioned above, it becomes less likely that a Nash equilibrium exists.

To see why, consider the effects of the first firm adopting a strategy  $\tilde{s}$ . This strategy involves advertising at  $t = 1$ , a set of prices for the week characterized as:  $\tilde{p}$  at  $t = 1$ ,  $p^*$  for the remainder of the week, and then using  $a^*$  for the remainder of the game. Define  $\tilde{p}$  as a price such that

$$\tilde{p}d(\tilde{p})\gamma(M, (v, \tilde{p}), (v, p^*), \dots, (v, p^*)) - C(d(\tilde{p})\gamma) - \frac{1}{T}A > 0. \quad 21$$

If such a price exists, there is no Nash equilibrium in this model. By adopting  $\tilde{s}$ , firm 1 earns positive profits. It does so because entry does not occur. At  $t = 2$ , the entrants know, because firm 1 adopted  $\tilde{s}$ , that there would be no "room" for their entry and they would not enter. Thus,

firm 1 earns positive profits at  $t = 1$ , and zero profits for the remainder of the week. The present value of the stream of profits when firm 1 uses  $\tilde{s}$  are positive. Thus,  $n^*$  firms using  $a^*$ , and the remainder using  $b^*$  is not a Nash equilibrium. Furthermore, because future entry can be successfully limited, there is no Nash equilibrium. Some active firm always has an incentive to alter the price it advertises at  $t = 1$  for any specification of strategies. The game has degenerated far enough so that the non-existence result of Edgeworth, Salop and Bryant is applicable.

The proposed strategy,  $\tilde{s}$ , precommits the incumbent to an action which, if entry occurs, causes the entrant to earn negative profits. Thus, entry is successfully blocked. Under the present assumptions, the strategy of placing a sequence of prices in a single ad, is no more costly than advertising one price good for the whole week. Within the structure of this model, the use of  $\tilde{s}$  represents an entry deterring action whose analogues have been discussed most recently by Salop [1979], Dixit [1979, 1981], and Schmalensee [1981]. In this model, the firm successfully blocks entry by credibly promising to "behave itself" in the future. It has altered its payoffs so that the entrant is persuaded not to enter.

A difficulty with the preceding analysis is the assumption that these "new" advertising strategies are no more costly than their simple counterparts. The firms can now commit to entry deterring strategies at no cost. I should mention that one can reinterpret the model used in Sections 2 and 3 as also permitting a form of commitment. In those sections, the firm can adopt a strategy of multiple advertisements. At  $t$  it could advertise  $p > p^*$  and in another ad  $p = p^*$  in the future. However, this strategy costs  $2A$  not  $A$ . A firm had to incur costs to precommit itself, and the costs were so large that the firms never found it profitable to make this type of commitment. The remainder of this section deals with the intermediate case

when the costs of precommitment are  $\Delta$  where  $0 \leq \Delta \leq A$ . Thus, if a firm chooses to advertise a non-constant sequence of prices at  $t$ , it costs  $A + \Delta$ .

From the preceding discussion, one observes that if there is a  $\tilde{p}$  such that

$$(1) \quad \tilde{p}d(\tilde{p})\gamma(M, (v, \tilde{p}), (v, p^*), \dots, (v, p^*)) - C(d(\tilde{p})\gamma) - \frac{1}{\tau} A - \Delta > 0,$$

no Nash equilibrium exists.<sup>22</sup> Thus, our first observation is that (1) cannot hold if there is to be a Nash equilibrium. If the cost of commitment are large enough, or if the consumers are flexible enough, or some combination of both occurs, equation 1 will not be satisfied.<sup>23</sup> As a result, a necessary condition for the existence of a Nash equilibrium is

$$C5: \quad \forall p, \quad pd(p)\gamma(M, (v, p), (v, p), \dots, (v, p)) - C(d(p)\gamma) - \frac{1}{\tau} A - \Delta \leq 0.$$

The existence of these precommitment advertising strategies allows a firm to adopt a strategy which can take the form of a threat. Suppose a firm chose to: advertise at  $t$ , some  $p > p^*$  good for  $\tau - 1$  periods and  $p^*$  in the  $t + \tau$  period. This firm has threatened potential entrants with future behavior that may make entry unprofitable. Whether or not these threat-like strategies will be used depends upon their profitability as well as the credibility of the threat.

While it is not always possible to find a threat strategy which yields positive profits if the threat is believed, the typical case does have threat strategies that yield positive profits if believed. The important question is when will the threat be credible. Since a threat requires charging  $p^*$  in the future,<sup>24</sup> the threat will not be credible if the incumbent cannot recoup part of its advertising costs in the periods when it charges  $p^*$ .

Consider the following scenario. The  $i^{\text{th}}$  firm (an active firm in  $s^*$ ) adopts  $s_i$  a strategy that advertises in the first period a sequence of prices  $\{p_j\}_{j=1}^{\tau}$ , and the others continue to use their part of  $s^*$ . For  $s_i$  to differ from  $s_i^*$ ,  $p_j > p^*$ , for some  $j$ . Note that  $p_1$  will be larger than  $p^*$  if any are because the entrant could not respond to  $p_1 > p^*$  until  $t + 1$  but can respond to  $p_j > p^*$  ( $j > 1$ ) in the period in which  $p_j$  is charged. To determine the credibility of the threat, I must find the number of shoppers in period  $j$ . To do so, let  $m$  be the first day that  $p_j = p^*$ . Construct

$$E = L - \gamma(M, (v, p_1), (v, p_2, \rho), \dots, (v, p_2, \rho)) - (n^* - 1)L/\tau n^*. \quad 25$$

Given the consumers beliefs about the future at  $t = 1$ ,  $E$  is the number of consumers that have chosen not to shop at  $t = 1$ . If entry occurs using a strategy that has the entrant advertising  $p^*$  good for a week, the consumers' beliefs are altered (potentially). Note that  $E > (\tau - 1)L/\tau$ . If  $p_2 > p^*$  and consumers expect entry then  $E/(\tau - 1)$  shop at  $t = 2, 3, \dots, \tau$ . If  $p_2 > p^*$  and the consumers do not expect entry then  $L - E$  shop at  $t = 2$ . Finally, if  $p_2 = p^*$  then  $E/(\tau - 1)$  shop at  $t = 2$ . This means that at  $m$ , the date that firm  $i$  reverts to  $p^*$ , the number of consumers who have not

shopped is known, and is represented by  $\beta$ . For firm  $i$ 's threat to be credible

$$p^*d(p^*)\beta/(n^*+1) - C(d(p^*)\beta/(n^*+1)) > 0.$$

If this is positive then remaining open in  $m$  permits  $i$  to recoup part of its advertising expenses. The firm is more than covering its variable costs and thus would remain open.<sup>26</sup> Thus, the threat is not credible if

$$C6: p^*d(p^*)\beta/(n^*+1) - c(\beta d(p^*)/(n^*+1)) \leq 0$$

It is immediate, then, that when the set of advertising strategies is enlarged, and C5 and C6 hold, a market outcome has  $n^*$  firms advertising  $p^*$  once a week.

Conclusions:

The purpose of this paper was to examine the advertising of prices by firms, from the point of view that advertising prices is a means of transmitting information to otherwise imperfectly informed consumers. To focus on the firms' incentives to transmit information, it was assumed that if they did not, the consumers shopped without any information about prices.

Within the explicitly dynamic model considered, I showed that when no advertising medium exists, the outcome was monopolistically competitive. When firms were unable to advertise (inform consumers), the unique market outcome had the product sold at the consumers' reservation price. When a medium was available, the firms chose to advertise and the market outcome had them advertising the price equal to marginal costs and average costs including the advertising expenditure. I also established that no equilibrium existed in which advertising did not take place.

The firms did have an incentive to transmit their prices to consumers, so that they shopped knowing the prices the firms charged. The model was then extended to incorporate sunk costs and the potential to threaten future entrants. I found that as long as sunk costs were not too large, they had no effect on the market outcome, since they do not pose a significant enough barrier to entry.

While the analysis of equilibria when sunk costs are positive was undertaken to provide a more complete understanding of this model, an additional benefit was obtained. I found that the model had a contestable equilibrium in spite of the positive sunk costs. Recognizing this allows one to generalize the effect that generated the outcome. It is well known that sunk costs reduce the potential entrant's ability to compete with an established firm. My analysis suggests that in any environment in which the current activities of an incumbent restrict its flexibility in responding

to future entry, a contestable equilibrium is possible. One obtains the contestable equilibrium if the established firm's competitive advantage (induced by the positive sunk costs) is offset by the firm's lost flexibility in responding to entry. In my model the established firm's flexibility is diminished because it must advertise again in order to respond to entry. In other words, it incurs larger advertising costs in order to continue to compete. If the additional advertising costs are too large, the established firms are disciplined by potential entry and the contestable outcome is obtained.

The incorporation of potential threats via "preemptive" advertising was also considered. Here, as one might expect, the credibility of the threat determined the market outcome. A condition under which the threat was not credible was presented, which required that the firm not cover its variable costs at  $p^*$  in the face of entry.

Lastly, I showed that this model can be thought of as providing a more solid theoretical foundation for the model first introduced by Salop and Stiglitz. I showed that if the consumers could be divided into those with low and those with high costs of obtaining the information contained in the firms' advertisements, equilibria of the type reported in Salop and Stiglitz were obtained. One way of viewing this is that I have provided the underpinnings for eliminating the exogeneity of the information source in Salop and Stiglitz. In doing so, I have not employed a number of assumptions which they used and which were part of the basis for the Wilde and Schwartz criticism of their model.

I should note that certain welfare conclusions are immediate within the simple version of the model. When all consumers costlessly learn the information contained in the firms' advertisements, how well the market provides information can be considered. If the government has no means of providing information to consumers which is both unavailable to

the firms, and cheaper, than the market has transmitted information efficiently and the consumers have paid for it.<sup>27</sup>

Throughout, equilibria existed only if the consumers had enough flexibility. This meant that it had to be possible for enough consumers to avoid a high price store. Conditions under which this held were examined. The crucial ones required rapid entry as a means of disciplining the incumbents. Within the context of my model, if the conditions were not satisfied, no equilibrium existed. This was a consequence of the simple strategy spaces provided to the firms. When the firms are at least partly insulated from the pressures of entry, one would expect them, in a dynamic model, to develop a "working relationship" with one another. Based upon the work of Green and Porter and the references cited therein, I would conjecture that with enlarged strategy spaces, this model would develop into a non-cooperative game that has a "collusive" outcome constrained by the potential for future entry.

## FOOTNOTES

- 1 This work includes Diamond [1971], Salop and Stiglitz [1977], Butters [1977], Wilde and Schwartz [1979] and Varian [1980].
- 2 "Too many" and "not too many" are explicitly defined in Section 4. Essentially "not too many" occurs when it is unprofitable for a firm to choose to specialize in the uninformed consumers.
- 3 This assumption is relaxed in Section 4.
- 4 Thus, at  $t$ , the firm may advertise a price that is in effect for a pre-specified number of periods. It also means that to specify a sequence of prices (not all the same) at  $t$  requires additional advertising expenditures.
- 5 Obviously,  $n_1$  need not be an integer. Throughout the paper, I will ignore the difficulty. As can be seen in Bagnoli [1982], accounting for it imposes additional complications without yielding any additional insight.
- 6 Again, I ignore the possibility that  $n^*$  is not an integer.
- 7 Some consumers can fail to obtain the product.
- 8 Where cost refers to both the consumer's monetary expenditures as well as the cost associated with a probability of obtaining the unit less than 1.
- 9 Standard rationing mechanisms satisfy this assumption. In particular, first come, first served rationing, quantities restricted rationing: and "efficient" rationing all satisfy this assumption.
- 10 Here as well as in the remainder of the paper, there is no equilibrium when future entry does not place "enough" discipline on the incumbents. As I alluded to before, from the work of Green and Porter [1982], I believe that enlarging the strategy space would generate a "collusive" equilibrium.
- 11 See the analyses of Bryant [1980], Salop [1977] and Edgeworth's criticism of Bertrand's model as discussed in Shubik [1981] for details on the non-existence of Nash equilibria if firms choose prices and have U-shaped average cost curves.

- 12 This also ignores the problems that occur when demand at this price is not an integer multiple of the minimum efficient scale of the firm.
- 13 Here, a firm facing excess demand does not say "sorry we are out of stock". Instead, it issues rainchecks. In some sense, the firm must be unable or unwilling to use quantity restrictions.
- 14 Note that this implies that rainchecks are not an "efficient" non-price rationing mechanism.
- 15 Obviously, this insight extends to any behavior by the established firms that reduces their ability to respond to entry.
- 16 This is a convenient specification of the distribution of consumers' information acquisition costs. It can be altered in exactly the ways Salop and Stiglitz alter it.
- 17 A more general analysis would incorporate a less stringent assumption on the distribution of costs of reading the newspaper, and/or the possibility of consumers gathering information as in Wilde and Schwartz [1979].
- 18  $n_2 - k^* > 0$  because  $C(q)$  exhibits a U-shaped average cost curve and  $r > p^*$ . Also, note that  $n_2 = n_1$ .
- 19 See Bagnoli [1981] or Wilde and Schwartz [1982] for a more complete discussion of the implicit requirement of an infinite number of firms in Salop and Stiglitz.
- 20 See Adams and Yellen [1976].
- 21 It is conceivable that such a price does not exist.  $\gamma$  may be smaller than the firm's "breakeven" quantity for all  $p > p^*$ . This can occur because  $\gamma$  is discontinuous at  $\tilde{p} = p^*$ . In other words,  $\lim_{p \rightarrow p^*} \gamma(M, (v, p), (v, p^*), \dots, (v, p^*)) < l/n^*$ . If there is no such  $\tilde{p}$ , a Nash equilibrium may exist. Since the present model will be a special case of the amended model to be discussed, I will postpone consideration of the situation when no such  $\tilde{p}$  exists.
- 22 Note that the firm must cover  $\Delta$  in the period in which it advertises  $\tilde{p}_1$  because in all periods when it charges  $p^*$ , it earns  $p^*l/n^* - C(l/n^*) = \frac{1}{\tau} A$ .

- 23 Recall from the explanation of C2 that one can guarantee that for any positive  $\Delta$ , (1) is not satisfied if entry occurs rapidly enough.
- 24 If a threatener did not charge  $p^*$ , an entrant can adopt a strategy of entering and advertising at  $t, p^*$  good for a week and guarantee itself zero profits.
- 25  $\gamma > 0$  is necessary for the threat to be profitable.
- 26 If it remains open, the entrant's profits are negative because  $\gamma > 0$ .
- 27 This result is unlikely to be robust because of the assumptions on the advertising medium employed. The firm, for a fee, informed all consumers. If the fee had depended upon the number of consumers informed and the firm could choose how many to inform, information would have been transmitted inefficiently because the low price equilibrium can be obtained without informing every consumer.

## REFERENCES

- Adams, W. and J. Yellen, "Commodity Bundling and the Burden of Monopoly," Quarterly Journal of Economics, August, 1976.
- Bagnoli, M., "Equilibrium with Uninformed Consumers," draft, 1981.
- Bagnoli, M., "A Note on Partially Informed Consumers," draft, 1982.
- Bagnoli, M., "On the Advertising of Prices," draft, 1982.
- Bryant, J., "Competitive Equilibrium with Price Setting Firms and Stochastic Demand," International Economic Review, October, 1980.
- Butters, G., "Equilibrium Distribution of Sales and Advertising Prices," Review of Economic Studies, October, 1977.
- Dixit, A., "A Model of Duopoly Suggesting a Theory of Entry Barriers," Bell Journal, Spring, 1979.
- Dixit, A., "Recent Developments in Oligopoly Theory," American Economic Review, May, 1982.
- Green, E. and R. Porter, "Noncooperative Collusion Under Imperfect Price Information," Center for Economic Research, Department of Economics, University of Minnesota, working paper no. 81-142.
- Salop, S., "Bertrand Revisited: The Non-existence of Purely Competitive Equilibrium Without an Auctioneer," Board of Governors of the Federal Reserve System working paper, 1977.
- Salop, S., "Strategic Entry Deterrence," American Economic Review, May, 1979.
- Salop, S. and J. Stiglitz, "Bargains and Ripoffs: A Model of Monopolistically Competitive Price Dispersion," Review of Economic Studies, October, 1977.
- Schmalensee, R., "Economies of Scale and Barriers to Entry," Journal of Political Economy, December, 1981.
- Varian, H., "A Model of Sales," American Economic Review, December, 1980.
- Wilde, L. and A. Schwartz, "Equilibrium Comparison Shopping," Review of Economic Studies, July, 1979.
- Wilde, L. and A. Schwartz, "Imperfect Information, Monopolistic Competition, and Public Policy," American Economic Review, May, 1982.



1

1

