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nter for Research on Economic and Social Theory
Research Seminar in Quantitative Economics

Discussion Paper

DEPARTMENT OF ECONOMICS

University of Michigan

Ann Arbor, Michigan 48109

A Note on Property Taxation of a Non-Renewable Resource

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C-62

March 1985

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ABSTRACT

Suppose that a property tax is imposed on the competitive owners of a non-renewable resource in a closed economy. This tax obligation is naturally capitalized into a reduction in the market value of the existing stock of the resource. In this paper, it is shown a) that under many circumstances the owners of the resource are necessarily made worse off by more than the total capitalized value of the tax payments, and b) that consumers are necessarily made better off by the imposition of a small tax. Thus, unlike most competitive situations in which taxes are paid partly by consumers and partly by producers, the property tax, as well as the allocative distortion due to the tax, may be born entirely by producers, and consumers may actually benefit--regardless of any expenditures that the tax may support.

A Note on Property Taxation of a Non-Renewable Resource

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Consider a closed economy that possesses a known stock of a non-renewable resource. This resource is extracted costlessly by a competitive industry and is sold to a market with known demand. The economy has an interest rate fixed at r. There exists an alternative technology that will replace all uses of the resource at a price P".

These conditions lead to the well-known Hotelling rule that the price of the resource will rise at the interest rate r, reaching P" at the moment that the stock of the resource is exhausted (Dasgupta and Stiglitz, 1981). It is furthermore the case that this dynamic path is welfare-maximizing (Sweeney, 1977). For example, if the income elasticity of demand for the resource were zero, this path would maximize the sum of producers' and consumers' surplus as conventionally measured.

Suppose that a property tax is imposed on the competitive owners of the resource. This tax is computed by multiplying a fixed rate q times the current price times the unexploited stock at any time t. In ordinary competitive environments, taxes of any sort tend to be shared by both producers and consumers: Taxes on output lead to increased prices (although in amounts less than the magnitudes of the taxes), while property and other fixed taxes

* I would like to express my thanks to T. Bergstrom, R. Porter, R. Mendelsohn, and two anonymous referees for their valuable comments on earlier drafts of this paper.

generally reduce the long run equilibrium number of suppliers so that the remaining firms operate at higher points on their long run marginal cost curves. As colloquially expressed, taxes, or a significant portion of them, are generally "passed on" to consumers.

Property taxes on natural resources, however, are capitalized into reductions in the value of those resources, so that there is a tendency, at least initially, for market prices to fall rather than rise. Of course, the timing of extraction is also affected, so that at some points in time, price is higher than it would otherwise have been. Nevertheless, it turns out that the competitive owners of the resource may be made worse off by more than 100% of the tax--which is to say that the current market value of the existing stock of the resource at the time that the tax is imposed falls by more than the present value of all the property taxes that are paid on it. Moreover, although general welfare (the sum of consumers' and producers' surplus) is reduced by the distortion due to the tax, the decline in market value of existing stocks of the resource may be greater, so that consumers' welfare is increased. Not only are these taxes not "passed on" to consumers in the form of higher prices, consumers are actually better off!

To the best of my knowledge, these two propositions were first outlined by Gamponia and Mendelsohn (1984). Their conclusions are drawn entirely from simulation experiments, however, and no formal proofs of the theorems are provided.

Moreover, their simulations suggested that the propositions are universally true when in fact they are not.

The purpose of this note is to provide some analysis of these questions for an important class of demand functions--those for which price elasticity does not decrease in absolute value as price is increased. Marshall (1920) argued that in fact all demand functions satisfy this condition: He believed that commodities with high prices (such as diamonds) would typically have high price elasticities while those with low prices (such as salt) would have low price elasticities. Demand curves that touch the axes (that have a finite price at which demand is zero, and a finite demand at a zero price) necessarily satisfy this condition in the vicinity of the end points, and all approximately linear demand curves satisfy it throughout. I will refer to demand functions that meet this condition as possessing the "Marshallian" property. This condition is convenient in many contexts: It guarantees that marginal revenue curves slope downward whenever MR>0, and it has been used in analyses of problems of monopolistic exploitation of non-renewable resources (Lewis, 1976; Bergstrom, Cross, and Porter, 1980).[1]

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1. By accepting this proposition, I am implicitly rejecting Stiglitz and Dasgupta's (1982) suggestion that constant elasticity demand curves represent a "central" case in any but a purely mathematical sense. A demand function whose elasticity is constant or increases with increases in quantity has the properties that a) the quantity demanded is unbounded at a zero price and b) there is no price so high as to choke off all demand. Neither of these conditions would be particularly plausible in this context, and constant elasticity is treated here as an extreme case.

Notation

We use the following notation:

t denotes time; $t=0$ describes the date at which the property tax is imposed.

$P(t)$ is the market price of one unit of the resource at time t.

$f(P(t),t)$ is the market demand for the resource at time t. Note that we do not assume demand to be stationary, but include t as a shift parameter.

$F(t) = f(P(t),t)$ is an abbreviation that we will use frequently for notational convenience.

r is the interest rate

T is the date of exhaustion of the resource

P'' is the price at which the alternative technology replaces use of the resource in question.

P_o is the price at time $t=0$

S is the stock of the resource available at time $t=0$

q is the property tax rate.

Equilibrium

Our framework is a simple extension of the conventional arbitrage model (Dasgupta and Heal, 1979). Equilibrium is characterized by the condition that the price of one unit of the resource at time t must be sufficient to cover the interest foregone and the property tax payments that have been made since time t=0:

$$P(t) = P_o e^{rt} + q\int_o^t P(u)e^{r(t-u)}du \qquad (1)$$

Differentiation with respect to t provides:

$$\dot{P}(t) = (r+q)P(t) \qquad (2)$$

or:

$$P(t) = P_o e^{(r+q)t} \qquad (3)$$

This equation reproduces the well-known observation that a property tax is equivalent to an increase in the interest rate.

At any time t, the stock of the resource still unexploited is given by

$$\int_t^T F(u)du \qquad (4)$$

and the date of exhaustion, T, is established by:

$$\int_o^T F(u)du = S \qquad (5)$$

Total property taxes paid at time t are equal to the tax rate, q, times the current price, times the unexploited stock of the resource:

$$R(t) = qP(t)\int_t^T F(u)du \qquad (6)$$

The present value of this stream of property tax revenues is:

$$R = q\int_o^T P(t)e^{-rt}\int_t^T F(u)du\ dt \qquad (7)$$

Reversing the order of integration:

$$R = q\int_o^T F(u)\ \int_o^u P(t)e^{-rt}dt\ du \qquad (8)$$

Using (3), equation (8) becomes:

$$R = P_o \int_o^T F(u)(e^{qu}-1)du, \tag{9}$$

and from (5) we have:

$$R = P_o \int_o^T F(u)e^{qu}du - P_o S \tag{10}$$

We will call Z the present value of the stream of sales from the perspective of t=0:

$$Z \equiv \int_o^T F(u)P(u)e^{-ru}du \tag{11}$$

Substituting from (3):

$$Z = P_o \int_o^T F(u)e^{qu}du \tag{12}$$

Profits to the owners of the resource at any time t are equal to the present value of sales, Z, minus the present value of the taxes levied on the stocks, R. The value of this net stream of returns at time t=0 describes the net asset value of the stock, and this is $P_o S$ because extraction cost is zero:

$$P_o S = Z - R \tag{13}$$

Substituting (12) into (13) reproduces equation (10).

Our first proposition is that when the property tax is imposed, the value of $P_o S$ may fall by more than the change in R, which is to say that the the present value of gross sales, Z, declines when q is increased.

Some Preliminary Results

It is easy to identify a simple situation in which the propositions are valid. Suppose the price elasticity of demand is zero. Then at each time t, F(t) is fixed at some Q(t) for all $t \leq T$, where the dependence of Q on t reflects the possibility that demand may not be stationary. Under this assumption, the date of exhaustion is unaffected by the price path. However, $P(t)=P''$ at $t=T$, and because the rate of increase in P(t) is increased by the tax, it must be the case that for all $t<T$, market price with a tax is lower than price without the tax. Thus the tax unambiguously reduces Z. Moreover, because consumers are buying the same quantities of the resource over the same period, and at lower prices, their welfare must have increased, and we have, besides the tax payments, a net transfer from producers to consumers.

More generally, we note from equation (2) that the rate of price increase rises with the tax rate. Because the price path must terminate at a price P", any increased value of q must bring the date of exhaustion forward in time: If it did not, then the higher rate of price increase would imply that prices with the tax would be lower than prices without the tax for all $t<T$; this would mean that consumption with the tax would be greater than consumption without the tax for all $t<T$, and this in turn would imply total consumption exceeding S. Therefore:

$$\frac{dT}{dq} \leq 0 \tag{14}$$

It is also the case that any increased value of q will reduce P_o. If it did not, then the higher rate of price increase would imply that price with the tax was greater than price without the tax for all $t>0$, and P" would be reached before S was exhausted. Therefore:

$$\frac{dP_o}{dq} < 0 \qquad (15)$$

It is easy to find other cases for which dZ/dq is unambiguously negative. For example, suppose the demand function is stationary over time and the price elasticity of demand is constant and equal to one. Then we have $F(t)P(t)=K$ for all values of $t \leq T$, and we can integrate (11) directly:

$$Z = \frac{K}{r}(1-e^{-rT}) \qquad (16)$$

In this case, it is immediately apparent that the reduction in T that follows from an increase in the tax must reduce Z.

It is also easy to show that $dZ/dq < 0$ is true in all cases of constant demand elasticity so long as that elasticity exceeds one. To see this, suppose that for the moment we replace the competitive industry with a monopolist, and permit extraction costs to be positive. A monopolist is always made worse off by an amount greater than the amount of a property tax. In the absence of a tax, the monopolist would choose an output path that maximized the present value of sales net of extraction cost. By taxing unexploited stocks of the resource, the property tax induces the monopolist to divert some output from the future toward the present, and this distortion in the output path reduces the present value of net sales. In addition to this net revenue loss, the monopolist must pay the tax, and hence his total wealth has fallen by more than the tax[2].

Now return to the competitive industry with zero extraction costs and suppose that the price elasticity of demand is constant and greater than 1 in absolute value. Then we know from Stiglitz (1976) that a monopolist will sell along the same price path as does a competitive market, and that the profit maximizing price path is unique (see also Sweeney, 1977). That is, because Z would be the monopolist's profit (in the absence of a tax) the competitive path maximizes Z. Then, because the imposition of the tax will alter the price path, the value of Z must be affected, Z is no longer maximized, and our competitive producers also pay more than the present value of the tax.

This same observation provides a clue to a source of counterexamples to any assertion that the proposition is universally true. Suppose the demand function has a constant elasticity less than one. In this case, the monopolist maximizes profits, not along the competitive price path, but at the corner $P(t)=P''-e$ for small e and all $t<T$. Because the competitive industry is not following this path, it is possible that a distortion might move the industry in a profitable direction, <u>increasing</u> Z. (It is not possible for such an increase to be so powerful as to provide net benefits to the industry, however,

2. This conclusion is not restricted to the case of the property tax. It applies also to the case of excise taxes.

because, as we have shown, dP_o/dq is always negative.) An example of this possibility is provided by the top panel of Figure 1. This is a graph of Z as a function of various tax rates, q, for the case of the stationary constant elasticity demand curve $Q=50P^{-0.5}$, where r=0.05, S=2000 and P"=1000. Over quite a wide range of values of q (from q=0.01 to q=0.25), Z increases with the tax.[3] Moreover, because P_o declines in response to any increase in q, it is clear from (13) that tax revenue must be increasing in q whenever $dZ/dq>0$. Thus the (quite reasonable) supposition that q would never be increased beyond a point at which $dR/dq\geq 0$ is not sufficient to restrict q to values within the range for which $dZ/dq<0$. The second panel of Figure 1 dipicts the behavior of tax revenue as the tax rate is varied. In this example, revenue increases with q up to q=0.25.

3. For this and other examples, the solutions provided are obtained numerically. (The technique is to evaluate the integral in (5) through either explicit integration or Simpson's Approximation, and then to find the value of T which satisfies (5) by means of an iterative bisection method for finding the roots of a function.) The existence of such counterexamples may be shown analytically, however, by substituting the constant elasticity demand function $Q=aP^b$ into (12), differentiating with respect to q, and considering the special simplifying case $b=-q/(r+q)$. It is then easy to see that if T is large enough (which means if S is large enough) the value of dZ/dq can be made positive.

The General Case

Using (3) we have:

$$P_o = P''e^{-(r+q)T} \tag{17}$$

Differentiating :

$$\frac{dP_o}{dq} = -P_oV \tag{18}$$

where we have defined:

$$V \equiv T + (r+q)\frac{dT}{dq} \tag{19}$$

and we know from (14) and (15) that

$$0 < V < T \tag{20}$$

Equation (5) establishes the date of exhaustion, T. Let us differentiate this equation with respect to q, using (3) and (17):

$$F(T)\frac{dT}{dq} + \int_0^T f_1(P,u)P(u)(u-V)du = 0 \tag{21}$$

where $f_1(P,u)$ is the derivative of the demand function with respect to its price argument.

If we multiply (21) through by (r+q), both add and subtract a term TF(T), and use (19), we may solve for V directly:

$$V = \frac{TF(T) - (r+q)P_o\int_0^T f_1(P,u)e^{(r+q)u}udu}{F(T) - (r+q)P_o\int_0^T f_1(P,u)e^{(r+q)u}du} \tag{22}$$

We notice that the expression $P_o(r+q)e^{(r+q)u}$ is equal to $dP(u)/du$, and thus the denomonator of (22) may be rewritten:

$$F(T) - \int_0^T f_1(P,u)\frac{dP(u)}{du}du = F(T) - \int_{P_o}^{P''} f_1(P,u)dP = F(0) \tag{23}$$

We construct the intermediate variable $Y = uf(P,u)$ and note that the derivative of Y with respect to u is $f(P,u)+P_o(r+q)f_1(P,u)ue^{(r+q)}$. Substituting this into the numerator of (22), we may integrate by parts and reduce the entire expression to S. Thus expression (22) finally becomes:

$$V = \frac{S}{Q_o} \tag{24}$$

where Q_o is the quantity demanded at time t=0.

To exploit the Marshallian property of demand, we rewrite (21) to take explicit account of the elasticity of demand:

$$F(T)\frac{dT}{dq} + \int_o^T F(u)\eta(u)(u-V)du = 0 \tag{25}$$

where η is the price elasticity of demand. Here elasticity is defined to have the negative sign, and is naturally a function of the time variable u.

We suppose that the demand function possesses the Marshallian property in the strong sense that higher prices are always associated with higher values of elasticity (in absolute value). Because price is monotone increasing in u, we can write:

$$\eta(u) = \eta_o e(u); \quad e'(u) \geq 0 \tag{26}$$

where η_o is the value of elasticity at time t=0.

We will rewrite (25) as:

$$F(T)\frac{dT}{dq} + \eta_o \int_o^T F(u)e(u)(u-V)du = 0 \tag{27}$$

From (19), V has a value between 0 and T. When the variable of integration, u, is less than V, the integrand in (27) is strictly negative. When u is greater than V, the integrand is strictly positive. Consider the expression $e(V)/e(u)$. Because $e(u)$ is positive and increasing in u, we note that $e(V)/e(u)\leq 1$ for all $u>V$, and $e(V)/e(u)\geq 1$ for all $u<V$. If we were to multiply the integrand of (27) by this ratio, the value of the integral must remain constant or fall (because the positive elements in the sum are all being multiplied by values less than one, and the negative elements by values exceeding 1). The integral as it appears in (27) is already negative, because $dT/dq<0$ and $\eta_o<0$. Therefore, we must have:

$$\int_o^T F(u)(u-V)du < 0 \tag{28}$$

Now consider the expression e^{qu}/e^{qV}. This ratio is greater than or equal to 1 for all values $u>V$ and is less than or equal to 1 for all values $u<V$. Thus if we were to multiply the integrand of (27) by e^{qu}/e^{qV}, the value of the integral must remain constant or rise (all positive elements in the sum are multiplied by more than 1, all negative elements by less than 1). Thus, because $\eta_o<0$, we obtain:

$$F(T)\frac{dT}{dq} + \eta_o e^{-qV} \int_o^T F(u)e^{qu}e(u)(u-V)du \leq 0 \tag{29}$$

or, because $dT/dq<0$, $\eta_o<0$, and $e^{qT}>e^{qV}$:

$$\int_o^T F(u)e(u)e^{qu}(u-V)du \geq -\frac{1}{\eta_o}F(T)e^{qT}\frac{dT}{dq} \tag{30}$$

We may now consider the value of gross sales, Z, which is our primary object of attention. If we differentiate (12) with respect to q, replace dP_o/dq with equation (18), and simplify, we obtain:

$$\frac{dZ}{dq} = P_o F(T) e^{qT} \frac{dT}{dq} + P_o \int_o^T F(u) e^{qu} [1+\eta(u)](u-V)du \qquad (31)$$

Given the sign of dT/dq from (14), we know that the first term on the right of (31) is always negative, and therefore it is obvious that $dZ/dq<0$ whenever the price elasticity is constant and equal to -1.

There are three circumstances under which the proposition $dZ/dq<0$ is necessarily true.

A) Demand elasticity is zero. This has already been demonstrated.

B) The tax is zero. At the point q=0, equation (31) becomes: (using 26):

$$\frac{dZ}{dq} = P_o F(T) \frac{dT}{dq} + P_o \int_o^T F(u)(u-V)du + P_o \eta_o \int_o^T F(u) e(u)(u-V)du \qquad (32)$$

Substituting from F(T)dT/dq from (25) this reduces to:

$$\frac{dZ}{dq} = P_o \int_o^T F(u)(u-V)du \qquad (33)$$

and this expression is negative by (28).

C) For any value of q, demand is elastic over "most" of its range. This condition may be formalized by either a) the requirement $\eta_o \leq -1$, which, given the Marshallian condition, guarantees that demand is elastic over all t, or, b) the less stringent requirement $\eta_o e(V) \leq -1$, which guarantees that elasticity exceeds one in absolute value at all times after the intermediate time given by V. The value of V is S/Q_o, and thus this second condition is that the demand be elastic at least by the time at which the stock would have been exhausted had demand remained at its initial level indefinitely. For most examples that come to mind, this is a modest requirement. To prove this third variation on the proposition, define K to be the integral expression in equation (31), and expand, using our definition (26):

$$K = \int_0^T F(u)e^{qu}(u-V)du + \eta_o \int_0^T F(u)e^{qu}e(u)(u-V)du \qquad (34)$$

Because e(u) increases in u, the expression $e(u)/e(V) \geq 1$ for all $u>V$, and $e(u)/e(V) \leq 1$ for all $u<V$. If we multiply the integrand in the first term of (34) by this expression, we will multiply all positive elements in the sum by numbers greater than or equal to 1 and all negative elements by numbers less than or equal to 1, and this cannot decrease the value of the integral as a whole. Thus:

$$K \leq \left[\frac{1}{e(V)} + \eta_o\right] \int_0^T F(u)e^{qu}e(u)(u-V)du \qquad (35)$$

Our condition that $\eta_o e(V) \leq -1$ guarantees $1/e(V)+\eta_o<0$, and substituting from (30), we obtain:

$$K < -\frac{1}{\eta_o} F(T)e^{qT} \frac{dT}{dq}\left(\frac{1}{e(V)} + \eta_o\right) \qquad (36)$$

and:

$$\frac{dZ}{dq} \leq P_o F(T)e^{qT} \frac{dT}{dq}\left[1 - \frac{1}{\eta_o e(V)} - 1\right] < 0 \qquad (37)$$

The top panels of Figures 1 and 2 provide some illustrations of these cases. The example described in Figure 1 satisfies neither conditions A nor C, and thus the slope dZ/dq may be positive at values of q which are not very close to 0 (at which point condition B would be satisfied). Figure 2, on the other hand, describes the case of an elastic stationary demand curve ($Q=5P^{-2.0}$), and this satisfies condition C for all values of q. As expected, then, $dZ/dq<0$ throughout.

Consumers' Surplus

In this section, we address the possibility that the property tax increases the welfare of consumers (without regard for how the revenue from the tax is spent). Suppose we accept the usual assumptions that a) the income elasticity of demand for the resource is zero, so that consumers' surplus becomes a valid measure of consumer welfare, and b) the discount rate which consumers apply to future welfare is the same as the market rate of interest r. Then at any price P(t) the consumers' surplus at time t (relative to having to pay the price of the alternative technology, P") is given by W(t) where:

$$W(t) = \int_{P(t)}^{P''} f(x,t)dx \tag{38}$$

This may be rewritten:

$$W(t) = \int_{t}^{T} f(P(u),u)\frac{dP(u)}{du}du \tag{39}$$

Now using (2), equation (39) becomes:

$$W(t) = (r+q)\int_{t}^{T} F(u)P(u)du \tag{40}$$

The present value of this stream of consumers' surplus is:

$$W = (r+q)\int_0^T e^{-rt} \int_t^T F(u)P(u)du \qquad (41)$$

Reversing the order of integration in (41):

$$W = (r+q)\int_0^T F(u)P(u) \int_0^u e^{-rt}dt\ du \qquad (42)$$

$$= \frac{(r+q)}{r}\int_0^T F(u)P(u)(1-e^{-ru})du \qquad (43)$$

Using (11) and (40):

$$W = \frac{r+q}{r}\int_0^T F(u)P(u)du - \frac{r+q}{r}Z \qquad (44)$$

$$= \frac{W_o}{r} - \frac{r+q}{r}Z \qquad (45)$$

where W_o is W(t) valued at t=0. In the absence of any tax, therefore, consumers' welfare is equal to $(W_o/r-Z)$: the present value of an indefinite stream of values W_o minus the present value of gross sales.

We know from Sweeney (1977) that the sum of Consumers' welfare and profits is maximized by the competitive price path in the absence of taxes. The imposition of the property tax distorts the path and therefore it must reduce this sum. It follows immediately that under any circumstances for which consumer welfare is increased by the tax, producers are made worse off by more than 100% of a tax, and that consumers are made worse off whenever the tax leads to an increase in the gross value of sales. As one might expect, however, the conditions for $dW/dq>0$ are more stringent than those for $dZ/dq<0$. The bottom panel in Figure 1 describes W as a function of q for the case of the inelastic demand curve used in our first example. Not only does W fall whenever dZ/dq is positive, but it continues to decline over a considerably wider range. (Indeed, it is positive only for values of q which are less than 0.003, and this range is too small to show clearly on the graph.) Even in cases that guarantee $dZ/dq<0$, we are not assured of $dW/dq>0$. Figure 2 contains an example. For this case, dW/dq is positive only over the range $q=0$ to $q=0.0005$, a range much too small to show on the chart.

These examples are not meant to imply that $dW/dq<0$ is in any sense the usual situation. Indeed Gamponia and Mendelsohn's belief that $dW/dq>0$ is the normal case is strongly encouraged in other situations. Figure 3 describes an example whose parameters are taken from their paper, and it is typical of the cases used in their study in that $dW/dq>0$ for all values of q for which dR/dq is also positive, and $dZ/dq<0$ throughout. These properties are even more strongly present in models which use more nearly linear demand functions, and it required some effort to obtain the counterexamples provided here. Nevertheless, it is clear that counterexamples exist in a wide class of cases and that a general analytic criterion which will guarantee $dW/dq>0$ will have to be stringent. In fact, we have only two candidates:

A) Demand has zero elasticity throughout. This case has already been demonstrated earlier.

B) The tax is zero, and the Marshallian condition is satisfied--that is, we are considering only the initial imposition of the tax. To prove this, we differentiate (44) with respect to q:

$$\frac{dW}{dq} = \frac{1}{r}\left[\frac{dW_o}{dq} - Z - (r+q)\frac{dZ}{dq}\right] \tag{46}$$

The value of W_o may be obtained from (40) with t=0.

Differentiating this expression with respect to q and using (18):

$$\frac{dW_o}{dq} = - Q_o \frac{dP_o}{dq} = P_o Q_o V \tag{47}$$

where Q_o is the value of f(P) at $P=P_o$.

Substituting into (46) and using (24) for the value of V, we obtain:

$$\frac{dW}{dq} = \frac{1}{r}\left[P_o S - Z - (r+q)\frac{dZ}{dq}\right]$$

$$= \frac{1}{r}\left[R - (r+q)\frac{dZ}{dq}\right] \tag{48}$$

At the point q=0, tax revenue, R, equals zero, and thus dW/dq has the opposite sign to dZ/dq. We have already shown that subject to our Marshallian condition dZ/dq<0 at q=0, and so we must have dW/dq>0. Thus we have our result that consumer welfare must be increased by the initial imposition of the property tax.

Dasgupta and Heal (1979) note that when extraction costs are zero, an <u>ad valorem</u> tax will have no effect upon the competitive price path of an exhaustible resource. Thus such a tax does not affect consumer welfare, and producers' loss is exactly 100% of the tax. The property tax goes further, lowering the current price by more than the tax so that consumers actually gain. In these cases, the property tax is redistributive even without consideration of the programs on which the tax revenues may be spent.

Extraction Costs

A detailed evaluation of extraction costs would go beyond the scope of this paper. Gamponia and Mendelsohn originally formulated their propositions in an environment of constant extraction costs, however, and it is worthwhile to include this special case in our model. We continue to use P to designate the value of the unextracted resource, so that the original arbitrage condition (1) still holds. If extraction cost is constant at some level C, then market price, M, is equal to P+C. If the cost of the alternative technology is M" then we have P"=M"-C. The variable Z is now the present value of the stream of sales revenue net of extraction cost. If the market demand function is Q=h(M,t), then the functions used in our equations are defined by f(P,t)=h(P+C,t). Given these definitions, our equations (1)-(37) are entirely unaltered. The function h(M,t) must be used to replace f(P,t) in equation (38). However, for constant extraction cost, we note that dM(u)/du = dP(u)/du, and with this condition, the derivation (40)-(45) stands, and equations (47) and (48) are unaltered. Thus all of our conclusions hold.

This modification does affect the interpretation of the elasticity conditions. If E is the market price elasticity at the market price, M, then the elasticity expression in our equations is given by:

$$\eta = E(1 - \frac{C}{P+C}) \qquad (49)$$

This adjustment may increase the likelihood of $dZ/dq<0$ because it reinforces the tendency for elasticity as used in our equations to increase in absolute value as price increases (and although the base value η_o is smaller, the value of $\eta_o e(u)$ is much less affected because at high prices, $C/(P+C)$ is relatively small). Even if E were to decrease slightly with price, our conditions would be satisfied.

Stationarity

The demand function used throughout this analysis has included a time shift argument, and this has enabled us to avoid assumptions of stationarity. However, non-stationary demand functions do add stringency to our version of Marshall's demand property because we require demand elasticity to rise with a price which is rising over time. If the demand function is not stationary, the passage of time itself might operate to lower elasticity and offset the effect of the higher price. Formally, if we differentiate demand elasticity with respect to time, we obtain:

$$\frac{d\eta}{dt} = \frac{\partial\eta}{\partial P}\frac{dP}{dt} + \frac{P}{f^2}(f\cdot f_{12} - f_1\cdot f_2) \tag{50}$$

where $f \equiv f(P,t)$, $f_1 \equiv \frac{\partial f(P,t)}{\partial P}$, $f_2 \equiv \frac{\partial f(P,t)}{\partial t}$, $f_{12} \equiv \frac{\partial^2 f(P,t)}{\partial P \partial t}$

The first term in (50) captures Marshall's property for a stationary demand function and thus we expect this term to be negative. The second term describes the action of time independent of price, and if this term is sufficiently strongly positive, our elasticity premise is not valid. This might be the case, for example, if the passage of time were to increase the dependence of the economic system upon this resource (perhaps by exhausting close substitutes), so that its demand elasticity falls. Figure 4 provides an example of this possibility. This case employs a demand function $Q=5P^{-1.25}+0.05t$ with $p''=20$, $S=600$, and $r=0.05$. It is clear from the Figure that the effect of the time shift is to reverse the signs of the slopes so that $dZ/dq>0$ and $dW/dq<0$ even in the vicinity of $q=0$.

If the passage of time made possible technological advances which identify possible substitutes for this resource, then the second term in (50) will become negative also and all of our theorems stand. It is interesting to note that if $f(P,t)$ is separable into the multiplicative form $g(P)h(t)$, which might be the case, for example, if population growth were responsible for the shift in demand, then the second term of (50) is precisely zero and Marshall's property applied to the demand at any point in time is sufficient for our more general conditions. Constant elasticity demand curves can only be non-stationary if time enters in this multiplicative form, of course, because otherwise the value of (50) would not be zero, and the constancy of elasticity would vanish.

Implementation

It might appear at first that a property tax could not reasonably be put into place, because it would require knowledge of the stock of the resource, and owners would have an incentive to conceal their supplies. Property containing the resource would be bought and sold surreptitiously, and reported selling prices might deviate significantly from true prices. (This problem is often alleged to arise in the case of property taxes on real estate.) In contrast, actual sales of the resource are much more readily observable, so that excise or _ad valorem_ taxes seem to be more easily applied. In fact, however, a modified dynamic excise tax can be devised that duplicates the effect of the property tax. Sale of a unit of the resource is _ex post_ evidence that the unit was always available, and one can charge the property tax, with interest, at the date of sale. Thus, if the tax is instituted at time t=0, an excise tax would be charged on all units sold at time t at a rate:

$$\tau = q\int_0^t P(u)e^{ru}du \tag{51}$$

In equilibrium, the price path is given by (3), and so the tax rate is equal to:

$$\tau = \frac{qP_0}{2r+q}(e^{(2r+q)t} - 1) \tag{52}$$

The variables in this equation are all known at time t, and, so long as the quantity of sales is known, the tax can be implemented. This system shares with other excise taxes the advantages that it is not imposed until revenue from sales is actually realized, and it is flexible enough not to impose the risks upon property owners that their stocks might prove to be much smaller (or larger) than anticipated.

It might even be possible to use this device to impose property taxes upon foreign suppliers. Suppose that country A possesses all of the known stocks of some resource, but that the consumption takes place entirely in country B. A tariff imposed by B that follows the path indicated by equations (51) or (52) would duplicate the effect of a property tax imposed directly upon the foreign stocks.

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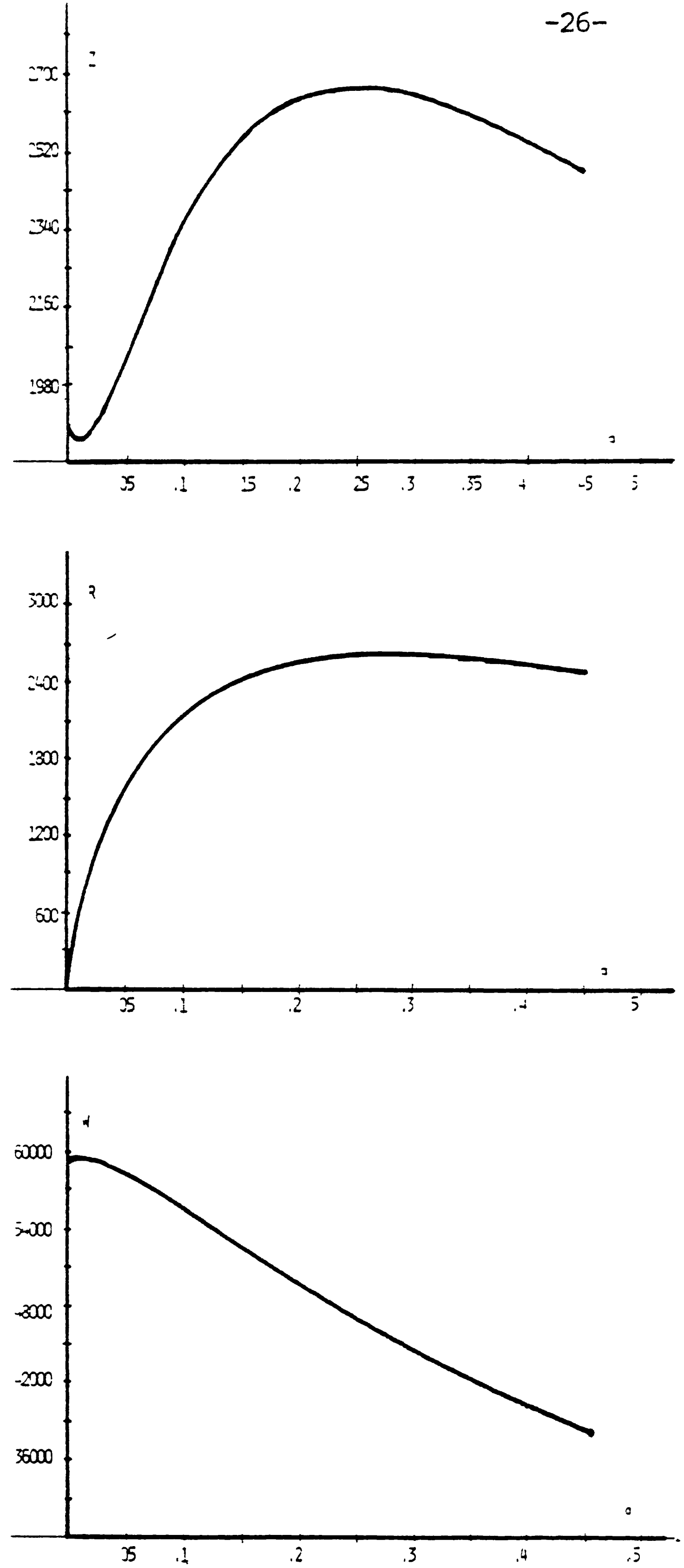

Figure 1

Demand: $Q = 50P^{-0.5}$
extraction cost = 0
$S = 2000$, $P'' = 1000$, $r = 0.05$

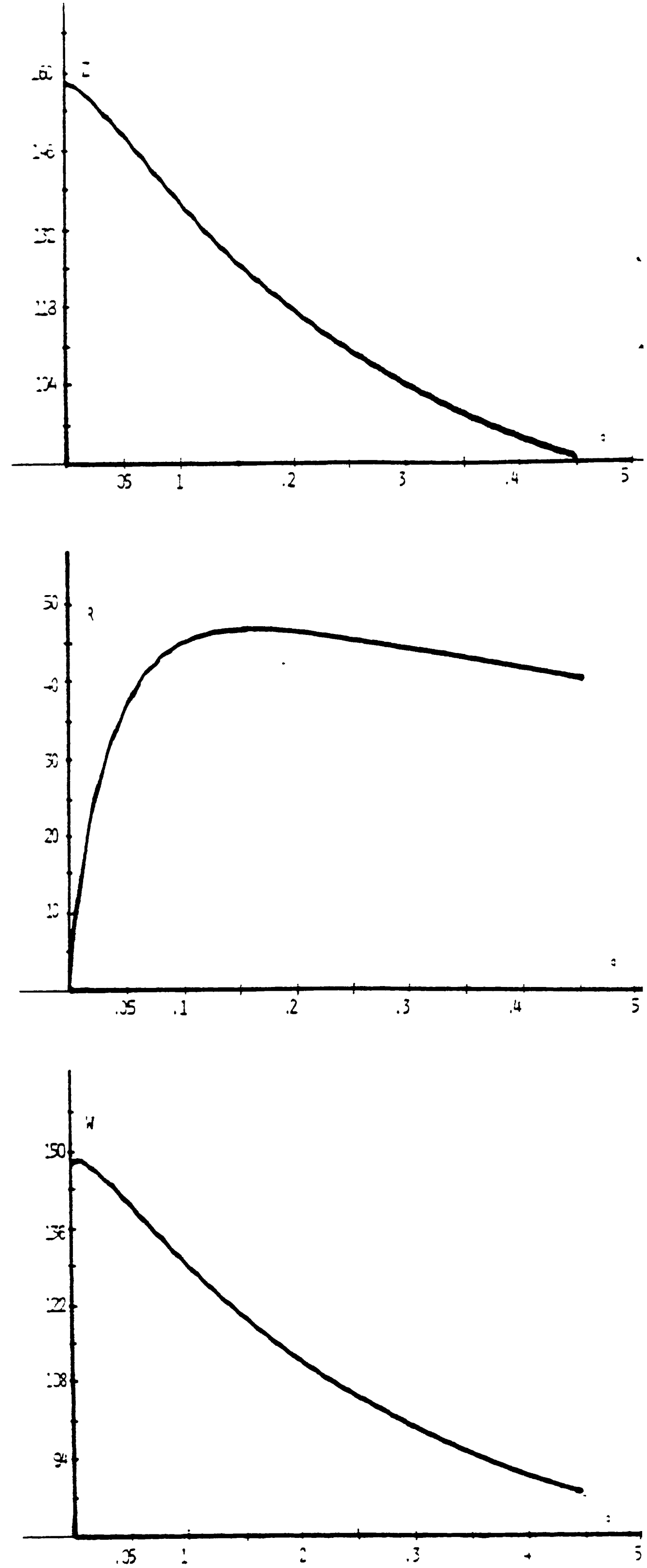

Figure 2

Demand: $Q = 5P^{-2}$
extraction cost = 0
$S = 500$, $P'' = 10$, $r = 0.05$

Values of gross sales (Z), tax revenue (R), and consumer surplus (W) as functions of the property tax rate (q)

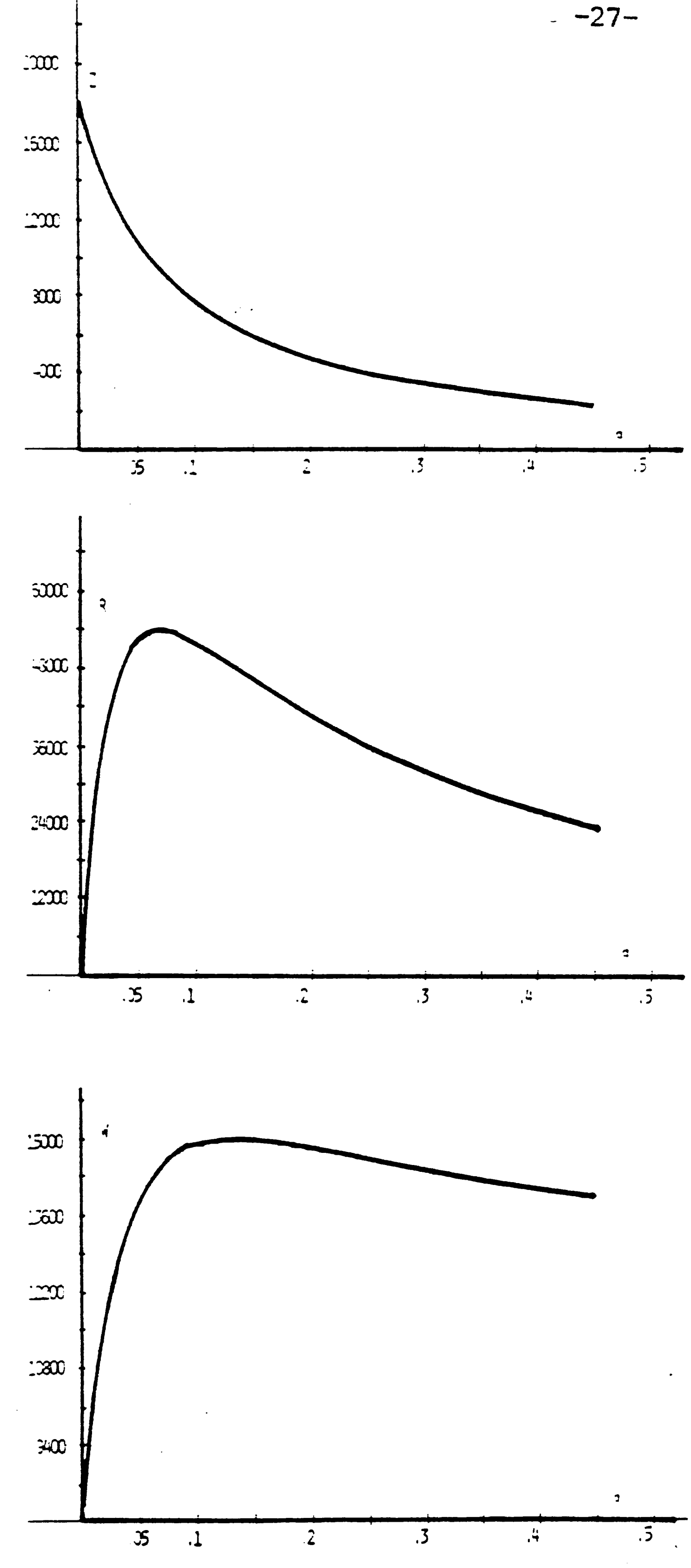

Figure 3

Demand: $Q = 200P^{-0.5}$
extraction cost = 1
S = 2000, P" = 20, r = 0.02

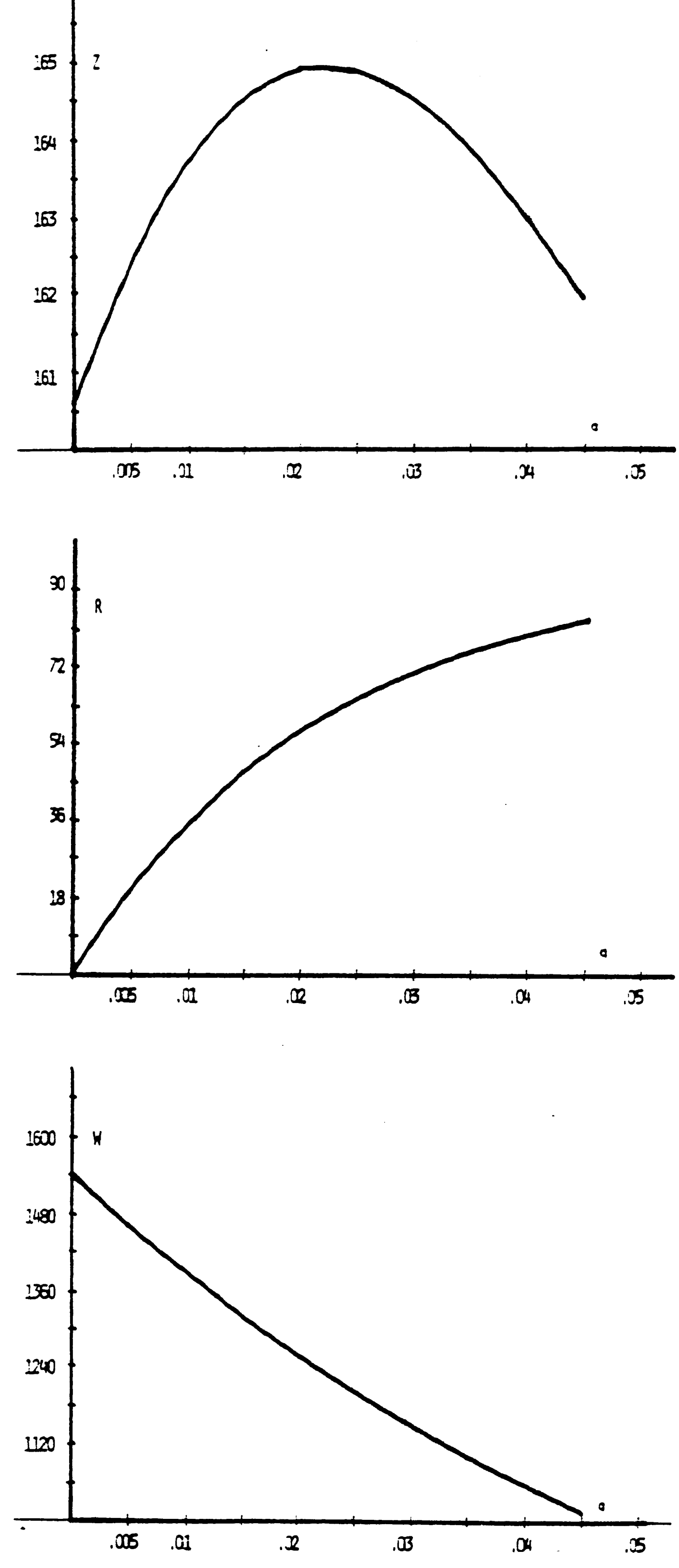

Figure 4

Demand: $Q = 5P^{-1.25} + 0.05t$
extraction cost = 0
S = 600, P" = 20, r = 0.05

Values of sales (Z), tax revenue (R), and consumer surplus as functions of the property tax rate (q)

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