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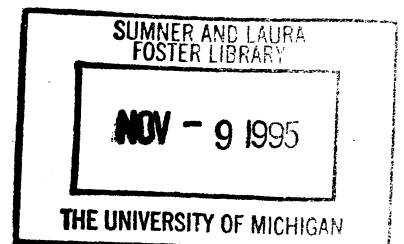
**Collapses of Fixed Exchange-Rate Regimes as  
Breakdown in Cooperation: the EMS  
in 1992-1993 and the Transition to EMU**

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Collapses of Fixed Exchange-Rate Regimes as  
Breakdown in Cooperation: the EMS in 1992-93  
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**Abstract**

In this paper the collapse of a bilateral fixed exchange-rate regime is described as the optimizing decision of the two countries' monetary authorities on when to break down the cooperative exchange-rate agreement. In particular, the two countries experience a trade-off between (a) fixing the nominal exchange rate, and therefore losing monetary independence, but having exogenous benefits from the agreement, and (b) letting the nominal exchange rate freely fluctuate so as to isolate the countries from asymmetric shocks on nominal variables. The paper derives the optimal exit decision in a stochastic framework when there are exogenous and irreversible benefits from the fixed exchange-rate regime. As a result, the agreement tends to last longer than it would in a deterministic framework even though big asymmetric shocks hit the two countries. This could well describe why the exchange-rate arrangement among the European countries (i.e., the Exchange Rate Mechanism of the European Monetary System) lasted so long with no realignments after 1987. In particular, the crisis occurred a few years after both German Monetary Unification and the burst of the last European recession.

**JEL Classification:** F33, F41, F42.

**Keywords:** Fixed Exchange Rates, Monetary Policy Coordination, European Monetary System, Optimal Stopping Rules.

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# Collapses of Fixed Exchange-Rate Regimes as Breakdown in Cooperation: the EMS in 1992-93 and the Transition to EMU<sup>1</sup>

Giuseppe De Arcangelis<sup>2</sup>

## 1 Introduction

The monetary history of Europe in the last two decades has been characterized by a succession of different exchange-rate regimes and it has been the object of many recent academic contributions.

Following its establishment in 1979, the Exchange Rate Mechanism (ERM) of the European Monetary System (EMS) was mainly an adjustable-peg with frequent realignments in the first eight years. With the Basle-Nyborg Agreement (1987) the system became a tightly fixed exchange-rate regime (see Giavazzi and Spaventa, 1990). In particular, the Agreement discouraged realignments and promoted intra- and infra-marginal interventions by lowering the cost of borrowing among central banks.<sup>3</sup> In other words, it was believed that enhancing *cooperation* among the central banks would avoid the occurrence of future currency crises.

And this actually worked for over five years. Suddenly, in the fall 1992 the English Pound and the Italian Lira abandoned the system due to the occurrence of huge speculative attacks. A few months later other currencies also (including the “healthy” French Franc) were under attack, but the system managed to survive. Finally, in the summer 1993 most currencies

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<sup>3</sup>See, in particular, the changes in the *very short-term financial facilities* as described, for instance, in De Grauwe (1994a).

were again under attack and the national authorities decided not to realign, but to widen their fluctuation bands from  $\pm 2.25\%$  to  $\pm 15\%$  around the declared parities (with the exception of the Dutch Guilder that maintained the original  $\pm 2.25\%$  band).

This recent crisis has greatly revived the debate on speculative attacks, balance-of-payments crisis and the long-run sustainability of fixed exchange-rate regimes with free capital mobility.

In particular, many authors interpreted all the major events that characterized Europe between 1987 and 1992 — i.e., German Monetary Unification, the Maastricht Treaty, the European recession that started in 1990-91 — in the framework of traditional speculative-attack models.

More exactly, the literature has mainly compared two classes of models hinged on the exhaustion of international reserves. The first one highlights that a misalignment in the fundamentals between the leading country (which has been identified as Germany in Europe) and the other countries leads to speculative attacks as a run on international reserves. For instance, Krugman (1979) and Flood and Garber (1984) interpret this misalignment as a divergence in the monetary policies of the two countries.

The second class stresses the importance of *self-fulfilling* speculative attacks. In this case speculators foresee a change in the monetary policy of the weak-currency country *as a consequence* of the currency crisis. Fundamentals do not need to be divergent *before* the crisis. Instead, speculators perceive that there is an incentive for the weak-currency authorities to change monetary policy *only after* that they will not fix the exchange rate anymore. As a consequence, the fundamentals diverge, the weak-currency actually depreciate and the speculators can make huge capital gains in the foreign exchange market (see Obstfeld, 1986).

Eichengreen and Wyplosz (1993) have underlined the gradualism “with no forgiveness” towards the European Monetary Union (EMU) of the Maastricht Treaty (i.e., strict requirement of no realignments in the past two years to enter the last phase of EMU, otherwise the country would be rejected from the EMU project) as the incentive for the national authorities to switch policy regime once they had to abandon the fixed-exchange-rate policy. According to Eichengreen and Wyplosz (1993), this triggered a self-fulfilling, Obstfeld (1986)-type of attack on most of the currencies especially because the expected probability of a change in the policy regime was particularly high due to the deep European recession, at its peak in 1992-93.

However, some recent alternative models have underlined that the exhaustion of reserves is rarely the effective cause of the regime abandonment (see, for instance, in Obstfeld, 1994, the evidence regarding the attack on

the Swedish Krona).

Instead, these recent contributions have stressed that collapses of fixed exchange-rate regimes can be the outcome of optimizing decisions. In particular, Ozkan and Sutherland (1994a, b and c) have shown how the abandonment of the fixed exchange-rate regime for a small country may be the optimizing decision of authorities whose welfare function is only based on aggregate output (which they more generally qualify as a monetary index) in a "keynesian" (fixed-price) world. When the conduct of monetary policy in the foreign leading country is very tight and the foreign interest rate rises, the authorities of the weak-currency country may find it optimal to leave the regime.

In this paper I propose the application of a similar optimizing approach, but in a two-country model and with a general asymmetric shock. In particular, the monetary authorities of both countries optimally decide to *what extent* to coordinate perfectly their monetary policies and successfully avoid speculative attacks. After all, this is what the Basle-Nyborg Agreement promoted.

In particular, differently from other models that have been applied to the ERM crisis, I consider the fixed exchange-rate regime as a fully *bilateral* regime where both countries receive benefits and pay some costs. The analysis endogenously determines how the costs are shared and, as a result, gives the trigger value of the driving asymmetric shock at which exchange-rate pegging is optimally abandoned by both countries.

During the years between 1990 and 1993 all the EMS countries were hit by two big asymmetric shocks pointing in the same direction: the German Unification, which greatly increased fiscal transfers and public expenditure in Germany; and a deep recession in all the other EMS countries. The model developed in this paper underlines the importance of such asymmetries and aims at pointing out the important factors that characterize the trigger value of such a strong shock. When that trigger value is reached, the weak-currency monetary authorities are no longer willing to restrict the money supply and the strong-currency monetary authorities are no longer willing to expand their money supply (for instance, by a supporting intervention in favor of the weak currency).

In the present model the burden for each country imposed by asymmetric shocks is endogenously determined. However, such a burden can be too high and the flexible exchange-rate regime may be desirable to both countries in order to reacquire the extra policy instrument needed to cope with the shock — i.e., the nominal exchange rate.

The decision-making problem of the monetary authorities has strong

similarities with the case of the firm that has to decide when to disinvest in a certain market: there is a value in waiting to see if better times come, and this may explain why disinvestment does not immediately occur (see Dixit, 1992). Similarly, there was a value in waiting for the monetary authorities of both countries to see if the asymmetric shock was reversing — i.e., if either the recession would get milder or the fiscal policy in Germany less expansionary. This may explain why the EMS crisis occurred so long after the asymmetric shock started, but certainly when it was at its peak.

In the next section I present the optimizing decision of the monetary authorities of the two countries in the case of perfect capital mobility. Section 3 contains an illustrative example that readers with knowledge of the optimal stopping-point problem may skip. Section 4 derives the crisis trigger point and its determinants are discussed in Section 5. Section 6 concludes with some implications for the future of the transition towards EMU.

## 2 An Optimizing Model of Exchange-Rate Regime Switching in a Two-Country World

Consider a two-country model with perfect capital mobility and perfect asset substitutability. Initially the two countries are in a fixed (nominal) exchange-rate regime and country-1 monetary authorities' welfare is represented by the following function:

$$W_1(p_1) = E \left[ \int_t^{\infty} [Z - p_1(\tau)^2] e^{-\delta(\tau-t)} d\tau \mid p_1(t) = p_1 \right] \quad (1)$$

In words, the country-1 monetary authorities want to maximize the expected discounted flow of net benefits with the discount rate  $\delta$ . The authorities have a strong preference for price stability. Actually, the quadratic term in the flow represents deviations of the country-1 (log) price level,  $p_1(\tau)$ , from a given price level normalized to one.  $Z$  instead represents an exogenous flow of benefits from being in a fixed exchange-rate regime; once the authorities decide to let the currency freely fluctuate, the flow  $Z$  is irreversibly lost for all the future periods.<sup>4</sup>

Country-2 monetary authorities have a similar welfare function:

<sup>4</sup>The most typical example of such benefits is the set of economic agreements in other areas (for instance, the Common Agricultural Policy) among the European countries, which highly recommends a stability in the nominal exchange rate (see also Giavazzi and Giovannini, 1989).

$$W_2(p_2) = E \left[ \int_t^{\infty} [Z - \alpha p_2(\tau)^2] e^{-\delta(\tau-t)} d\tau \mid p_2(t) = p_2 \right] \quad (2)$$

The only difference between the two countries' welfare is in the "price-instability aversion", which is represented by the parameter  $\alpha$ . When  $\alpha$  is less (greater) than one, then the country-2 domestic authorities care less (more) about price stability than country-1 authorities and this is represented by a lower (higher) impact on their welfare when there is departure from price stability.<sup>5</sup>

The price levels that the two countries are willing to stabilize are determined in the following two-country model:

$$m_1(\tau) - p_1(\tau) = \kappa y_1(\tau) - \frac{\lambda}{2} i_1(\tau) \quad (3a)$$

$$y_1(\tau) = \frac{\eta}{2} [s(\tau) - p_1(\tau) + p_2(\tau)] - \frac{\gamma}{2} [i_1(\tau) - G_1(\tau)] + \varepsilon_1(\tau) \quad (3b)$$

$$y_1(\tau) = \bar{y}_1 \quad (3c)$$

$$d\varepsilon_1 = \sigma_1 dw \quad (3d)$$

$$m_2(\tau) - p_2(\tau) = \kappa y_2(\tau) - \frac{\lambda}{2} i_2(\tau) \quad (4a)$$

$$y_2(\tau) = -\frac{\eta}{2} [s(\tau) - p_1(\tau) + p_2(\tau)] - \frac{\gamma}{2} [i_2(\tau) - G_2(\tau)] - \varepsilon_2(\tau) \quad (4b)$$

$$y_2(\tau) = \bar{y}_2 \quad (4c)$$

$$d\varepsilon_2 = \sigma_2 dw \quad (4d)$$

$$i_1(\tau) = i_2(\tau) + F(\tau) \quad (5)$$

$$\varepsilon(\tau) = \varepsilon_1(\tau) + \varepsilon_2(\tau) \quad (6a)$$

$$d\varepsilon = \sigma dw \quad (6b)$$

$$\sigma = \sqrt{\sigma_1^2 + \sigma_2^2 + 2\rho_{1,2}\sigma_1\sigma_2} \quad (6c)$$

<sup>5</sup>The flow of benefits  $Z$  is assumed to be equal for both countries, but the analysis can be extended to the case of different flows for each country.

All the variables are in log terms (except for the interest rates). The two countries are characterized by two distinct money markets for each currency (Eq. (3a) and (4a)). Each foreign currency (i.e., country-2 currency in country 1 and country-1 currency in country 2) is dominated by foreign bonds, since foreign money does not give any money services.<sup>6</sup> There is also a goods market in each country where the supply of goods is fixed (Eq. (3c) and (4c)) and the demand for goods (Eq. (3b) and (4b)) depends on the relative price of the two goods — i.e. the *real* exchange rate — and the real interest rate. The nominal exchange rate  $s$  is measured in units of country-1 currency per one unit of country-2 currency and Eq. (5) represents uncovered interest parity. Finally,  $F$ ,  $G_1$  and  $G_2$  are indices of (respectively) expected nominal exchange-rate depreciation, expected country-1 inflation and expected country-2 inflation.

The two economies are identical except for the different indices of expected inflation and the presence of two asymmetric shocks  $\varepsilon_1$  and  $\varepsilon_2$ . The two country-specific shocks are the driving processes and are distributed as driftless brownian motions respectively with standard deviations  $\sigma_1$  and  $\sigma_2$ . They can be combined in one "fundamental" asymmetric shock,  $\varepsilon$ , whose standard deviation depends on the standard deviations of the single shocks and on their correlation,  $\rho_{1,2}$  (Eq. (6a)-(6c)).

The model is characterized by the classical dichotomy between the real and the nominal side. Therefore, independently of the type of *nominal* exchange-rate regime in place, the shocks  $\varepsilon_1$  and  $\varepsilon_2$  affect the equilibrium values of the relevant *real* variables — i.e., the real exchange rate and the real interest rate.

In other words, a positive value in both shocks implies a relative increase in the demand for the country-1 good with respect to the country-2 good. As a consequence, the relative price of the country-1 good with respect to the country-2 good must increase — i.e., there must be a real appreciation of the country-1 currency.

By normalizing the aggregate supply of each good to one and solving for the country-1 real exchange rate, I obtain at all times  $\tau$ :<sup>7</sup>

$$s(\tau) - p_1(\tau) + p_2(\tau) = -\frac{\varepsilon(\tau)}{\eta} + \frac{\gamma}{2\eta} [F(\tau) - G_1(\tau) + G_2(\tau)] = -\frac{\varepsilon(\tau)}{\eta}$$

<sup>6</sup>However, domestic agents need foreign currency to buy foreign bonds.

<sup>7</sup>Notice that the term in squared brackets is the expected change in the real exchange rate. Since the driving process is an unregulated brownian motion, all the variables will be functions of such a process and their expected change is zero. Therefore, the expected real depreciation of the country-1 currency is zero.

Let me assume with no loss of generality that  $\varepsilon_1 > \varepsilon_2$ , so that  $\varepsilon > 0$ . Then, interest rates must rise and the demand for money in both countries must fall. Then, given the welfare functions of the two countries' monetary authorities, the available options for monetary policymaking depend on the type of nominal exchange-rate regime.

Under flexible exchange rates restrictive monetary policies can be undertaken in both countries and the real appreciation of country-1 currency will fully become a nominal appreciation. Price stability in both countries can be achieved via changes (and also high volatility) in the nominal exchange rate. However, by switching to the flexible exchange-rate regime the authorities forgo all the exogenous benefits  $Z$ .

The countries start in a fixed exchange-rate regime and in this case the monetary authorities of both countries have one instrument (i.e., monetary policy) for two objectives (i.e., price stability and a fixed exchange rate). In particular, given the necessary change in the real exchange rate, now both authorities cannot implement restrictive monetary policies to save price stability and have at the same time a fixed value for  $s$ . Once the pegged value of the exchange rate is normalized to one, then the following must hold:

$$p_2(\tau) - p_1(\tau) = m_2(\tau) - m_1(\tau) = -\frac{\varepsilon(\tau)}{\eta}$$

In other words, the price levels and, hence, the money stocks are interdependent: the model simply determines what must happen to the *relative* price level (and, hence, to the *relative* money stocks), but leaves indeterminate the adjustment that must take place in each country. This is the so-called *redundancy* or *n-1* problem that always arises in a fixed exchange-rate regime.<sup>8</sup>

Let me assume that the burden of the shock is split between the two monetary authorities according to the fraction  $\beta$ :

$$p_1(\tau) = \beta \frac{\varepsilon(\tau)}{\eta}$$

<sup>8</sup>Giavazzi and Giovannini (1989, chapter 4) have a similar two-country model, but with a different driving process. They show that if the authorities have different objectives in their welfare functions (in particular, one has the money stock and the other international reserves), then in the Pareto-optimal arrangement the country with international reserves in its objective function will always take the whole burden of adjustment in the driving process. However, empirical evidence seems to be weak on the idea that the European countries (other than Germany) want to stabilize international reserves as their main objective.

$$p_2(\tau) = -(1 - \beta) \frac{\varepsilon(\tau)}{\eta}$$

where  $0 < \beta < 1$ .

Hence, given the required adjustment in the real exchange rate, country-1 authorities increase the money stock in order to let the country-1 price level increase by  $\beta \frac{\varepsilon(\tau)}{\eta}$  and country-2 authorities decrease the money stock so that the country-2 price level decreases by  $(1 - \beta) \frac{\varepsilon(\tau)}{\eta}$  at every time  $\tau$ .

When the shock takes more and more positive values and price instability rises, the cost for staying in the fixed exchange-rate regime becomes higher and higher. Then, there will be a particular value of  $\varepsilon$  for which the authorities decide to switch to a flexible exchange rate and to irreversibly lose the benefits  $Z$ .

In other words, the decision when to end the regime can be derived as the solution of an optimal stopping-point problem. Before formally showing such a solution, in the next Section I present a discrete-time, discrete-space example with a three-period horizon to give some general intuition of the general problem. Readers that are familiar with the optimal stopping-point problem in continuous time and a stochastic setup may skip next Section.

### 3 A Three-Period, Two-State, One-Country Example

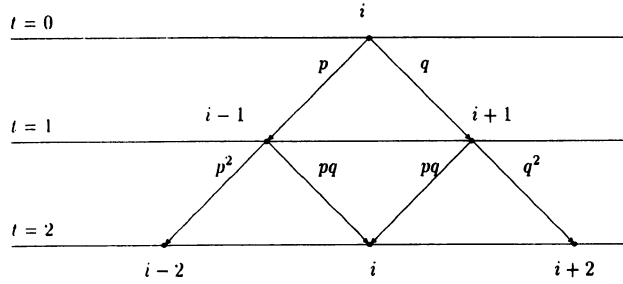
In this example let me neglect the interaction between the two countries by assuming that  $\beta = 1$ . The currency regime is then a *unilateral* exchange-rate regime where the burden of adjustment falls entirely on country 1 and its survival is only a decision of country 1.

Let me also assume that time is discrete, that there are three periods (0 to 2) and that the increments to the driving process  $\varepsilon$  follow a discrete random walk.

In particular,  $\varepsilon$  can take discrete values and starting from state  $i$  at time 0, it can be  $i - 1$  with probability  $p$  and  $i + 1$  with probability  $q = 1 - p$  at time 1. Therefore, at the final time 2,  $\varepsilon$  can take value  $i - 2$  with probability  $p^2$ ,  $i$  with probability  $2pq$  and  $i + 2$  with probability  $q^2$ . In other words, the increments of  $\varepsilon$  follow a random walk and the cumulative change of  $\varepsilon$  is a binomial random variable with transitional probabilities  $p$  and  $q$  since the increments are assumed to be independent. Figure 1 presents a diagram for this case.

In each period, first there is a realization of the stochastic shock, then

Figure 1: States of the Discrete Random Walk in the Three-Period Example



the country decides whether to still keep the exchange rate fixed or to leave the agreement.

Let me assume for simplicity that  $\eta = 1$ . Then the welfare function can be written as follows if the country stays in the fixed exchange-rate regime for all the three periods:

$$\begin{aligned} W(i; T=2) &= E \left[ \sum_{t=0}^T \left( \frac{1}{1+\delta} \right)^t (Z - \varepsilon_t^2) \mid \varepsilon_0 = i \right] = \\ &= (Z - i^2) + \left( \frac{1}{1+\delta} \right) \{ p[Z - (i-1)^2] + q[Z - (i+1)^2] \} + \\ &\quad \left( \frac{1}{1+\delta} \right)^2 \{ p^2[Z - (i-2)^2] + 2pq[Z - i^2] + q^2[Z - (i+2)^2] \} \end{aligned}$$

The expression above clearly shows that, given the binomial distribution of  $\varepsilon$ , the value of the welfare function depends only on the initial state of the shock ( $\varepsilon_0 = i$ ) and on the length of the horizon ( $T = 2$ ), apart from the exogenous  $Z$ , the given discount rate  $\delta$  and the probabilities  $p$  and  $q$ . I will derive a generalization of this formula in the next Section.

Let me assume that  $p = q = 1/2$ ; then, the welfare function becomes:

$$W_{0,2}(i; T=2) = (1 + R + R^2)Z - (1 + R + R^2)i^2 - R - 2R$$

where  $R = 1/(1 + \delta)$  is the discount factor.

Hence, when the initial value of the shock is in the following range:

$$-\sqrt{Z - R \frac{1+2R}{1+R+R^2}} < i < +\sqrt{Z - R \frac{1+2R}{1+R+R^2}}$$

then the welfare function when fixing the exchange rate for the remaining two periods is positive and the option to leave the exchange-rate agreement is not exercised at the end of time 0. Let me stress only the positive part of this interval and define:

$$i_{0,2} \equiv \sqrt{Z - R \frac{1+2R}{1+R+R^2}}$$

The above computation however does not take into account that at time 1, after the realization of the shock, the country can leave the agreement. This opportunity of letting the currency float after one period increases the overall value of the welfare function.

For instance, when sitting at time 0 and by assuming that at time 1 the exit option is exercised, the welfare function takes the following value:

$$\begin{aligned} W_{0,1}(i; T=2) &= (Z - i^2) + \left( \frac{1}{1+\delta} \right) \left\{ \frac{1}{2}[Z - (i-1)^2] + \frac{1}{2}[Z - (i+1)^2] \right\} \\ &= (1+R)Z - (1+R)i^2 - R \end{aligned}$$

Let me consider only positive realizations of the shocks. Then, the threshold value of  $i$  up to which it befits to stay in the agreement is:

$$i_{0,1} = \sqrt{Z - \frac{R}{1+R}}$$

which is always higher than the previous threshold  $i_{0,2}$ .

The two welfare functions,  $W_{0,1}$  and  $W_{0,2}$ , are shown in Figure 2 when the exogenous benefits  $Z$  is sufficiently high. When  $i < i^*$  the welfare for staying until period 2 is higher than the welfare from exiting at period 1.<sup>9</sup> On the contrary, when  $i > i^*$  then  $W_{0,1} > W_{0,2}$ .

<sup>9</sup>The value of  $i^*$  is  $\sqrt{Z - \frac{R}{1+R}}$ .



Since exiting before period 2 is optional, the welfare function at period 0 is then given by:

$$W_0 = \max [W_{0,1}; W_{0,2}]$$

which is represented by the thick line in Figure 2.

Moreover, the initial realization of the shock  $\varepsilon$  determines the expected exit time. When  $\varepsilon$  takes on a low value — i.e.,  $i < i^*$  —, then  $W_{0,2}$  holds and the expected ending time of the regime is period 2. Instead, if  $i > i^*$ , then  $W_{0,1}$  holds and the expected exit time is period 1.

The expected exit time strongly depends on the value of the exogenous benefit  $Z$ . When  $Z$  is low,<sup>10</sup>  $W_{0,1}$  is always higher than  $W_{0,2}$  and the expected exit time is always period 1 regardless of the initial realization of  $\varepsilon$ . This has an intuitive explanation: when the exogenous benefits to stay in the regime are low, then there is an incentive to bear only low adverse shocks for an expected shorter period of time.

Next, at period 1 after the realization of the shock, the option to leave can be exercised or not depending on the initial realization of the shock.

In case at period 1  $\varepsilon$  rises to  $i + 1$ , then the welfare function becomes:

$$W_{1,1}^+(i; T=1) = (Z - i^2) + R[Z - (i + 1)^2] + \frac{1}{2}R^2\{[Z - i^2] + [Z - (i + 2)^2]\}$$

This expression is greater than 0 when:

$$-\frac{R(1+R)}{(1+R+R^2)} - \sqrt{Z - \frac{R(1+2R)}{(1+R+R^2)} + \left[\frac{R(1+R)}{(1+R+R^2)}\right]^2} < i < \frac{R(1+R)}{(1+R+R^2)} + \sqrt{Z - \frac{R(1+2R)}{(1+R+R^2)} + \left[\frac{R(1+R)}{(1+R+R^2)}\right]^2}$$

It can be shown that the upper bound of this interval is lower than the upper bound of the interval obtained when considering the welfare function with no option to leave at time 1 — i.e.,  $i_{0,2}$ . Let me define such an upper bound  $i_{1,1}^+$ :

$$i_{1,1}^+ \equiv -\frac{R(1+R)}{(1+R+R^2)} + \sqrt{Z - \frac{R(1+2R)}{(1+R+R^2)} + \left[\frac{R(1+R)}{(1+R+R^2)}\right]^2}$$

<sup>10</sup>More exactly when  $Z < 2/R$ .

Instead, in case at period 1 the shock takes value  $i - 1$ , the welfare function is:

$$W_{1,1}^-(i; T=1) = (Z - i^2) + R[Z - (i - 1)^2] + \frac{1}{2}R^2\{[Z - (i - 2)^2] + [Z - i^2]\}$$

And the interval in which welfare is positive is the following:

$$\frac{R(1+R)}{(1+R+R^2)} - \sqrt{Z - \frac{R(1+2R)}{(1+R+R^2)} + \left[\frac{R(1+R)}{(1+R+R^2)}\right]^2} < i < \frac{R(1+R)}{(1+R+R^2)} + \sqrt{Z - \frac{R(1+2R)}{(1+R+R^2)} + \left[\frac{R(1+R)}{(1+R+R^2)}\right]^2}$$

Let me define the upper bound of this interval as  $i_{1,1}^-$ :

$$i_{1,1}^- \equiv \frac{R(1+R)}{(1+R+R^2)} + \sqrt{Z - \frac{R(1+2R)}{(1+R+R^2)} + \left[\frac{R(1+R)}{(1+R+R^2)}\right]^2}$$

In this case, depending on the value of  $Z$  and the discount rate  $\delta$  such an interval can either comprehend both  $i_{0,1}$  and  $i_{0,2}$  or not.

Let me first notice that for high values of  $Z$  the upper bound  $i_{1,1}^-$  of the interval is always higher than  $i_{0,1}$ . However, the country never gets to period 1 if  $\varepsilon$  does not take an initial value that is lower than  $i_{0,1}$ . Therefore, sitting at period 1, any value of  $i$  exceeding  $i_{0,1}$  can be disregarded.

Instead, when  $Z$  takes a low value<sup>11</sup> the highest threshold  $i_{1,1}^-$  at period 1 is lower than  $i_{0,1}$ .

Hence, the highest threshold at period 1 to exercise the exit option is:

$$i_{1,1} = \max [i_{1,1}^-, i_{1,1}^+]$$

I have already shown that  $i_{1,1}^+$  is lower than  $i_{0,1}$ ; moreover,  $i_{1,1}^-$  is always lower than  $i_{0,1}$  when  $Z$  is not too high.

In general,  $i_{1,1}$  must not be higher than  $i_{0,1}$  because only when the realization of the shock at period 0 is lower than  $i_{0,1}$  the authorities do not exercise the exit option already at the end of period 0. In addition, for low values of  $Z$  the threshold for which the option is exercised at time 1, i.e.  $i_{1,1}$ , is always lower than the threshold at time 0, i.e.  $i_{0,1}$ . Therefore, I can conclude that  $i_{1,1}$  is never higher than  $i_{0,1}$  and is strictly lower when  $Z$  is not too high as it is shown in Figure 3.

<sup>11</sup>But still high enough to assure a positive radicand.

In the end, this simple example highlights a few important points.

First, as already mentioned above, the opportunity of opting out of the agreement increases the welfare function. This is shown in Figure 2 where the final welfare function becomes a sort of envelope of the two welfare functions  $W_{0,1}$  and  $W_{0,2}$ .

Second, given the finite horizon, the remaining time for the option to be exercised is particularly relevant to determine the threshold values of the shock for which the fixed exchange-rate system survives from one period to the next. In general, these thresholds are lower and lower as the remaining time decreases:  $i_{1,1}$  cannot be higher than  $i_{0,1}$ , as explained above, and it is surely lower when  $Z$  is not very high (see Figure 3).

Intuitively, this happens because the remaining time to exercise the option goes to zero as time goes by and we are approaching the final period. Hence, since the welfare is increased by the opportunity of exiting, the welfare function is then raised by less and less as there are fewer and fewer periods in which the option can be exercised.<sup>12</sup> In terms of Figure 2 as we are approaching the final period there are fewer and fewer welfare functions of which to take the envelope. The example shows that the "degree of tolerance" towards adverse shocks lowers the value of the exit point as time goes by.

Moreover, the time when the option is exercised is a random variable and depends on both the initial realization of the shock and the remaining time. In particular, in the example above it is possible to obtain the collapse time of the regime in some particular cases. For instance, if  $i_{1,1} < i_{0,1}$  and the initial realization of  $\varepsilon$  is  $i_{1,1} < i_0 < i_{0,1}$ , then the exchange rate regime ends in period 1. In fact, at period 0  $\varepsilon$  is low enough that country 1 decides to stay one more period. However, since the initial realization  $i_0$  is higher than the threshold  $i_{1,1}$ , the country leaves the regime at the end of period 1. In other words, given the particular values of the parameters such that  $i_{1,1} < i_{0,1}$  and the initial realization  $i_0$ , even if  $\varepsilon$  goes down between period 0 and 1 its new value will not be low enough to assure a positive welfare for the case of staying one period — i.e., both  $W_{1,1}^+$  and  $W_{1,1}^-$  are less than zero.

Finally, it ought to be noticed that the finite-horizon case becomes particularly complex because of the presence of two state variables — i.e., remaining time to exercise the option and the initial state  $i$ .

<sup>12</sup>See Dixit (1993, p. 49-51) for a generalization.

## 4 The Optimal Switching States and the Cooperative Burden Shares

The example above has provided some intuition on the problem introduced in Sect. 2. However, it has neglected the interactive nature of the model between the two countries. Moreover, given the short-run horizon, the ending point of the regime could be obtained simply by considering all the possible states of the world at all time periods.

Let me now consider the general problem by initially reporting the welfare of the two countries for a generic burden share  $\beta$  and a generic ending point  $\bar{\varepsilon}$ :

$$W_1(\varepsilon; \beta, \bar{\varepsilon}) = E \left[ \int_t^{\hat{T}(\varepsilon)} \left[ Z - \beta^2 \frac{\varepsilon(\tau)^2}{\eta^2} \right] e^{-\delta(\tau-t)} d\tau \mid \varepsilon(t) = \varepsilon \right]$$

$$W_2(\varepsilon; \beta, \bar{\varepsilon}) = E \left[ \int_t^{\hat{T}(\varepsilon)} \left[ Z - (1 - \beta)^2 \frac{\varepsilon(\tau)^2}{\eta^2} \right] e^{-\delta(\tau-t)} d\tau \mid \varepsilon(t) = \varepsilon \right]$$

where  $\hat{T}(\cdot)$  is the random switching time to flexible exchange rates, which depends on the value of the switching state,  $\bar{\varepsilon}$ . I recall also that the shock  $\varepsilon(\tau)$  starts with value  $\varepsilon$  at time  $t$  and follows a driftless brownian motion.

After time  $\hat{T}(\bar{\varepsilon})$  both countries are in flexible exchange rates. Then, they no longer have the flow  $Z$ , but their monetary policies can be set independently and price stability can be always maintained. Hence, both welfare functions will be zero from time  $\hat{T}(\bar{\varepsilon})$  onwards.

The object of the analysis is to find the burden share and the value of the switching state(s) that will satisfy both criteria of cooperation and optimality for the two countries.

In particular, the outcome of the analysis should provide first a relationship between the current state and the burden share. This function should give the burden share which the two countries agree upon at different realizations of the shock.

Second, given the burden share at each state, the exit trigger point is decided so that it satisfies two conditions. First, at the switching state the two countries should be completely indifferent between fixing the nominal exchange rate and floating (*value-matching* condition). Second, among all the possible switching states that satisfy the value-matching condition, the

two countries will choose the first one that optimizes a certain cooperative criterion (*smooth-pasting* or optimality condition).

In particular, I consider two types of cooperation. First, I assume that welfare transfers are possible. Next, I consider a general Nash-bargaining setup where no welfare transfers are present.

#### 4.1 The Cooperative Setup with Welfare Transfers

In this section I assume that welfare transfers are possible between the two countries. An *efficient* solution can then be obtained in terms of the burden share and the exit trigger point.

Operationally, given the welfare possibility frontier that all the possible burden splits can generate, the availability of utility transfers allows to select a particular point on that possibility frontier so that the sum of the welfare is maximized. The value of  $\beta$  that corresponds to that point on the possibility frontier is the efficient share.

Such an efficient share could be directly obtained by maximizing (over the choice variable  $\beta$ ) the sum of the welfare.

In addition, a common trigger-exit point is selected by the two countries. At any exit trigger state the two countries must be indifferent between fixing the nominal exchange rate and letting it float. In a cooperative setup with transfers this means that, given the efficient redistribution of the asymmetric shock, transfers from the country with the positive welfare are just enough to compensate the country that is losing. An additional increase in the absolute value of the asymmetric shock would then make both countries worse off with respect to the floating-rate regime.

Then, among all these possible trigger-exit states, the two countries will select the one that will maximize the sum of their welfare.

Formally, the exit state and the burden share should be obtained as solutions to the following problem:

$$\begin{aligned} (\beta_c^*(\varepsilon), \varepsilon_c^*) &= \arg \max_{\beta, \bar{\varepsilon}} [W_1(\varepsilon; \beta, \bar{\varepsilon}) + W_2(\varepsilon; \beta, \bar{\varepsilon})] \\ \text{s. t.} \quad & [W_1(\varepsilon; \beta, \bar{\varepsilon}) + W_2(\varepsilon; \beta, \bar{\varepsilon})]_{\varepsilon=\bar{\varepsilon}} = 0 \end{aligned}$$

By substituting for the two welfare functions, the problem can be rewritten as follows:

$$\begin{aligned} (\beta_c^*(\varepsilon), \varepsilon_c^*) &= \arg \max_{\beta, \bar{\varepsilon}} E \left[ \int_t^{\bar{T}(\varepsilon)} \left\{ \left[ Z - \left( \beta \frac{\varepsilon(\tau)}{\eta} \right)^2 \right] + \right. \right. \\ &\quad \left. \left. \left[ Z - \alpha(1-\beta)^2 \left( \frac{\varepsilon(\tau)}{\eta} \right)^2 \right] \right\} e^{-\delta(\tau-t)} d\tau \mid \varepsilon(t) = \varepsilon \right] \\ \text{s. t.} \quad & E \left[ \int_t^{\bar{T}(\varepsilon)} \left\{ \left[ Z - \left( \beta \frac{\varepsilon(\tau)}{\eta} \right)^2 \right] + \right. \right. \\ &\quad \left. \left. \left[ Z - \alpha(1-\beta)^2 \left( \frac{\varepsilon(\tau)}{\eta} \right)^2 \right] \right\} e^{-\delta(\tau-t)} d\tau \mid \varepsilon(t) = \bar{\varepsilon} \right] = 0 \end{aligned}$$

Let me define  $V(\varepsilon)$  as the value function for this problem. It depends only on one state variable — i.e., the initial state  $\varepsilon$ . This occurs because although the expected integral to be maximized does not have an infinite horizon, the stopping time is a function of the optimal stopping state. But the optimal stopping state is a variable internally determined by the maximization problem. Therefore, “remaining time to exercise the option”<sup>13</sup> does not represent an extra dimension of the problem since it is continuously and internally determined by the optimization problem.

Hence, the value function  $V(\varepsilon)$  can be defined as follows:

$$\begin{aligned} V(\varepsilon) &= \max_{\beta, \bar{\varepsilon}} E \left[ \int_t^{\bar{T}(\varepsilon)} f(\varepsilon; \beta) e^{-\delta(\tau-t)} d\tau \mid \varepsilon(t) = \varepsilon \right] \\ \text{s. t.} \quad & V(\bar{\varepsilon}) = 0 \end{aligned}$$

where

$$f(\varepsilon; \beta) \equiv \left[ Z - \left( \beta \frac{\varepsilon}{\eta} \right)^2 \right] + \left[ Z - \alpha(1-\beta)^2 \left( \frac{\varepsilon}{\eta} \right)^2 \right]$$

Since the value function is obtained as the max value function over two variables,  $\beta$  and  $\bar{\varepsilon}$ , let me fix momentarily  $\bar{\varepsilon}$  at  $\varepsilon^a$  and solve for the optimal  $\beta^*$ , which should then be a function of  $\varepsilon$  and of the initially fixed  $\varepsilon^a$ .

<sup>13</sup>This is the additional state variable to include in these kinds of optimization problems. See for instance Dixit (1993), p. 49-51.

When fixing  $\varepsilon$  to  $\varepsilon^a$ , the problem becomes a finite-horizon one since the ending time is now fixed by  $\varepsilon^a$ . Therefore, the value function now depends also on time and must satisfy the following (Bellman) partial differential equation:

$$-V_t(\tau, \varepsilon) = \max_{\beta} \left[ f(\varepsilon; \beta) e^{-\delta(\tau-t)} + \frac{\sigma^2}{2} V_{\varepsilon\varepsilon}(\tau, \varepsilon) \right]$$

or

$$-V_t(\tau, \varepsilon) = \frac{\sigma^2}{2} V_{\varepsilon\varepsilon}(\tau, \varepsilon) + \max_{\beta} \left[ f(\varepsilon; \beta) e^{-\delta(\tau-t)} \right]$$

Hence, the function  $\beta_c^*(\varepsilon)$  can be easily obtained as:

$$\beta_c^*(\varepsilon) = \arg \max_{\beta} f(\varepsilon; \beta)$$

It turns out then that  $\beta_c^*(\varepsilon)$  does not depend on  $\varepsilon^a$ . Therefore, the burden share is stable over different states of the world and does not depend on the vicinity to the collapsing state. In other words,  $\beta_c^*(\varepsilon)$  is *state-consistent* to use the terminology of Ozkan and Sutherland (1994a, b and c).

Moreover, the choice of the burden share in this intertemporal framework is not different from the single period case. In fact,  $\beta_c^*(\varepsilon)$  is simply determined as the share that maximizes the *flow* of benefits at each time. This is a particular feature of the solution that depends mainly on the complete exogeneity of the driving process and the absence of any feedback from the burden share to the process  $\varepsilon$ .

As a result, the best policy for the two countries when deciding cooperatively the burden share is to choose a law for  $\beta$  that maximizes the joint welfare at each point in time.

By substituting back the definition for  $f(\varepsilon; \beta)$ , the result is even stronger in this case:

$$\beta_c^* = \frac{\alpha}{1 + \alpha}$$

The efficient share is constant over states in this cooperative setup and it depends only on the relative "inflation aversion" of the two countries: the lower the inflation aversion of country 2, then the lower the burden share of the more "inflation-averse" country 1.

Once the efficient burden share is determined, let me now obtain the "efficient flow" by substituting it back in the original flow function:

$$f_c^*(\varepsilon) \equiv f(\varepsilon; \beta_c^*) = 2Z - \frac{\alpha}{1 + \alpha} \frac{\varepsilon^2}{\eta^2}$$

Given the solution for the burden share, now I have to solve for the optimal exit point. In particular, once the solution for the solution for the burden share is substituted back in the value function, I now have:

$$V(\varepsilon) = \max_{\bar{\varepsilon}} E \left[ \int_t^{\hat{T}(\varepsilon)} f_c^*(\varepsilon) e^{-\delta(\tau-t)} d\tau \mid \varepsilon(t) = \varepsilon \right]$$

s. t.  $V(\bar{\varepsilon}) = 0$

Since the optimal exit point is endogenously determined, then also the stopping time is endogenous. Hence, the value function is now a function only of one state variable as originally stated above.

In particular, when  $\varepsilon$  is below an exit point, then the value function must satisfy the following Bellman equation for any (current) state  $\varepsilon$ :

$$V(\varepsilon) = (1 - \delta dt) E[V(\varepsilon + d\varepsilon)] + f_c^*(\varepsilon) dt \quad (7)$$

with the boundary condition that  $V(\bar{\varepsilon}) = 0$  for a generic exit point  $\bar{\varepsilon}$ .

In the Appendix A it is shown how the differential equation (7) is equivalent to the following ordinary differential equation by using Ito's lemma:

$$\delta V(\varepsilon) = \frac{\sigma^2}{2} V''(\varepsilon) + f_c^*(\varepsilon)$$

In addition, one boundary condition holds at  $\bar{\varepsilon}$  as an exit point. This means that when the asymmetric shock takes value  $\bar{\varepsilon}$ , the two countries are indifferent between the fixed and the floating regime, as explained above:

$$V(\bar{\varepsilon}) = 0$$

A second boundary condition states instead that cumulative negative shocks are neglected for simplicity.<sup>14</sup>

<sup>14</sup>In fact, under a quadratic welfare function I should assume that a persistent negative shock could cause an undesirable high deflation domestically. Here, I rule out the exit for such a case (or alternatively I assume that the lower exit point tends to  $-\infty$ ) for two main reasons. First, if country 1 is Germany and  $\varepsilon$  wants to represent the German Unification shock and the recession in Europe, then economically I am interested in the effect of the positive realizations of such a random variable, although the variable could

Hence, I can now obtain the value function for a generic exit point  $\varepsilon$  as the solution of the above differential equation with the two boundary conditions just discussed. This is a function of both the current state and the generic exit point, and actually represents the joint welfare of the two countries for any current state  $\varepsilon$  below the exit point  $\bar{\varepsilon}$ .<sup>15</sup>

$$V(\varepsilon, \varepsilon) = \left( \frac{\alpha}{(1+\alpha)\delta} \frac{\varepsilon}{\eta^2} - \frac{2Z}{\delta} + \frac{\alpha\sigma^2}{(1+\alpha)\eta^2\delta^2} \right) e^{\theta(\varepsilon-\bar{\varepsilon})} + \frac{2Z}{\delta} - \frac{\alpha\sigma^2}{(1+\alpha)\eta^2\delta^2} - \frac{\alpha}{(1+\alpha)\delta} \frac{\varepsilon^2}{\eta^2} \quad (8)$$

where  $\theta = \sqrt{\frac{2\sigma^2}{\delta}}$ .

Finally, optimization must take place: the authorities choose optimally the switching point  $\bar{\varepsilon}$  by selecting the value for that maximizes (8).

The optimality condition for  $\bar{\varepsilon}$  is given by the first-order condition in the maximization of (8) provided that the function is concave. This first order condition represents the *smooth-pasting condition* for this problem. In other words, the first order condition selects the optimal stopping point at which action takes place and the monetary authorities break the exchange-rate agreement.

In the firm problem of Dixit (1992) the smooth-pasting condition selects the optimal trigger point for which the value of the investment (or disinvestment) option is maximized. Here the smooth-pasting condition gives the optimality condition to be satisfied by the optimal switching state in order to maximize the welfare function of the authorities.

By differentiating with respect to  $\bar{\varepsilon}$  smooth-pasting requires:

$$2 \frac{\alpha}{(1+\alpha)\eta^2} \bar{\varepsilon} - \theta \frac{\alpha}{(1+\alpha)\eta^2} \bar{\varepsilon}^2 + \theta \left( 2Z - \frac{\alpha\sigma^2}{(1+\alpha)\delta\eta^2} \right) = 0 \quad (9)$$

It ought to be noticed that if  $Z$  is sufficiently high the welfare function is never *always decreasing* with respect to the switching point  $\bar{\varepsilon}$ . In other words, if the common benefits to the authorities are too low, then no optimal switching point can be found since the authorities can always increase their welfare by lowering the switching point. This is the same as to say that no

take also negative values. Second, even when including a lower exit point the qualitative results of the analysis would not change substantially. However, the presence of a lower exit point makes the algebra much more complex and the economic meaning of the results more difficult to analyze.

<sup>15</sup>See Appendix A.

fixed exchange-rate regime with large asymmetric shocks can be maintained unless there are sufficiently high benefits accruing to both countries.

By ruling out low values of  $Z$ , the optimal switching point is the following:

$$\varepsilon_c^* = \sqrt{\frac{\sigma^2}{2\delta}} + \sqrt{2 \frac{\eta^2(1+\alpha)}{\alpha} Z - \frac{\sigma^2}{2\delta}} \quad (10)$$

A discussion on the main determinants of this exit point is provided in Sect. 5.

In the next Section, I will present the solution to the bargaining problem according to the axiomatic bargaining approach.

## 4.2 The Nash Bargaining Solution

The Nash bargaining approach proposes a solution to the bargaining problem that satisfies four axioms: invariance to equivalent welfare representations, symmetry, independence of irrelevant alternatives and Pareto efficiency.

Such a solution can be obtained in the present case by maximizing the *product* of the welfare functions over the burden share  $\beta$  and the exit point  $\bar{\varepsilon}$ :

$$\begin{aligned} (\beta_c^*(\varepsilon), \varepsilon_c^*) &= \arg \max_{\beta, \bar{\varepsilon}} [W_1(\varepsilon; \beta, \bar{\varepsilon}) W_2(\varepsilon; \beta, \bar{\varepsilon})] \\ \text{s. t.} & \quad [W_1(\varepsilon; \beta, \bar{\varepsilon}) + W_2(\varepsilon; \beta, \bar{\varepsilon})]_{\varepsilon=\bar{\varepsilon}} = 0 \end{aligned}$$

Let me consider again the value function for this problem:

$$\begin{aligned} V(\varepsilon) &= \max_{\beta, \bar{\varepsilon}} E \left[ \int_t^{\bar{T}(\varepsilon)} f(\varepsilon; \beta) e^{-\delta(\tau-t)} d\tau \mid \varepsilon(t) = \varepsilon \right] \\ \text{s. t.} & \quad V(\bar{\varepsilon}) = 0 \end{aligned}$$

where

$$f(\varepsilon; \beta) \equiv \left[ Z - \left( \beta \frac{\varepsilon}{\eta} \right)^2 \right] \left[ Z - \alpha(1-\beta)^2 \left( \frac{\varepsilon}{\eta} \right)^2 \right]$$

The form of the problem is similar to the case of cooperation with welfare transfers.

In particular, since the function  $f(\cdot; \cdot)$  still depends only on the same two variables as in the previous case, the determination of the burden share will follow the same steps as above.

As a result, the burden share  $\beta_n^*(\varepsilon)$  results again from the pointwise decision of the two countries and it coincides with the Nash-bargaining solution for the period-by-period case:

$$\beta_n^*(\varepsilon) = \arg \max_{\beta} f(\varepsilon; \beta) = \arg \max_{\beta} \left[ Z - \left( \beta \frac{\varepsilon}{\eta} \right)^2 \right] \left[ Z - \alpha(1 - \beta)^2 \left( \frac{\varepsilon}{\eta} \right)^2 \right]$$

Therefore, quite surprisingly the intertemporal dimension does not play any role in determining the law for  $\beta$ . This is due to the various characteristics of the setup. First, the Markovian characteristic of the brownian motion gives an initial time separability to the problem. In addition, the time independence of the welfare flows and the absence of feedback from the current "control variable"  $\beta$  on the driving process isolates the max operator on the welfare flow.

Once the share  $\beta_n^*(\varepsilon)$  is determined, it can be substituted in the welfare flow of each country in order to obtain the welfare flow for each country as a function only of the current state.

Let me define:

$$f_i^*(\varepsilon) \equiv f_i^*(\varepsilon; \beta_n^*(\varepsilon))$$

for  $i = 1, 2$ , where

$$\begin{aligned} f_1(\varepsilon; \beta) &= \left[ Z - \left( \beta \frac{\varepsilon}{\eta} \right)^2 \right] \\ f_2(\varepsilon; \beta) &= \left[ Z - \alpha(1 - \beta)^2 \left( \frac{\varepsilon}{\eta} \right)^2 \right] \end{aligned}$$

Then, I can obtain the welfare functions for each country given a generic common exit state  $\bar{\varepsilon}$ . Analogously to the Sect. 4.1, the welfare function of country  $i$  must satisfy the following differential equation when the current state  $\varepsilon$  is below the exit state  $\bar{\varepsilon}$ :

$$\delta W_i(\varepsilon) = \frac{\sigma^2}{2} W_i''(\varepsilon) + f_i^*(\varepsilon)$$

The relative general solution is the following:

$$W_i(\varepsilon) = A e^{-\theta \varepsilon} + B e^{\theta \varepsilon} + \bar{W}_i(\varepsilon)$$

$A$  and  $B$  are the constants of integration to be determined by the boundary conditions and  $\bar{W}_i(\varepsilon)$  is the particular solution to the fundamental differential equation, which depends on the shape of the function  $f_i^*(\varepsilon)$ .

The boundary conditions  $V_i(\bar{\varepsilon}) = 0$  and the irrelevance of negative realizations of the asymmetric shock<sup>16</sup> can be considered as in Sect. 4.1.

Then, the solution takes the following shape for a generic exit point  $\bar{\varepsilon}$ :

$$W_i(\varepsilon; \bar{\varepsilon}) = -\bar{W}_i(\bar{\varepsilon}) e^{\theta(\varepsilon - \bar{\varepsilon})} + \bar{W}_i(\varepsilon)$$

Finally, the Nash-bargaining exit point can be obtained as the exit point  $\varepsilon_n^*$  that maximizes the product of the two welfare functions just obtained:

$$\varepsilon_n^* = \arg \max_{\varepsilon} W_1(\varepsilon; \bar{\varepsilon}) W_2(\varepsilon; \bar{\varepsilon})$$

By substituting from the welfare functions above, the first-order condition for the above problem has the following form:

$$\begin{aligned} & [\bar{W}_1'(\bar{\varepsilon}) \bar{W}_2(\bar{\varepsilon}) + \bar{W}_1(\bar{\varepsilon}) \bar{W}_2'(\bar{\varepsilon}) - 2\theta \bar{W}_1(\bar{\varepsilon}) \bar{W}_2(\bar{\varepsilon})] e^{\theta(\varepsilon - \bar{\varepsilon})} - \\ & [\bar{W}_1'(\bar{\varepsilon}) \bar{W}_2(\varepsilon) + \bar{W}_1(\varepsilon) \bar{W}_2'(\bar{\varepsilon})] + \theta [\bar{W}_1(\bar{\varepsilon}) \bar{W}_2(\varepsilon) + \bar{W}_1(\varepsilon) \bar{W}_2(\bar{\varepsilon})] = 0 \end{aligned}$$

Differently from the cooperative case with welfare transfers now state inconsistency is present since the exit trigger point varies with the different realizations of the shock. Hence, the first-order condition determines a relationship between the current state and the exit trigger point:

$$\bar{\varepsilon} = g(\varepsilon)$$

The Nash state-consistent exit point is then the first fixed point of the above condition.

## 5 The Determinants of the Optimal Trigger Points and the 1992-93 ERM Crisis

Eq. (10) shows the trigger value of the asymmetric shock  $\varepsilon$  for which it is no longer optimal for both authorities to fix the nominal exchange rate even with the presence of welfare transfers. In the current section I study

<sup>16</sup>See footnote 14.

the relevant characteristics of this trigger point and I relate them to the ERM crisis in 1992-93 by assuming that the EMS was fully cooperative exchange-rate regime with welfare transfers.

In undertaking the decision to break the agreement and not to defend the declared parities the analysis endogenously takes into consideration that both countries (and not only the weak-currency one) are forgoing the benefits from an area of exchange-rate stability. In Europe higher exchange-rate volatility means, for instance, to put pressure on the Community Budget due to the compensatory transfers that would come from the asymmetric design of the Common Agricultural Policy.

The value of the trigger point  $\varepsilon_c^*$  depends on some key parameters. It is then interesting to study the effect of each one of them on the value of the trigger point and, therefore, on the survival probability of the fixed exchange rate between two countries.<sup>17</sup>

By assuming that  $\varepsilon$  is the total asymmetric shock due to the German Unification ( $\varepsilon_1$ ) and the recession in Europe ( $\varepsilon_2$ ), then country 1 is the center country (i.e. Germany) and country 2 each one of all the other EMS members. Then, the model is able to point out some features in the bilateral relationship between Germany and all the other EMS countries that could explain why some countries abandoned the regime as early as September 1992 (i.e., Italy and UK) and some others surrendered to market speculation by changing the "rules of the game" almost a year later.<sup>18</sup>

Let me consider first the influence of the parameter  $\eta$ . The relationship with the exit trigger point is strongly positive. Intuitively, the higher the aggregate demand elasticity to the real exchange rate, the lower the needed adjustment in the real exchange rate (and, hence, in each domestic price level under fixed exchange rates) when an asymmetric aggregate demand shock occurs.

In other words, the parameter  $\eta$  measures a sort of degree of substitutability between the goods produced in the two countries. In the extreme case of perfectly substitutable goods (i.e.,  $\eta$  tends to infinity), purchasing power parity holds and the aggregate demand shock has only an effect on the real interest rate, but no effect on the real exchange rate. In this case the two countries do not lose any instrument (i.e., the nominal exchange

<sup>17</sup>The simulations are performed by using the following basic values:  $\sigma=0.1$ ,  $\delta=0.01$ ,  $\eta=1$ ,  $Z=2$ ,  $\alpha=0.95$ .

<sup>18</sup>The Spanish Peseta, the Portuguese Escudo and the Irish Punt managed to remain in the ERM, but with (sometimes multiple) realignments. In the present analysis a realignment can be considered an exit with immediate re-entry that reduces the exogenous benefits  $Z$ . Therefore, the realignment case will not be so different from the exit case.

rate) in a fixed exchange-rate arrangement, but gain the benefits  $Z$  in the current model.

An indicative measure of the relative interdependence between the European economies and their relative degree of openness can be obtained from the IMF Multilateral Exchange Rate Model.<sup>19</sup> By considering the impact of the changes in the relative prices of output among the main European economies, it is possible to obtain the weight of each relevant bilateral exchange rate when constructing the effective exchange rate of each economy. For instance, the weight of the Deutsche Mark in the effective French Franc is 0.236, whereas the weight of the French Franc in the effective Deutsche Mark is 0.201. By taking the simple averages of these bilateral weights as a rough reference, the 0.218 between France and Germany is almost double the value between Germany and UK (0.114).

Then, given the influence of the parameter  $\eta$ , the explanation of the earlier "exit" of UK with respect to France can hinge on the different degree of openness of the two economies with respect to Germany.

Next, I consider the relationship between the impatience rate of the authorities and the exit trigger point. It ought to be noticed first that the influence of the impatience rate on the trigger point is only present when there is uncertainty in the model, i.e. when  $\sigma$  is not zero. In fact, in the deterministic case changing values for  $\delta$  do not show up in the burden shares and the trigger point depends only on  $\eta$ ,  $Z$  and  $\alpha$ . Therefore, the intertemporal profile in the preferences of the monetary authorities plays a role exclusively in the current stochastic framework.<sup>20</sup>

Fig. 4 presents the relationship between the impatience rate and the trigger point. In particular, the graph shows that the more impatient the authorities, the lower the trigger point. In other words, when there is a high weight on the present negative values in the welfare functions, this can offset the possibility that in the future the shock can revert and the welfare flow could become positive again. In the extreme case of authorities that care only for the current value of the welfare flow — i.e.,  $\delta$  is extremely large — the trigger point is close to the deterministic trigger point.

Actually, the graph highlights that the negative relationship between the impatience rate and the trigger point is confined to low values of the discount

<sup>19</sup>The data are referred to the 1977 base year and are fully reported in Giavazzi and Giovannini (1989, p. 56).

<sup>20</sup>In the cooperative case (both with and without transfers, an asymmetry in the discount rates would make the problem no longer time-autonomous. Then, the efficient fraction  $\beta$  becomes time dependent and the problem in stochastic dynamic programming can be solved by finding the solution to a partial differential equation.

rate. The trigger point becomes very steady for values of the impatience rate that are greater than half the standard deviation of the total asymmetric shock — i.e., for a value of  $\theta$  higher than one — in the present framework.

Next, Fig. 5 shows the effect of uncertainty in the model as introduced with the brownian motion process. The dashed horizontal line represents the trigger point when there is zero variability in the stochastic process — i.e., the deterministic case.

The picture shows that the introduction of a stochastic element raises the value of the trigger point and the expected survival of the regime. In other words, when the variability of the process is sufficiently low, the authorities are more willing “to wait and see”. If negative offsetting shocks took place, the process would revert to zero and the welfare would become positive again. Therefore, the authorities do not have to forgo the benefits  $Z$ .

In terms of the original model the total variability  $\sigma$  is given by the sum of each country-specific shock and their relative correlation (see Eq. (6c)). As Fig. 5 highlights, differences in  $\sigma_2$  and in  $\rho_{1,2}$  (and therefore in the total  $\sigma$ ) between the country couples with Germany would identify different trigger points in the bilateral relationships and thus different exit times. This may represent an additional explanation why the various ERM countries experienced the exchange-rate crisis at different times.<sup>21</sup>

Next, the role of the exogenous benefits  $Z$  and of the parameter  $\alpha$  are analyzed in Fig. 6. The higher the benefits  $Z$ , then of course the higher the trigger point and the longer the fixed exchange-rate regime is expected to last. However, also the lower the parameter  $\alpha$  — i.e., the two countries have a very different preference towards price stability — then the higher the trigger point. This occurs because the country with the lowest preference towards price stability is available to take a higher share of the burden in the price adjustment.<sup>22</sup> Therefore, Fig. 6 shows how the relationship between

<sup>21</sup> Bayoumi and Eichengreen (1993) have obtained a measure of country-specific aggregate-demand and aggregate-supply shocks for the European countries, but only up to 1988 and therefore excluding the critical years for the EMS. Their results show higher correlations between the aggregate-supply shocks of Germany and the “core” countries (France, Belgium and the Netherlands), whereas much lower correlations between Germany and UK or Italy are present. The correlations between aggregate-demand shocks are less sharp and are all in the range 0.1-0.4.

An interesting update of this same study is presented by Whitt (1995) He includes the years after 1988 till 1992 and not surprisingly some of the correlations obtained by Eichengreen and Bayoumi (1993) have changed. In particular, the correlation between the aggregate demand shocks of Germany-France and Germany-UK are both significantly negative (respectively -0.32 with a standard error of 0.11 and -0.34 with a standard error of 0.11).

<sup>22</sup> Hence, the exchange-rate arrangement is naturally asymmetric because of the different

the trigger point and the exogenous benefits  $Z$  changes for different values of  $\alpha$ . In particular, the picture highlights that such a relationship is quite stable when  $\alpha$  is sufficiently high.<sup>23</sup>

In conclusion, the analysis points out that a fixed exchange-rate regime could collapse when a strong (composite) asymmetric shock occurs.

This confirms what the theory of optimum currency areas points out. However, the analysis also stresses that when introducing a fully stochastic shock, the crisis trigger point may be much higher than in the deterministic case, as Fig. 5 shows.

In other words, a sort of *hysteresis* effect is present. In the stochastic case the two countries' monetary authorities are willing to stay longer since offsetting shocks may occur in the future and re-establish a fully positive flow of benefits. Then, quitting the regime immediately when the benefit flow becomes negative may result in the permanent (and irreversible) loss of future benefits.

In the end, the analysis highlights couples of countries: (i) that are “less open” (i.e., for which the elasticity of the aggregate demand to the real exchange rate is lower), (ii) that experience composite asymmetric shocks with either too low or too high variability, (iii) that have very impatient authorities towards price stability and (iv) for which the exogenous benefits from the existence of a stable exchange rate are low in comparison with their willingness to absorb part of the asymmetric shock. Those are the couples of countries to experience first the collapse in their bilateral exchange-rate regime notwithstanding the cooperation between their monetary authorities.

attitudes towards inflation. When  $\alpha$  is not zero, the system would be perfectly asymmetric with an endogenous  $\beta = 0$  only if country 1 receives zero exogenous benefits  $Z$  from the nominal exchange-rate stability. In that case the model would describe unilateral pegging. Hence, the present model still saves asymmetries in the exchange-rate arrangement, but *a-priori* excludes unilateral pegging.

<sup>23</sup> In the graph  $\alpha$  takes values between 0.5 and 1. The relationship becomes less stable only when  $\alpha$  is very small. More exactly, the elasticity of  $Z$  with respect to  $\alpha$  is  $\frac{1}{1+\alpha}$ . This means that when comparing the effect of the same (and sufficiently high) percentage changes in  $Z$  and  $\alpha$  on the trigger point value, the effect is much higher when  $\alpha$  is very small than when it is close to one. In particular, given the nonlinearity of the elasticity, such an effect will be quite stable for values of  $\alpha$  sufficiently close to one.

An indicative study on the subject is Alesina and Grilli (1992). The authors actually compute indices of economic and political independence of the various national central banks. Germany stands first with a value of 14; the Netherlands follows with 10 and all the other ERM countries score between 5 and 7 (with the exception of Portugal with 3). Notice that a statutory requirement of monetary stability is present only for the Danish and the Dutch central banks besides the Bundesbank.



## 6 Conclusions and Lessons for EMU

This paper describes the collapse of a bilateral (but not necessarily symmetric), cooperative fixed exchange-rate regime.

In principle, such a fixed exchange-rate regime with the strong-currency central bank always ready to support the other currencies under attack could last forever, as pointed out in De Grauwe (1994a, p. 126-7).

History, however, shows that no declared bi- or multilateral exchange-rate regime has lasted successfully through turbulent periods. This has brought some scholars to question the real possibility of creating "true" bilateral exchange-rate arrangements and to conclude that all fixed exchange-rate regimes develop into Stackelberg-structured games more than cooperative games (see, for instance, Giavazzi and Giovannini, 1989, Ch. 8).

Moreover, it has also been shown that such an international arrangement where all the "wet" monetary authorities appoint the most "hard-nose" central bank to do the monetary policymaking for the whole area, can be mainly advantageous for the "wet" (periphery-follower) authorities. In other words, the fixed exchange-rate regime can substitute for the appointment of the Rogoff (1985) conservative central banker in the periphery countries. However, a fundamental difference is present: whereas the Rogoff conservative central banker is fully accountable to the domestic overall authorities, the center country is not. Then, since the central banker in the center country cannot be fired, the periphery country shows its disapproval towards the "appointed" center-country monetary authorities by exerting the exit option and letting the domestic currency freely fluctuate.<sup>24</sup>

As a consequence of this approach, balance-of-payments crises have all been described in a small-country framework as in Krugman (1979), Obstfeld (1986) or lately Ozkan and Sutherland (1994a and b).

However, although the internal arrangement of any fixed exchange-rate regime brings asymmetries among the members, it is not always necessary to design any asymmetric exchange-rate regime as *perfectly* asymmetric in a leader-follower(s) structure. Actually, in such a framework it is still puzzling why one country would like to take the role of the leader. For instance, Giavazzi and Giovannini (1989, Ch. 5) have shown that the center country would be always better off in the flexible exchange-rate regime unless: (i) the periphery country is particularly small and the center country is practically indifferent about the currency pegging by the periphery country; or (ii) there

<sup>24</sup>See Eichengreen (1994), Ch. 6. The contribution by Ozkan and Sutherland can be considered a formalization of this concept.

are some sort of exogenous benefits that occur to the center country when limiting the exchange-rate flexibility with the periphery country.

This paper starts from this latter point. In some declared bilateral exchange-rate arrangements there may be exogenous benefits in limiting the exchange-rate flexibility and this is what would qualify them as "bilateral" versus "unilateral". Then, this does not mean that the exchange-rate arrangement can last forever, but only that the dynamics of its crisis is different and must involve both countries.

In this paper I describe how the bilateral exchange-rate regime (which could be qualified "cooperative" in a game-theoretical sense, but is "asymmetric" in the adjustment to shocks) could also collapse when asymmetric stochastic shocks are particularly strong.

In particular, the model hinges on a very simplified framework where the interaction between the two countries is restrained to the interdependence of the inflation rates in the fixed exchange-rate regime. Then the trigger-crisis point will be more likely: (i) the less open the two economies, (ii) the more impatient the monetary authorities towards inflation, (iii) the lower (or sometimes too high) the variability of the asymmetric shock, and (iv) the lower the exogenous benefits from limited exchange-rate flexibility in comparison with the complementarity in the preferences of the two monetary authorities.

An example of such a "bilateral", but not necessarily "symmetric", fixed exchange-rate regime can be considered the ERM after 1987.

Since the EMS was created, there was an announced bilateral way of dealing with speculative attacks on the inside currencies: both the strong and the weak-currency authorities were committed to intervene whenever the exchange rate were to reach one of the band margins. The system enhanced its cooperative nature with the Basle-Nyborg Agreement by allowing financing facilities for the weak currencies even before the band limits were to be reached.

Given the anti-inflationary bias of the Bundesbank, the actual commitment of the German monetary authorities has always been questioned.<sup>25</sup> However, this has not at all prevented the Bundesbank from lending to other central banks in the occurrence of exchange rate crises, although mainly with

<sup>25</sup>Eichengreen and Wyplosz (1993) stress that the Bundesbank was never completely committed to this cooperative system, fearing for domestic price stability. This was achieved by means of a particular legal trick that held responsible the German government in case of not honoring the international EMS treaty, but not the Bundesbank. However, the Bundesbank is the authority in charge of printing and, in the case of the EMS treaty, lending Deutsche Marks to other countries.

sterilized interventions. In particular, during the 1992-93 crisis the Bundesbank (together with other central banks) gave financial support to the currencies under attack, but only up to a *certain amount* and not *unlimitedly* as the ERM rules prescribe.

In the present paper the financial support from the strong-currency central bank is endogenously determined for the cooperative case. It actually represents the degree of commitment in terms of sharing the burden of the limited nominal exchange-rate flexibility with the partner country.

Such a degree of commitment represents also the credibility of the exchange-rate agreement. Once the trigger point has been reached, the strong-currency authorities are no longer willing to support the weak currency by expanding their money supply (for instance, by lending domestic currency to the weak-currency authorities with no sterilization) and letting the domestic price level increase. At the same time, the weak-currency authorities are no longer available to implement a restrictive monetary policy.<sup>26</sup>

Once this is perceived in the foreign exchange market, the fixed exchange-rate regime loses its credibility and speculative attacks can successfully take place.

Finally, a question regarding the future of the EMU can naturally be raised since the fixed exchange-rate regime of this paper closely resembles a monetary union. If even a cooperative exchange-rate regime cannot handle such asymmetric shocks, would then EMU be viable? Does the model show that there may be situations for which the European Monetary Union could inevitably break down notwithstanding the cooperative agreement?

On the one hand, it ought to be noticed that the model actually shows the relationship between two independent central banks, each one of them with its own money supply and trying to stabilize its own domestic price level. In contrast, in a monetary union there will be just one money by definition and one general price level to stabilize by a central monetary authority.<sup>27</sup> However, in case of very adverse shocks, it could happen that the regional monetary authorities would start printing their own regional money by undertaking the very high (exit) costs. In terms of the present model the introduction of such exit costs would make the trigger exit point much higher and, therefore, much less likely to occur.

On the other hand, in the present setup fiscal policy is absent and it is a

<sup>26</sup>In the model this would bring undesired deflation, but it could be interpreted as a more realistic recessionary policy in the case of presence of nominal rigidities.

<sup>27</sup>Actually De Grauwe (1994b) argues that if the European Central Bank had been already established and operating during the period of the crisis as a super-central bank, probably there would have been no tensions on the exchange rates.

hope that a well-designed transfer system should be able to help cope with strong asymmetric shocks in the area.

However, besides the breakdown of the already established monetary union, it seems more important to consider the design of the transition towards EMU. Recently the European Commission has confirmed the gradualist approach towards EMU by allowing for a period of "irrevocably fixed" exchange rates between national currencies before switching towards a common legal tender. This means that the temptation of devaluing will have a much lower cost than in the case of an established common currency where a regional authority must start printing its own money from zero. In the transition period the regional money is already there.

Then, this paper warns that even in the best type of cooperation — i.e., cooperation with welfare transfers of Sect. 4.1 — asymmetric shocks can still disrupt the fixed exchange-rate regime.

However, the present model also indicates that the best way of implementing such a gradualist approach is by making a credible and irreversible change in the welfare function: the regional monetary policymaking should credibly be committed to an objective (for instance, price stability) for the *whole* monetary area.

In other words, a period of "irrevocably fixed" fixed exchange rates would fail to lead to tensions and speculative attacks only if there will be a central monetary authority already operating with a declared different objective from price stability in every region and full power over the regional monetary authorities. Only such an institution would internalize major asymmetric shocks.

## Appendix

### A A Formal Derivation of the Welfare Function

The objective of this Appendix is the derivation of the welfare as a function of the current state  $\varepsilon$  and of a general switching state  $\bar{\varepsilon}$  in the cooperative case with welfare transfers.

After obtaining the efficient burden share  $\beta_\varepsilon^*$ , the value function for a general exit point  $\bar{\varepsilon}$  represents the joint welfare function of the two countries.

Such welfare function must satisfy the differential equation 7 when the current state  $\varepsilon$  is not close to the exit point:

$$V(\varepsilon) = (1 - \delta dt)E[V(\varepsilon + d\varepsilon)] + \int_{\bar{\varepsilon}}^{\varepsilon} f_\varepsilon^*(\varepsilon)dt$$

Since  $E[V(\varepsilon + d\varepsilon)] = V(\varepsilon) + E[dV]$ , the above differential equation can be rewritten as follows:

$$\delta V(\varepsilon) = E[dV] + f_c^*(\varepsilon)dt \quad (11)$$

This is an arbitrage condition that the welfare function must satisfy. The right hand-side represents the expected return when staying in the fixed exchange-rate agreement at time  $t$  — obtaining the flow  $f_c^*(\varepsilon)$  — and leaving at time  $t + dt$  — collecting the expected capital gain (or loss)  $E[dV]$ . The left hand-side is the “normal” return between  $t$  and  $t + dt$  according to the discount rate of the monetary authorities.

The expected change in the welfare function represents the expected change in the differential of a function that depends uniquely on a brownian motion process. Hence, the simpler form of Ito's lemma can be applied:

$$E[dV] = \frac{\sigma^2}{2} V''(\varepsilon)dt \quad (12)$$

where  $V''(\varepsilon)$  stands for the second derivative of  $V(\varepsilon)$  with respect to  $\varepsilon$ .

Then, substituting (12) into (11) I obtain the time-autonomous second-order differential equation of the text, which takes the following form when substituting also for  $f_c^*(\varepsilon)$ :

$$\delta V(\varepsilon) = \frac{\sigma^2}{2} V''(\varepsilon) + \left( 2Z - \frac{\alpha}{1 + \alpha} \frac{\varepsilon^2}{\eta^2} \right)$$

The welfare function is given by the solution of this differential equation. In particular, the overall solution is given by the sum of the general solution and a particular solution.

The general solution is:

$$V^G(\varepsilon) = C_1 e^{-\theta\varepsilon} + C_2 e^{\theta\varepsilon}$$

where  $\theta = \sqrt{2\delta/\sigma^2}$  and the constants  $C_1$  and  $C_2$  have to be determined. As a particular solution, by trying the following general functional form

$$V^P(\varepsilon) = M_0 + M_1 \varepsilon^2$$

and applying the method of undetermined coefficients to pin down the  $M_i$ 's constants, I obtain the following:

$$V^P(\varepsilon) = \frac{2Z}{\delta} - \frac{\alpha\sigma^2}{(1 + \alpha)\eta^2\delta^2} - \frac{\alpha}{(1 + \alpha)\delta} \frac{\varepsilon^2}{\eta^2}$$

Hence,

$$V(\varepsilon) = V^G(\varepsilon) + V^P(\varepsilon) = C_1 e^{-\theta\varepsilon} + C_2 e^{\theta\varepsilon} + \frac{2Z}{\delta} - \frac{\alpha\sigma^2}{(1 + \alpha)\eta^2\delta^2} - \frac{\alpha}{(1 + \alpha)\delta} \frac{\varepsilon^2}{\eta^2}$$

The two constants can now be determined by assuming two boundary conditions.

First, it ought to be noticed that the component  $\frac{2Z}{\delta} - \frac{\alpha\sigma^2}{(1 + \alpha)\eta^2\delta^2} - \frac{\alpha}{(1 + \alpha)\delta} \frac{\varepsilon^2}{\eta^2}$  is the welfare function under fixed exchange rates and *ignoring any possible switch towards the flexible exchange-rate regime* (see Dixit, 1993, p. 22-5). In other words, this is the welfare function under *irrevocably* fixed exchange rates or a monetary union with infinite exit costs. Then, when  $\varepsilon \rightarrow -\infty$  and no expectations of leaving the regime arise, the welfare function has to tend to that component:

$$\lim_{\varepsilon \rightarrow -\infty} W_1(\varepsilon) = \frac{2Z}{\delta} - \frac{\alpha\sigma^2}{(1 + \alpha)\eta^2\delta^2} - \frac{\alpha}{(1 + \alpha)\delta} \frac{\varepsilon^2}{\eta^2}$$

This condition implies that  $C_1 = 0$ .

Second, a *value-matching condition* has to hold at the general switching state  $\bar{\varepsilon}$ : the value of the welfare function in fixed exchange rates at the switching state has to equal the value of the welfare function under flexible exchange rates, which is always zero:

$$\lim_{\varepsilon \rightarrow \bar{\varepsilon}^+} V(\varepsilon) = 0$$

This implies that

$$C_2 = - \left( \frac{2Z}{\delta} - \frac{\alpha\sigma^2}{(1 + \alpha)\eta^2\delta^2} - \frac{\alpha}{(1 + \alpha)\delta} \frac{\bar{\varepsilon}^2}{\eta^2} \right) e^{-\theta\bar{\varepsilon}}$$

and the welfare function (8) is then obtained.

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Figure 2: The Welfare Functions at Time 0.

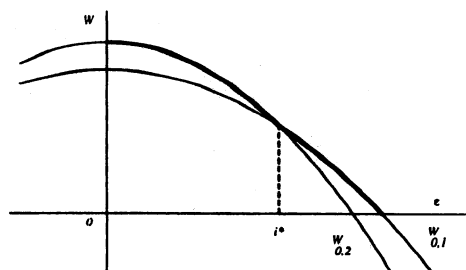


Figure 3: Threshold Values for the Exit Option when  $Z$  is Low

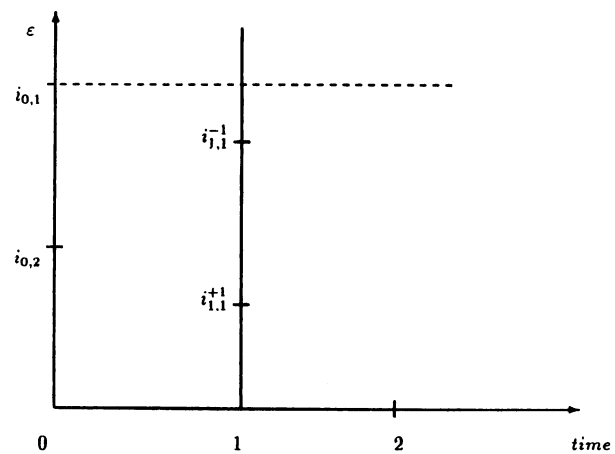


Figure 4: The Cooperative Exit Trigger Point and the Discount Rate,  $\delta$

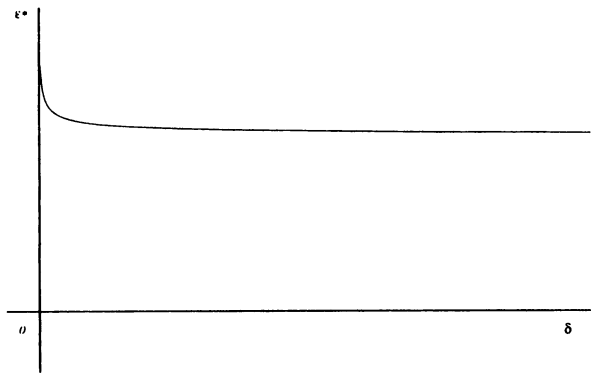


Figure 5: The Cooperative Exit Trigger Point and the Standard Error,  $\sigma$

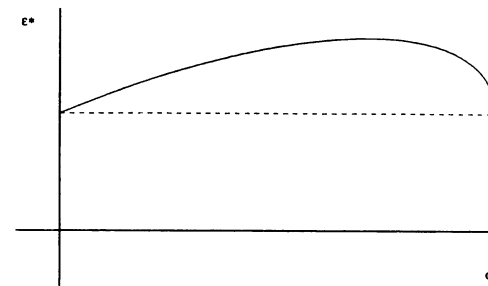


Figure 6: The Cooperative Exit Trigger Point, the Exogenous Benefits ( $Z$ ) and Relative Inflation Aversion ( $\alpha$ )

