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Determinants of Bilateral Trade: Does Gravity Work in a Neoclassical World?

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ABSTRACT

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This paper derives equations for the value of bilateral trade from two extreme cases of the Heckscher-Ohlin Model, both of which could also represent a variety of other models as well. The first case is free trade, in which the absence of all barriers to trade in homogeneous products causes producers and consumers to be indifferent among trading partners. Resolving this indifference randomly, expected trade flows correspond exactly to the simple frictionless gravity equation if preferences are identical and homothetic or if demands are uncorrelated with supplies, and they depart from that equation systematically when there are such correlations. The second case is of countries that each produce distinct goods, as in the H-O Model with complete specialization or a variety of other models. Expressions are derived for bilateral trade, first with Cobb-Douglas preferences and then with CES preferences. The standard gravity equation with trade declining in distance continues to be a central tendency for these trade flows, with departures from it that are easily understood in terms of relative transport costs. The main lessons from the paper are two. First, it is not all that difficult to justify even simple forms of the gravity equation from standard trade theories. Second, because the gravity equation appears to characterize a large class of models, its use for empirical tests of any of them is suspect.

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I. Introduction

It has long been recognized that bilateral trade patterns are well described empirically by a so-called gravity equation, which relates trade between two countries positively to both of their incomes and negatively to the distance between them, usually with a functional form that is reminiscent of the law of gravity in physics. It also used to be frequently stated that the gravity equation was without theoretical foundation. In particular, it was claimed that the Heckscher-Ohlin (H-O) Model of international trade was incapable of providing such a foundation, and perhaps even that the H-O Model was theoretically inconsistent with the gravity equation. In this paper I will take another look at these issues. It is certainly no longer true that the gravity equation is without a theoretical basis, since several of the same authors who noted its absence went on to provide one. I will briefly review their contributions in a moment. Since none of them build directly on an H-O base, it might be supposed that the empirical success of the gravity equation is evidence against the H-O model, as at least one researcher has implied by using the gravity equation as a test of an alternative model

*I have benefitted greatly from comments and conversations with Don Davis, Simon Evenett, Jeffrey Frankel, Jon Haveman, David Hummels, and Jim Levinsohn.

incorporating monopolistic competition. I will argue however that the H-O model, at least in some of the equilibria that it permits, admits easily of interpretations that accord readily with the gravity equation. At the same time, developing these interpretations can yield additional insights about why bilateral trade patterns in some cases depart from the gravity equation as well.

There are two keys to these results, which once I mention them may make the rest of the paper obvious to those well-schooled in trade theory. I will state them now so that such readers needn't waste time reading further. The two keys open doors to two different cases of H-O model equilibria, one with free trade and one without.

With free trade -- that is, literally zero barriers to trade of all sorts, including both tariffs and transport costs -- the key is that trade is just as cheap, and therefore no less likely, than domestic transactions. Therefore, instead of thinking as we normally do of countries first satisfying demands from domestic supply and then importing only what is left, we should think of demanders as being indifferent among all equally priced sources of supply, both domestic and foreign. Suppliers likewise shouldn't care about to whom they sell. The H-O model (and other models based solely on comparative advantage and perfect competition) is usually examined only for its implications for net trade, and we then jump to the conclusion that gross trade flows are equal to net. But with zero trade barriers, there is no reason for trade to be this small. If instead we allow markets to be settled randomly among all possibilities among which producers and consumers are indifferent, then trade flows will generally be larger and will fall naturally into a gravity-equation configuration, in a frictionless form without a role for distance. With identical preferences across countries, this configuration is particularly simple. With non-identical preferences it is a bit more complex, but it is also more instructive.

The other key is to the case of trade that is not free. If there exist positive barriers on all trade flows, however small, then the H-O model cannot have factor price equalization (FPE) between any two countries that trade with each other. For if they did have FPE, then their prices of all goods would be identical and neither could overcome the positive barrier on its exports to the other. Since we do observe trade between every pair of countries that we care about, it follows that the H-O equilibria we look at with non-free trade should be ones without FPE between any pair of countries. If we assume also that the number of goods in the world is extremely large compared to the number of factors, for almost all goods only one country will be the least-cost producer. With trade barriers this does not imply complete specialization by countries in largely different goods, but it makes such a case more plausible than might have been thought otherwise. In any case, motivated by this observation I will study bilateral nonfree trade under the assumption that each good is produced by only one country. With that assumption, bilateral trade patterns in the H-O model are essentially the same as in other models with differentiated products, and it is no surprise that the gravity equation emerges once again. My contribution here will be to derive bilateral trade in terms of incomes and trade barriers in a form that may be more readily interpretable than has been seen before.

None of this should be very surprising, though I admit that this is much clearer to me now than it was when I started thinking about it. All that the gravity equation says, after all, aside from its particular functional form, is that bilateral trade should be positively related to

the two countries' incomes and negatively related to the distance between them. Transport costs would surely yield the latter in just about any sensible model. And the dependence on incomes would also be hard to avoid. The size of a country obviously puts an upper limit on the amount that it can trade (unless it simply re-exports, which one normally excludes), so that small countries necessarily trade little. For income not to be positively related to trade it would therefore have to be also true that large countries also trade very little, at least on average. Therefore, the smaller are the smallest countries, the less must all countries trade in order to avoid getting a positive relationship between size and trade. Looked at in that way, it would therefore be very surprising if some positive relationship between bilateral trade and national incomes did not also emerge from just about any sensible trade model. The H-O model has some quirky features, but in this respect at least it turns out to be sensible.

As for the functional form, a simple version of the gravity equation — what I will call the standard gravity equation — is typically specified as

$$T_{ij} = A\frac{Y_iY_j}{D_{ij}} \tag{1}$$

where T_{ij} is the value of exports from country *I* to country *j*, the *Y*'s are their respective national incomes, D_{ij} is a measure of the distance between them[1], and *A* is a constant of proportionality. While this particular multiplicative functional form may not be obvious, the

[1]Clearly this measure should not go to zero for adjacent countries, or (1) would yield infinite trade between them. Empirical work typically uses distance between national capitols. For theoretical purposes below, it is convenient to use a measure that starts at one (such as one plus distance) to accommodate transactions of a country with itself.

easiest alternative of a linear equation clearly would not do, for trade between two countries must surely go to zero as the size of either goes to zero. None of this constitutes a derivation of the gravity equation, of course, but it does suggest why one would expect something like it to hold in any plausible model.

I turn now to a brief review of the literature in Section II, followed by the two cases just mentioned: free trade in Section III and nonfree trade in Section IV.

II. Theoretical Foundations for the Gravity Equation

As has been noted many times, the gravity equation for describing trade flows first appeared in the empirical literature without much serious attempt to justify it theoretically. Tinbergen (1962) and Pöyhönen (1963) did the first econometric studies of trade flows based on the gravity equation, for which they gave only intuitive justification. Linnemann (1966) added more variables and went further toward a theoretical justification in terms of a Walrasian general equilibrium system, although the Walrasian model tends to include too many explanatory variables for each trade flow to be easily reduced to the gravity equation. Leamer and Stern (1970) followed Savage and Deutsch (1960) in deriving it from a probability model of transactions, very similar to what I will suggest below but applied only to trade, not to all transactions, and without making any explicit connection to the H-O model. Leamer (1974) used both the gravity equation and the H-O model to motivate explanatory variables in a regression analysis of trade flows, but he did not integrate the two approaches theoretically.

These contributions were followed by several more formal attempts to derive the gravity equation from models that assumed product differentiation. Anderson (1979) was the first to

do so, first assuming Cobb-Douglas preferences and then, in an appendix, CES preferences. In both cases he made what today would be called the Armington Assumption, that products were differentiated by country of origin. His framework was in fact very similar to what I will examine here with nonfree trade, though I motivate the differentiation among products, as already noted, by the H-O model's case of non-FPE and specialization rather than by the Armington Assumption. Anderson also modeled preferences over only traded goods, while I will assume for simplicity that they hold over all goods. Anderson's primary concern was to examine the econometric properties of the resulting equations, rather than to extract easily interpretable theoretical implications as I seek here.

Finally, Jeffrey Bergstrand has explored the theoretical determination of bilateral trade in a series of papers. In Bergstrand (1985) he, like Anderson, used CES preferences over Armington-differentiated goods to derive a reduced form equation for bilateral trade involving price indices. Using GDP deflators to approximate these price indices, he estimated his system in order to test his assumptions of product differentiation. For richness his CES preferences were also nested, with a different elasticity of substitution among imports than between imports and domestic goods. His empirical estimates supported the assumption that goods were not perfect substitutes and that imports were closer substitutes for each other than for domestic goods.

In Bergstrand (1989, 1990) he departed even further from the H-O model by assuming Dixit-Stiglitz (1977) monopolistic competition, and product differentiation therefore among firms rather than among countries. This was imbedded however in a two sector economy in which each monopolistically competitive sector had different factor proportions, thus being a hybrid of the perfectly competitive H-O model and the one-sector monopolistically competitive model of Krugman (1979). In the first paper, Bergstrand used this framework to derive yet again a version of the gravity equation, and in the second he examined bilateral intra-industry trade.

Bergstrand's later work therefore serves to bring together the earlier Armington-based approaches to deriving the gravity equation with a second strand of literature in which gravity equations were derived from simple monopolistic competition models. Almost from the start of the New Trade Theory's attention to such models, it was recognized that they provided an immediate and simple justification for the gravity equation.[2] Indeed, Helpman (1987) used this correspondence between the gravity equation and the monopolistic competition model as the basis for an empirical test of the latter. That is, he interpreted the close fit of the gravity equation to data as supportive empirical evidence for the monopolistic competition model. For this to be correct, of course, it would need to be true, as Helpman apparently believed, that the gravity equation does not also arise from other models. He remarked (p. 63) that "The factor proportions theory contributes very little to our understanding of the determination of the volume of trade in the world economy, or the volume of trade within groups of countries," and he went on to demonstrate geometrically that the volume of trade under FPE in the 2x2x2 H-O model is independent of country sizes.[3] Helpman was, I would like to think, in good

[2]One such was apparently Krugman (1980), cited in Helpman (1987).

[3]This argument appeared first in Helpman and Krugman (1984). I would argue that Helpman's locus for comparisons, which are along straight lines parallel to the diagonal of a Dixit-Norman-Helpman-Krugman factor allocation rectangle, is inappropriate. Along these straight lines, the differences in relative factor endowments of the two countries also change,

company. No less an authority than Deardorff (1984, pp. 500-504) noted several of the empirical regularities that are captured in the gravity equation and pronounced them paradoxes, inconsistent with, or at least not explainable by, the H-O Model.

Helpman applied his test to data on trade of the OECD countries, where most would agree that monopolistic competition is plausibly present. Hummels and Levinsohn (1995) decided to attempt a sort of negative test of the same proposition by looking for the same relationship in the trade among a much wider variety of countries, including ones where monopolistic competition is less plausibly a factor. To their surprise, they found that the test worked just as well for that group of countries, thus leading one to suspect that perhaps the relationship represented by the gravity equation is more ubiquitous, and not unique to the monopolistic competition model. It might be thought that the work by Anderson and Bergstrand cited above would have already suggested this, since they derived gravity equations from a variety of models other than the monopolistic one that Bergstrand eventually incorporated into his analysis. But in fact the versions of the gravity equation that those authors obtained were somewhat complex and opaque, and it was not obvious that they would lead to the success of the very simple gravity equation tested by Helpman.

My point in this paper, of course, is that one can get essentially this same simple gravity equation from the H-O model properly considered, both with free trade and without. This

becoming more pronounced (and leading to greater trade) at the same time that countries are becoming more different in size (leading to less trade). A better comparison would have been along a locus for which the percentage difference in factor endowment ratios remains constant. This would be a curve bowed out from the diagonal of the box, and along this curve the trade volume would be largest where country incomes are equal, just as in the gravity equation.

does not mean that the empirical success of the gravity model lends support to the H-O model, any more than it does to the monopolistic competition model. For reasons I have already indicated, I suspect that just about any plausible model of trade would yield something very like the gravity equation, whose empirical success is therefore not evidence of anything, but just a fact of life.

III. Free Trade

Consider now an H-O model with any numbers of goods and factors. In fact, for most of what I will say in this section, the argument is more general and could apply to any perfectly competitive trade model with homogenous products, including a Ricardian model, a specific-factors model, a model with arbitrary differences in technology, and so forth. For this model, consider a free trade equilibrium, with each country a net exporter of some goods to the world market and a net importer of others. This equilibrium need not be unique, as it will not be in the H-O model with FPE and more goods than factors. If the model is H-O, then there may be FPE among some or all countries, but there need not be. We need merely have some vectors of production, consumption, and therefore net trade in each country that are consistent with maximization by perfectly competitive producers and consumers in all countries, facing the same (due to free trade) prices for all goods, and such that world markets clear.

It is customary to note that patterns of bilateral trade are not determined in such a model, and indeed they are not. But the reason for this indeterminacy is itself important: both producers and consumers are indifferent, under the assumption of free trade and homogeneous

products, among the many possible destinations for their sales and sources for their purchases. Therefore, while it is true that a wide variety of outcomes are possible, we can get an idea of the average outcome by just allowing choices among indifferent outcomes to be made randomly.

Thus, having already found the equilibrium levels of production and consumption, let the actual transactions be determined as follows: producers each put their outputs into a world pool for their industry; consumers then choose randomly their desired levels of consumption from these pools. If consumers draw from these pools in small increments, then the Law of Large Numbers will allow us to predict quite accurately what their total choices will be by using expected values. In general, these expected values will be appropriate averages of the wide variety of outcomes that are in fact possible in the model.

Homothetic Preferences

All of this works extremely simply if preferences of consumers everywhere are identical and homothetic, which I will now assume as a first case. Let x_i be country i's vector of production and c_i its vector of consumption in a free trade equilibrium with world price vector p.[4] It's income is therefore $Y_i = p'x_i = p'c_i$, where I also assume balanced trade so that expenditure equals income. Now consider the value of exports from country I to country j, T_{ij}. With identical, homothetic preferences all countries will spend the same fraction, β_k, of their incomes on good k, so that country j's consumption of good k is $c_{jk}=\beta_k Y_j / p_k$. Drawing

[4]All vectors are column vectors unless transposed with a prime, $'$.

randomly from the world pool of good k, to which country I has contributed the fraction $\gamma_{ik}=x_{ik}/\sum_h x_{hk}$, country j's purchases of good k from country I will be $c_{ijk}=\gamma_{ik}\beta_k Y_j/p_k$. Let $x_k^w=\sum_i x_{ik}$ be world output of good k. Note that, with identical fractions of income being spent on good k by all countries, that fraction must also equal the share of good k in world income, Y^w: $\beta_k=p_k x_k^w/Y^w$, the value of j's total imports from I is

$$\begin{aligned} T_{ij} &= \sum_k p_k c_{ijk} = \sum_k \gamma_{ik}\beta_k Y_j \\ &= \sum_k \frac{x_{ik}}{x_k^w}\frac{p_k x_k^w}{Y^w}Y_j = \sum_k p_k x_{ik}\frac{Y_j}{Y^w} \\ &= \frac{Y_i Y_j}{Y^w} \end{aligned} \tag{2}$$

Thus with identical, homothetic preferences and free trade, an even simpler gravity equation than (1) emerges immediately, with constant of proportionality $A=1/Y^w$. Distance, of course, plays no role here since there are no transport costs, and I will call (2) the simple frictionless gravity equation. To get this, all that is needed is to resolve the indeterminacy of who sells to whom by making that decision randomly.

Arbitrary Preferences

If preferences are not identical and/or not homothetic, then the equilibrium may have each country spending a different share of its income on each good, and the simple derivation above does not work. Let β_{ik} be the share of its income that country I spends on good k in the equilibrium, and also let α_{ik} be the share of country i's income that it derives from producing good k. The first and second equalities of (2) still hold, but with β_k replaced by β_{ik}. The

value of world output of good k is $p_k x_k^w=\sum_i \alpha_{ik}Y_i$, and therefore the fraction of world output of good k that is produced by country I is $\gamma_{ik}=\alpha_{ik}Y_i/\sum_h \alpha_{hk}Y_h$. Country j, drawing randomly from the pool for good k an amount equal to its demand $\beta_{jk}Y_j$, it will get that fraction from country I. Thus the value of sales by country I to country j of good k will be

$$T_{ijk} = \frac{\alpha_{ik}Y_i}{\sum_h \alpha_{hk}Y_h}\beta_{jk}Y_j \tag{3}$$

Summing across goods k, we get

$$T_{ij} = \sum_k T_{ijk} = \sum_k \frac{\alpha_{ik}Y_i}{\sum_h \alpha_{hk}Y_h}\beta_{jk}Y_j = Y_i Y_j \sum_k \frac{\alpha_{ik}\beta_{jk}}{p_k x_k^w} \tag{4}$$

This is not the gravity equation, since the summation could be quite different for different values of I and j. As an extreme example, if country I happens to specialize completely in a good that country j does not demand at all, then T_{ij} will be zero regardless of Y_i and Y_j.

However, it is possible to simplify (4) further if one can assume that the fractions that exporters produce and that importers consume are in some sense unrelated. Let $\lambda_k=p_k x_k^w/Y^w$ be the fraction of world income accounted for by production of good k. Then

$$T_{ij} = \frac{Y_i Y_j}{Y^w} \sum_k \frac{\alpha_{ik}\,\beta_{jk}}{\lambda_k} \tag{6}$$

Clearly, since each country's good-shares of both production (α_{ik}) and consumption (β_{jk}) sum to one, this will reduce to the simple frictionless gravity equation (2) if either the exporter produces goods in the same proportions as the world ($\alpha_{ik}=\lambda_k$) or if the importer consumes goods in the same proportion as the world ($\beta_{jk}=\lambda_k$, as was true in the case of identical, homothetic preferences), but not in general. If the λ_k were equal for all k, thus each being $1/n$ where n is the number of goods, we would also get back to (2) if α_{ik} and β_{jk} were uncorrelated. With goods having unequal shares of the world market, we can also get this if we define correlations on a weighted basis, using the λ_k as weights.

That is, let

$$\tilde{\alpha}_{ik} = \frac{\alpha_{ik}-\lambda_k}{\lambda_k} \quad , \quad \tilde{\beta}_{jk} = \frac{\beta_{jk}-\lambda_k}{\lambda_k}$$

be the proportional deviations of country i's production shares and of country j's consumption shares from world averages. Then

$$\sum_k \lambda_i \tilde{\alpha}_{ik}\,\tilde{\beta}_{jk} = \sum_k \frac{1}{\lambda_k}(\alpha_{ik}\,\beta_{jk} - \lambda_k\beta_{jk} - \lambda_k\alpha_{ik} + \lambda_k^2) = \sum_k \frac{\alpha_{ik}\,\beta_{jk}}{\lambda_k} - 1 \tag{7}$$

and we can rewrite (5) as

$$T_{ij} = \frac{Y_i Y_j}{Y^w}\,(1 + \sum_k \lambda_k\,\tilde{\alpha}_{ik}\,\tilde{\beta}_{jk}) \tag{8}$$

This is the main result of this section of the paper. The sign of the summation in (8) is the same as the sign of the weighted covariance between $\tilde{\alpha}_{ik}$ and $\tilde{\beta}_{jk}$. Thus if these deviations of exporter production shares and importer consumption shares from world averages are uncorrelated, then once again the simple frictionless gravity equation (2) will hold exactly.

Perhaps more importantly, equation (8) also states simply and intuitively when two countries will trade either more or less than the amounts indicated by the simple frictionless gravity equation. If an exporter produces above average amounts of the same goods that an importer consumes above average, then their trade will be greater than would have been explained by their incomes alone. On the other hand, if an exporter produces above average what the importer consumes below average, their trade will be unusually low. These statements presume that the simple frictionless gravity equation describes what is "usual." This is in fact the case here, since across all country pairs (i, j) the average of bilateral trade is equal to what the simple frictionless gravity equation prescribes:

$$\begin{aligned}\sum_{ij}\left(T_{ij} - \frac{Y_i Y_j}{Y^w}\right) &= \sum_{ijk}\frac{Y_i Y_j}{Y^w}\lambda_k\,\tilde{\alpha}_{ik}\,\tilde{\beta}_{jk} \\ &= \sum_{ik}\frac{Y_i}{Y^w}\tilde{\alpha}_{ik}\sum_j (Y_j\,\beta_{jk} - Y_j\,\lambda_k) \\ &= \sum_{ik}\frac{Y_i}{Y^w}\tilde{\alpha}_{ik}\left(\sum_j c_{jk} - \lambda_k Y^w\right) \\ &= 0\end{aligned} \tag{9}$$

To sum up, with free trade the values of bilateral trade are on average given by the simple frictionless gravity equation, $Y_i Y_j / Y^w$. If expenditure fractions differ across countries because preferences are not identical and/or not homothetic, then individual bilateral trade flows will vary around this frictionless gravity value. If one country tends to overproduce what another over consumes, then exports of the former to the latter will be above that value, and if one tends to underproduce what another over consumes, then these exports will be below that value.

It is important for these results that sales of a country to itself, T_{ii}, are included along with international trade. In this form the gravity equation holds on average even in the special case of countries who each demand only their own products. Their above average "exports" to themselves then offset their below average (zero) exports to each other to leave the average unaffected.

Combined with what we already know about the H-O model and what we may suspect about preferences, this also leads us loosely to a corollary that I suspect could be made more formal with additional effort. Suppose that preferences are internationally identical but not homothetic, and suppose further that high-income consumers tend to consume larger budget shares of capital-intensive goods. Then capital abundant countries will have high higher than average per capita incomes and will therefore consume disproportionately capital-intensive goods. At the same time, from the H-O Theorem they will also produce disproportionately these same goods. Therefore we would expect to find these countries trading more than average with each other and less than average with low-income labor abundant countries. This is the same result that Markusen (1986) found in his "eclectic" model and for essentially the

same reason. Although Markusen had increasing returns and monopolistic competition in his manufacturing sectors, this served primarily to generate intraindustry trade. His volume-of-trade result was driven by a high income elasticity for capital-intensive goods.

Such a disproportionately high volume of trade among high income countries happens to accord well with trade patterns in the real world. On the other hand, under the same circumstances the theory here also predicts that labor abundant (hence poor) countries will trade disproportionately with each other as well. This is the same conclusion that Linder (1961) came to from a quite different theoretical model, but the empirical evidence in its favor is less clear.[5]

IV. Non-free Trade

I turn now to the case of non-free trade, assuming instead that there not only exist barriers to trade but that these exist for every good. These barriers needn't be large, but I will assume them to be strictly positive on all international transactions. The case that I will consider will in addition have the property that every country produces and exports different goods. Indeed this extreme specialization is the only property that I need in this section — the trade barriers are incidental.[6] I thought briefly that this case was the only one that could arise

[5] As I understand it, Jeffrey Frankel and his co-authors have found in several studies, such as Frankel, Stein, and Wei (1994), that high-income countries trade disproportionately more than the gravity equation would suggest with all trading partners and not just among themselves, while low income countries trade less.

[6] Thus the results in this section would also obtain in a H-O model with free trade if factor endowments differed sufficiently to yield such specialization. They also hold in a Ricardian model with specialization and in any Armington model and any monopolistic-

with positive transport costs, but I now realize that my argument was flawed. I will nonetheless try to motivate the specialization assumption along the lines of that argument, but ultimately I can only claim to be considering a special case.

As mentioned in the introduction, the H-O Model has a striking implication in the presence of strictly positive transport costs: While in general the H-O Model permits equilibria with both FPE and non-FPE among groups of countries, no two countries that have the same factor prices can trade with each other. The reason is that with identical factor prices (recall that the FPE Theorem equates factor prices absolutely, not just relatively) they will have identical costs of production. With perfect competition, neither country's producers could compete with domestic producers in the other's market, since the exporters would have to overcome the positive transport cost and domestic suppliers would not.

Now this is not a very appealing property of the H-O model, I admit, and this by itself might be enough to make you prefer a model with some sort of imperfect competition. But it is a property of the H-O Model nonetheless, and I will take advantage of it. Since we do in the real world observe virtually every country trading with every other, if we are to give the H-O Model a chance to apply in the real world we must assume unequal factor prices in each pair of countries.

Now suppose also that there are many more goods than there are factors, perhaps even an infinite number of goods as in Dornbusch et al. (1977, 1980). If there were free trade, having unequal factor prices would severely limit the number of goods that any two countries

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competition model, where in effect product differentiation implies specialization.

could produce in common. With trade barriers this is no longer the case, since goods can become nontraded, and they can also compete in the same market if the difference in transport costs exactly equals the difference in costs. But if transport costs for a given good are constant between any pair of countries (not varying with the amount transported), then I think the case can be made that only a negligibly small subset of all goods will be sold by any two countries to the same market. Thus for almost all trade, a country's consumers will be buying each good from only a single country's producers, either their own domestic industry or from the industry of a single foreign exporter.

This is not quite the same as saying that there only exists a single exporter of each good anywhere in the world, but that is nonetheless the case that I will consider. Indeed, I will go one step further and assume that each good is not only exported by only one country but is also produced in only that one country. That being the case, the products of each country will be distinct in the eyes of consumers, not because of an Armington Assumption that national origin matters, but because there really are different goods. One could argue that this is just as unrealistic as the case I dismissed above of countries not trading with each other at all, since for any industrial classification one observes production in multiple countries of goods that are classed the same. However, just as in the debate over the existence of intra-industry trade, where the phenomenon is sometimes argued to be an artifact of aggregation, it may be that multiple producing countries may simply be producing different goods.

Suppose then that every good is produced by a different country in a particular international trading equilibrium. As long as we only consider that equilibrium, we can identify each good with the country that produces it and enter them into a utility function as imperfect substitutes. Let transport costs be of Samuelson's "iceberg" form, with the transport factor (one plus the transport cost) between countries i and j being t_{ij}. That is, a fraction $(t_{ij}-1)$ of the good shipped from country i is used up in transport to country j.

With perfect competition, sellers from country i will not discriminate among markets to which they sell, and they will therefore receive a single price, p_i, for their products in all markets. Buyers however must pay the transport cost, and therefore the buyers' price in market j will be $t_{ij}p_i$.

What can we say about the pattern of bilateral trade? That depends on preferences, which I will assume first to be identical and Cobb-Douglas. That is, consumers in each country spend a fixed share, β_i, of their incomes on the product of country I. Let x_i be the output of country I. Country i's income, Y_i, is

$$Y_i = p_i\, x_i = \textstyle\sum_j \beta_i Y_j = \beta_i Y^w \tag{10}$$

from which $\beta_i = Y_i / Y^w$. Trade can be valued either f.o.b. (exclusive of transport costs) or c.i.f. (inclusive of transport costs). On a c.i.f. basis we get immediately

$$T_{ij}^c = \beta_i Y_j = \frac{Y_i Y_j}{Y^w} \tag{11}$$

With Cobb-Douglas preferences, therefore, we once again get the simple frictionless gravity equation for c.i.f. trade, with no role for transport costs or distance. On an f.o.b. basis, however, these flows must be reduced by the amount of the transport cost:

$$T_{ij}^f = \frac{Y_i Y_j}{t_{ij} Y^w} \tag{12}$$

To the extent that transport cost is related to distance, this immediately gives a result very similar to the standard gravity equation, (1), which includes distance.

This Cobb-Douglas formulation is nonetheless not very satisfactory, because the bilateral expenditures on international trade do not decline with distance. To allow for that to happen, and as the last model I will consider here, let preferences be instead CES. Let consumers in country j maximize a CES utility function defined on the products of all countries i (including their own):

$$U^j = \left(\textstyle\sum_i \beta_i\, c_{ij}^{\left(\frac{\sigma-1}{\sigma}\right)}\right)^{\frac{\sigma}{\sigma-1}} \tag{13}$$

where $\sigma>0$ is the common elasticity of substitution between any pair of countries' products. Facing c.i.f. prices of the goods $t_{ij}p_i$ of the goods, j's consumers, maximizing this function subject to their income $Y_j=p_j x_j$ from producing x_j, will consume

$$c_{ij} = \frac{1}{t_{ij}\, p_i} Y_j\, \beta_i \left(\frac{t_{ij}p_i}{p_j^I}\right)^{1-\sigma} \tag{14}$$

where p_j^I is a CES price index of landed prices in country j:

$$p_j^I = \left(\sum_i \beta_i \, t_{ij}^{1-\sigma} p_i^{1-\sigma}\right)^{1/(1-\sigma)} \tag{15}$$

Therefore the f.o.b. value of exports from country i to country j is

$$T_{ij}^f = \frac{1}{t_{ij}} Y_j \, \beta_i \left(\frac{t_{ij}\, p_i}{p_j^I}\right)^{1-\sigma} \tag{16}$$

Note that the c.i.f. value of trade is this same expression multiplied by t_{ij}, which is therefore now decreasing in t_{ij} if $\sigma>1$.

The parameter β_i is no longer country i's share of world income, as it was in the Cobb-Douglas case, so this does not reduce as easily to the standard gravity equation. However, if we let θ_i be country i's share of world income, we can relate it to β_i as follows and then solve for β_i:

$$\begin{aligned} \theta_i &= \frac{Y_i}{Y^w} = \frac{p_i\, x_i}{Y^w} \\ &= \frac{1}{Y^w} \sum_j \beta_i \, p_j \, x_j \left(\frac{t_{ij}\, p_i}{p_j^I}\right)^{1-\sigma} \\ &= \beta_i \sum_j \theta_j \left(\frac{t_{ij}\, p_i}{p_j^I}\right)^{1-\sigma} \end{aligned} \tag{17}$$

from which

$$\beta_i = \frac{Y_i}{Y^w} \frac{1}{\sum_j \theta_j \left(\frac{t_{ij}\, p_i}{p_j^I}\right)^{1-\sigma}} \tag{18}$$

Using this in (16) we get

$$T_{ij}^f = \frac{Y_i Y_j}{Y^w} \frac{1}{t_{ij}} \left[\frac{\left(\frac{t_{ij}}{p_j^I}\right)^{1-\sigma}}{\sum_h \theta_h \left(\frac{t_{ih}}{p_h^I}\right)^{1-\sigma}}\right] \tag{19}$$

To simplify this and facilitate interpretation, first let each country's product price, p_i, be normalized at unity. Then p_j^I becomes a CES index of country j's transport factors as an importer, what I will call its average distance from suppliers δ^S:

$$\delta_j^S = \left(\sum_i \beta_i \, t_{ij}^{1-\sigma}\right)^{\left(\frac{1}{1-\sigma}\right)} \tag{20}$$

What matters for demand along a particular route is the transport factor t_{ij} relative to this average distance from suppliers, what I will call the relative distance from suppliers ρ_{ij}:

$$\rho_{ij} = \frac{t_{ij}}{\delta_j^S} \tag{21}$$

With this notation, the trade flow in (19) becomes

$$T_{ij}^{f} = \frac{Y_i Y_j}{Y^w} \frac{1}{t_{ij}} \left[\frac{\rho_{ij}^{1-\sigma}}{\sum_h \theta_h \rho_{ih}^{1-\sigma}} \right] \tag{22}$$

This is the main result of this section of the paper. It says the following: If importing country j's relative distance from exporting country i is the same as an average of all demanders' relative distances from i, then exports from i to j will be the same as in the Cobb-Douglas case. That is, c.i.f. exports will be given by the simple frictionless gravity equation, while f.o.b. exports will be reduced below that equation by the transport factor from i to j, much as in the standard gravity equation with transport factor (one plus transport cost) measuring distance. If j's relative distance from i is greater than this average, then c.i.f. (resp. f.o.b.) trade along this route will be correspondingly less than the simple frictionless (resp. standard) gravity equation, while if j's relative distance from i is less than this, trade will be correspondingly more. Since the transport factor for a country from itself is always unity and therefore less than any such average, countries' purchases from themselves will always be more than would appear warranted by the simple frictionless gravity equation.

The result also says that the elasticity of trade with respect to these relative distance measures is $-(\sigma-1)$. Thus, the greater is the elasticity of substitution among goods, the more will trade between distant countries fall short of the gravity equation and the more will trade among close countries (and transactions within countries themselves) exceed it.

Likewise, a general reduction in the transport factors themselves, such as might occur with an improvement in transportation technology, will pull trade closer to the amounts

predicted by the simple frictionless gravity equation. This does not therefore mean that all bilateral trade flows will expand with a drop in transport costs. Rather, trade between distant countries will expand, while trade between close countries will contract, since the latter lose some of their advantage relative to distant countries. Of course since purchases of a country from itself also contract, it must also be true that total international trade expands.

V. Conclusion

In this paper I have derived equations for the value of bilateral trade from two extreme cases of the H-O Model, both of which could also represent a variety of other models as well. The first case was free trade, in which the absence of all barriers to trade in homogeneous products causes producers and consumers to be indifferent among trading partners, including their own country, so long as they buy or sell the desired goods. Resolving this indeterminacy with a random drawing, I derived expected trade flows that correspond exactly to the simple frictionless gravity equation whenever preferences are identical and homothetic. Generalizing the result to arbitrary preferences, I found that this gravity equation would still hold on average, but that individual trade flows would exceed or fall short of it depending on a weighted correlation between the exporter's and the importer's deviations from the world average supplies and demands. This in turn was suggestive of how particular non-homotheticities in demand could interact with factor endowments and factor proportions to cause countries to trade excessively (compared to the simple frictionless gravity equation) with countries like themselves.

The second case considered was of countries that each produce different goods. This is also a possible equilibrium of the H-O Model, though of course it is also a property of other models that have been used in the literature to derive the gravity equation, such as models with Armington preferences and models with monopolistic competition. Here I derived expressions for bilateral trade, first with Cobb-Douglas preferences and then with CES preferences. The former is almost too simple, yielding the simple frictionless gravity equation exactly for trade valued c.i.f. and the standard gravity equation, with division by a transport factor, for trade valued f.o.b. The CES case is more cumbersome, but it too reduces to something not all that different: bilateral trade flows are centered around the same values found in the Cobb-Douglas case, but they are smaller for countries that are a greater than average distance apart as measured by transport cost, and larger for countries that are closer than average. The latter includes purchases of a country from itself, which are increased above the Cobb-Douglas case by the greatest amount. The extent of these departures from the simple Cobb-Douglas gravity equation depends on the elasticity of substitution among goods, being larger the greater is that elasticity.

The lesson from all of this is twofold, I think. First, it is not all that difficult to justify even simple forms of the gravity equation from standard trade theories. Second, because the gravity equation appears to characterize a large class of models, its use for empirical tests of any of them is suspect.

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