Directions of Lumpy Country Trade*

by

Alan V. Deardorff

March 22, 1991

*This paper arose from a suggestion of Dick Porter who, when told a little about another paper, Courant and Deardorff (1990), jumped to the conclusion that it was about this. It wasn't, but it was a great idea. Thanks, Dick.
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The University of Michigan

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Prepared for
The Conference to Celebrate the Sixtieth Birthday of
Ronald W. Jones
University of Pennsylvania

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ABSTRACT

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This paper shows that countries with an uneven distribution of factors across regions (lumpy countries) may export goods of different factor intensity to different trading partners. This phenomenon, here called Cross-Over Trade, has been observed in the empirical literature for a number of countries. Previous theoretical explanations have relied on transport costs. The current argument works with free trade.

Address correspondence to:

Alan V. Deardorff
Institute of Public Policy Studies
The University of Michigan
440 Lorch Hall
Ann Arbor, MI 48109–1220

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I. Introduction

The Heckscher-Ohlin Model of international trade makes a strong statement, in the form of the Heckscher-Ohlin (H-O) Theorem, about what goods a country will export and import. These and other implications of the H-O Model have been amply demonstrated by Jones (1956), as well as in his subsequent writings. The H-O Model has very little to say, however, about with whom these goods will be exchanged, even though there is evidence that the bilateral trade of at least some countries has a distinctive pattern that is reminiscent of the H-O Theorem, in that they export goods of different factor intensities to different trading partners. That is, countries are observed to export more capital-intensive goods to less capital-abundant countries, and less capital-intensive goods to more capital-abundant countries.¹

This behavior, which I will call “cross-over trade,” does not arise in the usual H-O model with free trade, although I have, in Deardorff (1987), explored how it can be obtained in such a model by introducing transportation costs of various sorts.² Here,

¹The seminal empirical observation of this phenomenon was for Japan by Tatemoto and Ichimura (1959). They compared the factor intensities of Japan’s exports to the two groups of developed and developing countries, rather than looking explicitly at the factor endowments of the trading partners. The pattern was reconfirmed for Japan by Heller (1976) and Urata (1983). However, Urata tested for the pattern using regression coefficients rather than actual factor intensities, and found it in data for 1975 but not 1967. Similar evidence for India is contained in Khanna (1982), which also contains other references to work on this subject.

²Deardorff (1987) adds transport costs to the model of Jones (1974), who had shown how a country of intermediate factor abundance will specialize in a small range of goods of
however, I will explore the fact that this same behavior is also likely to arise even with free trade within another modification of the H-O Model that Paul Courant and I have dealt with more recently. In Courant and Deardorff (1990) we show that if countries are "lumpy," in the sense of having intra-national regions that differ in their relative factor-endowments, then that lumpiness can contribute to a pattern of trade that is contrary to the H-O Theorem. It is straightforward that such lumpiness can also lead to cross-over trade, since a lumpy country is in effect an aggregate of more than one country, each with different factor endowments. Heckscher-Ohlin trade for each is then cross-over trade for the aggregate. I will explore this result in more detail, however, in order to delineate the combinations of factor endowments that will give rise to it.

The lumpy country model of Courant and Deardorff (1990) includes only two goods and two factors of production, identified there as labor and land. Here I will rename the factors labor and capital, so as to correspond to the empirical observation mentioned above, though as usual in models of this sort, the names of the factors have nothing to do with their characteristics. Both labor and land are simply generic factors of production, available in fixed supplies and entering symmetrically into the production functions for producing goods.

I will also allow here for three goods instead of two. With only two goods, as already explored in Courant and Deardorff (1990), while the two regions of the country may individually export different goods, the country as a whole exports only one. Therefore it cannot display the phenomenon of cross-over trade. With a third good, however, each region can export a different good while both import a third, and this pattern, as it turns out, may well have the property of cross-over trade. Therefore a

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intermediate factor intensity. Without transport costs, all goods in this range are exported to all trading partners. With transport costs goods near the ends of this range cannot be profitably exported to countries of very similar factor endowments, giving rise to differences in the country's exports to different trading partners.
three-good lumpy-country model seems to be an appropriate place to examine this phenomenon.

With three goods and two factors, there are two fundamentally different cases that may arise on the world market, and the behavior of any country within the world depends importantly on which of these cases obtains. These two cases are laid out in some detail in section II of the paper. On the one hand, the prices of the three goods may be perfectly aligned, such that production of all three goods is possible in any country or region with an appropriate factor endowment. Since in that case all such countries would have the same factor prices, I will call this the case of World Factor Price Equalization, or WFPE prices. Alternatively, if prices of the three goods do not line up exactly in this way, then any country confronting them with free trade will be able to produce at most only two of the three goods. In that case, which pair of goods is produced in a country, as well as the factor prices that support that production, will differ across countries depending on their factor endowments. I will call that the case of World Specialization, or WS prices.

After first elaborating on these two configurations of prices as they relate to homogeneous countries in Section II, I will then consider the case of a lumpy country facing WFPE prices first, in section III, since this is the case that seems to dominate international trade theory. I will then turn briefly in section IV to the case of a lumpy country.

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3 This is not to say that factor prices would in fact be the same in all countries in this case. FPE will hold only for those countries whose factor endowments are within the single diversification cone corresponding to these prices, and in a world of many countries there may well be many countries outside the cone. In fact, strictly speaking, there need be no more than one country inside the diversification cone, in which case factor prices would not have to be equalized across any pair of countries, though that possibility has a rather unlikely feel about it. The importance of WFPE, then, is simply the existence of a cone of factor endowments within which all three goods could be produced and factor prices would be equalized for any countries that happen to be in the cone.

4 Again, the label WS may suggest more specialization than will in fact occur. With free trade, it is still possible in the WS case for every country to produce two of the three goods, so long as their factor endowments lie in one of the two cones of diversification that I will identify below. And with trade impediments of sufficient size, WS prices could be consistent with every country producing every good. The point of WS prices, then, is that producers who face them unprotected by trade impediments will not be able to diversify completely into production of all three goods.
country facing WS prices, which is the case that I find to be the more plausible description of the world economy.

II. Price Configurations with Three Goods and Two Factors

The two possible configurations for world prices in the three-good, two-factor model are shown in panels A and B of Figure 1. Both panels show the familiar Lerner-Pearce diagram of trade theory adapted for the case of three goods. Exogenous prices of the goods, \( p_1, p_2 \) and \( p_3 \), enter the diagram by determining, together with a constant returns to scale technology for producing each good, a set of unit-value isoquants, \( X_1 = 1/p_1 \), \( X_2 = 1/p_2 \), and \( X_3 = 1/p_3 \).

In panel A of Figure 1, these three isoquants happen to line up exactly so that a single common tangent touches all three. This common tangent is the unit isocost line consistent with producing any more than one of the three goods, and its intercepts with the axes indicate the corresponding factor prices as \( 1/w^0 \) and \( 1/r^0 \). If these factor prices prevail, then potential producers in the three industries will employ capital and labor in the ratios \( k_1, k_2, \) and \( k_3 \) respectively, as shown by the rays from the origin through the respective tangencies with the unit isocost line.

As is well known from the literature on factor price equalization, whether these factor prices will in fact prevail in a particular country depends upon whether its factor endowment lies inside the diversification cone defined by the \( k_1 \) and \( k_3 \) rays. If endowments lie outside this cone, then only one good—either \( X_1 \) or \( X_3 \)—will be produced, and factor prices will be given not by the common tangent but rather by the slope of the one isoquant that is in use. If on the other hand endowments do lie inside the diversification cone, then the factor prices will indeed be \( w^0 \) and \( r^0 \) and production of more than one good will be possible. In this case of more goods than factors, however, the exact outputs of the goods, and even which of the three are produced at all, are indeterminate, as shown in Melvin (1968).
Panel B of Figure 1 is quite different. Here, the prices of the goods again determine three unit value isoquants, but they do not now share a common tangent. Instead, there is one unit isocost line tangent to the unit value isoquants for \( X_1 \) and \( X_2 \), with corresponding factor prices \( w^1 \) and \( r^1 \), and a second unit isocost line tangent to \( X_2 \) and \( X_3 \) with factor prices \( w^2 \) and \( r^2 \). At the first of these sets of factor prices, \( w^1 \) and \( r^1 \), goods 1 and 2 can both be produced—using capital-labor ratios \( k^1_1 \) and \( k^1_2 \)—but good 3 cannot. At the second set of factor prices, on the other hand, goods 2 and 3 can be produced together using techniques \( k^2_2 \) and \( k^3_3 \), but industry 1 shuts down.

Thus in panel B there are many different possible patterns of specialization and also of factor prices, and which obtains in a given country will depend on its factor endowments. If a country’s endowments lie in the first diversification cone, between \( k^1_1 \) and \( k^1_2 \), then it will have factor prices \( w^1 \) and \( r^1 \) and it will produce goods 1 and 2. If its endowments lie instead in the second diversification cone, between \( k^2_2 \) and \( k^3_3 \), it will have factor prices \( w^2 \) and \( r^2 \) and produce goods 2 and 3. Finally, if its endowments lie outside both of these cones, then it will produce only one good, exactly as in the extreme cases of specialization in panel A. However, these possibilities of specialization now include also the intermediate case in which endowments are not extreme at all, but rather lie between \( k^1_2 \) and \( k^2_2 \), and the country produces only the good of intermediate factor intensity, good 2.

What determines which of these cases occurs? If prices of goods were chosen randomly, then it would seem that the configuration in panel B would be much more likely, since panel A requires the coincidence of prices that line up exactly. However, in a world where all countries share these same technologies it is clear that prices cannot be chosen randomly. If they were, then another configuration, not shown in Figure 1, would be just as likely as panel B: a case in which the \( X_2 \) unit value isoquant lies further from the origin than the common tangent to the isoquants of \( X_1 \) and \( X_3 \). However with that price configuration good 2 would not be produced anywhere in the world, and the configuration cannot therefore represent an equilibrium.
Thus the requirements of world market equilibrium impose some constraints on what prices may arise, and these requirements may therefore make the configuration in panel A be much more likely than it at first appears. In fact it is the configuration that has been most commonly assumed in modern trade theory, where cases of international factor price equalization tend to dominate the literature. For notice that in panel A, applied now to all countries of the world, factor prices will be equalized for all countries whose factor endowments happen to lie within the single diversification cone, between $k_1$ and $k_2$, and this could conceivably include all countries of the world, if their factor endowments were not to differ too much compared to the differences in factor intensities of the industries. Therefore I have labelled this the case of world factor price equalization, or WFPE prices.

In contrast, in panel B, factor prices cannot be equalized for all countries. Factor prices are the same for any pair of countries whose endowments lie within the same one of the two diversification cones. But it is not possible for all countries in the world to lie within the same cone, since that would mean one of the goods could not be produced anywhere in the world. Thus for the prices in panel B to represent an equilibrium there must be at least two sets of factor prices prevailing in different countries of the world. It also follows that goods 1 and 3 cannot be produced in the same country, so that there must be at least a certain amount of specialization in the countries of the world. Therefore I have labelled this the case of world specialization, or WS prices.

There is not space here for me to discuss which of these two cases is the more likely. Twenty years ago there was a flurry of articles addressing this question, starting apparently with Johnson (1967) and culminating in a fascinating paper by Vanek and Bertrand (1971). The latter concluded—based upon a geometric extension of the world transformation curve to three, four, and then higher dimensions—that world factor price equalization becomes increasingly likely as the number of goods in the economy increases. Fascinating as this argument was, however, I have found myself increasingly skeptical
over the years that world factor price equalization provides the best description of the world economy. Therefore I will deal with each of the two cases in turn in the sections below.

III. The Case of World Factor Price Equalization

Suppose then that a small economy faces world prices that are aligned as in Figure 1A. To neutralize effects of factor endowments on the trade of the country as a whole, I will first consider the very special case of what I will call zero net factor-content trade. This is the case of a country whose total factor endowments would permit it to produce exactly what it consumes at free trade prices, if these factors were perfectly mobile within the country between regions. With the indeterminancy of output that arises when goods outnumber factors, as they do here, there may still be trade in goods, but the factor content of that trade will be zero. An advantage of this assumption is that any net trade in factor content that arises when lumpiness is introduced can then be attributed to that lumpiness, just as in Courant and Deardorff (1990). And, as will be the case, if lumpiness also forces a pattern of trade in goods that would not otherwise arise, that too can be attributed to the effects of lumpiness. How trade patterns in both goods and factor content will vary if this is not the case will be considered briefly later in this section.

Zero Net Factor-Content Trade

Figure 2 shows the production Edgeworth Box for this country, whose total factor endowments, $\overline{L}$ and $\overline{K}$, are at point E. Since it would be possible to produce all three goods only if this factor endowment lay within the diversification cone, $(\overline{L}, \overline{K})$ must be consistent, at world prices and hence at the factor prices shown in Figure 1A, with the use of the techniques of production in each industry indicated in Figure 1A as $k_1$, $k_2$, and $k_3$.

Assume identical and homothetic preferences for consumers throughout the country, so that total demand for each good is independent of the distribution of income within the
country between regions, so long as total income is constant. Let $\mathbf{X}_1$, $\mathbf{X}_2$, and $\mathbf{X}_3$ be these quantities demanded in the country at world prices. Also let the vectors $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ represent the amounts of factors needed to produce these quantities in each industry using the techniques of production $k_1$, $k_2$, and $k_3$. These vectors have the same slopes as $k_1$, $k_2$, and $k_3$ respectively, and their lengths are determined by the demands for the goods. Because we are in a special case of zero net factor-content trade, where the country is capable of producing exactly what it consumes in free trade, these three vectors must add up to the country's total factor endowment at $E$. This is indicated in Figure 2 by drawing them into the Edgeworth Box end to end starting with the origin and forming a path through the box that ends just at its upper right-hand corner, $E$. There are several such paths that could be drawn, depending on the order in which the vectors are selected. Of special interest are the two paths shown in Figure 2, $O\mathbf{v}_1\mathbf{v}_2\mathbf{v}_3$ and $O\mathbf{v}_3\mathbf{v}_2\mathbf{v}_1$, where the vectors are arranged in order of increasing and decreasing capital intensity respectively. These two paths, and the various factor rays of which their segments are a part, will make it possible to map out areas in factor space where various trade patterns arise once factors are lumpy.

Suppose now that the country is divided into two regions, A and B, each endowed with its own labor and capital in amounts that add up to the country's endowment at $E$. As in Courant and Deardorff (1990), one can represent the allocation of these factors between the two regions by points within the Edgeworth Box, measuring, say, Region A's endowment from the lower-left corner at $O$ and Region B's endowment from the upper-right corner at $E$. One can then compare the allocation point to the various factor rays and isoquants to determine the possible outputs of the three goods in the two regions.

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5Total income is constant only as long as factor prices remain equalized. See footnote 6 below.

6This is not strictly an Edgeworth Box, which would more commonly measure industry factor employments, rather than regional factor endowments, from these opposite corners. The use here is similar to the technique of Dixit and Norman (1980, pp. 110-122) for
Suppose first that the allocation point lies within what is labelled as area I of the box—the hexagonal area formed by the two paths from O to E just described. In that case both region's factor endowments lie within the diversification cone defined by $k_1$ and $k_3$, and they will therefore share the common world factor prices. Furthermore, it is easily verified that the vectors $\hat{v}_1$, $\hat{v}_2$, and $\hat{v}_3$ can be divided between them in such a way that the two together produce exactly what the country demands. Therefore area I corresponds to the possibility of no trade.\footnote{Throughout the diversification cone with more goods than factors, outputs are of course indeterminate. Therefore, while it is possible in area I for there to be no trade, it is also possible for there to be trade. I will be concerned here only with the trade that must arise due to lumpiness, not the trade that may arise solely because of the indeterminancy of output.}

Consider, at the opposite extreme, the area of the box labelled V. Here both regions of the country are outside of the diversification cone and cannot share factor prices that are equal to each other's or to those in the world. Instead, Region A must specialize completely in good 3, and have the rather high wage-rental ratio implied by the $X_3$ isoquant through its endowment point (not shown). Region B, in contrast, specializes in good 1 and has a much lower wage-rental ratio. Furthermore, because both (relative to their respective origins) are above the isoquants $\bar{X}_1$ and $\bar{X}_3$ for producing what the country demands, they must produce more of their respective goods than are needed domestically and must export the difference to the world market.\footnote{This conclusion requires the assumption made earlier that preferences in the two regions be identical and homothetic. It that were not the case, then different incomes in the two regions could lead to a level of total consumption of the goods that would differ from the levels $\bar{X}_1$ and $\bar{X}_3$. Furthermore, because of the inefficiency of the factor allocation in area V, national income will be somewhat lower than it could have been if factors were mobile between regions. Thus with homothetic preferences, and hence normal goods, the amounts "needed domestically" are actually lower than $\bar{X}_1$ and $\bar{X}_3$.} And since neither produces good 2 at all, they both must import it from the world market.\footnote{This conclusion requires the assumption made earlier that preferences in the two regions be identical and homothetic. It that were not the case, then different incomes in the two regions could lead to a level of total consumption of the goods that would differ from the levels $\bar{X}_1$ and $\bar{X}_3$. Furthermore, because of the inefficiency of the factor allocation in area V, national income will be somewhat lower than it could have been if factors were mobile between regions. Thus with homothetic preferences, and hence normal goods, the amounts "needed domestically" are actually lower than $\bar{X}_1$ and $\bar{X}_3$.}
Assuming that the rest of the world is composed of many countries of varying factor abundances for which the Heckscher-Ohlin Theorem applies conventionally, this then is a clear example of cross-over trade. Because of its lumpiness in area V of the box, this country exports both the most labor intensive and the most capital intensive goods, and imports the good of intermediate factor intensity. Since more labor abundant countries will also tend to export the labor intensive good, this country’s exports of good 1 will have to go instead to the more capital abundant countries of the world. And since the latter will export the capital intensive good, this country’s exports of good 3 will have to go to the more labor abundant countries. With good 2 imported from whoever produces it—presumably countries of intermediate factor abundance—this is exactly the phenomenon of cross-over trade that was identified above.

How general is this phenomenon? Does it only arise when regions of a country are so disparate that they specialize in this extreme fashion? The answer is no, as it turns out, as can be seen by looking at area II of the box in Figure 2.

In area II, both regions of the country have factor endowments within the \( k_1-k_3 \) diversification cone, and therefore both will share the world factor prices. That is, there is factor price equalization throughout the parallelogram formed by the two \( k_1 \) and \( k_3 \) rays, exactly as in the two-good case of Courant and Deardorff (1990). However, it is not true that there can be no trade at these points.

For an allocation in area II, Region A, for example, is so close to the \( k_3 \) ray that it must produce a very large proportion of good 3 in order to keep its factors fully employed, and in fact it must produce more of good 3 than \( \hat{X}_3 \). This is illustrated in Figure 3, where an Edgeworth Box for Region A alone is drawn assuming a regional factor allocation that corresponds to area II in Figure 2. As shown, in order to keep its factors fully employed, Region A could produce goods 3 and 2, with an output of the former of \( X_3 \). Or it could produce goods 3 and 1, with an output of the former of \( X_3 \). Or it could produce a convex
linear combination of these two possibilities. But in any case, its output of good 3 must be at least $X_3^1$, and this is clearly greater than $X_3$.

By the same reasoning, Region B in area II also must produce an amount of good 1 that exceeds $X_1$, the amount that the country as a whole demands of that good. Therefore, in area II, both regions produce more of a good of extreme factor intensity than can be absorbed at home, and both must therefore export these goods to the world market, exactly as in area V discussed above.

Continuing the argument, it is clear that areas III and IV also have this property, for a mixture of the reasons in areas II and V. Thus, throughout the shaded areas II-V of Figure 2, the country displays cross-over trade.

For other unmarked areas in the upper left of Figure 2, cross-over trade may still occur, though it is less likely because one of the regions of the country may not produce enough of any good to satisfy the country's demand and still have anything left to export. At the left of the Figure, for example, for allocations below the $X_3$ isoquant and above the $k_3$ ray, Region A produces only good 3, but it produces less of it than would have been demanded with mobile factors. This does not quite preclude its exporting, however, since the inefficiency of specialization will reduce national income somewhat, and also therefore reduce the country's demand for good 3 below $X_3$. Thus there will be a small part of the figure, just below the $X_3$ isoquant (and another above the $X_1$ isoquant) where cross-over trade will still take place. However, for most of the other allocations to the left and above area I there will not be cross-over trade because only one good will be exported, and only one region of the country will export it. Of course, these results for the North-West portion of the Box are duplicated in the South-East, where exactly symmetric results obtain.

Thus, for the special case considered so far of WFPE prices and zero net factor-content trade when factors are interregionally mobile, lumpiness of factors can indeed lead to trade. If factors are sufficiently lumpy—that is, if they are sufficiently unevenly
allocated across regions—and if both regions remain sufficiently large that they may need to trade with the outside world, then the phenomenon of cross-over trade arises quite naturally.

I would also note that this trade, caused by lumpiness, is trade in goods, and need not involve any net trade in factor content. In area II of Figure 2, for example, the factor content of production is equal to the factor content of consumption, and the net factor content of trade is therefore still zero. Thus lumpiness may lead to trade in goods that is contrary to the Heckscher-Ohlin model in another sense: it is not detectable by a Leontief-like factor-content test of trade patterns. ⁹ Likewise, outside of areas I and II, where at least one region of the country completely specializes, the factor content of trade may well be nonzero, though since different techniques of production are then optimal within different regions, this will depend in part on how one chooses to measure factor content. ¹⁰

### Nonzero Net Factor-Content Trade

Now consider briefly how results may change in the more general case in which the country would not be able to produce its consumption bundle under free trade if factors were mobile. Such a case is shown in Figure 4. Here the three vectors indicating the factor requirements of consumption, \( \mathbf{v}_1, \mathbf{v}_2, \) and \( \mathbf{v}_3 \), add up to point \( C \) that is different from the endowment point, \( E \). With balanced trade, the value of consumption must equal the value of income, and therefore point \( C \) must lie on the same isocost line as point \( E \), valued at the world factor prices shown in Figure 1. With a nonzero factor content of trade, however, \( C \) could lie either to the right or to the left of point \( E \), depending on the nature of the goods demanded and hence the factor content of consumption.

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⁹See Deardorff (1984) for a discussion of such tests.

¹⁰In Deardorff (1982) a version of the Heckscher-Ohlin Theorem is derived measuring factor content (and factor intensity) in terms of techniques actually used to produce each unit of a good, wherever that production may occur. That theorem continues to be valid here, though its usefulness is clouded by the variety of techniques that may be in use in different locations.
As drawn, point C lies to the right of E, so that the country (with free trade and mobile factors) is a net importer of labor and a net exporter of capital in factor content terms. One therefore expects a certain bias in favor of the country exporting the more capital intensive good. Of interest is whether lumpiness can nonetheless cause one of the regions of the country to export the labor intensive good.

Areas of the box that delineate various trade patterns are somewhat more complex than before, and I will therefore concentrate only on the top half of the box, above its (undrawn) diagonal. Symmetric remarks apply below.

The path $O\bar{v}_1\bar{v}_2\bar{v}_3$ from O to C now yields information about production and specialization only in Region A. To determine analogous information about Region B, a parallel path, $E\bar{v}_1\bar{v}_2\bar{v}_3$, needs to be drawn down and to the left from the country’s endowment point, E, which also is the origin for measuring Region B’s factor endowments. Inserting also the $\tilde{X}_3$ and $\tilde{X}_1$ isoquants relative to O and E respectively, one can identify additional areas of the box that bear discussing.

Consider area VI, for example. Here Region A is again in the situation of Figure 3, where it must produce more than $\tilde{X}_3$ of good 3 and must export it. However Region B is now not in that situation with respect to good 1, and in fact one cannot place any useful restrictions on Region B’s outputs of the three goods in area VI. Thus while it is possible that cross-over trade may occur here, it is by no means assured. And this continues to be the case even in area VII of the box, where Region A now completely specializes in good 3 in excess of $\tilde{X}_3$.

In areas VIII and IX, on the other hand, cross-over trade is assured. In area VIII, Region B is in the situation of Figure 3, not specialized but producing more than $\tilde{X}_1$ of good 1. In area IX, it is specialized in good 1, again in excess of $\tilde{X}_1$. In either case, since Region A is also producing more than $\tilde{X}_3$ of good 3 in both areas, cross-over trade must occur. Thus cross-over trade once again occurs in the shaded area of Figure 4.
Note that the interesting case of area II from Figure 2 has now been diminished in importance, and could be eliminated entirely if net factor trade is large enough. In Figure 4 the portion of the box in which both countries continue to have equal world factor prices but nonetheless produce enough of goods 1 and 3 to export has been reduced to a tiny sliver at the north-east tip of area VIII. And it could easily have been eliminated entirely. Thus the case of cross-over trade together with factor price equalization evidently depends upon the extent of Heckscher-Ohlin trade being weak. But as areas VIII and IX make clear, the possibility of cross-over trade by itself remains substantial.

IV. The Case of World Specialization

I turn now to the second price configuration discussed in section II. If, as in Figure 1B, world prices of the three goods fail to align so that all three can be produced in any country, then specialization in different countries is inevitable. Cross-over trade is still quite possible, however, as will be indicated in Figure 5.

In section III I first abstracted from Heckscher-Ohlin trade by considering a country that would not trade at all if factors were mobile within it. That is not possible here, since regardless of a country’s factor endowments it will not produce all three goods under free trade.

Nor can I, in this case, use vectors indicating the factor requirements of consumption to map out the Edgeworth Box of a country, since the factors used to produce a good at world prices are not unique, at least in the case of good 2. Therefore the techniques employed in section III will not work here. However, these difficulties are compensated for by the fact that outputs are no longer indeterminate.

Consider, then, a country whose factor endowments would lead it to specialize in the good of intermediate factor intensity, good 2, if factors were internally mobile. Thus its factor endowment lies between the rays $k_2^1$ and $k_2^2$ of Figure 1B. Figure 5 shows an Edgeworth Box for the two regions of such a country, with the various factor rays of Figure 1B drawn from both the lower-left-hand corner origin for Region A and from the
upper-right-hand corner origin for Region B. These rays divide the box into a large number of areas within which the pattern of production, and sometimes trade, can be observed.

Looking again only at the upper portion of the box, above the (undrawn) diagonal, nine areas are identified by Roman numerals. The pattern of production in these areas for the two regions is as follows:

<table>
<thead>
<tr>
<th>Area of Fig. 5</th>
<th>Region A</th>
<th>Region B</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>II</td>
<td>2,3</td>
<td>2</td>
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<tr>
<td>III</td>
<td>2</td>
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<tr>
<td>IV</td>
<td>3</td>
<td>2</td>
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<tr>
<td>V</td>
<td>2</td>
<td>1</td>
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<tr>
<td>VI</td>
<td>2,3</td>
<td>1,2</td>
</tr>
<tr>
<td>VII</td>
<td>3</td>
<td>1,2</td>
</tr>
<tr>
<td>VIII</td>
<td>2,3</td>
<td>1</td>
</tr>
<tr>
<td>IX</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

Only in area I does the country produce as it would if factors were internally mobile. Everywhere else in the box goods 1 and/or 3 are produced in some region of the country, and in one area, IX, those are the only goods produced.

Especially in this latter area, IX, the pattern of production is strongly suggestive of cross-over trade, though there is no guarantee throughout that area that both regions produce enough of their respective outputs to be able to export them out of the country. To further identify trade patterns it is necessary, as before, to enter certain isoquants into the figure.

Thus the isoquants for $\hat{X}_3$ in Region A and $\hat{X}_1$ in region B have been added to Figure 5 as well. These are the quantities of goods 3 and 1 that the country as a whole demands with free trade, assuming that it faces the prices illustrated in Figure 1B and that it has the income that would be earned at those prices if factors were mobile. Within

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11 At least one must do so, of course, to pay for the country’s imports of good 2.
area IX, then, any allocation of factors between these two isoquants must entail cross-over trade, just as was argued earlier.

In other areas of the figure these isoquants cannot be used quite so directly, since more than one good is being produced in at least one region. It is still possible to use the information that these isoquants provide, however.

Consider the intersection of the $\hat{X}_3$ isoquant with the $k_3$ ray out of $O_A$, labelled point $a$. By drawing another line, $acd$, from this point parallel to the $k_2^2$ ray one can isolate the factor allocations within areas VI and VIII where Region A produces more than $\hat{X}_3$ of good 3. This follows from the familiar construction of the Edgeworth Box for two goods, applied to Region A.

Similarly, one can construct the line $bee$, parallel to the $k_2^1$ ray and starting at point $b$, the intersection of the $\hat{X}_1$ isoquant with the $k_1$ ray from $O_B$. Together these two lines define the locus $ecd$, above and to the left of which, between the two drawn isoquants, there must be cross-over trade. Thus cross-over trade arises throughout the shaded area in Figure 5.

Thus, while the mechanics of tracing out production and trade patterns are rather different here from the previous case, the conclusion is largely the same. There is a sizable area of the Edgeworth Box, and hence a sizable range of inter-regional factor allocations, for which the country will engage in cross-over trade. These factor allocations are essentially those for which the factors are sufficiently lumpy, in the sense of being unevenly allocated across regions, with at the same time the two regions remaining of roughly comparable size.

V. Conclusion

This paper has shown how lumpiness of factors of production can lead to the phenomenon of cross-over trade. That is, if factors of production within a country are allocated sufficiently unevenly across regions, then those regions will specialize in different
products than the country as a whole would produce if factors were mobile. As a result, one region may well produce more of a labor intensive good than the country can absorb, and if so it will export the excess to more capital-abundant countries elsewhere in the world. At the same time, the other region may specialize in the capital intensive good and export it to more labor abundant trading partners. This is not a particularly surprising or subtle phenomenon, but it is worth understanding that lumpiness may have this effect. For it means that lumpiness can have a more distorting effect on behavior in the economy than may have been previously understood.

In Courant and Deardorff (1990) we showed that lumpiness could lead a country to trade somewhat differently than it would if factors were mobile. But we did not suggest, and I do not here, that lumpiness would lead to a major departure from the predictions of the Heckscher-Ohlin model regarding trade. However it appears now that, while the country’s overall trade in terms, say, of its factor content, may not be very much changed by lumpiness, a great deal else is. Lumpiness can completely alter the economic landscape, in terms of what individual goods are produced and where, compared to what would be observed if factors were mobile.

This conclusion must to some extent be tentative, for it has only been derived in the context of a far too simple model. In a world of many goods, countries, and factors, it may be that the stark effects of lumpiness seen here would tend to blur. However the analysis at least suggests that the issue is worth investigating.
References

Courant, Paul N. and Alan V. Deardorff 1990 "International Trade of Lumpy Countries," in process.


Figure 2