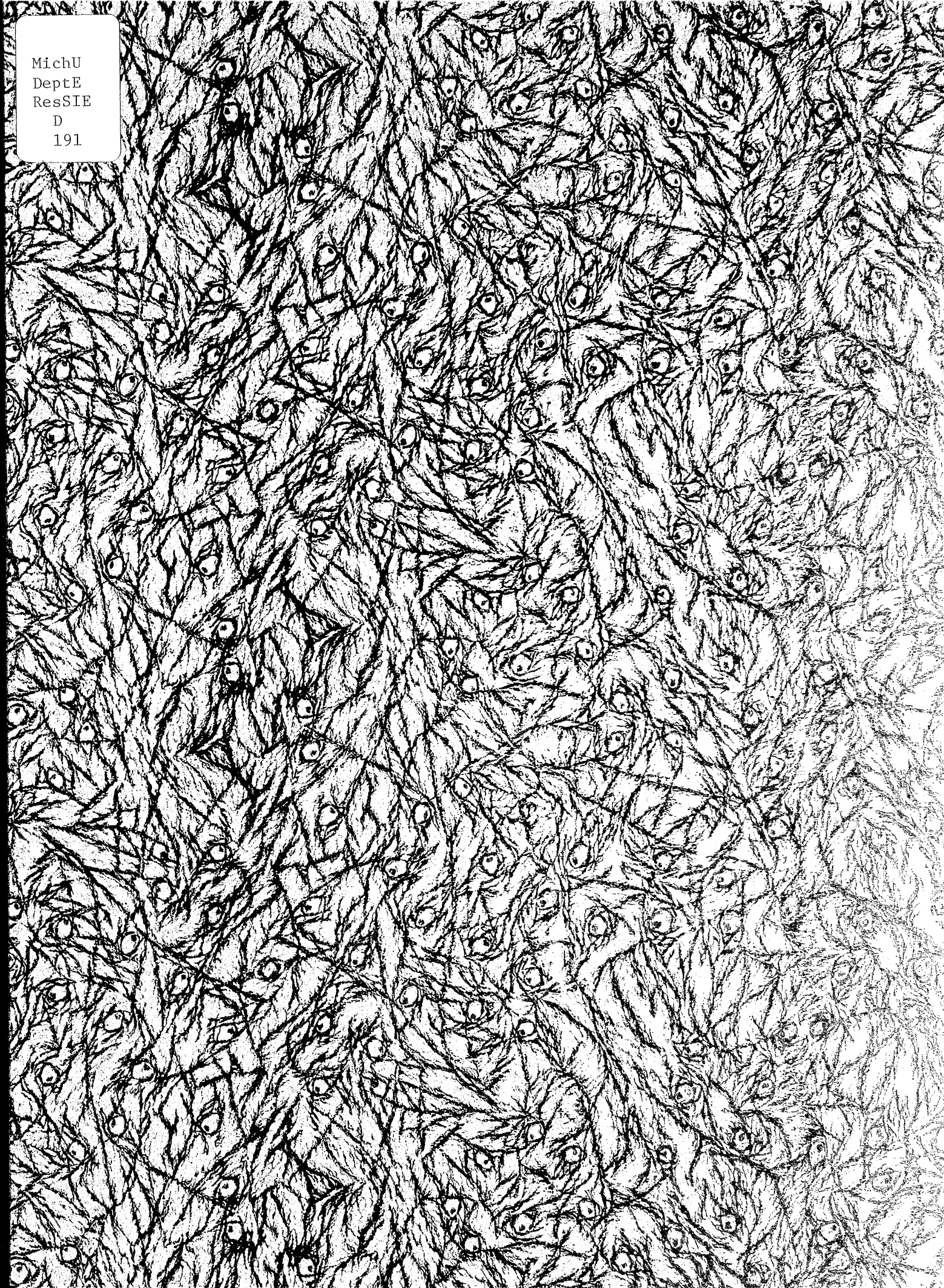
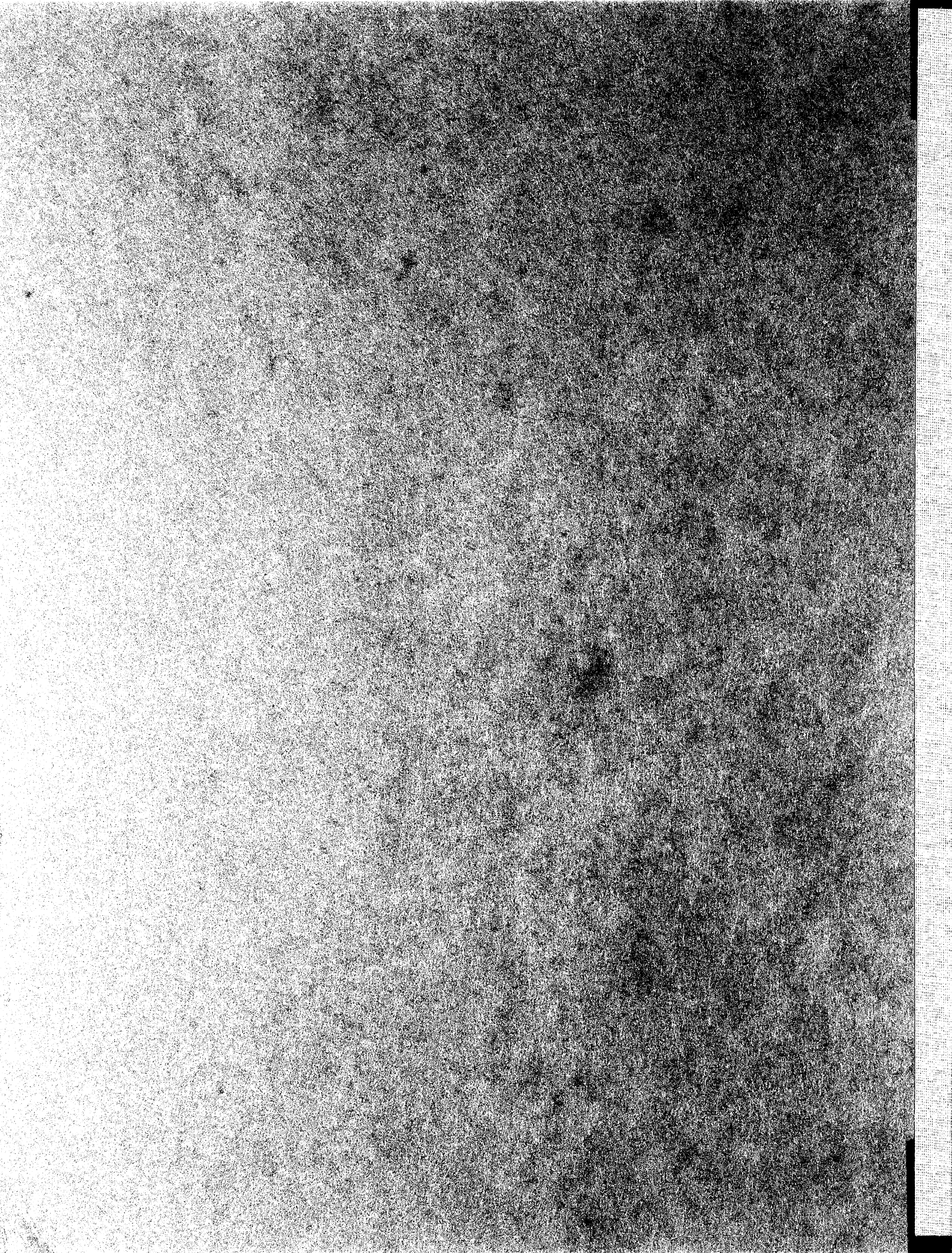


MichU  
DeptE  
ResSIE  
D  
191







**RESEARCH SEMINAR IN INTERNATIONAL ECONOMICS**

**Department of Economics  
The University of Michigan  
Ann Arbor, Michigan 48109-1220**

**SEMINAR DISCUSSION PAPER NO. 191**

**AN INTERPRETATION OF THE FACTOR CONTENT OF TRADE**

by

**Alan V. Deardorff  
The University of Michigan  
and  
Robert W. Staiger  
Stanford University**

**November 12, 1986**

This paper was written with partial support from the Ford Foundation. We would like to thank Bob Stern for his helpful comments on an earlier draft.

**The Sumner and  
Laura Foster Library**

FEB 20 1987



# **An Interpretation of the Factor Content of Trade\***

by

**Alan V. Deardorff**

**The University of Michigan**

and

**Robert W. Staiger**

**Stanford University**

**November 12, 1986**

**\*This paper was written with partial support from the Ford Foundation. We would like to thank Bob Stern for his helpful comments on an earlier draft.**

## **ABSTRACT**

### **An Interpretation of the Factor Content of Trade**

by

**Alan V. Deardorff and Robert W. Staiger**

This paper shows that the factor content of trade can be used as an indicator of the effects of trade on the relative prices of factors. The argument notes first that, under fairly general conditions, a trading equilibrium has the same factor prices as an equivalent autarky equilibrium that can be constructed by changing factor endowments by the amounts of the factor content of trade. Thus factor prices in two trading equilibria can be compared by comparing instead their two equivalent autarky equilibria. Since factor prices in autarky are related to factor endowments, it follows that factor prices with trade are related to its factor content. This relationship is formalized first in strong terms in a Cobb-Douglas model, and then in the weaker form of correlations in a more general model.

Address correspondence to:

Alan V. Deardorff  
Department of Economics  
The University of Michigan  
Ann Arbor, MI 48109-1220

November 12, 1986

# An Interpretation of the Factor Content of Trade

by

Alan V. Deardorff and Robert W. Staiger

## I. Introduction

The factor content of trade is well known to be useful as a way of formulating versions of the Heckscher-Ohlin Theorem that will be valid even under circumstances in which the commodity composition of trade is indeterminate.<sup>1</sup> This in turn has led to measurement of the factor content of trade as a means of applying and testing the Theorem empirically.<sup>2</sup> We argue in this paper that the factor content of trade is also useful as an indicator of the effects of trade on relative factor prices. Thus, for example, measurement of the factor content of trade can be used as an indicator of which factors of production have gained the most from trade relative to autarky, making it unnecessary actually to observe an autarky equilibrium in order to answer this question.

To show this, we note first in Section II that, under fairly general conditions, a trading equilibrium has the same factor prices as an equivalent autarky equilibrium that can be constructed by changing factor endowments by the amounts of the factor content of trade. Thus factor prices in two trading equilibria can be compared by comparing instead their two equivalent autarky equilibria. Since factor prices in autarky are related to factor endowments, it follows that factor prices with trade are related to its factor content.<sup>3</sup>

---

<sup>1</sup>See Melvin (1968) and Vanek (1968).

<sup>2</sup>Leontief (1954) actually measured the factor content of trade in obtaining his famous Paradox, which Stern and Maskus (1981) have reevaluated as suggested by Leamer (1980). Factor content has also been used more recently as the basis for tests of the Heckscher-Ohlin Theorem by Harkness (1983), Bowen, Leamer, and Sveikauskas (1984), and Staiger (1985).

<sup>3</sup>The relationship between post-trade factor prices and the factor content of trade has been explored by several authors. Helpman (1984) derives a relationship between bilateral post-trade factor price differentials and the factor content of bilateral trade. Neary and Schweinberger (1986) derive a relationship between *changes* in the factor content of a country's trade and the factor price vector in the *original* equilibrium. We focus here on the relationship between *changes* in the factor content of a country's trade

To derive a formal statement of this result, we first examine in Section III a model in which all preferences and production functions are Cobb-Douglas. In this model, the unitary elasticities of substitution that prevail in all activities imply a very strong relationship between autarky factor prices and endowments. This in turn gives us a similarly strong relationship involving the factor content of trade.

In Sections IV and V, then, we turn to the much weaker relationship that exists more generally between autarky factor prices and endowments: the fact, as shown for example by Dixit and Norman (1980), that the two are negatively correlated. In Section IV, we use this result to derive a similar relationship involving the factor content of trade and the effects of trade on factor prices. We then use this relationship in Section V to obtain several corollaries that establish correlations between these two variables. In our most general result, proved as corollary 3.4 in Section V, we establish a positive correlation between a measure we construct of the relative change in the factor content of trade, by factor, and the proportional changes in the corresponding factor prices.

Section VI concludes by noting certain limitations of this result.

## II. Equivalent Autarky Equilibria

Consider an economy capable of producing  $n$  goods,  $X_1 \dots X_n$ , using  $m$  primary factors,  $L_1 \dots L_m$ . Let technology be non-joint and linearly homogeneous, so that techniques of production can be characterized by  $m \times n$  matrices  $A = \{a_{ij}\}$ , whose elements are the quantities of factor  $i$  used in producing one unit of good  $j$ . Let  $F$  be the set of all such matrices of techniques that can be used.

A competitive production equilibrium for this economy can be defined for any given endowments of the factors,  $L^0 = (L_1^0, \dots, L_m^0)'$  and prices of the goods,  $p^0 = (p_1^0, \dots, p_n^0)'$ .

---

and *changes* in its factor prices. Hence, in contrast to previous work, our results allow inferences to be made concerning the effects of trade on relative factor prices.



Such an equilibrium will consist of a vector of factor prices,  $w^0$ , a vector of outputs,  $X^0 \geq 0$ , and a matrix of techniques  $A^0 \in F$  such that

$$(2.1) \quad A^0 X^0 \leq L^0$$

$$(2.2) \quad p^0 X^0 - w^0 A^0 X^0 \geq p^0 X - w^0 A X \quad \text{for all } X \geq 0, A \in F.$$

That is, (2.1) says that  $X^0$  is feasible given the factor endowments  $L^0$  and (2.2) says that it yields at least as great a profit as any other vector of outputs that competitive producers might attempt to produce. Since a strictly positive profit in any activity could always be improved upon simply by expanding output, while a negative profit could be eliminated by contracting output to zero, (2.2) implies the following:

$$(2.3) \quad w^0 A^0 \geq p^0$$

$$(2.4) \quad p^0 X^0 = w^0 A^0 X^0$$

so that unit cost equals price for each good that is produced and total profit is zero.

Now suppose that the economy is in a trading situation, and that a vector of goods  $C^0$  is being demanded. The difference between production and demand,  $T^0 = X^0 - C^0$ , is the net trade vector. We do not necessarily assume that trade is either balanced or free. Nor need we assume anything special about the properties of the rest of the world, eg., that technologies are internationally identical.

Define the factor content of trade,  $S^0$ , as the vector of factors needed to produce what is exported, less the factors needed to produce what is imported:

$$(2.5) \quad S^0 = A^0 T^0$$

Define an equivalent autarky equilibrium as an equilibrium that would arise if the original economy's factor endowments were changed by the amounts in  $S^0$ , net factor exports being removed and net factor imports being added, and the economy were not allowed to trade. We show, under one additional assumption, that an equivalent autarky

equilibrium exists in which outputs equal  $C^0$  and prices of both goods and factors are the same as prevailed in the original trading equilibrium. The additional assumption is that the economy is initially incompletely specialized, in the sense that it produces a positive amount of each good that it consumes. Since goods that are not consumed at all are not very interesting, we will simply assume that all outputs are strictly positive:

$$(2.6) \quad X^0 >> 0$$

**Proposition 1:** If factor endowments in the economy are  $B^0 = L^0 - S^0$ , then with the same goods-prices,  $p^0$ , as prevailed in the trading equilibrium, a competitive production equilibrium exists in which outputs are  $C^0$ , factor prices are  $w^0$  and techniques are  $A^0$ .

**Proof:** We need only establish that relationships analogous to (2.1) and (2.2) hold in the new equilibrium. These are established as follows:

$$(2.7) \quad \begin{aligned} A^0 C^0 &= A^0 [X^0 - (X^0 - C^0)] \\ &\leq L^0 - A^0 T^0 \\ &= L^0 - S^0 = B^0 \end{aligned}$$

where the inequality follows from (2.1). Thus with the new endowments it is feasible to produce the bundle of goods,  $C^0$ . To see that such production also maximizes profits, note that (2.6) together with (2.3) and (2.4) imply that price equals average cost in all industries:

$$(2.8) \quad p^0 = w^0 A^0,$$

and thus that production of  $C^0$  with techniques  $A^0$  yields zero profit, which is the best that one can do when facing  $p^0$  and  $w^0$ :

$$\begin{aligned}
(2.9) \quad p^0 C^0 - w^0 A^0 C^0 &= 0 \\
&= p^0 X^0 - w^0 A^0 X^0 \\
&\geq p^0 X - w^0 A X \quad \text{for all } X \geq 0, A \in F.
\end{aligned}$$

where the last inequality follows from (2.2). **QED**

Proposition 1 deals only with the production side of the economy. However, since the same bundle of goods is demanded in this new equilibrium as in the initial trading equilibrium, and the same domestic prices prevail, it is an equilibrium for demand as well.<sup>4</sup>

The intuition behind this result may be appreciated by noting in (1.7) that, with full employment of all factors,  $B^0$  is the factor content of consumption,  $A^0 C^0$ . Thus the equivalent autarky equilibrium merely endows the economy with the factors needed to produce what it consumes, and thus obviates the need for trade at prevailing prices of goods and factors.

The significance of this result should also be explained. It means that factor prices in any nonspecialized trading equilibrium can be inferred from an equivalent autarky equilibrium based on factor endowments  $L-S$ , where  $L$  are the endowments in the trading equilibrium and  $S$  is the factor content of trade in the trading equilibrium measured with the techniques of production used in the trading equilibrium. Since factor prices in autarky can be somewhat easier to specify than in trade, this result can be very

---

<sup>4</sup>One also needs to check that there is still income sufficient to purchase the goods. If in the initial equilibrium producers and consumers were faced with the same prices,  $p^0$ , then factor income in the equivalent autarky equilibrium would exactly equal  $p^0 C^0$ . Notice that any tariff revenue in the trading equilibrium can be safely ignored here, since direct production of the consumption bundle  $C^0$  generates earned income sufficient to purchase it, even if that was not the case with trade. On the other hand, if domestic taxes in the initial trading equilibrium caused producer and consumer prices to diverge, then appropriate transfers may be necessary.

useful. In the following sections we use it to relate factor prices to the factor content of trade.

### III. Strong Results with a Cobb-Douglas Model

Now consider an economy with the very strong property that all preferences and production functions are Cobb-Douglas. This assumption allows us to derive autarky factor prices explicitly as functions of factor endowments, and thus use the result of Section II to get a strong relationship between factor prices and the factor content of trade.

Cobb-Douglas production functions imply that each factor earns a constant share of the revenue in each industry. Cobb-Douglas preferences imply similarly that consumers spend a constant fraction of their total expenditure,  $E$ , on each good. In autarky, where consumer expenditure on a good equals the revenue of the industry producing that good, the two together imply that each factor's total income (from employment in all industries together) is a constant fraction of consumer expenditure. Letting that constant for factor  $i$  be  $c_i$ , we have

$$(3.1) \quad w_i L_i = c_i E$$

Were we to divide through by  $L_i$ , we would have immediately that autarky factor prices are inversely related to factor endowments.

Now consider two equilibria, numbered 1 and 2, that may involve trade. With trade, (3.1) does not apply directly. However, we can use the result of Section II to express factor prices with trade in terms of what they would be without trade in an equivalent autarky equilibrium, the factor endowments of which are  $B=L-S$ . Thus letting  $L^0$  be the actual factor endowments, assumed the same in both trading equilibria, factor prices can be expressed as

$$(3.2) \quad w_i^h = c_i E^h / (L_i^0 - S_i^h) \quad h=1,2.$$

If we now compare factor prices in the two equilibria, we have

$$(3.3) \quad \frac{w_i^2}{w_i^1} = \frac{E_i^2}{E_i^1} \cdot \frac{(L_i^0 - S_i^1)}{(L_i^0 - S_i^2)}$$

Thus, factor prices depend positively on levels of expenditure, and inversely on endowments net of the factor content of trade. The result can be simplified if we normalize prices in both equilibria so that total expenditure is unity. Then the expenditure ratio in (3.3) drops out, and we can express the relative change in factor prices between the two equilibria in terms of the change in the factor content of trade relative to the factor content of consumption:

**Proposition 2:** If preferences and technologies are Cobb-Douglas and identical in two trading equilibria for which factor endowments are also identical, then if prices are normalized to equate total expenditure to unity in both equilibria,

$$(3.4) \quad \frac{w_i^2 - w_i^1}{w_i^1} = \frac{S_i^2 - S_i^1}{B_i^2} .$$

Finally, instead of comparing two trading equilibria, we can compare a single trading equilibrium with the autarky equilibrium that would have obtained with the same endowments. Letting 1=t be the trading equilibrium and 2=a be the autarky equilibrium, and noting that trade and its factor content are zero in autarky, we get the following very simple relationship:

**Corollary 2.1:** Under the assumptions of Proposition 2, if one of the equilibria is itself an autarky equilibrium, then

$$(3.5) \quad \frac{w_i^t - w_i^a}{w_i^t} = \frac{S_i^t}{L_i^0} .$$



Thus if we consider the move from autarky to any trading equilibrium in this Cobb-Douglas world, the factor content of trade as a fraction of each factor endowment is an exact measure of the fraction of each factor price that can be attributed to that trade.

These are very strong results, but they are not quite as strong as they may at first appear. They do not indicate in any absolute or real sense what the effects of trade and changes in trade will be on factor prices. Instead, they indicate only the effects on factor prices *relative* to one another. That is, if (3.5) is negative, for example, this does not necessarily mean that the price of factor  $i$  has fallen in real terms or in any relevant nominal terms. It does however mean that the price of factor  $i$  has fallen relative to the prices of other factors which are determined by (3.5) to have increased.

What the factor prices here represent can be seen more clearly by considering the particular normalization we have chosen. By equating nominal aggregate expenditure,  $E$ , in the two equilibria being compared, we make factor prices reflect the shares of aggregate expenditure earned by a unit of each factor. If prices and aggregate expenditure happened to be such as to yield a constant level of aggregate welfare, then these factor prices would indeed measure welfare. But in general aggregate welfare does change between the two equilibria and this interpretation is generally not possible.

For example, in the comparison of autarky and trade in (3.5), it is normally the case that aggregate welfare rises with trade. Thus the effects on welfare of individual factors include both the effects on their shares of aggregate welfare described by (3.5), plus their shares of the usual gains from trade. It follows that, while we can be sure that factors for which (3.5) is positive do gain from trade, other factors for which (3.5) is negative may gain from trade as well, if the aggregate gain from trade is large enough to offset the relative loss indicated by (3.5). That a net loss is nonetheless possible, however, is well known from the Stolper-Samuelson Theorem.

#### IV. Weak Results with a More General Model

Without a strong assumption such as Cobb-Douglas preferences and production functions, there does not exist in general a relationship between factor endowments and autarky factor prices that is strong enough to hold for each and every factor. Variations of complementarity and substitutability among goods and factors — which are ruled out by the Cobb-Douglas assumption — can cause prices of particular factors to move in ways that contradict the changes in their own factor endowments. A factor that is closely substitutable for another factor, for example, may rise in price even though its endowment rises, if the endowment of its close substitute shrinks sufficiently. These problems with relating factor prices and endowments in autarky also make it difficult to derive any generally valid strong relationship between factor prices and the factor content of trade that holds for every factor of production.

However, a general result is possible if we look only for a relationship that holds on average across all factors. In a model like that of Section III but without the Cobb-Douglas restriction, Dixit and Norman (1980, p. 99) derived the following simple relationship between autarky factor prices and factor endowments:

$$(4.1) \quad (w^1 - w^2)'(L^1 - L^2) \leq 0 \quad ,$$

where factor prices,  $w^1$  and  $w^2$ , are those that correspond to endowments,  $L^1$  and  $L^2$  respectively, in autarky. For this result they required only that preferences be homothetic, that production functions be linearly homogeneous and non-joint, and that prices of goods be normalized so that aggregate nominal expenditure is the same in both equilibria, just as we did in Section III.

Dixit and Norman interpret (4.1) as showing that “autarky factor price differences are negatively correlated with endowment differences.” This is not strictly

correct, but it is true that (4.1) imposes an important restriction on the relationship between endowments and autarky factor prices that is related to a correlation.<sup>5</sup>

If we now draw on the result of Section II, we can compare factor prices in trading equilibria in terms of their equivalent autarky equilibria.

**Proposition 3:** Let  $w^i$  be the vector of factor prices in a nonspecialized trading equilibrium in which the factor endowments are  $L^0$  and the factor content of trade is  $S^i$ , for  $i=1,2$ . Then

$$(4.2) \quad (w^1 - w^2)'(S^1 - S^2) \geq 0 \quad .$$

**Proof:** From Section II, factor prices,  $w^i$  are the same as would obtain in an autarky equilibrium with factor endowments  $L^0 - S^i$ , and we can replace the factor endowments in (4.1) with these expressions,

$$(4.3) \quad (w^1 - w^2)'[(L^0 - S^1) - (L^0 - S^2)] \leq 0$$

and (4.2) then follows. **QED**

This is the basic general result of this paper. It establishes a simple correlation-like relationship between changes in factor prices and changes in the factor content of trade. If, between two trading equilibria, the factor content of trade rises for some factors and falls for others, this says that the prices of the former will tend to rise relative to the prices of the latter.

As in Section II, it is also possible to compare a trading equilibrium with autarky. Letting 1=t and 2=a in (4.2), and noting that the factor content of trade is zero in autarky, we have

---

<sup>5</sup>The inequality in (4.1) directly implies the sign of a correlation if either of the two vectors in the inner product has zero mean. See Deardorff (1980). In this case factor endowments could easily all move in the same direction, while the normalization on goods prices prevents us from imposing a similar normalization on factor prices. It remains to be seen what sort of true correlation results can be derived from (4.1).]

**Corollary 3.1:**

$$(4.4) \quad (w^t - w^a)'S^t \geq 0 \quad .$$

Thus the effect of trade relative to autarky is to tend to raise the returns to those factors which are exported in factor content terms relative to those which are imported.

**V. Corollaries Involving True Correlations**

The simple results just derived, as already noted, are not the same as true correlations. Further assumptions and further work are needed to derive true correlations from them, and to determine therefore whether the results tell us anything meaningful about the models from which they derive.

Such correlations are particularly easy to derive if it happens to be the case that there exist factor prices subject to which the factor content of trade is balanced. Thus we have:

**Corollary 3.2:** If the factor content of trade is balanced when valued at some vector of factor prices,  $w^0$ , then there is a nonnegative correlation between changes in the value of the factor content of trade at these factor prices and the changes in the factor prices themselves relative also to  $w^0$ :

$$(5.1) \quad \text{Cor}\left[\frac{(w_i^1 - w_i^2)}{w_i^0}, w_i^0(S_i^1 - S_i^2)\right] \geq 0$$

and, comparing a trade equilibrium with autarky,

$$(5.2) \quad \text{Cor}\left[\frac{(w_i^t - w_i^a)}{w_i^0}, w_i^0 S_i^t\right] \geq 0$$

These results follow immediately from the assumption of balanced factor trade, since it implies that the sums of the second arguments in these correlations are zero and thus have zero means.<sup>6</sup>

These results would be most useful only in cases where the factor content of trade actually is balanced at one of the prevailing sets of factor prices. Unfortunately only with free trade is this likely to be the case:

**Corollary 3.3:** If commodity trade is free and balanced at world prices,  $p^w$ , then

$$(5.3) \quad \text{Cor}\left[\frac{(w_i^f - w_i^a)}{w_i^f}, w_i^f S_i^f\right] \geq 0$$

where  $w^f$  and  $S^f$  are the vectors of factor prices and the factor content of trade that exist with free trade.

**Proof:** With incomplete specialization, so that (2.3) holds with equality, we have

$$(5.4) \quad w^f S^f = w^f A^f T^f = p^f T^f = 0$$

so that factor trade is balanced at free-trade factor prices and we can apply

Corollary 1. **QED**

This is a useful result for the theoretical world of factor price equalization, but unfortunately it does not apply to the more relevant world of impediments to trade. And in the latter world, there is in general no reason to expect factor trade to be balanced even when goods trade is. Tariffs, for example, permit countries to consume bundles of goods that are worth more, at domestic prices, than what they produce, and this in general causes the factor content of their trade valued at domestic factor prices to appear to be in deficit.

---

<sup>6</sup>See Deardorff (1980).



Indeed, it is possible in general for the factor content of trade to be negative for all factors, and it is possible similarly for changes in the factor content of trade to be in the same direction for all factors. In such cases the above corollaries, which rely at a minimum on trade in at least some factors going in opposite directions, are of little use. What is needed is a relationship between factor prices and factor trade that focuses instead on their *relative* magnitudes.

One such result can be obtained by comparing the value of the change in factor content of trade to what it would be if all factor contents were to change in the same proportion to factor consumption. That is, let

$$(5.5) \quad B^2 = L^0 - S^2$$

be the factor content of consumption in the second equilibrium, and compute the average change in factor content of trade relative to that consumption as follows:

$$(5.6) \quad \Gamma = \frac{w^1(S^2 - S^1)}{w^1 B^2}$$

Thus  $\Gamma$  is the change in the factor content of trade between the two equilibria expressed as a fraction of the factor content of consumption in the second equilibrium, both valued at the factor prices in the first equilibrium,  $w^1$ .

Now, as a measure of the relative change in the factor content of trade for each factor, we define

$$(5.7) \quad \Delta_i = w_i^1(S_i^2 - S_i^1) - \Gamma w_i^1 B_i^2$$

That is,  $\Delta_i$  is the difference between the actual change in factor content of trade for factor  $i$  and that which would have occurred instead if all factor trades had expanded or contracted in the same proportion to second-period factor consumption. Obviously, if it does happen to be the case that the change in factor trade is balanced, then  $\Gamma$  is zero and  $\Delta_i$  is just the

value of the change in factor trade itself. But if factor trade is not balanced, involving, say, a net expansion of factor imports, then  $\Delta$  will include both positive and negative elements even if  $(S^2 - S^1)$  does not, and the positive elements will include those factors for which factor imports have expanded the least relative to consumption. Indeed, it follows immediately from the definition that

$$(5.8) \quad \sum_i \Delta_i = 0$$

With this definition of the change in the relative factor content of trade at our disposal, we can now state the most general correlation result of the paper:

**Corollary 3.4:** If factor prices are normalized on the vector of factor content of consumption in equilibrium 2, then there is a nonnegative correlation between changes in relative factor prices and relative changes in the factor content of trade as defined in (5.7):

$$(5.9) \quad \text{Cor}\left[\frac{(w_i^2 - w_i^1)}{w_i^1}, \Delta_i\right] \geq 0$$

**Proof:** In addition to the two equilibria, 1 and 2, being compared, consider a third equilibrium for this economy, numbered 3, with the property that the factor content of consumption is proportional to that in equilibrium 2 but equal in value, at factor prices  $w^1$ , to that in equilibrium 1. That is, define a vector  $S^3$  such that

$$(5.10) \quad L^0 - S^3 = B^3 = \lambda B^2$$

and

$$(5.11) \quad w^1(L^0 - S^3) = w^1 B^3 = w^1 B^1 .$$

Substituting (5.10) into (5.11),  $\lambda$  can be determined as

$$(5.12) \quad \lambda = \frac{w^1 B^1}{w^1 B^2}$$

and thus

$$(5.13) \quad \begin{aligned} S^3 &= L^0 - \lambda B^2 \\ &= L^0 - B^2 + (1-\lambda)B^2 \\ &= S^2 + \frac{w^1 B^2 - w^1 B^1}{w^1 B^2} B^2 \\ &= S^2 + \frac{w^1 (S^1 - S^2)}{w^1 B^2} B^2 \\ &= S^2 - \Gamma B^2 \end{aligned}$$

Using the assumed homogeneity of technology and homotheticity of preferences, (5.10) implies that this new equilibrium has the same prices of both goods and factors as equilibrium 2,

$$(5.14) \quad w^3 = w^2, \quad p^3 = p^2$$

and that consumption is proportional to that in equilibrium 2,

$$(5.15) \quad C^3 = \lambda C^2.$$

Suppose now that the vector of factor content of consumption in equilibrium 2 is used as numeraire. That is, factor prices are normalized so that

$$(5.16) \quad w^2 B^2 = w^1 B^2$$

It follows that

$$\begin{aligned}
(5.17) \quad p^3, C^3 &= p^2, \lambda C^2 \\
&= \lambda w^2, A^2 C^2 \\
&= \lambda w^2, B^2 \\
&= \lambda w^1, B^2 \\
&= \frac{w^1, B^1}{w^1, B^2} w^1, B^2 \\
&= w^1, B^1 \\
&= w^1, A^1 C^1 \\
&= p^1, C^1
\end{aligned}$$

Thus we can apply proposition 3 to a comparison of equilibria 1 and 3:

$$(5.18) \quad (w^3 - w^1), (S^3 - S^1) \geq 0$$

Writing this inner product as a summation, using (5.13) and (5.14), this becomes

$$\begin{aligned}
(5.19) \quad 0 &\leq \sum_i (w_i^3 - w_i^1)(S_i^3 - S_i^1) \\
&= \sum_i (w_i^2 - w_i^1)(S_i^2 - S_i^1 - \Gamma B_i^2) \\
&= \sum_i \frac{w_i^2 - w_i^1}{w_i^1} [w_i^1 (S_i^2 - S_i^1) - \Gamma w_i^1 B_i^2]
\end{aligned}$$

$$= \sum_i \frac{w_i^2 - w_i^1}{w_i^1} \Delta_i$$

From (5.8) and the definition of a correlation, (5.9) then follows.<sup>7</sup> **QED**

## VI. Conclusion

We have already mentioned the fact that our results relate only to relative factor prices, and should not be viewed as indicating which factor prices go up and down absolutely or in real terms. This qualification needs to be repeated and stressed, since there will inevitably be a strong temptation to regard a positive or negative factor content of trade as automatically associated with an absolute gain or loss from trade. But this could be quite misleading.

An example of why it may be misleading arose in another paper, Staiger, Deardorff and Stern (1985), where we used the Michigan Computational Model of World Production and Trade to calculate changes in trade due to trade barriers and then computed the corresponding changes in the factor content of that trade. In one instance we found the factor content of a country's trade to rise for all factors when another country raised tariffs. This was true even though we required balanced trade in our calculations, and it seemed to suggest that all of the country's factors had gained from the foreign tariff. This is not the case, however.

What had happened, as we have already suggested in Section V, was that the foreign tariff had caused a worsening of the country's terms of trade — a fall in the prices of its exports relative to its imports — requiring a rise in the quantities of exports and/or a fall in the quantities of imports in order to keep trade balanced. Thus the factor content of trade increased. But this clearly does not imply a gain for all factor owners, since the worsening of the terms of trade implies that the country as a whole has suffered a loss.

---

<sup>7</sup>See Deardorff (1980).



However, this calculation also illustrates the usefulness of the result put forward in this paper. It is much easier to measure the factor content of trade, and even to estimate how the factor content of trade will change in response to policy, than it is to estimate empirically the complete effects of trade on factor markets. Thus the factor content of trade can be used as a manageable indicator of what the effects of trade and trade policy may be on factor rewards. Even though the effects that are captured are only relative, this can still be a very useful piece of information.

Just how complete this information is depends, clearly, on the nature of preferences and production functions. As Section III indicates, if the world is well approximated by Cobb-Douglas preferences and technologies, then the factor content of trade provides an accurate measure of relative effects on factor prices that holds for each individual factor. On the other hand, if variations in substitutability among factors are important, then only the average relationships of Sections IV and V can be established.

## REFERENCES

- Bowen, Harry P., Edward E. Leamer, and Leo Sveikauskas (1984) "Multicountry, Multifactor Tests of the Factor Abundance Theory," in process (October).
- Deardorff, Alan V. (1980) "The General Validity of the Law of Comparative Advantage," *Journal of Political Economy* 88 (October), pp. 941-957.
- Dixit, Avinash K., and Victor Norman (1980) *Theory of International Trade* (Cambridge University Press, London).
- Harkness, J. P. (1983) "The Factor-Proportions Model with Many Nations, Goods and Factors: Theory and Evidence," *Review of Economics and Statistics* 65 (May), pp. 298-305.
- Helpman, E. (1984) "The Factor Content of Foreign Trade," *The Economic Journal* 94, pp. 89-94.
- Leamer, Edward W. (1980) "The Leontief Paradox, Reconsidered," *Journal of Political Economy* 88 (June), pp. 495-503.
- Leontief, W. (1954) "Domestic Production and Foreign Trade: The American Capital Position Reexamined," *Economia Internazionale* 7.
- Melvin, James R. (1968) "Production and Trade with Two Factors and Three Goods," *American Economic Review* 58 (December), pp. 1248-1268.
- Neary, J. P., and A. G. Schweinberger (1986) "Factor Content Functions and the Theory of International Trade," *The Review of Economic Studies*, 53, pp. 421-432.
- Staiger, Robert W. (1985) "Testing the Predictions of the Heckscher-Ohlin Model," in process (March).
- Staiger, Robert W., Alan V. Deardorff, and Robert M. Stern (1985) "The Effects of Protection on the Factor Content of Japanese and American Foreign Trade," in process.
- Stern, Robert M. and Keith E. Maskus (1981) "Determinants of the Structure of U.S. Foreign Trade," *Journal of International Economics* 11 (May) pp. 207-224.
- Vanek, Jaroslav (1968) "The Factor Proportions Theory: The n-Factor Case," *Kyklos* 4 (October) pp. 749-756.



