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A QUANTITY-CONSTRAINED MACROECONOMIC MODEL WITH PRICE FLEXIBILITY

by

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A QUANTITY-CONSTRAINED MACROECONOMIC MODEL WITH PRICE FLEXIBILITY*

The large fluctuations in output and employment characteristic of modern economies remain a central issue in macroeconomics. Theories which model low employment periods fall into two general classes. One, termed equilibrium business cycle theories, posits continual market clearing in the classical sense, and views fluctuations in employment as movements along an intertemporal labor supply curve. Alternatively, there are theories which postulate that both price and quantity signals clear markets and consider periods of low employment or output in terms of quantity-constrained equilibria. That is, agents perceive constraints on the amount they can sell, which leads them to alter their demands. An equilibrium in *effective* demands is then established.

Though models which attempt to explain the existence of persistent low employment in terms of quantity-constrained equilibrium are widespread in macroeconomics they raise a basic question. If an agent faces a constraint on the amount he can sell (or buy), why doesn't he lower (or raise) the price at which he trades in the attempt to break the constraint? What, for example, prevents prices from falling in response to excess supply? One answer consists of purposely sidestepping the question by taking "sticky" prices as given and proceeding to investigate the implications. Quantity constraints are seen as the *result* of rigid or sluggish prices, with speeds of price adjustment taken as exogenous parameters.

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Even the proponents of such a view admit that this is not really an answer at all. The basic question remains: Are quantity constraints inconsistent with flexible prices? The purpose of this paper is to argue that they are by presenting a framework in which agents are fully rational and are able to lower prices when faced with sales constraints, but where an excess supply equilibrium may be established. That is, it will be demonstrated that the quantity constraints associated with fix-price models can arise in a model where prices are flexible, even when sellers are aware of the trading possibilities they face. It is hoped that the model advanced will not only illustrate why such a result may come about, but also present one possible general framework in which to study questions of price flexibility and unemployment. The model will further indicate why the response to a fall in demand may be largely an adjustment in quantities rather than prices.

I. Walrasian and Non-Walrasian Equilibrium

We want to consider an economy which possesses both a Walrasian equilibrium and a non-Walrasian equilibrium. In the non-Walrasian equilibrium, price is above the firm's marginal cost and the marginal utility of consumption at the going wage is above the marginal disutility of labor for workers. Agents on the long side are free to lower prices, but do not find it optimal to do so.

What characteristics should the two equilibria share? Consider the competitive equilibrium. The central assumption is that agents are price takers, believing they can buy or sell as much as they want at market prices. That is at the going price an agent acts on the assumption that he faces an infinitely elastic demand curve, this assumption being confirmed (in a sense) in equilibrium.

Since a rational competitive equilibrium is one where assumptions of infinite elasticity on the part of agents are confirmed (or, more precisely, self-confirming, since agents base their actions on this assumption and these actions determine, via general equilibrium, their trading possibilities), one may ask whether, for the same tastes and technology, the economy possesses a finite elasticity equilibrium of the above description. That is, is there an equilibrium in which agents formulate demands and supplies for an assumed finite elasticity of demand, and the demand functions which are generated have the conjectured elasticity? Such a solution would be a candidate for a rational non-Walrasian equilibrium.

What aspects of economic decision making are important for the existence of excess supply (or excess demand) equilibria in a world with price flexibility? Several appear important, and a number of somewhat restrictive assumptions are adopted not because they are necessarily more "realistic" than possible alternatives, but because they highlight these aspects of a flexible price non-Walrasian equilibrium.

To begin, the fact that supply of labor does not constitute demand for goods appears crucial in explaining equilibria with excess supply. That is, the hiring of workers and the payment of wages to labor does not insure that the firm will be able to sell the goods that labor produces. I model this by a simple temporal assumption that the labor market clears in the "morning" (that is, at the beginning of the period), but the goods market doesn't clear till the "afternoon" (at the end of the same period), after output has been produced.

Though the nonsynchronous nature of trading is important to an understanding of non-Walrasian equilibrium, one must distinguish this from the constraint that all transactions must use money. Though the constraint that "money buys goods, but goods do not buy goods" is a convenient way of modelling the uncoupling of supply of labor and demand for goods, one should recognize it as just that: a convenient device for studying economies with nonsynchronous trading, but <u>not</u> the underlying cause of this lack of synchronization. A multi-good barter economy would face the same problem. (See Drazen (1980)). Rather than impose a multi-good structure explicitly, I will use money as a third commodity, labor being sold for money in the morning, goods being bought with money in the afternoon. To make clear, however, that money as constraint on transactions is not the cause of effective demand shortfalls, I will assume that the firm has a sufficient quantity of money at the start of the period to allow it to purchase the amount of labor it desires. (One might want to imagine the transaction structure in terms of checks the firm writes on its account which do not clear until after goods are sold.)

As in any model, expectations play a crucial role in determining the nature of the equilibrium. Demand for labor by the firm is derived from demand for output, so that the expectation of an inability to sell output is translated into low demand for labor. Low current income of consumers will imply low current demand, depending on the individual's expectations of future income. To capture this dependence of current decisions on expectations of the future in a simple framework, we consider both firms and individuals as facing a two period decision problem. By use of value functions from dynamic programming this formalism could easily be extended to a multi-period problem. Since the purpose of this paper is simply to prove the existence of a current period non-Walrasian equilibrium and to study its properties, the two-period framework will be satisfactory.

The most important aspect, of course, is the price setting mechanism.

II. Price Setting under Excess Supply Conditions.

To demonstrate that exogenous price rigidity is not necessary for the existence of a non-Walraisn equilibrium, I want to consider an economy in which agents on the long side of the market are able to change prices to break constraints. Consider a situation of aggregate excess supply. By the principle of voluntary exchange, it is assumed that the actual level of transactions is the minimum of quantity demanded and quantity supplied. Hence, if we consider the short side of the market (in this case, demand), the price-quantity combination is a point on the notional demand curve. Since at the market price, the demander can buy the amount he wants, we retain the assumption that on the short side, agents are price takers.

Now consider the long side of the market. In a situation of aggregate excess supply, a seller, though one of many selling the same product, must abandon the competitive assumption that he can sell as much as he likes at the going price.[1] It is reasonable to argue that constrained agents no longer act like price takers. Instead, they become price setters, considering lowering the price they charge in order to sell more.

The price the constrained agent charges will depend on the demand curve he believes he faces. If each seller believes that an infinitesimal price cut will increase quantity demanded by a near infinite amount, individual behavior will, of course, lead to the Walrasian solution. Therefore, if we are to have an equilibrium where price is above marginal cost, sellers must believe they

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[1]The argument was presented by Arrow (1959).

face a downward sloping demand curve. How can this be, if the number of sellers is arbitrarily large? In the output market, the answer depends on the speed with which a single seller gets new customers when he cuts his price. The demand curve the seller faces is the industry demand curve multiplied by the fraction of the total number of customers buying from that seller. We usually argue that the firm's demand curve has close to infinite slope by arguing not only that its fraction of the market is small, but also that a small cut in his price relative to the market price will drive his fraction of total demand to unity in an infinitesimally small length of time. Once we abandon this assumption and assume that customers flow to the firm at a finite rate, a single seller, though infinitesimally small, will act as if he faces a downward sloping demand curve in the short run. Such an insight was the basis for the path-breaking paper of Phelps and Winter (1970), though some of the same ideas are contained in a less rigorous form in Sraffa (1926).

Following Phelps and Winter, we could consider a seller of goods in a situation where there is excess supply at the market price and where the flow of new customers to the firm in response to any price cut is finite within the period. If all customers are identical and if the firm acts on the assumption that prices of other firms are constant, the demand curve he faces can be written as a function of the price he charges

$$x(p) \equiv \mu(p)\eta(p)$$

where μ is the fraction of customers buying at his store in the current period as a function of the price he charges, and $\eta(p)$ is the industry demand curve. (We suppress the prices of other firms.) For simplicity, we may further assume (as in Phelps and Winter) that the "arrival" function $\mu(p)$ is independent of $\eta(p)$ and is known to the firm. Knowledge of the true $\eta(p)$ will translate directly into knowledge of $x(p)$. In summary, a finite slope of $\mu(p)$ within a period implies that an individual seller of output, though one of many, will rationally act as if he faces a downward sloping demand curve for his output.

Why might a similar type of result hold for the labor market in a situation of excess supply of labor? That is, though there may be a large number of homogeneous workers, why might an individual worker act as if he faces a downward-sloping labor demand curve? The situation is, in a certain sense, analogous to that of a seller of goods. For the demand curve facing the individual to have infinite elasticity, a seller of labor must by definition be able to regain employment (or increase hours worked) with an infinitesimal wage cut. This implies (in analogy with the seller of goods) higher employment at the expense of an employed worker, that is, by taking away his job. For this to be true, however, the cost to the employer of making such a "switch" must be zero (that is, less than the infinitesimal wage cut.

In fact, of course, such turnover costs (letting a worker go and replacing him with an identical worker; reducing hours of one worker, increasing those of another identical worker) are non-zero. These costs may be explicit (for example, training) or implicit. More specifically, consider a group of perfectly homogeneous workers. What is being argued is that there is a cost to the firm in treating identical workers unequally--either in loss of morale among workers who receive less favored treatment implying a probable loss in productivity (for example, Rees (1973)), or in a loss in "reputation" among new entrants if the firm is viewed as arbitrary. Shifting a fixed level of total employment from one worker to another identical worker in response to a very small offered wage cut might be seen by workers as arbitrary. The firm's willingness to turnover identical workers will be a function of the differntial between the wage offered by the worker seeking more employment and the market wage for identical workers. The above arguments imply that rather than being perfectly flat (the strictly competitive case), this function might be rather steep.

More rigorously, let $k(w)$ be the fraction of total employment of workers of a given type supplied by a representative worker of that type, as a function of the wage he quotes. (The market wage, which he treats as a constant, is suppressed.) If $n(w)$ is the market demand curve, then the demand curve faced by the individual is $k(w)n(w)$. As long as $k(w)$ has a non-zero slope, the individual worker will face a downward sloping demand curve. If no wage cut in a given range induces firms to turnover identical workers, the demand curve the individual faces will be a constant fraction of the representative firm's labor demand curve. Without loss of generality, this will be the case considered in the model below.

III. A General Equilibrium Model

To capture the above characteristics, the following model is proposed. At the start of the period, the representative firm hires labor from worker-consumers. Based on its expectations of demand for output (where the firm is a price setter), it maximizes expected profits by choosing for any nominal wage, a level of labor demanded (as well as a price it plans to charge, and a level of inventories it plans to carry over to next period). Therefore, in a situation of possible excess supply of goods, the firm announces a labor demand curve, conditional on the output it expects to sell, as a function of the nominal wage. The representative worker-consumer, as a result of his own optimization problem, chooses an optimal wage and, therefore, an optimal level of labor supplied.

The firm pays workers at the start of the period with money, and production takes place. At the end of the period, after output has been produced, the goods market opens. Based on the income he has received and his expectations of the future, the worker announces demand for output, as a function of price. The firm chooses the price that maximizes its profit, workers paying for goods with money balances.

A. Firm Behavior

At the beginning of the period the firm chooses a level of labor demand for each wage to maximize expected discounted profits. This is chosen subject to the production function and the level of product demand expected at the end of this period, as well as expectations of product demand and labor supplied next period. Since my purpose is to prove the existence of an equilibrium in which expectations of this period's product demand are $\eta(p)$ correct, and since it is not an uncertain level of product demand per se which is important for the existence of a non-Walrasian equilibrium, I will consider the case where the firm acts as if it has point expectations of quantity demanded it expects at each price.[2] This simplifies the problem and the definition of equilibrium without changing the basic results. In this case, the firm chooses, conditional on the demand curve it expects, a price it plans to charge, a level of inventories to be carried over, and a level of labor demand, all as functions of the wage. Mathematically, we have the beginning of current period problem of choosing actual n (and planned p and i) to

[2]Zabel (1970) considers a monopolistic firm with known costs but uncertain demand and compares its decisions to a monopolist facing a known demand curve. In the case where the expected value of demand at each price is equal to the known demand curve, the price-output combination chosen under uncertainty is not equivalent to that chosen under certainty. Certainty equivalence is not an adequate way to introduce monopoly. I will consider the case of more general uncertainty in a future paper. Obviously, our definition of equilibrium would have to be revised.

maximize

$$px - wn + \frac{1}{R} E(\hat{p}\hat{x} - \hat{w}\hat{n}) \tag{1}$$

such that

$$x + i = f(n) \tag{2}$$

$$i + f(\hat{n}) = \hat{x} \tag{3}$$

where

x = current period demand for output (a function of p)

$= \mu(p)\,\eta(p)$

p = current period price the firm plans to change (a function of w)

w = nominal wage in the current period

n = demand for labor (hours) in the current period (a function of w)

i = inventories of output the firm plans to carry over

R = interest factor

with circumflexed variables ("^") representing the second period analogues. The firm takes prices of other firms as given, and these are suppressed in the functional notation.

We assume the firm holds a stock of money M_0 at the start of the period (all prices are money prices), and that M is sufficiently large to finance the firm's purchases of labor. (Since prices are determined up to a factor of proportionality, such an assumption is not restrictive.) Any money balances in excess of M_0 at the end of the period (which will be shown to be equal to profits) will be paid out to owners of the firm.

The firm's maximization problem yields first-order conditions for the current period variables

$$x + x'\left(p - \frac{w}{f'(n)}\right) = 0 \tag{4}$$

$$\frac{w}{f'(n)} = \frac{1}{R}\,E\left(\frac{\hat{w}}{f'(\hat{n})}\right) \tag{5}$$

$$x + i = f(n) \tag{2}$$

which yield (conditional on $x(p)$ and on the distributions of future variables), n, i, and p for all values of w. (To conserve on notation, we will usually leave implicit the functional dependence of the variables on w, denoting specific values by p^* or n^*.)

Since the firm must hire labor at the start of the period, it announces its labor demand function $n(w)$. Workers will then choose an optimal n^*, which yields a level of hours hired n^*. The firm pays out nominal balances w^*n^*.

At the end of the period, the firm has an amount of product $f(n^*)$ on hand, to be divided between current sales and inventory. If the demand curve the firm anticipated is the actual demand curve, it will charge the price it planned p^* and sell what it planned $x(p^*)$. The condition that the firm's expectations about the demand curve are correct is the condition for equilibrium. More specifically, the firm's expectations about $\eta(p)$ yield an $x(p)$ which yields, in turn, a function $n(w)$ at the start of the period, which, via the worker-consumer's decision problem, yields an actual demand curve $\eta(p)$ at the end of the period. An equilibrium in this model will be an $\eta(p)$ which reproduces itself.

Let us now turn to the consumer side.

B. Consumer Behavior

At the beginning of the period, the worker-consumer faces a known demand curve for labor (hours), $n(w)$. This implies a downward-sloping demand function for his own labor, by the arguments in Section II. As long as $k(w)$ (fraction of total labor to the representative firm supplied by an individual worker) is not flat, the basic qualitative results do not depend on its exact slope. For convenience we will therefore assume k is constant and act as if the representative worker faces the representative firm's demand curve $n(w)$. He must therefore choose an optimal labor supply n^*, yielding a level of income w^*n^*, without knowing what the market price of consumption goods, p, will be at the end of the period. He chooses n^* by choosing the optimal w along the labor demand curve. He must also choose a consumption function $c(p)$, telling how much output he will buy at the end of the period at any price p. Both the consumption function and the price are announced only at the end of the period. The consumer is a price taker in output, the firm choosing the market price p^*.

The consumer has a two period utility function, defined over consumption and labor in each period.

$$U(c, n) + \beta U(\hat{c}, \hat{n}) \tag{6}$$

where U is this period's utility and $\beta U(\hat{c}, \hat{n})$ is the discounted value of next period's utility. The budget constraints are

$$pc + S = wn \tag{7}$$

$$RS + \hat{w}\hat{n} = \hat{p}\hat{c} \tag{8}$$

where S is nominal saving and, as before, variables with a circumflex represent the future period values. He chooses a single value for hours supplied[3] and a demand function for consumption, given distributions of future period variables to maximize constrained expected utility. Therefore let $W(\cdot)$ be the expected discounted value of next period's utility, this expectation taken over future $\hat{p}$ and future labor demand curves. Let $f(p)$ be the subjective distribution function of this period's goods price, as seen from the start of the period (needed for choice of n^*), and let $w(n)$ be the inverse demand curve.

The choice of $c(p)$ and n^* may be written as if it were a two-stage process, choosing optimal $c(p)$ for any value of n, and then choosing optimal n. We write the maximization problem as

$$\underset{n}{\text{Max}} \int_p \left[\underset{c(p)}{\text{Max}} \left\{U(c, n) + W\left(\frac{R(w(n) \cdot n - pc) + \hat{w}\hat{n}}{\hat{p}}, \hat{n}\right)\right\}\right] df(p) . \qquad (9)$$

The first order condition for $c(p)$ is then simply

$$\frac{U_c}{p} = R\, W_c \qquad (10)$$

which gives $c(p)$ for any n (where subscripted U and W indicate relevant partial derivatives). The first order condition for n is

$$E_p\left(\frac{U_c}{p}\right) \cdot \left[w + n \frac{dw}{dn}\right] = -E_p\, U_n \qquad (11)$$

where E_p is the expectation taken over p.

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[3]It should not be difficult to modify the model to interpret n as number of workers employed (rather than hours per worker) so that the model generates actual unemployment. $n(w)$ is the number of people hired at each wage. An individual worker faces a probability of employment schedule in that cutting his wage offer increases the probability that he will be employed. The firm, facing a wage w^*, chooses at random which workers will be employed. In an equilibrium which reproduces itself period after period, the worker chooses the w^* which maximizes (for example, for the case where $U_n = 0$) the expected value of labor income.

Solving (10) and (11) yields an optimal n (call it n^*) and $c(p)$ for expectations of future variables (including maximized future income). Hence, at the beginning of the period the consumer, facing a labor demand curve $n(w)$, chooses an optimal wage w^* implying sales of n^* to the firm, for which he receives money balances w^*n^*. Let us denote this level of labor income by y. He buys goods at the end of the period with these money balances. The consumer's decision problem may be seen as taking a function $n(w)$ into a function $c(p)$ (which is "announced" only at the end of the period).

C. Profit-Based Demand and Equilibrium

To close the model, we consider the profits paid out to owners of the firm. Such profits induce demand for output. To keep the model simple, we assume for now that profit-based demand is at a constant nominal level Z. The only restriction we will place on Z is that, for at least some values of the future variables, Z is such that a Walrasian equilibrium exists. Constant nominal Z is clearly a strong assumption. More complicated formulations do not change the basic results, but make the model much less tractable. (We will consider different formulations below). Given Z, total real demand for output becomes

$$\eta(p) = c(p) + \frac{Z}{p} . \tag{12}$$

The firm chooses the p which maximizes its profits for the actual $x(p)$ which arises from $\eta(p)$. If its assumption about the demand curve was correct, the price it actually charges on observing the demand curve is the price it planned to charge at the beginning of the period, p^*. If it sells an amount $x(p^*)$, it receives money balances M_1 equal to $p^*x(p^*)$. We assume the firm is constrained to end the period with the same level of money balances with which it began the day, the excess $M_1 - M_0$ being turned over to its owners. It is easy to see that this excess is just equal to current profits $px - wn$.

Since the firm's decision at the beginning of the period takes a conjectured output demand function $\eta(p)$ (yielding an $x(p)$) into a labor demand function $n(w)$ and the individual's decision takes $n(w)$ into $c(p)$, and therefore $\eta(p)$, a within-period rational equilibrium is an output demand function that reproduces itself. More formally we may define an equilibrium as:

DEFINITION: A within-period rational expectations equilibrium is a demand function $\eta(p)$ such that

$$\eta(p) = c(p;\ n(w^*;\ \eta(p)),\ w^*) + \frac{Z}{p}$$

for a given value of Z and distribution of future period variables $\hat{x}(\hat{p})$, $\hat{n}(\hat{w})$.

The $c(\cdot)$ function in the definition represents the dependence of $c(p)$ on $n(w)$ which, in turn, depends on $\eta(p)$. The actual decision problem is embodied in the firm's first-order conditions (2)-(5) and the consumer's first-order conditions (10) and (11).

For a given aggregate demand curve and associated individual demand curve, the representative firm chooses a price, conditional on the price other firms are charging. The individual firm's price cannot of course diverge from market price indefinitely. If it did the firm would either have to capture the whole market or disappear. Does this imply that the only single price equilibrium is the competitive one? It does not, as Phelps and Winter show rigorously. As long as customers flow to firms at a finite rate, a long-run single price equilibrium can be one with price above marginal cost. For intuition as to how this can be, consider the case where new customer flow is extremely slow, but the discount rate is high. Consider a short-run equilibrium where all firms charge the same price but price is above marginal cost. Why doesn't an individual firm shade its price by a small amount, trading off a short term loss for (possibly infinite) higher profits in the long run? Since customers flow to the firm very slowly the present discounted value of future profits may be smaller than the short-run cost. Phelps and Winter in fact show that the ratio of price to marginal cost in a long-run equilibrium will depend on customer arrival rate and the discount rate, and a range of values is possible. For our purposes, the Phelps and Winter paper insures the existence of a long run excess supply equilibrium if the short-run curve faced by the individual has a finite slope. Hence, we will concentrate on showing the existence of a self-replicating $\eta(p)$.

Before considering the existence of a non-Walrasian equilibrium in this price setting format, let us examine the competitive or Walrasian equilibrium which would arise from the same tastes and technology. The competitive firm is a price taker. Let us assume it knows current period prices with certainty, but not necessarily next period's prices. The firm maximizes (1) in the current period subject to (2) and (3) by choice of x, n, and i, treating p and w as given. This would yield

$$\frac{w}{p} = f'(n) \tag{13}$$

$$\frac{w}{f'(n)} = \frac{1}{R} E\left(\frac{\hat{w}}{f'(\hat{n})}\right) \tag{14}$$

$$x + i = f(n) \tag{2}$$

Not surprisingly, this is what the model above would yield if x' were equal to ∞ at some p.

Similarly, let us consider a price-taking consumer who must choose a level of labor supplied and consumption, facing known current period wages and prices, though next period prices may be uncertain. Choice of current c and n to maximize a two period utility function such as (6) subject to the same budget constraints would yield (for given w and p)

$$-\frac{U_n}{U_c} = \frac{w}{p} \qquad (15)^4$$

$$\frac{U_c}{p} = RW_c \,. \qquad (16)$$

As with firm behavior, the first-order conditions for the consumer in the competitive case are what the above model would yield if the representative worker-consumer faced an infinitely elastic demand for labor curve. Simultaneous solution of the competitive first-order conditions will yield a Walrasian equilibrium. (For future reference, let us denote the current period variables in this equilibrium by a superscript o, as in, for example, p^o.)

The Walrasian equilibrium can be interpreted as the equilibrium which would obtain, for these tastes and technology, when individuals act on the assumption of infinite elasticity of demand for the products they sell, and find they can in fact sell what they wish in equilibrium. The non-Walrasian equilibrium is one where sellers' actions are based on assumed finite elasticity of demand for the products they face, and these assumptions are correct.

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[4] If we maintained the same temporal structure as above so that the individual would choose n at the beginning of the period knowing w but not p, (15) would become

$$\underset{p}{E}\left(\frac{U_c}{p}\right) \cdot W \equiv - \underset{p}{E}(U_n) \qquad (15')$$

which would parallel (10).

We have now completed the specification of a simple framework in which price flexibility may be made consistent with constraints on sales, even when expectaions are rational. The existence of a non-Walrasian equilibrium does not depend on either mistaken expectations or on the impossibilty of agents on the long side changing prices. We will therefore want to consider the characteristics of a non-Walrasian equilibrium with the aim of understanding why in fact such an equilibrium might obtain. We will further want to consider how this framework might be useful in explaining why economies experience large fluctuations in output and employment and (relatively) small fluctuations in wages and prices. Before doing this, however, the actual existence of a non-Walrasian equilibrium should be demonstrated formally.

IV. The Existence of a Non-Walrasian Equilibrium

To show how price flexibility may be made consistent with non-Walrasian equilibrium, we must demonstrate the existence of a rational non-Walrasian equilibrium where prices are set by the long side of the market. The sequence of firm and consumer decisions in Section II defines a mapping from a finite elasticity output demand function perceived by the firm to an actual output demand function. A within-period rational expectations equilibrium was therefore defined as an output demand function $\eta(p)$ which, if perceived by the firm, reproduces itself. That is, the sequence of decision problems leads to an actual demand function identical to that perceived by the firm. The existence of a non-Walrasian equilibrium is the existence of such a self-reproducing $\eta(p)$.

The general idea of the proof is as follows. Consider the individual choice problem. A consumer with given expectations about the future, who faces a labor demand curve $n(w)$, chooses a utility maximizing level of labor supplied n^*, an associated wage w^* (both scalars), and a consumption function $c(p)$. We will show that for a given pair (n^*, w^*) arising from the maximization problem, there is associated with it a unique $c(p)$ and, therefore, for given Z, a unique $\eta(p)$ In this sense, $\eta(p)$ may be indexed by the ordered pair (n^*, w^*), and there corresponds to the mapping taking $x(p)$ into itself a mapping taking values of (n^*, w^*) into (n^*, w^*). By demonstrating the second mapping has a fixed point, we will therefore have demonstrated that the original mapping has a fixed point, which is equivalent to the existence of a non-Walrasian equilibrium.

Consider first the association of (n^*, w^*) and $x(p)$. A consumer has given expectations of next period's variables at the beginning of the period when he faces a labor demand curve $n(w)$ and an interest factor R. Via simultaneous solution of the first-order conditions (10) and (11) and the budget constraints (7) and (8), expected utility maximization yields an n^*, an associated wage rate w^*, and a consumption function $c(p)$ (as well as a saving function $S(p)$). Now, suppose two labor demand curves are such that under utility maximization they yield the same (n^*, w^*). Since the utility function is continuous, the first-order conditions will clearly yield the same c (for a given p) in each case. In other words, over all p, there is a unique $c(p)$ associated with a given (n^*, w^*). For a given value of Z, entrepreneurial nominal demand for output, there corresponds to $c(p)$ a total output demand function $\eta(p)$. In short, to any pair (n^*, w^*) which could arise from expected utility maximization for some $n(w)$, there corresponds a unique $\eta(p)$. If we denote by σ the pair (w^*, n^*), we may write this association as $\eta(p\colon \sigma)$. One must of course remember that $\eta(p\colon \sigma)$ does <u>not</u> mean that $\eta(p)$ is a function of σ in the economic sense (since σ and $c(p)$ are chosen simultaneously), but only that there exists a unique association.

This association between $\eta(p)$ and σ can then be thought of as defining a mapping of σ into itself corresponding to the mapping of $\eta(p)$ into itself defined in Section II. That is, since the sequence of decision problems takes an output demand function conjectured by the firm into an actual output demand function, it takes the value of σ associated with the conjectured $\eta(p)$ into the value of σ associated with the $\eta(p)$ actually faced by the firm at the end of the period. Let us call this mapping $A: \sigma \rightarrow \sigma$. The set over which A is defined, call it Σ, is the set of all (n^*, w^*) which could arise from utility maximization for some $n(w)$. That is, to say that an arbitrary σ_j is contained in Σ is to say that there exists some $c(p)$ (and therefore $\eta(p)$ which could arise from utility maximization whenever σ_j does.

To show that $A: \sigma \rightarrow \sigma$ has a fixed point, we present the following two lemmas, the proofs of which are presented in an appendix.

Lemma 3.1: Σ is compact and convex in E^2.

Proof: See Appendix.

Lemma 3.2: $A: \sigma \rightarrow \sigma$ is continuous.

Proof: See Appendix.

Since $A: \sigma \rightarrow \sigma$ is then a continuous mapping taking a compact, convex set in E^2 into itself, it has a fixed point, call it $\hat{\sigma}$.

The existence of a non-Walrasian equilibrium follows immediately from the existence of a fixed point $\hat{\sigma}$ for A and from the correspondence to each σ in Σ of an $\eta(p)$. More formally, we have

Proposition 1: The economy described in Section II has an equilibrium for given expectations of second period variables and Z.

Proof: Lemmas 3.1 and 3.2 guarantee the existence of a fixed point for $A: \sigma \to \sigma$ and the definition of A means that there corresponds to the fixed point $\hat{\sigma}$ an output demand function $\eta(p)$ which "reproduces" itself under the sequence of decision problems in Section II. The existence of such an $\eta(p)$ is equivalent to the existence of a rational non-Walrasian equilibrium.

V. Properties of the Non-Walrasian Equilibrium

We are now in a position to consider the characteristics of the non-Walrasian equilibrium of the economy in Section III comparing it to the Walrasian equilibrium, to discover why conceptually such an equilibrium exists. To begin, let us consider the sense in which there are quantity constraints on agents' sales. At the going price and wage firms would be willing to sell more (as price is above marginal cost) and workers would be willing to supply more labor (as the marginal disutility of labor is below the marginal utility of consumption). One may argue that this description can be applied to a regular monopolist facing a downward sloping demand curve. There is a crucial difference between the regular monopolist's situation and the situation of agents in this model. When the market is characterized by a small number of sellers, given tastes and technology admit only the monopolistic solution. One could then describe the desire to sell more without lowering prices as unrealistic.

The situation of sellers in the non-Walrasian equilibrium is not fully analogous. Since the model is one of inherent perfect competition, a Walrasian equilibrium exists. Therefore, not only do agents want to sell more at the going price (or, of course, a higher price) in the non-Walraisn equilibrium, but there, in fact is an equilibrium where they can sell more, for unchanged tastes and technology. (The question, of course, is how to reach it.) The possibility of an equilibrium where agents can sell more at a higher price (and of course the knowledge it exists) means that the desire to sell more is not unrealistic in the sense that it was for the monopolist.

Non-Walrasian equilibrium clearly does not depend on incorrect expectations. By the defintion of equilibrium, it is a rational expectatations equilibrium. Sellers are correct about the demand curve they face, and, therefore, fully aware of available trading possibilities.

Nor is non-Walrasian equilibrium due to the inability of agents to cut prices to break constraints. This is stressed simply because a major goal of this paper is to show how fix-price results could be consistent with flexible prices. Here, optimizing agents could lower prices to the level of marginal costs, but do not find it optimal to so.

In a very basic sense an equilibrium with price above marginal cost depends of course on sellers facing downward sloping demand curves. Though this sounds straightforward, it leads to two insights about the existence of non-Walrasian equilibrium. First, whenever suppliers face other than an infinitely elastic demand curve, their response to an inability to sell all they want at the market price will be partially a decrease in demand for inputs. In other words, what are called "quantity spillovers" in the fix-price literature are fully consistent with price flexibility, as long as the demand curve facing the individual has some finite elasticity. In this paper, such spellovers lead to the perceived demand curve replicating itself. The firm's perceptions are not simply justified, but, in fact, self-justifying.

Second, there are a number of reasons why we might expect an individual seller, even in a market with a large number of suppliers of the same product, to believe rationally that he faces a downward sloping demand curve for his own output. Slow flow of price information for goods, a reasonable unwillingness of firms to treat identical workers differently in response to small offered wage changes for labor were reasons put forward here. These are clearly not exhaustive. What is argued here is that finite elasticity demand curves to the individual may be sufficient (in the sense argued in the previous sections) in a model where agents are correct in their beliefs to yield an equilibrium where actual supply is less than notional supply. That is, rational perceptions of finite elasticity demand curves to the individual can be shown to yield a quantity constrained equilibrium even when constrained agents can set prices. Though some may argue that such a possibility is obvious on a conceptual level and has been often suggested, the purpose here was to formalize one's intuition and make it rigorous.

One other aspect is important in explaining the existence of non-Walrasian equilibrium, and in further clarifying the possibility of a Walrasian solution. This is the constraints which the market puts on the signals agents can send. In the non-Walrasian equilibrium the problem is not that prices are too high, but that expectations and income are too low. An increase in labor demand in aggregate (to the point where workers perceive no constraints on desired sales) is necessary, but no individual firm can send this message.

To understand why, consider a model where the existence of many sellers is explicit. If a single firm believed it could sell as much as it wanted at the current price, it would increase its demand for labor to equate price and marginal cost. This would lead to higher labor income and higher total demand for output. However, only a fraction of the increase in sales generated would return to the firm. The increase in income due to the single firm's actions may be seen as generating positive externalities. If all firms acted on the assumption that there are no sales constraints and put no quantity constraints on demand for labor, the economy could conceivably reach the Walrasian equilibrium. In a decentralized market, there is no incentive for an individual firm to act in this way, and no signal they can send indicating a desire for such action to be taken.

VI. Quantity and Price Fluctuations

The model presented so far indicates why, even if prices are flexible, a fall in demand for product (or labor) may not induce a fall in prices or wages sufficient to restore sales to their previous level. A seller facing a fall in the demand for the good he sells will respond partially by cutting demand for goods he buys and partially by cutting the price he charges. A further question concerns the relative magnitudes of these two responses. Why might a fall in demand be met largely by quantity adjustments rather than price cuts? Since business cycles display relatively large fluctuations in output and employment and relatively small fluctuations in prices and wages, this question is central.

The low amplitude of price fluctuations suggests low elasticities of the demand curves faced by sellers. Exogenously specifying low elasticities, however, gives little insight into price sluggishness and is really only a small step away from exogenous price rigidity.

The model presented here yields a far stronger result--there is an endogenous relationship between the level and the elasticity of demand for product (or labor). More specifically, the decision problems set out in Section III imply that the <u>price derivative</u> of the demand function for either labor or output increases (or decreases) as the <u>level</u> of demand increases (or decreases). This implies that a firm which experiences a fall in demand for its output perceives at the same time, a reduced incentive to cut prices in the attempt to restore sales, because the responsiveness of demand to price cuts has simultaneously fallen. Similarly, a worker facing a fall in demand for labor finds wage cuts of limited effectiveness as the elasticity of demand for labor has simultaneously fallen.

Consider first consumer behavior. An inward shift in this period's labor demand curve (or in next period's expected labor demand) will cause a fall in current (expected future) income. This fall in income will be associated with not only a fall in current consumption demand at each price level p, but also an increase in the slope of the consumption demand curve (that is, a fall in the absolute value of the derivative of consumption demand with respect to price). More formally we have

> Proposition 2: An inward shift in the current or the expected future labor demand curve (meaning a fall in current or expected future income) causes consumption at each price to fall. Moreover, if the utility function is such that saving increases with uncertainty about future income (which holds if the utility function displays decreasing relative risk aversion), then a fall in income will cause the consumption function to become steeper (that is, will cause $|\frac{\partial c}{\partial p}|$ to fall.)
>
> Proof: See Appendix.

The first result--that consumption falls as income falls--is not very surprising. The fall in the price derivative of consumption with a fall in income perhaps is. It is well known that in a two-period framework saving will will fall as future income becomes more risky, if the utility function displays decreasing relative risk aversion (Rothschild and Stiglitz (1971)). How is this related to price elasticity? An intuitive explanation runs as follows. Since a fall in income increases the coefficient of risk aversion, the individual requires a larger risk premium to undertake a given risk. Because future income is risky, increasing consumption today (thereby decreasing saving) increases risk in future consumption streams. Since a fall in the level of income increases the coefficient of risk aversion, the individual requires a larger risk premium to undertake a given risk. Therefore a larger decrease in price (which may be seen as a risk premium in this case) is necessary to induce an individual to increase his current consumption by a fixed amount.

Based on this result, a similar result for firm behavior and the demand for labor curve in this model may also be derived. A fall in the level and elasticity of demand for output will induce a fall in the level and elasticity of demand for labor by the firm. More formally, we may derive

Proposition 3: Let us index the demand for output curve to the firm $x(p)$ by a shift parameter α, so that an increase in α indicates an outward shift of the product demand curve and let x_p indicate the derivative of x with respect to p. Since by Proposition 2, we have that $\partial|x_p|/\partial\alpha > 0$, the following results hold:

i) The price the firm charges increases as the demand curve shifts out, unless the contemporaneous change in the elasticity of demand is large. ($\frac{dp}{d\alpha} > 0$, unless $\frac{\partial|x_p|}{\partial\alpha}$ is large.)

ii) The larger is the contemporaneous increase in elasticity, the smaller is the increase in price the firm charges in response to a higher level of demand. (The larger is $\frac{\partial|x_p|}{\partial\alpha}$, the smaller is $\frac{dp}{d\alpha}$.)

iii) Labor demand at each wage rises with an increase in output demand, the magnitude of the increase being larger the smaller is the price response to a shift in output demand and the steeper is the output demand curve. ($n(w)$ shifts out with increases in α, this effect being larger the smaller are $|x_p|$ and $\frac{dp}{d\alpha}$.)

iv) The derivative of the labor demand curve with respect to the wage is negative and rises in absolute value with α. An alternative way to put this is that $\frac{\partial n}{\partial w}$ rises with x_p.

Proof: See Appendix.

Proposition 3 characterizes the effects of shift in the price-setting firm's demand curve. Consider an adverse shift (a fall in α). The firm will lower the price it charges. However, if the price responsiveness of demand x_p falls as the level of demand falls, the price-cutting response of the firm will be lessened. For a firm facing a finite elasticity demand curve, the fall in demand for product will be partially reflected in a fall in input demand. The smaller the response of the firm in cutting output price (due to low elasticity of product demand), the larger their response in cutting input demand. Moreover, the fall in the level and price sensitivity of output demand will be reflected in a fall in wage sensitivity of labor demand. In other words, in addition to $x(p)$ and x_p moving in the same direction, so do $n(w)$ and $\frac{\partial n}{\partial w}$. As the demand curve for labor shifts in, it becomes steeper.

Some rough intuition for the final result is as follows. How much the firm will increase the quantity of labor demanded for a fall in the wage will obviously depend on the value to the firm of the output produced. This output has two "uses": current sales and inventory. If the firm's sales prospects worsen and if the value of inventory is concave in inventory, the firm clearly requires a larger fall in wage to induce it to increase output by the same amount. A lower input cost will of course induce the firm to hire more of the input and then cut its price to sell the increase output. The larger the price cut "necessary" to sell the increased output, the larger the wage cut necessary to induce the firm to hire more labor.

The general intuition of these results should not be surprising. A firm which perceives an inability to sell output (and cannot hold inventory costlessly) will require a large incentive (in terms of lower wages) to increase hiring of labor. An individual who because of anticipated low income in the future increases his saving today will need a larger incentive than normally (in terms of a fall in today's price level relative to tomorrow's) to increase current consumption.

These results suggest why economic cycles may be characterized by quantity rather than price adjustments. The larger is the simultaneous fall in the price (wage) derivative of the output (labor) demand curve when the level of demand falls, the more this will be reflected in quantity movements on both output and input sides. Partial empirical support may be found in Hamermesh and Obst (1977) who show that, for a number of types of workers, the absolute value of the elasticity of demand for labor falls as unemployment rises. Hence the model appears capable of replicating the broad empirical facts about cycles in terms of relative amplitudes of price and quantity movements, and the road by which such relative amplitudes are explained appears to have empirical validity.

One further hopes that it can give insight into the driving forces of business cycles. Two lines of research suggest themselves. First, expectations are crucial here in explaining the level of demand for output and labor. By considering various mechanisms by which expectaions might change over time, one should be able to generate a business cycle.

An alternative source of cycles is the profit-based output demand function Z. We assumed for convenience that Z was fixed. A more realistic formulation would be to allow Z to vary with changes in the level of profits. If the model were extended to include capital and Z were interpreted as investment demand, the dependence of Z on profits appears particularly attractive, for it gives a linkage between two of the empirically most volatile series observed over the cycle. For a reasonable investment demand function, the model should be able to generate cycles in output and employment.

VII. Some Observations on Governmental Policy

What implications does this model have for government policy? Two somewhat interesting conclusions may be drawn. The first is that any rationale for government policy may be seen strictly in terms of the externalities inherent in private behavior. The second concerns one form of optimal government policy and its apparent similarlity to pre-Keynesian notions of optimal policy.

To determine if there is a role for government policy, one must understand what is happening in the non-Walrasian equilibrium. As was discussed previously, the problem is insufficient demand due to low sales or income on the part of demanders. Though a sufficiently large increase in labor demand above the non-Walrasian level by all firms would repay itself in the form of higher sales, no individual firm perceives such an incentive. Its wage bill would rise without a concomitant increase in its sales, since (even if such a move were consistent with equilibrium) only a fraction of the higher demand induced would accrue to the initiating firm. Actions which increase demand by loosening the constraints on income which demanders face can therefore be thought of as public goods, the social benefit outweighing the benefit to the individual firm. Conversely, a firm's actions in reducing demand for labor in response to reduced demand for its output may be thought of as generating negative externalities. The rationale for government policy is then very simple: Are there actions which could, in some sense, offset these negative externalities?

Two sorts of policy could be considered. The first is straightforward. A given level of demand for output by government acts like an increase in Z, increasing equilibrium output and employment. If the government runs a current deficit to finance its purchases (rather than using taxes to reduce current private sector income), such spending is a public good (no pun intended), the benefits of which accrue to all sectors.

A comparison of the Walrasian and non-Walrasian equilibria indicates, however, that such countercyclical spending is, in a sense, unnecessary and suboptimal. Suppose the government announced that it stood ready to buy whatever output firms would wish to supply at a price p^0 and employ whatever laborers could not find work in the private sector at a wage w^0. Since firms perceive infinite elasticity of demand at p_0 and face a given market wage w_0, they will be induced to demand the Walrasian level of labor supplied, n^0. Workers, facing p^0 and w^0, will want to supply exactly n^0 (absolving the government of the need to make good on its promise to be employer of last resort) and demand x^0, the amount firms want to supply (absolving the government of the need to purchase output). Hence, by announcing its willingness to be a demander at the Walrasian prices, the government restores the private expectations necessary to support the Walrasian equilibrium, and, unlike the previous case, need make no purchases.

Though the policy experiment suggested may sound a bit whimsical, there is something to be learned. In a model such as this, varying government purchases to move from one non-Walrasian equilibirum to another can increase welfare, but is really a second-best policy. What the government really wants to do is to provide a framework which ensures that the private sector is able to reach the Walrasian equilibrium. The shift in focus for government action from continual adjustments in policy to provision of an environment conducive to attainment of the Walrasian equilibrium hearkens back to pre-Keynesian business cycle theory.

VIII. Conclusions

This paper had two basic aims. One was to address the central criticism of a wide class of macroeconomic models: why don't prices adjust in the face of apparent quantity constraints? To answer this question, I presented a model which makes price flexibility consistent with constraints on sales. Agents on the long side of the market are price setters. They are able to lower prices in the face of a fall in demand for their product but do not find it optimal to lower prices sufficiently to counteract the fall in demand completely. In the equilibrium which obtains the behavior of optimizing agents is fully rational and prices clear markets. At the same time however, demand is a constraint in the sense that at the going price agents would like to sell more and there exists and equilibrium where this is possible.

The second aim was to consider why in such a model prices might fall relatively little in response to adverse demand shifts. It was shown that as demand falls, the elasticity of demand also falls. Therefore, sellers may view price cuts as relatively ineffective in periods of low demand.

There probably are other ways of introducing price determination into this type of model. I chose this route not because I felt it was the only reasonable one, but only because it appeared both sensible an tractable. There are also a number of ways to specify the structure of production relations. Once again, the structure I chose was sensible but simple, allowing one to isolate precisely what is happening at the non-Walrasian equilibrium.

Several interesting papers deserve mention here. Hahn (1978, among others) has considered what he terms conjectural equilibria, where agents have conjectures about the prices at which they must trade to break constraints. In equilibrium, the price which they believe they must offer to execute their actual trade is the market price. However, agents' perceptions of demand curves are not correct globally in that model. Neary and Stiglitz (1979) consider the effects of current wage-price flexibility when agents anticipate constraints in the future and future prices are not flexible. Though current wage price flexibility will not guarantee full employment, the existence of a constrained equilibrium in their model does depend on some price being rigid. Therefore, the reasons why price flexibility does not ensure Walrasian equilibrium are rather different than the ones presented in this paper. Hart (1979) considers a model where there are a large number of buyers and sellers but where an excess supply equilibrium obtains, due to agents having some monopoly power in setting prices. Though close in spirit to this paper, there is a crucial difference. In the Hart paper, monopolistic price setting arises because of exogenous factors which make markets monopolistic in the usual sense of a small number of sellers. On the output side, there are a large number of distinct markets, each with a low ratio of buyers to sellers. The strict separation of markets is simply assumed. On the labor side, workers form labor syndicates giving them monopoly power. The existence of such syndicates is also simply assumed. This is in contrast to this paper, where price setting does not derive from explicit monopolistic considerations, and a Walrasian equilibrium exists.

The next step is to generate and investigate business cycles in this framework. A basic test of the model--its ability to replicate observed patterns of the phenomenon to be studied--indicates that this should be a worthwhile undertaking.

More generally, one hopes that this model will provide a possible, theoretically robust framework for the study of quantity-constrained macroeconomic equilibrium. It suggests one way to move beyond fixed priced models. I do not want to argue that this framework is "better" (whatever that means) than the Lucas-Sargent equilibrium paradigm. It represents, however, a step in trying to make quantity-constrained macroeconomic models of similar conceptual rigor to those models. And this is a step which adherents of both approaches agree is necessary.

APPENDIX

We here consider proofs of various lemmas and propositions in the text.

1. Lemma 3.1: Σ is compact and convex in E^2.

Σ is the set of all (n^*, w^*) pairs which could arise from consumer utility maximization for some labor demand curve given productive technology. There is clearly a maximum level of labor supplied. Call this level $\bar{n}$. Since n is bounded, $f(n)$ must be bounded. Together these imply that the wage w must be bounded. Σ is therefore bounded.

For each value of $n \in [0, \bar{n}]$ there is a maximum and a minimum value of w which could arise from the consumer's choice problem. Calling these $\bar{w}^*(n_j^*)$ and $\underline{w}^*(n_j^*)$, these are the boundaries of Σ and Σ is closed. Therefore Σ is compact.

For convexity, consider $\sigma_0 = (n_0^*, w_0^*)$ and $\sigma_1 = (n_1^*, w_1^*)$ both contained in Σ. In other words, there exist demand for labor functions $n(w)$ consistent with technology which would yield σ_0 and σ_1 under utility maximization. There should therefore be some "intermediate" $n(w)$ which will yield an intermediate value of σ, namely $(\lambda n_1^* + (1 - \lambda)n_0^*, \ \lambda w_1^* + (1 - \lambda)w_0^*)$ for $0 \leq \lambda \leq 1$.

2. Lemma 3.2: $A: \sigma \to \sigma$ is continuous.

We want to show that if two points σ_0 and σ_1 are close, then $A(\sigma_0)$ and $A(\sigma_1)$ are close. (For a metric, consider $d(\sigma_0, \sigma_1) = \max(|n_0^* - n_1^*|, |w_0 - w_1|)$.) The first step is to show that σ_0 close to σ_1 implies that $\eta(p: \sigma_0)$ is close to $\eta(p: \sigma_1)$. Suppose a consumer would choose the set $(\sigma_0, c(p: \sigma_0), S(p: \sigma_0))$ and if he faced a labor demand curve $n_0(w)$ and would choose $(\sigma_1, c(p: \sigma_1), S(p: \sigma_1))$ if he faced a curve $n_1(w)$. What is being claimed is that if $n_0(w)$ and $n_1(w)$ are such that σ_0 and σ_1 are close, then $c(p: \sigma_0)$ and $c(p: \sigma_1)$ will be close at all p (and their slopes will be close as well. [One can think of the norm as $d'(c(p: \sigma_0), c(p: \sigma_1)) = \max(\sup_p |c(p: \sigma_0) - c(p: \sigma_1)|, \sup_p |c'(p: \sigma_0) - c'(p: \sigma_1)|)$.] Consider the budget constraint for a given p.

$$c(p: \sigma) + \frac{S(p: \sigma)}{p} = \frac{wn}{p} .$$

If σ_0 is close to σ_1 then the right-hand side must be close, meaning the left-hand side is close. Since n_0^* is close to n_1^* and R is given, then not only are the sum of c and S/p close for σ_0 and σ_1, but each component is close for a concave utility function. This must be true for all p. (For small changes in p, clearly the "slopes" of the two curves will be close.) If $c(p: \sigma_0)$ is close to $c(p: \sigma_1)$ then $\eta(p: \sigma_0)$ is close to $\eta(p: \sigma_1)$.

If $\eta(p: \sigma_0)$ and $\eta(p: \sigma_1)$ are close (as are their slopes), then $n(w; \eta(p: \sigma_0))$ and $n(w; \eta(p: \sigma_1))$ must be close since the firm maximizes profits over a convex set $f(n) > x + i$. If the two induced labor demand curves are close, then the induced values of σ, $A(\sigma_0)$ and $A(\sigma_1)$ must clearly be close for a concave utility function.

3. Proposition 2 (on levels and slopes of output demand functions).

The first part of the proposition, that consumption falls as income falls, is straightforward. Formally, one could introduce a shift parameter into the labor demand functions and differentiate the first-order conditions with respect to the parameter. One could then show that for regular utility functions, consumption at each price must rise as the labor demand curve shifts out.

To demonstrate the second part of the proposition, we first must find the slope of $c(p)$, that is, how, for given values of n^* and future period variables, the amount consumed changes with changes in price at the end of the period. We therefore differentiate (10) with respect to p to obtain

$$\frac{\partial c}{\partial p} = \frac{RW_c - pcR^2W_{cc}}{U_{cc} + p^2R^2W_{cc}} . \tag{A.1}$$

We then want to consider how $|\frac{\partial c}{\partial p}|$ changes with changes in expected second period income. For simplicity, let's consider an additive shift in future income by a real amount γ, for a given distribution of $\hat{n}$. (That is, let us suppose that the individual anticipates that, for the same value of $\hat{n}$, real income will change by an amount γ). To show that $\frac{d}{d\gamma}|\frac{\partial c}{\partial p}| > 0$, we may show $\frac{d}{d\gamma}(\frac{\partial c}{\partial p}) < 0$. Differentiating (A.1) with respect to γ, we obtain (detailed calculations are available from the author)

$$\frac{d}{d\gamma}\left(\frac{\partial c}{\partial p}\right) = - \frac{\partial c/\partial \gamma}{p^2 \nu}\left[\left(cU_{ccc} + 2U_{cc}\right) + \frac{\partial s}{\partial p}\left(U_{ccc} + p^2R^2W_{ccc}\left(\frac{1}{\partial c/\partial \gamma} - pR\right)\right)\right] \tag{A.2}$$

where $\nu = - \frac{U_{cc}}{p} - pR^2W_{cc} > 0$ and $\frac{\partial c}{\partial \gamma} < \frac{1}{pR}$.

A sufficient condition for the expresstion to be negative is that the utility function displays decreasing relative risk aversion which will imply that $(cU_{ccc} + 2U_{cc})$, U_{ccc}, and W_{ccc} are all positive.

4. Proposition 3 (On firm behavior)

The first-order conditions for the firm's decision problem implicitly define n and p as functions of w. We can therefore differentiate equations (2)-(5) with respect to α and solve to derive $\frac{dp(w)}{d\alpha}$ and $\frac{dn(w)}{d\alpha}$.

We obtain (where we write the output demand function as $x(p, \alpha)$) first

$$\frac{dp(w)}{d\alpha} = \frac{\frac{\partial x}{\partial \alpha}\left(-1 - x_p(k_1 + k_2)\right) + \frac{\partial |x_p|}{\partial \alpha}\left(p - \frac{w}{f'(n)}\right)}{2x_p + \left(k_1 + k_2\right)x_p^2 + x_{pp}\left(p - \frac{w}{f'(n)}\right)} \tag{A.3}$$

where

$$k_1 = \frac{(f'(n))^3}{wf''(n)} < 0$$

and

$$k_2 = \frac{1}{R} \cdot \left(\frac{1}{E(w\hat{f}''(\hat{n})/(f'(\hat{n}))^3}\right) < 0 \ .$$

For x_{pp} small, the denominator of (A.3) is negative. In the numerator, the first term negative, the second, positive. Therefore $\frac{dp}{d\alpha} > 0$, unless $\frac{\partial |x_p|}{\partial \alpha}$ is highly positive. In other words, an outward shift of the output demand curve causes price charged to rise, unless the curve becomes significantly flatter as it shifts out. Clear, the larger is $\frac{\partial |x_p|}{\partial \alpha}$, the smaller is $\frac{dp(w)}{d\alpha}$.

We obtain for $\frac{\partial n(w)}{\partial \alpha}$

$$\frac{\partial n(w)}{\partial \alpha} = \frac{\frac{\partial x}{\partial \alpha} + x_p \frac{dp}{d\alpha}}{f'(n)\left(1 + \frac{k_2}{k_1}\right)} > 0 \ . \tag{A.4}$$

The smaller are $\frac{dp}{d\alpha}$ or x_p, the larger is $\frac{\partial n(w)}{\partial \alpha}$.

Differentiating (2)-(5) with respect to w and solving, we obtain for the slope of the current period labor demand curve

$$\frac{dn}{dw} = \frac{\dfrac{x_p}{2f'(n) + \dfrac{x_{pp}}{x_p}\left(p - \dfrac{w}{f'(n)}\right)} + \dfrac{k_2}{f'(n)}}{f'(n)\left[1 + \dfrac{1}{k_1}\left(k_2 + x_p\left[\left(2 + \dfrac{x_{pp}}{x_p}\left(p - \dfrac{w}{f'(n)}\right)\right]\right)\right]} \tag{A.5}$$

which, for x_{pp} small, is negative. For $x_{pp} \approx 0$, we may derive

$$\frac{d\left(\frac{dn}{dw}\right)}{d(x_p)} \approx \frac{2\left(\dfrac{f'(n)}{k_2}\right)^2}{\left(\dfrac{(f'(n))^2}{k_1} + \dfrac{2(f'(n))^2/k_2}{\dfrac{x_p}{k_2} + Z}\right)^2} > 0 . \tag{A.6}$$

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