Will Raising Wages in the High-Wage Sector Increase Total Employment?-A Critique of the Stewart-Weeks View on the Relationship between Wage Changes and Unemployment in LDC's

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ABSTRACT

Will Raising Wages in the High-wage Sector Increase Total Employment?

A Critique of the Stewart-Weeks View on the Relationship

between Wage Changes and Unemployment in LDC's

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Frances Stewart and John Weeks have recently argued that increasing the wage rate in the "controlled" sector of an underdeveloped economy may increase total employment in some plausible circumstances. This paper examines the problem using a fully specified general equilibrium model of a two sector closed economy with unemployment, deriving necessary and sufficient conditions for a rate in the controlled sector wage rate to lead to an increase in total employment. Assigning realistic values to all factors involved indicates a negative rather than a positive relationship between the controlled sector wage rate and total employment, contrary to Stewart and Weeks' assertion. Policy-makers considering raising wages in the controlled sector should take into account the known or likely values of all the factors shown to be involved -- not just those mentioned by Stewart and Weeks -- to assess the impact on total employment. In addition, they should consider the effect of the wage rise on output as well, since it is not in fact reasonable to suppose, as Stewart and Weeks do, that output will be unchanged.

† † †

Frances Stewart et John Weeks ont récemment soutenu que l'augmentation du taux de salaire dans le secteur "contrôlé" d'une économie sous-développée pourrait provoquer l'accroissement de l'emploi total dans quelques plausibles circonstances. Cet essai examine la question en utilisant un modèle d'équilibre général d'une économie fermée à deux secteurs avec chômage, trouvant des conditions nécessaires et suffisantes à un relèvement du taux de salaire dans le secteur contrôlé pour provoquer un accroissement de l'emploi total. Attribuer des valeurs réalistes à tous les paramètres déterminants indique un rapport négatif plutôt que positif entre le taux de salaire du secteur contrôlé et l'emploi total, contrairement à l'affirmation de Stewart et Weeks. Les responsables de la politique économique d'un pays se donnant l'intention d'augmenter les salaires dans le secteur contrôlé devraient tenir compte des valeurs connues ou probables de tous les éléments présentés comme devant être impliqués - pas uniquement ceux mentionnés par Stewart et Weeks - pour calculer l'impact sur l'emploi total. De plus, ils devraient aussi bien considérer l'effet de la hausse des salaires sur la production, puisqu'il n'est pas en fait raisonnable de supposer, comme le font Stewart et Weeks, que son niveau restera inchangé.

Will Raising Wages in the High-Wage Sector Increase Total Employment? - A Critique of the Stewart-Weeks View on the Relationship between Wage Changes and Unemployment in LDC's[1]

In a recent article in the Journal of Development Studies,[2] Frances Stewart and John Weeks argue that increasing the wage rate in the already high-wage, capital-intensive "controlled" sector of an underdeveloped economy may increase overall employment in some plausible circumstances [Stewart and Weeks, 1975, p. 93]. The political appeal of such a claim is self-evident, and makes it of more than merely theoretical interest. It seems a safe bet, therefore, that policy-makers in some less developed countries will, sooner or later, seize upon the authors' argument as economic justification for granting politically popular wage rises in the controlled sector. They would be ill-advised to do so, however, without closer examination of the problem, since a policy of raising controlled sector wage rates is likely in fact to produce results the opposite of those suggested by Stewart and Weeks.

In order to show this, a more systematic and rigorous approach than that employed by Stewart and Weeks is required. The general inadequacy of their analysis in this respect clearly precludes its use as a basis for policy formulation. Nowhere in their article is the case for a policy of wage rises for an already high-wage sector seriously analyzed in terms of concisely stated, verifiable conditions on parameter values on which information is available or on which educated guesses can be made, although such a formulation of the problem is possible, and will be presented in this note.

The Stewart and Weeks analysis hardly goes further than to assert what is fairly self-evident anyway, that for the controlled sector wage and total employment to be positively related: 1) the elasticity of substitution in controlled sector production between capital and labor must be low--(plausible empirically, but how low is low?); and 2) the outputs of the controlled and uncontrolled sectors must be highly substitutable in consumption and in intermediate uses--that is, the elasticity of substitution between controlled and uncontrolled sector output in consumption and intermediate uses must be high. High substitutability between controlled and uncontrolled sector outputs, however, is less plausible empirically than low substitutability between capital and labor in controlled sector production, and again, how high is "high"? There is no way of telling from Stewart and Weeks' presentation; they do seem to give the impression, from their choice of assumptions, of thinking it must be very high indeed, since they choose to treat the outputs of the two sectors as nearly perfect substitutes -- a procedure which is implausible and overly restrictive. Actually, it can be shown that even if sectoral outputs are considerably less than highly substitutable in consumption, and not at all substitutable in intermediate uses, the Stewart-Weeks result of a positive relationship between total employment and the controlled sector wage rate, while unlikely, is not necessarily ruled out.

Clearly if these two parameters alone mattered, all that would count would be their combined effect, so that the critical value of the one would depend on the actual value of the other exclusively. However, it is not true that they alone matter. Other factors must be taken into account as well, and Stewart and Weeks either neglect these totally or mention them only in passing.

The fact that the role of capital -- the scarce factor of production -- receives such summary attention as it does in Stewart and Weeks' article is disturbing when it is realized that not only is the elasticity of substitution between capital and labor in controlled sector production a critical parameter, but that the size of the gap between capital-intensities in the two sectors, the initial allocation of capital between the two sectors, the share of capital in controlled sector output, and the mobility of capital between sectors in response to a change in relative rates of return, are all crucial factors in determining whether a rise in the controlled sector wage will raise or lower total employment. Yet Stewart and Weeks give but scant attention to these factors; indeed they obscure their importance by focusing too exclusively on the degrees of substitutability in production and consumption. Realistic values for LDCs for these factors in their totality indicate a negative rather than a positive relationship between the controlled sector wage and total employment in the closed economy case, as will be shown.

A particularly disconcerting aspect of the Stewart-Weeks article is their totally unjustified contention that total output (as they define it) will remain unchanged in the face of a wage rise in the controlled sector. By their own admission, their demonstration that a rise in the controlled sector wage raises total employment hinges on this assumption [Stewart and Weeks, 1975, p. 100]. In fact, however, output may rise, fall or remain constant, depending on the combined effect of the values of the elasticities of substitution and the other factors mentioned above.

The impression conveyed by their assurance that constancy of output is a plausible assumption is that raising the controlled sector wage rate can increase employment with no sacrifice in real income or potential welfare. Of course, while there are conditions under which this may be true, the more likely outcome is a fall in potential welfare and/or real income if total employment rises (which it well may not do).

Finally, a practical note: the implicit assumption that the controlled sector is necessarily highly capital-intensive compared to the rest of the economy means in effect that government is implicityly excluded from it. Yet it's hard to conceive how government wages and wages in the private and para-public subsectors of the controlled sector can be treated as independent policy variables. And in some LDCs -- those of Africa, for example -- the bulk of the controlled sector labor force would be in government employment.[3]

Stewart and Weeks' Algebraic Analysis

Using the following notation,

L -- total employment

L_1, L_2 -- labor employed in controlled and uncontrolled sectors

X_1, X_2 -- outputs of the two sectors

w_1, w_2 -- wage rates in the two sectors

Stewart and Weeks summarize the effects of a wage rise on employment by writing

$$\begin{aligned}
\Delta L &= -\Delta L_1 + \Delta L_2 \\
&= -\left[\Delta w_1 \times \frac{\Delta L_1}{\Delta w_1} + \frac{\Delta O_1}{\Delta w_1} \times \frac{\Delta L_1}{\Delta O_1}\right] + \\
&\quad \left[\frac{\Delta O_2}{\Delta w_1} \times \frac{\Delta L_2}{\Delta O_2} - \Delta w_2 \times \frac{\Delta L_2}{\Delta w_2}\right]
\end{aligned}$$

Aside from deficiencies attributable to carelessness in proofreading (why does Δw_1 appear in the first term but not in the second or third?) their formulation suffers from a major conceptual defect: its use of the delta notation obscures what are partial derivatives and what are total derivatives -- a vital distinction. Properly stated, the formula should read

$$dL_1 + dL_2 = \frac{\partial L_1}{\partial w_1} dw_1 + \left[\frac{\partial L_1}{\partial O_1}\frac{dO_1}{dw_1} + \frac{\partial L_2}{\partial O_2}\frac{dO_2}{dw_1}\right] dw_1 + \frac{\partial L_2}{\partial w_2}\frac{dw_2}{dw_1} dw_1$$

Stated this way, the problem posed by the middle term, with its components

$$\frac{\partial L_1}{\partial O_1}\frac{dO_1}{dw_1} \text{ and } \frac{\partial L_2}{\partial O_2}\frac{dO_2}{dw_1} ,$$

becomes clear -- how are the total derivatives $\frac{dO_1}{dw_1}$ and $\frac{dO_2}{dw_1}$, determined?

The signs and magnitudes of total derivatives such as these can be determined only by resolution of a set of simulataneous equations corresponding to a model of the whole economy, in which X_1, X_2, L_1, and L_2 appear as endogenously determined variables. But rather than formulate and work through such a model, Stewart and Weeks make the totally unconvincing assertion that "in a closed economy no change in the level of output seems a reasonable assumption," claiming that therefore (with the marginal product of labor higher in sector 1 than in sector 2) the middle term is positive. [Stewart and Weeks, 1975, pp. 99-100.] Their result is thus seen to depend directly on the highly suspect assumption that total output will be unaffected by a change in the controlled sector wage rate.

More fundamentally, the result obtained by Stewart and Weeks depends on the two assumptions that they defend as reasonable:

(1) high substitutability in comsumption and intermediate uses between X_1 and X_2, and

(2) a low elasticity of substitution between labor and capital in production in sector 1.

However, there is no indication of what combinations of values for these parameters would assure a positive relationship between the controlled sector wage and total employment. More important, there is no hint of other factors which might be important to consider; and, as pointed out, the assumption of unchanged output makes the whole demonstration quite unsatisfactory.

In view of all this, Stewart and Weeks' claim in their analysis of the neoclassical, closed economy case that "the negative effects on employment of an increase in wages in the controlled sector -- resulting from reduction of output in that sector, and substitution for labour... will be offset (in part, in whole, or even exceeded) by positive effects on employment in the uncontrolled sector" [Stewart and Weeks, 1975, p. 99] is, to say the least, not well established by their own argument.

In this note, I show that a more than offsetting increase in employment in the uncontrolled sector, while possible, is unlikely; and that the negative effects on employment in the controlled sector may even be reinforced by a simultaneous decline in employment in the uncontrolled sector. (To the extent employment in the uncontrolled sector declines, the fall in employment in the controlled sector will be less than would otherwise be the case, since in this case capital would flow from the uncontrolled sector into the controlled sector, but the effect on total employment in the two sectors combined, of course, is negative. Readers may wish to dismiss this as an unlikely outcome. But what will be proved here is the stronger assertion that total employment is likely to fall when wages in the controlled sector are hiked, even if the capital flow is in the "right" -- e.g., that presupposed by Stewart and Weeks' argument -- direction.)

Reformulation of the problem

In what follows, the problem is formalized so as to focus explicitly on the role of the capital stock and its allocation between sectors. This leads to specification of the conditions under which a rise in the controlled sector wage will induce a rise in total employment. These conditions involve not only the parameter values already mentioned but also the initial capital-labor ratios, the initial share of wages in sector 1 output and the initial allocation of capital between sectors. It is arguable that realistic values for these variables and the elasticities of substitution in production and consumption would indicate a negative rather than a positive realtionship between L and w_1.

Using a two sector model, with constant returns to scale production functions in capital and labor, I derive necessary and sufficient conditions for a rise in the controlled sector wage to lead to a rise in total employment in the closed economy case. (A full description of the model and the derivations are given in the accompanying mathematical notes.)

The model consists of the following variables:

1)	X_1	output of sector 1 (controlled sector)
2)	X_2	output of sector 2 (uncontrolled sector)
3)	K_1	capital employed in sector 1 (controlled sector)
4)	K_2	capital employed in sector 2 (uncontrolled sector)
5)	L_1	labor employed in sectors 1 and 2
6)	L_2	
7)	w_1	wage rates in sectors 1 and 2 (given in terms of sector 2's good)
8)	w_2	
9)	r_1	rates of return on capital in sectors 1 and 2
10)	r_2	
11)	p	relative price of sector 1's good (given, in the open economy case) endogenously determined in the closed economy case by equalization of rates of return on capital employed in sectors 1 and 2.

Alternatively, in the more general case, rates of return on capital between sectors may not be equalized, and instead $\frac{K_1}{K_2} = G(\frac{r_1}{r_2})$. However, in the mathematical notes at the end of this paper, it is assumed that rates of return on capital are equalized between sectors.[4]

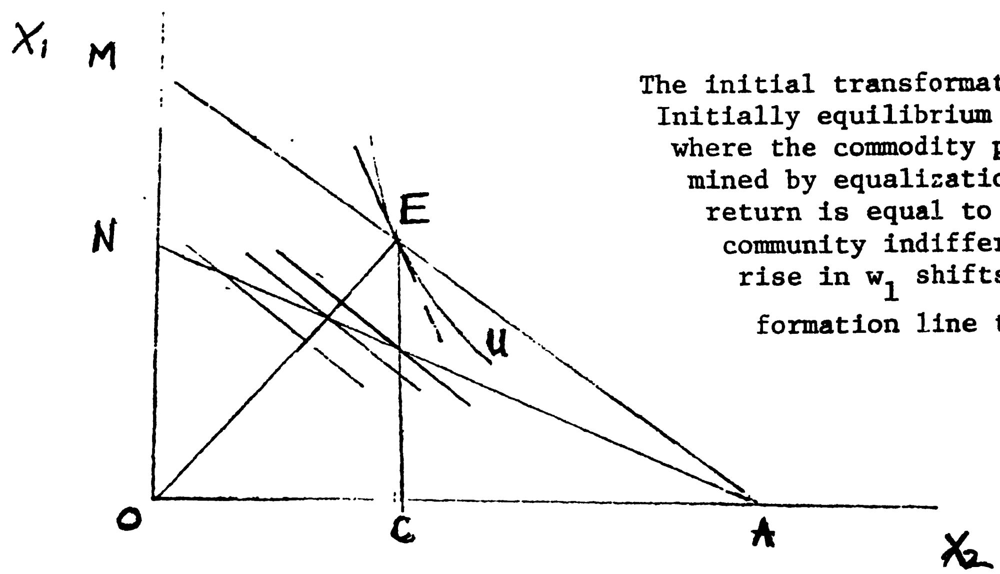
In the uncontrolled sector, the wage rate is exogenously given, and cannot be raised or lowered by government fiat, while in the controlled sector the wage rate is a policy variable. It is assumed that the economy is closed, and that capital is perfectly mobile between sectors.

Before stating the necessary and sufficient conditions for an increase in w_1 to raise total employment, it will be useful to look at the problem in diagrammatic terms, to see how Stewart and Weeks' qualitative requirements regarding substitutability of inputs in production and of outputs in consumption can be readily established graphically by using my approach.

With production functions showing constant returns to scale in capital and labor, with wage rates fixed, and with employers in each sector hiring labor up to the point where the value of labor's marginal product equals the sectoral wage, the transformation curve between the controlled sector good X_1 and the uncontrolled sector good X_2 is a straight line. Equalization of the rate of return between sectors determines the market price ratio p. It should be noted that in general the market price ratio will differ from the social marginal rate of transformation as given by the negative of the slope of the transformation line, $-\frac{dX_2}{dX_1}$. If sector 1 is capital-intensive, then for a certain range of $w_1 \geq w_2$, $p < -\frac{dX_2}{dX_1}$. (Thus there will be a divergence between the marginal rate of substitution between X_1 and X_2 and the social marginal rate of transformation between the two goods, as indicated in Figure I below;

hence a second-best situation even if wage rates are equalized between sectors. Equalization of the market price ratio and the social marginal rate of substitution requires $w_1 \neq w_2$. This is shown in accompanying mathematical notes at the end of this paper.)

If w_1 rises, p rises too, and so does the social marginal rate of transformation of X_2 into X_1. Therefore the transformation line shifts inward (rotating counter-clockwise in the accompanying diagram around point A which denotes the maximum rate at which the economy can produce X_2 with all its capital stock in sector 2). The extent to which the transformation line shifts inward depends on the magnitude of the elasticity of substitution of capital for labor in production in sector 1. The lower this is, the lower the loss of employment in sector 1 for any particular reallocation of capital between sectors.



The initial transformation line is AM. Initially equilibrium is at point E where the commodity price ratio determined by equalization of rates of return is equal to the slope of the community indifference curve U. A rise in w_1 shifts down the transformation line to AN and lowers $\frac{1}{p}$.

FIGURE I

It is not inconceivable that the equilibrium point may move to the southwest of E (without going to the left of OE). This will happen if capital moves into sector 1, so that less capital and labor are used in sector 2 (which is the labor-intensive sector).

It might seem that for this to happen X_1 has to be an inferior good. But this is not so.

Assuming identical homethetic utility functions for all individuals so that community indifference curves are uniquely defined and have the same slope along any ray through the origin such as OE, we see that in the region OCE, the marginal rate of substitution of X_2 for X_1, $\left(\frac{\frac{\partial U}{\partial X_1}}{\frac{\partial U}{\partial X_2}}\right)$, is greater than at E. In this area less is produced of each commodity. But less will be produced of X_2 only if K_2 has fallen, and if K_2 has fallen, L_2 will have fallen as well.[5]

Suppose more of X_2 is produced in the new equilibrium than at point E. $\frac{X_1}{X_2}$ will have fallen by the amount ζ (say by one percent). Suppose this is greater than is justified by the rise in p (which is uniquely determined, regardless of the final output mix). Then X_2 will have to fall and X_1 must rise until the ratio of demand prices equals that of supply prices. The final position on the new, lower, transformation line AN may easily be to the southwest of E. If it is, the total employment must have fallen, since output in the uncontrolled sector can fall only if the capital stock in this sector has fallen, and the controlled sector to which the capital moves is capital-intensive relative to the uncontrolled sector.

A little reflection will make it clear that the more easily X_2 can be substituted for X_1 in consumption, the more likely is a large expansion in demand for X_2, hence the better the prospect of success for a Stewart-Weeks policy of raising the wage in the high wage, capital-intensive sector.

In short, total employment will be more likely to rise, when the wage rate in the controlled, high wage sector is hiked, the greater the substitutability in consumption of X_1 and X_2 and the lower the substitutability of K for L in sector 1 production. But this is not all.

In an accompanying mathematical note it is shown that a necessary but not sufficient condition for total employment to rise in the case where demand is given by the function $\frac{X_2}{X_1} = p^{\eta}$ is that $\eta - \sigma_1 > 0$ -- where η is the elasticity of substitution in consumption between X_1 and X_2 and σ_1 is the elasticity of substitution in production between capital and labor in the production of X_1. (See Appendix I, p. 12.)

The sufficient condition can be written as follows:

$$\frac{(k_1 - k_2)}{k_2} \left(\frac{f - k_1 f'}{f}\right)\left(1 - \frac{K_1}{K}\right) (\eta - \sigma_1) - \sigma_1 > 0.$$

-- where η is the elasticity of substitution in consumption between X_1 and X_2, σ_1 is the elasticity of substitution in production between capital and labor in the production of X_1 (the controlled sector's output), k_i are the capital labor intensities, $\frac{f - kf'}{f}$ is the ratio of labor's marginal product to its average product, and $\frac{K_1}{K}$ is the controlled sector's share of the capital stock. (See Appendix I, p. 12.)

According to this condition, η must not only exceed σ_1, but must do so by a given margin.

Using the sufficient condition to write the critical value of η as a function of σ_1, w_1, w_2, K_1 and K, we have:

$$\eta = \frac{\sigma_1 + \frac{(k_1 - k_2)}{k_2}\left(\frac{f - k_1 f'}{f}\right)\left(1 - \frac{K_1}{K}\right)\sigma_1}{\frac{(k_1 - k_2)}{k_2}\left(\frac{f - k_1 f'}{f}\right)\left(1 - \frac{K_1}{K}\right)}$$

or

$$\eta = \left\{ \frac{1}{\frac{(k_1 - k_2)}{k_2}\left(\frac{f - k_1 f'}{f}\right)\left(1 - \frac{K_1}{K}\right)} + 1 \right\} \sigma_1$$

In this expression, w_1 and w_2 enter indirectly because (given the production function parameters) their values determine the values of k_1, k_2, f, f', etc. Note that p does not appear in this expression (although a given pair of w_1, w_2 does imply a particular p).

If sector 1 is capital-intensive ($k_1 > k_2$) η must exceed σ_1 <u>at least by</u> the margin $\dfrac{\sigma_1}{\frac{(k_1 - k_2)}{k_1}\left(\frac{f - k_1 f'}{f}\right)\left(1 - \frac{K_1}{K}\right)}$ for there to be a positive effect. Thus the sufficient condition is considerably more stringent than the necessary condition.

The margin by which η must exceed σ_1 will be larger, the smaller is the relative share of wages in sector 1, and the smaller is sector 2's share of the capital stock.

Stewart and Weeks assume that the products of the two sectors are very close substitutes in consumption [<u>Stewart and Weeks</u>, <u>1975</u>, p. 95]. Thus they consider the overall elasticity of substitution in consumption to be very high. Such a belief is certainly consistent with their view that raising the wage rate in the controlled sector will increase total employment in the controlled and uncontrolled sectors taken together. The conditions derived indicate that the outputs can be less than nearly-substitutable in consumption for the Stewart-Weeks result to hold. However, even so, given the likely values for the wage share in the controlled sector's output and the controlled sector's share of the capital stock, the critical value of η would have to be quite a bit above unity.[6] Opinions may differ on this but it seems likely that the elasticity of substitution in consumption between the outputs of the two sectors would be below the critical value for reasonable values of the other factors in the equation.

Employment, Potential Welfare, and Volume of Output: Some Further Observations

Looking at the sufficient condition on p. 11, we see that the greater the proportion of the capital stock in the controlled sector, the smaller will be the first term in absolute value, hence the more likely the expression as a whole is to be negative.

What can be said about the value of $\frac{K_1}{K}$ given η, σ_1, w_2 and the initial value of w_1? Given the demand specification $\frac{X_2}{X_1} = p^{\eta}$, it is clear that if p is high, $\frac{X_2}{X_1}$ will be high, hence $\frac{K_1}{K}$ will be low. A high value of $\frac{w_1}{w_2}$ implies a high value of p -- and also widens the gap between k_1 and k_2.

With a Cobb Douglas production function, the term $\frac{f - k_1 f'}{f}$ is a constant. But if the elasticity of substitution in the controlled sector, σ_1, is less than unity, $\frac{f - k_1 f'}{f}$ rises as w_1 rises, so that the value of the first term increases.

In the limit when $\sigma_1 = 0$, of course, total employment rises as w_1 is increased for any elasticity of substitution in consumption greater than zero.

Furthermore, the increase in employment may be accompanied by a rise in potential welfare. This possibility is illustrated below. (See Figure II).

In this limiting case, the rise in w_1 does not shift the transformation line inward at all.

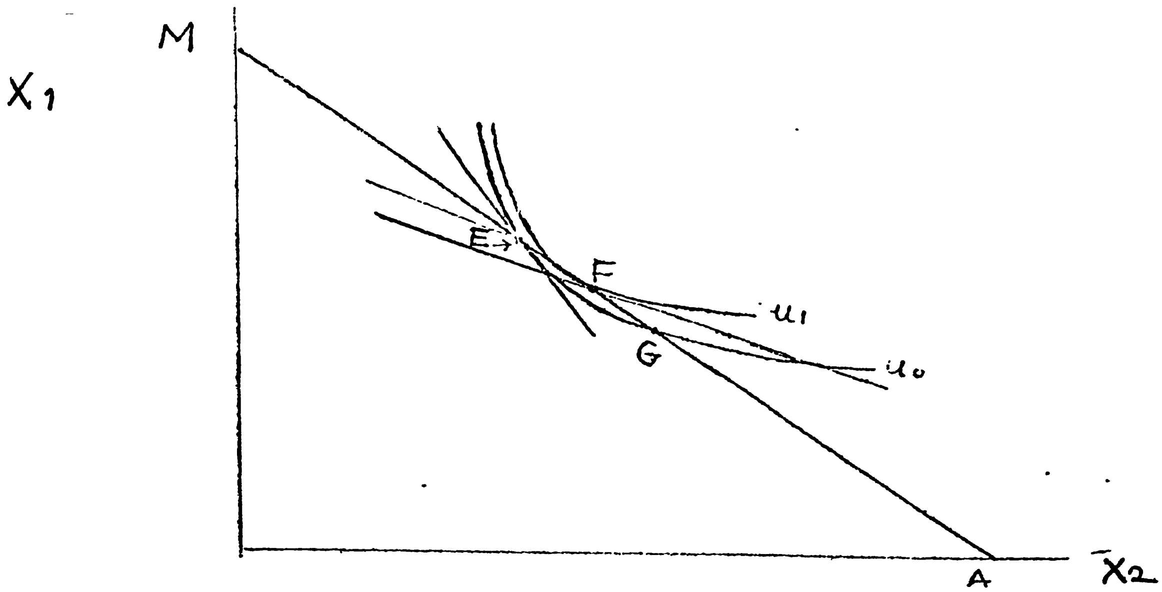


FIGURE II

The fall in $\frac{1}{p}$ encourages individuals to consume more of the uncontrolled sector good X_2 and less of the controlled sector good. This moves the economy to the southeast along the unshifted transformation line AM. The new point of equilibrium F <u>may</u> be on a higher community indifference curve. But then again, it may not be. If the rise in w_1 is too great, the fall in $\frac{1}{p}$ will be big enough to carry the economy beyond point G. This will mean a correspondingly greater increase in employment but at the expense of a fall in potential welfare. Hence, a threshold exists beyond which a tradeoff between potential welfare and employment is reestablished.

With a low elasticity of substitution in production in the controlled sector, any fall in employment in this sector following a rise in w_1 is likely to be associated more with a fall in sectoral output than with the substitution of capital for labor in the sector. Both capital and labor leave the controlled sector for employment in the other sector. If the capital-labor ratio in the uncontrolled sector is lower, more labor can be employed than leaves the controlled sector, so total employment rises. It may be noted additionally that if $\sigma_1 < 1$, the relative share of labor in sector 1 rises as w_1 rises. If further σ_1 is low enough relative to η that total employment increases, the relative share of labor in national income also rises.

Finally, consider Stewart and Weeks' argument that the total volume of output must remain constant when w_1 rises. According to Stewart and Weeks, the outputs of the controlled and uncontrolled sectors are near-substitutes in consumption, and initially p is close to unity (actually a little greater than unity because of the "superior quality" of sector 1's output). Now depending on the value of w_1, the marginal rate of transformation $-\frac{dX_1}{dX_2}$ may be less than, equal to, or greater than $\frac{1}{p}$. If it is greater than $\frac{1}{p}$, it may exceed unity -- and if it does, to the extent X_2 is substituted for X_1, total output clearly must fall -- contrary to Stewart and Weeks' assumption. In this case, total output will be unchanged only if η is equal to zero -- and, if this so, employment will be unchanged. On the other hand, suppose that the marginal rate of transformation is <u>less</u> than $\frac{1}{p}$. In this case the marginal rate of transformation $-\frac{dX_1}{dX_2}$, would be less than unity also, since $\frac{1}{p} < 1$. Hence if we assume $\sigma_1 = 0$ and $\eta > 0$, whatever the outcome for potential welfare (i.e., whether society ends up on a higher or lower community indifference curve) the total volume of output, measured at the initial <u>or</u> the final set of prices, unequivocally must rise rather than remain constant. The assumption that total output remains constant when w_1 rises is therefore equivalent in this case to assuming either that σ_1 is greater than zero, or that η is equal to zero (but this latter possibility is ruled out, for Stewart and Weeks, by their assumption that X_1 and X_2 are near-substitutes in consumption).

A Summing Up

To sum up briefly, a more systematic exploration of the question raised by Stewart and Weeks shows that the degrees of substitutability between capital and labor in production and between the outputs of the controlled and uncontrolled sectors in consumption are important in determining whether a rise in the controlled sector wage will lead to a rise or a fall in total employment, but that other factors intervene as well. Assigning realistic values to all factors involved indicates a negative rather than a positive relationship between total employment and the controlled sector wage. Furthermore, even if the relationship is positive, a rise in the controlled sector wage, by causing an inward shift of the transformation line between controlled and uncontrolled sector output will tend to lower potential welfare and/or real income, contrary to what is implied in Stewart and Weeks' unjustified assertion that total output will remain constant. Policy-makers considering granting a wage hike in the controlled sectors of the economy should take into account the known or likely values of all the factors shown to be involved in this note, not just the elasticities of substitution, in estimating the likely impact on total employment, and should consider the effect of the wage rise under consideration on real income as well.

FOOTNOTES

[1]This paper has grown out of discussions with Brendan Horton, who has contributed greatly by way of suggestions and comments on earlier drafts dealing with the Stewart-Weeks argument. I am also grateful to Al Saulniers, Peter Heller, Elliot Berg and other colleagues for reading and commenting on this paper. Needless to say, however, I am responsible for any errors. This hopefully represents only the first part of a more extensive critique which will also deal with the open economy case.

[2]"The Employment Effects of Wage Changes in Poor Countries," Journal of Development Studies, Vol. 11, No. 2, January 1975, Special Issue on Employment and Income Distribution, pp. 93-107.

[3]I am indebted to Elliot Berg for pointing this out.

[4]If instead of equalization, one assumed maintenance of a constant differential between rates of return, the conclusions of the model are not materially altered. The equation $pf' = \phi'$ in the mathematical appendix is replaced by $pf' = \alpha\phi'$ where p is the relative price of the controlled sector good, f' and ϕ' are the marginal productivities of capital in the two sectors and α is a constant. Differentiation of the modified system of equations with respect to w_1 leads to the same set of necessary and sufficient conditions as are obtained with the unmodified model. Initial values of key variables will be different, however.

[5]This is a crucial point. Clearly if capital is not mobile between sectors in response to differentials in rates of return, a rise in w_1 must cause total employment to fall. The transformation line will shift down, there will be some adjustment upward of p and the new point of equilibrium will be on CE below E.

[6]By way of example, suppose $\sigma_1 = 0.5$,

$$\frac{k_1 - k_2}{k_2} = 2,$$

$$\frac{f - k_1 f'}{f} = 0.2, \text{ and}$$

$$1 - \frac{K_1}{K} = 0.4.$$

Then, the margin $= \dfrac{0.5}{2(0.2)\ (0.4)} = 3.125$. If $\sigma_1 = 1$, other values staying the same, the margin would be 6.25 and the critical value of η would be 7.25.

Appendix I

Effect of a Wage Rise on Total Employment in a Two Sector, Closed Economy

The model consists of nine equations in nine endogenous variables:

1)	$X_1 = L_1 f(k_1)$	constant returns to scale production functions of sector 1 (controlled sector) and sector 2 (uncontrolled sector).
2)	$X_2 = L_2 \phi(k_2)$	
3)	$p(f - k_1 f') = w_1$	equalization of value of marginal product of labor to the sectoral wage rate in sectors 1 and 2.
4)	$\phi - k_2 \phi' = w_2$	
5)	$pf' = \phi'$	equalization of rates of return on capital across sectors.
6)	$K_1 + K_2 = \bar{K}$	full employment of the capital stock.
7)	$L_1 = \frac{1}{k_1} K_1$	employment of labor as a function of sectoral capital-labor ratios and distribution of capital between sectors.
8)	$L_2 = \frac{1}{k_2} K_2$	
9)	$\frac{X_2}{X_1} = p^{\eta}$	constant elasticity of substitution demand specification.

where

1) X_1 and X_2 are the quantities produced of the controlled sector (sector 1) good and the uncontrolled sector (sector 2) good.

2) L_1 and L_2 are the quantities of labor employed in the two sectors.

3) K_1 and K_2 are the quantities of capital employed in the two sectors. Capital, unlike labor, is fully employed.

4) k_1 and k_2 are the capital-labor ratios in the two sectors.

5) w_1 and w_2 are the wage rates. w_2 is given; w_1 is a policy variable. By hypothesis, $w_1 > w_2$.

6) p is the relative price of the sector 1 good.

Thus, there are nine endogenous variables: X_1, X_2, L_1, L_2, K_1, K_2, k_1, k_2, and p.

Two additional definitional equations are needed to incorporate explicitly the rates of return on capital into the model:

$$r_1 = pf'$$
$$r_2 = \phi'$$

With these two additional equations and variables, we have a system of eleven independent equations in eleven endogenous variables.

The demand function is what would prevail if all individuals in the economy had identical homethetic utility functions in X_1 and X_2 with constant elasticity of substitution in consumption equal to η.

† † †

Let us consider the effect of a rise in w_1 on k_1, k_2 and p. From equation (4), we know that the effect on k_2 is nul.

From equations (3) and (5), we have:

10) $$(f - k_1 f')\frac{dp}{dw_1} + p(-k_1 f'')\frac{dk_1}{dw_1} = 1$$

11) $$f'\frac{dp}{dw_1} + pf''\frac{dk_1}{dw_1} = \phi''\frac{dk_2}{dw_1} \quad (= 0)$$

Hence, by Cramer's rule:

$$\frac{dp}{dw_1} = \frac{\begin{vmatrix} 1 & (-pk_1f'') \\ 0 & pf'' \end{vmatrix}}{\begin{vmatrix} f - k_1f' & (-pk_1f'') \\ f' & pf'' \end{vmatrix}}$$

$$= \frac{pf''}{pf''(f - k_1f') + pf''k_1f'}$$

12) $$\frac{dp}{dw_1} = \frac{1}{f}$$

$$\text{i.e., } \frac{dp}{dw_1} > 0$$

Note that $$\frac{w_1}{p}\frac{dp}{dw_1} = \frac{w_1}{p}\frac{1}{f}$$

$$= \frac{p(f - k_1f')}{p}\frac{1}{f}$$

$$= \frac{f - k_1f'}{f}$$

that is,

13) $$\frac{w_1}{p}\frac{dp}{dw_1} = \frac{L_1}{X_1}\frac{\partial X_1}{\partial L_1}$$

. . . the ratio of the marginal product of labor in the controlled sector to the average product of labor in the controlled sector, or the elasticity of output with respect to labor in this sector, or the share of labor in controlled sector output.

Since $f - k_1 f' < f$, $\frac{w_1}{p}\frac{dp}{dw_1}$ is necessarily less than unity.

Also, by Cramer's rule:

$$\frac{dk_1}{dw_1} = \frac{\begin{vmatrix} f - k_1 f' & 1 \\ f' & 0 \end{vmatrix}}{\begin{vmatrix} f - k_1 f' & -k_1 p f'' \\ f' & p f'' \end{vmatrix}}$$

$$= \frac{-f'}{pf''(f - k_1 f') + pf'' k_1 f}$$

14) $$\frac{dk_1}{dw_1} = \frac{-f'}{pf'' f}$$

Thus, $\frac{dk_1}{dw_1}$ is necessarily greater than zero--which is only to be expected--the capital-labor ratio in sector 1 will rise since labor has become more costly to employ here.

Note that

$$\frac{w_1}{k_1}\frac{dk_1}{dw_1} = \frac{p(f - k_1 f')}{k_1}\left(- \frac{f'}{pf'' f}\right)$$

$$\frac{w_1}{k_1}\frac{dk_1}{dw_1} = -\frac{(f - k_1 f')f'}{k_1 f f''}$$

That is,

15) $$\frac{w_1}{k_1}\frac{dk_1}{dw_1} = \sigma_1$$

. . . where σ_1 is the elasticity of substitution between capital and labor in production in the controlled sector.

† † †

Now, turning to equations (1), (2), (7), and (8), we can write $\frac{X_2}{X_1}$ as

$$\frac{X_2}{X_1} = \frac{L_2 \phi(k_2)}{L_1 f(k_1)}$$

$$= \frac{k_1}{k_2}\frac{(K - K_1)}{K_1}\frac{\phi(k_2)}{f(k_1)}$$

But also $\frac{X_2}{X_1}$, from equation (9), equals p^η so we have:

16) $$\frac{k_1}{k_2}\frac{(K - K_1)}{K_1}\frac{\phi(k_2)}{f(k_1)} = p^\eta$$

Hence it must be true that

17) $$\frac{d\left\{\frac{k_1}{k_2}\frac{(K - K_1)}{K_1}\frac{\phi(k_2)}{f(k_1)}\right\}}{dw_1} = \frac{dp^\eta}{dw_1}$$

We know that k_2 and $\phi(k_2)$ don't change when w_1 rises --- only k_1, $f(k_1)$, p and K_1 can change. Therefore equation (17) can be rewritten as:

18) $$\frac{\phi}{k_2}\,\frac{d\left(\frac{k_1}{f}\,\frac{K-K_1}{K_1}\right)}{dw} = \frac{dp^{\eta}}{dw_1}$$

Applying the product rule of differentiation and simplifying, this becomes

19) $$\frac{\phi}{k_2}\,\frac{K-K_1}{K_1}\,\frac{d\,\frac{k_1}{f}}{dw_1} + \frac{\phi}{k_2}\,\frac{k_1}{f}\,\frac{d\left(\frac{K-K_1}{K_1}\right)}{dw_1} = \frac{dp^{\eta}}{dw_1}$$

Therefore we can write

20) $$\frac{d\left(\frac{K-K_1}{K_1}\right)}{dw_1} = \frac{\frac{dp^{\eta}}{dw_1} - \frac{\phi}{k_2}\left(\frac{K-K_1}{K_1}\right)\frac{d\,\frac{k_1}{f}}{dw_1}}{\frac{\phi}{k_2}\,\frac{k_1}{f}}$$

Now, applying the quotient rule of differentiation and simplifying the resulting expression, we can show that

21) $$\frac{d\,\frac{k_1}{f}}{dw_1} = \frac{-(f - k_1f')f'}{f^2pff''}$$

Dividing both sides of this equation by $\frac{k_1}{f}$, we have

22) $$\frac{d\,\frac{k_1}{f}}{dw_1} \div \frac{k_1}{f} = -\frac{(f - k_1f')f'}{k_1fpff''}$$

$$= \left(\frac{1}{pf}\right)\left\{-\frac{(f - k_1f')f'}{k_1ff''}\right\}$$

$$= \frac{1}{pf}\;\sigma_1$$

Substituting this back into the expression for $\frac{d\left(\frac{K-K_1}{K_1}\right)}{dw_1}$ (equation 20) we obtain

23) $$\frac{d\left(\frac{K-K_1}{K_1}\right)}{dw_1} = \frac{1}{\left(\frac{\phi}{k_2}\frac{k_1}{f}\right)}\frac{dp^{\eta}}{dw_1} - \left(\frac{K-K_1}{K_1}\right)\left(\frac{1}{pf}\right)\sigma_1$$

$$= \frac{X_1}{X_2}\frac{K-K_1}{K_1}\eta p^{(n-1)}\frac{dp}{dw_1} - \frac{K-K_1}{K_1}\frac{\sigma_1}{pf}$$

Since $\frac{X_2}{X_1} = p^{\eta}$, $\frac{X_1}{X_2} = p^{-\eta}$. Hence we can write, after some simplification,

$$\frac{d\left(\frac{K-K_1}{K_1}\right)}{dw_1} = \frac{K-K_1}{K_1}\left\{\frac{1}{p}\,\eta\,\frac{dp}{dw_1} - \frac{\sigma_1}{pf}\right\}$$

Recalling that $\frac{dp}{dw_1} = \frac{1}{f}$ (equation 12) we have

24) $$\frac{d\left(\frac{K-K_1}{K_1}\right)}{dw_1} = \frac{K-K_1}{K_1}\frac{1}{pf}(\eta-\sigma_1)$$

Therefore, it follows that

25) $$\frac{1}{\left(\frac{K-K_1}{K_1}\right)}\frac{d\left(\frac{K-K_1}{K_1}\right)}{dw_1} = \frac{1}{pf}(\eta-\sigma_1)$$

and that

$$26)\qquad \frac{w_1}{\left(\frac{K-K_1}{K_1}\right)} \frac{d\left(\frac{K-K_1}{K_1}\right)}{dw_1} = \frac{w_1}{pf}(\eta-\sigma_1)$$

$$= \frac{f - k_1 f'}{f}(\eta-\sigma_1)$$

In short, capital will flow into the uncontrolled sector when w_1 is raised only if the elasticity of substitution in consumption between X_1 and X_2 exceeds the elasticity of factor substitution in production in the controlled sector. By treating X_1 and X_2 as almost perfect substitutes (assigning a very high value to η), Stewart and Weeks virtually guarantee this result, if σ_1 is realistically taken to be less than or equal to unity.

The empirical question is, how justified is the assumption of near perfect substitutability in consumption of the outputs of sector 1 and sector 2? It would seem that Stewart and Weeks should have underlined more strongly the importance of the extreme substitutability hypothesis in their analysis.

We leave aside this question for the moment. The reader should note that the strength of the capital outflow effect, whatever its sign, depends on the share of wages in the value of output of the controlled sector. If $\frac{w_1}{pf}$ is very low, it will take a higher proportionate variation of w_1 to achieve a given percentage change in the ratio of K_2 to K_1 than if $\frac{w_1}{pf}$ were close to unity.

The Impact of a Wage Rise in Sector 1 on Employment

Even $\eta - \sigma_1 > 0$ does not guarantee that total employment must rise when the wage rate in the controlled sector is raised. The difference must be great enough so that a large enough volume of capital flows out of sector 1 into sector 2 so that the increase of employment in sector 2 more than compensates the fall in employment in sector 1.

From equations (7) and (8) we know that

27) $$\frac{dL_1}{dw_1} = \frac{1}{k_1}\frac{dK_1}{dw_1} - \frac{K_1}{(k_1)^2}\frac{dk_1}{dw_1}$$

and

28) $$\frac{dL_2}{dw_1} = \frac{1}{k_2}\frac{dK_2}{dw_1}$$

From equation (6) we have

29) $$\frac{dK_2}{dw_1} = -\frac{dK_1}{dw_1}$$

Now total employment, L, is the sum of employment in sector 1 (L_1) and employment in sector 2 (L_2). Therefore, it follows that the change in total employment with respect to a variation in w_1 can be written as

30) $$\begin{aligned}\frac{dL}{dw_1} &= \frac{dL_1}{dw_1} + \frac{dL_2}{dw_1} \\ &= \frac{1}{k_1}\frac{dK_1}{dw_1} + \left(-\frac{1}{k_2}\right)\frac{dK_1}{dw_1} - K_1\left(\frac{1}{k_1}\right)^2\frac{dk_1}{dw_1} \\ &= \left(\frac{1}{k_1} - \frac{1}{k_2}\right)\frac{dK_1}{dw_1} - K_1\left(\frac{1}{k_1}\right)^2\frac{dk_1}{dw_1}\end{aligned}$$

We have already seen that $\frac{dk_1}{dw_1} = -\frac{f'}{pf''f}$. This enables us to rewrite the expression for $\frac{dL}{dw_1}$ as follows:

31) $$\frac{dL}{dw_1} = \left(\frac{1}{k_1} - \frac{1}{k_2}\right)\frac{dK_1}{dw_1} - \frac{K_1}{(k_1)^2}\frac{(-f')}{pf''f}$$

We must now calculate $\frac{dK_1}{dw_1}$ explicitly.

From the result earlier obtained (equation 24) we know that

32) $$\frac{d\left(\frac{K-K_1}{K_1}\right)}{dw_1} = \frac{K-K_1}{K_1}\frac{(\eta-\sigma_1)}{pf}.$$

Using the quotient rule, and simplifying, we have

33) $$\frac{d\left(\frac{K-K_1}{K_1}\right)}{dw_1} = -\frac{K}{(K_1)^2}\frac{dK_1}{dw_1}$$

Therefore, we see that

34) $$\frac{dK_1}{dw_1} = -\frac{(K_1)^2}{K}\frac{d\left(\frac{K-K_1}{K_1}\right)}{dw_1}$$

or, by substitution,

35) $$\frac{dK_1}{dw_1} = -\frac{(K_1)^2}{K}\frac{(K-K_1)}{K}\frac{1}{pf}(\eta-\sigma_1)$$

$$= \frac{K_1(K-K_1)}{K}\frac{(\eta-\sigma_1)}{pf}$$

Substituting this value of $\frac{dK_1}{dw_1}$ into the expression for $\frac{dL}{dw_1}$, we have

36)
$$\frac{dL}{dw_1} = \left(\frac{1}{k_1} - \frac{1}{k_2}\right)\frac{dK_1}{dw_1} + \frac{K_1}{k_1}\;\frac{f'}{k_1 p f'' f}$$

$$= -\left(\frac{1}{k_1} - \frac{1}{k_2}\right)\frac{(K-K_1)K_1}{K}\;\frac{(\eta-\sigma_1)}{pf} + \frac{K_1}{k_1}\;\frac{f'}{k_1 p f'' f}$$

$$= -\left(\frac{k_2-k_1}{k_1 k_2}\right)\;\frac{(K-K_1)K_1}{K}\;\frac{(\eta-\sigma_1)}{pf} + \frac{K_1 f'}{k_1 k_1 p f'' f}$$

$$= K_1\left[\left(\frac{k_1-k_2}{k_1 k_2}\right)\left(1 - \frac{K_1}{K}\right)\frac{(\eta-\sigma_1)}{pf} + \frac{K_1 f'}{k_1 k_1 p f'' f}\right]$$

Writing this in elasticity terms, with appropriate substitutions (e.g., $L_1 = \frac{K_1}{k_1}$; $L_2 = \frac{K-K_1}{k_2}$), we have

37)
$$\frac{w_1}{L}\,\frac{dL}{dw_1} = \frac{p(f - k_1 f')}{\frac{K_1}{k_1} + \frac{K-K_1}{k_2}}\;\frac{dL}{dw_1}$$

$$= \frac{p(f - k_1 f')}{\left(\frac{k_2 K_1 + k_1(K-K_1)}{k_1 k_2}\right)}\;\frac{dL}{dw_1}$$

$$= \left\{\frac{(f - k_1 f')}{k_2 K_1 + k_1(K-K_1)}\right\} K_1(k_1-k_2)\left(1 - \frac{K_1}{K}\right)\frac{(\eta-\sigma_1)}{f}$$
$$+ \left\{\frac{(f - k_1 f')}{\frac{k_2 K_1 + k_1(K-K_1)}{k_2}}\right\} K_1 \left\{\frac{f'}{k_1 f'' f}\right\}$$

$$= \frac{K_1}{(k_2 - k_1)K_1 + k_1 K}\left\{(k_1 - k_2)\left(\frac{f - k_1 f'}{f}\right)\left(1 - \frac{K_1}{K}\right)(\eta - \sigma_1) - k_2\sigma_1\right\}$$

$$= \frac{1}{\frac{(k_2 - k_1)}{k_2} + \frac{k_1}{k_2}\frac{K}{K_1}}\left\{\frac{k_1 - k_2}{k_2}\left(\frac{f - k_1 f'}{f}\right)\left(1 - \frac{K_1}{K}\right)(\eta - \sigma_1) - \sigma_1\right\}$$

It is now clear that for $\frac{dL}{dw_1}$ to be greater than zero we must have

38) $$\frac{(k_1 - k_2)}{k_2}\left(\frac{f - k_1 f'}{f}\right)\left(1 - \frac{K_1}{K}\right)(\eta - \sigma_1) - \sigma_1 > 0.$$

Now let us use this condition to write the critical value of η as a function of σ_1, w_1, w_2, K_1 and K.

39) $$\eta = \frac{k_2\sigma_1 + (k_1 - k_2)\left(\frac{f - k_1 f'}{f}\right)\left(1 - \frac{K_1}{K}\right)\sigma_1}{(k_1 - k_2)\left(\frac{f - k_1 f'}{f}\right)\left(1 - \frac{K_1}{K}\right)}$$

$$= \left\{\frac{1}{\frac{(k_1 - k_2)}{k_2}\;\frac{(f - k_1 f)'}{f}\left(1 - \frac{K_1}{K}\right)} + 1\right\}\sigma_1$$

In this expression, w_1 and w_2 enter indirectly because (given the production function parameters) their values determine the values of k_1, k_2, f, f$'$, etc... Note that p does not appear in this expression (although a given pair of w_1, w_2 does imply a particular p).

If sector 1 is capital-intensive ($k_1 > k_2$), η must exceed σ_1 for an increase in w_1 to have a positive effect on overall employment, and it must exceed σ_1 at least by the margin $\dfrac{\sigma_1}{(k_1 - k_2)\left(\dfrac{f - k_1 f'}{f}\right)\left(1 - \dfrac{K_1}{K}\right)}$ for there to be a positive effect. Thus the sufficient condition is considerably more stringent than the necessary condition.

The margin by which η must exceed σ_1 will be larger, the smaller the relative share of wages in sector 1, and the smaller is sector 2's share of the capital stock.

Appendix II

Divergence Between the Marginal Rate of Transformation and the Market Price Ratio in a Two Sector Economy with Constant Wage Rates and Production Functions Having Constant Returns to Scale in Capital and Labor

The Model

The model is that of Appendix I except that it is not closed with a demand function specification.

1) $$X_1 = L_1 f(k_1)$$

2) $$X_2 = L_2 \phi(k_2)$$

3) $$p(f - k_1 f') = w_1$$

4) $$\phi - k_2 \phi' = w_2$$

5) $$pf' = \phi'$$

6) $$K_1 + K_2 = \overline{K}$$

7) $$L_1 = \frac{1}{k_1} K_1$$

8) $$L_2 = \frac{1}{k_2} K_2$$

Thus there are eight endogenous variables--X_2, L_1, L_2, K_1, K_2, k_1, k_2, and p--if we treat w_1, w_2, $\overline{K}$ and X_1 as exogenous.

Calculation of $-\frac{dX_2}{dX_1}$

Differentiating equations (3) through (5), we have

$$3)' \qquad (f - k_1 f')\frac{dp}{dX_1} + p(-k_1 f'')\frac{dk_1}{dX_1} = 0$$

$$4)' \qquad -k_2\phi''\frac{dk_2}{dX_1} = 0$$

$$5)' \qquad f'\frac{dp}{dX_1} + pf''\frac{dk_1}{dX_1} = \phi''\frac{dk_2}{dX_1}$$

This reduces to

$$9) \qquad (f - k_1 f')\frac{dp}{dX_1} - pk_1 f''\frac{dk_1}{dX_1} = 0$$

and

$$10) \qquad f'\frac{dp}{dX_1} + pf''\frac{dk_1}{dX_1} = 0$$

(9) and (10) can be rewritten as:

$$11) \qquad \frac{dk_1}{dX_1} = \frac{1}{pf''}\left(\frac{f}{k_1} - f'\right)\frac{dp}{dX_1}$$

and

$$12) \qquad \frac{dk_1}{dX_1} = \frac{1}{pf''}(-f')\frac{dp}{dX_1}$$

Both can be true only if

$$a) \quad \frac{f}{k_1} = 0$$

or

$$b) \quad \frac{dk_1}{dX_1} = \frac{dp}{dX_1} = 0.$$

So it follows that

$$\frac{dk_1}{dX_1} = \frac{dp}{dX_1} = 0.$$

Now turning to equations (1), (2), (6), (7), (8):

Differentiating these with respect to X_1 and making appropriate substitutions, we have

1)′ $$f \frac{dL_1}{dX_1} = 1$$

2)′ $$\frac{dX_2}{dX_1} = \phi \frac{dL_2}{dX_1}$$

6)′ $$\frac{dK_1}{dX_1} = - \frac{dK_2}{dX_1}$$

7)′ $$\frac{dL_1}{dX_1} = \frac{1}{k_1} \frac{dK_1}{dX_1}$$

8)′ $$\frac{dL_2}{dX_1} = \frac{1}{k_2} \frac{dK_2}{dX_1}$$

Hence it follows that

13) $$\begin{aligned} \frac{dX_2}{dX_1} &= \frac{\phi}{k_2} \frac{dK_2}{dX_1} \\ &= - \frac{\phi}{k_2} \frac{dK_1}{dX_1} \\ &= - \frac{\phi}{k_2} k_1 \frac{dL_1}{dX_1} \\ &= - \frac{\phi}{f} \frac{k_1}{k_2} . \end{aligned}$$

Thus, we have

14) $$- \frac{dX_2}{dX} = \frac{\phi}{f} \frac{k_1}{k_2} .$$

Relationship between $\frac{1}{p}$ and $-\frac{dX_2}{dX_1}$

According to (5)

15) $$p = \frac{\phi'}{f'}$$

Now, by (4)

16) $$p(f - k_1 f') = w_1$$

hence

17) $$f' = \frac{(f - \frac{w_1}{p})}{k_1}$$

Similarly, by (6), $\phi - k_2\phi' = w_2$, hence

18) $$\phi' = \frac{\phi - w_2}{k_2}$$

Thus, by substitution, using (15), (17) and (18),

19) $$p = \frac{\frac{\phi - w_2}{k_2}}{\frac{f - \frac{w_1}{p}}{k_1}}$$

$$= \frac{k_1}{k_2} \frac{\phi - w_2}{\left(f - \frac{w_1}{p}\right)}$$

From this, it is clear that

20) $$p(f - \frac{w_1}{p}) = \frac{k_1}{k_2}(\phi - w_2)$$

Thus we can write

$$21) \qquad pf = \frac{k_1}{k_2}\phi - \frac{k_1}{k_2}w_2 + w_1$$

$$22) \qquad p = \frac{k_1}{k_2}\frac{\phi}{f} - \frac{k_1 w_2 - w_1 k_2}{k_2 f}$$

$$= -\frac{dX_2}{dX_1} + \frac{w_1 k_2 - w_2 k_1}{k_2 f}$$

...so if $k_1(w) > k_2(w)$ (if sector 1 is capital-intensive)

$$23) \qquad p < -\frac{dX_2}{dX_1} \quad \text{when } w_1 = w_2$$

or equivalently

$$24) \qquad -\frac{dX_1}{dX_2} < \frac{1}{p}\,.$$

Case of a Constant Differential Between Rates of Return on Capital Between Sectors

The formulation of the problem thus far has presupposed that rates of return on capital are equalized between sectors. To examine the case where a constant differential between rates of return exists, we must replace equation (5) by the equality $pf' = \alpha\phi'$ where α is some positive constant different from unity.

It can be shown that $-\frac{dX_2}{dX_1}$ in this case is equal to $\frac{\phi}{f}\frac{k_1}{k_2}$, just as before.

However, equation (18) will be found to be replaced by the relationship

$$25) \qquad p = \frac{k_1\phi}{k_2 f} + (\alpha-1)\frac{k_1\phi}{k_2 f} - (\alpha-1)\frac{k_1 w_2}{k_2 f} + \frac{w_1 k_2 - w_2 k_1}{k_2 f}$$

$$= \frac{k_1\phi}{k_2 f} + (\alpha-1)\frac{k_1(\phi - w_2)}{k_2 f} + \frac{w_1 k_2 - w_2 k_1}{k_2 f}$$

$$= -\frac{dX_2}{dX_1} + (\alpha-1)\frac{k_1(\phi - w_2)}{k_2 f} + \frac{w_1 k_2 - w_2 k_1}{k_2 f} .$$

Therefore, if $\alpha > 1$, p is greater for a given value of $-\frac{dX_2}{dX_1}$ than it would be if $\alpha \leq 1$, and if $\alpha = 1$, $p = -\frac{dX_2}{dX_1} + \frac{k_2 w_1 - k_1 w_2}{k_2 f}$.