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Distance, Demand, and Oligopoly Pricing by

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Abstract. We demonstrate how to estimate a model of product demand and oligopoly pricing when products are multi-dimensionally differentiated. We provide an empirical counterpart to recent theoretical work on product differentiation. Using specifications informed by economic theory, we simultaneously estimate a demand system and price-cost margins for products differentiated in many dimensions.

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1. Introduction

Product differentiation plays an important role in many fields of economics. In industrial organization, for example, it is a necessary condition if prices are to exceed marginal costs with Bertrand competition. While recent empirical work in industrial organization has focused attention on estimating non-observable price-cost margins, detailed empirical treatments of product differentiation have been scant. ¹ An important exception to this is Bresnahan (1981, 1987). He has estimated a model of demand and oligopolistic pricing for products which are differentiated along one dimension as in a Hotelling (1921) model.

In international trade theory, too, product differentiation has been recently integrated into theoretic models. Much of this literature is well treated in Helpman and Krugman (1985,1989). One lesson that falls out of this literature is that the market conduct and product differentiation of firms are critical in determining the welfare impact of trade restrictions. The industries in which one might hope to evaluate the empirical relevance of these theories are often characterized by multi-dimensional product differentiation. Examples include the auto, aircraft, and computer industries. To test the hypotheses developed in this literature, it is therefore essential to have an econometric model incorporating both multi-dimensional product differentiation and oligopolistic pricing. ²

We thank Robert Pollak for early discussions on this topic, Gene Grossman for detailed and perceptive comments, and seminar participants at Stockholm, Hebrew and Tel Aviv Universities. This paper was completed (with the help of Bitnet) while Feenstra was a visitor at the Institute for Advanced Studies at the Hebrew University of Jerusalem and while Levinsohn was a visitor at the Institute for International Economic Studies, University of Stockholm. Each thanks his respective hosts.

Economic theorists have also recently turned their attention to the careful modelling of product differentiation. On the demand side, Anderson, de Palma, and Thisse (1989) investigate the conditions under which a demand system for multidimensionally differentiated products satisfy properties such as that of gross substitutes and symmetry. Adding a supply side, Caplin and Nalebuff (1988) provide conditions under which there exists a pure-strategy price equilibrium for firms producing multi-dimensionally differentiated products. It is of special note that this newer theoretical work models product differentiation as occurring in many dimensions.³ This is a welcome concession to reality.

In this paper, we demonstrate how to estimate a model of product demand and oligopoly prices when products are multi-dimensionally differentiated. We provide an empirical counterpart to the recent theoretical work on product differentiation. Using specifications informed by economic theory, we simultaneously estimate a demand system for differentiated products and price-marginal cost margins.

Our work may be seen as a generalization of Bresnahan's work. In the theory underlying his empirical methods, varieties of a product are arranged along a line of quality so that each model (except the lowest and highest) has two competitors on the line. Taking the quality of each model as exogenous (i.e. solved in the first stage of a two stage game), the demand and profit-maximizing prices for each variety of the product are simultaneously determined. A critical variable is the *distance* between each model and its competitors: as competitors get closer, the demand for a model becomes more elastic and its price-marginal cost markup decreases. Our generalization of Bresnahan's work allows us to drop the Hotelling set-up and instead allow products to vary over multiple dimensions. This means that each model of the product can have *many* competitors. In earlier work (Levinsohn and Feenstra, 1989), we have shown how the utility function of consumers can be used to obtain a metric on characteristics space, which makes it possible to identify competitors. In this paper, we use a particularly simple utility function which implies that the metric is (the square of) Euclidean distance after the units of each characteristic have been properly adjusted.

Since a utility function is used in identifying competitors, it should also place restrictions on the form of demand. However, demand for each model is evaluated as a multiple

integral over the market space, and we are not able to obtain a closed-form solution. Our central theoretical result evaluates the derivatives of demand, and so a first-order approximation to the demand function can be determined. We find that the elasticity of demand is inversely related to the “distance” between a model and its competitors, but that “distance” should be measured as the *harmonic mean of distances from a model to each of its competitors*.⁴ The harmonic mean has the property that if any one competing model is arbitrarily close, the harmonic mean approaches zero, so the elasticity of demand approaches infinity: when two models have the same characteristics, they are perfect substitutes. Thus, our theory gives us an economically meaningful way to measure “distance” when there are many competitors.

With this first-order approximation to demand, we solve for the profit-maximizing prices for firms under Bertrand competition, and these prices are directly related to the harmonic mean of distances to competitors. The pricing equation for each model takes a particularly simple form: price is a linear function of characteristics (reflecting marginal cost), the harmonic mean of distances from competitors, and a term which arises from the joint maximization of profits over all goods sold by a multi-product firm. If models are arbitrarily close, then the latter terms approach zero and price is just a function of characteristics: this corresponds to the price schedule derived by Rosen (1974) with a continuum of products. Thus, our analysis shows how the conventional hedonic regression must be modified when there are a discrete number of products and oligopoly pricing.

In Section 2, we derive the theoretical results of the paper. These are used to provide the econometric specification of the demand function and the pricing equation that are estimated in Section 3. In that section, we use a panel data set from the U.S. automobile market, and simultaneously estimate the demand and oligopoly pricing equations. We are able to test several interesting hypotheses that arise from our multi-dimensional set-up. For example, does the elasticity of demand for a model rise as competing models become more similar? Do oligopolistic firms really charge a higher price for their product as other competing products become more different? If the oligopolistic firms are multi-product firms, do they charge a higher price for products that compete primarily with other of their own products? A number of rather novel estimation issues arise, and these are

discussed in turn. Section 4 presents and interprets the estimation results. In Section 5, we conduct sensitivity analyses to investigate the robustness of our results. Conclusions are presented in Section 6, and the proofs of Propositions are gathered in the Appendix.

2. The Model

2.1 Utility and Competitors

While we formalize our ideas generally, we will use automobiles as a running example of our theory. We describe each car available by a vector of characteristics $\mathbf{z} = (z_1, \dots, z_K) \geq 0$. These characteristics are assumed to be exogenous, and we can think of them as being determined in the first stage of a two-stage game between firms. Consumers obtain utility $U(\mathbf{z}, \alpha)$ from purchasing a car, where $\alpha = (\alpha_1, \dots, \alpha_K)$ is a vector of taste parameters which varies across consumers. While some of our results can be obtained with a quite general form of utility (see Levinsohn and Feenstra, 1989), the estimation requires a specific form which we shall adopt now:

$$U(\mathbf{z}, \alpha) = \sigma \sum_{i=1}^K \ln(z_i - \alpha_i) \quad (1)$$

where $\sigma > 0$ is common across consumers.⁵ We shall assume that $\alpha \geq 0$, and this vector can be interpreted as the minimum acceptable characteristics for a consumer, since $z < \alpha$ would yield utility of $-\infty$. We assume that the taste parameters of all consumers are given by a compact set $A \subset R_+^K$.

The prices and products available to consumers are denoted respectively by p_m and z_m , for models $m=1, \dots, M$. In principle, this set of products should also include alternatives to purchasing a new car, such as used cars and alternative modes of transport. However, in our estimation we shall only use data on new cars. In this sense, our paper deals with the choice of which model to purchase, but not with the decision of whether to buy a car at all.⁶

We shall find it convenient to make a change of variables from the taste parameters α to the consumers' "ideal" product z^* . As in Lancaster (1979), the ideal product z^* is what each consumer would purchase if all models $z \geq 0$ were hypothetically available. However,

in order to determine this optimal choice, we must also specify what prices would be. To this end, we shall assume that if the continuum of products $z \geq 0$ were available, prices would equal marginal costs. After solving for the equilibrium of our model, we shall be able to return and justify this assumption (see section 2.3). We shall suppose that marginal costs $C(z)$ are a linear function of characteristics:

$$C(z) = \beta_0 + \beta'z, \quad z \geq 0. \quad (2)$$

When the continuum of products $z \geq 0$ are available, consumers face prices equal to costs $C(z)$, but when the discrete products z_m , $m = 1, \dots, M$ are available, consumers then face actual prices p_m . The actual price will generally not equal marginal cost, and the price-cost margin π_m is defined as,

$$\pi_m \equiv p_m - (\beta_0 + \beta'z_m), \quad m = 1, \dots, M. \quad (3)$$

When all products $z \geq 0$ are available consumers are faced with the prices in (2), so they will choose the ideal products:

$$z^* = \arg \max_{z \geq 0} \{U(z, \alpha) - (\beta_0 + \beta'z)\},$$

or, using the specific utility function in (1):

$$z_i^* = \alpha_i + \frac{\sigma}{\beta_i}. \quad (4)$$

Condition (4) shows how the ideal product z^* is determined by the taste parameters. We can think of it as establishing a one-to-one mapping from α to z^* . Then instead of identifying consumers by their tastes, we can identify them by their ideal products z^* . Inverting (4) shows the taste parameters which correspond to each choice of z^* :

$$\alpha_i = z_i^* - \frac{\sigma}{\beta_i}. \quad (4')$$

Substituting (4') into (1) we obtain utility as a function of the consumed characteristics z and the preferred product z^* :

$$V(z, z^*) = \sigma \sum_{i=1}^K \ln(z_i - z_i^* + \frac{\sigma}{\beta_i}). \quad (5)$$

We will use a second-order approximation to (5). Calculating the derivatives with respect to z , and evaluating at $z = z^*$, we find that $\frac{\partial V}{\partial z_i} = \beta_i$ and $\frac{\partial^2 V}{\partial z_i^2} = -\frac{\beta_i^2}{\sigma}$, using (4). We then have:

$$V(z, z^*) \approx V(z^*, z^*) + \beta'(z - z^*) - \frac{1}{2\sigma}(z - z^*)'B(z - z^*), \quad (6)$$

where

$$B \text{ is a diagonal matrix with elements } B_{ii} = \beta_i^2 \text{ and } B_{ij} = 0 \text{ for } i \neq j. \quad (6')$$

Using (5), we calculate that $V(z^*, z^*) = \sigma \sum \ln(\frac{\sigma}{\beta_i})$, which is constant. The second term in (6) shows that utility rises with the actual characteristics consumed, reflecting the fact that consumers prefer higher amounts of any z_i . From the last term in (6), we see that utility decreases with the “distance” between the actual and preferred products, where “distance” is measured as the square of Euclidean distance after each characteristic z_i is multiplied by β_i .

A utility function like (6) has been used elsewhere in the literature on product differentiation, and is quite conventional.⁷ Our derivation using the utility function in (1) shows exactly how the “weighting” matrix B is constructed (i.e. diagonal with elements β_i^2). Note that B depends on the parameters of marginal cost function in (2), so the utility function (6) combines elements of preferences and technology. This occurs because the mapping from α to z^* in (4) depends on the technology. Our theoretical results throughout section 2 are valid for *any* positive definite matrix B , but in our estimation we shall use the specific diagonal form in (6').

Using (6), we can define the “market space” of each product. First, note that the entire set of ideal products is given by:

$$S = \{z^* | z_i^* = \alpha_i + \frac{\sigma}{\beta_i}, i = 1, \dots, K, \alpha \in A\}. \quad (7)$$

using (4).⁸ Then the set of consumers with ideal products z^* who would choose model z_m , $m = 1, \dots, M$, is given by:

$$\begin{aligned} S_m &= \{z^* \in S \mid V(z_m, z^*) - p_m \geq V(z_n, z^*) - p_n, \quad 1 \leq n \leq M\} \\ &= \{z^* \in S \mid \frac{1}{2\sigma}(z_m - z^*)'B(z_m - z^*) + \pi_m \leq \frac{1}{2\sigma}(z_n - z^*)'B(z_n - z^*) + \pi_n\} \end{aligned} \quad (8)$$

where the second line of (8) follows from (3) and (6).

To illustrate the “market space” S_m , note that the inequality in (8) can be written as:

$$z^{*'}B(z_n - z_m) \leq \sigma(\pi_n - \pi_m) + \frac{(z_n'Bz_n - z_m'Bz_m)}{2}. \quad (9)$$

When (9) holds with equality it is the equation for a plane in z space, with the normal vector $B(z_n - z_m)$. In Figure 1, we show the planes between each pair of products A, B, and C (with characteristics z_a, z_b, z_c) as perpendicular to the dashed lines between these points. This illustration is valid so long as the characteristics z_i are first multiplied by β_i , as labelled on the axes. The planes we have drawn are the boundaries of the market spaces S_a, S_b , and S_c . The exact position of each plane is determined by the price cost margins π_m , as follows.

If $\pi_m = \pi_n$, then it is readily checked that a z^* satisfying (9) with equality is $z^* = \frac{(z_m + z_n)}{2}$, or the midpoint between z_m and z_n . Thus, if $\pi_a = \pi_b = \pi_c$ in Figure 1, then the boundaries of the market segments are equi-distant between each pair of products. Now suppose that π_a rises. The boundaries of the market space S_a are still planes, satisfying (9) with equality, but are drawn towards the point A as illustrated with the arrows.⁹ Consumers with preferred product z^* who were formerly indifferent between models A and B, and therefore on the boundary between S_a and S_b now purchase model B (and similarly for consumers formerly indifferent between A and C). Thus, as prices and price-cost margins rise, the market space of a model shrinks. We can use the market spaces to define the competitors of a product:

Definition: Models m and n are neighbors if $S_m \cap S_n \neq \emptyset$, $m, n = 1, \dots, M$.

We will let the integer set $I_m \subset \{1, \dots, M\}$ denote the competitors or neighbors of model m , which depends on prices and locations of products. In section 3, we shall discuss how the set of neighbors is determined empirically, but here suppose that I_m is known.

2.2 Demand

We shall make the strong assumption that the density of consumers over S is uniform with parameter ρ .¹⁰ Then demand for model m is:

$$Q_m = \int_{S_m} \rho dz^*. \quad (10)$$

Since the market spaces depend on prices, so does demand in (10). We are not able to obtain a closed-form solution for this multiple integral.¹¹ However, our central theoretical result, proved in the Appendix, derives the first derivatives of (10).¹² This allows us to compute a first-order approximation to demand, summarized as follows:

Proposition 1: There exist values $\theta_{mn} \geq 0$ such that a first-order approximation to demand Q_m around the point $\pi_m = \pi_n$, $n \in I_m$, is:

$$\ln Q_m \approx \ln Q_m^* - 2K\sigma \left(\frac{\pi_m}{H_m} \right) + 2K\sigma \sum_{n \in I_m} \frac{\theta_{mn} \pi_n}{B_{mn}}, \quad (11)$$

where

- (a) $B_{mn} = (z_m - z_n)' B (z_m - z_n)$;
- (b) $H_m = \frac{1}{\sum_{n \in I_m} \frac{\theta_{mn}}{B_{mn}}}$;
- (c) $\sum_{n \in I_m} \theta_{mn} = 1$ if S_m is in the *strict* interior of S .

To interpret this result, note that B_{mn} is the square of Euclidean distance from model m to n (after the characteristics are first adjusted by β_i). Then the derivative of $\ln Q_m$ with respect to π_n is inversely related to this distance: a change in the price of a competitor will have a smaller effect if it is farther away. The cross-price derivative is directly related to θ_{mn} , which is an unknown value. In the Appendix (Remarks 1 and 2), we provide an interpretation of θ_{mn} using Figure 2. In Figure 2, the market space S_d is divided into various sub-spaces as illustrated. Then we can interpret θ_{an} as equal to $\frac{\text{area}(S_{an})}{\text{area}(S_d)}$, for $n = a, b, c$. In other words, θ_{mn} is the volume of a triangle (or more generally a trapezoid) with vertex at z_m and base at the common boundary of S_m and S_n , relative to the entire volume of S_m .¹³ Turning to the derivative of $\ln Q_m$ with respect to π_m , we note that it is inversely related to H_m . From (b), H_m is interpreted as a *weighted harmonic mean* of the

distances from model m to its neighbors, with the weights θ_{mn} . If these weights sum to unity, then it is readily shown that H_m lies between the minimum and maximum values of B_{mn} , $n \in I_m$. In addition, if $B_{mn} = 0$ for any one neighbor, then $H_m = 0$: if two models have the same characteristics, then they are perfect substitutes, and the elasticity of demand is infinity.

We will not be able to estimate (11) directly, since the values θ_{mn} are unknown. For estimation, we shall let θ_{mn} take on the values:

$$\theta_{mn} = \left(\frac{1}{N_m}\right), \text{ where } N_m = \text{number of neighbors of model } m. \quad (12)$$

This choice for θ_{mn} will satisfy the restriction in (c) that $\sum \theta_{mn} = 1$ for S_m in the strict interior of S . In Figure 2, with four products and two characteristics, only the market space of model D is in the strict interior of S . However, as the number of products grows relative to characteristics, more models will be surrounded by neighbors, and then have their market spaces strictly interior to S . The choice of θ_{mn} in (12) means that we will be replacing the weighted harmonic mean in (b) by the simple harmonic mean.

2.3 Oligopoly Pricing

We will solve for the profit-maximizing prices for each firm under Bertrand competition. To illustrate the solution, let us initially suppose that each firm produces only *one* model, and then consider multiproduct firms. We assume that the marginal costs of producing a model with characteristics z_m are constant and equal to $\beta_0 + \beta'z_m$. The characteristics of the model are exogenous, i.e. solved in the first stage of a two-stage game. We will suppose that the profits available from each model are enough to cover fixed costs, but do not analyse these. Then each firm solves the problem:

$$\max_{p_m} p_m Q_m - (\beta_0 + \beta'z_m) Q_m. \quad (13)$$

Notice that choosing p_m is the same as choosing π_m , since they are related by (3). Then the solution to (13) is calculated using the demand function in (11):

$$p_m = \beta_0 + \beta'z_m + \frac{H_m}{2K\sigma}. \quad (14)$$

Thus, the optimal prices for each product increase according to the harmonic mean of distances to neighbors. The price-cost margin equals $\frac{H_m}{2K\sigma}$. The economic intuition behind this is appealing. If models are very close to each other then H_m approaches zero and the prices approach marginal cost. This justifies our assumption in section 2.1 that price would equal marginal cost if all products $z \geq 0$ were available. When the continuum of products is available, the equilibrium schedule $p_m = \beta_0 + \beta'z_m$ corresponds to that derived by Rosen (1974).¹⁴

When each firm produces multiple products, let $J_c \subset \{1, \dots, M\}$ be the set of models produced by company c . Then the profit-maximization problem is restated as:

$$\max_{p_m} \sum_{m \in J_c} p_m Q_m - (\beta_0 + \beta'z_m)Q_m. \quad (15)$$

The solution to (15) is derived in the Appendix, and using the same notation as in Proposition 1, we have:

Proposition 2: The profit-maximizing prices under Bertrand competition are:

$$p_m = \beta_0 + \beta'z_m + \frac{H_m}{2K\sigma} + \frac{\Gamma_m}{2K\sigma}. \quad (16)$$

where

- (a) $(\Gamma_1, \dots, \Gamma_M)' = (C + C^2 + C^3 + \dots)(H_1, \dots, H_M)'$;
- (b) C is an $M \times M$ matrix with $C_{mm} = 0$, and $C_{mn} = \theta_{mn} H_m / B_{mn}$ if m and n are neighbors *and* made by the same company ; $C_{mn} = 0$ otherwise.

We see that the optimal prices still increase with the harmonic mean, but (16) contains an extra term arising from the extra profits that the multi-product oligopolists earns from collusive within-firm pricing. The matrix C can be arranged to be block-diagonal in the products of each company, and its rows sum to less than unity so long as no model and all its neighbors belong to the same company. This will ensure that the infinite sum $C + C^2 + C^3 + \dots$ converges. When actually calculating Γ_m we continue the summation in (a) until $C^i(H_1, \dots, H_M)'$ becomes suitably small.

3. Data and Estimation Issues

3.1 Equations to be Estimated

Equations (11) and (16) provide the system comprised of a demand and a oligopoly pricing equation that is to be estimated. Simple substitutions using the definitions of the harmonic mean (from equations (11) and (12)), the weighting matrix B (from (6)), price-cost margins (from (3)), and the term arising from joint profit maximization (from (16)) give the estimating equations in terms of observable data and parameters to be estimated. With characteristics indexed by j , a model indexed by m and its neighbors by n , and time indexed by t , we have:

$$\ln Q_{mt} = d_0 + d_t + \gamma_1 \left(\frac{p_{mt} - \beta_0 - \sum_{j=1}^K \beta_j z_{mtj}}{\left(\sum_{n \in I_m} \frac{1}{N_m \sum_{j=1}^K ((z_{mtj} - z_{ntj}) \beta_j)^2} \right)^{-1}} \right) + \gamma_2 \left(\sum_{n \in I_m} \frac{p_{nt} - \beta_0 - \sum_{j=1}^K \beta_j z_{ntj}}{N_m \sum_{j=1}^K ((z_{mtj} - z_{ntj}) \beta_j)^2} \right) + \delta_{mt} \quad (17)$$

$$p_{mt} = \beta_0 + \beta_t + \sum_{j=1}^K \beta_j z_{mtj} + \lambda_1 \left(\sum_{n \in I_m} \frac{1}{N_m \sum_{j=1}^K ((z_{mtj} - z_{ntj}) \beta_j)^2} \right)^{-1} + \lambda_2 \Gamma_{mt}(\beta, z) + \epsilon_{mt} \quad (18)$$

where d_0 and d_t are a constant and coefficients on year dummies, respectively, and similarly for β_0 and β_t .

Note that Γ_{mt} in (18) is itself a function of H_{mt} 's which are themselves non-linear functions of characteristics z and the β 's. (See Proposition 2(b) for the exact definition.) Also, the summations over $n \in I_m$ are summations over the set of neighbors to a given model. This set is determined by (8) and the definition of neighbors given in section 2.1.

Before any detailed discussion of the data with which we estimate the system or the estimation techniques employed, first note that the data required to estimate the system are sales – the Q 's, prices – the p 's, and characteristics – the z 's. The β 's, γ 's, and λ 's are parameters to be estimated. The theory developed in Section 2 imposes a particular relationship between the λ 's and the γ 's. This relationship is given by:

$$-\gamma_1 = \gamma_2 = \frac{1}{\lambda_1} = \frac{1}{\lambda_2}$$

Rather than impose these restrictions from the outset, we will treat them as testable implications of the theory. Underlying these restrictions is some straightforward economic intuition. The restriction $-\gamma_1 = \gamma_2$ implies that if the prices of all models rise by one dollar, individual model demands are unaffected. This restriction is an implication of our assumption that there are no outside goods.

The restriction $\lambda_1 = \lambda_2$ is related to the pricing strategy employed by multi-product firms. To better understand this restriction, note that the oligopoly prices in (16) depend on both the harmonic mean of distances to neighbors H_m , and on the joint profit maximization term Γ_m . As the harmonic mean increases, the optimal price rises with the coefficient $\lambda_1 (= 1/2K\sigma)$. But now suppose that the harmonic mean for a neighboring model H_n rises, where models m and n are made by the same company. Then the increase in H_n leads to a rise in the neighbors price according to $\lambda_2 (= 1/2K\sigma)$. Of course, this increase in the neighbor's price would also affect the price of model m due to joint profit maximization. The restriction $\lambda_1 = \lambda_2$ simply says that a company will use the same rule for all of its products when converting harmonic means to optimal prices. We regard this as a quite reasonable consistency requirement on the pricing decisions of a multi-product firm.

The restriction $-\gamma_1 = \frac{1}{\lambda_1}$ implies that the demand elasticity resulting from consumer behavior is the same elasticity used by oligopolists in setting optimal prices.

We have added stochastic disturbance terms, δ_{mt} and ϵ_{mt} in (17) and (18). Comparing (11) and (17), we see that $\delta_{mt} = \ln Q_{mt}^* - (d_0 + d_t)$. The term $\ln Q_{mt}^*$ in (11) is interpreted as demand for each model if price-cost margins were equal (i.e. $\pi_m = \pi_n \forall m, n$). In this case, demand would depend on the locations of the products: models whose neighbors were farther away would have higher demand. We shall treat $\ln Q_{mt}^*$ as iid normal in each year. Interpreting $d_0 + d_t$ as the mean value of $\ln Q_{mt}^*$ for each t , we then obtain δ_{mt} as iid normal with mean zero.

Since equation (16) holds with equality, there should be no error in (18) if we had the "true" equilibrium prices and our model was an exact description of reality. However, we

shall be using the suggested retail prices (SRP), which may differ from the transactions prices paid by consumers. Then one interpretation of ϵ_{mt} is the measurement error arising from using SRP. We shall treat ϵ_{mt} as iid normal with mean zero, and independent of δ_{mt} . The independence assumption is needed for the system to be identified. Since $p_{mt} - \beta_0 - \beta'z_{mt}$ depends on ϵ_{mt} and appears on the right hand side of (17), we could not obtain unbiased estimates of that equation if δ_{mt} and ϵ_{mt} were correlated. The independence assumption is justified in our context by our use of suggested retail prices which are announced at the start of the model-year. In contrast, the quantity data for sales are over the entire year. This means that p_{mt} 's are announced before Q_{mt} 's are known.

Finally, the year dummies in (17) and (18) may be thought of as fixed effects in a panel context. These variables are included to pick up unmodelled components of the disturbance terms that are correlated with time. In (17), one might imagine that cyclical macro variables may effect auto demand in a given year. In (18), the year dummies are more likely to pick up inflationary trends.¹⁵

3.2 Data

We estimate (17) and (18) using a panel data set comprised of 86 models of automobiles sold in the United States during the period 1983 through 1987. We include all models sold for each of these five years except exotica (Lotus, Ferrari, Rolls Royce, and the like.) The complete list of models is included in Table 1.¹⁶

We model automobiles as differentiated over five dimensions. That is, the vector of characteristics for each model in each period, z_{mt} , has five elements. These differentiating characteristics are weight (in thousands of pounds), horsepower, aerodynamics (measured as the inverse of height in inches), and dummy variables for whether the car has air conditioning as standard equipment (a proxy for luxury) and whether the car is European.¹⁷ We choose to limit the product differentiation to five characteristics for computational reasons. In the sensitivity analyses, we check to see how robust results are to the choice of characteristics.

The sales data are sales by nameplate (measured in thousands), and the price data are list prices of the base models (in thousands of dollars.) While something like the average

transaction price for each model in each year is of course preferable to list prices, such data are simply not available on an all-encompassing basis. All data are from the *Automotive News Market Data Book* (annual issues.)¹⁸

3.3 Estimation Issues

Estimating (17) and (18) poses some unique econometric issues. The first of these involves estimating the set of neighbors for each model. The second issue arises because each observation is itself summed over a different set of neighbors. The third relates to the extensive non-linearity of the system. We elaborate on each of these in turn.

The simple harmonic mean of distances from a model to its neighbors appears in both (17) and (18). Before the system can be estimated, it is necessary to know which models neighbor which. The first step in estimation, then, is to determine I_m – the set of neighbors to each model.¹⁹ The theory developed in section 2.1 guides this process. Recall that two models m and n are neighbors if $S_m \cap S_n \neq \emptyset$, $m, n = 1, \dots, M$. We can interpret this definition as saying that two models are neighbors if consumers indifferent to these models prefer these models to all other available models. As noted in section 2.1, a particularly convenient feature of the utility function (1) is that the consumer whose ideal variety is the midpoint of a line drawn between two models will be indifferent to the two models. The metric by which the midpoint is determined is simply (the square of) Euclidean distance when each characteristic has been pre-multiplied by β_i . The vector β , though, is estimated in the system given by (17) and (18), and to estimate these, one must know the set of neighbors. We address this problem by applying OLS to (2) to get preliminary estimates of β . These β_i 's are then used to compute neighbors. In the sensitivity analyses in Section 5, we will take the β that results from estimation of the system, and use that β to recompute neighbors. With the new neighbors, the system can then be re-estimated.

The algorithm which computes neighbors is straightforward. We first take a pair of potential neighbors. We locate the midpoint of the line connecting these two models.²⁰ With this midpoint as the ideal variety, z^* , we then ask if any other available models are closer to z^* using the metric discussed above. If no available model is closer, the two models are, by our definition, neighbors. Conversely, if another available model is closer,

the two are not neighbors. We repeat this procedure for every possible pair of models within a year. (We do not model possible inter-temporal competition between models.) This procedure will identify neighbors in multi-dimensional characteristics space which is needed to form I_m in (17) and (18).²¹

Once the set of neighbors, I_m , has been determined, we turn our attention to estimating (17) and (18). Because the disturbance terms are additive in each equation, estimation by Non-linear Least Squares (NLS) and Maximum Likelihood (MLE) are asymptotically equivalent. Since each observation contains variables summed over sets unique to that observation (the I_m 's), though, standard NLS and MLE estimation programs are not suitable. We estimate (17) and (18) using a variant of the Gauss-Newton algorithm for NLS that was designed specifically for estimating systems with the properties of (17) and (18).²²

The Gauss-Newton algorithm is an iterative method. For the problem at hand, two issues deserve special note. First, in general, it is preferable to utilize analytic derivatives when using Newton-type methods.²³ Given the fairly extreme nonlinearity in our estimating equations, the advantages of analytic derivatives are magnified. Accordingly, our Gauss-Newton method employs analytic derivatives.²⁴ Second, note that with each iteration, the estimated values of β will typically change. As these change, the set of neighbors I_m might change. We do not allow this to occur. Rather, we assume that the set of neighbors is constant between iterations. The reason for this is that if the set of neighbors changed with each iteration, there is no reason to expect iterative methods to converge. As mentioned above, after obtaining NLS estimates of (17) and (18) using the neighbors identified by preliminary (OLS) values of β , we shall then re-compute neighbors and re-estimate the system.

4. Results and Interpretation

The first step in the estimation is identifying the set of neighbors for each of the 86 models in the sample. This is done using the 1985 cohort of models. We assume that the set I_m is constant over the period of estimation.²⁵ Prior to computing these sets of

neighbors, initial estimates of the β 's are required. Applying OLS to (2) for all years yields:

$$p_{mt} = -18.177 + 2.861 \textit{WEIGHT}_{mt} + .072 \textit{HP}_{mt} + 4.819 \textit{AIR}_{mt} + \\ 644.136 \textit{AERO}_{mt} + 6.887 \textit{EUROPE}_{mt} \quad R^2 = .775, N = 430 \quad (19)$$

Although only the parameter estimates are used to identify neighbors, all coefficients are significant at the 99 percent level. Determining neighbors is just the first step in estimating the system given by (17) and (18). We report the results of this initial step in Table 1.

Table 1 lists the 86 models in the sample and the model numbers of each neighbor of each model. For example, the first line of Table 1 indicates that the set of neighbors for the Toyota Tercel is comprised of the Nissan Pulsar (i.e. model number 12), the Mazda GLC, the Subaru DL/GL, and the Renault Alliance. Because the technique is new and not of the standard econometric variety, diagnostics are not developed. Readers can judge the validity of the technique by asking themselves whether the neighbors to the car they own are reasonably close substitutes. We believe that our technique, which relies solely on the primitives of utility maximization, gives quite reasonable results.

Once the sets (I_m 's) have been estimated, we are ready to estimate (17) and (18). As noted in footnote 16, the average squared Euclidean distance to a model's neighbors appears in the denominator of terms in (17) and (18). Identical models were combined to avoid division by zero. Some models, though, are almost identical in some but not all years. This gives rise to values of the terms that the λ 's and γ 's multiply that are several thousand times as large as all the other values. We could either combine these models with their near-twins for all years (even if the problem existed for only one year) or delete these outliers (26 out of 430 observations) from the sample. We adopt the latter option since combining models made by different firms poses difficulties when estimating a firm's joint profit maximization opportunities.

We estimate the system subject to various restriction, and these results are summarized in Table 2. The first column of Table 2 contains system estimates when no restrictions are imposed on the γ 's or λ 's. The signs of all coefficients are consistent with our theoretic model and all coefficients except γ_2 are statistically significant.²⁶

When we impose the restriction $-\gamma_1 = \gamma_2$, parameter estimates are those given in column 2 of the table. Using the likelihood ratio test, we find that the data do not reject

this restriction. The test statistic is .8831 and is distributed Chi-Square with one degree of freedom. Again, the signs are theory-consistent and all coefficients are now statistically significant.

We next impose the additional constraint that $\lambda_1 = \lambda_2$ and these results are presented in column 3 of Table 2. Again employing the likelihood ratio test, we find that the data do not reject this restriction as the test statistic is 1.718 and distributed Chi-Square with two degrees of freedom. When the restriction is imposed, signs remain consistent with theory and all coefficients are statistically significant.²⁷

The rest of this section discusses the interpretation of the results. Since the joint restrictions $\lambda_1 = \lambda_2$ and $-\gamma_1 = \gamma_2$ are suggested by the theory and accepted by the data, we shall focus on this case as we discuss results.²⁸

We find that γ_1 is negative (-.0099) and statistically significant. This supports the notion that the demand for a model, given the location of its neighbors, falls when the price of the model rises. The own elasticity of demand is easily computed using (17). One feature of our model is that every model in each year has a different own elasticity of demand, and this elasticity depends on the location of neighboring models. All else equal, models whose neighbors are quite close have more elastic own price responses. We calculate the elasticity of demand for every model in the sample.²⁹ We find that the own price elasticity of demand has an average value of -.516 and a median value of only -.211. These inelastic values are clearly incompatible with an oligopolistic equilibrium. In general, it appears that the magnitudes of the estimated γ 's are not theory-consistent. Our approach to the demand equation attempts to measure *levels* of demand using *derivatives* of demand. We adopt this strategy due to the difficulty of estimating the multiple integral in (10) directly. While every coefficient in our demand equation was of the correct sign and statistically significant, the "derivatives" approach does not appear to yield own price elasticities of demand that are consistent with an oligopolistic equilibrium.³⁰

Also γ_2 is positive (.0099) and statistically significant. As the prices of a model's competitors rise, the model's own sales also rise. Again, every model has a different cross-price elasticity in our set-up. This elasticity is greater the closer a model's competitors are (using the measure of distance provided by the theory in section 2.) The demand equation

also permits estimates of a very wide variety of elasticities. One can perturb the system on any of a number of margins, and compute how model demand changes. For example, automobile industry analysts could use (17) to compute the elasticity of demand for Fords with respect to a change in the price of General Motor's models. Trade economists could compute the demand elasticity for domestic cars with respect to a price change in Japanese models; and regulatory economists could compute the demand elasticity for light autos with respect to a price change (tax) on high horsepower models.³¹ In sum, the estimated demand equation is useful in a variety of interesting economic situations.

In the oligopoly pricing equation, (18), we find that λ_1 , the term that multiplies the harmonic mean of neighbors' distances, is positive (.410) and statistically significant. All else equal, as a model's competitors become farther away, the price-cost margin rises. Just as the estimated demand equation implies an own price elasticity of demand for every model, so does the pricing equation. It is straightforward to show that this elasticity for model m in period t is given by $\frac{P_{mt}}{\lambda_1 H_{mt}}$ where H_{mt} is the harmonic mean of distances to neighbors. The resulting elasticities may be interpreted as the ones used by firms in setting their prices whereas the elasticities from the demand equation may be thought of as resulting from consumer behavior. In a theoretically consistent world, these two elasticities would be the same.

We find that the demand elasticity implied by the pricing equation has a median value of -51.7. This value seems reasonable when we keep in mind this is the elasticity for a given model (of the almost 100 available) and that any model has many substitutes. The distribution of elasticities is illustrated in Figure 3. Figure 3 shows the wide range of elasticities. The figure also shows that many models have virtually perfectly elastic demands. These observations have at least one neighbor that is very similar. In general, the demand elasticities implied by the pricing equation appear reasonable and are compatible with an oligopolistic equilibrium.

By computing the harmonic means for each model using the simultaneously estimated β 's, we can also calculate the mean price-cost margin excluding any returns to collusion arising from the multi-product nature of the market. That is, we can compute the term $H_m/2K\sigma$ in (16). We find this mark-up has an average value of \$599 and a median value

of \$200. One interpretation of this figure is that it represents the mark-up that would result if the 86 models in the sample were produced by 86 separate firms. Again due to the presence of outliers, summary statistics may be misleading. We present the distribution of these mark-ups for each model-year in Figure 4. There we see that about half the sample (205 of 404 observations) have a mark-up of \$500 or less while 36 have a mark-up of over \$5000.

We find that λ_2 is also positive (.410) and is precisely estimated. The data strongly support the hypothesis that oligopolistic multi-product firms increase the prices of those products that have as competitors products made by the same parent firm. We find the magnitude of this within-firm collusion can be quite substantial. Using the simultaneously estimated β 's, we compute the term that λ_2 multiplies. The average value of this term for products that have at least one neighbor made by the same firm times the estimated λ_2 gives the extra price-cost markup. We find this additional markup due to joint profit maximization has a median value of \$144 and averages \$606. The distribution of the additional mark-ups due to joint profit maximization is illustrated in Figure 5. There we see that more than half (230 of 404) of the observations have at least one neighbor made by the own parent firm. Of these observations, about half (128 of 230) have extra mark-ups of \$500 or less while 24 observations have mark-ups greater than \$4000.

The estimates of β are also reported in Table 2. All of the β 's (except the constant) are positive which is what our theory predicts. Also, all the parameter estimates are statistically significant. The β 's that appear in (17) and (18) are used to measure distances. Positive β 's indicate that as a model's competitors become farther away in any of the five dimensions in which the products are differentiated, the own and cross price responses in demand are reduced and oligopoly markups are increased. We can also go back to equation (2) (which was substituted into (17) and (18)) to interpret the β 's. Then each β can be thought of as the marginal cost of producing an additional unit of a characteristic. Here, though, interpretation is tenuous since many of the characteristics are proxying for a variety of other characteristics. In the instance of air conditioning, $\beta_{air} = 4.894$. Clearly, it does not cost a firm almost \$5000 on the margin to add air conditioning to a model. Insofar as the dummy variable AIR is proxying for the wide range of luxury items associated with

air conditioning as standard equipment, the estimate is more reasonable. Similar caution should be exercised when interpreting the other β 's.

The next to last two lines of Table 2 report the R^2 for the demand and the pricing equations. We find that our specification explains about 81 percent of the variation in model prices and about 5.0 percent of the variation in model demands. While the data support the behavioral hypotheses suggested by our theory, a substantial amount of the variance in model demands remains unexplained.³²

The estimates of the parameters on the dummy variables for years are not reported in Table 2. We find that these parameters are close to zero and not statistically significant in the demand equation. In the pricing equation, the parameters are positive and significant. Their magnitude indicates that the average price of all models rises about \$500 each year.

5. Sensitivity Analyses

Ideally, the β 's reported in the first three columns of Table 2 should be used to estimate the sets of neighbors instead of the initial estimates generated by (19). (But then with different sets of neighbors, β will again change which leads to yet another set of neighbors and so on.) As a sensitivity exercise, we re-calculated the sets of neighbors using the NLS β 's in column 3 of Table 2. With these new sets of neighbors, we then re-estimated equations (17) and (18) (still imposing that $-\gamma_1 = \gamma_2$ and $\lambda_1 = \lambda_2$.) Column 4 of Table 2 presents the new system estimates. This experiment shows that the results are quite robust to our inability to simultaneously estimate distances and sets of neighbors. All coefficients except the γ 's that were statistically significant in the base case (column 3) remain so. (The γ 's are just statistically insignificant unless we impose our priors that $\gamma_1 < 0$ and employ a one-tailed test.) Further, the magnitudes of most coefficients in columns 3 and 4 are quite similar. Apparently the inability to simultaneously estimate distances from neighbors and the set of neighbors is not important to our results.

Leamer (1983, 1985) has argued in a persuasive and entertaining fashion for experiments which test the importance of ad hoc specification decisions. In our context, the precise list of characteristics may be viewed as "doubtful." In this spirit, Table 3 investigates how sensitive results are to the choice of characteristics. In that table, we drop one characteristic

and instead use a plausible alternative. For example, we drop horsepower and instead use engine displacement. The other substitutions are a dummy for foreign instead of the dummy for European, a continuous measure of luxury³³ instead of the dummy for air conditioning, and an alternative measure of aerodynamics (using the inverse of headroom instead of inverse height).³⁴ In each case, we re-estimate the system using non-linear least squares and imposing that $\lambda_1 = \lambda_2$ and $-\gamma_1 = \gamma_2$.

Table 3 reports the average own elasticity of demand, the average price-cost margin, and the average extra mark-up due to within firm joint profit maximization.³⁵ We find that using alternate sets of characteristics changes the magnitude of the elasticity and mark-ups, but never affects our qualitative results. Demand elasticities from the demand equation are all negative and remain fairly inelastic. Mark-ups are all positive and with the possible exception of when the SHORT proxy (the inverse of headroom) is used, are even similar in magnitude. With any of the sets of characteristics, the resulting average mark-ups lie in the central part of the distribution of base case mark-ups (as illustrated in Figures 4 and 5.) Overall, we conclude that our qualitative results do not depend on a fortuitous choice of characteristics.

6. Conclusions

Since theoretic models in many fields of economics assume product differentiation, and in the real world this differentiation is frequently multi-dimensional, econometric methods which might allow researchers to test the theories are needed. In this paper, we have developed a method for estimating demand and oligopoly pricing when products are multi-dimensionally differentiated.

Our technique is solidly rooted in consumer utility maximization and firm profit maximization, and this theoretic model directly guides our econometric specifications. We derive a demand system for multi-dimensionally differentiated products that has several testable hypotheses. Our theory, for example, predicts that both the price of a model and the appropriately measured distance from a model to its competitors will effect the demand for that model. Similarly, oligopoly pricing depends on the model's own characteristics, and how far away a model's neighbors are. Our theory also indicates how to estimate

the extra oligopoly rent multi-product firms accrue when their products compete with one another.

We estimate the equations generated by our theory using data from the U.S. automobile market. We find that the data broadly support the predictions of the model. For example, the estimated demand equation indicates that demand for a model falls as the model's own price rises and as the prices of competing models fall. Further, the own- and cross- price responses becomes more elastic as a model's competitors become closer. The magnitudes of these demand elasticities, though, are not consistent with an oligopolistic equilibrium. On the oligopoly pricing side of our model, the data support the notion that a firm will increase a model's price if that model's competitors are farther away. We also find support for the existence of extra oligopoly rents due to multi-product firms. The demand elasticities implied by the pricing equation are consistent with an oligopolistic equilibrium. Were our model completely supported by the data, the demand elasticities implied by the demand and by the pricing equations would be the same. They are not.

The within-equation restrictions on the demand and pricing equations were tested and readily accepted. There are, though, other restrictions which were implicitly imposed and not tested. For example, for more general utility functions than that in (1), it can be shown that the β 's which enter the harmonic means and marginal costs indeed differ. We have not pursued this generalization here since it would substantially increase the number of parameters to be estimated. Experimenting with more general utility functions, though, is a direction for further work. Along the same lines, relaxing our assumption of a uniform density of consumers may lead to a specification which is better supported by the data.

A more general assumption we have imposed on the model is that of Bertrand competition. Obviously, it would be desirable to extend the model to allow for other forms of market conduct and to see if market conduct has changed in response to specific policies. We are presently studying the effects of the "voluntary" export restraint with Japanese auto firms, initiated in 1981, on the market conduct of American firms.

Future research, then, is directed toward relaxing some of these restrictive assumptions while still deriving an empirically implementable model. Also, there are many other industries for which our approach is applicable. The promising results of this first attempt at

estimating the demand and oligopoly pricing for multi-dimensionally differentiated products have prompted us to research these extensions and further applications. We hope others might be similarly motivated.

Appendix

We shall prove Propositions 1 and 2 using results more general than those required in the text. Let π_m and π_n denote the price cost margins on model m and its neighbors, $n \in I_m$. We restrict these to satisfy:

$$\pi_n - \pi_m > \frac{-B_{mn}}{2\sigma} \quad (A1)$$

where $B_{mn} = (z_m - z_n)'B(z_m - z_n)$. From (8), it can be seen that (A1) ensures $z_m \in S_m$, so that the market space of a model contains the model itself.

Demand in (10) is obtained as a multiple integral over S_m . Since the boundaries of the market space in (8) vary continuously with prices, we shall treat demand as continuously differentiable. The derivatives of demand are given by:

Theorem 1

There exist values $\theta_{mn} \geq 0$, depending on prices, such that the derivatives of demand are:

$$\begin{aligned} (a) \quad & \frac{1}{Q_m} \frac{\partial Q_m}{\partial p_n} = \frac{2\sigma K \theta_{mn}}{B_{mn} + 2\sigma(\pi_n - \pi_m)} \\ (b) \quad & \frac{1}{Q_m} \frac{\partial Q_m}{\partial p_m} = -2\sigma K \sum_{n \in I_m} \frac{\theta_{mn}}{B_{mn} + 2\sigma(\pi_n - \pi_m)} \\ (c) \quad & \sum_{n \in I_m} \theta_{mn} = 1 \text{ if } S_m \text{ is in the strict interior of } S. \end{aligned}$$

Proof:

(a) This follows by defining θ_{mn} as,

$$\theta_{mn} \equiv \left(\frac{1}{Q_m} \frac{\partial Q_m}{\partial p_n} \right) [B_{mn} + 2\sigma(\pi_n - \pi_m)] / 2\sigma K. \quad (A2)$$

From (8), we see that the market space S_m becomes larger as π_n rises, so that $\frac{\partial Q_m}{\partial p_n} \geq 0$. Then using (A1), it follows that $\theta_{mn} \geq 0$.

(b) The market spaces in (8) depend on $(\pi_n - \pi_m) = [(p_n - p_m) + \beta'(z_m - z_n)]$, from (3). This means that raising p_m by an amount δ will have the same effect on demand as lowering p_n by δ for all $n \in I_m$. That is,

$$\frac{\partial Q_m}{\partial p_m} = - \sum_{n \in I_m} \frac{\partial Q_m}{\partial p_n}.$$

Then (b) follows directly from (a).

(c) Begin with some price-cost margins π_n^o and π_m^o satisfying (A1). Let S_m^o denote the market space of model m , with demand Q_m^o . We shall suppose that S_m^o is in the strict interior of S , and so it is defined by (8) without any reference to S :

$$S_m^o = \{z^* \mid \frac{1}{2\sigma}(z_m - z^*)'B(z_m - z^*) + \pi_m^o \leq \frac{1}{2\sigma}(z_n - z^*)'B(z_n - z^*) + \pi_n^o, 1 \leq n \leq M\}.$$

Then for all $n \in I_m$, consider the new price-cost margins:

$$\pi_n = \pi_n^o + \Delta_n, \quad \text{where } \Delta_n = \frac{\delta B_{mn}}{2\sigma} + \delta(\pi_n^o - \pi_m^o). \quad (\text{A3})$$

For δ sufficiently small, the new market space for model m will still be in the strict interior of S , and is given by:

$$S_m = \{z^* \mid \frac{1}{2\sigma}(z_m - z^*)'B(z_m - z^*) + \pi_m^o \leq \frac{1}{2\sigma}(z_n - z^*)'B(z_n - z^*) + \pi_n^o + \Delta_n, 1 \leq n \leq M\}.$$

Substituting for Δ_n from (A3) and simplifying, we can show that S_m equals:

$$S_m = \{z^* \mid z^* = \bar{z} + \delta(\bar{z} - z_m) \text{ and } \bar{z} \in S_m^o\}. \quad (\text{A4})$$

Thus, the new market space S_m is exactly an expanded (for $\delta > 0$) version of S_m^o . Demand with the price-cost margins π_m can then be evaluated as:

$$\begin{aligned} Q_m &= \int_{S_m} \rho dz^* \\ &= \int_{S_m^o} \rho \left| \frac{\partial z^*}{\partial \bar{z}} \right| d\bar{z} \\ &= (1 + \delta)^K Q_m^o. \end{aligned} \quad (\text{A5})$$

The second line of (A5) follows by making a change of variables from z^* to \bar{z} as indicated in (A4). The determinant of this Jacobian is $\left| \frac{\partial z^*}{\partial \bar{z}} \right| = (1 + \delta)^K$ where K is the dimension of characteristics space. Then the final line of (A5) follows from the definition of Q_m^o .

From (A5), we calculate that,

$$\frac{\partial Q_m}{\partial \delta} \Big|_{\delta=0} = KQ_m^o. \quad (A6)$$

However, using (A3), we calculate that,

$$\begin{aligned} \frac{\partial Q_m}{\partial \delta} \Big|_{\delta=0} &= \sum_{n \in I_m} \frac{\partial Q_m}{\partial p_n} \Big|_{\delta=0} \frac{\partial \pi_n}{\partial \delta} \Big|_{\delta=0} \\ &= \sum_{n \in I_m} \frac{\partial Q_m}{\partial p_n} \Big|_{\delta=0} [B_{mn} + 2\sigma(\pi_n - \pi_m)]/2\sigma. \end{aligned} \quad (A7)$$

Setting (A6) equal to (A7), and using (A2), it follows immediately that $\sum_{n \in I_m} \theta_{mn} = 1$ when evaluated at π_m^o and π_n^o . But since π_m^o and π_n^o were any price-cost margins satisfying (A1), this proves part (c). Q.E.D.

Evaluating the derivatives in Theorem 1 at the price-cost margins $\pi_m = \pi_n, n \in I_m$, we obtain Proposition 1 in the text. In order to prove Proposition 2, we need the following result.

Theorem 2

When demand is continuously differentiable, $\frac{\partial Q_m}{\partial p_n} = \frac{\partial Q_n}{\partial p_m}$.

Proof:

Total consumer surplus over the set of available products is,

$$W = \sum_{m=1}^M \int_{S_m} [V(z_m, z^*) - p_m] \rho dz^*. \quad (A8)$$

With each consumer maximizing surplus, W is also maximized. That is, the market spaces shown in (8) give the highest value of W compared to any other choices of $S_m \subseteq S$ with $S_m \cap S_n$ of measure zero. Then analogous to the envelope theorem, when differentiating (A8) with respect to prices, we can hold the market spaces S_m constant. Calculating this derivative,

$$\frac{\partial W}{\partial p_m} = - \int_{S_m} \rho dz^* = -Q_m.$$

Then by Young's Theorem,

$$\frac{\partial^2 W}{\partial p_m \partial p_n} = -\frac{\partial Q_m}{\partial p_n} = -\frac{\partial Q_n}{\partial p_m}.$$

Q.E.D.

Remark 1: Combining the immediately above result with part (a) of Theorem 1, we see that $Q_m \theta_{mn} = Q_n \theta_{nm}$ when $\pi_n = \pi_m$. This allows us to interpret the θ_{mn} in terms of Figure 2. The market space S_d for model D in Figure 2 is divided into S_{da} , S_{db} , and S_{dc} .

Define,

$$\theta_{dm} = \frac{\text{area}(S_{dm})}{\text{area}(S_d)}, \quad m = a, b, c. \quad (A9)$$

We clearly have $\theta_{da} + \theta_{db} + \theta_{dc} = 1$. But in addition, with $\pi_d = \pi_a$ in Figure 2, the triangles S_{ad} and S_{da} are identical, since their common boundary is perpendicular to and crosses the mid-point of a line between A and D. Then define $\theta_{ad} = \frac{\text{area}(S_{ad})}{\text{area}(S_a)}$. It follows that $\text{area}(S_a)\theta_{ad} = \text{area}(S_d)\theta_{da}$. Since demand is the integral over market spaces, we obtain $Q_a\theta_{ad} = Q_d\theta_{da}$ as required. We conclude that (A9) is a valid interpretation of θ_{mn} . While we have assumed that $\pi_m = \pi_n$, an extension of our argument can show that (A9) is still valid when $\pi_m \neq \pi_n$.

Remark 2: This interpretation of θ_{mn} suggests that for models m where S_m includes a boundary of S , we have,

$$\sum_{n \in I_m} \theta_{mn} < 1. \quad (A10)$$

This is demonstrated by model A in Figure 2, for which $\frac{\text{area}(S_{ab}) + \text{area}(S_{ac}) + \text{area}(S_{ad})}{\text{area}(S_a)} < 1$. We expect that (A10) is the counterpart to Theorem 1(c) when S_m is not in the strict interior of S , but do not prove this here.

Returning to Proposition 2, the first order conditions for (15) can be written as:

$$\pi_m = -\frac{Q_m}{\partial Q_m / \partial p_m} - \sum_n \pi_n \left(\frac{\partial Q_n / \partial p_m}{\partial Q_m / \partial p_m} \right), \quad m \in J_c, \quad (A11)$$

where the summation is over $n \in (J_c \cap I_m)$. We can use Theorem 2 to replace $\frac{\partial Q_n}{\partial p_m}$ by $\frac{\partial Q_m}{\partial p_n}$ in (A11). Then evaluating all derivatives of Q_m at the point $\pi_m = \pi_n$, $n \in I_m$, we can write (A11) as,

$$\pi_m = \frac{H_m}{2K\sigma} + \sum_n \pi_n \left(\frac{\theta_{mn} H_m}{B_{mn}} \right), \quad n \in J_c \quad (A12).$$

Using the notation of Proposition 2, (A12) can be written as $(I - C)\pi = \frac{H}{2K\sigma}$ where $\pi = (\pi_1, \dots, \pi_M)'$ and $H = (H_1, \dots, H_M)'$ are column vectors. It follows that $\pi = (I - C)^{-1} \frac{H}{2K\sigma}$. Assuming that $\theta_{mn} > 0$ for $n \in I_m$, the rows of C sum to less than unity so long as no model and its neighbors all belong to the same company. We then have $(I - C)^{-1} = I + C + C^2 + C^3 + \dots$, and so Proposition 2 is established.

Footnotes

1. A survey of the New Empirical Industrial Organization (NEIO), including some studies of product differentiation, is provided by Bresnahan (1988). A recent study by Trajtenberg (1989) models consumer preferences with products differentiated in many dimensions, as we consider here. His paper, though, is quite different from ours. He derives the demand and welfare gains from the introduction of a new product, while we, in contrast, derive demand and oligopoly pricing. Because Trajtenberg considers a product with few available varieties (CT scanners), he is able to estimate demand with a multinomial logit model. We shall consider a product with many varieties (autos), and this requires new and different functional forms and estimation techniques. See also footnote 11.

2. Examples of empirical trade papers which do model multi-dimensional product differentiation but do not model oligopolistic firms are Feenstra (1988) who investigates the gains from trade resulting from the introduction of new products, and Levinsohn (1988) who analyzes the effect of tariffs on the demand for differentiated products.

3. Our approach is compared to these papers in footnotes 7, 9, and 11

4. The harmonic mean of a series whose observations are denoted by x_n is given by:

$$H = \left(\frac{1}{N} \sum_{n=1}^N \frac{1}{x_n} \right)^{-1}$$

5. This utility function is a bit more general than it appears, since multiplying z_i by γ_i we can write $\sum \ln(\gamma_i z_i - \alpha_i) = \sum \ln(\gamma_i) + \sum \ln[z_i - (\alpha_i/\gamma_i)]$. The first term in this expression is constant and can be omitted, and the second term is already captured by (1.)

6. Bresnahan (1981, 1987) is able to estimate the price and quality of alternatives to purchasing a new car, as he locates the alternatives at the lower and upper ends of the quality line. With multiple characteristics, the same approach does not seem feasible.

7. See Lancaster (1979) who uses a single characteristic, and Anderson, Palma, and Thisse (1989) who use a multi-dimensional version very similar to (6). Caplin and Nalebuff adopt a general utility function which includes (6) as a special case.

8. Since we assumed that A is compact, so is S .

9. Note that the boundaries of the market spaces, where (9) holds with equality, vary continuously with prices. Our characterization of the market spaces is the same as in Caplin and Nalebuff (1988), who have a more general utility function. They also use a more general density function for consumers, and so their results on the existence of a pure-strategy price equilibrium apply to our model.

10. If we begin with a density function $f(\alpha)$ over taste parameters, then using (5), the density over characteristics is $g(z^*) = f(z_1^* - \frac{\sigma}{\beta_1}, \dots, z_k^* - \frac{\sigma}{\beta_k})$. Assuming that $g(z^*) = \rho$ is the same as assuming $f(\alpha) = \rho$. See also footnote 11.

11. Anderson, Palma, and Thisse (1989) do obtain closed form solutions for demand. They assume that the number of models, M , does not exceed the number of characteristics K by more than one ($M \leq K + 1$) and they need a special arrangement of models in characteristics space. They are then able to consider a wide range of density functions for consumers, including that which leads to the multinomial logit demand system (see also Anderson, de Palma, and Thisse (1988)). In contrast, we have many more models than characteristics, and we wish to derive the properties of demand with an arbitrary location of models, but we need a uniform density of consumers.

12. Note that in the Appendix we derive the first derivatives of (10) around any price-cost margins, π_m and π_n . In Proposition 1, we restrict our attention to the special case $\pi_m = \pi_n$. This greatly simplifies our estimation procedures.

13. Substituting this interpretation of θ_{mn} into (11), we can compute $d \ln Q_m / dp_n$ as:

$$\frac{1}{Q_m} \frac{dQ_m}{dp_n} = 2K\sigma \frac{\text{area}(S_{mn})}{\text{area}(S_m)B_{mn}}$$

Multiplying the top and bottom of the right by ρ , we see that Q_m will cancel with $\rho \text{ area}(S_m)$. In addition, the triangle S_{mn} has height $\frac{\sqrt{B_{mn}}}{2}$ and base $S_m \cap S_n$. It follows that $\text{area}(S_{mn}) = \frac{\sqrt{B_{mn}}}{4} \text{area}(S_m \cap S_n)$. Substituting this into the above, we obtain,

$$\frac{dQ_m}{dp_n} = \frac{\rho K \sigma}{2\sqrt{B_{mn}}} \text{area}(S_m \cap S_n).$$

Thus, the derivative of Q_m with respect to p_n is directly related to the size of the common boundary between S_m and S_n , which reflects the number of consumers who will switch products as prices change, and inversely related to the Euclidean distance between models.

14. The hedonic price schedule $p_m = \beta_0 + \beta'z_m$ is linear because we have assumed that marginal costs $C(z)$ are linear in characteristics and independent of quantity. Jones (1988) has recently examined conditions on consumer preferences which imply a linear hedonic price schedule, and found that these are very restrictive.

15. With the year dummies appearing in (18), an alternative way to measure the price-cost margins appearing in the numerators on the right of (17) is $(p_{mt} - \beta_0 - \beta_t - \beta'z_{mt})$. We chose to use $(p_{mt} - \beta_0 - \beta'z_{mt})$ in (17) to slightly simplify the estimation, but the two formulations are equivalent when $-\gamma_1 = \gamma_2$.

16. A measure of the distance between a model and its competitors is in the denominator of several of the terms in (17) and (18). A few models have as neighbors a twin model that is always made by the same parent firm and has absolutely identical characteristics (as we measure them.) Hence the distance between the model and the twin is zero. We combined the sales figures for these twin models and include them as one model.

17. The characteristics for each model are those that come standard with the base model.

18. All data are available as ascii files on floppy disk upon request to the authors.

19. We describe the identification of neighbors in much greater detail, using a more general utility function, in Levinsohn and Feenstra (1989).

20. As discussed in section 2.1, a consumer with ideal product $z^* = (z_m + z_n)/2$ is indifferent between purchased models m and n when their prices satisfy $\pi_m = \pi_n$. In our search for neighbors, we are implicitly assuming that $\pi_m = \pi_n \forall m, n$. Alternatively, we are assuming that σ is very small in (9), so that the term $\sigma(\pi_m - \pi_n)$ vanishes.

21. Since we do not map out the hyperplanes which serve as boundaries between market spaces, our procedure can falsely reject two models as neighbors, but can never falsely accept. This is illustrated in Figure 2, where the mid-point of a line between B and C lies

in S_a . This means that the consumer whose ideal product is midway between B and C would prefer model A, and our procedure would reject B and C as neighbors. This rejection is false, however, since S_b and S_c have a non-zero intersection as illustrated. Heuristically, the false rejection of neighbors seems more likely for models near the boundary of S .

22. The programs for Gauss-Newton are written in Fortran 77. The source code is available from the authors on request.

23. See Quandt (1983) for a discussion of why this is so.

24. It is not difficult to analytically compute the derivatives of (17) and (18) with respect to (β, γ, λ) , except for the derivatives with respect to the term $\Gamma_{mt}(\beta, z)$ defined in Proposition 2. We compute $d(\Gamma_1, \dots, \Gamma_m)/d\beta = (C + C^2 + C^3 + \dots)d(H_1, H_2, \dots, H_m)/d\beta$ for each t , and are therefore ignoring the change in C with respect to β . We believe this simplification is unimportant because the pattern of zero and positive elements in C is invariant to β (for given sets of neighbors I_m).

25. Experiments show that while the set I_m does change slightly from year to year, the average of the squared Euclidean distances, which is what matters, is quite stable.

26. We will use the term "statistically significant" to mean statistically different from zero at the 95 percent confidence level unless we indicate otherwise.

27. Although not reported in Table 2, we also estimate the model with the additional cross equation constraint that $-\gamma_1 = 1/\lambda_1$ imposed. As a quick glance at the estimates of the γ 's and λ 's indicates, this restriction is resoundingly rejected. Discussion of why this might be so and the economic implications of this rejection are discussed in Section 6.

28. The reader will note, though, that coefficient estimates in the restricted and unrestricted cases are all very similar.

29. These results are available on request to the authors.

30. Bresnahan (1981), in contrast, evaluates the simpler integral that results from his Hotelling model. He appears to impose the cross- and within-equation restrictions we test and arrives at reasonable demand elasticities averaging about -2.3.

31. See Levinsohn (1988) for a discussion of some of these issues.
32. The low R^2 in the demand equation is not surprising since the residual includes $\ln Q_{mt}^*$ from (11).
33. Our proxy for luxury is $(AIR + 1) * Legroom * Headroom$
34. We do not substitute for WEIGHT, since without it many of the distances between models are close to zero (i.e. products are not sufficiently differentiated.)
35. We report the elasticity of demand from the demand instead of pricing equation in column 1 of Table 3. This is because had we reported the more sensible elasticity from the pricing equation, we would have no information about how robust estimates of the γ 's were.

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Figure 1

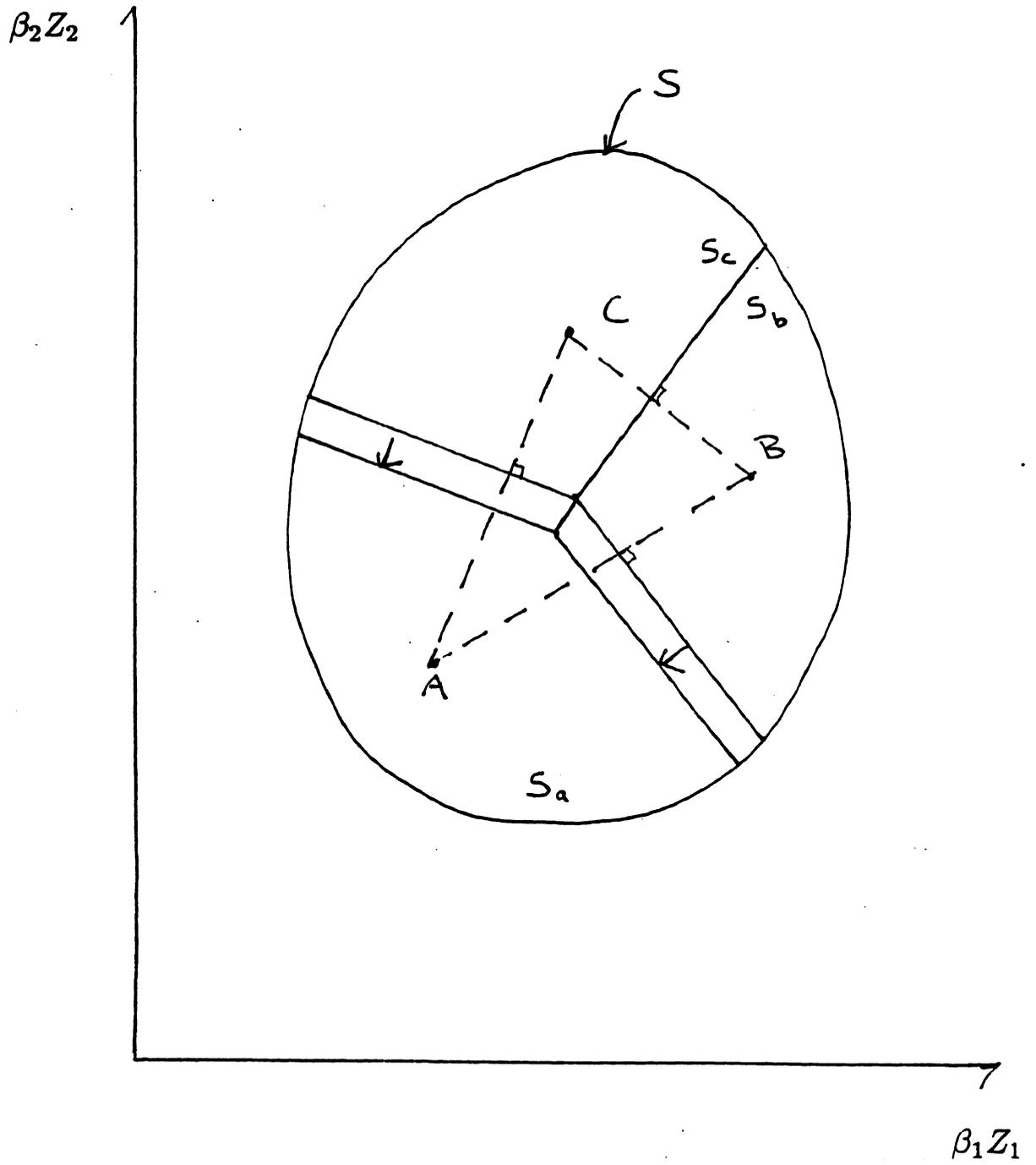


Figure 2

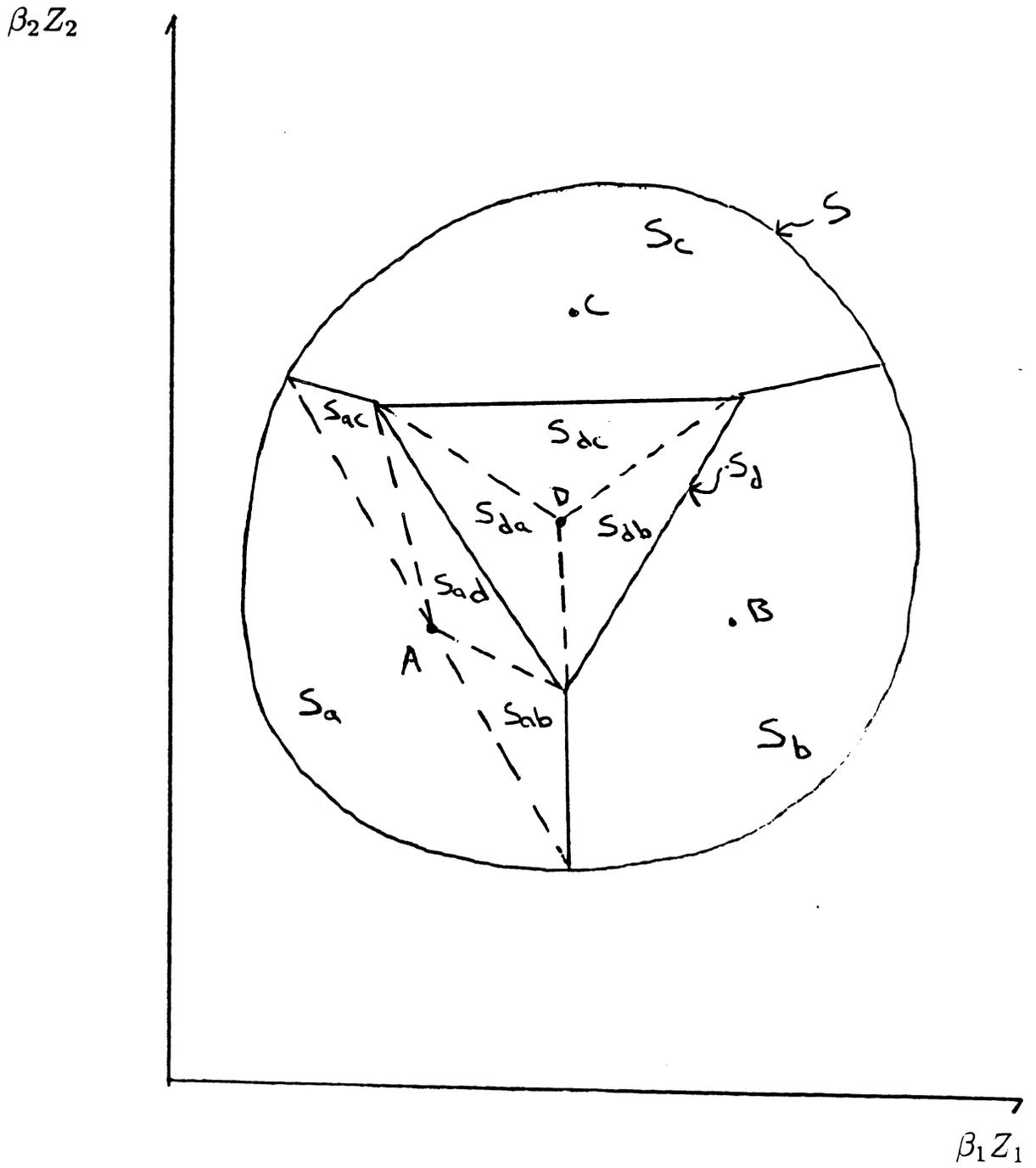


Figure 3

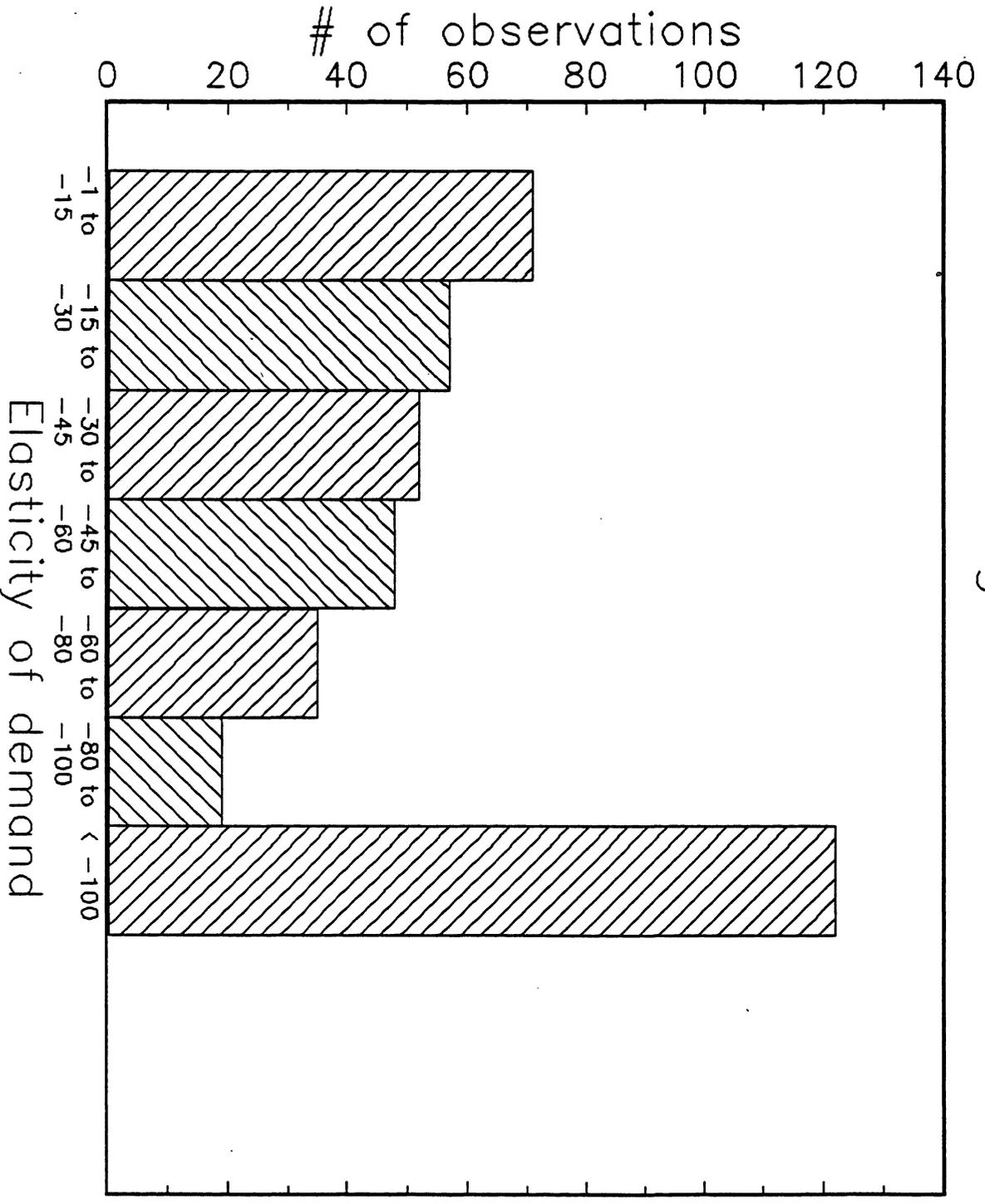


Figure 4

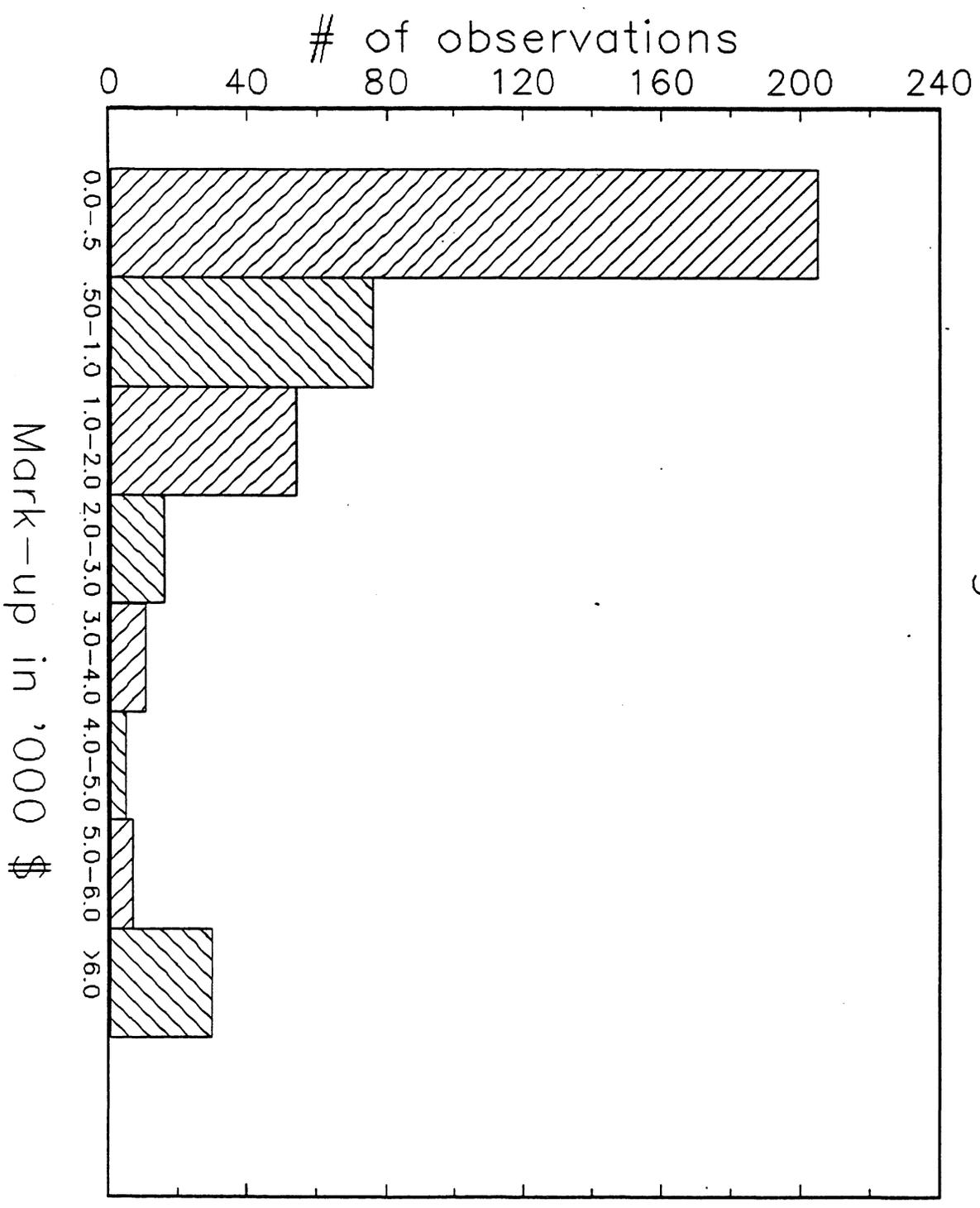


Figure 5

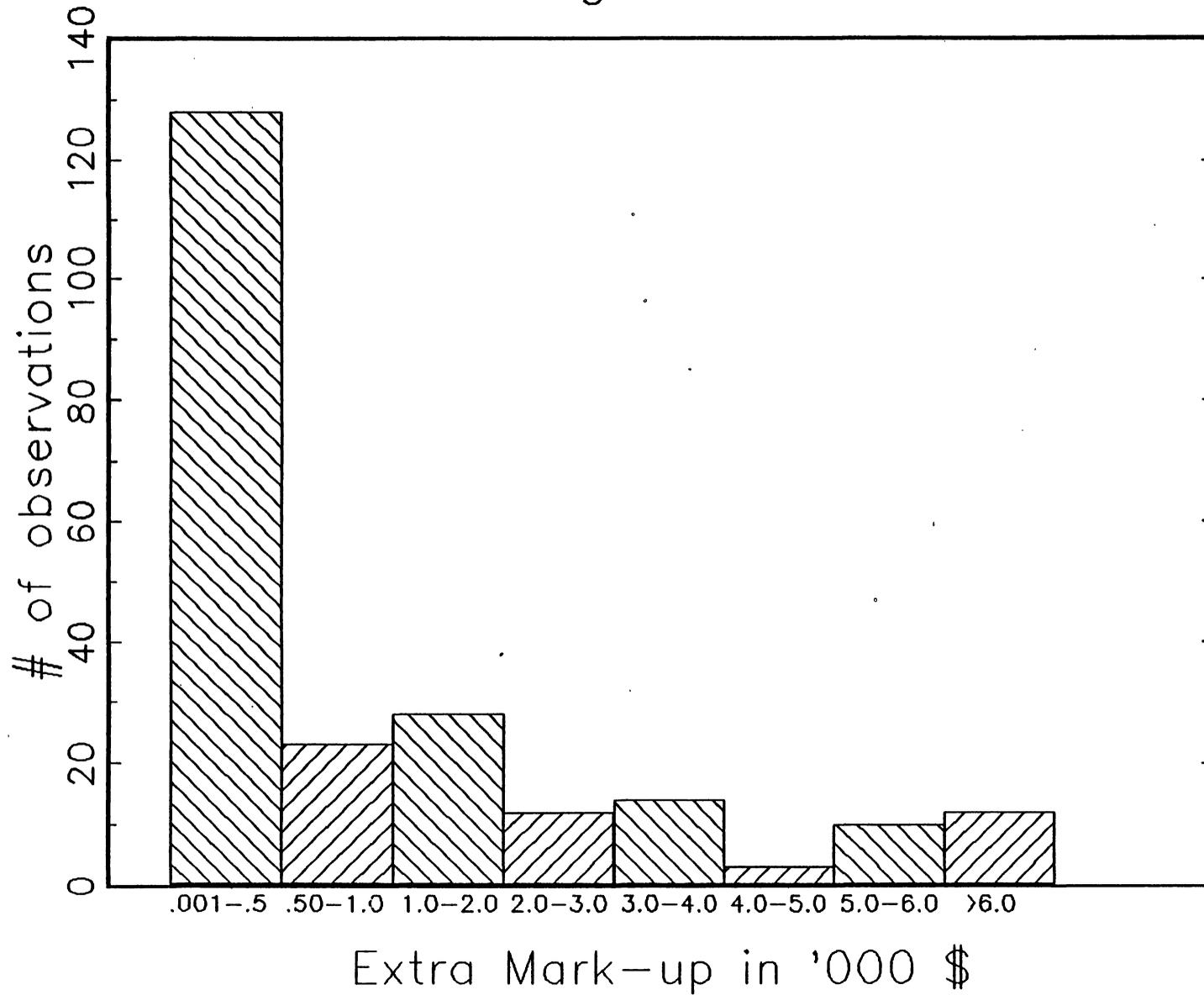


TABLE 1										
Models and Their Neighbors										
Model Number and Name		Model Number of Neighbors								
1	Toyota Tercel	12	18	19	43					
2	Toyota Corolla	40	49	53	81					
3	Toyota Celica	10	30	35	51					
4	Toyata Camry	11	13	23	32	50	62			
5	Toyota Cressida	6	8	27	37					
6	Toyota Supra	5	9	39	73					
7	Nissan Sentra	18								
8	Nissan Maxima	5	22	27	60	74	79			
9	Nissan 300zx	6	35	38	39	73				
10	Nissan 200SX	3	11	32	45	50				
11	Nissan Stanza	4	10	13	14	32	50	86		
12	Nissan PulsarNX	1	15	18	19					
13	Honda Accord	4	11	14	40	82	86			
14	Honda Prelude	11	13	33	34	45				
15	Honda Civic	12	19	20	43					
16	Mazda 626	62	82							
17	Mazda RX-7	34	45							
18	Mazda GLC	7	12	1	43					
19	Suburu DL/GL	12	15	1	53					
20	Chry/Ply Colt	15	43							
21	Volvo DL	23	26	31	42	56	71	72	84	
22	Volvo 760 GLE	8	25	26	35	37	38	74	79	
23	VW Jetta	4	21	24	32	42	62	82	86	
24	VW Quantum	23	31	32	36	41	42	65	72	
25	BMW 320/318	22	30	31	38	56				
26	BMW 530/528	21	22	28	31	56	79	84		
27	BMW 733	5	8	28	37	39	57	59	60	61 73
28	Mercedes 300D	26	27	29	47	52	60	64	68	75 78
29	Mercedes 300SD	28	47	59	66	67	68			
30	Mercedes 190E	3	25	36						
31	Audi 5000	21	24	25	26	36	41	72		
32	Audi 4000	4	10	11	23	24	36	50	65	
33	Mitsub Tredia	14	34	45	70	76				
34	Mitsub Cordia	14	17	33	49					
35	Mitsub Starion	3	9	22	38	56	74			
36	Saab 900 S	24	30	31	32	51	72			

TABLE 1 (continued)											
Models and Their Neighbors											
Model Number and Name		Model Number of Neighbors									
37	Saab 900 Turbo	5	22	27	38	39					
38	Porsche 944	9	22	25	35	37					
39	Porsche 911	6	9	27	37	73					
40	Isuzu I-mark	2	13	82							
41	Isuzu Impulse	24	31	54	65	71	79	83	84		
42	Peugeot 505	21	23	24	44	71	72	84			
43	Alliance	15	18	20	1	49	69				
44	Eagle	42	71	72	78	85					
45	Turismo	10	14	17	33						
46	LeBaron	50	51	54	63	65	76				
47	NewYorker/5thAv	28	29	52	57	58	59	66	68		
48	Omni	81									
49	Charger	2	34	43	76	81					
50	Aries	4	10	11	32	46	51	62	70		
51	Dodge600 4dr	3	36	46	50	56					
52	Diplomat	28	47	57	60	68	75	78			
53	Escort	2	19	69	86						
54	Mustang	41	46	63	65	72	76	83			
55	Tempo	70	76	82							
56	T-bird	21	25	26	35	51	64	71	72	74	79 85
57	Crown Victoria	27	47	52	58	59	60	68			
58	Grand Marquis	47	57	59	66	68	80				
59	Continental	27	29	47	57	58	60	66	68	80	
60	MarkVii	8	27	28	52	57	59	64	68		
61	Lincoln	27	67	80							
62	Skyhawk	4	16	23	50	63	65				
63	Skylark	46	54	62	65	72	76				
64	Electra	28	56	60	74	75	78	79	85		
65	Cimarron	24	32	41	46	54	62	63	76		
66	Seville	29	47	58	59	68	80				
67	Cadillac	29	61	80							
68	ElDorado	28	29	47	52	57	58	59	60	66	
69	Chevette	43	53	81							

TABLE 1 (continued)											
Models and Their Neighbors											
Model Number and Name		Model Number of Neighbors									
70	Cavalier	33	50	55	76						
71	Camaro	21	41	42	44	56	83	85			
72	Celebrity	21	24	31	36	42	44	54	56	63	84
73	Corvette	6	9	27	39						
74	MonteCarlo	8	22	35	56	64	75	79	85		
75	Chevrolet Impala	28	52	64	74	77	78				
76	Firenza	33	46	49	54	55	63	65	70		
77	Cutlass/Supreme	75	78	79	84	85					
78	Olds88	28	44	52	64	75	77				
79	Olds98	8	22	26	41	56	64	74	77	84	
80	Toronado	58	59	61	66	67					
81	1000	2	48	49	69						
82	Sunbird (2000)	13	16	23	40	55					
83	Firebird	41	54	71							
84	Bonneville	21	26	41	42	72	77	79			
85	GrandPrix	44	56	64	71	74	77				
86	Rabbit	11	13	23	53						

TABLE 2				
Estimates of Demand and Oligopoly Pricing Equations				
Restrictions Imposed				
	No Restrictions	$-\gamma_1 = \gamma_2$	$-\gamma_1 = \gamma_2$ $\lambda_1 = \lambda_2$	$-\gamma_1 = \gamma_2$ $\lambda_1 = \lambda_2$ Results with New β
γ_1	-.0149* (.0054)	-.0101* (.0047)	-.0099* (.0045)	-.0064 (.0040)
γ_2	.0063 (.0054)	.0101* (.0047)	.0099* (.0045)	.0064 (.0040)
λ_1	.387* (.083)	.451* (.090)	.410* (.071)	.405* (.075)
λ_2	.332* (.087)	.374* (.093)	.410* (.071)	.405* (.075)
β_0	-16.171* (.402)	-16.844* (.421)	-16.769* (.422)	-14.965* (.486)
β_{wt}	3.146* (.241)	2.987* (.238)	2.926* (.204)	2.675* (.214)
β_{hp}	.0441* (.0053)	.0398* (.0052)	.0416* (.0044)	.0474* (.0046)
β_{air}	4.579* (.398)	4.885* (.398)	4.894* (.398)	4.819* (.406)
β_{euro}	5.454* (.438)	5.337* (.438)	5.370* (.430)	5.185* (.444)
β_{aero}	561.67* (38.69)	635.33* (32.47)	631.72* (32.40)	544.00* (37.70)
d_0	3.9034* (.128)	3.899* (.128)	3.899* (.127)	3.907* (.127)
R^2_{demand}	.057	.051	.050	.044
$R^2_{pricing}$.815	.816	.816	.818
SSR_{demand}	518.36	522.02	522.15	525.62
$SSR_{pricing}$	3633.20	3618.34	3616.59	3578.24

Notes: The standard errors are reported in parentheses. An asterisk indicates significance at the 95 % level.

TABLE 3			
Sensitivity Analysis			
Re-estimation Of The System Using Different Characteristics $-\gamma_1 = \gamma_2$ and $\lambda_1 = \lambda_2$			
	Average Own Elasticity of Demand from Demand Equation	Average Mark-up	Average Extra Mark-up Due to Joint Profit Maximization
Base Case (NLS)	-0.516	\$599	\$606
Drop HP Use DISP	-0.159	\$717	\$543
Drop AIR Use LUXURY	-0.571	\$556	\$378
Drop EUROPE Use FORIEGN	-0.272	\$856	\$534
Drop AERO Use SHORT	-0.405	\$1363	\$998