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Different Environmental Services for Different Income Groups in LDC Cities: Second-Best Efficiency Arguments

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I. Introduction

The conventional arguments for providing basic environmental services to the poor in less developed country (LDC) cities run to externalities and merit-goods. These arguments urge that cities make some kind of basic service available to the poor because they will otherwise suffer socially unacceptable consequences to themselves and/or impose negative externalities on others.

We offer different kinds of arguments for providing basic environmental services to the poor in LDC cities: second-best efficiency arguments. When the poor are many and very poor, the city may maximize social welfare, within a budget constraint, by offering two kinds of basic services to its residents, a first-class service that the rich will want and a second-class service that the poor will be able to afford. The cases considered here can be added to the well-known merit-good and externality arguments. These arguments also give a theoretical foundation to the growing practice of offering different classes of service at different prices to different income groups in LDC cities.

This concern about differential provision of urban environmental services is not just a theoretical point. Consider three important examples where three different classes of service are possible. For drinking water, there are in-house taps, neighborhood taps or kiosks, and distant unclean rivers or pools. For sewage, there are in-house flush toilets, neighborhood flush or chemical latrines, and "bush toilets". And for solid waste collection, there are regular curbside pickups, neighborhood dumpsters, and private scavenging with on-street litter and degradation. In each case, the first-class and second-class methods are vastly superior to third-class "provision". Below, we will formally model the supply and demand -- and pricing -- of first-class and second-class service provision by the municipality.

What makes these models particularly relevant to LDC cities -- as opposed to cities in more prosperous countries -- are three things: 1) inadequate revenue sources and hence strict budget constraints on service provision, which forces LDC municipalities to charge prices above marginal costs; 2) sizeable income differentials between rich and poor in the cities, which leads to greatly different effective demands for basic urban services; and 3) large numbers of poor people, relative to the numbers of rich, which makes the total revenue earned from the poor and the consumer surplus derived by the poor quantitatively important magnitudes, despite the low effective demands of poor households.

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2 Department of Economics, Lorch Hall, The University of Michigan, Ann Arbor, MI 48109 (E-mail: CAROLYN.FISCHER@UM.CC.UMICH.EDU or GD46@UM.CC.UMICH.EDU). We are indebted to Jane Hall for suggesting that we formalize these arguments.

3 For a survey of the conventional issues and criteria for urban environmental service provision and pricing, see Bahl and Linn, 1992, Chapters 9 and 10.
What follows are three exercises in maximizing overall consumer surplus -- simply summed, across rich and poor groups, without income distribution weights -- subject to a constraint on the total revenue that needs to be generated. In this paper, we will not raise any issues of externalities or merit-good attributes. But in each of the three exercises, the possibility will arise that it is socially optimal to provide two types of service and to price these services in a manner that induces the rich to buy first-class service and the poor second-class service.

Briefly, the three different second-best efficiency arguments developed here:

1. **Price Discrimination.** If the municipality needs to raise prices above marginal costs, its ever higher prices gather revenues from the rich but drive the poor out of the market. If the poor are so driven out, they neither derive consumer surplus from consumption nor generate revenue for the city. A two-price system permits the municipality to charge high prices to the rich, and hence gain high revenues from them, while charging low prices to the poor, generating both consumer surplus and revenue there. (Section III)

2. **Marginal-Cost Differentials.** If the marginal costs of providing different types of service differ, then the city may be able to generate more consumer surplus for any given total revenue by offering different services to different income groups. (Section IV)

3. **Capital-Market Imperfections.** If there are once-and-for-all hookup costs to be covered, and the interest rates of the poor are distorted upward by capital-market imperfections, then the city may be able to increase total consumer surplus, for any given present value of total revenue, by offering the poor subsidized hookup. The subsidies would then be recaptured through either higher hookup fees on the rich or higher per-unit prices for all users. (Section V)

Section II develops the demand and cost framework that will be used for each of these three cases.

II. **The Demand for the Two Types of Service**

Consider two groups of municipal residents, which for brevity we will call the Rich and the Poor (with subscripts $i$ equal to $r$ and $p$, respectively). They have identical tastes for a particular urban service, though they do not have identical incomes with which to purchase it. The service can be made available in two different ways, which we will call "first-class" service and "second-class" service (with subscripts $j$ equal to 1 and 2, respectively). Households, whether Rich or Poor, must decide which of the two classes of services to purchase, and how much of that service to consume.\(^3\)

Since the purpose of the paper is simply to show the existence of these second-best arguments, any plausible demand structure will suffice. So we choose a very simple demand structure. All Rich households are identical, and all Poor households are identical. Demand by each household in each income group for each type of service is assumed to be linear, ranging from a willingness to pay for the first unit of the $j$th service of $A_jY_j$ (where $A_j$ is the fraction of income that each group is willing to pay and $Y_j$ is the income of each member of

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\(^3\) Of course, if the prices of the services are sufficiently high, a household may decide to purchase neither first-class service nor second-class service, relying either on no service at all or on some private-sector "third-class" delivery system -- which is not modeled here.
the group) down to a maximum consumption level of $B_j$ (where the household is satiated at a price of zero). In functional form, the quantity demanded of the $j$th service by a household in the $i$th income group ($Q_{ij}$) is

$$Q_{ij} = \begin{cases} 
0 & \quad \text{if } P_i > A_i Y_i \\
\frac{B_i(A_i Y_i - P_i)}{A_i Y_i} & \quad \text{if } 0 < P_i < A_i Y_i \\
B_j & \quad \text{if } P_i = 0 
\end{cases}$$

Note three things about the demand function:

1. The point of satiation (i.e. $B_j$) is independent of income. The reason for this is that, at a price of zero, income provides no constraint to consumption; since each income group has identical tastes, each reaches satiation at the same consumption level. But the point of satiation differs according to which service is received.

2. The relevant elasticities will depend upon where the household is located on the demand curve. The price elasticity of demand for the service ranges from infinity (when $Q_{ij}$ is zero) to zero (when $P_i$ is zero). The income elasticity also ranges from infinity (when $Q_{ij}$ is zero) to zero (when $P_i$ is zero).

3. Households never buy both types of service; they buy first-class service or second-class service or neither service, depending upon which choice maximizes their consumer surplus.

Some assumptions about parameters are sensible. Of course, $Y_r > Y_p$, by the definition of Rich and Poor. $A_r > A_p$, because every household is willing to pay more for the first unit of first-class service than for the first unit of second-class service, whatever its income. $B_r > B_p$, because the satiation point is reached at a higher consumption level for first-class service than for second-class service. We will also assume that the numbers of the Poor ($N_p$) are greater than the Rich ($N_r$).

Figure 1 illustrates the market demand curves for all the Rich and all the Poor for service type $j$.

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4. This means that, for the demand curve for either class of service, the price-axis intercept is assumed to depend upon income, but the quantity-axis intercept is assumed not to depend upon income. While this seems a reasonable simple characterization of the demand for many municipal environmental services, it does ignore complementarity -- the Rich own pools, gardens, and washing machines and hence would consume more water than the Poor (of either class of service) were it free. Most of the results below follow if $B_j$ is changed throughout to $B_j Y_p$, though many do not follow if $A_j Y_r$ is changed to just $A_r$. 

We also need some information about the costs of providing these services. Again, to keep things very simple, we assume that there are constant marginal costs to supplying each class of service, $C_j$. But there are also overhead costs to the system of providing these services. The municipality is assumed to be constrained fiscally, and it must raise revenues beyond its marginal costs to help meet these overhead costs -- or to help meet some other fiscal problem. We shall think of these overhead costs as simply being some unspecified lump-sum -- independent of the types of service provided. Both of these assumptions are, of course, arguable, but they will suffice to show the existence of the possibility that it is socially optimal to provide both types of service.

Each household, whether Rich or Poor, takes the price structure as given, calculates how much of each service it would buy if it bought that class of service, calculates the consumer surplus it would get from this optimal purchase in each class, and chooses the class of service (or neither service) that offers the higher consumer surplus. The municipality also measures welfare through calculation of consumer surplus, simply summed across the $N_R$ Rich households and the $N_P$ Poor households.

In the range of prices between zero and $A_j Y_i$, where consumption is positive but not satiated, the consumer surplus of a particular household of income $i$ buying service type $j$ ($CS_{ij}$) is

\[ CS_{ij} = \frac{Q_j (A_j Y_i - P_j)}{2} \]

or, substituting from equation (1),

\[ CS_{ij} = \frac{B_j (A_j Y_i - P_j)^2}{2A_j Y_i} \]

Finally, we must remember the municipality's need for net total revenue to be generated from the provision of these services -- where net total revenue means the revenue beyond that which covers marginal cost and which becomes available for overhead. In the range of prices between zero and $A_j Y_i$, the net total revenue contributed by a particular household of income $i$ buying service type $j$ ($TR_{ij}$) is

\[ TR_{ij} = (P_j - C)Q_j \]

or, substituting from equation (1),

\[ TR_{ij} = \frac{B_j (A_j Y_i - P_j)(P_j - C)}{A_j Y_i} \]

It is the relationship of consumer surplus ($CS$) and the net total revenue ($TR$), each summed over both $i$ and both $j$, that is the focus of the municipality's choice of price and service offerings.

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3 In the Sections III and V, we shall set $C_1 = C_2 = 0$ since marginal costs play no role in the argument there. In Section IV, however, they are important.
4 A first step toward variation in the overhead cost structure is taken in Section V, where "hookup" costs and fees are introduced.
III. The First Efficiency Case: Price Discrimination

The municipality is interested in expanding the aggregate consumer surplus (CS) of its residents, but it also needs to collect net total revenue (TR) in order to meet the overhead costs of the services. These two goals may conflict. Solving equation (3) for $P_j$ and

substituting that into equation (5) yields a quadratic relationship between $TR_j$ and $CS_j$:

$$TR_j = -2CS_j + \sqrt{(2A_jY_jB_j)CS_j^3}.$$  

Note that equation (6) ignores marginal costs (i.e. $C_j = 0$ for both $j$); their existence plays no role in this case, and they will be ignored throughout this section. $TR_j$ and $CS_j$ both rise as $P_j$ is reduced from $A_jY_j$ to $A_jY_j/2$; in this range of $P_j$ -- the price-elastic range -- there is no conflict between the increased generation of CS and the increased collection of TR. Thus, the municipality would always choose a value of $P_j$ between 0 and $A_jY_j/2$ if it were to offer the $j$th type of service just to households of the $i$th income level. All this is pictured in Figure 2.\footnote{The parameter values underlying Figure 2 are: $A_1=0.25; A_2=0.20; B_1=1.50; B_2=1.00; C_1=C_2=0; Y_1=6.00; Y_2=1.00; N_1=1; and N_2=4.}

The municipality has only one decision to make -- the prices of the two classes of services it will make available to the city's residents.\footnote{The assumption that both classes of service are offered to all residents is not restrictive, since the municipality can always charge a high (even infinite) price for a type of service, which is effectively the same thing as not offering the service at all. Furthermore, there is no change in overhead costs, by assumption in this section, from offering different types of service.} We will assume, in this section, that all residents face the same price structure; the municipality is unable or unwilling to charge different prices to different consumers depending on their incomes.\footnote{This assumption will be dropped in the next section.}
Once the municipality has set the prices of the two classes of service, households must decide which class of service they will buy. By equating $CS_h$ and $CS_0$ in equation (3), we can derive the switchover locus of prices ($P_1$ and $P_2$) that divides the selection of first-class service from the selection of second-class service for households in income group $i$:

\[
\frac{B_1(A_1Y_i-P_1)^2}{2A_1Y_i} = \frac{B_2(A_2Y_i-P_2)^2}{2A_2Y_i}.
\]

Manipulation of equation (7) permits $P_2$ to be written explicitly as a linear function of $P_1$:

\[
P_2 = \left[a - (a/b)^{1/2}\right]A_1Y_i + \left(a/b\right)^{1/2}P_1,
\]

again for each $i$, and where $a=A_1/A_2$ and $b=B_1/B_2$.\footnote{Both $a$ and $b$ are between zero and one, by our assumption of the nature of the demands for the two types of service.}

The intercept of equation (8) is negative for all income levels and is a larger negative the higher the group's income level. The slope of equation (8) is positive, being greater or less than one as $a$ is greater or less than $b$.

We can now picture the entire choice set for each income group. In principle, there are nine possible combinations of choices: the Rich buy type 1, type 2, or nothing; and the Poor buy type 1, type 2, or nothing. In fact, however, three of these nine possible combinations cannot be generated by any price combination: 1) the Rich buy second-class service while the Poor buy first-class; 2) the Rich buy nothing while the Poor buy second-class service; or 3) the Rich buy nothing while the Poor buy first-class service. The remaining six possible outcomes, and the combinations of $P_1$ and $P_2$ that induce them, are illustrated in Figure 3.\footnote{The parameter values in Figure 3 are the same as were used in Figure 2. The 0 in Figure 3 indicates that the income group buys neither first-class or second-class service in this range of prices.}

What type of service, and in what amounts, ends up being consumed by each income group clearly depends upon the price combination, $P_1$ and $P_2$, offered by the municipality. That combination in turn depends upon the net total revenue ($TR$) the municipality needs to collect. Let us start with zero need for net total revenue ($TR=0$). Clearly, the municipality will supply both services at zero prices (i.e. $P_1=P_2=0$), and all residents, Rich and Poor, will select the first-class service.\footnote{Since all residents will choose first-class service here, the price of the second-class service ($P_2$) is irrelevant. We arbitrarily start $P_2$ at zero.}

As the needed net total revenue becomes positive, the city's maximization of total consumer surplus, subject to this positive revenue constraint, will require the municipality to raise $P_1$ above zero. Both groups continue to demand first-class service. Thus, the first region of optimal pricing as the need for $TR$ grows from zero starts from the origin in Figure 4; it is the dark solid line where $P_2=0$ and $P_1>0$, labeled (1) in Figure 4.

At some positive $P_1$, if $P_2$ remains at zero, the Poor would choose to switch to second-class service. The exact switchover price for first-class service is found by solving equation (8) for $P_1$ at $P_2=0$ and $Y_i=Y_r$. This switchover price is...
Choices of Rich and Poor among Services
(for various values of P1 and P2)

Optimal Path of Prices
(as total revenue needs increase)
(9) \[ P_1 = (1 - \sqrt{a/b})A_1Y_p \]

But it is not socially desirable for the municipality to induce the Poor to switch. With the first-class demand curve to the right of the second-class demand curve at all prices (since both \(a\) and \(b\) are less than one), more revenue can be raised and more consumer surplus generated by continuing to provide first-class service to the Poor. To induce this, the municipality must begin to raise \(P_2\) as it raises \(P_1\). Thus, the second region of optimal pricing as the required TR increases is the upward-sloped stretch along the Poor's switchover (from first-class to second-class service) line, with the municipality keeping \(P_2\) just enough above the switchover locus to prevent the switch. This stretch is the dark solid line in Figure 4 that is labeled (2).

How far up the Poor switchover line does the optimal pricing locus go? Until no larger net total revenue can be obtained with both income groups consuming first-class service. Remember, at \(P_T = A_1Y/2\), the net total revenue extracted from sales to the Poor begins to decline, though the net total revenue gathered from the Rich continues to rise with increases in \(P_T\). At some point along this switchover line -- at some value of \(P_T\) above \(A_1Y/2\) -- it is socially optimal to set a price of second-class service low enough to induce the Poor, but not the Rich, to switch to second-class service. Thus, the third region of optimal pricing as the required TR increases is the dotted leap to the southeast -- labeled (3) in Figure 4 -- where the Poor switch to buying second-class service while the Rich continue to buy first-class service.\(^{13}\) This region (3) is a dotted line, rather than a solid line, because movement along it does not represent ever larger net total revenue -- the TR is equal at both ends of the dotted line.

Where in this region are the optimal \((P_1, P_2)\) price combinations? They are readily found. Write out total CS for both Rich and Poor (using equation (3) to form the sum, \(CS_r + CS_p\)), write out total TR (using equation (5) to form the sum, \(TR_r + TR_p\)), and maximize this CS subject to the achievement of a given level of TR.\(^{14}\) This yields the following optimal relationship between \(P_1\) and \(P_2\):

(10) \[ P_2 = (ay)P_1 \]

where \(y = Y/Y_p.\)\(^{15}\) The dotted line in Figure 4 therefore leaps to a point on this line.

As the required TR continues to grow, \(P_T\) and \(P_2\) continue to be raised along the line described by equation (10). This is the fourth region of optimal pricing, the dark solid line in Figure 4 that is labeled (4). In this region, the Rich continue to buy first-class service and the

\(^{13}\) It is possible that this dotted leap (i.e. region (3)) does not occur, with TR maximized at the end-point of region (2). To understand this intuitively, simply think of the numbers and the incomes of the Rich and the Poor being identical -- then there is no scope for increasing TR through price (and service type) discrimination. In LDC cities, where both the numbers and the incomes of the Poor and the Rich are very different, the existence of region (3) is eminently plausible.

\(^{14}\) Recall that \(C_r = C_p = 0\) throughout this section.

\(^{15}\) Equation (10) is simply the locus of points where the price elasticity of demand by the Rich for first-class service equals the price elasticity of demand by the Poor for second-class service. This equality of price elasticity often arises in price-discrimination situations. (Equation (10) is considerably more complex when \(C_r\) and \(C_p\) greater than zero are considered.)
Poor continue to buy second-class service, with \( P_1 \) and \( P_2 \) rising proportionately. This region continues to the northeast until either 1) \( P_1 \) and \( P_2 \) reach \( A_1 Y/2 \) and \( A_2 Y/2 \), respectively, at which point the maximum \( TR \) possible has been attained, or 2) \( P_1 \) and \( P_2 \) reach the switchover locus at which point the Rich begin to demand second-class service. If the Rich switchover locus is reached first, then a fifth and final region of optimal pricing follows this Rich switchover line to the northeast, the dark solid line in Figure 4 that is labeled (5). \( P_2 \) must be kept just above this switchover line, so as to keep the Rich from preferring second-class service. How far does this fifth region go? At \( P_1 = A_1 Y/2 \), the total revenue collected from the Poor reaches its maximum; region (5) certainly extends past this point. At \( P_1 = A_2 Y/2 \), the total revenue collected from the Rich begins to decline; region (5) certainly stops before this point. Somewhere between these two points, the municipality has exhausted its ability to increase \( TR \), at any cost in \( CS \).

Figure 5 illustrates all these regions, and more. For the same basic set of parameter values, the feasible \( CS \) and \( TR \) combinations are shown for over 400 pairs of values of \( P_1 \) and \( P_2 \).

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14 This proportionality only holds if marginal costs are zero. When marginal costs are brought in, it nevertheless can be shown that the optimal municipal markup over marginal cost in this region, \((P_1 - C_1)/C_1\), is smaller for the Poor (i.e. for second-class service) than for the Rich (i.e. for first-class service). Note that this differential markup does not occur for income distribution reasons.

17 The existence of this fifth region requires that

\[ y < 2 - \sqrt{2} \] .

In the numerical example being used, this condition is met, and hence there is a region (5).

18 Given the assumed shapes of the demand curves, there is never a reason to push the Rich into consumption of second-class service -- it is always possible to generate both more \( CS \) for the Rich and more \( TR \) from the Rich through provision of first-class service.

19 It is also possible that the dotted leap -- region (3) -- goes directly from region (2) to region (5).
Here, as along the dark solid lines of Figure 4, we see that two methods of providing the service dominate the others (in the sense that they can always generate more CS and more TR): 1) if the requirement for net total revenue is low, Rich and Poor are both provided with first-class service at low price; or 2) if the requirement for net total revenue is high, the prices are raised in a fashion that induces the Rich to buy first-class service and the Poor to buy second-class service.

IV. The Second Efficiency Case: Marginal-Cost Differentials

In Section III, the optimality of offering multiple tiers of service provision, along with different corresponding prices, resulted from the inability of the municipality to discriminate between Rich and Poor in other ways. If the city had been able to charge different prices to different income groups for the same service (i.e., $P_i$ rather than the same $P_i$ to each $i$), then it would have done so and continued to provide first-class service to both income groups. If the marginal cost of providing first-class service is less than or equal to the marginal cost of providing second-class service (i.e., $C_1 \leq C_2$), first-class service dominates second-class service in the sense that one can always get more of both TR and CS from any given household, regardless of income, by offering first-class service than by offering second-class service.

But when the marginal cost of delivering first-class service exceeds that of delivering second-class service (i.e., $C_1 > C_2$), this may no longer be true. In this case, depending on the income level, second-class service may yield greater TR for any given CS. The conditions for this are readily uncovered. Equation (8) gives us the iso-CS locus of prices for the $i$th income group:

$$ P_2 = \left[ a - (ab) \frac{1}{2} \right] A_i Y_i + (ab) \frac{1}{2} P_1 $$

Equation (5) tells us that $TR_H > TR_a$ for this $i$th income group as

$$ \left( \frac{A_i Y_i - P_1}{A_2 Y_i - P_2} \right) > \left( \frac{b}{a} \right) \left( \frac{P_2 - C_2}{P_1 - C_1} \right) $$

Substituting the iso-CS value of $P_2$ from equation (8) into this condition (11), we find that $TR_H < TR_a$ for a given CS as

$$ \left( \frac{A_i Y_i - C_2}{A_2 Y_i - C_2} \right)^2 > \left( \frac{b}{a} \right) $$

Only the greater-than sign was possible in inequality (12) when $C_1 = C_2 = 0$ (as in Section III), but either sign is possible when $C_1 > C_2 > 0$ is considered.

Furthermore, it is possible that, when only one service is to be offered to an income group, the Poor will generate more TR for given CS with second-class service while the Rich will generate more TR for given CS with first-class service. The conditions for this follow readily from conditions (12):

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20 Values of the prices range over a grid of $0 < P_1 < 0.80$ and $0 < P_2 < 0.60$. That there are far fewer than 400 observations visible in Figure 5 reflects the fact that different $P_1$ and $P_2$ combinations often generate the same CS and TR values.
Whether inequality (13) would be met in fact is an empirical question, but it is at least feasible. It requires, as a necessary but not sufficient condition, that

\[
\frac{A_1Y - C_1}{A_2Y - C_2} < (b/a) < \frac{A_1Y - C_1}{A_2Y - C_2}
\]

(13)

(14)

\[
\frac{C_2}{C_1} < \frac{A_2}{A_1}
\]

Condition (14) gains meaning by noticing that it must be fulfilled if, when the municipality practices marginal-cost pricing, households with higher incomes are more likely to prefer first-class service than households with lower incomes.

Even if the city could discriminate between the Rich and the Poor -- that is, charge them different prices for the same class of service (which we assumed was not possible in Section III) -- it might choose not to. Social welfare for given total revenue might be maximized by offering a single set of prices to both income groups and letting the Rich choose first-class service and the Poor choose second-class service.

All this, of course, is not necessarily a second-best argument. Even with marginal-cost pricing and no net total revenue requirement, social welfare might be higher under a two-tiered rather than first-class-only system of services. And when the city is revenue-constrained, providing both kinds of service enhances its revenue-gathering ability -- indeed, the municipality can collect more net total revenue for any given total consumer surplus by offering both classes of service. Thus, the marginal-cost differential itself, regardless of the revenue constraint, can justify offering two types of service.

V. The Third Efficiency Case: Capital-Market Imperfections

There are really three different kinds of costs associated with the delivery of most municipal services: 1) the basic capital costs of initiating the system -- the need to cover which has been the basis here for generating net total revenue; 2) the hookup costs for attaching any household to this overall system (K, not considered in the previous sections); and 3) the marginal delivery costs (Cp considered in the previous section). Here, let us begin to explore the effects of hookup costs and fees.

The municipality now has four prices to set: \( P_1 \) and \( P_2 \), the per-unit prices of the two classes of service, and \( H_1 \) and \( H_2 \), the once-and-for-all hookup fees for the two classes of service. The total price of service now involves an interest rate. However, in LDCs in particular, the Rich and the Poor do not face the same interest rate. Due to capital-market imperfections (such as asymmetric information or lemons problems), as well as the fact that administrative costs represent a larger percentage of smaller loans, the Poor have to pay higher interest rates than either the Rich or the municipality; namely, \( R_i > R_m = R_m \), where \( R_i \) is the interest rate of the ith group and the subscript, \( ms \), indicates the municipality.

The annualized net total revenue collected by the municipality when a household of income group \( i \) buys service type \( j \) is

\[
TR_{ij} = P_j H_j (1 + R_i)^{-t} - C_p - C_1 - C_2
\]

\[
TR_{ij} = P_j H_j (1 + R_i)^{-t} - C_p - C_1 - C_2
\]

\[
TR_{ij} = P_j H_j (1 + R_i)^{-t} - C_p - C_1 - C_2
\]

21 Again for simplicity, we assume that the Rich have access to capital at the same interest rate as the municipality.

22 In this section, all values of TR and CS are given as annualized flows, rather than present values, in order to keep the equations comparable to those of previous sections. The annualized flow is calculated in the text as simply the relevant interest rate times the present value -- mortality, depreciation, and horizons are all ignored.
The annualized "total consumer surplus" of the household of income group \( i \) buying service type \( j \), \( TS_i \), is the annualized value of the hookup fee subtracted from the annual consumer surplus:

\[
TS_i = -R_i H_j + CS_i
\]

where \( CS_i \) remains as defined in equation (3), a function of price and income.

Each household selects the service type that yields the larger \( TS \), provided that \( TS \) is positive. This lets us derive the locus of switchover prices \( (P_1, P_2, H_1, \text{ and } H_2) \) along which the household is indifferent between the two classes of service.\(^{23}\)

\[
P_2 = A_i Y_i - \sqrt{\frac{a}{b}} (A_i Y_i - P_j)^2 - \frac{(2A_i Y_i - H_i - H_2)}{B_i}
\]

where this switchover locus is constrained to the region where the \( TS \) is positive for each \( j \):

\[
0 < P_j < A_i Y_i - \sqrt{\frac{2A_i Y_i - H_i - H_2}{B_i}}
\]

When \((H_1 - H_2)\) rises above zero, the switchover locus shifts to the left, and it moves further to the left the lower is \( Y_i \), or the higher is \( R_i \). Since \( Y_p < Y_i \), and \( R_p > R_i \), the switchover locus moves further for the Poor than the Rich. This widens the region in which the Rich choose first-class service and the Poor second-class service. The upward-sloped portions of Figure 2 are drawn in Figure 6, for each of the Poor and the Rich, in their original positions (of Section III, where \( H_p = 0 \)) and in their positions with hookup costs added.\(^{24}\) Notice that the switchover locus hardly moves at all for the Rich, while it shifts significantly for the Poor.

The higher hookup costs of first-class service, together with the higher interest rates the Poor must pay, make the Poor even quicker, relative to the Rich, to opt for second-class service.

Can we say anything about the optimal set of the four prices and fees the municipality must set (i.e. \( P_1, P_2, H_1, \text{ and } H_2 \))? Efficiency in the service market requires marginal-cost pricing: \( P_1 = C_1 \) and \( P_2 = C_2 \). Were there no capital-market distortions, the \( H_j \)'s could be divided in any manner to make up the \( K_j \)'s and whatever additional revenue requirement exists, as long as \( H_j \) meets the switchover constraint and is less than \( CS_i / R_i \). \( H_p \), as a lump-sum fee, does not affect the quantity or service choice (within the given constraints). However, if the Poor face capital-market distortions, charging them a hookup fee creates deadweight loss. In this case, the municipality must balance the efficiency loss from raising prices above marginal costs against the capital-market distortions. Given our characterization of total surplus, a dollar transferred from the Rich to the Poor represents a social gain. To avoid such arbitrage opportunities involving hookup subsidies, we will restrict ourselves to non-negative hookup fees.

If the municipality can discriminate between Rich and Poor, it can achieve a first-best situation by setting \( P_1 = C_1 \) and essentially charging the Rich a lump-sum tax, \( H_p \), to make up

\[^{23}\text{See equations (8) and (9) in Section III -- the derivation here is identical except that } H_1 \text{ and } H_2 \text{ are added.}\]

\[^{24}\text{The parameters are all the same as before, plus } H_1 = .20, H_p = .10, R_p = .15, R_1 = .05, \text{ and } R_p = .05.\]
any additional revenue requirement. Thus, the capital-market imperfection is ameliorated by
loading all the hookup and overhead costs onto the Rich. The Rich can pay these hookup
costs at lower interest cost, and, as long as the total burden is not too high, the higher cost of
hooking up to first-class service does not drive the Rich to the use of second-class service.

The Poor will choose either first-class or second-class service, depending upon the conditions
outlined in the previous section. However, the switchover and maximum price constraints
become more important when the municipality cannot discriminate. The resulting optimal
pricing problem follows a path similar to that described in Section III.

For simplicity, let us again assume that $C_j = C_f = K_f = 0$ (to avoid some of the
complications we observed in Section IV). If net revenue needs are zero, $P_2 = C_f = 0$ and $H_f = 0$
for both $j$ and both groups consume first-class service. As net revenue needs rise, the
government raises revenue in the area of least distortion. Initially, raising $P_1$ above marginal
cost causes a smaller efficiency loss. Once $P_1$ reaches the level where a change is equally
distorting on the margin, any further increase in revenue must come from increases in $H_f$.

We shall refer to this point as $P_1^*$:

$$P_1^* = \frac{A_j N_j Y_j (N_f + N_j)(R - R_j)}{N_j Y_j^2 (N_f + N_j)^2 (2R - R_j)(N_f - N_j)}$$

At the point where the annualized hookup fee for the Poor equals their consumer
surplus differential between the two services (i.e., when $R_f H_f = CS_f(P_f) - CS_f(0)$), $H_f$ and/or
$P_2$ will have to rise as well to keep the poor from switching to second-class service. (Since

Figure 6

Service Choices of Rich and Poor
With and Without Hookup Fees

1.60
1.20
----------------- ------------ ---.
181x368
184x368
184x368
184x368

21
no one is consuming second-class service yet, we are not concerned with the efficiency of \( P_2 \) or \( H_2 \). This point may even be reached before \( P_1^* \) when \( H_1 = 0 \).

The maximum possible revenue the municipality could attain offering only first-class service would be reached when the annualized value of the hookup fee for the Poor equals their consumer surplus:

\[
R_y H^\text{max} = \frac{B_y (A_Y - P_1)^2}{2 A_y Y_p}
\]

However, the municipality would want to switch to two-tiered provision before it reaches this point of zero total surplus for the Poor.

Efficient pricing under two-tiered service calls for prices equal to marginal cost and all net revenue needs raised through \( H_y \), paid completely by the Rich. Social surplus is greater when both groups consume first-class service at \( P_1 = P_1^* \) and \( H_1 = 0 \) than under two-tiered service with \( P_1 = P_2 = 0 \), \( H_1 = 0 \), and \( H_2 = V/(R_y N_1) \), where \( V \) equals the annual revenue from \( P_1^* \) in the single service case. But raising an additional dollar of revenue in the first-class situation costs more in terms of social surplus than in the two-tiered case. Thus, there comes a value of \( H_1^* \) where social surpluses and net revenues are equal for both cases. At that point, the municipality must switch to two-tiered service if more revenue is required. The optimal pricing path will then jump to that equivalent revenue point where \( P_1 = C_1 = 0 \), \( H_1 = 0 \), and \( N_y H_i \) equals the previous net revenue:

\[
H_1 = \frac{(N_y + N_1) H_1^*}{N_y} \cdot \frac{V}{N_y R_y}
\]

As revenue requirements increase further, \( H_1 \) alone will continue being raised until the switchover constraint of the Rich is reached (if it is not already binding when the switch to two-tiered service is made). Then, to be able to raise \( H_1 \) further, either \( P_2 \) or \( H_2 \) must increase to keep the Rich from switching to second-class service. Here, the municipality must weigh the capital-market distortion against the efficiency loss of raising \( P_2 \) above marginal cost.

Maximizing the social \( TS \) subject to the budget constraint with the Rich’s switchover constraint binding reveals the optimal \( P_2^* \):

\[
P_2^* = \frac{A_y Y_p (N_y + N_1)(R_y - R_2)}{Y_p [2 R_y N_y - R_y (N_y + N_1) + N_y (R_y - R_2)]} > 0
\]

To keep the Rich from switching to second-class service while \( H_1 \) increases, the municipality raises \( P_2 \) from 0 (marginal cost) until it reaches \( P_2^* \), after which \( H_1 \) is increased instead.

Essentially, the Rich, who are not credit-constrained, do not mind paying a large lump-sum fee for the privilege of receiving first-class service; the Poor, however, are credit-constrained, and they prefer to pay in the form of per-unit charges which their cash flow can handle.

Under two-tiered service, the municipality is able to charge each group accordingly.

The maximum revenue possible under two-tiered provision will be attained when

\[\text{We do not immediately set } P_2 = P_2^*, \text{ since it would require } H_2 < 0.\]
Unlike in Section III, further net revenue increases are possible by allowing the Poor to drop out completely and jacking up the hookup fees of the Rich. Adding positive hookup costs \( K_p > 0 \) should not qualitatively change the analysis; since they are fixed costs, they should be treated in the same way as overhead, although total fixed costs will be different depending upon the type of provision. The result is that the starting revenue requirement will be positive, and the equal net revenue point under two-tiered service will be associated with a lower \( H_1 \) (if \( K_p > K_s > 0 \)). If \( K_p \) is large enough with respect to \( K_s \), two-tiered service may be preferred from the start, since the total fixed costs would be substantially less than with first-class service.

Therefore, the existence of differing hookup costs, like that of differing marginal costs, can provide grounds for offering two types of services. Furthermore, the option of charging hookup fees allows for greater revenue collection without distorting consumption decisions. Perhaps most importantly, when Rich and Poor consume different classes of services, the ability to charge different hookup fees allows the municipality to place the financial burden of the overhead costs squarely upon the shoulders of the Rich.

VI. Conclusion
Reference