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The Profitability of Exogenous Output Contractions: A Comparative-Static Analysis with Application to Strikes, Mergers and Export Subsidies

by

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ABSTRACT

We consider a Cournot equilibrium where firms with identical cost functions produce a homogeneous good. A subset of these firms faces an exogenously-induced marginal contraction of individual output. We show that for any given finite number of firms greater than one, each firm in the subset will gain (lose) if the number of firms in the subset is sufficiently large (small). With constant marginal costs of production and a linear inverse demand curve, the firms in the subset will gain if and only if they outnumber the firms outside it by more than one. In general, the firms in the subset will gain if and only if their number exceeds by more than one an "adjusted" number of outside firms, where the multiplicative adjustment factor depends on the curvatures of the cost and inverse demand curves. In a price-taking equilibrium, on the other hand, the firms in the subset will never lose from a marginal contraction of their output. Indeed, they will strictly gain if marginal cost is strictly increasing. These local results are used to extend the analysis to the effect on profit of exogenously-induced non-marginal changes in output.

These fundamental comparative-static properties have implications for the relationship of Stackelberg and Cournot equilibria. We show how they can be used to generalize some standard duopoly results on first-mover advantage to the case of N-player sequential-move oligopoly games. We also show how these properties of the Cournot model apply directly to the analysis of certain strike situations and underlie the results in the applied literature on gains from export subsidies and on losses from horizontal mergers.

Finally, we discuss how the results may be generalized to various assumptions about substitutability and complementarity and to strategic variables other than quantity.

I. Introduction

Consider an industry composed of N firms with identical cost functions. The Cournot equilibrium is displaced by an exogenously-induced marginal contraction of the output of a subset of these firms. Do profits of the firms in this designated subset increase or decrease as a result?

Readers whose point of reference is Cournot duopoly know that a marginal contraction of the output of a single firm in the neighborhood of the Cournot equilibrium will decrease its profits. On the other hand, readers whose reference point is monopoly know that when an N-firm industry is monopolized, a marginal contraction of the output of each "plant" will be profitable. Evidently, the answer to our question depends on the size of the designated subset relative to the size of the industry. It turns out also to depend on the curvature of the demand and cost functions.

In this paper, we answer the question posed at the outset and provide the underlying intuition. A marginal contraction is strictly beneficial (strictly harmful) if and only if the number of firms in the designated subset exceeds the "adjusted" number of firms outside it by strictly more (strictly less) than one. The adjustment factor is unity when cost and demand functions are linear but, more generally, depends on the convexity of the cost and demand curves. For example, a marginal contraction of two firms in a triopoly has no effect on the profits of firms in the subset if cost and demand functions are linear; if instead cost is linear but the demand function is strictly concave (strictly convex), a marginal contraction will strictly decrease (strictly increase) profits.

These results can be easily understood if they are viewed from the following perspective. Since, in the neighborhood of equilibrium, a marginal contraction in the output of any firm would have no effect on its profits (in the absence of other changes), the profit of a firm in the subset will increase in the new equilibrium if and only if the aggregate output of the other N-1 firms decreases. Such a decrease will occur if and only if the exogenous marginal contraction of all the other firms in the subset exceeds the induced expansion of the firms outside the subset. This perspective not only provides a precise explanation of the comparative statics results under Cournot competition mentioned above but also facilitates extension of these results to other forms of competition (e.g. Bertrand competition and price-taking behavior) and to situations where goods are either complements in demand or, alternatively, strategic complements.

These comparative-static results have many applications. We show, for example, that they underlie 1) the observations of Carter, Hueth, Mamer, and Schmitz (1981) (among others) that strikes may benefit struck firms; 2) the results in the strategic-trade literature, originating with Brander and Spencer (1985), that export subsidies may increase profits even if the subsidy receipts are taxed away from the export sector in a lump-sum; and 3) the results in the horizontal mergers (and cartel) literature originating with Salant, Switzer, and Reynolds (1983) that some mergers (or some cartelizations) may be unprofitable. In each of these cases, outputs of a designated subset of firms contract or expand as a result of some exogenous change, other firms best-reply, and the focus is on how profits of firms in the designated subset are affected.

More fundamentally, these comparative-static results have implications for the relationship of the strategic variables in simultaneous-move and sequential-move oligopoly games. In the case of quantity competition, for example, suppose a Stackelberg leader took over the operation of a subset of firms which were previously operated independently as part of a symmetric N-firm Cournot equilibrium. The conditions we derive indicate whether that leader would increase, decrease, or leave unchanged the output of each technology under his control. Contrary to the familiar but extreme case of duopoly, therefore, the Stackelberg leader may wish to contract outputs relative to their Cournot levels when he operates more than one technology. In such situations, standard results on first-mover advantage in quantity games, which have been derived assuming duopoly, are reversed. As before, our comparative-static results also have implications for Stackelberg games under other forms of competition (e.g. Bertrand competition or price-taking behavior by followers) and other assumptions about substitutability in demand or strategic substitutability.

The generalizations of the comparative-static results to market structures other than Cournot oligopoly also have important applications. For example, the generalization to Bertrand competition indicates how price ceilings or price floors imposed on a subset of firms in an industry will affect their profits when firms compete in price.

The remainder of the paper is organized as follows. In section II, the comparative-static properties of the Cournot and price-taking equilibria are derived. In section III, these results are applied to the cases of strikes, horizontal mergers, and export subsidies. In section IV we conclude the paper by discussing the generalization of our results under alternative sets of assumptions.

II. The Effect of an Exogenous Output Contraction

A. Marginal Changes

Consider an industry composed of N firms producing a homogeneous good with identical cost functions. Each firm in a subset $S \leq N$ is assumed to be exogenously induced to produce an identical output $\bar{q} > 0$. The output of each of the other firms is unconstrained. We will denote it q. Total industry output is then given by:

$$Q = S\bar{q} + (N-S)q \tag{1}$$

The inverse market demand is given by $p(Q)$. It is assumed there is a $Q^0 > 0$ such that $p(Q) > 0$ for $Q < Q^0$ and $p(Q) = 0$ for $Q \geq Q^0$ and that $p(Q)$ is twice continuously differentiable with $p'(Q) < 0$ for $Q < Q^0$. The typical firm's cost of production is $C(q)$, with

$C(q) > 0$ if $q > 0$ and $C(0) = 0$. It is assumed to possess continuous first and second derivatives, which satisfy $C' > 0$ and $C'' \geq 0$.

We further assume that $p'(Q) + qp''(Q) < 0$ for all $q \leq Q \leq Q^0$.[1] Thus a given firm's marginal revenue must fall when any rival firm increases its output. In addition to being substitutes in demand the goods are therefore also strategic substitutes: each firm's best-reply function is downward sloping.[2]

The firms select their output simultaneously in a one-stage game. Hence, in a Nash equilibrium, the output of each firm maximizes its profits given the output of the N-1 other firms. However, each of the S firms in the subset is assumed to be constrained to the level of output $\bar{q}$, which may or may not differ from q. Thus the Cournot equilibrium will satisfy the following set of equations:[3]

$$p + qp' - C'(q) = 0 \tag{2}$$

$$p + \bar{q}p' - C'(\bar{q}) = \mu \tag{3}$$

$$p = p(S\bar{q} + (N-S)q) \tag{4}$$

Equation (2) is the first-order condition for profit maximization of the typical unconstrained firm and equation (3) is that of the typical constrained firm. The variable μ is the shadow cost of the output constraint to the typical constrained firm. Whenever $\bar{q} \neq q$, then $\mu \neq 0$ and the constraint displaces the Cournot equilibrium; whenever $\bar{q} = q$, then $\mu = 0$ and the constraint does not displace the equilibrium. We will refer to this latter situation as the "unconstrained Cournot equilibrium".[4]

The equilibrium profits of each firm will be a function of $\bar{q}$, N and S. In particular, the profit of a typical firm in the subset of constrained firms is:

$$\pi^s(\bar{q},N,S) = p(S\bar{q} + (N-S)q)\bar{q} - C(\bar{q}) \tag{5}$$

where $q = q(\bar{q},N,S)$ is the equilibrium output of the unconstrained firm, obtained by solving (2), (3) and (4) for (p,q,μ).

Differentiating (5) with respect to $\bar{q}$ and making use of (3), we get:

$$\frac{\partial \pi^s}{\partial \bar{q}} = \mu + \left[(S-1) + (N-S)\frac{\partial q}{\partial \bar{q}}\right]p'\bar{q} \tag{6}$$

The first term represents the effect on the profit of the typical firm in the subset of a marginal variation in $\bar{q}$ if the output of the N-1 other firms were to remain unchanged. The second term captures the effect attributable to the change in the equilibrium output of the N-1 other firms induced by the change in $\bar{q}$. Recall that at an unconstrained Cournot equilibrium, $\bar{q} = q$ and $\mu = 0$. Hence, the effect on the profit of the typical firm in the subset of a marginal variation of its own output, holding constant the outputs of all the other firms, is negligible.[5] As a result, equation (6) implies that:

$$\left.\frac{\partial \pi^s}{\partial \bar{q}}\right|_{\bar{q}=q} \lesseqgtr 0 \quad \text{iff} \quad \left[(S-1) + (N-S)\frac{\partial q}{\partial \bar{q}}\right] \gtreqless 0 \tag{7}$$

The term in brackets in (7) is simply the derivative with respect to $\bar{q}$ of the aggregate output of the N-1 other firms, S-1 of which are in the subset and N-S of which are outside it. It follows that a marginal change (an increase or decrease) in $\bar{q}$, in the neighborhood of the unconstrained equilibrium, will raise the profit of each firm in the subset if and only if the equilibrium aggregate output of the N-1 other firms declines as a consequence.[6]

To investigate $\partial\pi^s/\partial\bar{q}$ in more detail, we need an explicit expression for $\partial q/\partial\bar{q}$. By total differentiation of the equilibrium conditions (2), (3) and (4) and application of Cramer's rule (see Appendix A), we get:

$$\frac{\partial q}{\partial \bar{q}} = \frac{-[p' + qp'']S}{\Delta} < 0 \tag{8}$$

where $\Delta = (N-S)[p' + qp''] + p' - C'' < 0$. Not surprisingly, a marginal reduction in $\bar{q}$ from $\bar{q} = q$ leads to an increase in the output of each of the N-S unconstrained firms. Upon substituting from (8) and using the fact that $\Delta < 0$ and $p' - C'' < 0$, condition (7) may now be rewritten:

$$\left.\frac{\partial \pi^s}{\partial \bar{q}}\right|_{\bar{q}=q} \lesseqgtr 0 \quad \text{iff} \quad S - \alpha(N-S) \gtreqless 1 \tag{9}$$

where:

$$\alpha = \alpha(\bar{q},N,S) = \frac{p' + qp''}{p' - C''} > 0 \tag{10}$$

In the case where $\alpha = 1$ (for example when $C'' = p'' = 0$) condition (9) says that the firms in the subset will strictly gain from an exogenously-induced marginal contraction (expansion) of their output if and only if they outnumber the firms outside the subset by more than one (less than one). Refer to the term $\alpha(N-S)$ as the "adjusted" number of firms outside the subset. Then, more generally, a marginal output contraction (expansion) is profitable for the firms in the subset if and only if they exceed the adjusted number of firms outside the subset by more than one (less than one).

It is clear by inspection of (10) that if marginal cost is constant ($C'' \equiv 0$), we have $\alpha = 1$ when the inverse demand curve is linear, $\alpha < 1$ when it is strictly convex and $\alpha > 1$ when it is strictly concave. More generally, with nondecreasing marginal cost ($C'' \geq 0$), convexity of the inverse demand curve is sufficient (but not necessary) for $\alpha \leq 1$. It is sufficient (but not necessary) for $\alpha < 1$ if marginal cost is strictly increasing ($C'' > 0$). Concavity of the inverse demand curve is necessary (but not sufficient) for $\alpha \geq 1$.

The curvatures of the cost and demand functions thus affect the magnitude of α and hence the comparative static results on π^s. In particular, if $\alpha \leq 1$ and the number of firms in the subset exceeds the actual (and hence the adjusted) number of outside firms by more than one, then $\partial\pi^s/\partial\bar{q} < 0$ at $\bar{q} = q$. If instead $\alpha \geq 1$ and the number of firms in the subset exceeds the actual (and hence the adjusted) number of firms outside it by less than one, then $\partial\pi^s/\partial\bar{q} > 0$.

Notice that as monopoly theory would lead one to expect, regardless of the curvatures of costs and demand curves, a monopolist always wants to reduce aggregate output below the level at which its multiple plants would operate in an unconstrained Cournot equilibrium. That is, since $\alpha > 0$, if $S = N \geq 2$, then $\partial\pi^s/\partial\bar{q} < 0$ at $\bar{q} = q$.

If, at the other extreme $S = 1$ and $N \geq 2$, then the result is reversed: $\partial\pi^s/\partial\bar{q} > 0$ at $\bar{q} = q$ and an exogenous expansion of its output increases the profit of the designated firm. The intuition behind this latter result is that, when $S = 1$, an expansion in $\bar{q}$ must induce a decrease in the aggregate output of the N-1 other firms and this will be profitable to the expanding firm. When $S > 1$, the argument is of course invalid: the direction of change of the aggregate output of the N-1 other firms is ambiguous since S-1 of them are also increasing their output. A marginal expansion in $\bar{q}$ may then be unprofitable to the firms in the subset.

This last implication of condition (9) is often unappreciated and merits emphasis. Conventional comparisons of Cournot and Stackelberg equilibria, for example, assume that the Stackelberg player controls a single technology. Similarly, games where a government policy in the first stage alters the payoff functions of a subset of noncooperative players in the second stage often assume that the subset contains a single firm. Such analyses in effect assume $S = 1$. This assumption may seem convenient since it permits the use of two-dimensional reaction-function diagrams. Moreover, the analyses may appear general since, when $S = 1$, the results hold for a wide class of cost and demand functions. But, as will become evident in the next subsection and beyond, such analyses are misleading, since their results can be reversed when $S > 1$.

The same comparative-static analysis applied to the situation where firms are price-takers rather than Cournot competitors will show that it could also be misleading to infer results for the price-taking equilibrium from those of the Cournot equilibrium. It is already clear from (9) that for the Cournot equilibrium, given the number of firms in the industry, the firms in the subset will gain from an exogenously-induced marginal output contraction if their number is large enough and will lose if their number is small enough. This is not the case in the price-taking equilibrium, where the constrained firms never lose from an exogenously-induced marginal output contraction.

The solution to the price-taking equilibrium will satisfy:

$$p - C'(q) = 0 \tag{2'}$$

$$p - C'(\bar{q}) = \mu \tag{3'}$$

as well as (4). Equations (2') and (3') have the same interpretation as (2) and (3), except for the assumption of price-taking behavior on the part of the firms. The equilibrium profits of the typical constrained firm may still be written as in (5), except that $q = q(\bar{q},N,S)$ is now the price-taking equilibrium output obtained from the solution to (2'), (3') and (4).

The derivative of equilibrium profits of the constrained firm with respect to $\bar{q}$ can be written as:

$$\frac{\partial \pi^s}{\partial \bar{q}} = \mu + q\frac{\partial p}{\partial \bar{q}} \tag{11}$$

The first term is zero when $\bar{q} = q$, at the unconstrained price-taking equilibrium. It captures the (negligible) effect of the marginal contraction in output on the profits of the constrained firm if price did not change. The second term captures the effect attributable to the equilibrium price change, as a consequence of the net change in aggregate output. Unless marginal cost is constant, in which case aggregate output and price remain unchanged, aggregate output will decrease and price will increase as a result of the marginal output contraction by the firms in the subset. Thus, after substituting for the value of $\partial p/\partial \bar{q}$ (see Appendix A), we may write:

$$\frac{\partial \pi^s}{\partial \bar{q}}\bigg|_{\bar{q}=q} = Sp'\bar{q}C''\left[1 - \frac{(N-S)p'}{(N-S)p' - C''}\right] \leq 0 \tag{12}$$

The constrained firms therefore never lose from an exogenous marginal contraction of output in a price-taking equilibrium and never gain from a marginal expansion. Indeed, if marginal cost is strictly increasing and the number of firms is finite, they always gain from a marginal contraction and always lose from a marginal expansion. When marginal cost is constant, the marginal gain (or loss) is zero.[7]

B. Non-Marginal Changes

We have so far restricted our attention to the effect on the profit of a subset S of constrained firms of a marginal contraction of their outputs in the neighborhood of the unconstrained equilibrium. The analysis can now be extended to the case of non-marginal contractions of their outputs. We will assume henceforth that the equilibrium profit of the S firms in the subset, $\pi^s(\bar{q},N,S)$ (see equation 5), is a single-peaked function of $\bar{q}$.[8]

Consider first the case of Cournot oligopoly. Our results imply that a marginal contraction is strictly beneficial to each firm in the subset if their number exceeds the (adjusted) number of firms outside it by more than one (i.e., $S - \alpha(N-S) > 1$). Under the same circumstances, a non-marginal contraction will also be beneficial as long as the output of each constrained firm is not forced below a critical level, $\gamma < q^*$, defined by:

$$p(S\gamma + (N-S)q(\gamma))\gamma - C(\gamma) = p^*q^* - C(q^*) \tag{13}$$

where p^* and q^* are the price and outputs in the unconstrained Cournot equilibrium and $q(\gamma)$ is defined implicitly, for given N and S, by $p(S\gamma + (N-S)q) + qp'(S\gamma + (N-S)q) = C'(q)$. If, on the other hand, the number of firms in the subset exceeds the (adjusted) number of outside firms by less than one (i.e., $S - \alpha(N-S) < 1$), then any reduction in output, either marginal or non-marginal, will be harmful to the constrained firms. Finally, if the number of firms in the subset happens to exceed the (adjusted) number of firms outside it by exactly one (i.e., $S - \alpha(N-S) = 1$), a marginal contraction in the neighborhood of the unconstrained Cournot equilibrium has no effect on the profit of the constrained firm but any non-marginal contraction is unprofitable.

The panels of Figure 1 illustrate each of these situations. In each panel, the equilibrium profit of the typical firm in the subset is plotted as a function of their exogenous output constraint ($\bar{q}$). Output level $\bar{q}^K$ would maximize the profits of each firm in the subset given that the outside firms best-reply. Points C^1, C^2 and C^3 correspond to the unconstrained Cournot equilibrium in each of the alternative situations just described.

[Figure 1 goes here]

Panel A illustrates the first case, where the number of firms in the subset exceeds the (adjusted) number of outside firms by more than one. As a result, from (9), the slope of π^s with respect to $\bar{q}$ is negative at C^1 and $\bar{q}^K$ must be smaller than q^*. It follows that an exogenous contraction to any level of output in the interval (γ, q^*) would be beneficial. Panel B illustrates the second case. The inequality in (9) is now reversed and therefore the slope of π^s is positive at C^2. Since $\bar{q}^K$ must then be greater than q^*, any contraction in the output of the constrained firms reduces their profits. The third situation, in which N and S are such that $\bar{q}^K = q^*$, is illustrated in panel C. Clearly any non-marginal contraction is then unprofitable to the S firms in the subset.

These results have implications for the relationship of Cournot to Stackelberg equilibrium. By Stackelberg equilibrium, we refer here to the situation where a leader controls the S firms in the subset and determines their outputs in order to maximize his joint profits, assuming that the N-S firms outside the subset will best reply. It is shown in Appendix C that for all N and S such that (7) (and hence (9)) holds with equality, the vector of price and outputs in the unconstrained Cournot equilibrium coincides with the vector of price and outputs in the Stackelberg equilibrium. With π^s a single-peaked function of $\bar{q}$, this Stackelberg equilibrium is unique. Thus, a marginal reduction in $\bar{q}$ at the unconstrained Cournot equilibrium will move the output of the firms in the subset toward the Stackelberg output if $\partial\pi^s/\partial\bar{q} < 0$ at the equilibrium and away from it if $\partial\pi^s/\partial\bar{q} > 0$. Clearly, the Stackelberg leader who gained control of the S technologies would always set their outputs at $\bar{q}^K$.[9] Since:

$$\bar{q}^K \lesseqgtr q^* \quad \text{iff} \quad S - \alpha(N-S) \gtreqless 1 \tag{14}$$

the relationship of the leader's output under Stackelberg and Cournot equilibrium has been characterized precisely. The conventional result that the Stackelberg leader produces more than in the unconstrained Cournot equilibrium (i.e., $\bar{q}^K > q^*$) is true if $S = 1$ (and $N \geq 2$) but not in general.

These results in turn have implications for the literature on "first-mover advantage". There are two distinct definitions of first-mover advantage in the literature. Under a first definition (see Shapiro, 1987, p. 67-68), a Stackelberg leader is said to have an advantage if his profits are strictly larger than they would be in a Cournot equilibrium. If we apply this definition, a Stackelberg leader has a first-mover advantage if and only if $S - \alpha(N-S) \neq 1$; he has none when $S - \alpha(N-S) = 1$.

Dowrick (1986) and Gal-Or (1985), among others, have investigated the advantage which a Stackelberg leader has over the follower in a Stackelberg duopoly game. This leads to a second and distinct definition of first-mover advantage: a Stackelberg leader is said to have an advantage if his profit is larger than the profit of the follower in the Stackelberg equilibrium. A natural generalization of this definition to the case where the subset moving first (or second) contains several firms is to replace "profit" by "profit per firm in the subset". Under this definition, a Stackelberg leader will have a first-mover advantage if and only if $S - \alpha(N-S) < 1$, since in that circumstance he will choose to expand output relative to the unconstrained Cournot output ($\bar{q}^K > q*$) and that will depress profit per firm of the followers. When $S - \alpha(N-S) > 1$, the Stackelberg leader will contract output ($\bar{q}^K < q*$). Since his profit per firm then rises less than profit per firm of the followers (see Appendix B), the leader would have a first-mover disadvantage.

Notice that under either of those two definitions, with $\alpha > 0$ the leader always has a first-mover advantage in the special case of a duopoly and indeed anytime $S = 1$ and $N \geq 2$, since that insures $S - \alpha(N-S) > 1$.

We conclude this section by considering the effects of non-marginal output contractions if the firms are price-takers. If all firms have constant marginal costs ($C'' = 0$), each then earns zero profits. As long as there is any firm outside the designated subset ($S < N$), neither a marginal nor a non-marginal contraction will have any effect on profits. On the other hand, in the more interesting case where marginal cost is strictly increasing ($C'' > 0$), a marginal contraction of $\bar{q}$ will always strictly increase the profit of the S firms. Assuming π^s is a single-peaked function of $\bar{q}$, this implies that the maximum of π^s always occurs at a smaller $\bar{q}$ than that associated with the unconstrained price-taking equilibrium. Any contraction therefore strictly benefits each constrained firm as long as its output is not forced below a critical level. This critical level is defined as in (13), with p^* and q^* reinterpreted as the unconstrained price-taking equilibrium price and outputs and $q(\gamma)$ redefined implicitly by $p(S\gamma + (N-S)q) = C'(q)$.

Again the results have implications for the relationship of the unconstrained equilibrium to the Stackelberg equilibrium (see Appendix C). Since the price-taking equilibrium level of output is never lower than the level of $\bar{q}$ which maximizes π^s, a Stackelberg leader who gained control of S firms previously in a price-taking equilibrium and anticipated that the remaining N-S firms would continue to be price-takers would never want to expand output of the S firms. He would maintain the price-taking equilibrium output level if marginal cost were constant and would contract the common output of the S firms under his control (or, equivalently, raise the price) if marginal costs were strictly increasing. Finally, if we adapt the two definitions of first-mover advantage to the case where the followers are price-takers, we get the following implications (for the case where $C'' > 0$): under the first definition, the leader has a first-mover advantage since his profits increase relative to the unconstrained price-taking equilibrium; however, under the second definition, the leader always has a first-mover disadvantage since his profit per firm increases less than the profit per firm of the followers (see Appendix B).

III. Some Applications

A. Strikes

The comparative-static results we have been investigating have many applications. Consider first the effect of a labor strike on the profits of the struck firms. It is often taken for granted that a struck firm must be harmed by the forced reduction in its output. However, our analysis delineates circumstances in which, under either an oligopolistic or a competitive market structure, each targeted firm will in fact benefit from a strike.[10] The 1979 lettuce strike against selected producers in the Imperial Valley of California appears to have illustrated this perverse result.[11]

In the case of Cournot oligopoly, our results imply that a marginal strike is strictly beneficial to each struck firm if and only if the number of struck firms exceeds the (adjusted) number of nonstruck firms by more than one. Under the same circumstances, a non-marginal strike will also be beneficial as long as the output of each struck firm is not forced below the critical level, $\gamma < q^*$, defined in (13).

In the price-taking equilibrium case, a marginal strike is strictly beneficial if firms have strictly convex costs and has no effect if firms have linear costs. These results again extend if we consider the case of a non-marginal strike. With linear costs, no reduction in the output of a struck firm affects its profits (as long as there exists some firm in the industry which is not struck). With strictly convex costs, each struck firm will strictly benefit from a non-marginal strike as long as its output is not forced below the appropriate critical level defined in the previous section.

B. Export Taxes and Subsidies

A rationale for subsidizing the exports of domestic firms has recently been proposed by Brander and Spencer (1985). They consider a model where Cournot duopolists, one in each country, export to consumers in a third country. They show that imposition of a marginal subsidy, in the neighborhood of free trade equilibrium, will always increase the profit of the domestic firm net of the subsidy.[12] These results rely again on the comparative-static properties of the Cournot model derived in section II. Our analysis makes clear that the Brander-Spencer result is a special case: regardless of the curvatures of the cost and demand curves, if there is only one firm in the subset and one or more firms outside the subset, then a marginal expansion of the output of the firm in the subset in the neighborhood of Cournot equilibrium must increase its profits. This clearly follows from (9). In an N-firm oligopoly, the domestic attractiveness of the subsidy will depend, given N, on the number of domestic firms.[13] In fact, the optimal policy may instead be a tax.[14]

Consider an industry composed of N identical firms. Partition it into S domestic firms and N-S foreign firms. Let t denote the per-unit tax (if positive) or subsidy (if negative) of the domestic firms and let q_d denote their individual output. The first-order condition of the typical domestic firm is then:

$$p + p'q_d - C'(q_d) = t \tag{15}$$

If condition (3) is now replaced by (15), the new system of equations defines the values of (p,q,q_d) in the Cournot equilibrium with export tax t; q now denotes the typical foreign firm's output.

Now by the implicit function theorem, there exists in the original problem a function $\mu(\bar{q},N,S)$ which solves (2), (3) and (4) and whose derivative $\partial\mu/\partial\bar{q}$ can be evaluated by Cramer's rule (see

Appendix A). Set $t = \mu(\bar{q},N,S)$ and it is clear that for any exogenous t, there is an endogenous q_d (set equal to $\bar{q}$), which will exactly duplicate the equilibrium obtained with $\bar{q}$ as an exogenous parameter and μ as an endogenous variable. Note that:

$$\mu(\bar{q},N,S) = t \gtreqless 0 \quad \text{for} \quad \bar{q} \lesseqgtr q \tag{16}$$

Moreover, since $\partial q_d/\partial t = 1/(\partial\mu/\partial\bar{q})$, the comparative-static properties of the Cournot equilibrium with respect to $\bar{q}$ derived in section II can be used to analyze the effects of changes in t. We simply have to treat an increase (decrease) in t as a decrease (increase) in $\bar{q}$.

Note also that the after-tax equilibrium profit of the domestic firm is now $\pi^s(q_d,N,S) - tq_d$, where $\pi^s(q_d,N,S)$, given by (5), is the equilibrium profit before payment to the government. It is assumed that the per-firm tax revenue, tq_d, is redistributed in a lump-sum. Thus an increase in π^s means a net welfare gain for the domestic country and whatever trade policy induces this gain is advantageous.

The results of section II for a contraction of output apply directly to the case of the export tax ($t > 0$) or subsidy ($t < 0$). Thus, if the number of domestic firms exceeds the (adjusted) number of foreign firms by more than one, a marginal increase of the export tax from $t = 0$ will cause an increase in π^s, which equals the sum of the domestic firms' after-tax equilibrium profits and the rebated tax revenues. In such a case, the initial Cournot equilibrium is a point such as C^1 in panel A of

Figure 1. The equilibrium profit function is negatively sloped with respect to $\bar{q}$ at C^1 (positively sloped with respect to t) and the equilibrium level of output exceeds the joint Stackelberg level. A tax therefore exogenously supports a move toward the joint Stackelberg level of output by the domestic firms and is therefore attractive from this point of view. An export subsidy in such circumstances would, of course, adversely affect the home country.

If instead the number of domestic firms exceeds the (adjusted) number of foreign firms by less than one, a marginal increase of the export tax from $t = 0$ would cause a reduction in π^s. The initial equilibrium is then a point such as C^2 in panel B, where $q^* < \bar{q}^K$, and the optimal trade policy for the home country is an export subsidy. Such circumstances inevitably arise when the domestic sector consists of a single firm ($S = 1$, $N \geq 2$).[15]

Consider finally a price-taking equilibrium. Since the price-taking equilibrium output of the domestic firms is never lower than the relevant Stackelberg output, an export subsidy will never be an attractive policy from the domestic point of view. But, if costs are strictly convex, a tax always will be.

C. Horizontal Mergers

As a final application, suppose the S firms in the subset now represent firms which are part of a merger or, equivalently, members of a cartel (with perfect enforcement). In general, merging or cartelizing will cause price to rise, aggregate output to fall, the outputs and profits of the (N-S) outside firms to rise, and the outputs of the merged entity to fall. Salant et al. (1983) and others have pointed out, however, that the profits of the merged entity (or cartel) may fall. A merger would of course increase profits if output of the N-S outside firms remained unchanged. But their output will not remain unchanged; it will increase. As a result of outsider expansion, merging or cartelizing some subsets of firms exogenously may be disadvantageous; if the merger decision were endogenized, such mergers would presumably not occur.

This "losses from merger" result is yet another consequence of properties of the Cournot model investigated in section II. The pre-merger equilibrium is simply the "unconstrained Cournot equilibrium" of that section. Price and outputs in the post-merger equilibrium are determined by the following three equations:

$$p + qp' - C'(q) = 0 \tag{17}$$

$$p + Sq_m p' - C'(q_m) = 0 \tag{18}$$

$$p = p(Sq_m + (N-S)q) \tag{19}$$

where q_m denotes the output of each technology operated by the merged entity and q denotes the output of the (N-S) outsiders.

As (18) reflects, each technology in the merged entity is operated for the joint profit of the entity. It will be easier to compare this set of equations to (2)-(4) if we rewrite (18) as follows:

$$p + q_m p' - C'(q_m) = -(S-1)q_m p' \tag{18'}$$

Suppose we solve this new set of equations for p ,q ,and q_m. If we now set $\bar{q} = q_m$, then (2)-(4) will reproduce the merger equilibrium. That is, the endogenous variables (p, q, μ) will equal (p, q, $-(S-1)q_m p'$). Hence, any merger equilibrium can be regarded as a special case of (2)-(4) for a particular setting of the constraint ($\bar{q}$).

Since the merger equilibrium is a special case, the results of section II can be applied. Whenever the number of firms in a merger exceeds the (adjusted) number of outside firms by less than one, a merger will cause a loss, as is illustrated in panel B of Figure 1. Since the Cournot output is smaller than the Stackelberg output in this case, the merger reduces output even further below the Stackelberg point and profits fall. Whenever the number of firms in a merger exceeds the (adjusted) number of outside firms by more than one, a marginal contraction of output is profitable. But a merger results in a non-marginal contraction in output and this may or may not be profitable. This case is illustrated in panel A of Figure 1. If $q_m < \gamma$ the merger will cause a loss.[16] If, on the other hand, $\gamma < q_m < q^*$, the merger will cause a gain.

The merger equilibrium can also be regarded as a special case of the equilibrium with export taxes. If we set

$t = -(S-1)q_m p'$, the endogenous variables in the export-tax equilibrium will equal those in the merger equilibrium. Thus, for any merger equilibrium, there is a tax rate which will generate the same outputs and price. Profits of the taxed firms will coincide with those of the merged firm provided the tax revenues are returned to the firms in a lump sum. Alternatively, if the tax revenues are not returned, profits to the taxed firms plus government receipts from the export tax will equal profits of the merged firm.

To emphasize the relationship of the seemingly dissimilar literatures on export subsidies and on horizontal mergers, we state the following propositions:

1) Whenever an export subsidy would increase the profits of the home country, merging the entire export sector must cause a loss.

2) Whenever merging the entire export sector would cause a gain, an export subsidy must reduce the profits of the home country.

IV. Concluding Remarks

In this paper we have analyzed a property of the Cournot model which underlies several important but seemingly disparate results. We have proved that a marginal output reduction by each of a subset of S firms in an industry is beneficial to them if S is sufficiently large relative to N-S and harmful if S is sufficiently small.

To this point, our analysis has been predicated on the assumption that firms 1)compete in quantity, 2)produce strategic substitutes (reaction functions slope downward) and 3)sell them to consumers who regard the goods as substitutes in demand. Since each of these three assumptions could have been specified in either of two different ways, there are in fact eight cases in all, seven of which have so far received no consideration. In this final section, we wish to comment informally on the implications of our analysis for these other cases. A more formal generalization is relegated to Appendix D.

We can dispel at the outset the notion that our results apply only to quantity competition. Sonnenschein (1968) has pointed out a result implicit in Cournot (1838): the situation where firms sell perfect substitutes and compete in quantities is dual to the case where firms sell perfect complements and compete in prices. Any result in one model can be reinterpreted as a result in the other. Consider then an industry of N firms with identical costs, each selling inputs which are perfect complements. Competition is in prices. We again impose the condition that the goods are strategic substitutes (the reaction functions in price space slope downward). Suppose a subset of S firms are forced to reduce their prices marginally, as would result if they were subject to a price ceiling. Our results in section II and duality then jointly imply that the subset of firms would benefit if S is sufficiently close to N. Indeed, the precise conditions we derived still hold, with the roles of quantities and prices interchanged. It also follows that if S is sufficiently small, a price ceiling imposed on the S firms would reduce their profits, as would a horizontal merger. Therefore, in contrast to the case of strategic complements in Davidson and Deneckere (1985), losses from horizontal mergers can occur when price is the strategic variable if goods are strategic substitutes.

More generally, whenever the goods are strategic substitutes, an exogenous marginal decrease in the strategic variables of a subset of S firms in a symmetric equilibrium will induce a marginal increase in the strategic variables of the remaining N-S firms. Viewed from the perspective of a single firm in the subset, the marginal change in its own variable has negligible effect and the changes in the other N-1 variables go in conflicting directions: S-1 of the variables decrease and N-S of them increase. The net effect will then depend on the size of the two subsets.[17] Whenever we have strategic substitutes, results which depend on the sizes of the two subsets will appear.

On the other hand, whenever the goods are strategic complements, an exogenous marginal decrease in the strategic variables of a subset of S firms in a symmetric equilibrium will induce a reinforcing marginal decrease in the strategic variables of the remaining N-S firms. Viewed from the perspective of a single firm in the subset, the marginal change in its own variable has negligible effect, but since all of the other N-1 changes are in the same rather than in offsetting directions, the comparative-static results will not depend at all on the relative size of S.

To illustrate, if quantity (price) is the strategic variable and the goods are substitutes in demand then a marginal contraction in the strategic variable will always increase (decrease) profits of each firm in the subset in the case of strategic complements; in the case of strategic substitutes, the same results will hold if S is sufficiently close to N and the reverse results will hold otherwise. Similarly, if quantity (price) is the strategic variable and the goods are complements in demand then a marginal contraction in the strategic variable will always decrease (increase) profits of each firm in the subset in the case of strategic complements; in the case of strategic substitutes, the same results will hold if S is sufficiently close to N and the reverse results will hold otherwise.

FOOTNOTES

1. This assumption is equivalent to $p'(Q) + Qp''(Q) < 0$ for all $Q \leq Q^0$ (see Shapiro, 1987). Although not necessary for existence, it does guarantee that a Cournot equilibrium exists in the case of homogeneous goods under weaker requirements on cost functions than those we assume (see Novshek, 1985 or Frayssé, 1986). Under our assumptions on costs and demand, it in fact is sufficient for uniqueness of Cournot equilibrium (see Kolstad and Mathiesen, 1987).

   The assumption of convex costs could be relaxed and the results derived in this section extended to a wider class of cost functions. If marginal costs were decreasing, our results would continue to hold as long as $C'' > p'$.

2. See Bulow, Geanakoplos and Klemperer (1985) for a general definition of strategic substitutes and complements. We discuss briefly the implications of our analysis for strategic complements (upward sloping reaction functions) in the concluding section.

3. The possibility clearly exists that $\bar{q}$ is large enough that each of the firms outside the constrained subset chooses to produce zero output; that is, for some values of $\bar{q}$, there is no positive value of q which satisfies (2). We will ignore this case here, by restricting our attention to situations where (2), (3) and (4) yield an interior solution for q. Since we have assumed zero fixed cost, this also means that no firm will earn negative profits in equilibrium.

4. If $\bar{q} < q$, then $\mu > 0$. Unless explicitly stated otherwise, this is the case we have in mind in most of section II. However, one can obviously think of important cases where $\bar{q} > q$, and as result $\mu < 0$. One such case is the export subsidy problem, which we discuss in section III. It will become clear further on that the results for $\bar{q} < q$ will always carry through for $\bar{q} > q$ if we simply interchange everywhere the words gain and lose.

5. This is simply an application of the envelope theorem to each of the S firms in the subset.

6. We show in Appendix B that the firms outside the subset always gain (lose) from a marginal contraction (expansion) of the outputs of the firms in the subset. Furthermore, when the firms in the constrained subset gain from a contraction of their output, those outside always gain more and when they lose from an expansion, those outside always lose more.

7. It also approaches zero with increasing marginal cost as the number of firms approaches infinity. This is also the case in Cournot equilibrium, as may be verified from equation (6). At the limit, as N tends to infinity, equations (2) and (3) coincide with (2') and (3') and the Cournot equilibrium coincides with the price-taking competitive equilibrium. However, it is noteworthy that, for any given S, $\partial \pi^s / \partial \bar{q}$ is positive for a sufficiently large finite N in Cournot equilibrium (condition 9), whereas it is negative in the competitive price-taking equilibrium (or zero with marginal cost constant). It therefore goes to zero from positive values in Cournot equilibrium and from negative values in price-taking equilibrium and the qualitative comparative static results of the two models coincide only in the limit, as N goes to infinity in both models.

8. It is easily verified that one situation among others where this assumption is satisfied is when marginal cost is constant and inverse demand is linear.

9. Since cost functions are convex and identical, the leader would have no incentive to operate two technologies differently.

10. A strike might not only reduce output but might in addition increase the cost of producing that reduced output. We ignore this additional effect but note that, even in its presence, each targeted firm might still benefit from the strike.

11. See Carter et al. (1981) for an empirical analysis of the effects of this strike.

12. Eaton and Grossman (1986) also consider Bertrand competition in their analysis of optimal trade policy under oligopoly. They show that whether the duopolists compete in price or quantity, an export tax (rather than subsidy) is optimal if the goods are strategic complements (upward sloping reaction functions) but not complements in demand. The export subsidy is optimal if the goods are strategic substitutes. The intuition behind this result will become clear in the next section, when we discuss extensions of our analysis to price competition, strategic complementarity and demand complementarity.

13. On this point, see also Dixit (1984) and Salant (1984). Referring to these papers, Brander and Spencer acknowledge that "adding more domestic firms weakens the case for domestic subsidies" (footnote 6, page 85). Indeed, it can easily destroy the case.

14. An optimal subsidy means here one that realizes the Stackelberg output for the domestic sector, since by assumption there is no domestic consumption of the good.

15. The literature on two-stage games (e.g. Dixit (1980)) also typically assumes that there exists a single incumbent whose prior positioning (expanded capacity, etc.) is designed to affect the subsequent simultaneous-move Nash equilibrium. If instead there were multiple incumbents supported by some form of prior government policy or otherwise, the opposite results (e.g. diminished capacity) could occur.

16. A particularly striking example has been worked out by Zachau (1987). With quadratic costs and proportional demand ($p = v/Q$), for any N and any $1 < S < N$, a merger causes a loss; the only merger which is profitable is merger to monopoly. In this example, whenever a marginal contraction would be profitable, the merger results in a non-marginal contraction which is so large that $q_m < \gamma$ and a loss results.

17. In the extreme case of monopoly ($S = N$), all of the variables will decrease; in the opposite extreme ($S = 1$) all of the variables will increase.

APPENDIX A

The Comparative Statics of the Equilibrium Systems

The Cournot equilibrium conditions (2), (3) and (4) and the price-taking equilibrium conditions (2'), (3') and (4) are systems of equations in p, q and μ with $\bar{q}$, N, S as parameters. The Jacobian of the system is $\Delta = (N-S)[p' + qp''] + p' - C'' < 0$ for the Cournot equilibrium and $D = (N-S)p' - C'' < 0$ for the price-taking equilibrium, since $p' < 0$, $p' + qp'' < 0$ and $C'' \geq 0$. The Jacobian being nonvanishing in both cases, there exists, by the implicit function theorem, functions $p(\bar{q},N,S)$, $q(\bar{q},N,S)$ and $\mu(q,N,S)$ which solve each system and whose derivatives with respect to the parameters may be obtained by applying Cramer's rule. In particular, for the Cournot equilibrium, the derivatives with respect to $\bar{q}$ are given by:

$$\frac{\partial p}{\partial \bar{q}} = \frac{[p' - C'']Sp'}{\Delta} < 0 \tag{A1}$$

$$\frac{\partial q}{\partial \bar{q}} = \frac{-[p' + qp'']S}{\Delta} < 0 \tag{A2}$$

$$\frac{\partial \mu}{\partial \bar{q}} = \left[1 + \frac{S[p' + \bar{q}p'']}{\Delta}\right][p' - C''] < 0 \tag{A3}$$

For the price-taking equilibrium, the same derivatives with respect to $\bar{q}$ are:

$$\frac{\partial p}{\partial \bar{q}} = \frac{-Sp'C''}{D} \leq 0 \tag{A4}$$

$$\frac{\partial q}{\partial \bar{q}} = \frac{-Sp'}{D} < 0 \tag{A5}$$

$$\frac{\partial \mu}{\partial \bar{q}} = -\left[1 + \frac{Sp'}{D}\right]C'' \leq 0 \tag{A6}$$

APPENDIX B

The Effect on the Profits of the Firms Outside the Subset

The profit of the typical firm outside the subset is:

$$\pi^n(\bar{q},N,S) = p(S\bar{q} + (N-S)q)q - C(q) \tag{B1}$$

where $q(\bar{q},N,S)$ is obtained from the solution to (2), (3) and (4) in the case of the Cournot equilibrium or (2'), (3') and (4) in the case of the price-taking equilibrium. In the case of the Cournot equilibrium, we verify, after making use of the envelope theorem and substituting for $\partial q/\partial\bar{q}$ from (A2), that:

$$\frac{\partial \pi^n}{\partial \bar{q}} = Sp'q\left[\frac{2p' + qp'' - C''}{\Delta}\right] < 0 \tag{B2}$$

Therefore, independently of N, S and $\bar{q}$, a marginal contraction of the outputs of a subset of the firms ($S > 0$) increases the profit of each of the firms outside the subset. This is because they end up producing more at a higher price (see Appendix A) and a greater spread between price and marginal cost, since $p' + qp'' < 0$. It can also be shown that they will always gain more than the firms in the subset. This is obvious if the firms in the subset lose. But suppose they gain and consider the following thought experiment. Take a firm inside the subset and expand its output until it is as large as the typical firm outside the subset. Simultaneously, take a firm outside the subset and contract its output until it produces as little as the typical firm within the subset. At no point would any other firm wish to deviate from its equilibrium output. Moreover, the price would remain at its equilibrium level. Clearly, this interchange will not be in the interest of the firm outside the subset. Since it is already producing at a price greater than marginal cost, any reduction in output with the price unchanged results in a loss of profit. The actual loss is given by:

$$\int_{\bar{q}}^{q} [p - C'(x)]dx$$

which is clearly positive when $p - C'(q)$ is positive and $\bar{q} < q$. Notice that if $\bar{q} > q$ the loss becomes a gain, i.e. if the firms in the subset lose from a marginal expansion of output, the outsiders lose even more. It is clear from (B2) that the firms outside the subset always lose from a marginal expansion of the outputs of the firms in the subset.

The equivalent to (B2) in the price-taking equilibrium is, after substituting for $\partial q/\partial\bar{q}$ from (A5):

$$\frac{\partial \pi^n}{\partial \bar{q}} = \frac{-Sp'qC''}{(N-S)p' - C''} \leq 0 \tag{B3}$$

With constant marginal costs, there is no effect on the profits of the firms outside the subset from either a marginal contraction or expansion of the outputs of the firms in the subset. But with strictly increasing marginal costs, the same results as for the Cournot equilibrium carry through, by similar reasoning.

APPENDIX C

The Stackelberg Levels of Output

In the Stackelberg equilibrium, the output of each of the S firms controlled by the leader solves:

$$\max_{\bar{q}} \left[p(S\bar{q}+(N-S)q)S\bar{q} - SC(\bar{q})\right] \tag{C1}$$

subject to q satisfying equation (2) when the N-S followers are Cournot players or equation (2′) when they are price-takers. The first-order condition to problem (C1), with S given, may be written:

$$\left[p + p'\bar{q} - C'(\bar{q})\right]S + \left[(S-1) + (N-S)\frac{\partial q}{\partial \bar{q}}\right]p'\bar{q}S = 0 \tag{C2}$$

Assume first a Cournot fringe. If N and S are such that condition (7), with $\partial q/\partial \bar{q}$ given by (A2), is satisfied with equality, the second term vanishes and condition (C2) reduces to $p + p'\bar{q} - C'(\bar{q}) = 0$. This is exactly the first-order condition at an unconstrained Cournot equilibrium (i.e., $\bar{q} = q$ and $\mu = 0$). For those combinations of N and S, the Stackelberg leader facing Cournot followers would therefore choose to operate the S firms under his control exactly as they would have been at an unconstrained Cournot equilibrium.

Now assume a fringe of price-takers. If N and S are such that condition (12) is satisfied with equality, then $[S + (N-S)\partial q/\partial \bar{q}] = 0$, where now $\partial q/\partial \bar{q}$ is given by (A5). The second term of (C2) therefore reduces to $-p'\bar{q}S$ and condition (C2) reduces to $p - C'(\bar{q}) = 0$. This is exactly the first-order condition at an unconstrained price-taking equilibrium. For those combinations of N and S, the Stackelberg leader facing a price-taking fringe would therefore choose to operate the S firms under his control exactly as they would have been at an unconstrained price-taking equilibrium.

APPENDIX D

Generalization of the comparative statics result on profits

Consider a symmetric game where each of N players simultaneously selects a scalar strategy, a_i, $i = 1,N$, and payoffs are collected. The payoff of player i, $i = 1,N$, is given by:

$$\pi^i(a_1,\ldots,a_i,\ldots,a_N) \tag{D1}$$

We assume π^i is twice continuously differentiable. We also assume that there exists a unique pure-strategy Nash equilibrium and that it is symmetric. Denote it $a_i = a$, $i = 1,N$.

A subset of S players is exogenously imposed the constraint $a_i = \bar{a}$. Without loss of generality we may assign the subscripts $1,2,\ldots,S$ to these players. The initial equilibrium will be displaced if and only if $\bar{a} \neq a$. We refer to the situation where $\bar{a} = a$ as the unconstrained equilibrium. Given N and S, at the new equilibrium we will have $a_i(\bar{a})$, $i = S+1,N$, as an implicit solution to the first-order condition of the N-S players outside the subset.

The effect of a marginal change in $\bar{a}$ on the equilibrium profit of the typical firm in the subset ($i = 1,S$) is given by:

$$\frac{d\pi^i}{d\bar{a}} = \frac{\partial\pi^i}{\partial a_i} + \sum_{\substack{j=1 \\ j\neq i}}^{S} \frac{\partial\pi^i}{\partial a_j} + \sum_{j=S+1}^{N} \frac{\partial\pi^i}{\partial a_j}\frac{da_j}{d\bar{a}} \tag{D2}$$

When $da_j/d\bar{a}$ is positive (respectively, negative), we have "strategic complements" (respectively, strategic substitutes). We consider the case where at the unconstrained equilibrium $\partial\pi^i/\partial a_j$ has the same sign for all $j \neq i$, $i = 1,S$ and $da_j/d\bar{a}$ has the same sign for all $j = S+1,N$.

When evaluated at the unconstrained equilibrium, $\partial\pi^i/\partial a_i = 0$ for all $i=1,N$. Hence, for strategic complements, (D2) implies $\mathrm{sgn}(d\pi^i/d\bar{a}) = \mathrm{sgn}(\partial\pi^i/\partial a_j)$ in the neighborhood of the unconstrained equilibrium, independently of N and S. On the other hand, for strategic substitutes, $\mathrm{sgn}(d\pi^i/d\bar{a})$ clearly depends on (N,S). For example, when S = 1, the first summation is zero and $\mathrm{sgn}(d\pi^i/d\bar{a}) = -\,\mathrm{sgn}(\partial\pi^i/\partial a_j)$. When N = S, the second summation is zero and $\mathrm{sgn}(d\pi^i/d\bar{a}) = \mathrm{sgn}(\partial\pi^i/\partial a_j)$.

These results are general. They include price and quantity competition as important special cases. Hence $\partial\pi^i/\partial a_j < 0$ arises in quantity competition if the goods are substitutes in demand and in price competition if the goods are complements in demand. $\partial\pi^i/\partial a_j > 0$ arises in quantity competition if the goods are complements and in price competition if the goods are substitutes.

REFERENCES

Brander, James A., and Spencer, Barbara J. "Export Subsidies and International Market Share Rivalry." Journal of International Economics 17 (1985): 83-100.

Bulow, Jeremy I.; Geanakoplos, John D.; and Klemperer, Paul D. "Multimarket Oligopoly: Strategic Substitutes and Complements." Journal of Political Economy 93 (1985): 488-511.

Carter, Colin A.; Hueth, Darrell L.; Mamer, John W.; and Schmitz, Andrew. "Labor Strikes and the Price of Lettuce." Western Journal of Agricultural Economics (July 1981): 1-14.

Cournot, Antoine A. Recherche sur les principes mathématiques de la théorie des richesses. Paris: Hachette, 1838.

Deneckere, Raymond, and Davidson, Carl. "Incentives to Form Coalitions with Bertrand Competition." Rand Journal of Economics 16 (1985): 473-486.

Dixit, Avinash K. "The Role of Investment in Entry Deterrence." Economic Journal 90 (1980): 95-106.

Dixit, Avinash K. "International Trade Policy for Oligopolistic Industries." Economic Journal 94 (1984) Supplement: 1-16.

Dowrick, Steve. "Von Stackelberg and Cournot Duopoly: Choosing Roles." Rand Journal of Economics 17 (1986): 251-260.

Eaton, Jonathan, and Grossman, Gene M. "Optimal Trade and Industrial Policy under Oligopoly." Quarterly Journal of Economics 101 (1986):383-406.

Frayssé, Jean. "Existence des équilibres de Cournot: un tour d'horizon." Annales d'économie et de statistique No 1 (1986): 9-33.

Gal-Or, Esther. "First Mover and Second Mover Advantages." International Economic Review 26 (1985): 649-653.

Kolstad, Charles D., and Mathiesen, Lars. "Necessary and Sufficient Conditions for Uniqueness of a Cournot Equilibrium." The Review of Economic Studies 54 (1987): 681-690.

Novshek, W. "On the Existence of Cournot Equilibrium." Review of Economic Studies 52 (1985): 85-98.

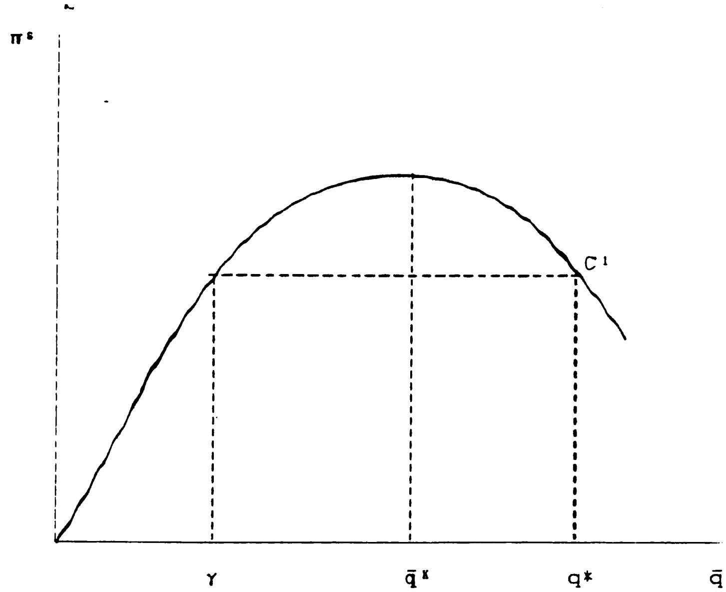
Salant, Stephen W. "Export Subsidies as Instruments of Economic and Foreign Policy." Rand Note N 2120-USDP. Santa Monica: The Rand Corporation, June 1984.

Salant, Stephen W.; Switzer,Sheldon; and Reynolds, Robert J. "Losses from Horizontal Merger: The Effects of an Exogenous Change in Industry Structure on Cournot-Nash Equilibrium." Quarterly Journal of Economics 98 (May 1983): 185-199.

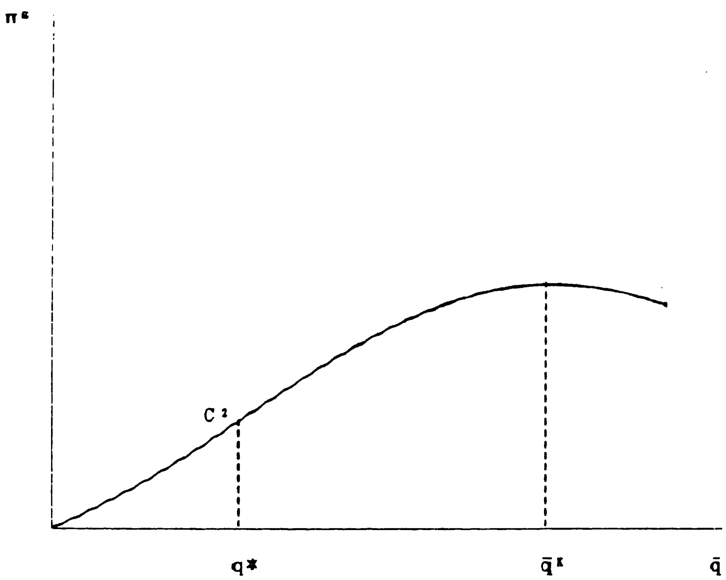
Shapiro, Carl. "Theories of Oligopoly Behavior." Discussion Paper no. 126. Princeton: Princeton University, Department of Economics, 1987.

Sonnenschein, Hugo. "The Dual of Duopoly is Complementary Monopoly: Or, Two of Cournot's Theories are One." Journal of Political Economy 76 (1968): 316-318.

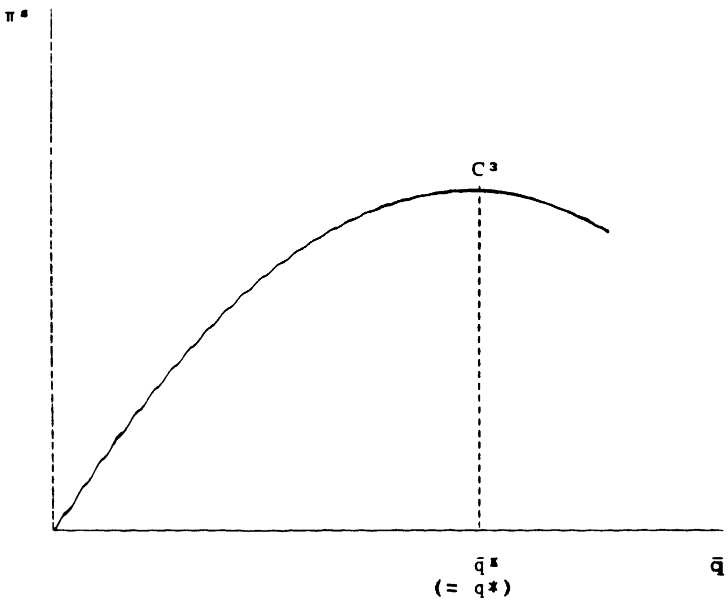
Zachau, Ulrich. "Mergers in the Model of an R&D-race." Mimeo. Bonn, Germany: Universitaet Bonn, Institut fuer Gesellschafts-und Wirtschaftswissenschaften, 1987.



Panel A



Panel B



Panel C

FIGURE 1

Recent Crest Working Papers

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88-13: Jerry A. Hausman, Jeffrey K. MacKie-Mason, "Price Discrimination and Patent Policy" February, 1988.

89-01: Mark Bagnoli, Severin Borenstein, "Carrot and Yardstick Regulation: Enhancing Market Performance with Output Prizes" October, 1988.

89-02: Ted Bergstrom, Jeffrey K. MacKie-Mason, "Some Simple Analytics of Peak-Load Pricing" October, 1988.

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89-10: Gerard Gaudet, Stephen W. Salant, "Uniqueness of Cournot Equilibrium: New Results from Old Methods" August, 1988.

89-11: Hal R. Varian, "Goodness-of-Fit in Demand Analysis" September, 1988.

89-12: Michelle J. White, "Legal Complexity" October, 1988.

89-13: Michelle J. White, "An Empirical Test of the Efficiency of Liability Rules in Accident Law" November, 1988.

89-14: Carl P. Simon, "Some Fine—Tuning for Dominant Diagonal Matrices" July, 1988.