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Efficiency requires that any risk in the economy be shared among individuals in such a way that each individual charges the same risk premium for an additional share in each lottery. If such risk sharing does not occur, then in principle income can be reallocated stochastically so as to make everyone better off. In particular, if certain risks are uncorrelated across individuals, everyone can be made better off by pooling these risks.

However, under existing institutions, individuals may not be able to pool all idiosyncratic risks. For example, a number of authors have explored the implications of the fact that individuals cannot easily avoid bearing the risk in their own labor income. Varian[1980] and Eaton and Rosen[1980] have shown that a personal income tax can result in a more efficient allocation of risk in the economy in this setting. Even when fluctuations in labor income are shared by all workers, Merton[1984], Fischer[1982], and Enders and Lapan[1982] have shown that when there is no market in a security corresponding to aggregate labor income then the government can still reallocate risk to nonworkers through a labor income tax in a Pareto-improving way.1

While markets may or may not adequately spread risk among individuals alive at any date, however, they certainly cannot spread risk to members of later generations for the simple reason that later generations are not alive to contract ex ante to bear some share of a lottery which resolves before they are born. Yet efficiency would require that each later generation bear some share of current lotteries.2 While the market does not provide a mechanism for current lotteries to be shared with later generations, however, the government easily can, through use of stochastic debt or a stochastic tax-transfer scheme, along the lines of Social Security.3 Debt can be issued (retired) in response to unfavorable (favorable) events, e.g. wars or recessions, and paid off (reissued) gradually over later generations. (Diamond[1965] worked out in a very general setting just how debt issues result in a reallocation of wealth between generations.) The objective of this paper is to examine the characteristics of an efficient risk-sharing policy.

Most models of risk-sharing examine the division of a set of lotteries among different individuals alive at a single point in time. In contrast, sharing risk between different generations raises a variety of new complications. For one, while earlier lotteries can in principle be shared with later generations, later lotteries cannot be shared with earlier generations. This complication alone, however, does not change the analysis in any fundamental way. Based on the intuition derived from static models, we might expect the efficient degree of risk sharing among different generations to involve each later generation bearing a relatively equal share of any given past lottery. This intuition has been stated occasionally in the literature (see, for example, Stiglitz[1983a, 1983b] and

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1 While there may be no explicit mechanism available to trade risk in labor income, firms may be able to reallocate risk between workers and capital owners through implicit labor contracts, an idea explored by Azariades[1975] and Bailey[1974].

2 For an implicit demonstration of this, see Bhattacharya[1982].

3 In principle, the family might also reallocate wealth ex post in response to events. If transfers occur anyway and are large relative to the risk, then they can easily adjust in response to the outcome. If not, and we assume not in this paper, then the family would face the same problems faced by the market in agreeing ex post on terms of risk sharing.
Gordon(1985)), but its implications have not been pursued. In section 1, we present a simple model which does imply equal sharing of earlier lotteries with later generations in steady state. Since there are many (an infinite number?) of future generations, we conclude that those alive at the time of the stochastic event (e.g. depression) should bear a very small share of any unexpected change in wealth. They ought to be almost fully compensated by the government, e.g. through a Social Security program, and the cost of the compensation should be paid off only gradually by later generations.

A variety of other complications arise, however, when sharing risk between different generations. In a static setting, the outcome of a lottery can be divided by any rule among different individuals while leaving the statistical distribution of the total resources unchanged. However, when some share of the outcome of a lottery is transferred from an earlier to a later generation, it earns a perhaps stochastic rate of return in the interim so that the later generation receives a different distribution of resources than the earlier generation originally passed forward. These more complicated terms of trade between generations have important effects on the efficient degree of risk-sharing.

In addition, when risk is shared in a static setting, each party can bind itself by contract to accept some share of any given lottery. However, a future generation cannot explicitly bind itself to any policy until it is alive to do so. But by that date, it knows the outcome of past lotteries that it is expected to share in, and, if they are sufficiently unfavorable, it may wish to renege on the policy. The government may not necessarily be able to precommit later generations to participate in the risk-sharing scheme. If not, the efficient design of the policy would look very different.

The organization of the paper is as follows. In section 1, we develop the characteristics of an efficient risk-sharing policy in a setting where these latter complications are ignored. This model of risk-sharing is very similar in form to that used in a static setting. In section 2, we then examine how the argument changes when the terms of trade between generations are less simple, while in section 3, we consider the possibility that later generations will renege on the policy, and the implications of this possibility for the design of the policy. Section 4 includes a brief discussion of a few other issues, while section 5 concludes the paper.

1. Risk Sharing Between Generations: Base Case

A. Steady State Analysis

We begin by exploring risk sharing in a very simple two period overlapping generations model. For simplicity, we assume there is no population growth and no technological change. Generations are therefore identical except for the period in which they are born. The generation born in period \( t \), and its earnings, are denoted through use of a subscript \( t \).

Members of each generation are assumed to work only while they are young and to consume only while they are old. We assume they work a fixed amount while young, earning a nonstochastic wage \( w \). They save this labor income and earn a stochastic return, \( \epsilon_t \), on it of mean zero, and variance \( s \), allowing them to consume \( w + \epsilon_t \) when they are old. Their utility depends solely on their consumption, which is stochastic. We assume, for simplicity, that their ex ante utility can be expressed as a function of their expected income and the variance of this income, and denote their utility by \( U(w) - V(s) \).

The outcome of the stochastic event is revealed "between periods", so after the previous generation has died but before the next generation is born. For simplicity, we assume that each stochastic

\[ \text{4 Most of our results generalize easily, though exposition is simpler in a mean-variance setting.} \]
event is identically distributed and independent of all others. Because there are never two genera-
tions alive both before and after a stochastic event, there is no possibility of sharing risk between
generations through the market. We assume, though, that risk is efficiently shared among members
of each generation. Each individual therefore has expected utility of $U(w) - V(s)$. When it adds
clarity, we include a subscript on the utility function indicating the generation number.

Since two generations are always alive simultaneously, however, the government can transfer
income from one to the other, and base this transfer on the outcome of past stochastic events.
For example, if the government transferred $e_{t-1}/2$ from the old to the young in each period $t$,
then the young in period $t$ receive net income of $w + e_{t-1}/2$. This income is saved, providing
$w + e_{t-1}/2 + e_t$ the next period.\(^5\) Part of this is paid as a transfer to the next generation, however,
leaving $w + (e_{t-1} + e_t)/2$ for consumption. Expected utility of generation $t$ would therefore be
$U(w) - V(\text{var}((e_{t-1} + e_t)/2)) = U(w) - V(s/2)$. Each lottery is shared between two generations,
allowing a pooling of risks and an increase in expected utility.

This tax-transfer scheme can easily be designed so as to share each lottery between $n$ genera-
tions. For example, the transfer from old to young in period $t$ can be specified to be $\sum b_i e_{t-i}$, for
some series of weights $b_i$. If this policy has been in effect for at least $n - 1$ generations, the utility
of generation $t$, expected as of $n$ periods earlier, can be expressed as $U(w) - V(\text{var}(\sum_{i=0}^{n-1} a_i e_{t-i}))$,
where $a_0 = 1 - b_1$, $a_i = b_i - b_{i+1}$ for $i = 1, \ldots, n - 2$, and $a_{n-1} = b_{n-1}$. If all resources are consumed
then it must be that $\sum_{i=0}^{n-1} a_i = 1$.

If $a_i = 1/n$ for all $i$, then each lottery is shared equally among $n$ generations. More importantly,
symmetry and risk aversion imply that this is the choice of the $a_i$ which maximizes steady state
expected utility. (The first $n - 2$ generations are clearly made better off by this policy, but do not
necessarily bear an efficient share of the risk.) As risk is shared among more generations — as
$n$ becomes larger — expected utility of each generation increases as well. In fact, as $n$ increases
without bound, each generation’s utility converges to $U(w)$ and the costs of bearing the collective
risks drop to zero.

If we were to allow for population growth, then even a larger share of each lottery ought
to be passed forward to future generations. In particular, if each lottery were to be shared by
$n$ generations, then we would conclude that each individual in the $n$ generations ought to share
equally in the initial lottery. But given population growth, later generations as a whole would bear
a larger share of the lottery than would earlier generations.

We therefore conclude from this model that the government ought to pass on most all the costs
of each stochastic event to later generations. Intuitively, risk ought to be shared equally among
all current and future individuals, and there are many future individuals relative to the current
number of individuals. Thus almost all risk should be passed forward to future generations.

In this section, we described the government policy as a tax-transfer scheme. It could equival-
ently have been described as a particular stochastic government debt policy. If each lottery is to be
shared equally between $n$ generations then the government upon seeing the outcome $e_t$ can retire
$[(n - 1)/n]e_t$ dollars of debt, funded by a tax on the old (those who received $e_t$), then reissue $e_t/n$
dollars of debt during each of the following $n - 1$ periods, paying the proceeds of each debt issue
to the old in that period. This policy results in the same redistribution of risk as the tax-transfer
scheme described originally.

\(^5\) For the moment, we assume that the same random income $e_t$ is received independently of how
much is saved from the previous period. In a later section we allow returns to be multiplicative.
B. The transition period

We have just described the risk-sharing policy which maximizes steady state expected utility. During the transition period after the policy is introduced, however, the optimal rule for sharing risks between generations is more complicated.

In general the government can allocate a share of the risk $a_{ij}$ occurring in period $i$ to generation $j$, where $\sum_{j=0}^{n-1} a_{ij} = 1$. In addition, it can impose a nonstochastic tax of $c_j$ on generation $j$, where budget balance requires that $\sum_{j=0}^{\infty} c_j = 0$. A policy is Pareto optimal when it maximizes the utility of the first generation while providing at least a prespecified level of utility to each later generation, and satisfies the sets of constraints that $\sum_j b_j = 0$ and that $\sum_j a_{ij} = 1$ for each $i$. The resulting first-order conditions imply that

$$V'_j a_{ij} / U'_j = z_i$$

for some $z_i$ and for all values of $i$ and $j > i$. Several conclusions follow easily. First, if the utility function depends linearly on mean and variance, then the ratio $V' / U'$ will be the same for every generation $i$, and by equation (1) each lottery will be shared equally among all $n$ eligible generations.

Second, each of the $a_{ij}$ must be strictly positive. By equation (1), each generation sharing in a lottery must charge the same risk premium for bearing a marginal share in the lottery. At least one of the $n$ generations must bear some share in the lottery, so be risk averse at the margin. It follows that $a_{ij} > 0$ for all relevant generations $i$.

Third, if at the optimum, one generation is less risk averse at the margin than another ($V'(.)/U'(.)$ is smaller), then it ought to be bearing more of each lottery that they both participate in. Since earlier generations during the transition period share in fewer lotteries than later generations, they might be expected to be less risk averse at the margin, so ought to bear a larger share of each common lottery. This follows strictly if both earlier and later generations are constrained to have the same utility, and if it is true globally that

$$(V'' / V') > (U'' / U')(V'/U')$$

To see this, note that $U(C_t) - V(R_t)$ by assumption is the same for all generations except the first, where $C_t$ is expected income and $R_t$ is the variance of income of generation $t$. If, to the contrary at the optimum, earlier generations are not less risk averse at the margin than later generations, then they certainly bear less risk (they bear no larger a share of fewer lotteries), but also have a smaller $C_t$ to leave utility at the common level. Can they still be no less risk averse? Differentiating utility once with respect to $R_t$, allowing $C_t$ to increase to leave utility constant, we find $\partial C_t / \partial R_t = V'/U'$, which we referred to as the marginal risk premium. Differentiating utility a second time with respect to $R_t$, we find that $\partial^2 C_t / \partial R_t^2 > 0$, implying that generations bearing less risk must be less risk averse, as long as equation (2) is satisfied. Therefore, given equation (2), earlier generations during the transition period must bear a larger share of any common lottery than do later generations.

C. More General Utility Functions

We have so far assumed that individuals consume only in their second period, and that their utility depends only on the mean and the variance of their consumption. Consider now the more general model in which individuals can consume in both periods, and in which their utility function equals $E[W(C_1, C_2)]$. While the same qualitative conclusions about risk-sharing still follow, specific conclusions do not. For example, consider the tax-transfer scheme which shares each lottery between two generations by transferring $a_{t-1} c_{t-1}$ from old to young in each period $t$. The value of $a$ which maximizes steady-state utility is still non-zero, but it no longer equals .5. The utility of the steady
state generation can be expressed as $E[W(C_1, w + ae_{t-1} + (1 - a)e_t - C_1)]$. Differentiating with respect to $a$, we find that

$$E[(W_1 - W_2)\partial C_1/\partial a] + E[(e_{t-1} - e_t)W_2] = 0,$$

where $W_1$ and $W_2$ represent the partial derivatives of utility with respect to $C_1$ and $C_2$ respectively. The first-order conditions for the individual’s choice of utility with respect to $C_1$ and $C_2$ respectively. The first-order conditions for the individual’s choice of utility, however, imply that the first term in equation (3) equals zero, so that at the optimal value of $a$, $E[e_{t-1}W_2] = E[e_tW_2]$. As long as $C_1$ is positively correlated with $e_{t-1}$, as it must be unless first period consumption is an inferior good, then this condition implies that the optimal value of $a$ is greater than .5. Intuitively, it is less costly to absorb a random event if one knows the outcome earlier in life. In this more general setting, therefore, even more risk is passed forward to future generations.

2. Risk Sharing Between Generations: Complicating Factors

A. Positive Real Return to Savings

So far, we have assumed that when a contingent dollar is transferred to future generations, then consumption of future generations increases by the same contingent dollar. This is not a reasonable description, however, of the process by which resources are passed on to future generations. In this section, we consider in turn a series of models which explore the implications of the more complicated intertemporal terms of trade.

To begin with, we assume that income saved in the first period of life earns a positive, constant, and nonstochastic rate of return of $r$. To simplify the discussion, we return to our original assumptions that individuals consume only when old and that they care only about the mean and the variance of their consumption, and reexamine the policy sharing risk among $n$ generations which maximizes steady-state utility. If generation $t$ is still allotted the share $a_i$ of the lottery which occurred $i$ periods earlier (in steady state $a_i$ cannot depend on $t$), then its consumption in the second period would be $w + \sum_{i=0}^{n-1} a_i e_{t-i} (1 + r)^i$, yielding expected utility of

$$U(w) - V(\sum_{i=0}^{n-1} a_i^2 (1 + r)^{2i} s).$$

Maximizing steady state utility with respect to the $a_i$, subject to the constraint that $\sum_i a_i = 1$, we find at the optimum that

$$a_i = \frac{(1 + r)^{-2i}}{\sum_{i=0}^{n-1} (1 + r)^{-2i}}$$

and utility equals $U(w) - V(s/\sum_{i=0}^{n-1} (1 + r)^{-2i})$. As before, utility increases as $n$ increases, and in the limit utility converges to $U(w) - V(s(r^2+2r)/(1+r)^2)$, while $a_i$ converges to $(r^2+2r)/(1+r)^2(1+i)$. Unlike in the base case, each generation still bears some risk and, in particular, can pass forward to later generations only a fraction $1/(1+r)^2$ of its own lottery. When risk is passed to the next generation, the size of the lottery is increased by a factor $(1 + r)$, increasing its variance by a factor $(1 + r)^2$, making it more expensive to pass risk forward to later generations.

B. Compounding of lotteries

For simplicity of notation, let us assume again that $r = 0$, but now assume that each lottery involves a stochastic proportional, rather than additive, return on savings. As risk is passed forward to later generations, savings become stochastic, and lotteries compound.

If each individual is still allotted the share $a_i$ of the lottery occurring $i$ periods earlier, then due to compounding of lotteries, the extra risk it would bear as a result of this lottery would be
\(a_i^2 s(1+s)^i\). Steady state utility becomes \(U(w) - V(s \sum_{i=0}^{n-1} a_i^2 (1+s)^i)\). This expression is very close to that derived assuming \(r > 0\), and we can conclude directly that the optimal values of the \(a_i\) are

\[a_i = (1+s)^{-i}/(\sum_{i=0}^{n-1}(1+s)^{-i}),\]

and in the limit, as \(n\) increases, \(a_i = s/(1+s)^{1+i}\). When risk is passed to a later generation, it grows in variability due to the random return it earns each period. As a result, it is more "expensive" than in our base case to pass risk on to later generations, and less ought to be done.

C. Diminishing returns to savings

When the policy proposed in section 1 is used to share risk among \(n\) generations, the savings done by a steady state generation would equal \(w + \sum_{j=1}^{n-1} ((n-j)/n) \epsilon_{t-j}\). There is nothing in the model which assures that savings is positive. Moreover, as \(n\) gets larger, savings follows close to a random walk from one generation to the next, and a random walk has probability one of hitting any boundary (e.g. a requirement that savings be nonnegative) in finite time. Our model therefore does not guarantee that net assets will remain positive.

This is not a logical problem with the model if the economy is open, and if the government can borrow from foreigners at an exogenous interest rate. The domestic capital stock can be large and positive even when the net assets of individuals in the country is negative – foreigners can own large amounts of both corporate and government bonds. One question this raises, however, is whether risk can be shared with the rest of the world, as well as among various generations. We assume that the amount of risk shared with foreigners is exogenous to government policy, and if some risk sharing occurs, that \(\epsilon_t\) represents the residual risk borne by generation \(t\).

If the economy is closed, however, then negative net assets imply a negative capital stock, a logical impossibility. One simple way to proceed is to impose a constraint that net assets must always be positive, and resolve for the optimal policy.\(^6\) The prime force keeping the capital stock large, however, is presumably the concavity of the production function and so the large marginal product of capital when the capital stock is small.

Stochastic models with a nonlinear return to savings quickly become very messy, however. So far, we have made little progress characterizing efficient risk-sharing policies in this setting except in very special cases. We sketch one such case here.

To keep the analysis simple, we maintain the assumptions of the base case that individuals consume only when old and care only about the mean and the variance of their consumption. We also examine risk-sharing between just two generations, and assume that the government transfers \(ae_{t-1}\) from old to young in each period \(t\). The resulting consumption of generation \(t\) is assumed to equal \(f(w + ae_{t-1}) + (1-a)\epsilon_t\), where \(f(\cdot)\) is a concave function. Given the policy, utility of a generation in steady-state can be expressed as

\[U(Ef(w + ae_{t-1})) - V((1-a)^2 s + \text{var}(f(w + ae_{t-1}))).\] (6)

The first derivative of utility with respect to \(a\) equals

\[U' \partial Ef/\partial a - V'(\partial \text{var}(f)/\partial a - 2(1-a)s].\] (7)

\(^6\) Such a model would be related to models of optimal savings as in Foley–Hellwig[1975], models of optimal commodity stockpiles, and also models of lifetime consumption and portfolio rules under uncertainty, as in Merton[1971].
How does the optimal policy change when the function \( f(\cdot) \) becomes more concave? If the function were linear, so that \( f(K) = (1 + r)K \), then the first-derivative simplifies to

\[
-2sV'[\{(1 + r)^2a - (1 - a)\}].
\]

Comparing equation (7) with equation (8) we see that allowing the function \( f(\cdot) \) to be concave changes the first-derivative of utility with respect to \( a \) in two qualitative ways. First, when \( f(\cdot) \) is concave, adding random fluctuations to the capital stock causes expected capital income to fall. In addition, the variance of capital income may change as \( f(\cdot) \) becomes concave. We have so far been unable to make any general statement about the direction of the change. As one example, if \( \epsilon_t \) is distributed normally, if \( f(\cdot) \) is a quadratic function, and if we allow it to become more concave, holding its first-derivative constant, then the variance of \( f(\cdot) \) increases. In this special case, the first-derivative of utility with respect to \( a \) is absolutely smaller when \( f(\cdot) \) is more concave, implying that the optimal degree of risk-sharing with the next generation is smaller.

3. Time Consistency of the Policy

In analyzing the government policy so far, we have assumed that what is optimal ex ante will in fact be done ex post. However, each generation inherits a history of past lotteries in which it is asked to share, in exchange for having future generations share in its own uncertainties. If the past history is unfavorable enough, however, it might wish to renege on its obligations to its parents, even at the price of foregoing risk sharing with its children. If reneging is otherwise costless, we show that children would wish to renege at least whenever they are asked to make a payment to their parents. Whether they would succeed in repealing the policy, however, depends on our model of political decision-making. In this section, we examine when children would wish to renege, and explore the efficient design of the policy given two alternative models of political decision-making.

In our first model of political decision-making, we assume that the younger generation is more numerous and that the median voter, who would be young, controls the government. In addition, we assume, to avoid obvious problems, that the median voter can repeal an existing risk-sharing policy, but cannot otherwise change its design.

Given this model, when would the policy be repealed? Clearly the incentive to repeal the policy depends on what penalties are incurred in doing so. We assume for now that the only penalty is that the risk-sharing plan cannot be reenacted for at least one generation, so that the young, if they repeal it, cannot share their own lottery with later generations. Assume that each generation’s lottery is bounded below by \(-b\), and that the proposed policy involves a transfer from children to parents which is a strictly decreasing function of the outcome of each of the previous \( n - 1 \) lotteries. Assume as well that the policy involves a net payment from children to parents when the worst outcome occurs during each of the previous \( n - 1 \) lotteries. But if all these worst outcomes do occur, the children will surely vote for repeal. They must make the maximum possible payment to their parents for sure, and they can receive back the same amount from their own children only if they also suffer the worst outcome on their lottery. With any other outcome of their own lottery, they lose on net from the policy. Assume then that there is some maximum payment that children would willingly make to their parents without voting for repeal. But if children are asked to make this payment, by the same argument they will refuse. At best, they can just recoup their losses from their own children, and most of the time they lose on net. Therefore children would repeal the policy whenever they are asked to make a payment to their parents.

Once the policy has been repealed, however, the next generation would have no interest in restarting the policy, since regardless of its own luck, its children would never pay them anything, whereas the policy would still involve its paying money to the next generation under some outcomes.
They would be better off with no policy in effect. Therefore, even if the policy were in effect initially, once children are asked to make a payment to their parents, the policy is repealed, and the policy is never reestablished. \(^7\)

The policy will not be repealed under all contingencies, however. Consider the situation where the outcome has been the best possible during each of the previous \(n-1\) generations, and that the policy implies a transfer from parents to children in this situation. The children would surely accept the transfer even though this obliges them to make the appropriate transfer to their own children – the transfer they must make can be no larger than the transfer they receive for sure, and will normally be smaller.

One possible mechanism to handle this likelihood of repeal is to build into the policy a large enough penalty on any generation that chooses to repeal to eliminate any incentive to do so. This penalty could involve either a tax, the proceeds from which could be used to retire government debt, or merely the administrative and social disruption that would ensue if the government were to default on its legal obligations (as would happen if past transfers were funded by government debt rather than by a tax-transfer scheme, such as Social Security). To guarantee that repeal would never occur, this penalty must be large enough so that even if the worst outcome has occurred during the previous \(n-1\) generations the next generation would rather follow through on the policy, receiving in partial compensation the right to share its own lottery with future generations, rather than face the penalty.

A second-best policy for the government to follow in setting up the rules for the policy would be to allow repeal to occur when a generation chooses it, but then to provide the funds necessary to induce the next generation to restart the policy. Since, under the policy, this generation can receive nothing from its children but will likely have to pay them something, they must be paid something ex ante to induce them to accept the obligation.

These government expenditures must be financed somehow. Can the government borrow to fund these costs of restarting the policy whenever repeal occurs, and impose taxes in some form to retire this debt so as to leave all generations better off than they would have been without the policy? \(^8\) Consider a policy, which shares risk between just two generations, where the policy will be repealed for the worst \(p\) percent of the outcomes of the parents' lottery. If the policy is not repealed, the average payment from parents to children is \(y\). Given this policy, if repeal does occur at some point, the government must make a payment of at most \((1 - p)y\) to the next generation to induce them to restart the policy. If the children were risk neutral, then they would require a payment of precisely \((1 - p)y\), since this is the expected amount they have to pay their children, whereas if they are risk averse, they would settle for a smaller inducement, since they have to pay under the policy only under the most favorable circumstances, whereas they receive funds from the government under all outcomes.

The government must therefore pay out at most \((1 - p)y\) during \(p\) percent of the periods, for an average payment of \((1 - p)py\). But whenever the policy is not repealed, parents pay their children \(y\) on average, whereas the children would settle for at most \((1 - p)y\) and still not vote for repeal. Therefore the government can tax away the difference and still leave the children better off under the policy. Tax revenues are therefore at least \(py\) during \((1 - p)\) percent of the periods, which is sufficient to fund expenses of at most \((1 - p)y\) during \(p\) percent of the periods. Some self-sustaining

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\(^7\) This of course raises the question of why the debt of the Social Security program has been able to grow so large.

\(^8\) We assume in this argument that the government does not renege on this debt, even when the policy is repealed.
policy is therefore possible, even with the possibility of repeal.9

The median-voter model is not the only possible model of government decision-making, however. As one simple alternative, let us assume that the policy would be repealed whenever those in favor of repeal are willing to pay more towards this effort than those against repeal are willing to pay to avoid it.10 Given this model of government decision-making, the analysis of when repeal occurs is very simple. At any date, the risk-sharing policy involves a nonstochastic transfer between old and young plus a promise of a stochastic transfer to the current young during the next period. Ignoring the next period’s transfer, both generations have exactly the same income at stake, and their political influences would exactly counterbalance. If the presence of the stochastic transfer during the next period raises the expected utility of the current young, conditional on knowledge of the outcome of past lotteries, then they would be more in favor of maintaining the policy, or less inclined to oppose it. Given that otherwise, preferences were exactly counterbalanced, we conclude that the policy will remain in force whenever the young gain by the presence of the stochastic transfer during the next period.

When do the young gain from the next period’s transfer? They certainly gain if the policy always involves a (stochastic) payment from young to old, as under Social Security. If the policy were to require payments from parents to children, then it would certainly be repealed. In general repeal would occur whenever, given knowledge of the outcome of past lotteries, the expected payment from parents to children during the next period is large enough to offset any gain from sharing risk with future generations.

Thus, in sharp contrast to the results of the previous model, we now find that a tax-transfer program like Social Security has a most stable design, and according to our model would never be repealed. In addition, if it did not exist, any generation would be very much in favor of enacting it, since the first generation receives a transfer from its children while making none to its parents. It should be noted, however, that every generation would like to be the first generation. If a generation could repeal the policy long enough to avoid making any payment to its parents, yet reenact it in time to receive a payment from its children, it would be another first generation. We ruled this behavior out by assumption, though further effort is needed to justify this assumption.

4. Further Issues

A. The Family as an Alternative Risk-Sharing Institution

Why cannot the family provide the same type of risk sharing between generations of the family that the above government policies would provide? If parents are altruistic and would always wish to leave a positive bequest, regardless of events, then they could provide the same risk-sharing. In general, however, the family would face a number of added difficulties.

If parents do expect to be able to share their lotteries with future generations within their family, then they would wish to maintain close to their normal consumption level even during a very unfavorable event, e.g. depression. However, they may not at that point have the financial assets to fund this consumption level. Under the government risk-sharing policy, the government would issue debt and use it to finance transfer payments. Under the analogous family policy, the parents would try to borrow funds to finance their consumption. However, they would lack collateral for the loan, and could not legally commit their children to pay it back. In contrast, the

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9 There would remain some chance that the debt accumulates beyond some acceptable level, but this chance quickly gets small as the expected net income each period increases above zero.

10 See Wellisz and Wilson[1984] for an extended application of this model.
government would not need explicit collateral and does legally bind future generations whenever it issues debt.

As we saw in the previous section, a political coalition might well develop under certain circumstances to repeal a government risk-sharing policy. Within a family risk-sharing policy, each generation would face similar incentives to "repeal" the policy. However, within the family, the paying generation can repeal the policy on its own regardless of the preferences of the receiving generation. Repealing a family risk-sharing policy is much easier than repealing a government policy.

When would a generation within the family wish to "repeal" the risk-sharing policy? When considering the preferences of a generation under a government risk-sharing policy, we assumed that there was a clear link between its willingness to participate in the policy when it is young and its likelihood of participating in the policy when it is old. In particular, we assumed that if the policy were repealed now, it would not exist during the next period. Within the family, however, there is no contractual link between transfers now and transfers next period. Children may well hope that their children will help them out, independently of whether they help their parents out. Without this link between transfers now and transfers next period, every generation would face an incentive to renege. Altruism may overcome this, but without sufficient altruism the family risk-sharing policy is likely to break down.

B. Moral Hazard

Whenever insurance is provided, we would expect moral hazard to be a problem. In this context, however, we are providing insurance to a generation rather than to an individual. Transfer payments to any individual could easily be specified to be independent of that individual's actions, depending only on the outcome of the "market" as a whole. In this case, each individual's incentives are left unchanged by the transfer payment, even though risk is being shared with later generations. Of course, the policy might be designed to depend on each individual's position, and so create a moral hazard problem, but this is not an essential characteristic of the policy.

Even if the transfer depends solely on "market" outcomes, however, some moral hazard problem may still exist. Social decisions could well change if the benefits from an activity are distributed across generations differently than the costs. For example, if the financial costs of a war are heavily passed on to future generations, but if the political benefits of the war are mainly received more quickly, then inefficient decisions may well be made about how to conduct the war. Similarly, if the risk-sharing policy is based on what happens to labor income, then current generations may face an incentive to enact other programs which shift income from labor to capital, thereby aiding current generations at the expense of future generations. All these examples, however, involve a mismeasurement of the outcome of the social lottery, and are not fundamental to the idea of sharing risk across generations.

3. Conclusions

In this paper, we have argued that in designing government debt and tax-transfer policies, it is important to consider their implications for the allocation of risk between generations. There is no reason to presume that the market or the family can allocate risk efficiently to future generations, implying that stochastic government policies have the potential to create first-order welfare improvements. The model supports the use of such policies as debt-finance of wars and recessions, or Social Security type tax-transfer schemes which aid unlucky generations, e.g. the Depression generation, at the expense of luckier generations.

It is premature, though, at this point to draw any conclusions concerning the degree to which
government debt or tax-transfer policy ought to be modified in light of risk-sharing considerations. In future work, we will attempt to set up a computational overlapping generations model of the economy and simulate the characteristics, both the design and the extent, of an optimal risk-sharing policy under various plausible parameter values.
REFERENCES


C-1 Lawrence E. Blume

C-2 John G. Cross

C-3 John G. Cross

C-4 Hal R. Varian

C-5 Hal R. Varian

C-6 John P. Laitner

C-7 John G. Cross

C-8 Carl P. Simon
Ellet's Transportation Model of an Economy with Differentiated Commodities and Consumers, I: Generic Cumulative Demand Functions, June 1978.

C-9 Theodore C. Bergstrom

C-10 Theodore C. Bergstrom

C-11 Hal R. Varian

C-12 Lawrence E. Blume

C-13 Lawrence E. Blume
Consistent Expectations, January 1979.

C-14 John G. Cross
On Baubles and Bubbles.

C-15 Theodore C. Bergstrom
<table>
<thead>
<tr>
<th>Paper</th>
<th>Authors</th>
<th>Title</th>
<th>Journal</th>
<th>Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>C-17</td>
<td>John P. Laitner</td>
<td>The Natural Rate of Unemployment</td>
<td></td>
<td>August 1979</td>
</tr>
<tr>
<td>C-18</td>
<td>Lawrence E. Blume and David Easley</td>
<td>Learning to be Rational</td>
<td>Journal of Economic Theory</td>
<td>vol. 26, no. 2, April 1982</td>
</tr>
<tr>
<td>C-19</td>
<td>Hal R. Varian</td>
<td>Notes on Cost-Benefit Analysis</td>
<td></td>
<td>October 1979</td>
</tr>
<tr>
<td>C-21</td>
<td>Theodore C. Bergstrom</td>
<td>Cournot, Novshek and the Many Firms</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C-22</td>
<td>Hal R. Varian</td>
<td>The Nonparametric Approach to Demand Analysis</td>
<td>Econometrica</td>
<td>50, (1982), 945-972</td>
</tr>
<tr>
<td>C-23</td>
<td>John P. Laitner</td>
<td>National Debt, Social Security, and Bequests</td>
<td></td>
<td>October 1980</td>
</tr>
<tr>
<td>C-26</td>
<td>John P. Laitner</td>
<td>Monopoly and Long-Run Capital Accumulation</td>
<td>Bell Journal of Economics</td>
<td>Spring 1982</td>
</tr>
<tr>
<td>C-27</td>
<td>David Sappington</td>
<td>Information Asymmetry and Optimal Inefficiency Between Principal and Agent</td>
<td></td>
<td>November 1981</td>
</tr>
<tr>
<td>C-29</td>
<td>Allan Drazen</td>
<td>A Quantity-Constrained Macroeconomic Model with Price Flexibility</td>
<td></td>
<td>Aug., 1980</td>
</tr>
<tr>
<td>C-30</td>
<td>David Sappington</td>
<td>Limited Liability Contracts Between Principal and Agent</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
C-32 Hal R. Varian

C-33 Hal R. Varian

C-34 Hal R. Varian
   Indirect Utility and Nonparametric Demand Analysis, March 1981.

C-35 Hal R. Varian

C-36 Hal R. Varian

C-37 Hal R. Varian

C-38 Theodore C. Bergstrom and Richard Cornes

C-39 Theodore Bergstrom and Richard Cornes

C-40 John G. Cross

C-41 Lawrence E. Blume and David Easley

C-42 Hal R. Varian and E. Philip Howrey

C-43 Theodore C. Bergstrom
   Lectures on Public Economics, November 1981.

C-44

C-45 David Sappington
   Optimal Regulation of Research and Development Under Imperfect Information, December 1981.

C-46 John P. Laitner
   Oligopoly Behavior When Conjectural Variations are Rational, Feb., 1982.
C-47 Theodore C. Bergstrom

C-48 Carl P. Simon

C-49 Theodore C. Bergstrom and Carl P. Simon

C-50 Lawrence E. Blume
On Settling Many Suits, April 1983.

C-51 Theodore C. Bergstrom
On the Theory of Cash Flow Taxes on "Rents".

C-52 Theodore C. Bergstrom

C-53 Theodore C. Bergstrom and Hal R. Varian

C-54 Mark Bagnoli

C-55 Lawrence E. Blume and David Easley

C-56 Theodore C. Bergstrom, Lawrence E. Blume and Hal R. Varian

C-57 Hal R. Varian

C-58 Hal R. Varian

C-59 Hal R. Varian and William Thomson
Theories of Justice Based on Symmetry, in Social Goals and Social Organization, Hurwicz and Sonnenschein (eds.) 1985.

C-60 Lawrence E. Blume and David Easley
Implementation of Rational Expectations Equilibrium with Strategic Behavior, revised August 1984.

C-61 N. Sören Blomquist

C-62 John G. Cross

C-63 Hal R. Varian and Theodore C. Bergstrom
C-64  Theodore C. Bergstrom and Hal R. Varian

C-65  Hal R. Varian
Observable Implications of Increasing or Decreasing Risk Aversion,

C-66  Roger H. Gordon and Hal R. Varian