

MichU
DeptE
CenREST
W
87-13

The Suzanne and
Laura Foster Library

SLP 201907

Center for Research on Economic and Social Theory
CREST Working Paper

Successful Takeovers without Exclusion

Mark Bagnoli
Barton L. Lipman

August, 1987
Number 87-13



DEPARTMENT OF ECONOMICS
University of Michigan
Ann Arbor, Michigan 48109

Successful Takeovers without Exclusion

by

Mark Bagnoli
University of Michigan

and

Barton L. Lipman
Carnegie-Mellon University

Revised

August 1987

We would like to express our appreciation to Ted Bergstrom, Rick Green, Phil Howrey, Dan Kovenock, Michael Rosenzweig, Steve Salant, and Chester Spatt for their comments and advice. As usual they are not responsible for any errors that we may have made. This work was partially supported by National Science Foundation Grant SES-8520296.

Section 1. Introduction.

Virtually all of the theoretical models of takeovers produced in recent years assume atomistic stockholders.¹ Analogous to standard “price-taking” assumptions, the idea is that there are many, small stockholders, none of whom can affect the outcome of a bid. Informal analyses of models with a single bidder (or raider) and atomistic stockholders have concluded that there is a free rider problem associated with takeovers. In the presence of this free rider problem, certain exclusionary devices are necessary for successful takeovers.

Clearly, the atomistic stockholder assumption is intended to model a very widely held firm. Do similar conclusions hold for firms which are not as widely held? The importance of this question is underscored by the fact that, as Demsetz and Lehn [1985] note, there are many very large firms which are controlled by a relatively small number of stockholders. In their sample of 511 large firms, the 20 largest stockholders own controlling interest in 22% of the firms and 37% of the stock on average. To answer this question, we consider a model with a finite number of stockholders. We will show that the outcome with a finite number of stockholders is quite different from the atomistic stockholder outcome. In fact, with a finite number of stockholders, successful takeovers are possible without exclusionary devices. Hence it is important to know when the conclusions of the atomistic stockholder model do hold, in order to determine when exclusionary devices are necessary for successful takeovers.

The assumption of atomistic stockholders can mean one of two things. Either it is actually true that no stockholder can affect the outcome or else some stockholders believe they have no effect when, in fact, they do. Since the latter is inconsistent with rational agents, we will identify the atomistic stockholder assumption with the former. We discuss two ways to formalize this assumption. The standard formalization of many, small agents who cannot individually influence aggregate outcomes is the infinite player game.² We will show that the infinite stockholder outcome is quite different from the atomistic stockholder outcome. An alternative approach is to study the

¹ A survey of this area is provided by Spatt [1986]. Examples of models with a single bidder and atomistic stockholders include Grossman and Hart [1980], Bradley [1980], and Bradley and Rosenzweig [1985]. The atomistic stockholder assumption is used differently in models with multiple raiders, a case we do not consider.

² See Green [1984].

limiting game, rather than the game at the limit. We will consider a sequence of firms, each of which is more widely held than its predecessor. We will give conditions under which the atomistic stockholder outcome obtains in the limit—that is, when exclusionary devices are necessary for successful takeovers.

The atomistic stockholder models conclude that exclusionary devices are socially desirable as they are necessary for successful takeovers. Since we have shown that they often are not necessary, we reconsider their effects. We show that, with a finite number of stockholders, these devices can lead to very undesirable consequences.³

To give the intuition for our results, we must first give an overview of the atomistic stockholder models. These models⁴ have shown that atomistic stockholders have an incentive to free ride on the improvements brought about by the raider. Since no stockholder can affect the outcome of the takeover bid, then, assuming stockholders have rational expectations, if the bid is going to succeed, no stockholder will sell unless he is offered at least the post-takeover value of his stock. Consequently, the raider cannot purchase a share unless he pays at least what the share is worth to him if the bid succeeds. If he does so, then even ignoring any costs of making a bid, he cannot earn profits by taking over the firm.⁵ Since stockholders will not tender for less than the expected post-takeover value of their shares to them, successful takeovers require a divergence between this value and the value of these shares to the raider. We will call such a divergence exclusion. The idea behind these mechanisms is that “The only way to create proper incentives for the production of a public good (*i.e.*, guaranteeing that the firm is efficiently run) is to exclude non-payers (minority stockholders) from enjoying the benefits of the public good.”⁶

Several different exclusionary devices have been discussed in the literature. Grossman and Hart focus on dilution. The idea is that prior to a takeover, the shareholders voluntarily accept a dilution of their property rights in the event of a takeover by adopting rules which allow the raider to exclude them from some of the increase in the value of a share.⁷ Bradley and Bradley and Kim focus on front-loaded two-tier bids. These bids specify two prices, the front-end and back-end

³ The further policy implications of our model are discussed in detail in Bagnoli and Lipman [1986a].

⁴ See, for example, Grossman and Hart, Bradley, Vermaelen [1981], and Bradley and Kim [1984].

⁵ This form of the argument is taken from Grossman and Hart.

⁶ Grossman and Hart, page 59. The details in parentheses have been added.

⁷ For example, the raider may do this by selling some of the firm's assets to another firm that he controls at less than the market value of these assets.

prices. The raider offers to pay the front-end price for controlling interest in the firm. If he takes over, then the minority stockholders are forced to sell their shares for the back-end price. As long as the back-end price is less than the value of share under the raider's management, this is a form of exclusion.

Note that the crucial part of the argument that exclusion is necessary for successful takeovers is that no stockholder perceives the effect his decision has on the outcome of the bid. Since this is precisely the atomistic stockholder assumption, one would expect a different outcome without atomistic stockholders. In fact, we will show that when there is a finite number of stockholders, some stockholders must be pivotal in the sense that they do recognize that they may affect the outcome. Making some stockholders pivotal is crucial because it forces them to choose whether or not the bid succeeds. Hence, they cannot free ride, so exclusion is not necessary for successful takeovers.

There are a few papers on takeovers which do not assume atomistic stockholders.⁸ Shleifer and Vishny [1986], Bebchuk [1985], and Hirschleifer and Titman [1987] consider models in which not all stockholders are atomistic. None of these papers focus on the same issues we are interested in. The paper most closely related to ours is a mimeo by Kovenock [1984]. He calculates the same mixed strategy subgame equilibrium we calculate below and makes some points related to ours. However, he does not consider the raider's optimal strategy.

Our analysis is divided into three parts. After defining the game more precisely in Section 2, we turn to an analysis of the differences between the finite and atomistic stockholder outcomes with nonexclusionary bids. In Section 3A, we consider any-and-all bids. These bids are very commonly used,⁹ possibly because they are less regulated than conditional bids. We show that if the raider is more efficient than current management, then all equilibria have a probability of a successful takeover strictly larger than the fraction of the per share increase in the value of the firm which the stockholders receive. In fact, some equilibria have the takeover succeeding with probability 1. In Section 3B, we consider conditional bids. With this broader strategy set for the raider, there is a unique equilibrium¹⁰ in which the raider takes over with probability 1 if he is more efficient than

⁸ A related paper which does not explicitly consider takeovers is Palfrey and Rosenthal's [1983] participation game.

⁹ For example, in a sample analyzed by Desai [1985], 79 out of 170 bids analyzed were any-and-all bids.

¹⁰ Since conditional bids include any-and-all bids as a special case and since the unique equilibrium does not have an any-and-all bid made, the model does not predict the coexistence of both types of bids. However, it is clear that including

current management and fails with probability one otherwise.

Intuitively, one would expect that as the firm becomes more widely held, it becomes harder for the raider to make stockholders pivotal and thus exclusion becomes necessary. In Section 4, we ask if and when this is true. More precisely, under what conditions is it true that the atomistic stockholder outcome obtains when the firm is sufficiently widely held? We first show that the infinite stockholder game will never yield the atomistic stockholder outcome. We then give conditions under which the limiting equilibria of the finite game is the same as the outcome with atomistic stockholders. Thus if a firm is sufficiently widely held and these conditions hold, exclusion is necessary for successful takeovers. However, even if the firm is very widely held, exclusion is not necessary if these other conditions do not hold. In Section 5, we show that with a finite number of stockholders, exclusion can lead to inefficient outcomes. Thus in Section 6, we conclude that the differences between the finite and atomistic stockholder games are substantial, important, and may not vanish in the limit.

Section 2. Notation and Definitions.

Except for the finite stockholder assumption, our game uses assumptions which are quite common in this literature. For example, our assumptions are virtually the same as those made by Grossman and Hart. We consider a two-stage game with I stockholders who own all N shares of stock in the firm. The i^{th} stockholder has h_i shares of stock where h_i is an integer. We assume that $0 < h_i < N - K$ for all i , where $K < N$ is the number of shares needed to control the corporation. Otherwise, some shareholder can prevent the takeover from succeeding even if everyone else sells. We will let p_0 be the value¹¹ of a share of stock when the firm is run by the current management and p_1 the value of a share if the firm is controlled by the raider. If the raider is more efficient than current management, then $p_1 > p_0$. All of these parameters are common knowledge.

In the first stage of the game, the raider will choose a strategy t from his strategy set \mathcal{T} . Since we will consider several different strategy sets, the precise definition of \mathcal{T} will be postponed.

the differences in regulatory treatment of these bids or other more realistic considerations would alter this conclusion.

¹¹ By "value," we mean the per share expected discounted value of the firm's profits under a given manager. Thus p_1 may not be the same as the pre-takeover price of a share of stock if a takeover bid is anticipated.

However, one element of the raider's strategy choice will always be a price per share he offers to pay, which we will call b .

Given any t , the second stage is a subgame played by the stockholders in which each shareholder simultaneously chooses (possibly via a mixed strategy) a number of shares to tender to the raider.¹² We require that the i^{th} shareholder offer an integer number of shares between 0 and h_i . A pure strategy by shareholder i will be denoted $\sigma_i(t)$ and we will let $\sigma(t) = (\sigma_1(t), \sigma_2(t), \dots, \sigma_I(t))$. A mixed strategy for shareholder i , denoted $F_i(\cdot | t)$, will be a choice of a (cumulative) probability distribution over the set of integers between 0 and h_i . Let

$$F(\cdot | t) = (F_1(\cdot | t), F_2(\cdot | t), \dots, F_I(\cdot | t)).$$

We will further restrict the stockholders' strategy sets by prohibiting any stockholder from tendering shares to the raider if $b \leq p_0$. That is, for any t such that $b \leq p_0$, $F_i(\sigma | t) = 1$ for all $\sigma \geq 0$.¹³ We will frequently omit the argument t for notational simplicity.

We take all players to be risk neutral. Hence the expected payoff to the i^{th} stockholder is the expected value of his h_i shares. The payoff to the raider is his expected profit on the takeover bid ignoring any costs associated with actually making a bid. We assume that if the takeover bid fails, each share continues to be worth p_0 .

We consider the subgame perfect equilibria of this game. On occasion, we will require that no stockholder play a weakly dominated strategy. The reason for this will be explained when we impose this requirement. An equilibrium is a vector $(F(\cdot | t), t)$ such that

- (i) for each t , $F_i(\cdot | t)$ maximizes the expected payoff to shareholder i given $F_j(\cdot | t)$ for $j \neq i$ (i.e., $F_i(\cdot | t)$ is a best reply for shareholder i) and
- (ii) t maximizes the raider's expected profits (over the set of allowed strategies for the raider) given $F(\cdot | t)$.

¹² Notice that the management is not able to fight the takeover in our model. This assumption is quite common in this literature. For an examination of defensive tactics, see Bradley and Rosenzweig or Bagnoli, Gordon, and Lipman [1987].

¹³ In general, there are equilibria in which stockholders do tender some shares and the raider may take over when $b < p_0$. In these cases, however, there is another equilibrium in which the bid fails with probability one. The latter equilibrium seems intuitively much more sensible and so we focus on it.

That is, $F(\cdot | t)$ is an equilibrium in the subgame induced by the raider's choice of t and the raider's choice of t is optimal for him given the set of equilibria induced by the possible choices of t .

Section 3. The Finite Stockholder Game without Exclusion.

A. Any-and-All Bids.

In this section, we restrict the raider to any-and-all bids. In this case, \bar{T} will be taken to be the real line. The raider's strategy choice will be a bid price, b , a price per share which will be offered for any and all shares the stockholders tender to him. In models with atomistic stockholders, no bid can succeed with $b < p_1$. In our model, in each of the equilibria in the subgame induced by such a b , the bid may succeed even if b is made arbitrarily close to p_0 . Before presenting our general characterization of equilibria in this game, we present two examples of equilibria in the subgame induced by a given $b \in (p_0, p_1)$.

First, many pure strategy equilibria exist in which exactly K shares are tendered to the raider and so the bid succeeds with probability 1. In fact, any σ such that $\sum_j \sigma_j = K$ and $\sigma_i \leq h_i$ for all i is an equilibrium. Since the bid succeeds, a shareholder who tenders an additional share foregoes p_1 to earn $b < p_1$. Since the bid would fail if fewer shares are tendered a shareholder who tenders fewer shares earns p_0 on each rather than $b > p_0$. Therefore no shareholder has an incentive to deviate from his proposed equilibrium strategy. Furthermore, there is no pure strategy equilibrium in which the number of shares tendered is not equal to K . If less than K are sold in some proposed equilibrium, then any shareholder choosing $\sigma_i < h_i$ prefers to choose a larger σ_i . If more than K are sold, then any shareholder choosing $\sigma_i > 0$ prefers to choose a smaller σ_i .

To understand these equilibria intuitively, notice that, because exactly K shares are sold in these equilibria, each seller is pivotal. By this we mean that if any of them tenders any fewer shares, the bid will fail. Thus, given the strategies of the other shareholders, each seller, in effect, must choose whether or not the bid succeeds. Hence, no seller can free ride.

It is easily shown that these equilibria are strong Nash equilibria.¹⁴ As van Damme [1983] shows, such equilibria are very robust. For example, one could add a “small” amount of uncertainty and find an equilibrium “close” to one of this type.¹⁵ It is also easily shown that any subgame equilibrium which is not in pure strategies is Pareto dominated by some pure strategy equilibrium.

It is also easy to show that there will often exist symmetric pure strategy equilibria. By “symmetric,” we mean that stockholders who are identically situated—*i.e.*, have the same number of shares—behave identically. While a rigorous formulation and proof is quite complicated, it is clear that a symmetric pure strategy equilibrium exists whenever there is sufficient heterogeneity in the holdings of stockholders. (A trivial case of this is when all stockholders own different numbers of shares. In this case, symmetry imposes no restrictions.)¹⁶

There are often also symmetric equilibria in mixed strategies.¹⁷ These are much easier to calculate when all stockholders have the same number of shares. The example we present has $h_i = 1$ for all i . Suppose that

$$F_j(\sigma_j) = \begin{cases} 1 - \gamma & \sigma_j = 0, \\ 1 & \sigma_j = 1, \end{cases}$$

for all $j \neq i$. That is, each shareholder other than i sells his share with probability γ . Then the i^{th} shareholder will also randomize if and only if

$$(1) \quad b = \sum_{j=0}^{K-1} \binom{N-1}{j} \gamma^j (1-\gamma)^{N-1-j} p_0 + \sum_{j=K}^{N-1} \binom{N-1}{j} \gamma^j (1-\gamma)^{N-1-j} p_1.$$

Notice that the first term on the right is the probability the bid fails given that i does not tender his share times the value of the share in this event. Similarly, the second term is the probability the takeover succeeds if i does not tender his share times the value of the share in this event. That is, the right-hand side is the expected value of i 's share if he does not tender. For any b strictly between p_0 and p_1 , the right-hand side of the above is strictly smaller than b at $\gamma = 0$ and strictly

¹⁴ A strong Nash equilibrium is a pure strategy equilibrium in which each player is using his unique best response. See van Damme [1983].

¹⁵ Some possible forms for this uncertainty would include uncertainty about the number of shareholders “loyal” to current management, the number who don’t find out about the bid, or other unknown heterogeneity among the shareholders.

¹⁶ As an example, suppose there are three stockholders with one share, two with two shares, and two with three shares. Then there are 13 shares outstanding. If $K = 7$, then a symmetric equilibrium would have all stockholders with one or two shares tendering all their shares. As an example of when there is no symmetric pure strategy equilibrium, suppose all stockholders have exactly one share. More generally, if all stockholders have the same number of shares, then there exists a symmetric pure strategy equilibrium if and only if K/I is an integer.

¹⁷ There are values of K and I such that the only symmetric equilibrium is in pure strategies.

larger at $\gamma = 1$. Since the right-hand side is continuous and strictly increasing in γ , there is a unique $\gamma \in (0, 1)$ satisfying equation (1). Therefore, an equilibrium has each stockholder j choosing $F_j(\sigma_j)$ as given above where γ satisfies (1).

Clearly if the pure strategy equilibrium is the one which occurs in the subgame, then the raider earns strictly positive profits for any $b \in (p_0, p_1)$. It is less obvious but also true that this holds if this mixed strategy equilibrium ensues instead. To see this, note that the raider's expected profits are

$$\sum_{j=0}^{K-1} \binom{N}{j} \gamma^j (1-\gamma)^{N-j} p_0 + \sum_{j=K}^N \binom{N}{j} \gamma^j (1-\gamma)^{N-j} p_1 - N\gamma b,$$

which is simply the probability of the takeover failing times the value of the acquired shares plus the probability of the takeover succeeding times the value of the acquired shares in this event less the expected costs of acquiring the shares. Substituting for b from (1) and rearranging shows that this is in fact equal to

$$(2) \quad \binom{N}{K} \gamma^K (1-\gamma)^{N-K} (p_1 - p_0) K > 0.$$

This expression has an interesting interpretation because $\binom{N}{K} \gamma^K (1-\gamma)^{N-K}$ is the probability that exactly K shares are tendered. Thus, the raider's profits are proportional to the probability that each seller is pivotal. In other words, his ability to earn profits hinges on his ability to make each seller pivotal with positive probability.¹⁸

So we have shown that in two types of equilibria in the induced subgame, the raider earns strictly positive expected profits and the bid succeeds with strictly positive probability because he makes some stockholders pivotal. In fact, it is not hard to see that the raider must always be able to make at least some stockholders pivotal with positive probability. Consider any bid strictly between p_0 and p_1 . If no stockholder is pivotal with positive probability, then there is no set of possible choices for the other stockholders such that the choice of the i^{th} stockholder affects the outcome. But if the takeover succeeds with probability strictly between zero and one, this is impossible. Consider all the outcomes which have positive probability in which the takeover

¹⁸ Interestingly, if the mixed strategy equilibrium arises in the subgame induced by each b , the raider chooses b to maximize (2). This is equivalent to choosing γ to maximize (2). This means that he chooses γ to maximize the probability that each seller is pivotal.

succeeds and choose the one with the smallest number of shares tendered. Any stockholder who sells fewer shares with positive probability would cause the takeover to fail if he chose a smaller number. Thus such a stockholder is pivotal at this outcome and hence is pivotal with positive probability. There must be at least one shareholder in this position as there is a positive probability that the bid fails. It is easy to see that the takeover cannot fail with probability one. Furthermore, the only way it can succeed with probability one is if we have a pure strategy equilibrium as discussed above. Hence there must be at least one stockholder who is pivotal in any equilibrium.

In Theorem 1, we show that this ability ensures that the raider earns strictly positive expected profits and that the bid succeeds with strictly positive probability in *any* equilibrium.

Theorem 1: *If the raider is more efficient than current management and is restricted to any-and-all bids, then, in every equilibrium, (i) $p_0 < b < p_1$, (ii) the raider earns strictly positive expected profits, and (iii) the bid succeeds with probability $\Phi \geq \frac{b-p_0}{p_1-p_0}$.*

Proof: Obviously, if the raider can earn strictly positive profits from some bid strictly between p_0 and p_1 , (i) must follow. So consider any bid in this range. It is easy to see that it is not an equilibrium for the bid to fail with probability one. The only equilibria where it succeeds with probability one are the pure strategy equilibria discussed above. It is easy to see that the theorem holds for these equilibria.

So suppose that the bid succeeds with probability strictly between zero and one. Clearly, then, some stockholder must associate positive probability with two different numbers of shares to tender. Let this stockholder be i and let s be the smallest and ℓ the largest number of shares i sells with positive probability. Define $\phi_i(k)$ as the probability that the bid will succeed given that i sells k shares. Since i associates nonzero probability with selling s shares and ℓ shares, he must receive the same expected payoff from either choice. Hence we must have

$$\ell b + (h_i - \ell)[\phi_i(\ell)p_1 + (1 - \phi_i(\ell))p_0] = sb + (h_i - s)[\phi_i(s)p_1 + (1 - \phi_i(s))p_0]$$

which can be rewritten as

$$(\ell - s)[b - \phi_i(s)p_1 - (1 - \phi_i(s))p_0] + (h_i - \ell)[\phi_i(\ell) - \phi_i(s)](p_1 - p_0) = 0$$

Since ℓ must be no larger than h_i and since $p_1 > p_0$, the second term must be nonnegative as $\phi_i(s) \leq \phi_i(\ell)$. Therefore,

$$b \leq \phi_i(s)p_1 + (1 - \phi_i(s))p_0.$$

Let Φ be the probability the takeover succeeds. Since s is the smallest number of shares i may sell, $\Phi \geq \phi_i(s)$. Therefore,

$$b \leq \Phi p_1 + (1 - \Phi)p_0$$

which immediately yields the lower bound in the theorem.

Let n be the (random) number of shares tendered to the raider. Then we must have $0 < \Pr[n \geq K] = \Phi < 1$. Then we can write the raider's expected profits as

$$(1 - \Phi)\mathbb{E}[n \mid n < K](p_0 - b) + \Phi\mathbb{E}[n \mid n \geq K](p_1 - b).$$

Using the inequality above, we can substitute in and rearrange to show that this is at least

$$\Phi(1 - \Phi)(p_1 - p_0) \left\{ \mathbb{E}[n \mid n \geq K] - \mathbb{E}[n \mid n < K] \right\}.$$

Since neither probability is zero, $p_1 > p_0$, and the term in brackets must be strictly positive, we see that the raider's expected profits are strictly positive. The fact that profits per share and the number of shares tendered are positively correlated implies that the raider's expected profits are strictly positive even if his expected profit per share is zero. ■

It is important to point out that this result does require the raider to be more efficient than current management—i.e., that $p_1 > p_0$. Otherwise, the bid cannot succeed, as there is no $b \in (p_0, p_1)$. Though this condition seems obvious, we will see in Section 5 that for some strategy

sets for the raider, there are equilibria in which less efficient raiders take over. In fact, we could restate Theorem 1 as: if the raider is restricted to any-and-all bids, he has a nonzero probability of taking over and will earn strictly positive profits in the attempt if and only if he is more efficient than current management.

B. Conditional Bids.

In this section, we will consider conditional bids. With such a bid, the raider chooses three objects. First, he chooses a number b which he will pay for shares offered to him. Second, he chooses an integer number of shares he wishes to acquire; that is, he announces he will pay b for κ shares. If fewer than that number is offered to the raider, he buys no shares at all. If that number is offered, he buys all of them and pays b for each. The third object he chooses is a rationing device which determines how many he buys from whom in the event that more than κ shares are offered. As an example of such a device, the Williams Act of 1968 requires that in such an event, the raider must buy the same percentage of shares from each person who offers shares to him and that he buy κ shares in total. This is commonly referred to as purchasing on a *pro rata* basis. We will not restrict the raider to this specific device. However, our results hold if such a device is used. Notice that if $\kappa = 1$ and the raider chooses to buy all shares offered above κ , then the conditional bid is identical to an any-and-all bid. Hence the strategy set considered here strictly contains the set considered in Section 3A.

Consider the following strategy for the raider. Suppose he offers infinitesimally more than p_0 for all N shares, *i.e.*, $\kappa = N$. Clearly, one equilibrium in the induced subgame has all shareholders offering all their shares. Any deviation from the proposed equilibrium strategy causes the shareholder to earn only $h_i p_0$, while selling all shares earns h_i times a number strictly larger than p_0 . Notice also that another equilibrium has no one offering any shares for sale. If any shareholder other than i chooses to withhold some shares, then i is indifferent between offering all his shares and offering any number less since the raider will not purchase regardless of what i does. Note, though, that withholding some shares is a weakly dominated strategy. Thus if there is even a tiny probability that all other stockholders will tender all their shares, each stockholder wishes to tender all of his. In this sense, the subgame equilibrium where players choose weakly dominated strategies

is not stable.¹⁹

Suppose we focus on equilibria in the two-stage game where no player uses a weakly dominated strategy. In this case, if the raider offers the bid defined above, he will take over the firm with probability one and earn profits which can be made arbitrarily close to $(p_1 - p_0)N$. Thus he can obtain an amount arbitrarily close to the increase in the value of the firm. As is intuitively clear, this is the best he can do unless he is able to somehow take some of the firm's initial value from the stockholders—that is, unless exclusion is allowed. So if exclusion is not allowed, we have the following theorem.

Theorem 2: *If the raider is more efficient than current management and is restricted to conditional bids, the unique equilibrium has $t = (b^*, \kappa^*, \cdot)$,²⁰ where (i) $b^* = p_0 + \delta$ for δ arbitrarily close to but strictly greater than 0,²¹ (ii) $\kappa^* = N$, and (iii) the bid succeeds with probability 1.*

The intuition of this theorem is not difficult to see. A conditional bid with $\kappa = N$ makes every stockholder pivotal and thus willing to sell even for $p_0 + \delta$. The ability to make every stockholder pivotal enables the raider to extract all of the gains from better management of the firm.

Clearly, the outcome in the finite game with conditional bids is quite different from the outcome with atomistic stockholders. But, of course, conditional bids for 100% of the shares are rarely observed for some obvious reasons. For one, not all stockholders will even realize that a bid has been made. Furthermore, stockholders loyal to current management, including possibly the managers themselves, can easily block the takeover. Some factors like these are easily incorporated into the analysis. For example, if it is common knowledge that $M \leq N - K$ shares are owned by stockholders who cannot be induced to tender to the raider for whatever reason, the raider can set $\kappa = N - M$ and the bid will succeed with probability one.²² Unobserved heterogeneity of the

¹⁹ More precisely, this subgame equilibrium is not trembling-hand perfect.

²⁰ The rationing device is irrelevant because $\kappa^* = N$ implies that it is impossible to have more than κ^* shares offered for sale to the raider.

²¹ Technically, the raider's best strategy is undefined, since he must choose $\delta > 0$, but is always better off choosing it smaller. Since there is no smallest number strictly larger than zero, this leaves δ undefined. We will ignore this problem in all that follows and will often use δ to denote a strictly positive, but arbitrarily small number.

²² Other kinds of complicating factors, when the raider has complete information about them, are also easily dealt with. For

stockholders is much more complex.

Theorem 2 implies that if conditional bids are allowed, then the raider's profits are arbitrarily close to the increase in the value of the firm. Clearly, this is the best he can do unless he is able to take some of the firm's initial value from the minority stockholders. A necessary condition for this is that he be able to exclude.

Corollary: If the raider is allowed any strategy set containing conditional bids but not exclusion, an equilibrium of the game is the equilibrium given in Theorem 2. Furthermore, any other equilibrium²³ has the same outcome.

In short, we have seen that there are very major differences between the finite and atomistic stockholder outcomes. The existence of these differences does not depend on the type of bid the raider can make. Instead, these differences are present because, with a finite number of stockholders, the raider can always make some stockholders pivotal. The important distinction between the different types of bids is how easily the raider can make stockholders pivotal.

example, if there is a nonzero cost to the stockholders of selling their shares, the raider can adjust his bid up accordingly if these costs are known to him.

²³ More precisely, this is true for any other equilibrium in which no player chooses a weakly dominated strategy.

Section 4. Widely Held Firms without Exclusion.

In Section 3, we showed that the outcome with a finite number of stockholders is different from the atomistic stockholder outcome. However, it seems more plausible that no individual stockholder will be able to affect the outcome of a takeover bid for a very widely held firm. Does this imply that exclusion is necessary for successful takeovers of very widely firms? In other words, does the atomistic stockholder outcome obtain for very widely held firms?

For most of this section, we will consider the case where the raider is restricted to any-and-all bids. We will show that the atomistic stockholder outcome does not obtain in the infinite stockholder game. If any equilibrium exists in this game, then the raider earns strictly positive expected profits from the takeover attempt. Thus either there is no equilibrium and so the infinite stockholder game yields no predictions or there is an equilibrium in which the free rider problem does not prevent successful and profitable takeovers without exclusion. The alternative approach is to consider the limiting outcome of the finite stockholder game for a sequence of ever more widely held firms. We will show that the limiting outcome may be, but is not necessarily, the same as the atomistic stockholder outcome. We give conditions under which these outcomes are the same, thus delimiting the cases where exclusion is necessary for successful takeovers. The analysis of conditional bids is far simpler and concludes the section.

First consider the infinite stockholder game. We will reinterpret our notation slightly to analyze this game. In particular, we will let I be the set of stockholders, which we will take to be the unit interval. Let $h(i)$ be the holdings of stockholder i , K be the measure of shares which must be tendered for the raider to succeed, and N be the measure of shares outstanding. That is, N is the integral over I of $h(i)$. We will let $\sigma(i)$ be the portion of his holdings tendered by stockholder i where $0 \leq \sigma(i) \leq h(i)$. Finally, the bid price, b , and p_0 and p_1 are "per unit" values. That is, the value of a stockholder's shares if the bid is successful and he tendered $\sigma(i)$, for example, is $\sigma(i)b + (h(i) - \sigma(i))p_1$.

Consider what happens if the raider makes an any-and-all bid of b where $p_0 < b < p_1$. Can we have an equilibrium in which the bid succeeds with probability one? Clearly, the answer is

no. No stockholder can influence the outcome because we can always remove one point from the integral over $\sigma(i)$ without affecting the value of the integral. Hence if the bid is going to succeed with probability one, then a given stockholder would prefer to tender nothing as $b < p_1$. Hence this is not an equilibrium. Can we have an equilibrium in which the bid succeeds with probability zero? Again, the answer is no. Since no stockholder can influence the outcome, if the bid is going to succeed with probability zero, each stockholder will wish to tender $h(i)$ as $b > p_0$. Hence if there is an equilibrium, the takeover must succeed with probability strictly between zero and one.

If only a finite number of stockholders randomize, then the outcome is determined by those who do not randomize and hence is certain. Thus we must have an infinite number randomizing. But with an infinite number of stockholders randomizing, it is not obvious that the aggregate outcome can be stochastic.²⁴ Rather than determining when (or if) the outcome here can be stochastic, let us consider what happens for either case. If the analogue of the strong law of large numbers holds, the aggregate outcome will be nonstochastic. In this event, there is no equilibrium. On the other hand, suppose that the aggregate outcome is stochastic. It is straightforward to imitate the proof of Theorem 1 to show that the raider's expected profits are strictly positive. In that proof we showed that the strictly positive correlation between profits per share and the measure of shares tendered implies that the raider's expected profits are strictly positive even if his expected profit per share is zero.

Hence the raider's profit from any given $b \in (p_0, p_1)$ must be strictly positive if an equilibrium exists in the induced subgame. For an equilibrium to exist in the two-stage game, there must be an equilibrium in each possible subgame. Hence if an equilibrium exists, it must have the raider earning strictly positive profits. Thus we see that either there is no equilibrium or the equilibrium is quite different from the atomistic stockholder outcome. Either way, the infinite stockholder game does not yield the atomistic stockholder prediction.

Another way to determine if the atomistic stockholder outcome obtains for a firm which is sufficiently widely held is to consider a sequence of firms each of which is more widely held than the previous firms in the sequence. Associate an equilibrium and thus an equilibrium outcome with each firm in the sequence. We can then analyze the sequence of equilibrium outcomes to determine

²⁴ Judd [1985] and others have analyzed when a continuum of independent random variables can generate a stochastic outcome.

if this sequence converges to the atomistic stockholder outcome. To do this formally, we must consider the following three issues.

First, what does it mean to say that one firm is more widely held than another? Intuitively, a firm with more stockholders than another is more widely held. Thus we would like to choose our sequence of firms by increasing the number of stockholders along the sequence holding everything else constant. Unfortunately, this cannot be done. If the stock price is held fixed, then the total value of the firm must increase along the sequence. If the value of the firm is fixed, the stock price must be falling. One would expect that whether or not we converge to the atomistic stockholder outcome depends on which extreme the sequence is closer to—a fixed stock price with the value of the firm going to infinity or a fixed value with the stock price going to zero. In the fixed stock price case, the size of the “pie”—the value created by the takeover or $N(p_1 - p_0)$ —is going to infinity. Hence even if the raider’s share of the pie falls because it becomes harder to make stockholders pivotal, successful and profitable takeovers without exclusion may still be possible. In the fixed value case, the size of the pie is constant so that if the free rider problem shrinks the raider’s share of the pie, it must also shrink his profits, so that exclusion becomes necessary.

Second, once the sequence of firms is chosen, we must associate an equilibrium with each firm. Since there are many equilibria for each firm, this choice may also affect the conclusion.

Third, we must precisely define what is meant by convergence to the atomistic stockholder outcome. In models with atomistic stockholders, any nonexclusionary takeover bid which succeeds (with positive probability) must yield zero profits for the raider absent any costs of bidding. Consequently, we consider whether or not the raider’s expected profits from a bid, absent such costs, converge to zero.

We will show that in the fixed stock price case, the raider’s profits do not go to zero, regardless of how we choose the equilibrium for each firm in the sequence. We will also show, by example, that in the fixed value case, the raider’s profits may or may not go to zero, depending on how we choose the equilibrium for each firm. In short, the general conclusion depends on which extreme the sequence of firms is closer to and, given the sequence of firms, which sequence of equilibria is chosen. Intuitively, a firm which is more valuable relative to the dispersion of ownership

of its stock is, in general, more appropriately described by the finite stockholder model than the atomistic stockholder model.

We consider a sequence of firms, one for each N . We will let $K(N)$ be the number of shares the raider must acquire to control the N^{th} firm in the sequence. For simplicity, we will assume $K(N)$ is the smallest integer larger than αN , where α is some fixed fraction of the firm which must be acquired for the raider to gain control. We assume throughout that the value of a share under the raider's management is strictly greater than the value under current management.

First, we consider the fixed stock price extreme. Here, for each firm in the sequence, p_0 is the value of a share if the firm remains under current management and p_1 is the value of a share under the raider's management. For this case, we have the following theorem.

Theorem 3: *If the raider is restricted to any-and-all bids, then for any sequence of equilibria with a fixed stock price, the limit of the raider's profits is strictly positive.*

Proof: We will show that there exists a $b \in (p_0, p_1)$ such that if the raider makes this bid for each firm in the sequence, the limit of his profits from this bid must be strictly positive. Hence the limit of his profits from the optimal sequence of bids must also be strictly positive. Clearly, if the probability of success goes to zero for this bid, the raider's profits cannot be strictly positive in the limit. However, recall that Theorem 1 showed that the probability of success must be strictly larger than $(b - p_0)/(p_1 - p_0)$. Hence as long as $b > p_0$, the probability of success must be bounded away from zero for each N . Therefore, it cannot go to zero.

So suppose that the probability of success goes to 1. Clearly, then, the probability of any outcome with fewer than $K(N)$ shares tendered must go to zero so that the raider's profits for N sufficiently large must be at least $K(N)(p_1 - b) > 0$.

Suppose, then, that the probability of success does not converge to zero or one.²⁵ It is

²⁵ This possibility may seem peculiar. However, we have produced an example in which the equilibrium probability of success converges to e^{-1} .

straightforward to imitate the proof of Theorem 1 to show that profits are greater than a number which does not go to zero.■

We now turn to the opposite extreme—the fixed value case. Here we will provide two examples which demonstrate that the raider’s profits may or may not go to zero in the limit depending on the exact sequence of equilibria chosen. We now assume that the value of a share in the N^{th} firm in the sequence is v_0/N if the firm remains under current management and v_1/N if the raider gains control. Of course, $v_1 > v_0$.

In our first example, we focus on the case where all players always choose pure strategies. In this case, for any N , for any $b \in (v_0/N, v_1/N)$, the bid succeeds with probability one and the raider’s profits are $K(N)(v_1/N - b)$. Hence, the raider chooses²⁶ $b = v_0/N + \delta$ and the raider’s profits for a given N are arbitrarily close to $[K(N)/N](v_1 - v_0)$, which converges to $\alpha(v_1 - v_0) > 0$. Notice that the raider’s limiting profits are just the fraction of the firm he acquires times the total increase in the value of the firm from the change in management.

For our second example, suppose that each stockholder in each firm in the sequence has exactly one share. Suppose that the symmetric mixed strategy equilibrium obtains in each subgame for every N . As shown in Section 3A, the raider’s profits for a given N and b are

$$(3) \quad \binom{N}{K(N)} \gamma^{K(N)} (1 - \gamma)^{N-K(N)} \frac{K(N)}{N} (v_1 - v_0).$$

Since the raider chooses b to maximize his profits given the γ it induces in the subgame, the optimal b maximizes profits subject to the constraint given in (1). Since (3) is obtained by substituting the constraint into the objective function, we can analyze the raider’s choice of γ to maximize this expression. It is easy to show that the raider chooses $\gamma = \frac{K(N)}{N}$, which converges to α as $N \rightarrow \infty$.

Stirling’s formula (see, e.g., Feller [1966]) states that

$$n! \approx \sqrt{2\pi n} n^{n+(1/2)} e^{-n}.$$

²⁶ Recall our use of δ described in the last section.

Substituting for γ into (3) and using Stirling's formula shows that the raider's profits are approximately

$$\left(\frac{1}{1 - \frac{K(N)}{N}}\right)^{\frac{1}{2}} \left(\frac{v_1 - v_0}{\sqrt{2\pi}}\right) \left(\frac{\sqrt{K(N)}}{N}\right).$$

The first term converges to the constant, $\sqrt{\frac{1}{1-\alpha}}$. The second term is a constant and the third term goes to zero so that the raider's expected profits converge to zero for this sequence of equilibria.

Hence we see that the sequence of equilibria corresponding to a sequence of more and more widely held firms converges to the atomistic stockholder outcome only if we are "close enough" to the fixed value case and then only for certain sequences of equilibria.²⁷ Thus the fact that one firm is more widely held than another does not necessarily imply that it is better modelled by the atomistic stockholder assumption. Which model is more appropriate for a given takeover depends on the value of the firm relative to how widely held it is. The atomistic stockholder model is more appropriate when the firm is not very valuable relative to the dispersion of ownership of its stock. The finite stockholder model is more appropriate in the opposite situation.²⁸ Hence we conclude that neither model is always appropriate for any-and-all bids.

The analysis of conditional bids is much easier. Recall that the equilibrium of Theorem 2 is the unique equilibrium for any finite number of stockholders and any distribution of the shares. Hence the particular sequence of firms or the particular sequence of equilibria chosen—that is, whether the stock price or value is fixed or which intermediate case we consider—is not relevant when conditional bids are allowed.²⁹ The atomistic stockholder outcome does not obtain in the limit.

To summarize, with a finite number of stockholders, exclusion is not necessary for successful takeovers. Whether or not exclusion becomes necessary as we consider more widely held firms

²⁷ Recall that we have ignored costs associated with making a takeover bid. If these costs increase with N , we could restate our results as follows. When profits as we have defined them go to infinity, net profits—that is, profits minus costs—are bounded away from zero only if costs rise sufficiently slowly. When profits converge to some positive but finite number, net profits are bounded away from zero only if costs are bounded below this limit. As noted, when profits as we have defined them go to zero, successful and profitable takeovers without exclusion are impossible for firms with a finite but sufficiently large number of stockholders.

²⁸ Recall from the introduction that there are many large firms controlled by a small number of stockholders.

²⁹ We do not discuss the infinite game with conditional bids because of some formal problems. In particular, consider the bid discussed in Theorem 2. If the bid is conditioned on every stockholder tendering, then this is still an equilibrium. However, if the bid is conditioned only on the measure of shares tendered, it is no longer an equilibrium because any single stockholder can withhold his shares without affecting the measure tendered.

depends in part on the types of strategies the raider can use and the equilibria focused on. It also depends on the value of the firm relative to how widely held it is.

Section 5. The Finite Stockholder Game with Exclusion.

Having shown that exclusion is unnecessary, a logical question to ask is what its effects are with a finite number of stockholders. As we will show in this section, exclusion can have harmful effects if conditional bids are allowed.³⁰

Let ψ be the amount of exclusion. Following Grossman and Hart, this means that in the event of a takeover a minority shareholder receives $p_1 - \psi$ rather than p_1 . Thus if the raider takes over the firm by purchasing, say, $k \geq K$ shares at a price of b , then his profits are $(p_1 - b)k + (N - k)\psi$.

Suppose $\psi \geq p_1 - p_0$.³¹ In this situation, the raider is not only able to exclude minority stockholders from all of the increase in the value of their shares, he is able to take some of the original value of these shares. As one might expect, this can enable an inefficient raider to take over profitably. An equilibrium in which this occurs is presented in the following theorem. First, we will define a particular class of rationing devices which may be used by the raider in the event that more shares are tendered than he sought. An encouraging device will be defined as one such that for all i , if $\sum_{j \neq i} \sigma_j \geq \kappa$, then the i^{th} shareholder's best strategy is $\sigma_i = h_i$. In other words, if the number of shares tendered by the other stockholders is at least the number requested by the raider, then each stockholder's best strategy is to tender all his shares. It is easy to show that an example of such a device is precisely the device required by the Williams Act, namely, purchasing on a *pro rata* basis.

Theorem 4: *If the raider can make conditional bids and if $p_1 - \psi \leq p_0$, then an equilibrium has the raider offering the conditional bid $b = p_0 + \delta$ for $\kappa = K$ shares with any encouraging device. In this equilibrium, the bid succeeds with probability 1.*

³⁰ It can be shown that exclusion does not have harmful effects in our model when only any-and-all bids are allowed. With an any-and-all bid, a stockholder can always tender his share and avoid the losses imposed by exclusion.

³¹ If ψ is smaller than this quantity, the equilibrium of Theorem 2 is still the unique equilibrium.

Proof: First, we will show that given the raider's bid, it is an equilibrium in the induced subgame for all stockholders to tender all their shares. This follows because if all shareholders other than i tender all their shares, then at least $N - h_i$ shares will be tendered. Since $N - h_i > K = \kappa$, this means that the number of shares tendered exceeds the number the raider will purchase. Since the rationing device is an encouraging device, by definition, this implies that the i^{th} stockholder will choose $\sigma_i = h_i$.

Given this equilibrium in the subgame, the raider's expected profit from this strategy is arbitrarily close to $K(p_1 - p_0) + (N - K)\psi$. But the raider cannot earn any higher profit than this from any alternative strategy. ■

It is important to note that this equilibrium is not unique because the equilibrium in the subgame induced by the raider's choice of this strategy is not unique. For example, as with the bid made in Theorem 2, another equilibrium in the subgame would have all shareholders refusing to tender any shares. Unlike the situation in that section, though, this equilibrium can be stable. There, the shareholder would gain δ per share if everyone offered all their shares and would gain nothing otherwise. Hence, with even a tiny probability that everyone else tenders all their shares, the remaining shareholder would tender all of his. In the equilibrium of Theorem 4, if everyone offers all their shares, each shareholder receives $p_0 + \delta$ for some of his shares and the smaller amount $p_1 - \psi$ for each of his remaining shares. Hence he is worse off if everyone offers all their shares and so not tendering is not a weakly dominated strategy. However, it is easily shown that all stockholders tendering is a strong Nash equilibrium and the other subgame equilibria are not.

Notice that this can be an equilibrium even if the raider is less efficient than current management—that is, even if $p_1 \leq p_0$. The following theorem elucidates this point.

Theorem 5: *If the raider is less efficient than current management and is allowed conditional bids, then for any $\psi > 0$, there is a lower bound on p_1 which is strictly smaller than p_0 for which the bid succeeds with probability 1 in an equilibrium.*

Proof: Since $p_1 - \psi < p_0$, the conditions for Theorem 4 are met.³² The raider's profit

³² Note that if $p_1 \leq p_0$, then $p_1 - \psi < p_0$ for any $\psi > 0$.

in the equilibrium discussed there is arbitrarily close to $K(p_1 - p_0) + (N - K)\psi$. This is strictly positive if and only if

$$p_1 > p_0 - [(N - K)/K]\psi.$$

The right-hand side is strictly smaller than p_0 for any $\psi > 0$. Hence the lower bound is the right-hand side, which will complete the proof. ■

One could interpret the statement that the raider is less efficient in either of two ways. First, one could interpret this as saying that the raider cannot control the managers of the firm as well as current stockholders. In this case, we can have $p_1 < p_0$ and the takeover moves assets to *less* efficient use. Alternatively, one could interpret less efficient as saying that the raider cannot run the firm as well as current management. In this case, he could retain current management, so that $p_1 = p_0$. If so, the takeover does not result in less efficient use of the firm's assets. However, if there is any cost to the takeover process (such as costs of making a bid, legal fees, etc.), then social resources are used up in generating a simple transfer of wealth. In this case, the takeover is an example of pure rent-seeking.

Numerous authors have suggested that competition among raiders or defensive tactics by current management will prevent inefficient takeovers.³³ However, it is not obvious that there will always be a more efficient raider to compete for the firm. As to current management, any expenditure of resources by the manager in an attempt to block the takeover must reduce the value of the firm.³⁴ Thus even if the takeover is prevented, the attempt would still lead to pure social waste. Hence unless the threat of defensive tactics completely deters takeover attempts by inefficient raiders, the possibility of such tactics does not solve this problem. Furthermore, defensive tactics may block some efficient takeovers. Clearly, more research is needed on this subject.³⁵

³³ See, for example, Bradley and Rosenzweig.

³⁴ A similar problem is noted by Bradley and Rosenzweig in the context of defensive stock repurchases.

³⁵ Some of these issues are addressed by Bagnoli, Gordon, and Lipman [1987].

Section 6. Conclusion.

We noted at the outset that most of the literature on takeovers assumes atomistic stockholders. As we pointed out, however, there are many large firms for which this assumption is obviously inappropriate. This led us to consider the finite stockholder game. We showed that there are substantial differences between the finite game and the atomistic stockholder models. In particular, because some stockholders must be pivotal and hence cannot free ride, successful takeovers are possible without exclusion.

Since the equilibrium outcome in the finite stockholder game is quite different from the atomistic stockholder outcome, the natural question to ask is under what conditions the atomistic stockholder outcome obtains for firms which are sufficiently widely held. We showed that the atomistic stockholder outcome does not obtain in the infinite stockholder game. We also showed that the difference between the finite and atomistic stockholder outcomes may not vanish in the limit. We argued that atomistic stockholder models may provide a reasonable approximation to the outcome for takeovers with any-and-all bids if the firm is not sufficiently valuable relative to the dispersion of stock ownership. Otherwise, the finite stockholder model is likely to provide a more accurate prediction, so that exclusion is not necessary for successful takeovers. Since, all else equal, stockholders generally benefit more from takeovers without exclusion, our analysis suggests that stockholders would prefer to invest in firms which are valuable relative to the dispersion of stock ownership. This, in turn, suggests that a given firm's stock will not be "too" widely held relative to its value. This seems like an interesting topic for future research.

After delineating the conditions under which exclusion is not necessary, we turned to an analysis of its effects in this situation. When exclusion is not necessary, it is no longer clear that it is socially desirable. In fact, exclusion can have very negative consequences if conditional bids are allowed.

In short, the finite stockholder and atomistic stockholder models yield dramatically different outcomes and different conclusions about the social consequences of exclusion. The fact that these differences are so strong and important together with the fact that these differences need not vanish in the limit suggests that the finite stockholder analysis should be carefully considered.

References.

- Bagnoli, M., Gordon, R., and B. Lipman, "Takeover Bids, Defensive Stock Repurchases and the Efficient Allocation of Corporate Control," University of Michigan working paper, 1987.
- Bagnoli, M., and B. Lipman, "An Economic Analysis of Restrictions on Takeover Bids," University of Michigan working paper, 1986.
- Bebchuk, L., "A Model of the Outcome of Takeover Bids," draft, November, 1985.
- Bradley, M., "Interfirm Tender Offers and the Market for Corporate Control," *Journal of Business*, 53, 1980, 345-376.
- Bradley, M., and E. H. Kim, "Prisoner's Dilemma and Competition in the Market for Corporate Control," University of Michigan working paper, 1984.
- Bradley, M., and M. Rosenzweig, "The Law and Economics of Defensive Stock Repurchases and Defensive Self-Tender Offers," University of Michigan working paper, 1985.
- van Damme, E., *Refinements of the Nash Equilibrium Concept*, Berlin: Springer-Verlag, 1983.
- Demsetz, H., and K. Lehn, "The Structure of Corporate Ownership: Causes and Consequences." *Journal of Political Economy*, December, 1985.
- Desai, Anand, "Acquisitions Initiated by Tender Offers and Stockholder Wealth," unpublished dissertation, University of Michigan, 1985.
- Feller, William, *An Introduction to Probability Theory and Its Applications*, New York: John Wiley & Sons, Inc., 1966.
- Green, E., "Continuum and Finite Player Noncooperative Models of Competition," *Econometrica*, July, 1984.
- Grossman, S, and O. Hart, "Takeover Bids, the Free Rider Problem, and the Theory of the Corporation," *The Bell Journal of Economics*, Spring, 1980.

Judd, K., "The Law of Large Numbers with a Continuum of IID Random Variables," *Journal of Economic Theory*, February, 1985.

Kovenock, D., "A Note on Takeover Bids," Purdue University, mimeo, 1984.

Palfrey, T., and H. Rosenthal, "The Strategic Calculus of Voting," *Public Choice*, 41, no. 1, 1983.

Shleifer, A., and R. Vishny, "Large Shareholders and Corporate Control," *Journal of Political Economy*, June 1986.

Spatt, C., "Strategic Analyses of Takeover Bids," Carnegie-Mellon University, working paper, 1986.

Vermaelen, T., "Common Stock Repurchases and Market Signalling: An Empirical Study," *Journal of Financial Economics*, 9, 1981, 139-183.

Recent CREST Working Papers

- 87-1: Jeffrey K. MacKie-Mason, "Nonlinear Taxation of Risky Assets and Investment, With Application to Mining" September, 1984.
- 87-2: Jeffrey K. MacKie-Mason, "Sequential Decision Problems and Asymmetric Information" September, 1985.
- 87-3: Michelle J. White, "Contract Breach and Contract Discharge due to Impossibility: A Unified Theory" July 29, 1987.
- 87-4: Ted Bergstrom, "Systems of Benevolent Utility Interdependence" May 19, 1987.
- 87-5: Ted Bergstrom, "A Fresh Look at the Rotten Kid Theorem—and Other Household Mysteries" November, 1986.
- 87-6: Michelle J. White, "The Corporate Bankruptcy Decision" July, 1987.
- 87-7: Michelle J. White, "Location Choice and Commuting Behavior in Cities with Decentralized Employment" July, 1987.
- 87-8: Lawrence E. Blume and David Easley, "Implementation of Walrasian Expectations Equilibria" December, 1985.
- 87-9: Lawrence E. Blume, "Lexicographic Refinements of Nash Equilibrium" April, 1986.
- 87-10: David Lam, "Lorenz Curves, Inequality, and Social Welfare Under Changing Population Composition" June 16, 1987.
- 87-11: Mark Bagnoli and Naveen Khanna, "Equilibrium with Debt and Equity Financing of New Projects: Why More Equity Financing Occurs When Stock Prices are High" June, 1987.
- 87-12: Mark Bagnoli and Barton L. Lipman, "Provision of Public Goods: Fully Implementing the Core through Private Contributions" March, 1987.
- 87-13: Mark Bagnoli and Barton L. Lipman, "Successful Takeovers without Exclusion" August, 1987.
- 87-14: Mark Bagnoli and Michael McKee, "Controlling the Game: Political Sponsors and Bureaus" May, 1987.
- 87-15: Mark Bagnoli and Michael McKee, "Can the Private Provision of Public Goods be Efficient?—Some Experimental Evidence" March, 1987.
- 87-16: Mark Bagnoli, "Non-Market Clearing Prices in a Dynamic Oligopoly with Incomplete Information" January, 1986.
- 87-17: John Laitner, "Bequests, Gifts, and Social Security" February 28, 1986.
- 87-18: John Laitner, "Dynamic Determinacy and the Existence of Sunspot Equilibria" May 12, 1986.
- 87-19: David Lam, "Does a Uniform Age Distribution Minimize Lifetime Wages?" August 12, 1987.
- 87-20: David Lam, "Assortative Mating with Household Public Goods" April, 1987.
- 87-21: Jeffrey A. Miron and Stephen P. Zeldes, "Production, Sales, and the Change in Inventories: An Identity that Doesn't Add Up" June 1987.
- 87-22: Jeffrey A. Miron and Stephen P. Zeldes, "Seasonality, Cost Shocks, and the Production Smoothing Model of Inventories" December, 1986.
- 87-23: Hal R. Varian, "Differences of Opinion in Financial Markets" March, 1985.
- 87-24: Roger H. Gordon and Hal R. Varian, "Taxation of Asset Income in the Presence of a World Securities Market" August, 1986.

- 87-25: Hal R. Varian, "Measuring the Deadweight Costs of DUP and Rent Seeking Activities" November, 1982.
- 87-26: Hal R. Varian, "Price Discrimination" January, 1987.
- 87-27: Roger H. Gordon and Hal R. Varian, "Intergenerational Risk Sharing" October, 1985.
- 87-28: Hal R. Varian, "Three Papers on Revealed Preference" August, 1987.
- 87-29: Hal R. Varian, "Optimal Tariffs and Financial Assets" April, 1987.
- 87-30: Jonathan Cave and Stephen W. Salant, "Cartels That Vote: Agricultural Marketing Boards and Induced Voting Behavior" August, 1987.
- 87-31: Stephen W. Salant and Donald H. Negri, "Pastures of Plenty: When is the Standard Analysis of Common Property Extraction Under Free Access Incorrect?" July 10, 1987.
- 87-32: Stephen W. Salant, "When is Inducing Self-Selection Sub-optimal for a Monopolist?" February, 1987.
- 87-33: Stephen W. Salant, "Treble Damage Awards in Private Lawsuits for Price-Fixing" August, 1987.
- 87-34: Stephen W. Salant and Roy Danchick, "Air Force Academy Attrition: A New Perspective on the College Dropout Problem" August, 1987.
- 87-35: Stephen W. Salant and Eban Goodstein, "Committee Voting Under Alternative Procedures and Preferences: An Experimental Analysis" April 20, 1987.
- 87-36: Robert B. Barsky and Jeffrey A. Miron, "The Seasonal Cycle and the Business Cycle" June, 1987.
- 87-37: Robert B. Barsky, N. Gregory Mankiw, Jeffrey A. Miron and David N. Weil, "The Worldwide Change in the Behavior of Interest Rates and Prices in 1914" July, 1987.
- 87-38: Jeffrey K. MacKie-Mason, "Taxes, Information and Corporate Financing Choices" April 1986.

