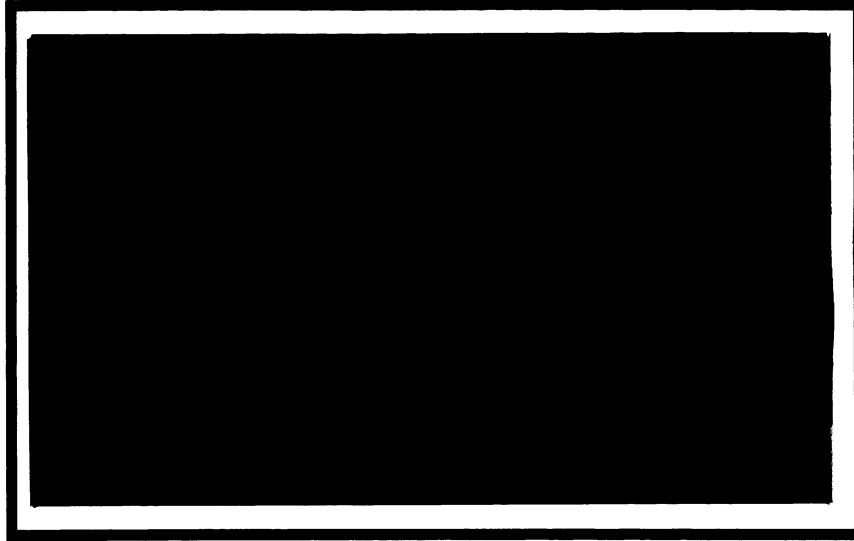


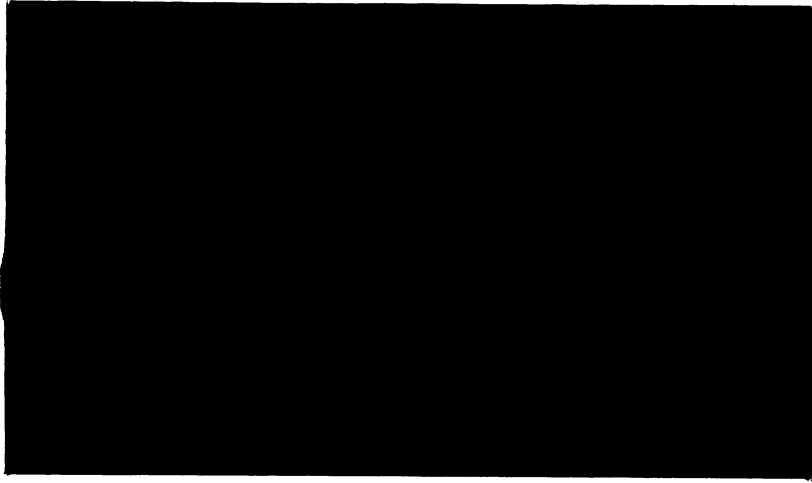
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MEDIAN VOTER MODELS AND  
THE GROWTH OF GOVERNMENT SERVICES\*

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In recent years economists have shown a more intense interest in matters having to do with the impact of government activities on the structure and performance of the economy. Some of this is the natural result of long tradition in the development of such areas as public finance, stabilization policy, and industrial organization. In addition, though, the profession at large has been prodded by special factors — e.g., the rising share of government in the GNP, the "taxpayer revolt", alleged regulatory excesses — to look much more broadly at what kinds of effects and side-effects government activities have on economic conduct.

This paper deals with the growth of government spending and, specifically, with the often-heard claim that government spending is "out of control" or has been growing "too rapidly". Nearly fifteen years ago William J. Baumol published a seminal paper which contained the provocative prediction that units of government would be unable to provide even a constant flow of typical governmental services without channeling "an ever increasing proportion of the labor force ... into these activities" (Baumol, 1967, p.420). This fundamental proposition -- derived from the observation that the technology of public service production is typically and inherently subject to relatively low productivity growth -- has played a prominent part in many subsequent analyses of the growth of the government sector. Recent papers by Break (1980), Gramlich (1981), Heller (1981), and Oates (1981) are but a few examples of the growing literature dealing with one or another of the implications of the basic proposition enunciated by Baumol.

One wonders whether the so-called taxpayer revolt — typified by the passage of the Proposition 13 tax cut in California in 1978 — may be

essentially a reaction against the Baumol prediction come true. In other words, were voters unfavorably surprised by the growing cost of providing the public services which they had initially sought? If voters were aware of the true cost dynamics, would they choose (vote for) fewer or slower growth of public services? These are precisely the kinds of questions raised by E.M. Gramlich and W.E. Oates who analyze the provision of public services in their 1981 papers. In the next section of this paper, I develop a fairly general model which may be used to analyze and evaluate the growth of public services. In the process, I shall compare and contrast some of the findings of Gramlich and Oates and show that, qualitatively at least, their conclusions hold up in the more general model of expenditure determination developed here.

In Section III of the paper I return to the issue of the taxpayer revolt. I attempt both to relate it to the model developed in Section II and to suggest why the model may be inadequate to a full understanding of taxpayer displeasure.

## II

An obvious first step in considering the claim that government spending has been growing excessively is to derive a standard or norm with which to compare the actual growth of government spending. A measure of desired growth of government spending can be derived by the application of median voter theory. The basic notion is that government activity should reflect the preferences of the median, or even more aptly, the decisive voter in the community. If the provision of government services actually reflects the preferences of the decisive voter, how rapidly will the government sector

grow and what share of GNP will it absorb? The following notation is used in specifying a median voter model to deal with these issues.

$X_i$  = Utils of government-provided services demanded by individual  $i$ , the median (decisive) voter.

$Y_i$  = Real income of the median voter.

$P_x$  = The relative price the median voter pays for a unit of  $X_i$  (the "tax-price") <sup>1/</sup>

$G$  = The quantity of government services provided.

$P$  = The relative price of providing government services.

$N$  = Total population.

$n$  = The population size of the target group receiving the government services.

$Y$  = Aggregate real income (= aggregate real tax base).

The median voter's demand function for  $X_i$  is assumed to be given by

$$(1) X_i = A_1 Y_i^\alpha P_x^\beta, \alpha > 0, \beta < 0.$$

The variable  $X_i$  is intended to reflect the value of government services as perceived by the median voter, and these value units are the direct objects of voter demand. For want of a better term, I refer to  $X_i$  as the utils attached to the services provided by government.

The aggregate services provided by government are assumed to be measured in real terms and are denoted by the variable  $G$ .<sup>2/</sup> These services are of

<sup>1/</sup>

In this paper a relative price is always to be understood as having the GNP deflator in its denominator; i.e., all relative prices are defined relative to the aggregate price level.

<sup>2/</sup>

To be absolutely clear, if  $G$  were all purchases by government, real GNP, say  $Q$ , would be given by  $Q=C+I+G$ . Nominal GNP would be given by  $P_q Q = P_c C + P_i I + P_g G$ , where  $P_c$ ,  $P_i$  and  $P_g$  are the deflators for consumption, investment, and government purchases, respectively, and  $P_q$  is the GNP deflator.

"value" to the median voter via a relation to be specified between  $X_i$  and  $G$ , but are provided by government to a (low income) target population. The relation between  $X_i$  and  $G$  is assumed to be

$$(2) \quad X_i = A_2 \left[ \frac{G/n}{Y_i} \right]^\theta \frac{G}{n}$$

in the relevant range  $0 < \frac{G/n}{Y_i} < \rho < 1$ , with  $\rho$  assumed known and  $-1 < \theta < 0$ . The median voter derives utils from the provision of real per-capita "income",  $G/n$ , to the target group. With  $\theta=0$  the utils are directly proportioned to  $G/n$ . With  $-1 < \theta < 0$  the median voter receives more utils from a given  $G/n$  the larger is  $G/n$  relative to  $Y_i$ , which is indicative of real per-capita income in the non-target group, but  $\partial^2 X_i / \partial (G/n)^2 < 0$  so that the marginal utility (to the median voter) of  $G/n$  declines as  $G/n$  rises toward  $Y_i$ . Equation (2) defines the utils function only for relative income  $\frac{(G/n)}{Y_i}$  no greater than the limit value  $\rho$ . In other words, there is an assumed limit to the median voter's altruism towards or empathy for the target group. It will be seen that it is not necessary to define the utils function for relative income in excess of  $\rho$ . The parameter  $\theta$  may be thought of as an indicator of the median voter's "intensity of altruism". For given values of  $G/n$  and  $Y_i$ ,  $\partial X_i / \partial \theta = X_i \ln \left[ \frac{G/n}{Y_i} \right] < 0$  so that higher values of  $\theta$  reduce the utility of government services to the median voter.

If  $P$  is the relative price level for government services,  $PG$  is the real total tax bill for the provision of  $G$ .<sup>3/</sup> If  $Y_i/Y$  measures the median voter's

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<sup>3/</sup> In the notation of footnote 2,  $P_g G$  would be the dollar cost of providing government services and  $\frac{P_g G}{P_q}$  would measure the real cost of government services in terms of the claim on society's output. Of course  $\frac{P_g}{P_q} = P$ , the relative price level for government services, so that  $PG$  is the real cost as a claim on society's output.

share of taxes collected,  $\frac{Y_i}{Y}$  PG measures the median voter's real tax payment to finance G. If  $P_x$  is the relative price the median voter pays for a unit of  $X_i$  through taxation, then by definition

$$P_x X_i = \frac{Y_i}{Y} PG.$$

Upon dividing both sides of the above equation by  $X_i$ , we derive the following representation for the relative tax price facing the median voter:

$$(3) \quad P_x = \frac{Y_i}{Y} \frac{PG}{X_i}$$

Equations (1) - (3) can be employed to eliminate  $P_x$  and  $X_i$  and leave a single equation relating G to the variables  $Y_i$ , Y, P, n and the parameters  $A_1$ ,  $A_2$ ,  $\alpha$ ,  $\beta$ , and  $\theta$ . Taking natural logarithms and differentials then yields

$$(4) \quad (1+\theta+\beta\theta)d \ln G = (\alpha+\beta+\theta+\beta\theta)d \ln Y_i - \beta d \ln Y + \beta d \ln P \\ + (1+\beta+\theta+\beta\theta)d \ln n$$

Employing (4) and the approximation

$$(5) \quad d \ln Y_i = d \ln Y - d \ln n$$

then yields the following statements:

$$(6) \quad d \ln \left[ \frac{G}{Y} \right] = \lambda(\alpha-1)d \ln \frac{Y}{N} + \lambda\beta d \ln P + (1+\lambda\beta)d \ln \frac{n}{N},$$

$$(7) \quad d \ln \left[ \frac{PG}{Y} \right] = \lambda(\alpha-1)d \ln \frac{Y}{N} + (1+\lambda\beta)d \ln P + (1+\lambda\beta)d \ln \frac{n}{N} \\ = d \ln \left[ \frac{G}{Y} \right] + d \ln P,$$

and

$$(8) \quad d \ln \left[ \frac{G/n}{Y_i} \right] = \lambda(\alpha-1)d \ln \frac{Y}{N} + \lambda\beta d \ln P + \lambda\beta d \ln \frac{n}{N} = d \ln \left[ \frac{G}{Y} \right] - d \ln \frac{n}{N},$$



where  $\lambda = (1+\theta+\beta\theta)^{-1}$ .

Equations (6) - (8) provide predictions about the desired growth of government services relative to income, but in each case the measure of "government services relative to income" differs. Equation (6) focuses on real government expenditures (the "quantity" of government services) as a share of real income (or real GNP). This is the concept to which Gramlich refers when he states that "... the growth rate of government spending will always be less than the growth rate of GNP" (Gramlich, 1981, p.5).

i.e., Gramlich's claim is that  $d \ln \left[ \frac{G}{Y} \right] < 0$ .<sup>4/</sup>

Equation (7), on the other hand, focuses on the cost to society of providing the government services in question since  $\frac{PG}{Y}$  measures the share of total product required to provide (pay for) the government services. This is the concept to which Oates refers when he states that "... in the long run desired public spending is likely to grow at a more rapid rate than total income" (Oates, 1981, p. 27). i.e. Oates's claim is that  $d \ln \left[ \frac{PG}{Y} \right] > 0$ .

Before considering the consistency of these two claims, consider first a dynamic version of the Baumol prediction discussed earlier. If the relative price of providing government services rises over time ( $d \ln P > 0$ ), then even if the quantity of services provided grows at the same rate as real output ( $d \ln \frac{G}{Y} = 0$ ), it follows that a growing fraction of society's output will have to be devoted to providing that flow of services ( $d \ln \frac{PG}{Y} > 0$ ). This follows directly from equations (6) and (7). If the median voter desires a decline in G relative to Y, then  $d \ln P > 0$  is not sufficient to establish

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<sup>4/</sup>

Throughout this paper, differentials of logarithmic expressions should be thought of as percent changes per unit of time of the argument of the logarithm, i.e.,  $d \ln(Z) = \frac{1}{Z} \frac{dZ}{dt}$ .

that PG will rise relative to Y.

Gramlich and Oates employ virtually identical median voter models which are special cases of the one used in this paper such that  $\theta=0$  and  $n = N$ .<sup>5/</sup> These conditions imply the following special cases of equations (6) and (7):

$$(6.1) \quad d \ln \left[ \frac{G}{Y} \right] = (\alpha - 1) d \ln \frac{Y}{N} + \beta d \ln P$$

and

$$(7.1) \quad d \ln \left[ \frac{PG}{Y} \right] = (\alpha - 1) d \ln \frac{Y}{N} + (1 + \beta) d \ln P.$$

If one accepts the common econometric estimates of the relevant demand functions for government services, then  $\alpha \cong 1$  and  $\beta \cong -.5$ .<sup>6/</sup> In that case (6.1) implies  $d \ln \frac{G}{Y} \cong -.5 d \ln P$  and (7.1) implies  $d \ln \frac{PG}{Y} \cong .5 d \ln P$  and  $d \ln P > 0$  insures that Gramlich and Oates are both right:

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<sup>5/</sup>

Gramlich's model comes closer to dealing with a collective or public good provided for the entire population, but with a congestion or crowding effect. Gramlich's version of equation (2) is  $X_i = G N^{-\delta}$  but he concludes on empirical grounds that  $\delta \cong 1$ . See Gramlich (1981, pp. 3-8). It is not readily apparent that Oates's model is in all essential respects equivalent to Gramlich's. The key to this realization is Oates's footnote 11 which implies that all the relevant variables are defined in per-capita terms. i.e., Oates's Y and X are to be thought of as per-capita real income and the per-capita level of public services provided, etc. Oates's variable E is therefore equivalent to  $\frac{PG}{N}$  in the notation of this paper. See Oates (1981, especially pp. 23-26).

<sup>6/</sup>

See the discussions of these estimates in Gramlich (1981) and Oates (1981). The qualitative conclusions of this paper do not require the exact values assumed for  $\alpha$  and  $\beta$ .

The median voter desires a declining trend in the provision of real government services relative to real GNP, but is willing to devote a growing share of society's output to providing those services.

We can attempt to generalize this conclusion in two steps. First, suppose  $\theta=0$ , but the government services being provided are not pure public goods. Rather the government is providing support services to a (low income) target group of size  $n < N$ . Then equations (6) and (7) become

$$(6.2) \quad d \ln \left[ \frac{G}{Y} \right] = (\alpha-1) d \ln \frac{Y}{N} + \beta d \ln P + (1+\beta) d \ln \frac{n}{N}$$

and

$$(7.2) \quad d \ln \left[ \frac{PG}{Y} \right] = (\alpha-1) d \ln \frac{Y}{N} + (1+\beta) d \ln P + (1+\beta) d \ln \frac{n}{N}$$

If we assume again that  $\alpha \cong 1$  and  $\beta \cong -.5$ , then  $d \ln \frac{G}{Y} \cong -.5 d \ln P + .5 d \ln \frac{n}{N}$

and  $d \ln \frac{PG}{Y} \cong .5 d \ln P + .5 d \ln \frac{n}{N}$ . If  $d \ln \frac{n}{N}$  is non-negative, which I take to be the empirically relevant case for recent history, then surely the real cost of providing government services must rise relative to real GNP ( $d \ln \frac{PG}{Y} > 0$ ) and the sign of  $d \ln \frac{G}{Y}$  itself could be positive. Note, however, that under the assumptions now being considered, equation (8) becomes

$$d \ln \frac{G/n}{Y_i} = -.5 d \ln P - .5 d \ln \frac{n}{N} < 0$$

if  $d \ln P > 0$  and  $d \ln \frac{n}{N} > 0$ . Thus, if the relative income ratio  $\frac{G/n}{Y_i}$  begins below the limit value  $\rho$  (see the discussion of equation (2)), it will remain below  $\rho$  even if the size of the target population rises relative to the total population.<sup>7/</sup> Indeed, if the target group is being allowed to

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<sup>7/</sup>

It is for this reason that the utils function, equation (2), need not be defined for relative income greater than  $\rho$ .

increase in relative size the tax-price rises too rapidly to prevent a decrease in the relative real income of the target group!

Finally, suppose  $-1 < \theta < 0$  and again  $\alpha \cong 1$  and  $\beta \cong -.5$ . Equations (6) and (7) now become

$$(6.3) \quad d \ln \left[ \frac{G}{Y} \right] = -.5\lambda d \ln P + (1-.5\lambda) d \ln \frac{n}{N}$$

and

$$(7.3) \quad d \ln \left[ \frac{PG}{Y} \right] = (1-.5\lambda) d \ln P + (1-.5\lambda) d \ln \frac{n}{N},$$

with  $1 < \lambda < 2$ . Clearly, the qualitative implications of equations (6.3) and (7.3) are the same as those of (6.2) and (7.2): if the relative price of providing government services rises, a desired increase in the relative size of the target population implies a rise in the share of society's output absorbed by the government sector, and may even imply an increase in real government services relative to real GNP. But again, equation (8) would guarantee a decrease in the relative real income of the target group. In essence, the median voter model implies a trade-off between quantity and quality in the provision of government services. If we -- the electorate -- choose to provide government services to a growing fraction of the population, it will be at the cost of a reduction in the relative per-capita income provided to the target group, and will still absorb an increasing share of society's output.

It should be noted that the foregoing references to  $G/n$  as "income" cannot be taken literally. If the government were really providing income in the sense of direct purchasing power (transfer payments) it would be difficult to justify the claim that  $d \ln P > 0$ . Without a rise in the relative price  $P$ , equations (6) and (7) are identical and there is no distinction between the

real services provided and the real cost of providing those services.

### III

What are the facts; how do they compare with predictions derived from the median voter model?<sup>8/</sup> George Break's extensive survey of government spending and taxing provides conclusions of special interest in light of the preceding analysis. Break states for example that:

"The government sector is larger than it was (just after World War II) but in many dimensions it has been growing recently less rapidly than the economy as a whole"

and

"The public sector's share of national output has not grown significantly since 1953... "<sup>9/</sup>

These facts would constitute no surprise relative to the predictions of the median voter model. Break goes further:

"The federal government's tax-transfer programs have grown rapidly, especially in the domestic program sector"

and

" ... state and local governments have replaced the federal government as the major partner (in the provision of public services)"<sup>10/</sup>

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<sup>8/</sup>

Gramlich (1981, pp. 5-8) provides an extensive review of the facts of this matter

<sup>9/</sup>

Both quotes from Break (1980) p. 654, parentheses added.

<sup>10/</sup>

Both quotes, *ibid*, p. 654, parentheses added

Here, then, may be the clue. Aggregate government behavior may appear to be consistent with voter preferences while the composition of government services is inconsistent with voter preferences. In recent years, income-support and income-supplement programs have proliferated at, perhaps, accelerating cost while the provision of defense and even some educational services has declined relative to GNP. Indeed, Courant, Gramlich and Rubinfeld (1980) have found that the displeasure which surveyed voters express toward the size and growth of government programs is aimed almost exclusively at welfare programs.

Ronald Reagan's successful presidential campaign of 1980 claimed precisely that the composition of government activity had strayed from voter preferences: too much welfare, not enough defense. But Reagan claimed further that the government sector as a whole had grown more than the public wanted. If the median voter model is both descriptive and normative, proving that  $\frac{PG}{Y}$  has grown too rapidly compared to voter tastes might require fairly subtle data measurement and econometrics.

It may also be that the median voter model is not descriptive. If some process other than the satisfaction of voter preferences is a significant determinant of the level and growth of aggregate government activity, the result might well be inconsistent with voter preferences. Thus the "tax revolt" may simply be misplaced aggression: the voters striking out where they can, whether or not the target of their remedy is the source of their displeasure.<sup>11/</sup>

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<sup>11/</sup>

See Courant, Gramlich, Rubinfeld (1979) for a discussion of "Public Employee Market Power" as the determinant of government spending.

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