RESEARCH SEMINAR IN INTERNATIONAL ECONOMICS

Department of Economics
The University of Michigan
Ann Arbor, Michigan 48109-1220

SEMINAR DISCUSSION PAPER NO. 180

A STRATEGIC APPROACH TO THE PRODUCT LIFE CYCLE

By

Richard Jensen*
Ohio State University

and

Marie Thursby
Ohio State University and
The University of Michigan

January, 1986

his paper analyzes the steady state open-loop Nash equilibrium of a game in which a orthern monopolist devotes resources to new product development and a Southern lanner diverts resources into reverse engineering to learn the technology to produce hese. A steady state equilibrium technology gap exists, showing that a constant echnology gap over time may be explained by optimal strategic behavior of decision nakers in a product cycle model. Given resource costs, neither the Northern monopolist nor the Southern planner wants to alter the gap. Finally, analysis of comparative steady states shows the importance of strategic versus non-strategic behavior.

Forthcoming in *Journal of International Economics*.

*We would like to acknowledge particularly insightful suggestions from Richard Brecher, eonard Cheng, Ronald Jones, David Richardson, and two anonymous referees. Financial support was provided by National Science Foundation Grant IST-8510068.

All correspondence should be addressed to the authors at Department of Economics, Arps Hall, 1945 North High Street, Ohio State University, Columbus, OH 43210-1172, USA.

Trade which is generated by new technology has been a topic of major interest in international trade since the pioneering work of Posner (1961), Hufbauer (1966), and Vernon (1966) in the sixties. Posner and Hufbauer posited not only that advanced countries would be the first exporters of high technology goods (i.e., either new goods or goods produced by superior technology) but also that they would "extend their leadership from one decade to the next (Hufbauer, p. 86)." Hence high technology trade would be "relatively disadvantageous to the less technologically advanced countries (Johnson, 1975, p. 41.)." Vernon's work concentrated on the trade pattern of "new" products over their life cycle. He hypothesized a trade pattern in which advanced countries would develop and initially export goods, and less advanced countries would produce and export goods in the later stages of their life cycle.

Several recent papers have developed formal dynamic models of high technology trade in order to rigorously examine these hypotheses. Cheng (1984a) has found some support for the Posner-Hufbauer hypothesis in his model of R and D rivalry. Krugman (1979) and Jensen and Thursby (1985) have developed models which predict Vernon's product life cycle trade pattern. Krugman obtains this pattern in a dynamic model where the North develops new products at an exogenously given rate and the South acquires the technology for producing them at an exogenously given rate. Jensen and Thursby examine the trade pattern and welfare when the rate of product innovation is endogenously chosen by profit maximizing firms in the North. R and D expenditure, and hence the rate of innovation, are determined optimally in the North given the rate at which technology is transferred to the South. As suggested by Posner and Hufbauer, they find that in the steady state the North maintains a technological lead. However, the rate of innovation chosen in the North is lower than that which would maximize Southern welfare. Since the South clearly benefits from the transfer of technology in this case, it is natural to ask whether or not it is optimal for the South to divert resources from current production in order to further increase the rate at which it learns new technology. Given the observed behavior of many LDC's, this is a question of practical as well as theoretical interest. This implies a strategic analysis in which the rate of innovation in the North and the rate of technology transfer to the South are optimally chosen. While for a given rate of innovation the South benefits from increasing the transfer rate, it is not clear what the North's best response to such a change would be.

In this paper we analyze the steady state open-loop Nash equilibrium of a game in which a Northern monopolist and a Southern planner choose the rates of innovation and technology transfer (respectively). What distinguishes our model from all others of the product life cycle is that the game theoretic approach allows us to analyze how innovation and technology transfer are strategically related in a dynamic framework. It should be noted that the use of open-loop stategies restricts the monopolist and planner to make their decisions for the entire game, ex ante. This means they do not take into account the effect of their current decision on their rival's future decisions, but the complexity of the underlying trade model prevents us from determining if closed-loop equilibria involving such feedback even exist.[1] Moreover, the complexity of even the open-loop analysis is such that we have specified the simplest model of innovation in the North. That is, we assume a Northern monopoly rather than the Cournot oligopoly of Jensen and Thursby. We also depart from their approach by assuming that rate of technology transfer is not given, but can be chosen (within certain bounds) by the Southern planner. We choose a Southern planner rather than a market model in order to focus on the socially optimal outcome for the South (the technologically backward country).

Assuming a Northern monopoly and Southern planner, aside from its tractability, has the advantage of at least loosely conforming to conventional notions about how large firms tend to develop new goods while technologically backward countries plan concerted efforts to try to catch up.

In Section 2 we describe the model and prove the existence of a steady state Nash equilibrium. Conditions are given under which the Southern planner will devote resources to increasing the rate of transfer over and above that which would occur costlessly. Section 3 presents the analysis of the steady state reaction functions and how changes in parameters affect the equilibrium rates of innovation and technology transfer. It is shown that in equilibrium the reaction functions are positively sloped. Section 4 is devoted to the technology gap, Section 5 concludes, and all proofs are in the Appendix. The two major results are: (1) a steady state equilibrium technology gap can be explained by optimal strategic behavior of decision makers in a product cycle model, and (2) equilibrium rates of innovation and technology transfer, and hence the technology gap, are affected by whether or not the South augments technology transfer. For example, when the South is passive, a change in the Southern labor supply does not affect the rate of innovation or gap, but the rate of innovation rises and the gap declines when the South augments technology transfer.

2. The Model

Assume a world with two countries, called North and South. Consumers in both are identical and have the utility function

$$U = \int_0^n c(\eta)^\gamma d\eta, \quad 0 < \gamma < 1 \tag{1}$$

where U is utility, n is the number of goods available for consumption, and $c(\eta)$ is the amount of the η^{th} good consumed. Workers in both are equally productive with units of measurement chosen so that one worker can produce one unit of any good (assuming the technology for producing it is known). There are M_N workers in the North and M_S in the South. The essential difference between the two is that only Northern workers can develop new goods (or, precisely, develop the technology required to produce new goods). Hence new product development can occur only in the North. All of these assumptions are standard for this literature (e.g., Krugman, Feenstra and Judd (1982), and Jensen and Thursby).

We define new goods as those recently developed goods whose production technology is known only in the North and old goods as those whose production technology is common knowledge. We assume that initially all goods are old goods. However, if development occurs in the North, then the stocks of new and old goods are determined over time by the rates of innovation and technology transfer. Innovation refers to the development of new goods and technology transfer to the process by which the South learns the technology required to produce new goods. All technological progress in this model occurs in the form of either the development of new goods or the South's learning how to produce recently developed goods; no technological change in the form of increases in productivity in the production of any given goods will occur.

In the spirit of Feenstra and Judd and Jensen and Thursby we assume that innovation requires the use of resources (labor) in R and D. We specify the output of the R and D process as the instantaneous rate of increase in new goods and the inputs as labor in R and D and the current total number of goods. That is,

$$\dot{n}(t) = h[D_N(t)]n(t), \; n(0) = n_0 > 0 \qquad (2)$$

where a dot over a variable denotes the derivative with respect to time, t, and $D_N(t)$ is the amount of labor in R and D at t.[2] The function $h(\cdot)$

is assumed to be twice continuously differentiable, bounded above by the social rate of discount, r, and satisfy $h(0) = 0$, $h' > 0 > h''$. Hence the marginal products of both inputs are positive, with that of labor decreasing and that of the total number of goods constant. Recalling that only the North can develop new goods, this specification simply says that the more new goods which have been developed, the more new goods can be developed at any given date by a fixed supply of Northern workers. That is, there is learning by doing in new product development.

If development and trade occur, we assume that the relative wage $w \equiv w_N/w_S > 1$, so the North completely specializes in the production of new goods and the South completely specializes in the production of old goods. If $n_S(t)$ is the number of old goods at t, then the number of new goods is $n_N(t) \equiv n(t) - n_S(t)$. Technology transfer is the process by which the South learns the production technology for new goods. We assume that all technology is eventually transferred through reverse engineering, the process in which the South learns how to produce a new good by dismantling and studying it (see Mansfield and Romeo (1980) for examples of this process). There is some basic rate of technology transfer, say v $(0 < v < 1)$, which will exist even if the South does not divert workers from current production. That is, after consuming any new good for a long enough (but finite) period of time, Southern workers become familiar enough with it to learn how to produce it. In addition, this rate of transfer can be augmented by devoting Southern workers to reverse engineering. It seems reasonable to assume that the number of new goods the South can learn to produce at any time t is a fraction of the total number of new goods available then.[3] Hence we assume the technology transfer process is given by

$$\dot{n}_S(t) = g[D_S(t)]n_N(t),\ n_S(0) = n_0 \tag{3}$$

where $D_S(t)$ is the number of Southern workers in reverse engineering and $g[D_S(t)]$ is the rate of transfer at t (i.e., the fraction of new goods the South learns to produce). The function $g(D_S)$ is nonnegative, strictly increasing, concave, and bounded below by v and above by r (i.e., labor has positive but declining marginal productivity in reverse engineering as well). Note that this specification precludes instantaneous technology transfer.

The Southern planner insures full employment and perfect competition. This, together with the assumptions on production, the utility function, the balance of payments, and full employment in the North, imply that the terms of trade (the price of a new good in terms of an old good) are

$$P = [n_N(M_S-D_S)/n_S(M_N-D_N)]^{1-\gamma} \tag{4}$$

This expression for the terms of trade differs from that derived by Krugman because it takes into account the fact that the development of new goods and the augmentation of technology transfer require the use of workers who must be taken out of current production. Finally, the North's share of total world output is

$$\alpha = \frac{P(M_N-D_N)}{P(M_N-D_N)+(M_S-D_S)}\ , \tag{5}$$

so that Southern utility per period is

$$U_S = (1-\alpha)^{\gamma}[n_N^{1-\gamma}(M_N-D_N)^{\gamma}+n_S^{1-\gamma}(M_S-D_S)^{\gamma}]. \tag{6}$$

Monopoly profit per period is

$$\Pi = P\ (M_N-D_N) - wM_N. \tag{7}$$

Under the assumption of full employment in the North, the Northern wage always adjusts instantaneously so that it does not exceed the marginal revenue product of labor in production. Hence, the wage bill wM_N is not essential, and we can view the monopolist's objective as the maximization of the present value of either profit or revenue (the equivalence of these is most easily seen from (A1) in the Appendix).

Formally, the monopolist's problem is to choose $D_N(t)$ for $t \geq 0$ to maximize the present discounted value of its stream of future profits subject to the innovation process (2) and the technology transfer process (3). The planner's problem is to choose $D_S(t)$ for $t \geq 0$ to maximize the present discounted value of the stream of Southern utilities subject to (2) and (3). Since the first order necessary conditions for these problems are not informative, we confine our analysis to the steady state Nash equilibrium introduced by the following proposition.

<u>Proposition 1</u>: There exists a unique, locally stable steady state Nash equilibrium in which the amounts of labor in innovation and reverse engineering are the constants $D_N^* \in (0, M_N)$ and $D_S^* \in (0, M_S)$ defined implicitly by

$$T_N(D_N^*, D_S^*) = -\gamma P^* + \frac{(1-\gamma)(\theta^*+1)h'(D_N^*)(M_N-D_N^*)P^*}{\theta^*(r+g(D_S^*))} = 0 \tag{8}$$

and

$$T_S(D_N^*, D_S^*) = \frac{-\gamma[1-(1-\gamma)\alpha^*]}{(M_S-D_S^*)} + \frac{(1-\gamma)[\theta^*-(1-\gamma)(1+\theta^*)\alpha^*]}{r+g(D_S^*)} g'(D_S^*) = 0 \tag{9}$$

whenever

$$\left| \frac{\partial T_N}{\partial D_S} \frac{\partial T_S}{\partial D_N} \middle/ \frac{\partial T_N}{\partial D_N} \frac{\partial T_S}{\partial D_S} \right| < 1 \tag{10}$$

and

$$g'(0) > \frac{\gamma[1-(1-\gamma)\alpha](v+r)}{(1-\gamma)[\theta-(1-\gamma)\alpha(1+\theta)]M_S} . \tag{11}$$

Moreover, the ratio of new to old goods is also a constant, $\theta^* = h(D_N^*)/g(D_S^*)$, in equilibrium.

Equations (8) and (9) characterize the steady state Nash equilibrium in open loop strategies for this dynamic (differential) game between the monopolist and the planner. The strategic interdependence between them can be seen from the fact that D_N^* and D_S^* enter both equations (indicating that each has taken the other's actions into account in maximizing). Equation (8) indicates that, given D_S^*, the best the monopolist can do is to choose D_N^*; and (9) indicates that, given D_N^*, the best the planner can do is to choose D_S^*. Intuitively, (8) says that the monopolist allocates labor between current production and development so as to equate its marginal revenue product in the two activities. Analogously, (9) states that the planner allocates labor between current production and reverse engineering so as to equate its marginal contribution to utility in the two activities.

As shown in the Appendix, $h'(0) > 0$ is sufficient to insure that the monopolist will innovate for any choice of D_S. Moreover, the condition given by (11) is sufficient to insure that the South reverse engineers for any $D_N \in (0, M_N)$. That is, all that is required for the North to innovate is that the marginal product of the first Northern worker in R and D be positive. However, since we have assumed that some technology transfer always occurs, the Southern planner will not augment that transfer unless the marginal product of labor in reverse engineering is high enough to offset the reduction in current utility. As one might expect (see (11)), the lower bound on that marginal product increases as the costless rate of transfer, v, increases. Finally equation (10) insures uniqueness and local stability of the equilibrium.

Two final remarks about the model are worthwhile. First, a steady state Nash equilibrium exists when (11) does not hold as well. It is $(\overline{D}_N, 0)$ where $T_N(\overline{D}_N, 0) = 0$ and $\overline{D}_N \in (0, M_N)$. In this case the South's equilibrium strategy is to be passive (i.e., devote no resources to reverse engineering and let technology transfer at the basic rate v). Although our analysis focuses on the case where the South augments technology transfer, we do note some interesting differences between these cases. Second, because the monopoly represents a distortion in the North, resources are underallocated to R and D compared to the level that would maximize Northern discounted utility. This follows from the fact that Northern consumers benefit from new product development after the technology has transferred and the monopolist has ceased to profit from it.

3. Reaction Functions and Comparative Steady States

An alternative way of looking at equations (8) and (9) is that they define, respectively, the steady state best response, or reaction, functions for the Northern monopolist, $r_N(D_S)$, and the Southern planner, $r_S(D_N)$. That is, $r_N(D_S)$ shows the optimal constant per period amount of labor that the monopolist should devote to R and D given that the South devotes D_S workers to reverse engineering. A similar interpretation holds for $r_S(D_N)$, and the equilibrium values D_N^* and D_S^* are defined by the intersection of these reaction functions. One of the primary advantages to our game-theoretic approach to innovation and technology transfer is that it shows us how strategic interdependence between the North and South could affect innovative behavior.

Proposition 2: The steady state reaction functions of the monopolist and the planner are positively sloped $(r_N'(D_S) > 0,\ r_S'(D_N) > 0)$ in equilibrium.

This result says that, in equilibrium, innovation and reverse engineering are directly related to each other. For example, suppose the South increases reverse engineering and so the rate of technology transfer. Then the monopolist's ability to profit from a given level of innovative effort declines because the length of time over which it has a monopoly on new goods is shorter. It follows from $r'_N(D_S) > 0$ that the monopolist's best response is to try to offset this erosion of its monopoly power by increasing the number of new goods it develops. To be precise the increase in reverse engineering decreases the marginal revenue product of labor in current production relative to that in R and D. Hence the equilibrium rate of new product development when the South augments reverse engineering exceeds that when it does not. Analogously, if the rate of innovation were to increase, then the South's best response is to increase reverse engineering since $r'_S(D_N) > 0$. This occurs because an increase in the rate of innovation increases the marginal contribution of Southern labor in reverse engineering to utility relative to that in current production.

The results of Propositions 1 and 2 are summarized conveniently by Figure 1, which is particularly useful in analyzing how the equilibrium values D_N^* and D_S^* vary with changes in parameters.

Proposition 3: An increase in M_S increases the equilibrium rates of innovation and technology transfer, but a change in M_N has an ambiguous effect. An increase in v also increases the equilibrium rates of innovation and technology transfer, although the effect on the amount of labor in reverse engineering is ambiguous.

The comparative dynamics results for this model are particularly illustrative because they show precisely how behavior when the South augments transfer may differ from that when it does not. Consider, for example, a change in the South's labor/supply. For a given rate of transfer, the monopolist will not respond to a change in the Southern labor supply because it does not affect the marginal revenue product of labor in production relative to development. Hence $r_N(D_S)$ does not shift when M_S changes. The South, however, will respond to its labor supply increase by increasing the amount of labor in reverse engineering for any given (positive) rate of development (that is, $r_S(D_N)$ shifts to the right). Since the monopolist responds to the increase in reverse engineering by increasing its new product development efforts, in equilibrium both the rate of innovation and the rate of technology transfer will be higher after an increase in the Southern labor supply.

Interestingly, an increase in the Northern labor supply will have an ambiguous effect on both Northern and Southern behavior. For a given transfer rate, the monopolist will increase the amount of labor in R and D when M_N increases (that is, $r_N(D_S)$ shifts up). This leads the South to respond by increasing the amount of labor in reverse engineering. However, the increase in M_N moves the terms of trade in the South's favor and induces it to increase current production (i.e. $r_S(D_N)$ shifts to the left). This leads the monopolist to respond by reducing the amount of labor in R and D. Hence, these conflicting effects make it impossible to determine how the equilibrium rates of innovation and technology transfer change.

Finally, an increase in the costless rate of transfer, v, causes the reaction functions to shift in the same directions as a change in M_N. However, the magnitude of the increase in $r_N(D_S)$ is sufficiently large relative to the decrease in $r_S(D_N^*)$ that D_N^*, and thus the equilibrium rate of innovation, must increase. And although the effect on D_S^* is ambiguous, we can show that the equilibrium rate of technology transfer must increase. To be precise, in order to analyze the effects of change in the costless transfer rate $v = g(0)$, we have written the rate of technology transfer as $g(D_S) = v+\tilde{g}(D_S)$, where $\tilde{g}(0) = 0$ and $\tilde{g}$ has the properties imposed on it by the assumptions on g. Then we can show that $\frac{\partial g(D_S^*)}{\partial v} = 1 + g'(D_S^*)\frac{\partial D_S^*}{\partial v} > 0$.

Hence, even if an increase in the costless transfer rate leads the South to devote fewer workers to reverse engineering, this will be done so that in equilibrium the rate of technology transfer still increases.

4. Technology Gap

The natural measure of the North's technological lead, or the technology gap between the North and South, in this model is the ratio of new to old goods, $\theta(t)=n_N(t)/n_S(t)$. In the proof of Proposition 1 we show that a steady state Nash equilibrium exists if and only if this ratio is the constant, $\theta^*=h(D^*_N)/g(D^*_S)$. Two remarks about this result are in order. First, because there is always a latent demand for new product development, a steady state equilibrium will not exist in this product cycle model unless this ratio becomes a constant (i.e., independent of t) in the limit. Next, the exact functional form of the steady state gap θ^* comes directly from the assumed forms of the innovation and transfer processes in (2) and (3). We chose these specifications because their justifications are firmly rooted in the R and D literature. If one were to choose alternative specifications for other sound reasons, it would still be the case that a steady state Nash equilibrium existed if and only if the ratio of new to old goods was a constant in the steady state. Such alternative processes would yield a different form of θ^*, however, and thereby possibly different comparative statics results. What is important about this result in our model is that it shows the equilibrium technology gap is positive but finite for these innovation and transfer processes. In particular, it is positive because the marginal product of the first worker in R and D is positive and because technology cannot be transferred instantaneously. It is finite because marginal productivity in R and D is diminishing and because some technology is transferred even if the South is passive.

Hence, when the monopolist and planner optimize strategically in this product cycle model, the long-run outcome is a constant technology gap which the North does not want to try to extend and the South does not want to try to close. Whether the South augments transfer or not, it is not optimal for the monopolist to try to keep increasing its lead because the opportunity cost (in foregone current profit) is too high. And even if the South augments transfer, it is not optimal for the planner to try to eliminate the gap because the cost (in foregone current utility) is too high. Our final result shows how this equilibrium technology gap varies with the labor supplies and the costless transfer rate.

Proposition 4: An increase in either M_S or v will reduce the equilibrium technology gap, but a change in M_N has an ambiguous effect.

Because both D_N^* and D_S^* increase with an increase in M_S, they generate conflicting effects on θ^*. However, we can show that θ^* decreases in this case. The easiest way to see this is to recall that $r_N(D_S)$ does not shift when M_S changes, so that the increase in the equilibrium rate of reverse engineering exceeds the increase in the rate of innovation. This is not surprising since it simply says that the South can take advantage of an increase in its labor supply in order to reduce the equilibrium technology gap. It is interesting to note that if the South does not devote resources to reverse engineering, then the gap would not be affected by a change in M_S because the rate of innovation would not change. Hence, an increase in the Southern labor supply induces increases in the rates of innovation and technology transfer and a decrease in the technology gap only when the South is not passive.

In much the same way, an increase in the costless transfer rate, v, reduces θ^*. That is, even though the increase in v increases both the rate of innovation and the rate of technology transfer, the South can take advantage of this change so as to reduce the equilibrium technology gap. Even if the South were passive, an increase in v would increase the rate of innovation and decrease the gap; however, the magnitudes of the resulting changes would be smaller.

Finally, note that an increase in M_N would cause an increase in the rate of innovation and the technology gap if the South were passive. However, when it is not, an increase in M_N has an ambiguous effect on both equilibrium rates and the equilibrium technology gap. That is, a Northern monopolist may not be willing or able to use an increase in its labor supply to increase the gap when the South devotes some resources to reverse engineering.

5. Conclusion

In this strategic product cycle model the steady state Nash equilibrium is characterized by a constant ratio of new to old goods. The Posner-Hufbauer hypothesis holds in that the North has a technological lead which it does not want to try to widen and the South does not want to try to reduce. However, trade is not relatively disadvantageous to the South. In this model there are two reasons why the South always benefits from Northern new product development and trade, whether it augments technology transfer or not. One is that both Northern and Southern utility at any date are higher when there are more goods available, and the development cost is borne by the North. The other is that the South eventually can learn to produce any given new good and use its comparative advantage in production to export that good to the North in exchange for recently developed goods.

Since Cheng also found some support for the Posner-Hufbauer hypothesis, it is important to point out two major differences between his analysis and ours. He models international rivalry between firms in R and D, whereas we model new product development and imitation. Hence his model might be viewed as being more appropriate for describing R and D rivalry between developed countries, whereas ours might be more appropriate in describing product life cycle trade between developed and less developed countries. The other major difference is that his model has a finite horizon (equal to the period in which profits can be earned on the innovation), whereas ours has an infinite horizon. Hence he finds some support for the Posner-Hufbauer hypothesis as a short-run phenomenon, but does not explore whether or not it can persist in the long-run.

Finally, since the Posner-Hufbauer hypothesis has been used to justify restrictive trade policies by technologically backward countries, it is important to note that these results suggest that such policies may be inappropriate if they reduce the rate of innovation in the North. Although we have not formally analyzed the effects of commercial policy in this model, our results suggest that such an extension would be interesting. Another nontrivial extension would be to examine when, if ever, the South would be willing to pay a fee for the technology and how this would affect innovation, transfer, and the technology gap.

Footnotes

1. Since there are no general existence theorems for differential games with closed loop strategies, such an equilibrium may not exist in this model.

2. The difference between this specification and that of Jensen and Thursby is that in the Cournot oligopoly the rate of innovation is function not only a of the amount of labor devoted to R and D by each firm and the number of goods available, but also of the extent to which firms can benefit from each other's R and D efforts.

3. This is analogous to the technology-push theory of innovation, which argues that the pace of innovation activity is related positively to advances in the underlying scientific base (see Nelson (1959), Phillips (1966), and Merton (1973), for example). In our view of reverse engineering the number of new goods is the underlying base upon which the South can draw.

References

Cheng, Leonard, "International Competition in R and D and Technological Leadership: An Examination of the Posner-Hufbauer Hypothesis," Journal of International Economics (August 1984): 15-40.

Cheng, Leonard, "Portection of 'High Technology' Industries, Countervailing Subsidies, and Welfare," Gainesville, Florida: Mimeo, October, 1984.

Feenstra, Robert and Judd, Ken, "Tariffs, Technology Transfer, and Welfare," Journal of Political Economy 90 (December 1982): 1142-1165.

Hufbauer, Gary C., Synthetic Materials and the Theory of International Trade, Cambridge, Mass.: Harvard University Press, 1966.

Jensen, Richard and Thursby, Marie, "A Decision-Theoretic Model of Innovation and Technology Transfer," Columbus, Ohio: mimeo, June, 1985.

Johnson, Harry G., Technology and Economic Interdependence, New York: St. Martin's Press, 1976.

Krugman, Paul, "A Model of Innovation, Technology Transfer, and the World Distribution of Income," Journal of Political Economy 87 (April 1979): 253-266.

Mansfield, Edwin and Romeo, Anthony, "Technology Transfer to Overseas Subsidiaries by U.S. Based Firms." Quarterly Journal of Economics (December 1980): 737-750.

Merton, R.K. The Sociology of Science. Chicago: University of Chicago Press, 1973.

Nelson, Richard. "The Simple Economics of Basic Scientific Research," Journal of Political Economy 67 (1959): 297-306.

Phillips, A. "Patents, Potential Competition, and Technical Progress." American Economic Review 56 (1966): 301-310.

Posner, M.V., "International Trade and Technical Change," Oxford Economic Papers 31 (October 1961): 323-34.

Vernon, Raymond, "International Investment and International Trade in the Product Cycle," Quarterly Journal of Economics 80 (May 1966): 190-207.

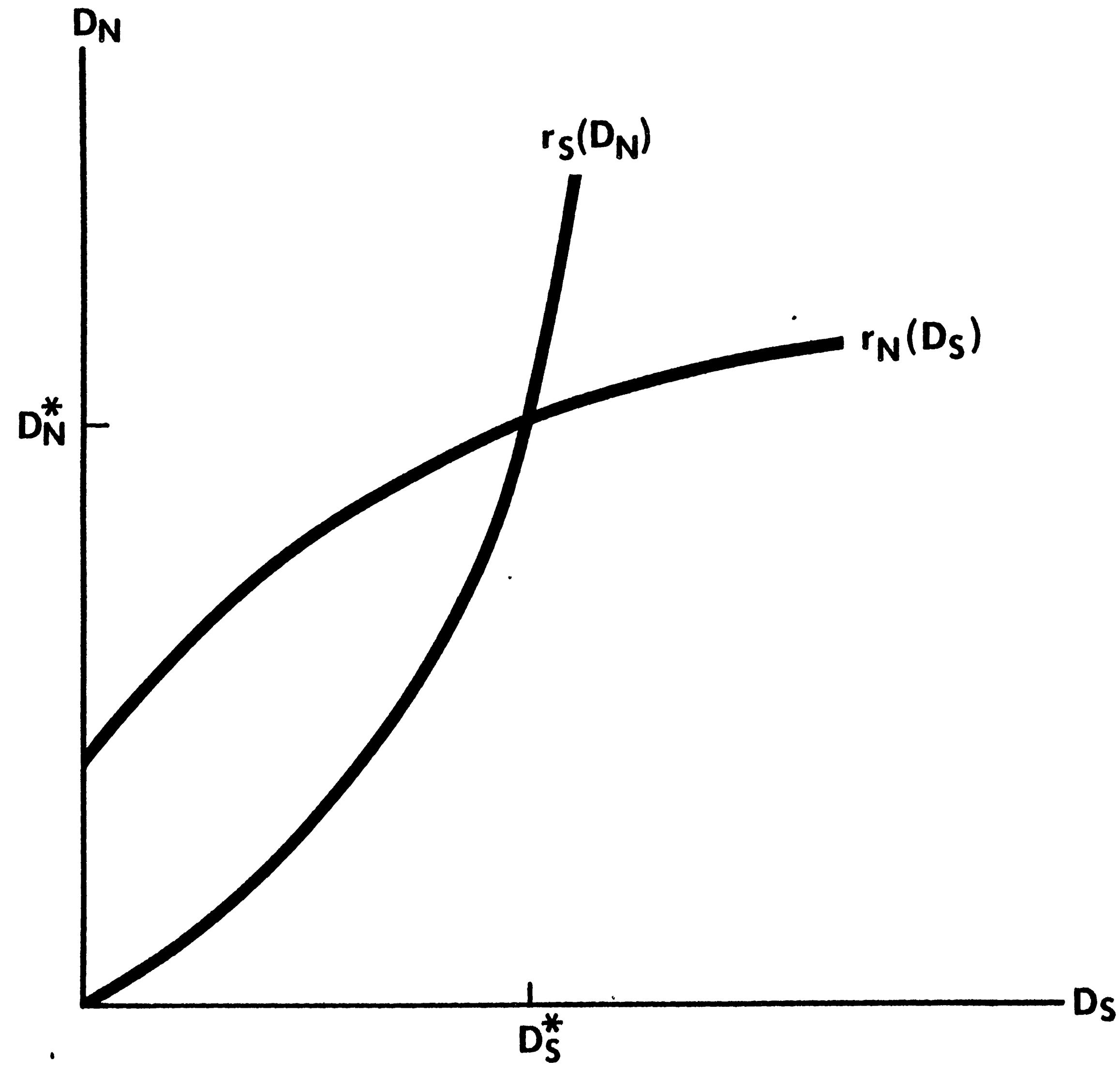

Figure 1

Appendix

A. Proof of Proposition 1

The monopolist's problem is to choose $D_N(t)$ for $t > 0$, given any $D_S(t) \varepsilon [0,M_S]$ for $t > 0$, so as to maximize $\int_0^{\infty} e^{-rt}\Pi(t)dt$ subject to (7), (2), (3), and (4). The current value Hamiltonian is $H_N(D_N,D_S,n,n_S,\lambda_N,\mu_N,t) = \Pi + \lambda_N h(D_N)n + \mu_N g(D_S)(n-n_S)$ and the first order necessary conditions are (2), (3),

$$\frac{\partial H_N}{\partial D_N} = -\gamma P + \lambda_N h'(D_N)n = 0 \tag{A1}$$

$$\dot{\lambda}_N = r\lambda_N - \frac{(1-\gamma)P(M_N - D_N)}{n-n_S} - \lambda_N h(D_N) - \mu_N g(D_S) \tag{A2}$$

$$\dot{\mu}_N = r\mu_N - \frac{(\gamma-1)Pn(M_N - D_N)}{(n-n_S)n_S} + \mu_N g(D_S) \tag{A3}$$

and the usual transversality contitions.

The Southern planner's problem is to choose $D_S(t)$ for $t > 0$, given any $D_N(t) \varepsilon [0, M_N]$, to maximize $\int_0^{\infty} e^{-rt}U_S(t)dt$ subject to (6), (5), (2), (3), and (4). The current value Hamiltonian is $H_S(D_N,D_S,n,n_S,\lambda_S,\mu_S,t) = U_S + \lambda_S h(D_N)n + \mu_S g(D_S)(n-n_S)$ and the first order necessary conditions are (2), (3),

$$\frac{\partial H_S}{\partial D_S} = \frac{\partial U_S}{\partial D_S} + \mu_S g'(D_S)(n-n_S) = 0 \tag{A4}$$

$$\dot{\lambda}_S = r\lambda_S - \frac{\partial U_S}{\partial n} - \lambda_S h(D_N) - \mu_S g(D_S) \tag{A5}$$

$$\dot{\mu}_S = r\mu_S - \frac{\partial U_S}{\partial n_S} + \mu_S g(D_S) \tag{A6}$$

and the usual transversality conditions.

To determine that a steady state interior equilibrium can exist and is characterized by (8) and (9), set $\dot{\lambda}_N = \dot{\mu}_N = 0$ in (A2) and (A3), solve these expressions for λ_N and μ_N, and substitute these into (A1).

Set $\dot{\lambda}_S = \dot{\mu}_S = 0$ in (A5) and (A6), solve these for λ_S and μ_S, and substitute into (A4). The resulting equations depend on t only through $D_N(t)$, $D_S(t)$, and $\theta(t)$, so a steady state equilibrium can exist if these are all constant. From (2) and (3), $\dot{\theta} = 0$ if and only if $\theta = h(D_N)/g(D_S)$ so the equations obtained are independent of t, namely (8) and (9).

It can be shown that $T_N(D_N,D_S) \gtreqless 0$ if and only if $\tilde{T}_N(D_N,D_S) \gtreqless 0$, where $\tilde{T}_N(D_N,D_S) = -\gamma\theta[r + g(D_S)] + (1-\gamma)(1+\theta)(M_N-D_N)h'(D_N)$. Then $\tilde{T}_N(0,D_S) = -(1-\gamma)M_N h'(0) > 0 > \tilde{T}_N(M_N,D_S) = -\gamma\theta[r + g(D_S)]$ for all D_S, which with the continuity of $\tilde{T}_N$ in D_N insures that there exists at least one $D_N^* \varepsilon (0,M_N)$ such that $\tilde{T}_N(D_N^*,D_S) = 0$ for any D_S. The fact that $\frac{\partial \tilde{T}_N}{\partial D_N} < 0$ when $\tilde{T}_N(D_N^*,D_S) = 0$ insures that the value D_N^* is unique, and hence that the reaction function $r_N(D_S)$ is well-defined and $r_N(D_S) \varepsilon (0,M_N)$ for all D_S since $h'(0) > 0$.

Next observe that (11) simply says $T_S(D_N,0) > 0$ for $D_N \varepsilon (0,M_N)$. This, plus the continuity of T_S in D_S and the fact that $\lim_{D_S \to M_S} T_S(D_S,D_N) = -\infty$ for all D_N, insures the existence of at least one $D_S^* \varepsilon (0,M_S)$ such that $T_S(D_N,D_S) = 0$ for any $D_N^* \varepsilon (0,M_N)$. Since $\frac{\partial T_S}{\partial D_S} < 0$ when $T_S(D_N,D_S^*) = 0$, it follows that this value of D_S^* is unique and thus $r_S(D_N)$ is well-defined for all $D_N \varepsilon [0,M_N)$, $r_S(0) = 0$, and $r_S(D_N) \varepsilon (0,M_S)$ for all $D_N \varepsilon (0,M_N)$ whenever (11) holds. To insure the existence of an interior equilibrium it is sufficient to assume that (11) holds evaluated at $(\bar{D}_N, 0)$ where $\bar{D}_N$ is defined by $T_N(\bar{D}_N,0) = 0$. (This will become obvious in the next paragraph). Finally, since it is not obvious we note here that the first order necessary conditions for the planner's problem require $\theta > (1-\gamma)\alpha(1+\theta)$ in the steady state.

Now consider the composite reaction function $f(D_S) = r_S(r_N(D_S))$. This function is well-defined and continuous for all $D_S \in [0, M_S]$. Under (11), $f(0) > 0$ since $r_N(0) \in (0, M_N)$ and (11) implies $r_S(r_N(0)) \in (0, M_S)$. Moreover, since $r_S(D_N) < M_S$ for any D_N, $f(M_S) < M_S$. Hence, there must exist at least one $D_S^* \in (0, M_S)$ such that $f(D_S^*) = D_S^*$. This proves the existence of an interior equilibrium (D_N^*, D_S^*) where D_S^* is computed as above and $D_N^* = r_N(D_S^*)$. If (11) does not hold, then the equilibrium is $(\bar{D}_N, 0)$ where $\bar{D}_N$ is defined by $T_N(\bar{D}_N, 0) = 0$. In both cases, if (10) also holds, then the equilibrium is unique and locally stable.

Note well that this existence proof does <u>not</u> invoke the inequality (10). Since $r_N'(D_S) = -\frac{\partial T_N}{\partial D_S} \Big/ \frac{\partial T_N}{\partial D_N}$ and $r_S'(D_N) = -\frac{\partial T_S}{\partial D_N} \Big/ \frac{\partial T_S}{\partial D_S}$, (10) requires that the slope of the composite reaction function be less than 1 in absolute value, or $|f'(D_S)| < 1$ for all D_S. Hence, assuming (10) insures that $f(D_S)$ is a contraction mapping, which implies that its fixed point D_S^* is unique and so the equilibrium (D_N^*, D_S^*) is unique. Assuming (10) also insures, by definition, that (D_N^*, D_S^*) is locally stable.

B. Proof of Proposition 2

By differentiating T_N with respect to D_N and D_S and using the fact that $T_N(D_N^*, D_S^*) = 0$, one can show $\frac{\partial T_N}{\partial D_N} < 0 < \frac{\partial T_N}{\partial D_S}$ in equilibrium, so $r_N'(D_S^*) > 0$. Similarly, using $T_S(D_N^*, D_S^*) = 0$ one can show $\frac{\partial T_S}{\partial D_S} < 0 < \frac{\partial T_S}{\partial D_N}$ in equilibrium and thus $r_S'(D_N^*) > 0$.

C. Proof of Proposition 3

By a standard application of the implicit function theorem, for each $x = M_N, M_S, v$

$$\frac{\partial D_N^*}{\partial x} = \left[\frac{\partial T_N}{\partial D_S}\frac{\partial T_S}{\partial x} - \frac{\partial T_S}{\partial D_S}\frac{\partial T_N}{\partial x}\right] \Big/ H \quad \text{and} \quad \frac{\partial D_S^*}{\partial x} = \left[\frac{\partial T_S}{\partial D_N}\frac{\partial T_N}{\partial x} - \frac{\partial T_N}{\partial D_N}\frac{\partial T_S}{\partial x}\right] \Big/ H$$

where $H = \frac{\partial T_N}{\partial D_N}\frac{\partial T_S}{\partial D_S} - \frac{\partial T_N}{\partial D_S}\frac{\partial T_S}{\partial D_N} > 0$ at (D_N^*, D_S^*) follows from (10) and the fact that $\frac{\partial T_N}{\partial D_S} > 0 > \frac{\partial T_N}{\partial D_N}$ and $\frac{\partial T_S}{\partial D_N} > 0 > \frac{\partial T_S}{\partial D_S}$ at (D_N^*, D_S^*). Now, since $\frac{\partial T_N}{\partial M_S} = 0$ and $\frac{\partial T_S}{\partial M_S} > 0$ at (D_N^*, D_S^*), it follows that $\frac{\partial D_N^*}{\partial M_S} > 0$ and $\frac{\partial D_S^*}{\partial M_S} > 0$. However, since $\frac{\partial T_N}{\partial M_N} > 0 > \frac{\partial T_S}{\partial M_N}$ at (D_N^*, D_S^*), we cannot determine the signs of $\frac{\partial D_N^*}{\partial M_N}$ and $\frac{\partial D_S^*}{\partial M_N}$. As noted in the text, we let $g(D_S) \equiv v + \tilde{g}(D_S)$ (where $\tilde{g}(0) = 0$ and $g' = \tilde{g}' > 0 > g'' = \tilde{g}''$) to examine the effects of a change in $v = g(0)$. Because $\frac{\partial T_N}{\partial v} > 0 > \frac{\partial T_S}{\partial v}$, the sign of $\frac{\partial D_S^*}{\partial v}$ is ambiguous. Nevertheless, using the facts that $\frac{\partial T_N}{\partial D_S} = g'\frac{\partial T_N}{\partial v}$ and $\frac{\partial T_S}{\partial D_S} - g'\frac{\partial T}{\partial v} < 0$ we can show $\frac{\partial D_N^*}{\partial v} > 0$. Since $h'(D_N) > 0$, this proves the reults stated about the equilibrium rate of innovation. And since $g'(D_S) > 0$, this proves the results stated about the equilibrium rate of technology transfer for changes in M_S or M_N. However, given that $g(D_S) = v + \tilde{g}(D_S)$, the change in the rate of transfer caused by a change in v is $\frac{\partial g(D_S^*)}{\partial v} = 1 + g'(D_S^*)\frac{\partial D_S^*}{\partial v}$. Again the facts that $\frac{\partial T_N}{\partial D_S} = g'\frac{\partial T_N}{\partial v}$ and $\frac{\partial T_S}{\partial D_S} - g'\frac{\partial T_S}{\partial v} < 0$ allows us to show $\frac{\partial g(D_S^*)}{\partial v} > 0$.

D. Proof of Proposition 4

The effect of a change in M_S is given by

$$\frac{\partial \theta^*}{\partial M_S} = \left[h'(D_N^*) \frac{\partial D_N^*}{\partial M_S} g(D_S^*) - h(D_N^*) g'(D_S^*) \frac{\partial D_S^*}{\partial M_S} \right] \Big/ g(D_S^*)^2.$$

One can show that the sign of this expression is the same as that of

$$h'g \frac{\partial T_N}{\partial D_S} + hg' \frac{\partial T_N}{\partial D_N} = -\gamma\theta g h' g' - (1-\gamma)(\theta+1) h h' g' + (1-\gamma)(\theta+1)(M_N - D_N) h'' < 0, \text{ so}$$

$\frac{\partial \theta^*}{\partial M_S} < 0$. Similarly,

$$\frac{\partial \theta^*}{\partial v} = \left[(h'(D_N^*) \frac{\partial D_N^*}{\partial v} g(D_S^*) - h(D_N^*) \frac{\partial g(D_S^*))}{\partial v} \right] \Big/ g(D_S^*)^2 < 0$$

can be shown by noting that $\frac{\partial D_N^*}{\partial v} = -\frac{\partial T_N^*}{\partial v} \frac{\partial g(D_S^*)}{\partial v} \frac{\partial T_N^*}{\partial D_N}$, so that $\frac{\partial \theta^*}{\partial v} < 0$ if and only if $h' \frac{\partial T_N}{\partial v} + \theta \frac{\partial T_N}{\partial D_N} < 0$, and then showing that $h' \frac{\partial T_N}{\partial v} + \theta \frac{\partial T_N}{\partial D_N}$

$= [\gamma + (1-\gamma)(\theta+1)]\theta h' + (1-\gamma)\theta(\theta+1)(M_N - D_N)h'' < 0.$

Remark: Often in the discussions of Propositions 3 and 4 we refer to results which would obtain in the case where the South is passive. As noted above, the equilibrium in this case is $(\bar{D}_N, 0)$, where $T_N(\bar{D}_N, 0) = 0$, and the technology gap is $\bar{\theta} = h(\bar{D}_N)/v$. Since $\frac{\partial T_N}{\partial D_N} < 0$, $\frac{\partial T_N}{\partial M_S} = 0$, $\frac{\partial T_N}{\partial M_N} > 0$, and $\frac{\partial T_N}{\partial v} > 0$ when $T_N(\bar{D}_N, 0) = 0$, it follows that $\frac{\partial \bar{D}_N}{\partial M_S} = \frac{\partial \bar{\theta}}{\partial M_S} = 0$, $\frac{\partial \bar{D}_N}{\partial M_N} > 0$ and $\frac{\partial \bar{\theta}_N}{\partial M_N} = h' \frac{\partial \bar{D}_N}{\partial M_N} v > 0$, and $\frac{\partial \bar{D}_N}{\partial v} > 0$. However, $\frac{\partial \bar{\theta}}{\partial v} = (h' \frac{\partial \bar{D}_N}{\partial v} v - h)/v^2 < 0$ since, as noted above for the case of $\frac{\partial \theta^*}{\partial v}$, $h' \frac{\partial T_N}{\partial v} + \theta \frac{\partial T_N}{\partial D_N} < 0$.