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Department of Economics
The University of Michigan
Ann Arbor, Michigan 48109-1220

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A DECISION THEORETIC MODEL OF INNOVATION, TECHNOLOGY TRANSFER, AND TRADE

by

Richard Jensen* and Marie Thursby**

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Abstract

We analyze a dynamic North-South model of innovation, technology transfer, and trade. Northern firms conduct R&D using labor which has alternative uses producing in the R&D sector or a nontraded good sector. Since technology transfer prevents the North from fully appropriating benefits of R&D, the optimal rate of innovation for either profit maximizing firms or a utility maximizing Northern planner is less than globally optimal. An increased transfer rate intensifies competition of lower wage Southern workers with Northern workers in production, so profit maximizing Northern firms (irrespective of their number or cooperation in R&D) reallocate labor toward R&D.

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*University of Kentucky
**Ohio State University and The University of Michigan

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1. Introduction

One of the major contributions to the industrial organization literature in the past two decades has been the treatment of innovation as a type of production process in which resources must be devoted to R&D in order to develop an innovation.[1] It is somewhat surprising, then, that this aspect of innovative behavior has been largely ignored in theoretical models of innovation, technology transfer, and international trade (see Feenstra and Judd (1982), Pugel (1982), Spencer and Brander (1983), Cheng (1984) and Jensen and Thursby (1986) for exceptions). In this paper we incorporate this notion of R&D into a dynamic North-South trade model where the pattern of trade is similar to Vernon's (1966) product life cycle. Innovation in the model is new product development which can be accomplished only by a Northern industry devoting labor to R&D. Initially this industry has a monopoly position in production of the new goods, but after some time a lower wage competitive Southern economy learns to produce these goods so that a product cycle trade pattern emerges.

The Northern industry is assumed to be imperfectly competitive, so that firms develop new products in order to maximize a discounted stream of positive profits. Development requires labor input, and the productivity of this input reflects learning-by-doing in product development. The incentive for firms to devote this input to product development depends both on competition in the domestic market in the North and, in an open economy, on foreign (Southern) competition. In this model, foreign firms are incapable of developing new products, but once they learn how to produce the newly developed Northern products, lower wages allow them to take over the market. The virtues to this model are that it conforms (albeit loosely) to stylized facts and it allows us to examine the interaction between the rate at which the technology for producing new products transfers to the South and the rate of innovation which is *optimal* for Northern industry. This is an important aspect of the product cycle since the transfer of technology to the South is an example of the well-known appropriability problem in innovation (Arrow (1962)). It has not been analyzed in most models of the product cycle (Dudley (1974), Krugman (1979), Dollar (1986))

[1]See Kamien and Schwartz (1982) for discussion and additional references.

because, with the exception of Feenstra and Judd (1982) and Jensen and Thursby (1986), the rate of innovation is not chosen optimally by profit maximizing firms.[2] Moreover, recent studies examining the impact of cooperative research ventures as a solution to the appropriability problem have been in a partial equilibrium framework and have abstracted from international aspects of the problem (Ruff (1969),[3] Katz (1984), Spence (1984), Grossman and Shapiro (1985), and Ordover and Willig (1985)). An important feature of our general equilibrium model is the inclusion of a competitive nontraded good sector in the North. This allows us to examine the effect of technology transfer not only on R&D, *per se*, but also on the size of the R&D sector within the economy.

The major results of the analysis are (1) the profit maximizing rate of innovation for Northern firms is lower than the rate which is socially optimal for either the North or the world; (2) the socially optimal rate of innovation from the North's point of view is lower than the rate of innovation which would maximize world or Southern welfare; and (3) profit maximizing behavior of Northern firms is associated with an increase in the size of the R&D sector in the North whenever the rate of technology transfer increases. These results are shown to hold irrespective of the number of firms in the Northern industry and whether or not these firms cooperate in R&D. The first two results occur because of the appropriability problem and because the South bears none of the resource cost of innovation. The South benefits from R&D both in terms of consumption of products it does not yet know how to produce and in terms of increased income once it learns the technology to produce new products. The intuition behind the third result is that, because an increased rate of technology transfer reduces the potential profits from sale of any given product, firms have an incentive to reduce the amount of production of any given product. But because consumer's utility (in this as well as other product cycle models) is increased

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[2]Feenstra and Judd incorporate the resource cost of R&D in an optimizing framework, but they model technology transfer as the sale of technology so that they abstract from appropriability issues, and their model is static so that they cannot examine how the timing of transfer affects the Northern industry's choice.

[3]Ruff's model is general equilibrium, but his economy is closed.

by greater variety of new products, firms attempt to offset this loss in potential profits by increasing expenditure on R&D. In this model the latter effect is more important so that the size of the R&D sector within the economy increases.

A related result of interest is that increased R&D associated with an increased transfer rate may or may not prevent a deterioration in the Northern terms of trade. In Krugman's original model, Northern terms of trade deteriorated with an increase in the transfer rate because the rate of innovation was fixed. Here the rate of innovation increases, and whether or not the terms of trade decline depends on magnitudes of the production and development effects. Our inclusion of the nontraded good sector is thus important since these effects depend, in part, on the opportunity cost of labor in terms of foregone production of the nontraded good. Finally, the Cournot industry analysis allows us to examine spillover effects within the North.

2. The Basic Model

Assume a world with two economies, called North and South. There are two sectors in the North, an imperfectly competitive traded good sector which develops and produces new goods and a perfectly competitive sector which produces a nontraded good. Northern consumers have identical homothetic preferences over these goods given by

$$U_N = c_{nt}^{\gamma} \int_0^n c_N(\eta)^{\gamma} d\eta, \quad 0<\gamma<1 \tag{1}$$

where U_N is Northern utility, n is the number of traded goods available for consumption, $c_N(\eta)$ is the amount of the ηth traded good consumed, and c_{nt} is consumption of the nontraded good. (1) is increasing in the number of traded goods (for given income and prices), and $1/(1-\gamma)$ is the elasticity of substitution among traded goods. The equation differs from those of Krugman, Feenstra and Judd, Dollar, and Jensen and Thursby by the inclusion of the nontraded good, which allows us to examine the resource cost of developing a new good not only in terms of foregone consumption of other R&D-related goods, but also in terms of another type of good. The form of (1) implies an elasticity of substitution between a traded and nontraded good of unity, and it guarantees that with free trade production in the North will be incompletely specialized. The South is perfectly competitive, and its consumers have identical, homothetic preferences given by

$$U_S = \int_0^n c_S(\eta)^\gamma d\eta. \quad 0<\gamma<1 \tag{2}$$

where U_S is Southern utility.

Labor is the only factor used to develop and produce traded goods. In order to focus on technologically related trade, we assume a worker in either country can produce one unit of any traded good, assuming the technology for producing it is known by the worker. The essential difference between the two is that only Northern workers can develop new goods, where new goods are defined as newly developed traded goods whose production technology is known only in the North. Old goods are those traded goods whose production technology is common knowledge. Initially all traded goods are old goods. If development occurs, then the stocks of new and old goods are determined over time by the rates of innovation and technology transfer. Innovation refers to the development of new goods and technology transfer to the South learning the technology required to produce new goods. No technical change in the form of increased productivity in production occurs in either the traded or nontraded good sectors. The nontraded good is produced by labor and a fixed factor (e.g. land). $\phi(L_{nt})$ denotes output of the nontraded good as a function of labor, where $\phi(0)=0$, $\phi'>0$, and $\phi''<0$ because of the fixed factor. These assumptions allow us to incorporate wage effects of hiring decisions in the R&D sector.

In this section we assume a monopolist in the R&D sector. This allows us to focus on the appropriability problem between the two countries without the complication of possible spillover effects of R&D within the North. Those effects will be considered in Section 4 where we assume a Cournot oligopoly in the North. In the simpler case of monopoly, the innovation process is given by

$$\dot{n}(t) = h[D_m(t)]n(t), \; n(0) = n_0 > 0 \tag{3}$$

where a dot over a variable indicates a derivative with respect to time t, $D_m(t)$ is the number of workers used by the monopolist in R&D at t, and the function h is assumed to be twice continuously differentiable, satisfy $h(0)=0$, $h'>0>h''$, and be bounded above by the social rate of discount, r. Three characteristics of this specification merit discussion. First, it models innovation as a production process in which the output is the instantaneous rate of increase in new goods and the inputs are labor in R&D and the current total number of traded goods. The marginal products of both inputs are positive, with that of labor decreasing and that of the total number of traded goods constant. It is impossible to develop any new goods without devoting some resources to R&D. Given that some labor is put into R&D, innovation is proportional to the number of traded goods currently in existence. Recalling that only the North develops new goods, this says that the more new goods have been developed, the more new goods can be developed at any given date by a fixed supply of Northern workers. This characteristic of our specification of innovation simply assumes there is learning-by-doing in the development of new goods.[4] Finally, we also assume that innovation cannot be too rapid in the sense that the rate of growth of the total number of traded goods ($h=\dot{n}/n$) cannot exceed the social rate of discount in the North. This assumption is needed to insure the convergence and existence of the objective functionals examined.

Once the monopolist has developed and exported new goods, the process by which the South learns the technology required to produce these goods is given by

$$\dot{n}_S(t) = vn_N(t),\ n_S(0) = n_0 \tag{4}$$

where v is a constant fraction and n_N is the number of Northern goods exported. This specification is used by Krugman (1979) and can be justified in terms of "reverse engineering," where Southern workers eventually learn how to produce imported goods by dismantling and studying them. This type of process has been shown to account for a substantial amount of technology transfer (Mansfield and Romeo (1980) and Mansfield, et al. (1982)). It seems

[4]See Arrow (1962) and Fellner (1969) and references therein for discussion of learning by doing in production processes.

reasonable that the more new goods there are, the greater the South's opportunity, and thus its ability, to learn to produce new goods through reverse engineering. Since (4) implies that the number of goods which the South learns to produce at any date is positively related to the number of goods available to them, it can be viewed as a formalization of the process of reverse engineering. One could argue that the transfer rate should be endogenized, but doing so would unnecessarily complicate the model and obscure comparative dynamics results. Jensen and Thursby (1986) endogenize the rate of transfer in a simpler model with only traded goods and a Northern monopolist. Dollar (1986) also endogenizes the rate of transfer in a competitive model with only traded goods by arbitrarily assuming that it is a function of the relative wage.

The form of the utility functions and the unitary marginal product of labor in producing traded goods insure that all goods produced by the monopolist sell at the same price, P_N, and all Southern goods sell at the same price, P_S. Perfect competition and full employment in the South insure $P_S=w_S$ where w_S is the wage of a Southern worker. Full employment in the North and perfect competition in the nontraded good sector imply the supply of labor facing the monopolist can be written as $w_N(D_m+L_m)\equiv\phi'(M_N-D_m-L_m)$ where w_N is the Northern wage, M_N is the amount of labor in the North, L_m is the amount of labor employed by the monopolist in production, ϕ' is the marginal product of labor in producing the nontraded good, and the price of the nontraded good is set to one. Finally, we assume $w_N'=-\phi''>0$, so that if new product development and trade occur, the terms of trade $P=P_N/P_S$, will exceed the relative wage, $w=w_N/w_S$.

Following Krugman we assume $w>1$, so that as soon as the South learns the technology to produce a traded good it steals the monopolist's market. Hence the number of traded goods produced by the North is the number of new goods, $n_N=n-n_S$, and the number of goods produced by the South is the number of old goods, n_S. The balance of payments in addition to our other assumptions imply that the terms of trade and wage can be written as

$$P=[\theta M_S/L_m]^{1-\gamma} \quad \text{and} \tag{5}$$

$$w = \phi'/P_S = PL_m \phi'/\phi \tag{6}$$

where M_S is amount of Southern labor and θ is the ratio of new to old goods, n_N/n_S. (5) differs from Krugman and Dollar's expressions for the terms of trade because it takes into account the fact that the development of new goods requires the use of workers who must be taken out of current production of other goods. (6) is derived in the Appendix and differs from wage expressions in the existing literature because of the imperfect competition in the R&D (traded good) sector and the fact that Northern labor has alternative uses in producing nontraded as well as traded goods.

The monopolist's profit per period is

$$\pi_m = PL_m - w(L_m + D_m) \tag{7}$$

and its problem is to choose $L_m(t)$ and $D_m(t)$ for all t to maximize $\int_0^\infty e^{-rt}\pi_m(t)dt$ subject to the innovation process (3), the technology transfer process (4), and $L_{nt} = M_N - D_m - L_m$. The current value Hamiltonian (see Section 8 of Part II of Kamien and Schwartz (1981) for a definition and discussion) is

$$H_m = \pi_m + \lambda_m h(D_m)n + \mu_m v n_N \tag{8}$$

and the first order necessary conditions are

$$\partial\pi_m/\partial L_m = 0 \tag{9}$$

$$(\partial\pi_m/\partial D_m) + \lambda_m h'(D_m)n = 0 \tag{10}$$

$$\dot{\lambda}_m = r\lambda_m - (\partial\pi_m/\partial n) - h(D_m)\lambda_m - \mu_m v \tag{11}$$

$$\dot{\mu}_m = r\mu_m - (\partial\pi_m/\partial n_S) + \mu_m v \tag{12}$$

and the usual transversality conditions (again see Kamien and Schwartz). Since the necessary conditions for this problem are not informative, we confine our analysis to the steady state.

Differentiating π_m with respect to L_m shows that the necessary condition for choice of L_m in the steady state is (9) written as

$$\gamma P - \gamma P\phi'\phi^{-1}(L_m+D_m) - w[1+\delta(L_m+D_m)] = 0 \tag{13}$$

where $\delta=(\phi'/\phi)\text{-}(\phi''/\phi')>0$ and ϕ', ϕ^{-1}, and δ are evaluated at $L_{nt}=M_N-D_m-L_m$.

We must have $\dot{\lambda}_m=0$ and $\dot{\mu}_m=0$ in a steady state, which with (11) and (12) implies $\lambda_m=\left[(\partial\pi_m/\partial n)+(\frac{v}{r+v})\,(\partial\pi_m/\partial n_S)\right]/[r-h(D_m)]$. Substituting this into (10) and computing all relevant derivatives (using (5), (6), and (7)) shows that the necessary condition for optimal choice of D_m in the steady state can be written as

$$\text{-}w[1+\delta(L_m+D_m)] + \frac{(1-\gamma)h'(D_m)(1+\theta)\{rPL_m[1-\phi'\phi^{-1}(L_m+D_m)]-v\theta\pi_m\}}{(r+v)\theta[r-h(D_m)]} = 0 \tag{14}$$

where, again, ϕ', ϕ^{-1}, and δ are evaluated at $L_{nt}=M_N-D_m-L_m$.

Dividing (13) by P and (14) by PL_m simplifies the necessary conditions to

$$H^L_m(L_m,D_m) = \gamma[1-\phi'\phi^{-1}(L_m+D_m)] - L_m\phi'\phi^{-1}[1+\delta(L_m+D_m)] = 0 \tag{15}$$

$$H^D_m(L_m,D_m) = \frac{(1-\gamma)h'(D_m)(1+\theta)}{(r+v)\theta}[1-\phi'\phi^{-1}(L_m+D_m)] - \phi'\phi^{-1}[1+\delta(L_m+D_m)] = 0. \tag{16}$$

The condition $H^L_m(L_m,D_m)=0$ has the standard interpretation that the Northern monopolist hires workers to produce new goods up to the point where the marginal revenue product of the last one hired is equal to the marginal expense of labor in production. The condition $H^D_m(L_m,D_m)=0$ says to hire workers for R&D up to the point where the increase in the present discounted value of the stream of future profits from the last one hired is equal to the marginal expense of labor in R&D. To ensure that the marginal expense of labor in the production or development of new goods is increasing in L_m or D_m, we shall assume $\partial\delta/\partial D\geq 0$ and $\partial\delta/\partial L\geq 0$ throughout.

Proposition 1: Under our assumptions there exists a unique and stable steady state equilibrium characterized by a positive rate of Northern innovation, $h(D^*_m)$, a positive ratio of new to old goods, $\theta_m^* = h(D_m^*)/v$, and positive amounts of both traded and nontraded goods produced by the North, L_m^* and $\phi(M_N - D_m^* - L_m^*)$, where D_m^* and L_m^* are defined by $H_m^L(L_m^*, D_m^*) = 0$ and $H_m^D(L_m^*, D_m^*) = 0$.

Proof: Since (13) and (14) (and hence (15) and (16)) depend on t only through D_m, L_m, and θ, we must show that constant D_m and L_m imply constant θ in order to prove that a steady state can exist (i.e., that (13) and (14) can be written as autonomous equations). However, since $\dot{\theta} = (1+\theta)[h(D_m) - v\theta]$, it follows that $\dot{\theta} = 0$ if and only if $\theta = h(D_m)/v$, which is constant whenever D_m is. To prove that $L_m^* + D_m^* < M_N$, note that $-\phi'(M_N - D_m - L_m)\phi^{-1}(M_N - D_m - L_m)$ approaches -∞ as $L_m + D_m$ approaches M_N while the remaining terms in (13) and (14) are finite. To prove $L_m^* > 0$, note that P approaches +∞ as L_m approaches 0, so the left hand side of (13) approaches +∞ as L_m approaches 0 for any $D_m < M_N$. To show $D_m^* > 0$, multiply (14) by θ and note that the left hand side of this new equation is positive as D_m approaches 0 as long as $h'(0) > 0$. This plus the continuity of (13) and (14) prove the existence of L_m^* and D_m^* such that (13) and (14) hold simultaneously. In order to do comparative dynamics, we shall assume that the matrix of second partial derivatives of H_m^L and H_m^D is negative definite, which also ensures the uniqueness and local stability of (L_m^*, D_m^*).

Implicit in equations (14) and (16) is the fact that in the steady state the ratio of new to old goods, θ_m^*, must be the constant $h(D_m^*)/v$. Accordingly the steady state values of the rate of innovation, employment and output in each sector in the North, the terms of trade, and the relative wage will be constant. The utility functions insure that the nontraded good sector in the North does not vanish. The additional assumption of no threshold effects in R&D ($h'(0) > 0$) insures that new goods are developed and produced in the steady state.

Note that (15) depends on L_m and D_m, but not θ. Solving (15) for $\hat{L}_m(D_m)$ and substituting into (16) allows us to depict the steady state in D_m-θ space. In Figure 1, the function $\theta = h(D_m)/v$ shows all combinations of D_m and θ such that $\dot{\theta} = (1+\theta)[h(D_m) - v\theta] = 0$, while the function $\theta = g(D_m)$ show combinations of D_m and θ such that $H_m^D(\hat{L}_m, D_m) = 0$. The intersection of these two determine the steady state values of D_m^* and $\theta_m^* = h(D_m^*)/v$, and the arrows indicate the path by which D_m and θ return to the steady state if there is a marginal displacement from it.

To compare D_m^* and L_m^* to the socially optimal allocation from the North's perspective, consider the problem of a Northern planner who chooses labor in development D(t) and labor in production of new goods L(t) to maximize the present value of Northern utility. The planner will allocate L/n_N workers to the production of each new good since the demand and cost conditions for each new good are the same. In particular, D(t) and L(t) are chosen to maximize $\int_0^\infty e^{-rt}U_N(t)dt$ subject to (3) and (4) where U_N is given by (1) evaluated at $c_{nt} = \phi(M_N - D - L)$, $c_N(\eta_N) = \alpha L/n_N$, $c_N(\eta_S) = \alpha M_S/n_S$, $\alpha = PL/(PL+M_S)$, and $P = [n_N M_S/n_S L]^{1-\gamma}$. Following the same procedure used to determine the steady state for the monopoly, we find the planner's steady state is characterized by $L^*>0$, $D^*>0$, $L^*+D^*<M_N$, and $\theta^* = h(D^*)/v$. Now L^* and D^* are implicitly defined by $H_N^L(L^*,D^*)=0$ and $H_N^D(L^*,D^*)=0$ where

$$H_N^L(L,D) = [1-(1-\gamma)(1-\alpha)]/L - \phi'\phi^{-1}, \tag{17}$$

$$H_N^D(L,D) = \frac{(1-\gamma)h'(D)(1+\theta)}{\gamma(r+v)\theta[r-h(D)]}[r-(1-\gamma)(1-\alpha)(r-h(D))] - \phi'\phi^{-1}, \tag{18}$$

and $\theta = h(D)/v$. Multiplying these by PL shows that in the planner's steady state optimum D^* and L^* satisfy

$$P[1-(1-\gamma)(1-\alpha)] - w^* = 0 \tag{19}$$

$$PL^*\frac{(1-\gamma)h'(D^*)(1+\theta^*)}{\gamma(r+v)\theta^*[r-h(D^*)]}[r-(1-\gamma)(1-\alpha)(r-h(D^*))] - w^* = 0 \tag{20}$$

where $w^* = PL^*\phi'(M_N - D^* - L^*)\phi^{-1}(M_N - D^* - L^*)$. Hence the planner allocates workers to the traded goods sector up to the point where the marginal contribution to the present value of Northern utility of the last worker in development and in production of new goods equals the wage w. From (13) and (14) recall that the monopolist hires workers up to the point where the marginal contribution to the present value of profit of the last workers in development or production of new goods equals the marginal expense of labor $w+w\delta(L+D)>w$. The essential difference is that the Northern planner will not exploit labor at all, and, in addition, will take into account utility from traded goods even after technology is transferred and they are old goods. Since the firm does not gain after technology transfer, it follows that it will hire fewer workers for both development and production. That is, assuming the matrix of second partial derivatives of H_N^D and H_N^L is negative definite (so a unique maximum exists and comparative dynamics can be done) it follows that $D^*>D_m^*$ and $L^*>L_m^*$.

Since Southern consumers get no utility from Northern nontraded goods, a Southern planner would allocate Northern workers to the development and production of new goods up to the point where the marginal contribution to the present value of Southern utility of the last worker in each activity are equal (i.e., all M_N workers are allocated between development and production of new goods such that the contribution at the margin is equal in both activities). Hence D^* and L^* must be lower than those which maximize Southern utility. A global planner whose objective is to maximize a weighted average of Northern and Southern utility will then choose D and L less than those the South prefers, but greater than those which maximize Northern utility or the monopolist's profit. Since Southerners bear none of the cost of R&D, they will prefer a higher rate of innovation than either a Northern planner or monopolist would choose. Alternatively put, given the inability to fully appropriate the benefits from its R&D, any Northern agent will underinnovate from a world point of view.

These results for the Northern and Southern planners prove the following proposition.

Proposition 2: Under our assumptions

(i) from both the North's and South's perspectives, the profit maximizing monopolist will innovate less than is socially optimal; and

(ii) the optimal rate of innovation from the North's perspective is lower than the rate which would maximize world or Southern utility.

3. Acceleration in Technology Transfer

Like other models of the product life cycle, this model predicts a trade pattern in constant flux. The North develops new goods and exports them until the technique for producing them becomes common knowledge, at which point it imports them from the South. The North's absolute advantage in developing new goods allows $w>1$; but the transfer of technology dictates that to maintain this wage, the North must continually innovate. In such a world it is natural for the North to be concerned when the South's ability to learn the new technology improves. For a constant rate of innovation, it is clear that the Northern wage and terms of trade decline with an exogenous increase in the rate of transfer (Krugman (1979), Dollar (1986)). The question the current model allows us to address is what happens to the profit maximizing rate of innovation when the transfer rate increases. In addition, the inclusion of the nontraded good sector allows us to examine the effect of an increased transfer rate on the size of the R&D (traded good) sector within the North.

Proposition 3: An increase in the transfer rate increases the steady state employment of labor in R&D and the rate of innovation. Employment of labor in production of new traded goods decreases, nonetheless total employment in the R&D sector increases. The effects on the ratio of new to old goods, the terms of trade, the wage, and the rate of innovation which is socially optimal for the North are ambiguous.

To see the effect of an accelerated transfer rate on D_m^* recall that an increase in v affects the monopolist's discounted future profits in two ways: it reduces the profitability of the last worker in R&D through $(r+v)$ in (16), and it increases it through the term $(1+\theta)/\theta$ in (16). As shown in the Appendix, the latter effect outweighs the former, so that D_m^* increases. The resulting increase in the marginal expense of labor in production of traded goods leads to a decrease in L_m^*, but this reduction is not enough to outweigh increased employment in R&D because direct effects outweigh indirect effects in this firm model. A Northern planner maximizing utility has the option of offsetting any reduction in utility caused by an increase in v by moving resources into the nontraded good sector. This extra degree of freedom not available to the profit maximizing monopolist leads to the ambiguous effect of a change in v on the socially optimal rate of innovation.[5]

The ambiguous effect of an increase in v on θ_m^* is most easily seen with reference to Figure 1. The increase in v shifts both the $\dot{\theta}=0$ and $g(\cdot)$ schedules to the right so that we know D_m^* increases, but without knowing the magnitudes of the two shifts we cannot tell whether θ_m^* increases or decreases. This ambiguity raises the possibility that the Northern terms of trade may not decrease with an increased transfer rate in a model with endogenous innovation. Recalling (5) and the fact that $\partial L_m^*/\partial v < 0$, the North's terms of trade must increase whenever θ_m^* does not decrease. If θ_m^* decreases, a sufficient condition for P^* to increase is that the elasticity of the demand for labor in production of traded goods with respect to the transfer rate be greater than the elasticity of the new to old good ratio with respect to the transfer rate (i.e., $-(v/L_m^*)(\partial L_m^*/\partial v) > -(v/\theta_m^*)(\partial \theta_m^*/\partial v)$). Finally, the increase in D_m^* and the decline in L_m^* imply that any decline in P^* will be less than the decline predicted with a fixed rate of innovation.

4. A Cournot Traded Good Industry

Given the vast R&D literature devoted to the effects of market structure on the rate of innovation and the popularity of oligopoly models in international trade,[6] it is natural to ask how firm rivalry within the North would affect new product development, technology transfer, and trade. In this section we assume there are an arbitrary number K of firms, each of which develops its own line of new traded goods. The goods in the product line of a given firm are differentiated from those of any other firm, but all traded goods are assumed to enter the utility

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[5]Jensen and Thursby (1985) show that with full employment, no nontraded good, and a CES utility function, a utility maximizing planner would increase the allocation of labor to R&D with an increase in v.

[6]On market structure and innovation see Kamien and Schwartz (1982) and references therein; for oligopoly in international trade see Spencer and Brander (1983) and Dixit (1984).

function of a representative consumer in the same fashion. Letting k index both the firms and their product lines, the utility function of every Northern consumer is now

$$U_N = c_{nt}^{\gamma}\left[\sum_{k=1}^{K} \int_0^{n_k} c_N(\eta_k)^{\gamma} d\eta_k\right] \tag{21}$$

and that of each Southern consumer is

$$U_S = \sum_{k=1}^{K} \int_0^{n_k} c_S(\eta_k)^{\gamma} d\eta_k \tag{22}$$

The fundamental properties of these utility functions are the same as those used in the analysis of a Northern monopoly. There are two reasons for allowing each firm to develop its own product line. One is that it loosely corresponds to the realistic notion that a firm would rather develop its own goods over which it has monopoly power than develop exactly the same goods its competitors do. The other is that it avoids certain nondifferentiability problems that would arise if the firms tried to develop the same new goods at different rates.[7]

Although the firms are developing different product lines, it is plausible that their R&D efforts are interrelated by an unavoidable transmission of information. For example, workers may exchange experience or ideas at formal or informal gatherings or even move between firms. This transmission of information is a spillover effect of R&D which seems quite common in practice. One natural way to model this spillover effect is to specify the "effective" development effort of a given firm as a function of its own development and that of its competitors. We define the effective research effort of firm k at time t as

$$E_k(t) = D_k(t) + \sum_{j \neq k} \beta_{kj} D_j(t) \tag{23}$$

where $D_k(t)$ is the number of workers hired by firm k for R&D and β_{kj} is the spillover coefficient determining the extent to which the research conducted by firm j enhances the level of

[7]If firms try to develop the same new goods, then each firm's profit as a function of the number of goods it can produce will be nondifferentiable at the point where it takes a lead over other firms (i.e. at the maximum number of goods all other firms can produce).

development actually attained by k (where $\beta_{kj}\epsilon[0,1]$ for all k,j). The actual rate of innovation for firm k is then

$$\dot{n}_k(t) = h[E_k(t)]n_k(t),\ n_k(0) = n_0. \tag{24}$$

This specification therefore allows for different rates of innovation and different product lines among firms.

Now the ability of the South to learn how to produce new goods may vary among product lines since the goods in one firm's line may be easier to reverse engineer than those in another firm's line. Thus we specify the technology transfer process for a given product line k as

$$\dot{n}_{Sk}(t) = v_k[n_k(t)-n_{Sk}(t)],\ n_{Sk}(0) = n_0 \tag{25}$$

where $v_k\epsilon(0,1)$ is the transfer rate for line k. Hence, even if the rate of innovation is the same for all product lines, different transfer rates can lead to different new-to-old good ratios $\theta_k(t)=n_{Nk}(t)/n_{Sk}(t)$ for each k, where $n_{Nk}(t)=n_k(t)-n_{Sk}(t)$.

Given any vector of old goods $(n_{S1},...,n_{Sk})$, the fraction of all old goods which belong to line k is $n_{Sk}/\sum_{j=1}^{K} n_{Sj}$ and so the total production of old goods in line k in the South is $n_{Sk}M_S/\sum_{j=1}^{K} n_{Sj}$. Total Southern production of any one old good in line k is thus $(n_{Sk}M_S/\sum_{j=1}^{K} n_{Sj})/n_{Sk} = M_S/\sum_{j=1}^{K} n_{Sj}$, which is the same for all k (i.e., $M_S/\sum_{j=1}^{K} n_{Sj}$ is the amount of any old good produced in the South). If L_k is the amount of labor hired by firm k to produce the n_{Nk} new goods in its line, then L_k/n_{Nk} is the amount produced of each new good in its line. The relative price (terms of trade) for each new good in line k is therefore

$$P_k = [(M_S/\sum_{j=1}^{K} n_{Sj})/(L_k/n_{Nk})]^{1-\gamma} = [n_{Nk}M_S/L_k\sum_{j=1}^{K} n_{Sj}]. \tag{26}$$

Letting $D=\sum_{k=1}^{K} D_k$ and $L=\sum_{k=1}^{K} L_k$, it then follows from full employment in the North and perfect competition in the nontraded goods sector that the relative wage of a Northern worker is now

$$w = \Big(\sum_{k=1}^{K} P_k L_k\Big)\phi^{-1}(M_N - D - L)\phi'(M_N - D - L). \tag{27}$$

The easiest way to see this is to recall (6) and note that now total revenue from the sale of all new goods is $\sum_{k=1}^{K} P_k L_k$.

The profit per period of firm k is then

$$\pi_k = P_k L_k - w(L_k + D_k) \tag{28}$$

and its problem is to choose $L_k(t)$ and $D_k(t)$ for all t to maximize $\int_0^\infty e^{-rt}\pi_k(t)dt$ subject to the innovation process (24) and the technology transfer process (25). We confine our analysis to the steady state Nash equilibrium in open-loop strategies, primarily to insure the analysis is tractable. The general specification of this industry model not only allows for asymmetric equilibria, but also makes them more likely than a symmetric equilibrium. Nevertheless, we can provide conditions sufficient to ensure that the equilibrium is symmetric. This is particularly useful since it allows us to provide comparative dynamics results. Before doing so, however, it is worthwhile to note several necessary conditions which any steady state equilibrium must satisfy.

As before, existence of a steady state requires that $\dot{\theta}_k = (1+\theta_k)[h(E_k) - v_k\theta_k] = 0$ for all k. However, it is now also necessary that the ratio $\sigma_k(t) = n_{Nk}(t)/\sum_{j=1}^{K} n_{Sj}(t)$ also be constant in the steady state (recall the expression for P_k). Since $\dot{\sigma}_k = \sigma_k[v_k\theta_k - \sum_{j=1}^{K} v_j\sigma_j]$, it follows that $h(E_k) = v_k\theta_k = \sum_{j=1}^{K} v_j\sigma_j$ for all k is necessary in a steady state in which all K firms operate. Since the term $\sum_{j=1}^{K} v_j\sigma_j$ is the same for every k, we must have $h(E_k) = v_k\theta_k = h(E_i) = v_i\theta_i$ for any firms i and k in such a steady state. This proves the following result.

Proposition 4: If there exists a steady state Nash equilibrium in open-loop strategies in which all firms operate, then it is necessary that the effective research effort of every firm is the same and the rate of innovation in every product line is the same.

The intuition for this result comes from observing that P_k is a function of $\sigma_k = n_{Nk} / \sum_{j=1}^{K} n_{Sj}$. Suppose that the rate of innovation of firm 1 is greater than that of any other firm. Due to technology transfer, this implies that the number of old goods of line 1 will eventually become large enough to drive σ_k to 0 for all $k \neq 1$. This drives P_k toward 0 for $k \neq 1$ and forces these other firms out of the new goods market.

The description of the steady state equilibrium is completed by specifying the optimal choices of labor in the production of new goods, L_k^*, and in R&D, D_k^*, for each firm. Using standard techniques these choices can be shown to require choosing L_k^* and D_k^* to satisfy $H_k^L(L_k^*, D_k^*) = 0$ and $H_k^D(L_k^*, D_k^*) = 0$ where

$$H_k^L(L_k, D_k) = \gamma P_k - \gamma P_k \phi^{-1} \phi'(L_k + D_k) - w[1 + \delta(L_k + D_k)] \tag{29}$$

$$H_k^D(L_k, D_k) = \frac{(1-\gamma)h'(E_k)(1+\theta_k)\{rP_k L_k[1 - \phi^{-1} \phi'(L_k + D_k)] - v_k \sigma_k \pi_k\}}{[r - h(E_k)](r + v_k)\theta_k} - w[1 + \delta(L_k + D_k)] . \tag{30}$$

Equations (29) and (30) are interpreted for an oligopolist as (15) and (16) for the monopolist.

To see that employment of firms can differ in the steady state consider a duopoly ($K=2$) example. If $v_1 = v_2 = v$, then $n_{S1} = n_{S2}$ as well as $n_1 = n_2$, so we must have $\theta_1 = \theta_2$ and $\sigma_1 = \sigma_2$. However, since $E_1 = E_2$ implies $D_1 = (1-\beta_{12})D_2/(1-\beta_{21})$, we have $D_1 = D_2$ if and only if $\beta_{12} = \beta_{21}$. Suppose instead $0 < \beta_{12} < \beta_{21} < 1$, so firm 2 gains more from spillovers in R and D than firm 1. Then it is necessary that $D_1 > D_2$, or 1 must hire more workers for R&D to attain the same effective research effort as 2. In fact, if $\beta_{12} = 0$ and $\beta_{21} = 1$, then $E_1 = E_2 > 0$ requires that $D_2 = 0$ and $E_1 = E_2 = D_1 > 0$. If 2 can learn everything about 1's R&D but 1 learns nothing from 2, then 2 can free ride completely on 1's R&D. Now suppose $\beta_{12} = \beta_{21} < 1$ but $v_1 > v_2$. Then $E_1 = E_2$ implies $D_1 = D_2$, but $v_1 > v_2$ implies $n_{S1} > n_{S2}$ and so $\theta_1 < \theta_2$ and $\sigma_1 < \sigma_2$. Suppose L_2 and D_2 satisfy $H_2^L(L_2, D_2) = H_2^D(L_2, D_2) = 0$. Since $\sigma_1 < \sigma_2$, we must have

$(\sigma_1 M_S/L_2)^{1-\gamma} < (\sigma_2 M_S/L_2)^{1-\gamma}$ which implies $H_1^L(L_2,D_1) = H_1^L(L_2,D_2) < 0$. Because $\partial H_k^L/\partial L_k < 0$, this implies $H_1^L(L_1,D_1) = 0$ if and only if $L_1 < L_2$. If 1's product line is more susceptible to reverse engineering, then it has monopoly power over fewer new goods than 2 and therefore produces less (hires fewer workers to produce its new goods).

Proposition 5. If transfer rates are identical but spillovers are not, then it is necessary in a steady state equilibrium that the firm which gains the least from spillovers also hire the most workers for R&D. If spillovers are identical but transfer rates are not, then it is necessary that the firm whose product line is the most susceptible to reverse engineering also hires the fewest workers for production.

5. Symmetric Cournot Equilibrium and Comparative Steady States

The preceding analysis shows that if the transfer rates and spillovers are identical ($v_k = v$ and $\beta_{kj} = \beta < 1$), then for every firm k and j we must have $E_k = E_j$, $D_k = D_j$, $n_k = n_j$, $N_{Sk} = n_{Sj}$, $\theta_k = \theta_j$, and $\sigma_k = \sigma_j$. Given these results, it follows that the only way the equations $H_k^L(L_k,D_k) = 0$ and $H_k^D(L_k,D_k) = 0$ can hold simultaneously for all k is if $L_k = L_j$ for all firms k and j. That is, the steady state equilibrium must be symmetric.

Proposition 6. If the transfer rates and spillovers among firms are identical, then the steady state equilibrium exists and must be symmetric. That is, for every firm k, $D_k = D_c^*$, $L_k = L_c^*$, $\theta_k = \theta_c^*$, and $\sigma_k = \sigma_c^*$ where $\theta_c^* = h(E_c^*)/v$, $E_c^* = [1+\beta(K-1)]D_c^*$, $\sigma_c^* = \theta_c^*/K$, and D_c^* and L_c^* satisfy $0 < K(L_c^* + D_c^*) < M_N$, $H_c^L(L_c^*,D_c^*) = 0$, and $H_c^D(L_c^*,D_c^*) = 0$ where

$$H_c^L(L_c,D_c) = \gamma[1-\phi'\phi^{-1}(L_c+D_c)] - KL_c\phi'\phi^{-1}[1+\delta(L_c+D_c)] \tag{31}$$

$$H_c^D(L_c,D_c) = \frac{(1-\gamma)h'(E_c)(1+\theta_c)}{(r+v)\theta_c}\left[\frac{Kr-h(E_c)}{K(r-h(E_c))} - \phi'\phi^{-1}(L_c+D_c)\right] -$$

$$K\phi'\phi^{-1}[1+\delta(L_c+D_c)]. \tag{32}$$

If the matrix of second derivatives of H_c^L and H_c^D with respect to L_c and D_c is negative definite (as we shall assume hereafter), then this symmetric equilibrium is unique and locally stable. Northern terms of trade in this steady state is $P_c^* = (\sigma_c M_S/L_c^*)^{1-\gamma} = (\theta_c^* M_S/KL_c^*)^{1-\gamma}$.

Proposition 7. In the symmetric steady state equilibrium the oligopoly hires more workers for production of new goods than a monopoly, but its rate of innovation may be higher or lower than that in monopoly. The oligopoly hires fewer workers for both production and development of new goods, and has a lower rate of innovation than is socially optimal from either the Northern or global point of view.

As for the monopolist, each oligopolist's choice of L_c^* does not depend directly on θ_c, v, or M_S, so that in the steady state the choice of labor in production is a static one. Hence, as in a standard static model, the oligopoly hires more workers (and produces more) than does the monopolist because the monopolist is better at exploiting its market power in the labor market. Naturally the same effect tends to make the oligopoly hire more workers for R&D as well. However, since the oligopoly exerts less market power in the new product market as well, and so has less to gain from new product development than a monopoly, it may or may not hire more workers for R&D. Moreover, even if we knew it hired more workers, this would not guarantee a higher rate of innovation because the effective rate of research effort is less than total employment in R&D whenever spillovers are not complete ($\beta < 1$). Welfare results follow the same reasoning as in Proposition 2.

Final results for the symmetric oligopoly deal with how the steady state varies with changes in the parameters.

Proposition 8. An increase in the transfer rate increases labor in R&D and the rate of innovation, reduces labor in production of new goods, and increases the size of the traded goods sector. If diminishing marginal productivity in R&D is substantial, then an increase in spillovers reduces labor in R&D, increases labor in production of new goods, and decreases the size of the traded goods sector.

The intuition for the effect of an increase in v is the same as for the monopolist. Since β does not enter H_c^L, the effects of a change in this common spillover coefficient are similar to those for the transfer rate. If H_c^D is increasing in β, then an increase in spillovers increases labor in R&D and the rate of innovation, reduces labor in production, but increases the size of the traded goods sector. A sufficient condition for this is $h''=0$ and $r/2 \leq h(E_c^*) \leq (K-1)r\theta_c^*$ (which is less likely the smaller the number of firms). Conversely, if H_c^D decreases with β, then an increase in spillovers in R&D reduces labor in R&D and the rate of innovation, increases labor in production, but reduces the size of the traded goods sector. A sufficient condition for this is substantial diminishing marginal productivity of labor in R&D (h'' large in absolute value).

6. Cooperative Research Lab

Given the existence of spillovers in R&D, it is natural to ask whether or not allowing the oligopolists to form a cooperative research lab can benefit the firms and/or the Northern consumers. Questions of the formation and effects of such labs have been studied by Ruff (1969), Katz (1984), Spence (1984), Grossman and Shapiro (1985), and Ordover and Willig (1985). However, all of these have been in a partial equilibrium framework or have abstracted from international aspects of the problem. In this model of product cycle trade, there is an appropriability problem both within the Northern economy due to spillovers and between the Northern and Southern economies due to technology transfer. Hence the analysis in this subsection contributes to the literature on cooperative research ventures by examining their impact in a general equilibrium model of product cycle trade when there are spillovers in R&D both between the Northern firms and between the Northern and Southern economies.

We maintain the assumption that spillovers among the firms and transfer rates are identical. Consider allowing the firms to form a cooperative lab in which all share equally in both the costs and the benefits of the lab. Then the profit of each firm is

$$\pi_l = P_k L_k - w[L_k + (E_l/K)]$$

where E_l is the jointly chosen level of employment in the lab. The rate of innovation of each firm is now $\dot{n}_k(t) = h[E_l(t)]n_k(t)$ for every k. Each firm's problem is then to choose $L_k(t)$ and $E_l(t)$ to maximize $\int_0^\infty e^{-rt}\pi_l(t)dt$ subject to the innovation and technology transfer processes.

Proposition 9. If spillovers and transfer rates are equal, then all firms forming a cooperative research lab in which they share equally in both its costs and benefits is a Nash equilibrium (in open-loop strategies). The resulting steady state rate of innovation and total employment in the production of new traded goods are higher than those in a monopoly or symmetric oligopoly without a lab, but lower than those which maximize Northern and global welfare. An increase in the transfer rate has the same qualitative effects as in the monopoly or symmetric oligopoly.

Formation of the lab improves the welfare of both Northern and Southern consumers by increasing the rate of innovation and production of new traded goods closer to the levels which are socially optimal. The reasons for these increases are essentially the same as those which explain why the firms form a lab. Formation of the lab increases firm profit in two ways. First, it allows the firms to collude in the choice of labor in R&D, thereby improving their ability to exploit labor and reducing the marginal expense of labor in R&D. Second, it internalizes the spillover externalities in the lab, thus increasing the effective level of R&D, the rate of innovation, the ratio of new-to-old goods, and the terms of trade. Conversely, the formation of the lab is a Nash equilibrium because a firm that drops out of the lab must suffer a decrease in profit. Consumers also gain from the lab because the reduction in the marginal expense of labor in R&D ensures that production of new traded goods increases as well as the rate of innovation.[8] However, because

[8]This tends to lower the terms of trade and so partially offset the increase from an increased rate of innovation. Katz finds a similar result in his partial equilibrium static model.

the firms still exploit labor and ignore the effect of old goods on utility, the rate of innovation and production of new goods are still less than those which maximize utility. The rationale for the effects of a change in the transfer rate is the same as in the case of monopoly.

7. Conclusion

We have analyzed several variants of a dynamic model of innovation, technology transfer, and international trade. The analysis extends previous work by endogenizing the rate of innovation in a model where profit maximizing firms must devote resources to R&D in order to innovate and these resources have alternative uses in production within the R&D sector or in a competitive nontraded good sector. In doing so we combine well known results from the R&D literature (i.e. learning by doing and spillover effects) with the trade structure of the Krugman-Vernon product life cycle. The transfer of technology inherent in the cycle prevents the Northern industry from fully appropriating the benefits of its R&D so that the rate innovation is lower than the globally optimal rate. Nonetheless, with an acceleration in the transfer rate, the optimal response of the Northern industry is to increase its product development. This occurs because the North has a comparative disadvantage in *producing* traded goods. An increase in v accentuates this disadvantage so that Southern workers displace Northern workers in producing traded goods to a greater extent. With this intensified competition from Southern labor in production, the North will reallocate labor toward product development where it has an absolute and comparative advantage.

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Appendix

Derivation of the Terms of Trade and Relative Wage

In the presence of both new and old goods, the utility of a Northern consumer becomes

$$U_N = c_{nt}^{\gamma}\left[\int_0^{n_S} c_N(\eta_S)^{\gamma} d\eta_S + \int_{n_S}^{n} c_N(\eta_N)^{\gamma} d\eta_N\right] \tag{A1}$$

where $c_N(\eta_N)$ is consumption of new good η_N and $c_N(\eta_S)$ is consumption of old good η_S. The budget constraint is

$$I_N = c_{nt} + \int_0^{n_S} p(\eta_S)c_N(\eta_S)d\eta_S + \int_{n_S}^{n} p(\eta_N)c_N(\eta_N)d\eta_N \tag{A2}$$

where I_N is Northern nominal income, $p(\eta_N)$ is the nominal price of new good η_N, $p(\eta_S)$ is the nominal price of old good η_S, and the price of a nontraded good is one. Necessary conditions for utility maximization imply

$$p(\eta_N)/p(\eta_S) = [c_N(\eta_S)/c_N(\eta_N)]^{1-\gamma} \tag{A3}$$

$$1/p(\eta_S) = c_N(\eta_S)^{1-\gamma}\left[\int_0^{n_S} c_N(\eta_S)^{\gamma} d\eta_S + \int_{n_S}^{n} c_N(\eta_N)^{\gamma} d\eta_N\right] c_{nt}^{-1} \tag{A4}$$

$$1/p(\eta_N) = c_N(\eta_N)^{1-\gamma}\left[\int_0^{n_S} c_N(\eta_S)^{\gamma} d\eta_S + \int_{n_S}^{n} c_N(\eta_N)^{\gamma} d\eta_N\right] c_{nt}^{-1} \tag{A5}$$

Solving (A4) or (A5) for c_{nt} and substituting from (A3) gives $c_{nt} = \int_0^{n_S} p(\eta_S)c_N(\eta_S)d\eta_S + \int_{n_S}^{n} p(\eta_N)c_N(\eta_N)d\eta_N$, so from (A2) we have $I_N = 2c_{nt}$ (i.e., Northern consumers divide their income equally between traded and nontraded goods). Note that, because utility is homothetic, the proportion of income spent on traded and nontraded goods is independent of the distribution of income among factor owners. Necessary conditions for maximization of Southern utility imply

$$p(\eta_N)/p(\eta_S) = [c_S(\eta_S)/c_S(\eta_N)]^{1-\gamma}. \tag{A6}$$

Therefore the demand functions for each new good are all identical and those for each old good are identical. Given the assumptions on production, it follows that a Northern monopolist will produce the same amount of each new good and the South will produce the same amount of each old good, so that the nominal price of each new good is the same and the nominal price of each old good is the same.

If L is total labor in production of new goods in the North (and also total production of all new goods), then $c_N(\eta_N)+c_S(\eta_N)=L/n_N$ and $c_N(\eta_S)+c_S(\eta_S)=M_S/n_S$. Total nominal income in the North is therefore $I_N=c_{nt}+n_Np(\eta_N)L/n_N=2p(\eta_N)L$ and that in the South is $I_S=p(\eta_S)M_S$. Using the fact that $c_N(\eta_N)=c_S(\eta_N)I_N/2I_S$ and $c_N(\eta_S)=c_S(\eta_S)I_N/2I_S$ (derived from comparing Northern and Southern demand functions), it follows that the share α of total production of any new good or any old good which is consumed in the North is

$$\alpha = I_N/(I_N+2I_S) = p(\eta_N)L/[p(\eta_N)L+M_S]. \tag{A7}$$

Therefore $c_N(\eta_N)=\alpha L/n_N$ and $c_N(\eta_S)=\alpha M_S/n_S$, and so the terms of trade between any new good and any old good given by (A3) becomes

$$p(\eta_N)/p(\eta_S) = [n_NM_S/n_SL]^{1-\gamma}, \tag{A8}$$

which is the same as (5) in the text since $P\equiv p(\eta_N)/p(\eta_S)$.

Now let ϕ be the production function for nontraded goods and L_{nt} be the amount of labor in the nontraded good sector. Then (A4) becomes

$$1/p(\eta_S) = \alpha[M_S+(n_N/n_S)^{1-\gamma}L^{\gamma}M_S^{1-\gamma}]/\phi(L_{nt}) \tag{A9}$$

which after substitution of (A7) and (A8) becomes

$$1/p(\eta_S) = [p(\eta_N)/p(\eta_S)]L/\phi(L_{nt}). \tag{A10}$$

As noted in the text, the relative wage $w\equiv w_N/w_S$ must be $[1/p(\eta_S)]\phi'(L_{nt})$, so

$$w = [p(\eta_N)/p(\eta_S)]L\phi'(L_{nt})/\phi(L_{nt}) \tag{A11}$$

which is equivalent to (6) in the text.

B. Proof of Proposition 3

The effect of a change in v on L^*_m and D^*_m is found by totally differentiating $H^L_m(L^*_m,D^*_m)=0$ and $H^D_M(L^*_m,D^*_m)=0$ with respect to L^*_m, D^*_m, and v. Since $(\partial H^L_m/\partial v)=0$, it follows that

$$\partial D^*_m/\partial v = -((\partial H^L_m/\partial L_m)\,(\partial H^D_m/\partial v))/|H|$$

$$(\partial L^*_m/\partial v) = ((\partial H^L_m/\partial D_m)\,(\partial H^D_m/\partial v))/|H|$$

$$(\partial D^*_m/\partial v) + (\partial L^*_m/\partial v) = (\partial H^D_m/\partial v)((\partial H^L_m/\partial D_m) - (\partial H^L_m/\partial L_m))/|H|$$

where $|H| = (\partial H^L_m/\partial L_m)(\partial H^D_m/\partial D_m) - (\partial H^L_m/\partial D_m)\,(\partial H^D_m/\partial L_m) > 0$.

Since
$$\partial H^D_m/\partial v = (1-\gamma)h'(D^*_m)[(r+v)\text{-}v(1+\theta^*_m)][1-\phi'\phi^{-1}(L^*_m+D^*_m)]/v(r+v)^2\theta^*_m$$
$$= (1-\gamma)h'(D^*_m)[r-h(D^*_m)][1-\phi'\phi^{-1}(L^*_m+D^*_m)]/v(r+v)^2\theta^*_m > 0,$$

$$\partial H^L_m/\partial L_m = -\gamma\delta-\gamma\phi'\phi^{-1}-\phi'\phi^{-1}[1+\delta(L^*_m+D^*_m)]$$
$$-L^*_m\phi'\phi^{-1}[\delta+(\partial\delta/\partial L_m)(L^*_m+D^*_m)] < 0,$$

$$(\partial H^L_m/\partial D_m) = -\gamma\delta - \gamma\phi'\phi^{-1} - L^*_m\phi'\phi^{-1}[\delta+(\partial\delta/\partial D_m)] < 0,$$

and, since $\partial\delta/\partial L_m=\partial\delta/\partial D_m$, $(\partial H^L_m/\partial D_m) - (\partial H^L_m/\partial L_m) = \phi'\phi^{-1}[1+\delta(L^*_m+D^*_m)] > 0$, we have $(\partial D^*_m/\partial v)>0>(\partial L^*_m/\partial v)$ and $\partial(L^*_m+D^*_m)/\partial v>0$. (Note that the assumption $\partial\delta/\partial L_m=\partial\delta/\partial D_m\geq 0$ is not needed to obtain $\partial(L^*_m+D^*_m)/\partial v>0$.)

Since $(\partial\theta^*_m/\partial v)=[h'(D^*_m)(\partial D^*_m/\partial v)v- h(D^*_m)]/v^2$ where $(\partial D^*_m/\partial v)>0$, the sign of $(\partial\theta^*_m/\partial v)$ is ambiguous. However, $(\partial P^*/\partial v)$ can be shown to have the sign of $(\partial\theta^*_m/\partial v)L^*_m-\theta^*_m(\partial L^*_m/\partial v)\gtreqless 0$ if and only if $-(\partial L^*_m/\partial v)v/L^*_m \gtreqless -(\partial\theta^*_m/\partial v)v/\theta^*_m$. Since the wage depends on θ^*_m and P^*, the effect of a change in v on it is also ambiguous.

Proofs of remaining propositions are straightforward and are available from the authors upon request.

C. Proof of Propositions 3, 4, and 5

Using the same procedure as in the proof of Proposition 1, one can show that the terms of trade of any new good in line k relative to any old good is given by (15) and that the relative wage is given by (16). The current value Hamiltonian for firm k is

$$H_k = \pi_k + \lambda_k h(E_k) n_k + \mu_k v_k n_{Nk}$$

and the first order necessary conditions are

$$\partial\pi_k/\partial L_k = 0 \tag{A21}$$

$$(\partial\pi_k/\partial D_k) + \lambda_k h'(E_k) n_k = 0 \tag{A22}$$

$$\dot{\lambda}_k = r\lambda_k - (\partial\pi_k/\partial n_k) - h(E_k)\lambda_k - \mu_k v \tag{A23}$$

$$\dot{\mu}_k = r\mu_k - (\partial\pi_k/\partial n_{Sk}) + \mu_k v \tag{A24}$$

and the usual transversality conditions. To determine whether or not a steady state can exist, set $\dot{\mu}_k = 0$ in (A24) and $\dot{\lambda}_k = 0$ in (A23), solve these for λ_k, and substitute this into (A22). Taking derivatives and collecting terms then gives (19) in the text. Computing the derivative in (A21) gives (18). These equations depend on t only through L_k, E_k, θ_k, and σ_k, all of which must be constant if there is a steady state. If L_k and D_k are constants for all firms, then we need only show that this implies $\dot{\theta}_k = 0$ and $\dot{\sigma}_k = 0$ for all k to show a steady state can exist. However, as noted in the text $\dot{\theta}_k = 0$ if and only if $h(E_k) = v_k\theta_k$ and $\dot{\sigma} = 0$ if and only if $\Sigma_{j=1}^{K} v_j\sigma_j = v_k\theta_k$ for all k. Therefore $h(E_k) = v_k\theta_k = \Sigma_{j=1}^{K} v_j\sigma_j$ for all k is necessary in any steady state in which all K firms survive. This proves Proposition 3 since the summation term does not depend on k. Proposition 4 is essentially proved in the text.

To prove Proposition 5, first note that $v_k = v$ for all k implies that $h(E_k) = v\theta_k = \Sigma_{j=1}^{K} v\sigma_j$ for all k is necessary in the steady state, so $\theta_k = h(E_k)/v$ for all k. Since $E_k = E_j$ for all k and j is also necessary, it follows that $n_k = n_j$, $n_{Nk} = n_{Nj}$, and $n_{Sk} = n_{Sj}$ for all k and j and for all t. Therefore $\sigma_k = \sigma_j$ for all k and j and all t. If $\beta_{kj} = \beta < 1$ for all k and j also, then $E_k = E_j$ implies $D_k = D_j$ for all k and j as well in the steady state. Hence, if we consider the necessary conditions (18) and (19) for two different firms k and j, the only possible difference between these pairs of equations is the value of L_k and L_j. Suppose that $L_j < L_k$. Then since H^L_k is decreasing in L_k, it follows that $H^L_k(L_k, D_k) = 0$ if and only if $H^L_j(L_j, D_k) > 0$. Therefore (18) and (19) can be satisfied simultaneously for all k if and only if $L_k = L_j$ for all k and j. This proves that when $v_k = v$ for all k and $\beta_{kj} = \beta < 1$ for all k and j, the steady state equilibrium must be symmetric if it exists. The proof of the existence of symmetric equilibrium which is interior is essentially the same as that for the monopoly. To show that the necessary conditions (18) and (19) become (20) and (21) in the symmetric steady state, first note that $n_{Nk} = n_{Nj}$ and $n_{Sk} = n_{Sj}$ for all k and j imply that $\sigma_k = \sigma^*_c$, and hence $\theta_k = \theta^*_c = K\sigma^*_c$, for all firms k. Substituting $L_k = L_c$, $D_k = D_c$, $\theta_k = \theta_c$, and $\sigma_k = \theta_c/k$ into (18) and (19) and then dividing (18) by $P_c = [\theta_c M_S / KL_c]^{1-\gamma}$ and (19) by $P_c L_c$ gives (20) and (21).

D. Proof of Proposition 6

Given identical transfer rates, the levels D* and L* which maximize the present value of Northern utility when there are K product lines are again given by (A19') and (A20'), although now L* is the total amount of labor in the production of all new goods (i.e., production in each product line is L*/K). Since w* evaluated at $L^* = KL^*_c$ and $D^* = DK^*_c$ is less than $w^*[1 + \delta(L^*_c + D^*_c]$ from (18) or (19), it follows that $KL^*_c < L^*$ and $KD^*_c < D^*$. Since $H^L_m(KL^*_c, E^*_c) < 0$ but the sign of $H^D_m(KL^*_c E^*_c)$ is ambiguous, we can only conclude (using stability) that $L^*_m < KL^*_c$.

E. Proof of Proposition 7

Totally differentiating $H^L_c(L^*_c, D^*_c) = 0$ and $H^D_c(L^*_c, D^*_c) = 0$ shows

$$\partial D^*_c / \partial v = -((\partial H^L_c / \partial L_c)\ (\partial H^D_c / \partial v)) / |H_c|$$

$$\partial L^*_c / \partial v = ((\partial H^L_c / \partial D_c)(\partial H^D_c / \partial v)) / |H_c|$$

$$(\partial D^*_c / \partial v) + (\partial L^*_c / \partial v) = (\partial H^D_c / \partial v) \left[(\partial H^L_c / \partial D_c) - (\partial H^L_c / \partial L_c) \right] / |H|$$

where $|H| = (\partial H_c^L/\partial L_c)(\partial H_c^D/\partial D_c)-(\partial H_c^L/\partial D_c)(\partial H_c^D/\partial L_c)>0$. Since

$$\partial H_c^D/\partial v = \frac{(1-\gamma)h'(E_c^*)[r-h(E_c^*)]}{v(r+v)^2\theta_c^*}\left[\frac{Kr-h(E_c^*)}{K[r-h(E_c^*)]}-\phi'\phi^{-1}(L_c^*+D_c^*)\right]>0,$$

$$\partial H_c^L/\partial L_c = -(\gamma K\delta-\gamma\phi'\phi^{-1}-K\phi'\phi^{-1}[1+\delta(L_c^*+D_c^*]$$

$$-KL_c^*\phi'\phi^{-1}[\delta+(\partial\delta/\partial L_c)(L_c^*+D_c^*)] < 0,$$

$$\partial H_c^L/\partial D_c = -\gamma K\delta-\gamma\phi'\phi^{-1}-KL_c^*\phi'\phi^{-1}[\delta+(\partial\delta/\partial D_c)(L_c^*+D_c^*)],$$

$(\partial H_c^L/\partial D_c)-(\partial H_c^L/\partial L_c) = K\phi'\phi^{-1}[1+\delta(L_c^*+D_c^*)]>0$ (where ϕ', ϕ^{-1}, and δ are evaluated at $M_N-KD_c^*$), it follows that $(\partial D_c^*/\partial v)>0>(\partial L_c^*/\partial v)$ and $\partial(L_c^*+D_c^*)/\partial v>0$.

Following the same procedure for β,

$$\partial D_c^*/\partial\beta = -((\partial H_c^L/\partial L_c)(\partial H_c^D/\partial\beta))/|H|$$

$$\partial L_c^*/\partial\beta = -((\partial H_c^L/\partial D_c)(\partial H_c^D/\partial\beta))/|H|$$

$$(\partial D_c^*/\partial\beta)+(\partial L_c^*/\partial\beta) = (\partial H_c^D/\partial\beta)\left[(\partial H_c^L/\partial D_c)-(\partial H_c^L/\partial L_c)\right]/|H_c|.$$

Hence, $(\partial H_c^D/\partial\beta)>0$ implies $(\partial D_c^*/\partial\beta)>0>(\partial L_c^*/\partial\beta)$ and $\partial(L_c^*+D_c^*)/\partial\beta>0$ while $(\partial H_c^D/\partial\beta)<0$ implies $(\partial D_c^*/\partial\beta)<0<(\partial L_c^*/\partial\beta)$ and $\partial(L_c^*+D_c^*)/\partial\beta<0$. The statement concerning β in the Proposition then follows from observing that $\partial H_c^D/\partial\beta=$

$$\frac{(1-\gamma)(K-1)D_c^*}{(r+v)\theta_c^*}\left[h''(1+\theta_c^*)-\frac{(h')^2}{v\theta_c^*}\right]\left[\frac{Kr-h}{K(r-h)}-\phi'\phi^{-1}(L_c^*+D_c^*)\right]+\frac{(1-\gamma)(h')^2(1+\theta_c^*)(K-1)^2rD_c^*}{(r+v)\theta_c^*K(r-h)^2}$$

where the first term is negative and the second positive. If h" is very large in absolute value, the first term will outweigh the second and $(\partial H_c^D/\partial\beta)<0$. If $h''=0$, then collecting terms shows

$(\partial H_c^D/\partial\beta)>0$ if $Kr(2h-r)+h[(K-1)\theta_c^* r-h]>0$, which holds if $h\geq r/2$ and $h\leq(K-1)\theta_c^* r$. Also note that $(\partial H_c^D/\partial\beta)>0$ at $K=1$ (since the second term vanishes).

F. Proof of Proposition 8

The current value Hamiltonian for a firm when there is a cooperative lab is

$$H_1 = \pi_1+\lambda_1 h(E_1)n+\mu_1 v n_N$$

since $n_k=n$ and $n_{Sk}=n_S$ for all k. First order necessary conditions are

$$\partial\pi_1/\partial L_1 = 0$$

$$(\partial\pi_1/\partial E_1) + \lambda_1 h'(E_1)n = 0$$

$$\dot{\lambda}_1 = r\lambda_1-(\partial\pi_1/\partial n)-h(E_1)\lambda_1-\mu_1 v$$

$$\dot{\mu}_1 = r\mu_1-(\partial\pi_1/\partial n_S)+\mu_1 v$$

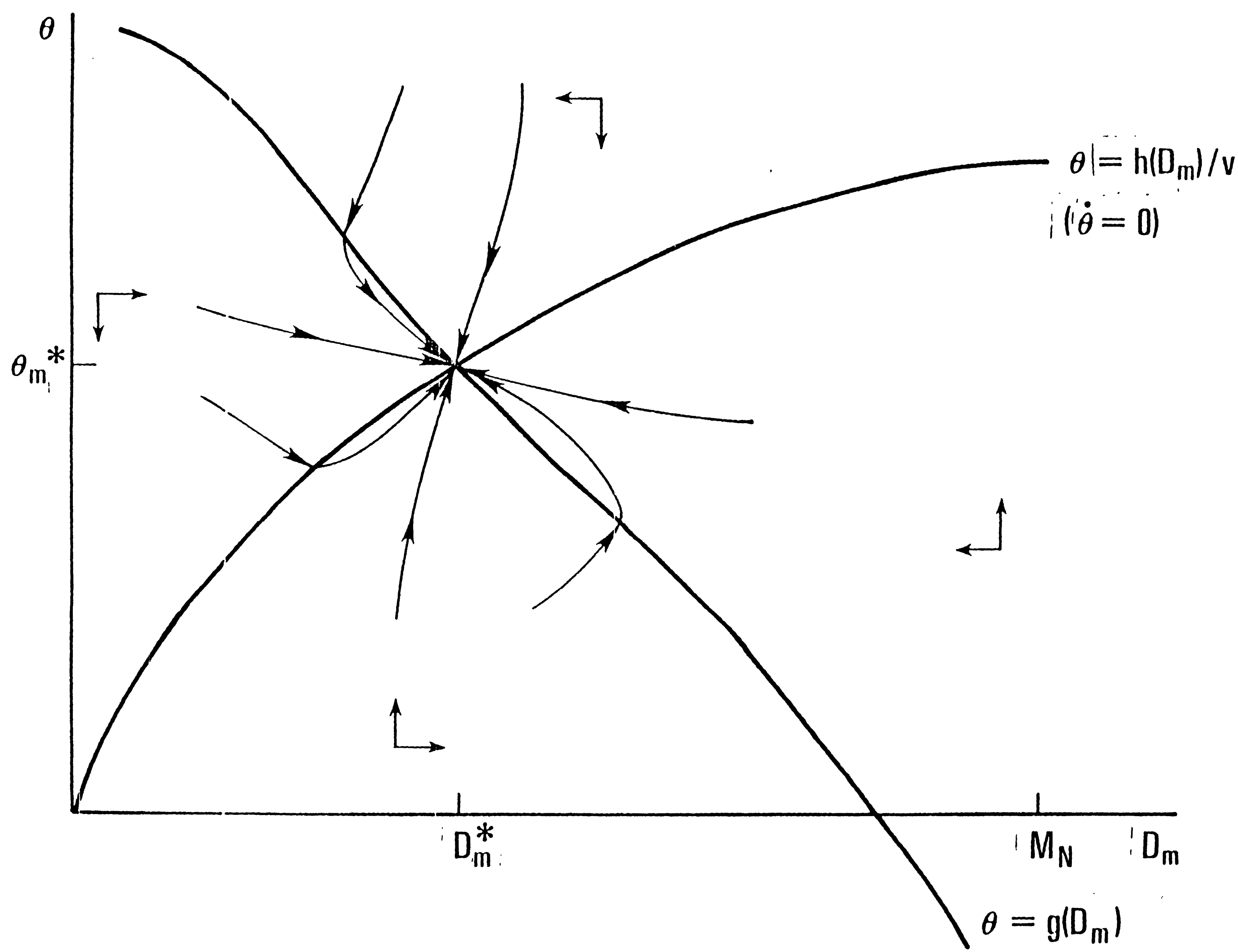
and the usual transversality conditions. Using these as before, one can show that necessary conditions for a steady state equilibrium are $E_1=E_1^*$, $L_k=L_1^*$ for all k, $\theta_1^*=h(E_1^*)/v$, and $\sigma_1^*=\theta_1^*/K$, where L_1^* and E_1^* are defined by $H_1^L(L_1^*,E_1^*)=0$ and $H_1^E(L_1^*,E_1^*)=0$ and

$$H_1^L(L_1,E_1) = \gamma[1-\phi'\phi^{-1}(L_1+(E_1/K))] - KL_1\phi'\phi^{-1}[1+\delta(L_1+(E_1/K))] \tag{A24}$$

$$H_1^E(L_1,E_1) = \frac{(1-\gamma)h'(E_1)(1+\theta_1)}{(r+v)\theta_1}\left\{\frac{Kr-h(E_1)}{K[r-h(E_1)]}-\phi'\phi^{-1}(L_1+(E_1/K))\right\}. \tag{A25}$$

The derivation of θ_1^*, σ_1^*, (A24), and (A25) and the proof that $L_k=L_1^*$ for all k is entirely analogous to that for the symmetric oligopoly.

Let $f(L,E)=\phi'(M_N-L-E)\phi^{-1}(M_N-L-E)(L+(E/K))$ and $g(L,E)=f(L,E)\delta(M_N-L-E)$ (i.e., δ evaluated at M_N-L-E). Then $(\partial f/\partial E)>0$ and $(\partial g/\partial E)>0$. Notice that total employment in R&D in the Cournot economy is $KD_c^*=E_c^*/b$ where $b\equiv[1+\beta(K-1)]/K<1$ since $\beta<1$. Rewriting H_c^L



θ
θm*
θ = h(Dm)/v
(θ̇ = 0)
Dm*
MN
Dm
θ = g(Dm)

DATE DUE
