LEARNING BY DOING WHILE REMEMBERING FORGETTING, WITH REMINDERS FROM PAKISTAN MANUFACTURING DATA

by

A.R. Kemal
Richard C. Porter
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of these papers.

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* The authors are Senior Research Economists at the Pakistan Institute of
  Development Economics (Islamabad) and at the Center for Research on Economic
  Development (Ann Arbor), respectively. We are indebted to Genevieve Kenney for
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ABSTRACT

The process of subsidized industrialization in many LDCs has been taking place for a sufficiently long time that it is becoming possible to test empirically whether these new and initially inefficient industries have been "learning by doing". Such tests are beginning to appear. Unfortunately, they are bedeviled by the difficulty of quantifying the concept, "learning". The surrogates usually used lead to the strange implication that learning is unbounded and hence that the merest iota of capital and labor is ultimately capable of learning how to produce infinite outputs (and doing it). Furthermore, the learning variable usually selected is highly collinear with time and hence is econometrically quite unsatisfactory. This paper explores these demerits and presents a new method that overcomes these problems by remembering forgetting. This new approach is employed to examine various learning hypotheses for sixteen manufacturing industries and the total manufacturing sector of Pakistan.

RESUME

Le processus qui consiste à subventionner l'industrialisation de nombreux pays en voie de développement a été suffisament utilisé pour qu'il soit possible de vérifier empiriquement si ces nouvelles industries, à l'origine inefficaces, ont bien profité de "l'expérience par la pratique". Des vérifications de ce genre commencent donc à être entreprises. Malheureusement, elles se trouvent contrecarrées par la difficulté de quantifier le concept de "l'expérience". Les succédanés généralement employés conduisent à l'étrange implication voulant que cette dernière soit illimitée et que, par conséquent, la réunion de la moindre quantité de capital et de main-d'oeuvre peut, en fin de compte, apprendre à produire à l'infini (et le faire). En outre, la variable expérience généralement sélectionnée est intimement liée au facteur temps et, de ce fait, insatisfaisante sur le plan économétrique. Ce document examine ces défauts et présente une nouvelle méthode capable de surmonter ces problèmes grâce à la prise en considération de la différence existant entre ce qui est appris et de qui est retenu. Cette nouvelle approche est employée pour examiner plusieurs hypothèses d'expérience applicables à seize industries de transformation ainsi qu'à tout ce secteur au Pakistan.
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I  Production Function Parameter Estimates (Equation 21)  
II $R^2$ Between "Knowledge" and "Time"  
III Production Function Parameter Estimates (Equation 22)
I. Introduction

In the development strategies of most LDCs, industrialization has played an important part. Subsidies of myriad kinds have been lavished, sometimes indiscriminately, upon manufacturing firms in the hope or expectation that, despite their initial inefficiency by world standards, they would "learn by doing" and eventually form the competitive vanguard of development. This process of subsidized industrialization has been taking place in many LDCs for a sufficiently long time that it is becoming possible to test empirically whether the anticipated learning by doing has in fact occurred.

Such tests are beginning to appear. Unfortunately, they are bedeviled by the difficulty of quantifying the concept, "learning by doing". The tendency so far has been to proxy the vague idea of how much has been learned by the more measurable concept of how much has been done -- usually how much has been produced or how much invested. While some such surrogate is necessary, the precise way in which this substitution has been carried out in the past is seriously flawed. Conceptually, it leads to the strange implication that learning is unbounded and hence that the merest iota of capital and labor is ultimately capable of learning how to produce infinite outputs (and doing it). Furthermore, the learning variable usually selected is highly collinear with time and hence is econometrically quite unsatisfactory.
In this paper, in the process of exploring the demerits of the traditional measures of learning, we present a new method that overcomes these problems. In essence, the new method differs in that it explicitly recognizes that knowledge is a stock, that learning is a flow, and that forgetting may play an important role in the whole process. The plan of the paper is as follows. In Section II, we discuss "learning" in principle, and the concept of forgetting is introduced. In Section III, the econometric troubles with learning are examined. In Section IV, this new approach is employed to explore various learning hypotheses for sixteen manufacturing industries and the total manufacturing sector of Pakistan.
II. "Learning," in Principle

Learning is to human capital as investment is to physical capital. And if one thinks of investment, one thinks of depreciation -- that is, the difference between gross and net investment. This basic idea seems to have eluded every empirical study of "learning by doing." In this section, we will show that this neglect of "forgetting" leads to absurd theoretical implications, and that a proper definition of accumulated learning provides plausible implications.

Learning is usually introduced into the production function as an independent variable, called accumulated knowledge -- and represented, from tradition and for reasons now unknown, by the letter $G$. Thus,

$$Y_t = F(K_t, L_t, G_t, t),$$

where $Y$ is output (usually value added), $K$ physical capital, $L$ labor, $t$ time, $F$ the production-function relationship, and the subscripts date the variables.

For empirical work, a specific functional form of $F(.)$ must be chosen; to make the point here, a simple Cobb-Douglas function suffices:

$$Y_t = A e^{\lambda t} K_t^\alpha L_t^\beta G_t^\gamma,$$

where $A$, $\alpha$, $\beta$, $\gamma$, and $\lambda$ are the usual parameters of such a function. In order to focus on the influence of learning, we will hold physical capital and labor constant (at $K_t = \bar{K}$ and $L_t = \bar{L}$), set $\lambda$ equal to zero, and scale

---

1For examples, see Rapping [7], Sheshinsky [8], David [2], Thomas [9], and Kemal [5]. The idea of "forgetting" has been used by Bardhan [1] in an interesting, but thoroughly theoretical analysis of the optimum subsidy to "learners."

2The inclusion of $t$ may play two different roles. One, it may capture the influence of steady, disembodied technological progress. And two, it may contain elements of learning by doing which depend upon how long the factors ($K$ and $L$) have been producing rather than how much they have produced. See Fellner [3].
the units of output (Y) so that

\[ A (\bar{K})^\alpha (\bar{L})^\beta = 1. \]  

(3)

Thus, 

\[ Y_t = (G_t)^\gamma. \]  

(4)

Accumulated knowledge (G) is variously defined in empirical studies, but the two most common formulations are i) the sum of past levels of output or ii) the sum of past levels of gross physical investment. Here, largely for convenience, we will examine the former definition, although the same results will be shown to follow from the definition of learning that involves accumulated gross physical capital formation1. With this definition, accumulated knowledge becomes

\[ G_t = \int_0^t Y_\tau d\tau. \]  

(5)

This definition, however, poses two minor, technical problems. One, at the moment when output begins, at \( t = 0 \), accumulated output and hence accumulated knowledge are both, by definition, zero; use of the Cobb-Douglas form of the production function, equation (4), then suggests that output can never get under way, since it requires some past output (embodied in \( G_t \)) to produce current output \( (Y_t) \). The escape from this dilemma may be philosophically difficult to some, but it is easy in practice. We consider \( Y_0 \) and \( G_0 \) to be exogenously given, with \( G_0 \) equal to \( Y_0^{1/\gamma} \), so that equation (4) holds even at \( t = 0 \) and equation (5) becomes

\[ G_t = G_0 + \int_0^t Y_\tau d\tau. \]  

(5')

The choice of definition is made largely for convenience, but it should be noted that it is also the most widely used and empirically most satisfactory definition.
The second problem is that, if equation (5) is taken literally, output in time \( t \) depends partly on itself — since \( Y_t \) is part of \( G_t \) which in turn determines, through the production function, \( Y_t \). This is not only awkward conceptually but also invites bias in the later econometric work. Again the practical solution is straightforward, to define the upper bound of the integral in equation (5) as \( t-\varepsilon \), where \( \varepsilon \) is some small, positive number.\(^1\) This adjustment, and the choice of \( \varepsilon \), are made simple in practice, owing to the fact that output and input data are only available for finite time periods.

To examine the time path of \( G_t \) and \( Y_t \), take the derivative of equation (5), with respect to \( t \), and substitute from equation (4) to get:

\[
\dot{G}_t = Y_t = (G_t)^Y,
\]

where the dot on top indicates the time derivative.\(^2\) This non-linear differential equation is readily solved, and the initial, exogenous value of knowledge \((G_0, equal to Y_0^{1/Y} by equation (4))\) can be used to eliminate the constant of integration:

\[
G_t = \left\{ \begin{array}{l}
(1-\gamma)t + G_0^{1-\gamma} \\
\frac{1}{1-\gamma}
\end{array} \right., \quad \text{and}
Y_t = \left\{ \begin{array}{l}
(1-\gamma)t + Y_0^{1-\gamma} \\
\frac{\gamma}{1-\gamma}
\end{array} \right. \tag{7}
\]

The first derivative (with respect to time, \( t \)) of output \((Y_t \text{ in equation (8)})\) is positive for all values of \( t \).\(^3\) Despite diminishing returns to knowledge

---

\(^1\)Now, the initial values of output from \( Y_0 \) through \( Y_\varepsilon \) must be taken as exogenous and the production function (4) applied only thereafter (for \( t > \varepsilon \)).

\(^2\)Note that the \( \varepsilon \) is being assumed to be infinitesimally small.

\(^3\)We assume, of course, that \( 0 < \gamma < 1 \). Values of \( \gamma \) less than zero would mean that learning was counterproductive, and values greater than one that it yielded increasing returns. The mathematical implications for equations (7) and (8) and their time derivatives would then be quite unrealistic.
(i.e., $0 < \gamma < 1$), output growth due to learning can continue forever and without limit.¹

That learning by doing can by itself lead to output growth forever without limit is the curious theoretical implication of the model. The assumption of the model that brings about this result is not hard to find. Knowledge is never forgotten. Thus, learning in each period boosts output by the same amount in every succeeding period -- forever. Workers die, managers retire, plants decay, technology obsolesces; but the learning embodied in them lives on -- and on. This might make sense if we thought of learning as "book-learning", for then the dead managers and workers would leave updated manuals behind as a legacy for their successors. And the tendency to forget can always be forestalled by a quick review of the manual. But that kind of knowledge is usually counted in "technical progress." What we usually call "learning by doing" is the mental and physical dexterity that people develop through the repetitive performance of a task. And then, clearly if learning is introduced as an ingredient in the production function, forgetting must also be recognized as a potential concomitant.

A simple model that incorporates learning and forgetting is readily constructed by extension of the previous model. We continue to utilize the simple Cobb-Douglas production function, equation (4), but we now

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¹ Indeed, if $\gamma > \frac{1}{2}$, $Y$ grows at an increasing rate, i.e. $\ddot{Y}_t > 0$. But even $\ddot{Y}_t < 0$ does not imply a limit to $Y_t$ as $t \to \infty$. Note that the same result follows if knowledge is defined as cumulated gross investment. Assume conservatively that the initial investment ($K_0$) does not cause learning and that thereafter investment simply replaces depreciated capital (i.e., equals $\mu K_0$). After $t$ periods, knowledge is $\mu K_0 t$ and (recall definition (3)) output is

$$Y_t = (\mu K_0 t)^\gamma.$$  

$Y_t$ is clearly positive for all $t$ and, although $\ddot{Y}_t$ is negative (for $\gamma < 1$), there is still no limit to $Y_t$ as $t \to \infty$. 

visualize two forces on accumulated knowledge ($G_t$). One, current output adds to knowledge as before, on a one-to-one basis; and two, the existing stock of knowledge depreciates at rate $\phi$. Thus

$$G_t = Y_t - \phi G_t, \quad (10)$$

or substituting for $Y_t$ from equation (4).

$$G_t = G^Y_t - \phi G_t. \quad (11)$$

Notice how this (also non-linear) differential equation differs from the earlier one, equation (6); this difference is essentially that $\phi$ is no longer assumed to be zero.

The solution to equation (11) and its implication for the time paths of $G_t$ and $Y_t$ are very different from the previous model, i.e., the one without forgetfulness. The exact equations for $G_t$ and $Y_t$ are complex, but mere examination of equation (11) is sufficient to show that, if $G_0$ and $Y_0$ are small, both grow (i.e. $G$ and $Y > 0$) until accumulated learning reaches a limit,

$$G = \left(\frac{1}{\phi}\right)^{1-\gamma}, \quad (14)$$

and output its limit,

$$Y = \left(\frac{1}{\phi}\right)^{1-\gamma}, \quad (15)$$

---

The solution to equation (11), in terms of output (converted by equation (4)), is:

$$Y_t = \left[\frac{1}{\phi}\right]^{1-\gamma} \left[1 - \left(1-\phi Y_0\right) e^{-\phi(1-\gamma)t}\right]^{1-\gamma} \quad (12)$$

$Y_t$ rises to its limit from $Y_0$ provided that

$$Y_0 < \left[\frac{1}{\phi}\right]^{1-\gamma}, \quad (13)$$

This condition (13) simply assumes that $G_0$ is positive.
If there is no new addition of capital, labor, or technology, learning by itself will raise output, but not without limit. Eventually, the addition to learning provided by current doing is offset by the forgetting of previous learning. At that time, learning by doing no longer raises output. Gross learning continues as long as output is produced, but net learning has ceased.

Only when depreciation of knowledge -- forgetting -- is recognized in a learning-by-doing model are plausible hypotheses about the effect of learning on productivity generated.
III. "Learning," in Practice

We now turn to the problem of econometric estimation of the effect of "learning by doing." Again, for simplicity, consider first the Cobb-Douglas form of the production function:

\[ Y_t = A e^{\lambda t} K_t^{\alpha} L_t^{\beta} G_t^\gamma. \]  

(2)

The usual econometric procedure is to assume an error term of the form, \( e_t \), and to linearize the regression by taking natural logs of equation (2):

\[ \ln Y_t = \ln A + \lambda t + \alpha \ln K_t + \beta \ln L_t + \gamma \ln G_t + u_t. \]  

(16)

Various kinds of samples have been used to provide estimates of the parameters of equation (16), \( A, \lambda, \alpha, \beta, \gamma, \) and \( \sigma_u^2 \) (i.e., the variance of the error term); but the most common, in the LDC context, is a time series of observations for manufacturing, in the aggregate or by particular sectors, as, for an example from Pakistan, the data we use here, in Section IV.

The first problem with the estimation of equation (16) derives from the multicollinearity of the independent variables. \( \ln K_t \) and \( \ln L_t \) are particularly likely to be correlated, but this can be handled by assuming constant returns to scale in capital and labor (i.e., \( \beta = 1 - \alpha \)) and by changing the dependent variable to the log of output per worker (i.e., \( \ln Y_t/L_t \)). More serious, especially owing to our present concern with learning by doing, is the high multicollinearity of \( t \) with \( \ln G_t \). Such high correlation clearly must be anticipated: no matter what path output \( (Y_t) \) follows over time, the cumulated sum of all past outputs \( (G_t) \) rises over time.

\[ \text{With the single term, } \alpha \ln K_t/L_t, \text{ replacing the two terms in } K_t \text{ and } L_t. \]
The result typically is that, when both of these potential independent variables, $\ln G_t$ and $t$, are included in the regression, neither is significant, and the null hypotheses of no disembodied technical progress and no learning by doing has to be accepted. The obvious -- and usual -- procedure at this point is to remove one of the two independent variables, $\ln G_t$ or $t$. If the former is removed, there is no clear way left to test for learning by doing, so the $t$ variable exits.

The regression that actually ends up being fitted is

$$\ln Y_t/L_t = \ln A + \alpha \ln K_t/L_t + \gamma \ln G_t + u_t.$$ \hspace{1cm} (17)

Two independent variables are left. But one of them is not likely to add much to the fit; since $K_t$ and $L_t$ are multicollinear and the two variables (especially $K_t$) are measured with error, there is probably little variation in $\ln K_t/L_t$ and what little there is is largely noise. Sensible estimates of $\alpha$ may or may not emerge, but much explanation of the variance of the dependent variable cannot be expected from this source.

From $\ln G_t$, however, a great deal of explanation can be expected. $\ln G_t$ rises over time, and $\ln Y_t/L_t$ will generally rise over time as well, for any of four important reasons:

1) disembodied technical progress is occurring through exogenous improvements in the quality of management, labor, or the infrastructure, raising productivity;

ii) the effectively utilized capital is increasing relative to the measured capital stock as initial excess capacity gradually disappears so that capital intensity is rising more rapidly than the measured $K_t/L_t$ indicates;

iii) increasing returns to scale -- although assumed away in the formulation of (17) -- are occurring, causing per-unit labor
needs to decline over time as demand and output expand; and/or

iv) learning by doing is taking place.

But all of these influences can bring about a high, positive, and significant estimate of \( \gamma \) in equation (17). If \( \ln Y_t/L_t \) is rising over time, there is only the (necessarily) rising independent variable, \( \ln G_t \), there to "explain" it.¹ The resulting significance of \( \gamma \) is consistent with, but does not necessarily imply, learning by doing.

It is sometimes thought that the importance of time and knowledge as separate influences on output, can be rank-ordered by fitting not only equation (17) but also

\[
\ln Y_t/L_t = \ln A + \alpha \ln K_t/L_t + \lambda t + u_t, \quad (18)
\]

and comparing the correlation coefficients of the two regressions. Or, alternatively, a single regression containing both independent variables, \( t \) and \( \ln G_t \), might be estimated despite their collinearity and the coefficient "forced to insignificance" declared the less important. There are two objections to these procedures, one of practice and one of principle. In practice, the multicollinearity between \( t \) and \( \ln G_t \) may be so great that there is little to choose between the estimated equations (17) and (18)². And where both variables are included, the coefficients generally become

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¹ As reported in Kemal [5], for most sectors, the "growth of productivity explained by learning" (with a Cobb-Douglas production function and cumulative output as the learning index) is very close in magnitude to the "growth rate of productivity". It is as high as 80 percent. This indicates that most of the variance of the productivity variable \( (Y_t/L_t) \) is being "explained" by the learning \( (G_t) \) variable.

² In Kemal [5], Table 4, a comparison of \( R^2 \) declares \( \ln G_t \) the winner in 14 of the 16 industries, but the median difference between the two \( R^2 \)'s is only .025.
insignificant and/or nonsensical\textsuperscript{1}. The objection in principle is that all these machinations constrain the disembodied technical progress to be exponential. If such progress is occurring, but in other than exponential fashion, the \( t \) coefficient will not "pick it up". Indeed, the \( \ln G_t \) coefficient may be reporting such progress, even in a situation in which no learning is taking place!

An extended, though simple, arithmetical example is warranted to show this point. Assume that \( L_t, K_t, \) and \( A \) are each unity (for all \( t \)), that \( \gamma = 0 \) (i.e., learning doesn't matter), that technical progress is occurring smoothly over time, and that the production function is

\[
Y_t = a t^b, \tag{19}
\]

where \( a \) and \( b \) are positive parameters. Though we should know that learning doesn't matter, we naively proceed to calculate \( G_t \):

\[
G_t = \frac{a}{b+1} t^{b+1}. \tag{20}
\]

Now, \( \ln Y_t \) is linearly related to \( \ln G_t \), and a regression of the former on the latter will, if unbiased, yield an estimate of \( \left( \frac{b}{b+1} \right) \) which could easily be thought to be an estimate of \( \gamma \). Thus, there would appear "strong" evidence" for learning even when we know by assumption that no learning is occurring (or, if learning is occurring, that it is irrelevant for production). Thus we see that significant, positive estimates of \( \gamma \) in equation (17) may mean no more than that there has been technical progress -- without learning.\textsuperscript{2}

Of course, it must be recognized that any independent variable, if it is

\textsuperscript{1}In Kemal [5], Table 5, it is counted as an advantage that the "learning coefficients remain positive" in 12 of the 16 industries. But for 10 of those 12, the coefficients become greater than one, implying increasing returns to learning (see Section II).

\textsuperscript{2}The formulation of learning-with-forgetting, from Section II, suffers some of this same difficulty, but to a much smaller extent. The correlation between \( \ln G_t \) and \( t \) is reduced once depreciation of knowledge (\( \phi \)) is introduced.
positively correlated with time, may be "picking up" some of the effect of technical progress (assuming that such progress is in fact taking place). And even a variable that measures learning-cum-forgetting (such as that developed in Section II) will be positively correlated with time. But once depreciation of knowledge is introduced, the close, almost linear relation between ln $G_t$ and $t$ begins to disappear, and the two variables become empowered with the ability to segregate the effects of learning and progress on the volume of output.

Indeed, the formulation of equation (17) may still invite greater multicollinearity of the independent variables than it should. It is not obvious that the assumption of constant returns to scale in capital and labor -- the assumption that permitted the move from equation (16) to equation (17) -- is the correct one to make. Does, at a moment of time, the doubling of capital and labor permit a doubling of output? Or is it a doubling of capital, labor, and knowledge? If the latter, then constant returns to sale means that $\alpha + \beta + \gamma = 1$, and the $\gamma \ln G_t$ term in equation (17) must be replaced by $\gamma \ln \frac{G_t}{L_t}$. This treatment of knowledge implies that it is more a private than a public good, and may be conceptually unusual, but it should reduce collinearity of independent variables. In the next section, we shall explore both knowledge formulations -- $\ln G_t$ and $\ln \frac{G_t}{L_t}$ -- using data from Pakistan manufacturing and remembering forgetting.
IV. Application to Pakistan Manufacturing

Real values over time for gross output, value added, capital stock, and employed labor have been estimated by Kemal [5], and the sources cited therein, for 16 sectors of Pakistan manufacturing. These data, for the years from 1960-61 to 1969-70, are now utilized to further explore the implications for learning by doing while remembering forgetting.

Superficial analysis of these data indicates that something -- technical progress, learning, or some other "residual-like" activity -- is occurring during the 1960s. Real value added per worker is rising at an annual rate of 3.08% for the total manufacturing sector, and the median rate of growth of real value added per worker for the 16 separate manufacturing sectors is 1.28%.¹ Moreover, increasing capital-intensity does not provide much explanation of this rise. Regression of the log of real value added per worker (ln \( Y_t/L_t \)) on the log of real capital per worker (ln \( K_t/L_t \)) yields an \( R^2 \) of only 0.16 for the total manufacturing sector and a median \( R^2 \) of only 0.09 for the 16 sectors.

To explore whether time or learning could be identified as a significant cause of the rising value added per worker, we fitted the following production function:

\[
\ln Y_t/L_t = \ln A + \alpha \ln K_t/L_t + \lambda t + \gamma \ln G_t(\phi).
\] (21)

Note that a Cobb-Douglas function with constant returns to scale in capital and labor is assumed, and that the learning variable, \( G_t \), is now written

¹ These rates of growth are the time (t) coefficients in the simple regression of the natural log of value added per worker (ln \( Y_t/L_t \)) on time for the ten observations of the 1960s. These coefficients are reported in column 7 of Table 1.
explicitly as a function of the rate of forgetting, $\phi$.\(^1\) In the regressions, an additive error term with the usual properties is assumed. Before reporting the regressions, we need to point out two econometric problems. One, we are assuming that the independent variables of equation (21) are independent of the error term, and this assumption is susceptible to argument, especially with respect to the capital-labor ratio \((\ln K_t/L_t)^2\) and knowledge \((\ln G_t)\).\(^3\) And two, the forgetting parameter, $\phi$, does not enter equation (21) linearly, so that a direct search for the value of $\phi$ that maximizes the $R^2$ is necessitated.\(^4\)

The coefficient estimates for the regressions of equation (21) are reported in columns 2-5 of Table 1, and the $R^2$ in column 6, for the total manufacturing sector and for each of the 16 sectors within it. As is readily seen, these results are not very satisfying. While the $R^2$ values are generally high, only about one third are significant in the sense that the addition of time and knowledge (i.e. parameters $\lambda$, $\gamma$, and $\phi$) adds significantly to the explanation of the variance of the dependent variable, $\ln Y_t/L_t$. An even smaller fraction of the parameter estimates are significant. Moreover, the learning coefficient, $\gamma$, taken literally, implies that learning has a negative effect on labor productivity in four sectors and that there is increasing returns to scale from learning in four others.

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1 The $G_t$ variable to be considered in this section is the cumulated, depreciated (for forgetting) sum of past gross output.

2 A fuller study would treat value added, capital, and/or labor as simultaneously determined and fit a profit, rather than a production, function.

3 Knowledge is here defined in terms of cumulated past gross output, properly depreciated for forgetting, rather than value added, but the relationship between the two is clear.

4 The search is carried out over the following values of $\phi$: zero, 0.05, 0.10, 0.20, and 0.30. The best fit among these is reported (in Table 1) -- no further search was conducted.
Furthermore, technical progress, $\lambda$, is negative in over the half the sectors; forgetting, $\phi$, is never significantly greater than zero; and the output elasticity of capital is negative much of the time and insignificant almost all of the time.

Thus, despite the sizeable rate of growth of labor productivity in Pakistan’s manufacturing, we cannot in general reject the null hypotheses that time ($\lambda$) and learning by doing ($\gamma$ and, and with forgetting, $\phi$) do not cause it. One need not look far to discover the cause of this failure. Partly, it is the short data series -- ten years do not provide many degrees of freedom. But mostly, it is the high multicollinearity between time ($t$) and knowledge ($\ln G_t$). Without allowance for forgetting, the $R^2$ between $t$ and $\ln G_t$ is rarely below 0.99 -- see Table 2. Unfortunately, the introduction of the concept of forgetting does little in a pragmatic way to overcome this problem. With $\phi$ set at 0.30, the $R^2$ between $t$ and $\ln G_t$ falls, but rarely below 0.90, which is not far enough to create sufficient independent movement of the two series.

Treating knowledge not as a public good that accrues simultaneously to management, capital, and labor but as a private good captured by specific factors is conceptually less traditional but does ease this problem of multicollinearity. This means fitting the following equation:

$$\ln Y_t/L_t = \ln A + \alpha \ln K_t/L_t + \lambda t + \gamma \ln G_t(\phi)/L_t,$$  \hspace{1cm} (22)

The results are reported in Table 3. Again, they are most unsatisfactory, for two reasons. One, the $R^2$ between time and knowledge per worker (i.e. between $t$ and $\ln G_t(\phi)/L_t$) remains high. And two, high collinearity between capital-intensity and "knowledge-intensity" (i.e. between $\ln K_t/L_t$ and $\ln G_t(\phi)/L_t$) begins to appear. While there are more significant parameter
estimates in Table 3 than in Table 1, many of them are attached to estimates that are, by hypothesis, of incorrect sign.

So, we arrive after a fairly long journey at a rather disappointing destination. Short time series just do not provide a sufficiently rich (i.e. non-collinear) data set to separate learning by doing from other forces on labor productivity. At the least, we shall have to look to longer series and disaggregated data -- perhaps to the level of the firm, and perhaps to case studies rather than econometrics. Even this course of research, however, is not without pitfalls. As one moves toward the level of the firm, "personality" becomes important, and it will increasingly become difficult to tell whether a firm is highly productive because it has learned a lot or has learned (i.e. produced) a lot because it is highly productive.

We conclude where we began. More than a quarter century of industrialization has been occurring in LDCs, and it is essential to begin assessing whether, where, and how the new industries have become efficient by international standards.
<table>
<thead>
<tr>
<th>Sector</th>
<th>( \alpha )</th>
<th>( \lambda )</th>
<th>( \gamma )</th>
<th>( \phi )</th>
<th>( R^2 )</th>
<th>Growth Rate of Value Added per Laborer</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
<td>(6)</td>
<td>(7)</td>
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<tr>
<td>Food Processing</td>
<td>-0.25</td>
<td>0.04*</td>
<td>0.27*</td>
<td>0.30</td>
<td>0.76*</td>
<td>6.62%</td>
</tr>
<tr>
<td>Tobacco Manufacturing</td>
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<td>0.48</td>
<td>-1.01*</td>
<td>0.00</td>
<td>0.00</td>
<td>0.92</td>
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<tr>
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<td>-0.01</td>
<td>-0.04</td>
<td>0.91</td>
<td>0.30</td>
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<td>4.96</td>
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<tr>
<td>Footwear and Wearing Apparel</td>
<td>0.33</td>
<td>-0.25</td>
<td>1.66</td>
<td>0.00</td>
<td>0.08</td>
<td>1.02</td>
</tr>
<tr>
<td>Paper and Paper Products</td>
<td>-0.09</td>
<td>-0.33</td>
<td>1.00</td>
<td>0.20</td>
<td>0.94*</td>
<td>-11.06</td>
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<tr>
<td>Printing and Publishing</td>
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<td>0.53</td>
<td>-2.12</td>
<td>0.05</td>
<td>0.89</td>
<td>-6.06</td>
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<td>Leather and Leather Products</td>
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<td>2.57</td>
<td>0.20</td>
<td>0.38</td>
<td>1.43</td>
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<tr>
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<td>-2.15*</td>
<td>0.00</td>
<td>0.88*</td>
<td>17.03</td>
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<td>0.35*</td>
<td>0.30</td>
<td>0.77</td>
<td>-3.43</td>
</tr>
<tr>
<td>Non-Metallic Mineral Products</td>
<td>0.18</td>
<td>-0.06</td>
<td>0.30</td>
<td>0.30</td>
<td>0.30</td>
<td>-1.72</td>
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<td>0.06</td>
<td>0.14</td>
<td>0.10</td>
<td>0.69</td>
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<td>Metal Products Manufacturing</td>
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<td>0.11</td>
<td>0.00</td>
<td>0.28</td>
<td>0.72</td>
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<td>Non-Electrical Machinery</td>
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<td>0.27*</td>
<td>0.30</td>
<td>0.49</td>
<td>1.13</td>
</tr>
<tr>
<td>Electrical Machinery</td>
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<td>-0.52*</td>
<td>1.99*</td>
<td>0.00</td>
<td>0.89</td>
<td>6.81</td>
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<tr>
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<td>0.35</td>
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<td>0.30</td>
<td>0.57</td>
<td>0.51</td>
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<td>0.00</td>
<td>0.60</td>
<td>7.92</td>
</tr>
<tr>
<td>Total Manufacturing Sector</td>
<td>0.91*</td>
<td>0.01</td>
<td>0.28</td>
<td>0.05</td>
<td>0.90*</td>
<td>3.08</td>
</tr>
</tbody>
</table>

**NOTES:** * indicates significance at 5% level.

In column 6, the * indicates that the addition of the time and knowledge variables adds significantly to the variance explained by capital-intensity alone (at the 5% level).
TABLE 2

$R^2$ BETWEEN "KNOWLEDGE" AND "TIME"

<table>
<thead>
<tr>
<th>Sector</th>
<th>for $\phi = 0$</th>
<th>for $\phi = .30$</th>
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<tr>
<td>Food Processing</td>
<td>.996</td>
<td>.978</td>
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<tr>
<td>Tobacco Manufacturing</td>
<td>.956</td>
<td>.895</td>
</tr>
<tr>
<td>Textile Manufacturing</td>
<td>.996</td>
<td>.960</td>
</tr>
<tr>
<td>Footwear and Wearing Apparel</td>
<td>.992</td>
<td>.958</td>
</tr>
<tr>
<td>Paper and Paper Products</td>
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<td>.984</td>
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<tr>
<td>Printing and Publishing</td>
<td>.998</td>
<td>.990</td>
</tr>
<tr>
<td>Leather and Leather Products</td>
<td>.998</td>
<td>.964</td>
</tr>
<tr>
<td>Rubber and Rubber Products</td>
<td>.986</td>
<td>.949</td>
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<tr>
<td>Chemicals</td>
<td>.931</td>
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<tr>
<td>Non-Metallic Mineral Products</td>
<td>.998</td>
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<tr>
<td>Basic Metals Industries</td>
<td>.996</td>
<td>.970</td>
</tr>
<tr>
<td>Metal Products Manufacturing</td>
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<td>.970</td>
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<td>Non-Electrical Machinery</td>
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<tr>
<td>Electrical Machinery</td>
<td>.994</td>
<td>.986</td>
</tr>
<tr>
<td>Transport Equipment</td>
<td>.996</td>
<td>.978</td>
</tr>
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<td>Miscellaneous Industries</td>
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<td>.893</td>
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<td>Total Manufacturing Sector</td>
<td>.994</td>
<td>.955</td>
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</table>
# TABLE 3

PRODUCTION FUNCTION PARAMETER ESTIMATES (EQUATION 22)

<table>
<thead>
<tr>
<th>Sector</th>
<th>α</th>
<th>λ</th>
<th>γ</th>
<th>φ</th>
<th>R²</th>
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<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
</tr>
<tr>
<td>Food Processing</td>
<td>-0.38*</td>
<td>0.06*</td>
<td>0.40*</td>
<td>0.30</td>
<td>0.79*</td>
</tr>
<tr>
<td>Tobacco Manufacturing</td>
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<td>-1.15*</td>
<td>0.00</td>
<td>0.78*</td>
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<tr>
<td>Textile Manufacturing</td>
<td>-2.46*</td>
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<td>2.17*</td>
<td>0.05</td>
<td>0.92</td>
</tr>
<tr>
<td>Footwear and Wearing Apparel</td>
<td>-0.10</td>
<td>0.02</td>
<td>0.53*</td>
<td>0.00</td>
<td>0.11*</td>
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<tr>
<td>Paper and Paper Products</td>
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<td>Printing and Publishing</td>
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<td>-0.04*</td>
<td>0.32</td>
<td>0.20</td>
<td>0.19</td>
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<td>Rubber and Rubber Products</td>
<td>1.23</td>
<td>0.25*</td>
<td>-1.62*</td>
<td>0.30</td>
<td>0.89</td>
</tr>
<tr>
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<td>0.74</td>
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<td>-0.43</td>
<td>0.00</td>
<td>0.73</td>
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<td>0.79</td>
</tr>
<tr>
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<td>0.13</td>
<td>0.02</td>
<td>-0.06</td>
<td>0.00</td>
<td>0.31</td>
</tr>
<tr>
<td>Electrical Machinery</td>
<td>0.72*</td>
<td>0.06*</td>
<td>-0.13</td>
<td>0.00</td>
<td>0.72</td>
</tr>
<tr>
<td>Transport Equipment</td>
<td>1.71</td>
<td>0.17</td>
<td>-1.54*</td>
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<td>0.64</td>
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<td>-0.00</td>
<td>0.00</td>
<td>0.56</td>
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<tr>
<td>Total Manufacturing Sector</td>
<td>0.55</td>
<td>0.02</td>
<td>0.31</td>
<td>0.05</td>
<td>0.90*</td>
</tr>
</tbody>
</table>

NOTES: * indicates significance at 5% level.
In column 6, the * indicates that the addition of the time and knowledge variables adds significantly to the variance explained by capital-intensity alone (at the 5% level).
REFERENCES


