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Abstract

We consider a monopolist selling durable goods to consumers with unit demands but different preferences for quality. The seller can offer items of different quality at the same time to induce buyers to self-select, as in Mussa-Rosen (1978), but is not artificially constrained to offer only one such menu. Instead the seller can offer a *sequence* of menus over time. In the two-buyer case, the seller finds it optimal to *abandon* multi-item menus with their quality distortions and instead induces self-selection intertemporally. In the unique subgame-perfect equilibrium of the finite-horizon game and the particular equilibrium that we consider in the infinite-horizon game, the monopolist offers in succession single items of efficient quality. In the continuous-time limit of the infinite-horizon game (under both complete and incomplete information), the monopolist approximates the present value of perfect price discrimination. All of our qualitative results for the two-buyer case continue to hold with an arbitrary, finite number of buyers of different types in some equilibria of the complete-information, infinite-horizon game .

1. Introduction

Consider the seller of a durable good such as a microwave oven. Buyers want at most one item but differ in their willingness to pay for extra features ("quality"). Since buyers differ, the seller has an incentive to price discriminate; but he cannot offer an item to any buyer without making it available to everyone else. To price discriminate, the seller must induce buyers of one type to purchase a particular item while buyers of a different type purchase a different item.

In principle, the seller can induce such self-selection in two ways. At any given time, he can offer items of different quality which will have differing appeal to the heterogeneous buyers. Alternatively, the seller may be able to induce *intertemporal* self-selection since anticipation of his future offers may lead some types of buyers to purchase a current item while others prefer to wait for something more appealing.

Each of these forms of self-selection has been studied extensively in the literature—but only in isolation. For example, Coase's conjecture about the unprofitability of durable-goods monopoly spawned a literature on intertemporal self-selection; but since the monopolist in this literature offers items of only a single quality, he cannot induce self-selection at a point in time.[1] Mussa and Rosen (1978), on the other hand, examine self-selection at a point in time; but by constraining their monopolist to offer only a single menu of items, they eliminate the possibility of intertemporal self-selection.[2]

In reality, a seller of durable goods has the opportunity to induce *both* forms of self-selection. Whether he utilizes one form exclusively or the two forms in combination depends in principle on their relative merits.

We consider here the problem of a durable-goods monopolist who can make a *sequence* of offers to two heterogeneous buyers with unit demands. However, our complete-information, infinite-horizon results hold for any finite number of buyers.

If both buyers remain in the final period of our finite-horizon game, the equilibrium is identical to what Mussa-Rosen describe: in the absence of a corner solution, the monopolist finds it optimal to offer a menu of items to induce self-selection. The optimal menu necessarily contains items of inefficient quality and fails to extract the entire surplus.

In *every* period prior to the last, however, the monopolist finds it optimal to abandon menus. Instead, he offers a single item of *efficient* quality based on his observation or—in the absence of direct observation—his beliefs about the marginal valuations of the remaining buyers.

Inducing self-selection over time turns out in the models we investigate to dominate inducing self-selection at a point in time. The monopolist offers a single item of a particular quality for sale and replaces it with an item of a different quality if and only if a sale occurs. Although the next item offered is more attractive to the previous purchaser than the item he actually purchased, that buyer is nonetheless rational to purchase the previous item. For he anticipates that the seller would not have made the subsequent item available until the previous item was purchased. By utilizing intertemporal self-selection—instead of point-in-time self-selection—the monopolist avoids having to distort the quality of the second item in order to make it *unattractive* to the first buyer; instead, he makes the second item *unavailable* (to either buyer) until the prior item is purchased.

The benefit of using intertemporal self-selection is that the monopolist can dispense with sales of inefficient quality. The cost is that he must space out his sales rather than make them at the same time. However, this cost becomes arbitrarily small if the interval between offers can be made sufficiently short. As long as the time interval between successive offers is sufficiently small, the monopolist offers only one item at a time and does not distort quality.

In this case, the monopolist increases the present value of his profits relative to Mussa-Rosen's single-offer case. In the continuous-time limit, the monopolist approximates the present value of perfect price discrimination if the time-horizon is sufficiently long.

These results differ so markedly from the received wisdom on second-degree price discrimination that readers may wonder whether they are artifacts of some assumption about the time-horizon or information structure. For this reason, we also consider infinite-horizon games where the monopolist can make an unlimited number of offers in succession and has either complete or incomplete information about the marginal valuation of a particular buyer. Even in those cases, the equilibria which we consider have the same characteristics: no multi-item offers, no quality distortions, and—in the continuous-time limit—the present value of perfect price discrimination.

In section 2, we review Mussa-Rosen's single-offer case, which is also the two-buyer subgame in the final period of our finite-horizon model. In section 3, we examine the multi-offer, complete-information game with a finite or unbounded horizon. Section 4 extends our infinite-horizon results to incomplete information and section 5 contains some concluding remarks.

The points we emphasize here continue to hold in complete-information, infinite-horizon games with arbitrary, finite numbers of buyers of each of a finite number of types. In the appendix of this paper we show that in any such game there always exists a subgame-perfect equilibrium in which the monopolist offers a sequence of single-item menus and obtains a present value that approaches the profit obtained by a perfectly discriminating monopolist in the continuous-time limit. Moreover, the strategies used by each player in this equilibrium are virtually identical to the ones we discuss in section 3.

2. The Optimal Single Offer of Prices and Qualities

In this section, we review the optimal policy developed by Mussa and Rosen (1978) for a monopolist who sells a good to buyers who value quality differently. We also establish notation which will be useful in the following section.

In Mussa and Rosen's model, the monopolist is restricted to making a *single* offer to the buyers. An offer specifies a menu of items, with each item fully characterized by its quality level and price. Each buyer is free to select any item from the menu.

Buyers are assumed to have unit demands; hence, each will select at most one item from the menu. For simplicity, we review the two-buyer case.

We regard the problem as a two-stage, single-period game. In the first stage, the monopolist makes an offer taking into account that in the second stage buyers will simultaneously respond optimally to it. If buyer α (for $\alpha = 1,2$) pays p_α to purchase an item of quality q_α, then the buyer's utility payoff is $U_\alpha = v_\alpha q_\alpha - p_\alpha$, where $v_1 > v_2 > 0$. A buyer who chooses not to purchase an item receives a utility of zero.

The buyers' marginal valuations for quality, v_1 and v_2, are assumed to be common knowledge. We assume further that the monopolist is either unable to identify which marginal valuation belongs to which buyer or, alternatively, that although he has this information he is barred legally from using it to offer an item to one type of buyer which is unavailable to the other type.[3]

When buyer α pays p_α to purchase an item of quality q_α, the payoff to the monopolist is $(p_1 - c(q_1)) + (p_2 - c(q_2))$, where $c(q)$ is the constant marginal cost (with respect to volume) of a unit with quality q. We adopt Mussa and Rosen's assumptions that $c'(q),\ c''(q) > 0$ for all q. We also assume that $c(0) = 0$ and that $v_2 > c'(0)$.[4]

In the subgame-perfect equilibrium of the game described above, it is optimal for the monopolist to adopt the following strategy.

The monopolist offers (p_1, q_1) and (p_2, q_2) in order to:

$$\max\ (p_1 - c(q_1)) + (p_2 - c(q_2)) \tag{1a}$$

subject to the constraints:

$$\begin{aligned} p_\alpha\,,\, q_\alpha &\geq 0 \quad \text{for } \alpha = 1,2 \\ v_1 q_1 - p_1 &\geq 0 \qquad \text{(1b)} \\ v_2 q_2 - p_2 &\geq 0 \qquad \text{(1c)} \\ v_1 q_1 - p_1 &\geq v_1 q_2 - p_2 \qquad \text{(1d)} \\ v_2 q_2 - p_2 &\geq v_2 q_1 - p_1 \,. \qquad \text{(1e)} \end{aligned}$$

The optimal strategy for each buyer in the second stage is to choose the best item offered in the first stage that generates nonnegative utility. The constraints (1b) through (1e) indicate how the buyers' strategies restrict the monopolist's equilibrium offers. Equations (1b) and (1c) are "participation" constraints indicating that buyers will only purchase items which generate nonnegative utility. Equations (1d) and (1e) are "self-selection" constraints. Equation (1d) states that (p_1, q_1), the item intended for buyer 1, will only be purchased by buyer 1 if it generates utility for him that is at least as high as the utility generated by the alternative item. Equation (1e) has a similar interpretation for buyer 2.

It is straightforward to verify that the optimization problem specified by equations (1a) through (1e) has a unique solution.

The two self-selection constraints can be re-written as follows:

$$v_1(q_1 - q_2)\ \geq\ p_1 - p_2\ \geq\ v_2(q_1 - q_2)\,. \tag{2}$$

If the monopolist's offer is optimal, at least one of the two participation constraints must bind. For if neither were binding, the monopolist could raise the two prices on the qualities offered by a small, common amount and still satisfy the participation constraints. Equation (2) indicates that as long as the *difference* in the two prices in the new program is unchanged, each self-selection constraint would also still hold. Hence the new program would still be feasible and would result in higher profits.

In equilibrium, buyer 2 will never be the only buyer to purchase. For any item (with $q_2 > 0$) which gives him nonnegative utility would produce strictly positive utility for buyer 1, and he would also purchase. There are therefore only two possibilities: sell to both buyers or sell to buyer 1. These correspond, respectively, to the interior ($p_\alpha > 0$ and $q_\alpha > 0$ for $\alpha = 1$ *and* $\alpha = 2$) and corner solutions ($p_2 = 0$ and $q_2 = 0$) of the monopolist's constrained optimization problem.

Since buyer 1 receives positive untility from any item that provides buyer 2 with positive utility, buyer 1's participation constraint cannot bind if buyer 2's participation constraint is not binding. Since at least one of the participation constraints must bind, the seller always extracts all of the surplus from buyer 2, equation (1c) holds with equality, and, hence, $p_2 = v_2 q_2$. The self-selection constraint of buyer 1 must also bind. For if it did not, then the monopolist could increase p_1 while leaving his other decision variables unchanged. This would generate a feasible but more profitable offer.

By solving the binding constraints (equations (1c) and (1d)) for p_1 and p_2 and substituting the resulting expressions in equation (1a), we obtain a strictly concave objective function in q_1 and q_2 which is maximized subject only to the nonnegativity constraints for q_1 and q_2. Solving the resulting optimization problem, we obtain for q_1: $v_1 - c'(q_1) = 0$. As for q_2, there are two possibilities: Either $q_2 > 0$ and $2v_2 - v_1 - c'(q_2) = 0$ or, alternatively, $q_2 = 0$ and $2v_2 - v_1 - c'(0) \leq 0$.[5]

If $2v_2 - v_1 - c'(0) > 0$, then it is optimal for the monopolist to sell to both buyers. Let $(p_\alpha^{MR}, q_\alpha^{MR})$ denote the item purchased by a type α buyer in this case. Denote the inverse of the marginal cost by $f(v) \equiv c'^{-1}(v)$. The optimal offer for the Mussa-Rosen monopolist is to set:

$$\begin{aligned} q_1^{MR} &= f(v_1) \qquad \text{(3a)} \\ q_2^{MR} &= f(2v_2 - v_1) \qquad \text{(3b)} \\ p_1^{MR} &= v_1 q_1^{MR} - (v_1 - v_2) q_2^{MR} \qquad \text{(3c)} \\ p_2^{MR} &= v_2 q_2^{MR}\,. \qquad \text{(3d)} \end{aligned}$$

It will be convenient to define the following additional variables for the case where the monopolist sells to both buyers: π_α^{MR}, V^{MR}, and U_α^{MR}.

Let π_α^{MR} denote the profit the monopolist derives from the type α buyer when both types buy as specified in equations (3a) through (3d):

$$\pi_\alpha^{MR} = p_\alpha^{MR} - c(q_\alpha^{MR})\,. \tag{4}$$

Let V^{MR} denote the sum of the payoffs which the monopolist collects from the buyers in this case:

$$V^{MR} = \pi_1^{MR} + \pi_2^{MR}\,. \tag{5}$$

Let U_α^{MR} denote the equilibrium payoff of buyer α in this case. That is, $U_2^{MR} = v_2 q_2^{MR} - p_2^{MR} = 0$ and $U_1^{MR} = v_1 q_1^{MR} - p_1^{MR}$.

If instead $2v_2 - v_1 - c'(0) \leq 0$, then a corner optimum occurs. In this circumstance, it is optimal for the Mussa-Rosen monopolist to offer only a *single* item which only buyer 1 accepts: $q_2 = p_2 = 0$ and $q_1 = f(v_1)$, $p_1 = v_1 q_1$.

In discussing the case of the Mussa-Rosen monopolist, it is useful to keep in mind the benchmark of perfect or first-degree price discrimination. Recall that a perfectly discriminating monopolist can offer an item on a take-it-or-leave-it basis to each buyer in isolation—that is, he can offer an item to one buyer while making it unavailable to the other buyer. Therefore his objective function is the same as (1a) but he is restricted *only* by the two participation constraints (1b and 1c)—and not by either self-selection constraint. The profit obtained from each type α buyer will be $v_\alpha q_\alpha - c(q_\alpha)$. The optimal item for such a monopolist to offer to a type α buyer has a price and quality level denoted by $(p_\alpha^{1st}, q_\alpha^{1st})$ and specified by the equations:

$$p_\alpha^{1st} = v_\alpha q_\alpha^{1st}, \tag{6a}$$
$$q_\alpha^{1st} = f(v_\alpha). \tag{6b}$$

Note that since the monopolist can extract all of the surplus from each buyer, he offers each the "efficient" quality level (the level which maximizes total surplus).

It will be convenient to define the following additional variable for the case of perfect price discrimination. Let π_α^{1st} denote the profit that the monopolist obtains when a type α buyer accepts the optimal take-it-or-leave-it offer specified in equations (6a) and (6b).

$$\pi_\alpha^{1st} = p_\alpha^{1st} - c(q_\alpha^{1st}). \tag{7}$$

Equations (3a) and (6b) indicate that buyer 1, who has the higher marginal valuation for quality, always receives the same quality level from the Mussa-Rosen monopolist as he would from a perfectly discriminating monopolist. This is true whether or not the Mussa-Rosen monopolist chooses to sell to buyer 2. On the other hand, equations (3b) and (6b) indicate that buyer 2, with a lower marginal valuation for quality, receives a quality level that is always less than the efficient level for buyer 2. (Buyer 2 receives either $q_2^{MR} < q_2^{1st}$ or makes no purchase.)

The equilibrium in the Mussa-Rosen, single-period game also has the following features. The lower valuation buyer, never obtains positive surplus. When it is optimal to sell only to buyer 1, then that buyer also receives only zero utility. However, when the monopolist sells to both buyers, then the higher valuation buyer receives strictly positive utility ($U_1^{MR} > 0$).

The Mussa-Rosen monopolist is unable to achieve the profits of the perfectly-discriminating monopolist even when it is optimal to serve both customers ($V^{MR} < \pi_1^{1st} + \pi_2^{1st}$). The offer of the perfectly discriminating monopolist must therefore violate one or more of the four constraints of the Mussa-Rosen monopolist. It must satisfy the two participation constraints and it happens to satisfy buyer 2's self-selection constraint. However, it violates the self-selection constraint of buyer 1.

Figure 1 summarizes the results. The qualities offered by the monopolist are measured along the horizontal axis, and the prices charged for each quality are measured along the vertical axis. Since buyers prefer higher quality and lower prices, indifference curves are positively sloped; points southeast of a given indifference curve are strictly preferred to points on the curve.

Given our assumptions, each indifference curve for the type α buyer is linear with slope v_α. The steeper upward-sloping line labeled $U_1 = 0$ and the flatter upward-sloping line labeled $U_2 = 0$ indicate the combinations of price and quality that provide zero utility for buyer 1 and buyer 2 respectively. The optimal offer of a perfectly discriminating monopolist to buyer 1 occurs at the intersection of the vertical line labeled q_1^{1st} and the line $U_1 = 0$. Similarly, the optimal offer that a perfectly discriminating monopolist would make to buyer 2 occurs at the intersection of the vertical line $q = q_2^{1st}$ and the line $U_2 = 0$.

The dilemma of Mussa-Rosen's monopolist is the following. He cannot offer an item to one buyer which is unavailable to the other. But if he were to offer simultaneously the two items sold separately by a perfect price discriminator, buyer 1 would strictly prefer (p_2^{1st}, q_2^{1st}) to the alternative item (p_1^{1st}, q_1^{1st}) since the former item would give him a strictly positive surplus. Since any item (with $q_2 > 0$) yielding zero surplus to buyer 2 will yield strictly positive surplus to buyer 1, the Mussa-Rosen monopolist cannot offer items to each *simultaneously* which extract all of their surplus.

As we have seen, the best that a monopolist can do if constrained to make every item offered available to both buyers is either to offer two items (the interior case) or to offer a single item (the corner case). In the interior case, the two items would both generate the same positive surplus (U_1^{MR}) for buyer 1 and therefore must lie on the same indifference curve (as illustrated by the dashed line in figure 1). The quality of the higher quality item would be efficient (q_1^{1st}) but the quality of the lower quality item—although strictly positive—would be below the efficient level ($q_2^{MR} < q_2^{1st}$). The corner case occurs when the dashed line coincides with buyer 1's indifference curve through the origin. Given buyer 1's self-selection constraint, the best *single* offer the monopolist can devise contains either two items and earns V^{MR} or contains one item and earns π_1^{1st}. Both of these payoffs are strictly smaller than $\pi_1^{1st} + \pi_2^{1st}$, the profits of perfect price discrimination.

But as we shall illustrate in the next section, even if the monopolist must make each item available to all buyers his difficulties can be alleviated provided he may make *multiple* offers. Indeed, if offers can be made in sufficiently rapid succession, the monopolist can often approximate the profits of perfect price discrimination. The next section exposits these results for the case of two buyers. The appendix of our paper generalizes the results to the case of a complete-information, infinite-horizon game where there are initially a finite number of buyers of each of a finite number of types.

3. Optimal Sequential Offers

In this section, we consider the gains which the monopolist can achieve by making a *sequence* of offers to the two buyers instead of a single offer. Making a sequence of offers is advantageous because it induces buyers of different types to purchase in different periods. For discount factors sufficiently close to one, we will show that the monopolist can always increase the present value of his profits by making a succession of single-item offers—each based on his observation or beliefs about the marginal valuations of the remaining buyers. In the continuous-time limit of the infinite-horizon game, the monopolist can approximate the profits of a perfectly discriminating monopolist.

We model the problem as a multi-period game, with the periods indexed by $t = 1, 2 \ldots T$. As in the previous section, there are two stages in each period. In the first stage, the monopolist makes an offer consisting of a menu of items, with each item fully characterized by its quality level and price. In the second stage, each buyer simultaneously chooses whether to select a single item from among those offered or to continue to the next period. Buyers are assumed to have unit demands and select at most one item during the course of the game. Since the monopolist makes an offer in each period, the horizon, T, corresponds to the maximum number of distinct offers that the monopolist can make during the game. When T is infinite, the number of offers that the monopolist can make is unbounded. In each game analyzed in this section there are only two buyers each of whom makes at most one purchase during the game; hence, the monopolist sells at most two units of the good no matter what we assume about the horizon length.

If buyer α purchases an item of quality q_α in period t_α and pays price p_α, then he obtains the utility (discounted to $t = 1$):

$$U_\alpha = \beta^{t_\alpha - 1} (v_\alpha q_\alpha - p_\alpha) , \tag{8}$$

where β is a discount factor assumed to be the same for all players and v_α is a positive constant. We assume that $1 > \beta > 0$ and that $v_1 > v_2 > 0$. A buyer who chooses not to purchase in any period receives a utility of zero.

If buyer 1 accepts (p_1, q_1) in period t_1 and buyer 2 accepts (p_2, q_2) in t_2, then the present value received by the monopolist is:

$$V = \beta^{t_1 - 1} (p_1 - c(q_1)) + \beta^{t_2 - 1} (p_2 - c(q_2)) . \tag{9}$$

If only the single buyer α purchases during the game (in period t_α), then the monopolist receives $\beta^{t_\alpha - 1} (p_\alpha - c(q_\alpha))$, and if neither buyer makes a purchase then the monopolist receives zero. As in the previous section, we assume that $c'(q)$, $c''(q) > 0$ for all q, that $c(0) = 0$, and that $v_2 > c'(0)$.

The marginal valuations for quality, v_1 and v_2, are assumed to be common knowledge. Throughout section 3, we assume the monopolist has complete information and knows the type (marginal valuation) of each specific buyer.

In section 4, we assume instead that the monopolist has incomplete information and does not know which marginal valuation belongs to which buyer. When the monopolist does not observe the marginal valuation of a buyer who accepts an offer, he tries to infer but cannot observe the marginal valuation of the remaining buyer.

A player's strategy in each period specifies his action in that period as a function of time and anything else that he can observe — for example, the sequence of previous offers, the number of buyers purchasing specific price-quality pairs, the periods in which various items were purchased, the marginal valuations of the buyers who purchased each item, and, for the buyers, the menu of prices and qualities offered in the first stage of the current period. The monopolist's strategy in each period specifies his offer in the first stage of the period as a function of these things. Similarly, a buyer's strategy specifies which price-quality pair (if any) to accept in the second stage of each period. The monopolist chooses his strategy to maximize the present value of his profits. Each buyer chooses a strategy to maximize his own utility.

3.1. Equilibrium in Complete-Information Games

We now consider both finite- and infinite-horizon games with complete information. We consider in turn the two parameter regimes which gave rise in Mussa-Rosen's single-offer case, respectively, to corner and interior equilibria: $2v_2 - v_1 - c'(0) \leq 0$ and $2v_2 - v_1 - c'(0) > 0$. In each case, we begin by describing the subgame-perfect equilibrium strategies for each of the three players. Having described them, we then verify that each strategy is optimal in every subgame given the other strategies.

In the regime where $2v_2 - v_1 - c'(0) \leq 0$, the buyers use the following strategy in each period. In the current period t, each buyer accepts the best offer (if any) of those which generate nonnegative utility for him. We refer to this strategy as the "get-it-while-you-can" strategy since the buyer seizes the first opportunity for surplus as if no future opportunities might present themselves.

The monopolist uses the following strategy in period t. If buyer 1 or both buyers remain in the game at the beginning of the period, then the monopolist offers the single price-quality pair (p_1^{1st}, q_1^{1st}) defined by equations (6a) and (6b). If only buyer 2 remains, then the monopolist offers the single price-quality pair (p_2^{1st}, q_2^{1st}). Using the above strategy, the monopolist "eats his way down" the distribution of buyers' marginal valuations for quality, extracting the surplus in each period from the remaining buyer with the highest marginal valuation for quality. We refer to the monopolist's strategy as the "Pacman strategy."

Next, we verify that these strategies are subgame perfect. In both finite- and infinite-horizon games, no buyer can ever obtain positive utility when the monopolist uses the Pacman strategy. Hence, it is optimal for each buyer to use the get-it-while-you-can strategy in each period.

Finally, we verify the optimality of the seller's strategy when buyers use the get-it-while-you-can strategy in each period. In any period with only one remaining buyer, the Pacman strategy is clearly optimal for the monopolist since it extracts the maximum possible surplus from this buyer.

Suppose instead that both buyers remain. Any price-quality pair (with $q > 0$) that generates nonnegative utility for buyer 2 generates strictly positive utility for buyer 1. Hence, it is infeasible—given the buyers' strategies—for the monopolist to sell only to buyer 2. This leaves the monopolist two choices: sell immediately to both buyers or sell only to buyer 1. Among all the offers which only buyer 1 would accept, the Pacman offer is clearly optimal for the monopolist since it extracts the maximum possible surplus from buyer 1. Moreover, the analysis in the previous section indicates that this profit strictly exceeds the profit from selling simultaneously to the two buyers.

Indeed, if at least one future period remains the Pacman strategy is even more profitable than in the single-offer case since the opportunity to make future offers enables the seller to extract the entire surplus of buyer 2 in the next period. Hence, the Pacman strategy results in the following present value for the monopolist:

$$V^P = \pi_1^{1st} + \beta\pi_2^{1st} . \tag{10}$$

The above arguments demonstrate that when $2v_2 - v_1 - c'(0) \leq 0$, the use of the Pacman strategy by the monopolist and the get-it-while-you-can strategy by each buyer constitutes a subgame-perfect equilibrium for the complete-information game with any horizon length, either finite or infinite. Moreover, these strategies constitute a subgame-perfect equilibrium for any discount factor between zero and one. In any game in which the monopolist can make two or more offers ($T \geq 2$), the monopolist receives the equilibrium present value specified in equation (10). This strictly dominates the profit obtained by a static Mussa-Rosen strategy.

We now consider the regime where $2v_2 - v_1 - c'(0) > 0$. Since we are ultimately interested in the play of the game when the time interval between successive offers becomes negligible, we focus on the equilibrium strategies when the discount factor pertaining to the interval between offers approaches one.

For discount factors sufficiently close to one, the subgame-perfect equilibrium strategies in the finite-horizon game may be described as follows. Buyer 2 uses the get-it-while-you-can strategy in each period. Buyer 1 uses the get-it-while-you-can strategy in any subgame where he is the sole remaining buyer. In addition, buyer 1 always uses the get-it-while-you-can strategy in the final period of the game ($t = T$) whether or not buyer 2 remains. Finally, if both buyers remain prior to T, buyer 1 uses the get-it-while-you-can strategy in the current period whenever he anticipates being the only remaining buyer from the next period onward; buyer 1 will anticipate being isolated if he observes that some item currently offered by the monopolist will provide nonnegative utility for buyer 2.

If both buyers remain prior to T and if buyer 1 observes that no item offered will yield nonnegative utility to buyer 2, then buyer 1 is more selective: in the current period t he will accept the best price-quality pair (if any) of those that generate utility (discounted to period 1) greater than or equal to the reservation utility (also discounted to period 1) $\beta^{T-1}U_1^{MR}$.[6] Buyer 1's strategy in this case is a generalization of the get-it-while-you-can strategy; we refer to it as the "generalized get-it-while-you-can strategy with discounted reservation utility $\beta^{T-1}U_1^{MR}$."

As for the description of the seller's strategy, he will use the Pacman strategy defined in the previous section in any period where only one buyer remains. If both buyers remain in the final period, the monopolist will make the Mussa-Rosen offer specified in equations (3a) — (3d). If both buyers remain prior to T, the monopolist

will offer the single quality q_1^{1st} at a price $p(t)$ which is given by the equation:

$$p(t) = (1 - \beta^{T-t})p_1^{1st} + \beta^{T-t}p_1^{MR} . \tag{11}$$

In equilibrium, buyer 1 accepts ($p(1)$, q_1^{1st}) in the first period and buyer 2 accepts (p_2^{1st}, q_2^{1st}) in the second period.

To verify that these strategies are subgame perfect, note first that the get-it-while-you-can strategy is optimal for buyer 2 since he can never obtain positive utility when the monopolist uses his equilibrium strategy. For the same reason, the get-it-while-you-can strategy is optimal for buyer 1 when he is the sole remaining buyer. Since a failure to accept an offer in the final period results in zero utility, the get-it-while-you-can strategy is also optimal for buyer 1 in the final period whether or not buyer 2 remains. Finally, if the monopolist offers a price-quality pair that provides nonnegative utility for buyer 2, then buyer 2, using the get-it-while-you-can strategy, will accept it; buyer 1 can therefore expect only zero utility in future periods, and the get-it-while-you-can strategy is also optimal for him in this case.

If both buyers remain prior to T and the monopolist offers no item in period t which gives buyer 2 nonnegative utility, then buyer 1 anticipates that buyer 2 will remain and that the seller will offer q_1^{1st} in the next period at a price of $p(t+1)$ defined in equation (11). For all $t + 1 < T$, equation (11) implies that $p(t+1) > p_1^{MR}$. Hence, the analysis of the single-offer game in the previous section implies that buyer 2 will reject such an offer. More generally, buyer 1 can anticipate that as long as he declines to purchase, the monopolist will continue to offer ($p(s)$, q_1^{1st}) in any period $t < s < T$ and that buyer 2 will continue to reject these offers. Moreover, by substituting equations (6a) and (11) into equation (8) and using the fact that $U_1^{MR} = p_1^{1st} - p_1^{MR}$, it is straightforward to verify that buyer 1 obtains the same discounted utility, $\beta^{T-1}U_1^{MR}$, by accepting any of the offers $(p(s), q_1^{1st})$.

Finally, suppose that buyer 1 declines to purchase until the final period. In this case, he can anticipate that as his final act the monopolist would offer the Mussa-Rosen menu and that buyer 1 would again earn the discounted utility $\beta^{T-1}U_1^{MR}$ by accepting an item from this menu. Hence, in any future period including the last, the buyer expects to obtain the discounted utility $\beta^{T-1}U_1^{MR}$ if he fails to accept the monopolist's offer in period t when both buyers remain. It is thus optimal for buyer 1 to accept the best offer in period t of those that provide discounted utility greater than or equal to $\beta^{T-1}U_1^{MR}$; that is, it is optimal for buyer 1 to use the generalized get-it-while-you-can-strategy with discounted reservation utility $\beta^{T-1}U_1^{MR}$.

To complete the verification of subgame perfection, we consider the optimality of the seller's strategy. As in the other parameter regime, the Pacman strategy is clearly optimal for the monopolist in any subgame with one remaining buyer since it extracts the maximum possible surplus from such a buyer.

If two buyers remain in the final period, then the game from that point onward is identical to the single-period game analyzed in section 2. Hence, the Mussa-Rosen strategy described in equations (3a) — (3d) is clearly optimal for the monopolist in the final period when both buyers remain.

If two buyers remain prior to the final period, the monopolist again has two choices: sell immediately to both buyers or sell only to buyer 1. It is infeasible to sell only to buyer 2, since any item (with $q > 0$) giving him nonnegative utility would give buyer 1 strictly positive utility; hence, buyer 1 would purchase either that item or some more attractive alternative and sales to both buyers would have occurred.

We now verify that, for sufficiently high discount factors, the seller will *never* make an offer containing two items prior to the final period. To sell simultaneously to both buyers, the monopolist must provide buyer 2 with nonnegative utility. But this will make buyer 1 anticipate isolation and he too will accept the best offer generating nonnegative utility. The analysis in the previous section demonstrates that when both buyers require nonnegative utility to participate and $2v_2 - v_1 - c'(0) > 0$, the static Mussa-Rosen offer is the most profitable two-item offer for the seller and results in a payoff of $\pi_1^{MR} + \pi_2^{MR}$.

The monopolist can easily dominate this best two-item offer provided he can make at least one future offer (that is, if $t < T$). For, suppose the monopolist simply deleted the lower quality item from the Mussa-Rosen offer. Buyer 2 will not accept the remaining item since it provides him with negative utility (as it did when it was part of the Mussa-Rosen menu). On the other hand, buyer 1 can obtain the discounted utility of $\beta^{t-1}U_1^{MR}$ by accepting the remaining item in period t. Since $\beta^{t-1}U_1^{MR} > \beta^{T-1}U_1^{MR}$, buyer 1—using his equilibrium strategy—would purchase the item. Hence, the monopolist can anticipate that buyer 1 will accept the offer and buyer 2 will reject it. The seller could then offer (p_2^{1st}, q_2^{1st}) next period and, since it would provide buyer 2 with nonnegative utility, could anticipate that the offer would be accepted. This strategy yields the seller $\pi_1^{MR} + \beta\pi_2^{1st}$. For $\beta > \beta^*$ where $\beta^* = \pi_2^{MR}/\pi_2^{1st}$ this payoff strictly exceeds the payoff from the two-item Mussa-Rosen offer.

Of course, this strategy—while superior to *any* two-item menu—is not the best single-item offer. Recall that in any period t prior to the last period ($t < T$), if both buyers remain and buyer 1 does not anticipate that buyer 2 will make a purchase in period t, then buyer 1 will accept the best offer yielding discounted utility $\beta^{T-1}U_1^{MR}$. The monopolist should therefore continue to offer q_1^{1st} since that maximizes the total surplus; but he should price it so as to extract all but the minimum surplus necessary for buyer 1 to purchase. The price of the proposed dominating offer (p_1^{MR}) is suboptimal since it gives U_1^{MR} to buyer 1 when he requires, in terms of period t utility, only $\beta^{T-t}U_1^{MR}$. The optimal price should therefore be $p(t) = p_1^{MR} + (U_1^{MR} - \beta^{T-t}U_1^{MR})$. Using the fact that $U_1^{MR} = p_1^{1st} - p_1^{MR}$, we conclude that the optimal price for the item of quality q_1^{1st} is given by equation (11).

The above discussion demonstrates that the proposed strategies constitute a subgame-perfect equilibrium in the two-buyer, finite-horizon game with complete information for any $\beta > \beta^*$. In equilibrium, the monopolist sells a unit of quality q_1^{1st} to buyer 1 in the first period at a price given by $p(1)$ in equation (11) and makes the offer (p_2^{1st}, q_2^{1st}) to buyer 2 in the second period. Substituting these prices and qualities into equation (9) and collecting terms using equations (4) and (7), we obtain the following expression for the equilibrium present value obtained by the monopolist in the play of the T-period game:

$$V_T = (1 - \beta^{T-1})\pi_1^{1st} + \beta^{T-1}\pi_1^{MR} + \beta\pi_2^{1st} . \tag{12}$$

For infinite-horizon games with complete information and sufficiently high β, the analysis when $2v_2 - v_1 - c'(0) > 0$ is virtually identical to the analysis for the previous parameter regime. In the infinite-horizon game, the use in each period of the get-it-while-you-can strategy by each buyer and the Pacman strategy by the monopolist constitutes a subgame-perfect equilibrium. The equilibrium present value obtained by the monopolist, V^P, is given by equation (10).

As in the previous parameter regime, the get-it-while-you-can strategy is optimal for each buyer since the buyers can anticipate only zero utility in future periods. The Pacman strategy is optimal for the monopolist if $V^P > V^{MR}$. A comparison of equations (5) and (10) indicates that $V^P > V^{MR}$ if $\beta > \beta^*$.[7] In the play of the equilibrium, the monopolist makes the offer (p_1^{1st}, q_1^{1st}) in the first period and receives the equilibrium present value, V^P, given in equation (10).

It can be verified by working backwards that whenever the horizon is finite the subgame-perfect equilibrium is essentially unique.[8] In contrast, there are a continuum of equilibria in the infinite-horizon case. The particular infinite-horizon equilibrium we discussed was chosen because the strategies supporting it are the limiting strategies in the finite-horizon case for a sufficiently long horizon. Buyer 2 in either case accepts the best offer in a period yielding nonnegative utility. Buyer 1 in the infinite-horizon case also accepts such offers and in the finite-horizon case adopts the identical strategy prior to T except when two buyers remain and no current item would give buyer 2 nonnegative utility. In such circumstances, buyer 1 purchases the best item offered provided it yields $\beta^{T-1}U_1^{MR}$. But this strategy becomes virtually the same as "get-it-while-you-can" as T becomes large. Recall that when the monopolist is restricted to a maximum of T offers, he makes the offer $(p(1), q_1^{1st})$ in the first period, with $p(1)$ given in equation (11), and obtains the equilibrium present value, V_T, given in equation (12). For each fixed value of β greater than β^*, equations (10), (11), and (12) indicate that the monopolist's first-period offer and present value in the T-period game approach the infinite-horizon first-period offer, (p_1^{1st}, q_1^{1st}), and the present value obtained in the infinite-horizon game as T becomes large. ($p(1) \to p_1^{1st}$ and $V_T \to V^P$ as $T \to \infty$.)

We conclude this subsection by putting the multi-offer case in the context of figure 1. In the parameter regime where $2v_2 - v_1 - c'(0) > 0$, the monopolist finds it optimal to make single-item offers (prior to any final period) as long as the discount factor is sufficiently high. The heavy dots in figure 1 represent the offers made to buyers 1 and 2 in the play of the multi-offer game with two buyers. The offer purchased by buyer 1 in the first period lies above the dashed indifference curve of buyer 1 and so provides lower utility for him than he receives in the single-offer game studied by Mussa and Rosen. In the second period, buyer 2 accepts the offer that he would receive from a perfectly discriminating monopolist.

Since this second offer lies below the self-selection constraint for buyer 1, buyer 1 would prefer to receive the offer accepted by buyer 2 rather than the offer which buyer 1 actually accepts. Why, then, does buyer 1 accept the inferior offer rather than wait for something better? Buyer 1 accepts the inferior offer because he anticipates that as long as he waits the monopolist would decline to offer anything better. This is not an "incredible threat" by the monopolist but instead his optimal behavior in the remainder of the game as long as both buyers remain.

3.2. The Continuous-Time Limit

One way to relate the periods in our game to calendar time is to assume that a fixed, finite interval of calendar time, δ, must elapse between successive offers. If it is also assumed that there is a maximum amount of time, Γ, after which the game must end, then T, the maximum number of offers which the monopolist can make, is related to the time horizon, Γ, via the equation:

$$T = \frac{\Gamma}{\delta} . \tag{13}$$

If the game has an infinite time horizon, then there is no apriori bound on the maximum number of offers; that is, $T = \infty$.

If ρ is a fixed parameter expressing the common instantaneous rate of time preference of the buyers and the monopolist, then β, the discount factor *for the time interval between successive offers* is related to ρ and δ via the equation:

$$\beta = \exp[-\rho\delta] . \tag{14}$$

Using the above formulation, we can define a continuous-time limit to our game by allowing the interval between successive offers δ to approach zero. In this case, although the discount factor for any fixed time interval (e.g. one year) remains constant, β must approach one.

In the infinite-horizon game, the monopolist obtains the present value V^P in both parameter regimes. As the time between successive offers shrinks to zero, the monopolist receives a present value equal to that obtained by a perfectly discriminating monopolist, $\pi_1^{1st} + \pi_2^{1st}$.

In finite-horizon games (with $T \geq 2$), the monopolist also receives the present value V^P when $2v_2 - v_1 - c'(0) \leq 0$. Hence, in the continuous-time limit, he also receives the present value obtained by a perfectly discriminating monopolist.

For the parameter regime $2v_2 - v_1 - c'(0) > 0$, however, the monopolist cannot in the continuous-time limit achieve the present value attainable under perfect price discrimination when the horizon length is fixed. For, in that case, buyer 1 can always hold out for the monopolist's final offer and be assured a strictly positive surplus.

To analyze the behavior of the initial price and the monopolist's present value of profit in this parameter regime as a function of the fixed time horizon and the interval between offers, we proceed as follows. As the interval between successive offers (δ) shrinks, β strictly increases while the threshold level β^* remains constant. Thus, for any fixed time horizon Γ, deviating from the static Mussa-Rosen strategy becomes optimal for the monopolist if the time interval between offers is sufficiently short. The monopolist then receives the present value V_T specified in equation (12). By using equations (13) and (14) to substitute for T and β in equations (11) and (12), we can rewrite the equations for $p(1)$, the price at which the monopolist sells to buyer 1 in the first period, and for V_T in terms of the "time parameters" δ, Γ, and ρ. Let V_Γ denote the present value obtained by the monopolist when the time horizon

is Γ, $2v_2 - v_1 - c'(0) > 0$, and $\beta > \beta^*$.

$$p(1) = (1 - e^{-\rho[\Gamma-\delta]})p_1^{1st} + e^{-\rho[\Gamma-\delta]}p_1^{MR} . \tag{15}$$

$$V_\Gamma = (1 - e^{-\rho[\Gamma-\delta]})\pi_1^{1st} + e^{-\rho[\Gamma-\delta]}\pi_1^{MR} + e^{-\rho\delta}\pi_2^{1st} . \tag{16}$$

It is straightforward to set $\delta = 0$ in equations (15) and (16) to determine the limiting expressions for $p(1)$ and Γ as the interval between offers shrinks to zero. For a finite time horizon, Γ, $p(1) < p_1^{1st}$ and $V_\Gamma < V^P$, even when $\delta = 0$.

However, for any fixed δ (and for the limiting case where $\delta = 0$), $p(1) \to p_1^{1st}$ and $V_\Gamma \to V^P$ as $\Gamma \to \infty$. When the time horizon becomes large, the monopolist in the finite-horizon game uses almost the same strategy in the first period and obtains almost the same present value as in the infinite-horizon game. Moreover, as $\delta \to 0$ *and* $\Gamma \to \infty$, $p(1) \to p_1^{1st}$ and $V_\Gamma \to \pi_1^{1st} + \pi_2^{1st}$. As the interval between successive offers becomes small and the time horizon becomes large, the monopolist makes the same offers and receives the same present value as the perfectly discriminating monopolist.

4. Extension to Incomplete Information

In the previous section, we assumed that the monopolist could observe the marginal valuation of each buyer but was barred from utilizing this information to make an offer to one buyer which was unavailable to a buyer with a different marginal valuation.

Suppose instead that the monopolist knows only the initial *distribution* of the buyers' marginal valuations but does not know the marginal valuation of any particular buyer. Limited information would then prevent the monopolist from offering an item to one buyer which was unavailable to a buyer with a different marginal valuation even if such discrimination were permitted.

In the single-offer case of Mussa-Rosen, this change in information structure does not alter the analysis. But as we shall see, it does affect the analysis in our multi-offer case. Nevertheless, the infinite-horizon example in this section demonstrates that the novel results which we have ascribed to relaxing Mussa-Rosen's assumption of a single offer do not depend on the assumption of complete information. Our example shows that the traditional results (the optimality of two-item offers, the inefficiency of the lower quality item, etc.) do not necessarily return in the multi-offer case once incomplete information is introduced.

To solve our game of incomplete information, we convert it to a game of imperfect information in the standard way. We introduce an initial, chance move which determines which particular buyer has the higher valuation[9] and examine a perfect Bayesian equilibrium (PBE) of this game.[10]

As in the complete information case, the monopolist offers single items of efficient quality in the first parameter regime ($2v_2 - v_1 - c'(0) \leq 0$) for any β and in the second parameter regime ($2v_2 - v_1 - c'(0) > 0$) for any $\beta > \beta^*$. In either circumstance, the payoff of the monopolist is V^P. As before, the present value of this payoff approaches that achieved under perfect price discrimination in the continuous-time limit.

To describe a "perfect Bayesian equilibrium"(PBE) we must once again specify the strategy of each of the three players; in addition, however, we must specify the *beliefs* of a given player about the marginal valuation of each buyer when it is that given player's move.

The PBE solution concept requires that the *strategy* of each player is optimal from every information set onward given that player's beliefs and the strategies of the other players. Moreover, along the equilibrium path, the beliefs must be *consistent* with the equilibrium strategies of the players and Bayes rule.

In the game that we consider, it is common knowledge that one buyer has marginal valuation v_1 and the other buyer has marginal valuation v_2. Since each buyer is assumed to know his own type, each buyer knows with certainty the type of the other buyer.[11] At the outset, the monopolist assigns probability 1/2 to the event that one particular buyer has the higher valuation and the other the lower valuation. If a single buyer accepts an offer, the monopolist is unable to observe the type of that buyer but revises his belief about the remaining buyer's marginal valuation and acts optimally given his revised beliefs.

We begin by describing the *beliefs* of the monopolist following the purchase of a single item. The seller concludes that buyer 2 remains whenever the single item purchased would give buyer 2 negative utility; otherwise, he concludes that buyer 1 remains.

Next we describe the equilibrium *strategies* of each player. When both buyers remain, each accepts the best item yielding nonnegative utility. Buyer 2 also employs this get-it-while-you-can strategy if he is the only buyer. If only buyer 1 remains and the item previously purchased gave buyer 2 nonnegative utility, then buyer 1 accepts the best item which gives him nonnegative utility. If, however, the item previously purchased gave buyer 2 negative utility, then buyer 1 accepts the best item yielding a current utility larger than $\beta[v_1 q_2^{1st} - p_2^{1st}]$.

The monopolist's equilibrium strategy is as follows. In periods where both buyers remain, the monopolist offers the *single* price-quality pair (p_1^{1st}, q_1^{1st}) just as he would do in a game with complete information. In periods where only one buyer remains, the monopolist offers the price-quality pair (p_2^{1st}, q_2^{1st}) if and only if the item previously purchased would have given buyer 2 negative utility. If it would have given buyer 2 nonnegative utility, then the monopolist offers the remaining buyer (p_1^{1st}, q_1^{1st}). We refer to the monopolist's equilibrium strategy as the "incomplete-information Pacman strategy."

It is straightforward to verify the optimality of the monopolist's strategy. Suppose a single buyer remains. If the item previously purchased would have given buyer 2 negative utility, then the monopolist assesses at one the probability that the remaining buyer has the lower valuation. Since such a buyer will in equilibrium always use the get-it-while-you-can strategy, the proposed strategy of offering (p_2^{1st}, q_2^{1st}) would extract all of buyer 2's surplus and is clearly optimal for the monopolist.

If, on the other hand, the item previously purchased would have given buyer 2 nonnegative utility, then the monopolist assesses at one the probability that the remaining buyer has the higher valuation. Since buyer 1 will in equilibrium use the get-it-while-you-can strategy following a purchase with this characteristic, the proposed strategy of offering (p_1^{1st}, q_1^{1st}) would extract all of buyer 1's surplus and is clearly optimal for the monopolist.

In any period when two buyers remain, each uses the get-it-while-you-can strategy. The proof of the optimality of the seller's strategy when two buyers remain is then identical to the case of complete information both in the first parameter regime for any β and in the second for $\beta > \beta^*$. In equilibrium, the monopolist's payoff is again V^P.

When the monopolist uses the incomplete-information Pacman strategy, buyer 2 can never obtain positive utility in equilibrium; hence, the get-it-while-you-can strategy is always optimal for him.

Consider the optimality of the strategy for buyer 1. If the best current item for buyer 2 yields nonnegative utility, buyer 1 anticipates that buyer 2 will purchase and that the seller will offer the single price-quality pair (p_1^{1st}, q_1^{1st}) repeatedly in the future until he purchases; it is, therefore, optimal for him to accept the best current item yielding nonnegative utility. If instead every current item would yield negative utility for buyer 2, then buyer 1 would anticipate that buyer 2 would remain. Since if buyer 1 also remains, the seller will offer the single price-quality pair (p_1^{1st}, q_1^{1st}) repeatedly until a purchase occurs, the get-it-while-you-can strategy is again optimal for buyer 1.

When buyer 1 is alone, it is clearly also optimal for him to use the get-it-while-you-can strategy if buyer 2 previously purchased an item yielding nonnegative utility; for, following that event, the monopolist concludes that buyer 1 remains and repeatedly offers the price-quality pair (p_1^{1st}, q_1^{1st}) until it is purchased. If, however, buyer 2 previously (in error) purchased an item yielding negative utility, the monopolist concludes (erroneously) that buyer 2 remains and repeatedly offers the item (p_2^{1st}, q_2^{1st}) which is tailored to extract all of the surplus from buyer 2. In that case, buyer 1 should accept the best current offer yielding a current utility of at least $\beta[v_1 q_2^{1st} - p_2^{1st}]$.

The definition of perfect Bayesian equilibrium places some restrictions on the monopolist's beliefs. On the equilibrium path, the monopolist's belief's must accord with the consequences of the equilibrium strategies and Bayes' rule.[12] To verify that the beliefs are "consistent" in this sense, note that in the first period of equilibrium play, the monopolist offers the single price-quality pair (p_1^{1st}, q_1^{1st}) and buyer 1 accepts it—precisely as the seller would surmise following such a purchase. This concludes the demonstration that the strategies and beliefs postulated form a perfect Bayesian equilibrium.

To summarize, the characteristics of this perfect Bayesian equilibrium are the same as in the complete information case: the monopolist offers single items of efficient quality in successive periods and, in the continuous-time limit, approximates the present value of perfect price discrimination.

We certainly do not contend that this perfect Bayesian equilibrium is unique. Indeed, by changing the beliefs and strategies off the equilibrium path, a continuum of other equilibria can be constructed.[13] Although each of these equilibria happen to have those characteristics mentioned above, other equilibria may exist which do not. What we have asserted and now proved is that the introduction of incomplete information is *insufficient* to restore the familiar results of the single-offer case.

5. Concluding Remarks

The literature on second-degree price discrimination has focused on the benefits to a monopolist of inducing buyers to self-select at a point in time by offering a single menu of items with different characteristics. As our paper shows, when the monopolist is allowed to offer a sequence of menus over time, the opportunity to induce intertemporal self-selection provides the monopolist with a potent additional means of extracting surplus. Indeed, in the examples we consider, it is optimal for the monopolist to abandon multi-item menus altogether and rely solely on intertemporal self-selection (except perhaps in the last period, if one exists). More generally, we expect that there will also be cases where it is optimal for the monopolist to utilize *both* point-in-time and intertemporal self-selection by offering a sequence of different multi-item menus over time.

The model in our paper is closely related to the one in Bagnoli et al. (1989) although our focus there was entirely different. In the previous paper, we assumed that the monopolist could not alter the quality of the durables he produced. Hence, every item offered at a point in time had the same price, and inducing point-in-time self-selection was infeasible. Our focus in that paper was instead on Coase's famous conjecture that in the continuous-time limit, buyers always receive the entire social surplus. We showed that this conjecture is false when the number of buyers is finite.

Taken together, our two papers provide a unified theory of durable-goods monopoly. In the rest of this section we wish to mention two issues common to both papers which deserve further thought.

The first issue concerns the domain of our two analyses. The structure of demand in both models is identical. Buyers want at most one unit of a good and demand nothing further once they have purchased it. This standard assumption is intended to capture in a stylized way the characteristics of durables (e.g. microwaves). In contrast, the demand for nondurables (e.g. hamburgers) drops to zero immediately after a purchase but fully recovers after some time interval. For example, the consumer of one hamburger at lunchtime today—although temporarily stuffed—might be equally ready to indulge in a hamburger at lunchtime tomorrow.

Either of our models can be adapted to this nondurable case by regarding it as a *sequence* of fixed-horizon durable-goods markets each of which occurs daily. To illustrate, suppose for simplicity that there will be a finite number of lunches. At the last lunch, the monopolist would make a sequence of price (or price-quality) offers to the buyers during the fixed-horizon called lunchtime. Since demand in this final market has the characteristics of a durable in our models, our results hold and the equilibrium payoffs (unique in the two-buyer case) will be anticipated by the players at the penultimate lunchtime. Since strategies at the penultimate lunchtime will have no effect on the payoffs anticipated at the final lunchtime, these values (properly discounted) can be added to payoffs at the penultimate stage and our models again can be used to predict the monopolist's offers and the buyers' responses .... Given this adaptation of our model, it would seem mistaken to regard our results as applicable only to durables.

The second issue we wish to raise concerns welfare. In our two papers, we exhibited

a number of examples in which the monopolist extracted all of the surplus in the continuous-time limit. In such examples, of course, the equilibrium allocation is efficient. However, there were three situations in which the monopolist extracted only a portion of the available surplus in the continuous-time limit. Recall from section 3, for example, that if the Mussa-Rosen menu is optimal in the final period when both buyers remain, then the buyer with the higher valuation inevitably gets some of the surplus and this surplus is not negligible even in the continuous-time limit provided the time horizon is short (see p. 23). Similarly, in the fixed quality case of the previous paper, the higher valuation buyer gets some surplus in the analogous circumstance—if the seller would prefer to sell to both buyers in the last period. Finally, the monopolist fails to extract the entire surplus in the continuous- time limit in a three buyer example in the fixed quality case.

We find it striking that even in these three examples, whatever surplus the durable-goods monopolist fails to capture goes to the consumers: there is negligible deadweight loss in the continuous-time limit![14] Moreover, if one accepts the adaptation of our model to the sale of some nondurables (e.g. hamburgers) as outlined above, then some nondurable monopolies are also efficient![15] We do not contend that monopoly—even durable-goods monopoly—inevitably maximizes social surplus. Indeed, we know that it *is* possible to construct nonstationary equilibria in games with unbounded horizons which are inefficient even in the continuous-time limit. However, our two papers have identified a collection of cases where monopoly is asymptotically efficient. Further research is needed to characterize the circumstances which insure that monopoly is efficient.

References

Ausubel, L. and R. Deneckere (1986) "Reputation in Bargaining and Durable Goods Monopoly," *Econometrica*, **57**, 511-31.

Bagnoli, M., S.W. Salant, and J. Swierzbinski (1989) "Durable-Goods Monopoly with Discrete Demand," *Journal of Political Economy*, **97**, 1459–1478.

Bulow, J. (1982) "Durable-Goods Monopolists," *Journal of Political Economy*, **90**, 314–332.

Coase, R. (1972) "Durability and Monopoly," *Journal of Law and Economics*, **15**, 143–149.

Fudenberg, D. and J. Tirole (1990) "*A Course in Game Theory*," forthcoming.

Gul, F., H. Sonnenschein, and R. Wilson (1986) "Foundations of Dynamic Monopoly and the Coase Conjecture," *Journal of Economic Theory*, **39**, 155–190.

Kahn, C. (1986) "The Durable Goods Monopolist and Consistency with Increasing Costs," *Econometrica*, **54**, 275–294.

Mussa, M. and S. Rosen (1978) "Monopoly and Product Quality," *Journal of Economic Theory*, **18**, 301–317.

Sobel, J. and I. Takahashi (1983) "A Multistage Model of Bargaining," *Review of Economic Studies*, **50**, 411–426.

Stokey, N. (1981) "Rational Expectations and Durable Goods Pricing," *Bell Journal of Economics*, **12**, 112–128.

Footnotes

1. Contributions to this literature include Ausubel and Deneckere (1986), Bagnoli, Salant, and Swierzbinski (1989), Bulow (1982), Coase (1972), Gul, Sonnenschein and Wilson (1986), Kahn (1986), Sobel and Takahashi (1983) and Stokey (1981).

2. The constraint that the monopolist offer only a single menu is unimportant as long as the willingness of each buyer to purchase when the next menu would be offered is independent of whether he purchased in the past. For simplicity, we refer to goods with this characteristic as "nondurables" and to all other goods as "durables." Note that, as discussed in the final section, this classification depends on the time interval of the service flow from a purchased product *relative* to the time interval between the seller's offers. In the case of nondurables, it would be optimal for the monopolist to offer the Mussa-Rosen menu in every period. However, many—including Mussa-Rosen (1978, p.316) themselves—have applied their model to conventional durables (e.g. cars). When he sells a durable, constraining the seller to offer only a single menu turns out to distort his behavior.

3. Each of these distinct information assumptions results in the same equilibrium in the single-offer case studied by Mussa and Rosen (1978). However, as will be shown in section 4, they have different implications in the multi-offer case. For if the seller lacks complete information about a given buyer's marginal valuation, then his assessment of the valuations of the buyers remaining after some purchases have occurred may be incorrect.

4. The assumption that $v_2 > c'(0)$ is sufficient to insure that both buyers are served under perfect price discrimination and that at least buyer 1 is served in Mussa-Rosen's problem.

5. Since by assumption $v_1 > v_2 > c'(0)$, $q_1 > 0$ and there is no need to write the complementary slackness condition for q_1. The expression $2v_2 - v_1 - c'(q_2)$ can be re-written as $v_2 - [c'(q_2) + (v_1 - v_2)]$ and given the following interpretation. A marginal increase in the quality offered buyer 2 increases the surplus the monopolist can extract from buyer 2 by v_2 but results in two types of costs: it increases his production costs by $c'(q_2)$ and forces him to reduce his charge to buyer 1 by $v_1 - v_2$. At an interior optimum, these marginal benefits and costs balance; at a corner, the marginal costs are at least as high as the marginal benefits.

6. If an item available in period t has a utility (discounted to period 1) of $\beta^{T-1}U_1^{MR}$, its value capitalized to period t is magnified by the factor β^{1-t}: in current value terms, it is worth $\beta^{T-t}U_1^{MR}$.

7. A comparison of equations (5) and (10) indicates that in order for the Pacman strategy to be optimal for the monopolist in the infinite-horizon game, it is sufficient for β to be greater than β^{**}, where

$$\beta^{**} = \frac{\pi_1^{MR} + \pi_2^{MR} - \pi_1^{1st}}{\pi_2^{1st}} < \beta^*.$$

8. A trivial multiplicity of equilibria arises because the seller can always add items to the menu which are unacceptable to every remaining buyer. Even in the absence of such items, a second trivial multiplicity of equilibria arises because of buyer 2's

indifference about accepting offers off the equilibrium path yielding zero utility. That is, in every subgame perfect equilibrium the strategies of the seller and buyer 1 are unique. Moreover, when buyer 2 alone remains, he must play get-it-while you-can. However, if both buyers remain it would also be optimal for buyer 2 to reject an offer yielding zero utility instead of accepting it as he would if he played get-it-while-you-can. Since when both buyers remain the seller would make an offer acceptable only to buyer 1, the play of the game and the payoffs which result are the same in both of these equilibrium strategy combinations.

9. We utilize the Harsanyi transformation because of its familiarity to readers. In our particular problem, however, an alternative transformation suggests itself. Suppose the monopolist knew the marginal valuation of each player but could not observe the identity of the purchaser because, for example, goods are ordered by electronic mail and must be left for the purchaser in a place the seller cannot observe. This too is a game of imperfect rather than incomplete information and the strategies and beliefs we discuss constitute a perfect Bayesian equilibrium for this game as well.

10. The solution to a game of incomplete information is often defined in terms of a sequential equilibrium, which is a refinement of the perfect Bayesian equilibrium concept. However, in our game there is a continuum of price-quality pairs from which the monopolist constructs his offers and the sequential equilibrium concept is not defined for such games. See Fudenberg and Tirole (1990) for a discussion of the concept of perfect Bayesian equilibrium and its refinements as well as its relationship to sequential equilibrium.

11. In a game with incomplete information and more than two buyers, a buyer would not be able to infer the type of every other buyer from a knowledge of the aggregate distribution of types.

12. Indeed, the beliefs in our equilibrium can be shown to satisfy an additional restriction which, although not required of perfect Bayesian equilibria, seems sensible. Starting at any information set off of the equilibrium path and using the beliefs there as priors, the subsequent beliefs must be *consistent* with Bayes' rule when the equilibrium strategies are played.

13. To construct different equilibria, consider the set of price-quality pairs which would simultaneously give buyer 2 nonnegative utility and buyer 1 at least $\beta[v_1 q_2^{1st} - p_2^{1st}]$. Arbitrarily pick any subset of these items and assume that if the monopolist observes a single purchase of an element in this subset when two buyers remain that he believes buyer 2 remains. In all other cases, retain the beliefs described in the text. Correspondingly, change the strategy of buyer 1 so that when the best current offer for buyer 2 falls in this subset, buyer 1 accepts the best current item yielding current utility of at least $\beta[v_1 q_2^{1st} - p_2^{1st}]$. In all other cases, retain the strategies described in the text. Then it is straightforward to verify that these new strategies and beliefs form a perfect Bayesian equilibrium (and also satisfy the additional restriction mentioned in footnote 12). Since there are a continuum of possible subsets which can be designated, there are a continuum of equilibria. In each, the monopolist offers single items of efficient quality in successive periods and, in the continuous-time limit, approximates the present value of perfect price discrimination.

14. Our examples of efficient monopolies also assume that there are at least

as many periods as there are distinct marginal valuations. The single-period game studied by Mussa-Rosen demonstrates that inefficient outcomes can easily result if this assumption is not satisfied.

15. It might be thought that the efficiency of the nondurable monopoly in this adaptation of our model is attributable to the assumption that buyers demand at most one unit at any given lunchtime. However, as the reader can easily verify, the identical result holds if the monopolist faces a single buyer with marginal valuation of v_1 for the first unit and v_2 for the second unit.

Appendix

This appendix extends our model to the case of an arbitrary, finite initial number of buyers. For the infinite-horizon game with complete information, we show that the use of the Pacman strategy by the monopolist and the get-it-while-you-can strategy by each buyer constitutes a subgame-perfect equilibrium when the discount factor is sufficiently close to one. In this equilibrium, the present value obtained by the monopolist approaches the profit obtained by a perfectly discriminating monopolist as the discount factor approaches one.

The timing of the game is the same as in the text. There are an infinite number of periods indexed by $t = 1, 2 \ldots, \infty$. In the first stage of period t, the monopolist offers a menu $M(t) = \{(p_i(t), q_i(t))\}$ which is a finite set of ordered pairs indicating the quality levels, $q_i(t)$, available in period t together with the price, $p_i(t)$, of a unit with a given quality. In period t, let $i = 1, \ldots, I(t)$ and suppose that $q_i > q_j$ when $i < j$. In the second stage of each period, buyers simultaneously choose whether to accept an item from the monopolist's menu or continue to the next period. Buyers choose at most one item during the game.

There are a finite number of distinct buyer types indexed by $\alpha = 1, 2 \ldots A$. A type α buyer who accepts the item $(p_i(t), q_i(t))$ in period t obtains the utility:

$$U_\alpha(p_i(t), q_i(t)) = \beta^{t-1}\left[v_\alpha q_i(t) - p_i(t)\right] , \tag{A.1}$$

where $1 > \beta > 0$ is a discount factor which is assumed to be the same for all players. As in the text, we refer to v_α as the marginal valuation for quality of a type α buyer. Let $v_1 > v_2 > \ldots > v_A > 0$. Buyers who do not accept an offer receive a utility of zero.

Initially, there are assumed to be a positive, finite number of buyers of each type. Let $n_\alpha(t)$ denote the number of type α buyers who have not yet purchased a unit of the monopolist's good at the beginning of period t. No new buyers enter the market after the game begins; hence, the number of type α buyers who have not yet accepted an offer remains constant or decreases over time.

As in the text, $c(q)$ denotes the constant marginal cost of producing a unit with quality level q. We continue to assume that $c'(q), c''(q) > 0$ for all q, that $c(0) = 0$, and that $v_A > c'(0)$.[1]

If $b_i(t)$ denotes the number of buyers accepting item i in period t, then the present value, V, of the monopolist's profits is given by the equation:

$$V = \sum_{t=1}^{\infty} \beta^{t-1} \left\{ \sum_{i=1}^{I(t)} b_i(t)[p_i(t) - c(q_i(t))] \right\} . \tag{A.2}$$

We restrict attention to games with complete information where all players can observe the type of each buyer who accepts an offer. Hence, in particular, the numbers $n_\alpha(t)$ are common knowledge for all t. As in the text, a player's strategy in each period specifies his action in that period as a function of time and anything else that he can observe — for example, the sequence of previous offers, the number of buyers purchasing specific price-quality pairs, the periods in which various items were purchased, the marginal valuations of the buyers who purchased each item, and, for the buyers, the menu of prices and qualities offered in the first stage of the current period. The monopolist's strategy in each period specifies his offer in the first stage of the period as a function of these things. Similarly, a buyer's strategy specifies which price-quality pair (if any) to accept in the second stage of each period. The monopolist chooses his strategy to maximize the present value of his profits. Each buyer chooses a strategy to maximize his own utility.

We now show that in the infinite-horizon game with complete information, the following strategies constitute a subgame-perfect equilibrium when the discount factor is sufficiently close to one. Suppose that in each period t a buyer accepts the best offer of those which provide him with nonnegative utility. As in the text, we refer to this strategy for the buyer as the get-it-while-you-can strategy.

Let $(p_\alpha^{1st}, q_\alpha^{1st})$ denote the take-it-or-leave-it offer specified in equations (6a) and (6b) of the text which a perfectly discriminating monopolist would make to a type α buyer. Let $\alpha(t)$ denote the type of the buyer with the highest marginal valuation for quality, $v_{\alpha(t)}$, of all those buyers who have not yet accepted an offer at time t. That is, in each period t, suppose that $n_{\alpha(t)}(t) > 0$, and that $n_\alpha(t) = 0$ for all $\alpha < \alpha(t)$. With this notation, a monopolist who uses the Pacman strategy offers a menu in period t consisting of the single item: $(p_{\alpha(t)}^{1st}, q_{\alpha(t)}^{1st})$.

The following proposition asserts that the Pacman strategy is optimal for the monopolist when all buyers use the get-it-while-you-can strategy and the discount factor is sufficiently close to one. Since no buyer can ever achieve positive utility when playing against the Pacman strategy, it is clearly optimal for all buyers to use the get-it-while-you-can strategy in each period when the monopolist uses the Pacman strategy. It follows as a corollary of these two assertions that the use of the Pacman strategy by the monopolist and the get-it-while-you-can strategy by each buyer constitutes a subgame-perfect equilibrium of our infinite-horizon game. This corollary, which represents the main result of the appendix, is stated formally at the end of the appendix.

Proposition 1 (Pacman) *If all buyers use the get-it-while-you-can strategy in each period, then for any initial set of buyers, there exists a discount factor β_P with $0 \le \beta_P < 1$ such that for all discount factors satisfying the inequality: $1 > \beta > \beta_P$, the*

Pacman strategy is optimal for the monopolist in the infinite-horizon market game with complete information. For initial sets of buyers that contain only one type of buyer, let $\beta_P = 0$. Otherwise, let β_P be defined by equation (A.11) below.

In the subgame-perfect equilibrium that we consider, the present value obtained by the monopolist using the Pacman strategy, V^P, is given by the equation:

$$V^P = \sum_{\alpha=1}^{A} \beta^{\alpha-1} n_\alpha(1) \pi_\alpha^{1st} , \tag{A.3}$$

where π_α^{1st} is the profit obtained from a type α buyer by a perfectly discriminating monopolist. An expression for π_α^{1st} is specified in equation (7) in the text. Note that as β approaches one, the monopolist's present value approaches the total surplus, S, obtained by a perfectly discriminating monopolist.

The rest of this appendix is a proof of proposition 1. The proof consists of three parts. First, we introduce some notation. Next, we show that if a certain inequality (equation A.7) holds for all $\beta > \beta_P$, then starting with an arbitrary subgame and any nonempty set of remaining buyers, the present value obtained from any monopolist's strategy that generates a pattern of sales different from the pattern obtained by the Pacman strategy can be strictly improved by replacing the deviant strategy with the Pacman strategy. Finally we show that the assumed inequality is indeed valid.

Consider the following notation for describing the present value obtained when various monopoly strategies are applied to a generic subset of buyers who use the get-it-while-you-can strategy.

Let Γ denote the name of a generic subset of the initial set of buyers. Let $A(\Gamma)$ denote the number of distinct buyer types in Γ and let n_α^Γ denote the number of type α buyers in Γ.

One important benchmark is the present value obtained by a perfectly discriminating monopolist who sells to the buyers in Γ. To each type α buyer, a perfectly discriminating monopolist makes the take-it-or-leave-it offer $(p_\alpha^{1st}, q_\alpha^{1st})$ and obtains the surplus, S_Γ, given by the equation:

$$S_\Gamma = \sum_{\alpha=1}^{A} n_\alpha^\Gamma \pi_\alpha^{1st} . \tag{A.4}$$

Note that $n_\alpha^\Gamma = 0$ for buyer types α that are not contained in Γ.

A second important benchmark is the supremum of the present value obtained by a monopolist who cannot make separate take-it-or-leave-it offers to each individual buyer and who sells to *all* the buyers in Γ in a *single* period. We refer to this present value as V_Γ^{All}.

A third important benchmark is the present value obtained by a monopolist who sells to the buyers in Γ using the Pacman strategy. Denote this present value by V_Γ^P.[2]

Now consider the proof of proposition 1.

If the initial set of buyers are all of the same type, then proposition 1 follows immediately since the Pacman strategy maximizes the surplus obtainable from that single type of buyer for all values of the discount factor. We set $\beta_P = 0$ in this case.

Henceforth, we restrict attention to initial collections of buyers with two or more distinct types of buyer.

Now consider an arbitrary, nonempty subset Γ of the initial set of buyers. We envision Γ as being the subset of initial buyers who have not yet accepted an offer at the beginning of an arbitrary subgame and consider an arbitrary monopoly strategy for selling to the buyers in Γ in this subgame. Since we are at present concerned only with the optimality of the monopolist's strategy, we need only consider subgames that begin with the monopolist's choice in the first stage of some period.

When all buyers use the get-it-while-you-can strategy, any offer that causes a type i buyer to accept some item will also cause all type j buyers with $v_j \geq v_i$ to accept some item in the offered menu. Hence, the response of buyers to a general monopoly strategy involves in each period either i. no sales, ii. sales to all the buyers with the single highest marginal valuation for quality, or iii. sales to a "clump" of buyers where a clump consists of all the buyers with marginal valuations for quality (two or more) higher than some threshold level, v_i. We will call the act of selling to a "clump" of buyers "clumping". The main idea of the proof is that if $\beta > \beta_P$, then clumping is never optimal.

Without loss of generality, we can restrict attention to monopoly strategies that deviate from the Pacman strategy in the first period of a subgame. For suppose that some monopoly strategy specifies the use of the Pacman strategy for a number of periods before deviating. In this case, we need only proceed to the subgame which begins with the monopolist's first deviation from the Pacman strategy and consider the subset of buyers remaining at the beginning of that subgame to be our starting set of buyers Γ.

If the set Γ contains buyers of only one type, then replacing a deviant offer by the Pacman offer strictly improves the monopolist's profits because it increases the surplus extracted from these buyers and has no other effect. (The game ends after the single type of buyer accepts the Pacman offer.) Hence, we restrict attention in what follows to subsets Γ that contain two or more buyer types.

Now consider a subgame with an arbitrary set of remaining buyers Γ that contains two or more types of buyer and an arbitrary monopoly strategy that deviates from the Pacman strategy in the first period of the subgame.

Suppose first that the initial deviation from the Pacman strategy involves zero sales in the first period. Since delay is costly, any strategy with zero sales in the first period can be strictly improved by making the Pacman offer to those buyers in Γ with the highest marginal valuation for quality and using the original strategy in future periods. This replacement advances forward the purchase of the buyers with the highest marginal valuation and may also increase the surplus obtained from these buyers. The replacement has no effect on the present value of the surplus obtained from other buyer types since the get-it-while-you-can strategy is stationary and since the choices which it specifies for type α buyers are not contingent on the presence or absence of other buyer types.

Although the set Γ is assumed to contain more than one type of buyer, suppose next that the initial deviation involves sales only to the buyers in Γ with the single highest marginal valuation for quality. In this case, replacing the deviating offer by

the Pacman offer in the current period and offering the same menus as previously in future periods clearly improves profits. The Pacman offer increases the surplus obtained from the buyers with the highest marginal valuation for quality while the present value obtained from the other buyers is not affected.

Suppose finally that the initial deviation involves selling to a "clump" of buyers with marginal valuations for quality greater than or equal to v_i. Let $\Gamma_i \subset \Gamma$ be the set of all buyers in Γ with marginal valuations greater than or equal to v_i and let Γ_{i-} be the set of all buyers in Γ with marginal valuations less than v_i. In this case, $A(\Gamma_i)$ denotes the number of buyer types in Γ with marginal valuations for quality greater than or equal to v_i. By hypothesis, $A(\Gamma_i) \geq 2$.

Let W_{i-} be the present value of the surplus obtained by making the offers originally specified by the monopoly strategy to the buyers in Γ_{i-}. Of course, Γ_{i-} may be empty, in which case $W_{i-} = 0$. Since $V_{\Gamma_i}^{All}$ represents an upper bound on the present value that can be obtained by selling immediately to all the buyers in Γ_i, an upper bound on the present value obtained by the original monopoly strategy (including the first period "clumping") is:

$$V_{\Gamma_i}^{All} + \beta W_{i-} \,. \tag{A.5}$$

The present value obtained by selling first to the buyers in Γ_i using the Pacman strategy and then making the offers specified by the original monopoly strategy to the buyers in Γ_{i-} is given by the expression:

$$V_{\Gamma_i}^{P} + \beta^{A(\Gamma_i)} W_{i-} \,. \tag{A.6}$$

Replacing the first period clumping by the use of the Pacman strategy is optimal if the present value specified in equation (A.6) exceeds the present value in equation (A.5). This condition can be written in the form:

$$V_{\Gamma_i}^{P} - V_{\Gamma_i}^{All} > \beta W_{i-}(1 - \beta^{A(\Gamma_i)-1}) \,. \tag{A.7}$$

The term on the left-hand side of equation (A.7) is the difference of the first terms in equations (A.6) and (A.5). It represents a lower bound on the additional value obtained by selling to the buyers in Γ_i using the Pacman strategy rather than the original deviant strategy. It is proved below in lemma 1 that for all subsets containing two or more buyer types and all $\beta > \beta_P$, the left-hand side of equation (A.7) is greater than a strictly positive quantity $\Delta/2$ which does not depend on the discount factor β.

The term on the right-hand side of equation (A.7) is the difference of the second terms in equations (A.5) and (A.6) and represents the cost of selling to the buyers in Γ_i using the Pacman strategy rather than the original monopoly strategy. This cost consists of the interest lost by delaying the purchases of the buyers in Γ_{i-}. For $\beta > \beta_P$, the value of the lost interest is sufficiently small that the inequality in equation (A.7) is satisfied, and it is optimal to replace the initial clumping with the offers specified by the Pacman strategy.

It is proved below that if $\beta > \beta_P$, then the inequality in equation (A.7) is satisfied for any subset of the initial set of buyers that contains two or more buyer types.

Subject to the validity of the inequality in equation (A.7), the previous arguments demonstrate that starting in an arbitrary subgame with an arbitrary set of remaining buyers who use the get-it-while-you-can strategy, the present value obtained by a monopolist can be strictly increased by proceeding to the first period in which a proposed monopoly strategy involves a pattern of sales that deviates from the pattern induced by the Pacman strategy and replacing this initial deviation with the offers specified by the Pacman strategy. If the original monopoly strategy deviates in more than one period from the Pacman strategy, then the modified strategy (with the first deviation replaced) can be strictly improved by replacing the second deviation by the Pacman strategy and so on for as many deviations as might be involved in the original proposed strategy. Since there are only a finite number of buyer types and each plays get-it-while-you-can, there can be only a finite number of deviations for the seller and the proposed procedure will terminate in a finite number of steps.

Any proposed monopoly strategy with a pattern of sales that deviates from the pattern induced by the Pacman strategy can be strictly improved by replacing successive deviations with the Pacman strategy. Hence, no departure from the Pacman strategy can (strictly) improve the present value obtained by the monopolist and the Pacman strategy must itself be optimal.

This establishes proposition 1 subject to the validity of equation (A.7).

Now consider lemma 1 which provides a lower bound for the left-hand side of equation (A.7).

Lemma 1 *For any subset of the initial set of buyers $\hat{\Gamma}$ that contains two or more distinct buyer types and any $\beta > \beta_P$, with β_P as defined in equation (A.11) below, $V_{\hat{\Gamma}}^P$, the present value obtained from selling to the buyers in $\hat{\Gamma}$ using the Pacman strategy, and $V_{\hat{\Gamma}}^{All}$, the supremum of the present values obtainable by selling in a single period to all the buyers in $\hat{\Gamma}$, satisfy the following inequality:*

$$V_{\hat{\Gamma}}^P - V_{\hat{\Gamma}}^{All} > \frac{\Delta}{2}, \tag{A.8}$$

where Δ is a strictly positive constant that does not depend on the discount factor and is defined in equation (A.10) below. It is assumed that the buyers in $\hat{\Gamma}$ use the get-it-while-you-can strategy in each period.

To prove lemma 1, consider a subset of the initial set of buyers $\hat{\Gamma}$ that contains two or more buyer types. Assume that the buyers use the get-it-while-you-can strategy in each period.

Define $\Delta_{\hat{\Gamma}}$ via the equation:

$$\Delta_{\hat{\Gamma}} = S_{\hat{\Gamma}} - V_{\hat{\Gamma}}^{All}. \tag{A.9}$$

For any subset $\hat{\Gamma}$ containing two or more buyer types, $\Delta_{\hat{\Gamma}}$ is *strictly* greater than zero. For suppose that $v_i > v_j$ represent the marginal valuations of two buyer types contained in the subset $\hat{\Gamma}$ and consider the strategy of a monopolist who cannot make separate take-it-or-leave-it offers to each buyer. It is not possible for such a

monopolist to make the offers (p_i^{1st}, q_i^{1st}) and (p_j^{1st}, q_j^{1st}) simultaneously and expect that type i buyers will accept the first offer and type j buyers will accept the second. Type i buyers will also prefer the second offer, which provides them with strictly positive rather than zero utility. Hence, a monopolist who wishes to sell to all the buyers in $\hat{\Gamma}$ in a single period but who cannot make separate take-it-or-leave-it offers to each buyer type cannot extract all of the type i buyers' surplus, and this implies that $S_{\hat{\Gamma}} > V_{\hat{\Gamma}}^{All}$.

Let Δ denote the minimum of the $\Delta_{\hat{\Gamma}}$ as $\hat{\Gamma}$ ranges over all the subsets of the initial set of buyers that contain two or more distinct buyer types, that is,

$$\Delta = \min_{\hat{\Gamma}} \Delta_{\hat{\Gamma}}. \tag{A.10}$$

The previous discussion implies that Δ is strictly greater than zero as long as the initial set of buyers contains two or more types of buyer. Note that Δ does not depend on the discount factor.

Finally, consider discount factors, β, that satisfy the following inequality:

$$1 > \beta > \beta_P = \left(1 - \frac{\Delta}{2S}\right)^{\frac{1}{A-1}}, \tag{A.11}$$

where S denotes the surplus obtained by a perfectly discriminating monopolist who sells to the initial collection of buyers. Since neither S, Δ, nor A depend on the discount factor, β_P is a well-defined number that is strictly between 0 and 1 and has a value that depends only on the characteristics of the initial collection of buyers.

For any subset $\hat{\Gamma}$ and any β satisfying equation (A.11), we have the inequalities:

$$\beta^{A-1} S_{\hat{\Gamma}} > S_{\hat{\Gamma}} - \frac{\Delta}{2}\frac{S_{\hat{\Gamma}}}{S} \geq S_{\hat{\Gamma}} - \frac{\Delta}{2}. \tag{A.12}$$

For any subset $\hat{\Gamma}$, the definition of the Pacman strategy also implies that:

$$V_{\hat{\Gamma}}^P > \beta^{A-1} S_{\hat{\Gamma}}. \tag{A.13}$$

A monopolist using the Pacman strategy obtains the same surplus from the type α buyers in $\hat{\Gamma}$ as that obtained by a perfectly discriminating monopolist but discounted by a factor that depends on the delay in serving these buyers. Since it never takes more than A periods to serve any buyer type in $\hat{\Gamma}$, the discount factor multiplying this surplus is always greater than or equal to β^{A-1}. Therefore, the sum of the discounted surpluses that produces the present value $V_{\hat{\Gamma}}^P$ must be greater than the undiscounted sum of these surpluses, $S_{\hat{\Gamma}}$, multiplied by the discount factor β^{A-1}. This is what equation (A.13) asserts.

Combining the inequalities in equations (A.12) and (A.13), we observe that for any β satisfying equation (A.11) and any subset $\hat{\Gamma}$ containing two or more distinct buyer types,

$$V_{\hat{\Gamma}}^P + \frac{\Delta}{2} > S_{\hat{\Gamma}}. \tag{A.14}$$

Using equation (A.14) to substitute for $S_{\hat{\Gamma}}$ in equation (A.9) and recalling that $\Delta_{\hat{\Gamma}} \geq \Delta$ demonstrates that for any subset $\hat{\Gamma}$ containing two or more distinct buyer types and any β satisfying equation (A.11), the inequality in equation (A.8) must be satisfied.

This completes the proof of lemma 1.

For any subset Γ_i containing two or more distinct buyer types and any $\beta > \beta_P$, lemma 1 can be used to rewrite equation (A.7) in the form:

$$V_{\Gamma_i}^P - V_{\Gamma_i}^{All} > \frac{\Delta}{2} > S(1-\beta^{A-1}) > \beta W_{i-}(1-\beta^{A(\Gamma_i)-1}) \,. \tag{A.15}$$

Proceeding from left to right, the first inequality in equation (A.15) follows from equation (A.8) and lemma 1. The second inequality follows from the definition of β_P in equation (A.11) and the assumption that $\beta > \beta_P$. The final inequality in equation (A.15) follows from i. the definition of W_{i-} which implies that $S > \beta W_{i-}$ and ii. the observation that $A \geq A(\Gamma_i)$.

This completes the proof of proposition 1.

Proposition 1 has the following corollary.

Corollary 1 *For any finite initial set of buyers and any discount factor $1 > \beta > \beta_P$ with $\beta_P = 0$ if the initial set of buyers contains only one type and with β_P otherwise given in equation (A.11), the use of the Pacman strategy by the monopolist and the get-it-while-you-can strategy by each buyer constitutes a subgame-perfect equilibrium of the infinite-horizon, complete-information market game.*

Footnotes to the Appendix

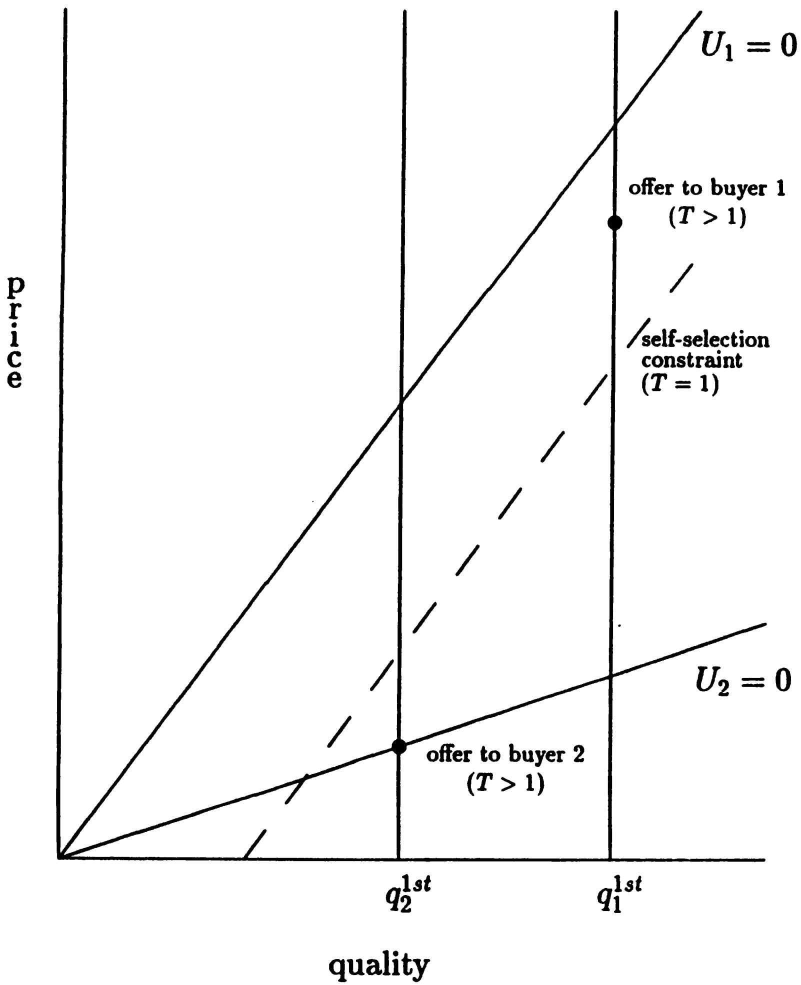
1. The assumption that $v_A > c'(0)$ is sufficient to insure that all buyer types would be served by a perfectly discriminating monopolist.

2. The value V_Γ^P can be written in a form that closely resembles the formula for V^P in equation (A.3). It is only necessary to renumber the buyer types in Γ so as to delete those buyer types which are not represented in Γ. For this purpose define a function $\alpha_\Gamma(\alpha)$ that maps the set $\{1, 2, \ldots, A\}$ to the set $\{1, 2, \ldots, A(\Gamma)\}$. If there are no type α buyers in the set Γ, then $\alpha_\Gamma(\alpha) = 0$. Otherwise, $\alpha_\Gamma(\alpha_1) = 1$ if α_1 is the buyer type in Γ with the highest marginal valuation for quality, $\alpha_\Gamma(\alpha_2) = 2$ if α_2 is the buyer type in Γ with the 2nd highest marginal valuation for quality, and so on.

Using the function α_Γ to re-index the buyers in the subset Γ, we can write the following expression for V_Γ^P:

$$V_\Gamma^P = \sum_{\alpha_\Gamma=1}^{A(\Gamma)} \beta^{\alpha_\Gamma - 1} n_{\alpha_\Gamma}^\Gamma \pi_{\alpha_\Gamma}^{1st} \,.$$

Figure 1. Optimal Price-Quality Offers



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